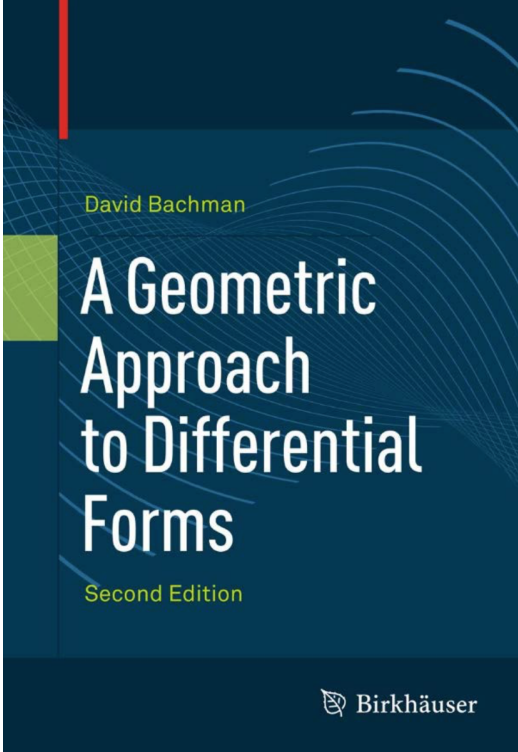


Exploring Math Σ *math* with EIGENMATH

Eigenmath Solutions Manual to DAVID BACHMAN's

A Geometric Approach to Differential Forms



```

-- Dr. W. Lindner 1/2023
-- Bachman p. 91
-- 6. STOKES' Theorem
-- 6.2 The generalized Stokes' Theorem

"EXERCISE 6.20"
-- Suppose w = x dx + x dy is a 1-form on R2.
-- Let C be the ellipse x^2/4+ y^2/9=1.
-- Determine the value of ∫w|C by integrating
some 2-form over the
-- region bounded by the ellipse.

-- plot ellipse
a = 2 -- horizontal radius.
b = 3 -- vertical radius.
x0 = 0 -- x0,y0 ellipse centre coordinates.
y0 = 0
x = x0 + a*cos(t)
y = y0 + b*sin(t)
xrange = (-4,4)
yrange = (-4,4)
trange = (-pi, pi)
draw((x,y),t)

-- Parametrization of Ellipse = Phi(-pi..pi)
Phi(t)=(2*cos(t), 3*sin(t)) -- t: -pi..pi
Phi

-- DEF of 'wedge' product w in R^2
extl2(w, v) = eval(w, dx,v[1], dy,v[2]) --
evaluation of w at v

-- Tangential vector in TpC
V = d(Phi,t)

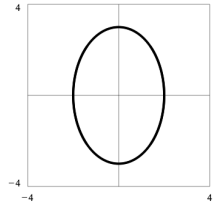
-- integrand
omega = x*extl2(dx,V) + x*extl2(dy,V)
omega

-- use Parametrization
x = 2 cos(t)
y = 3 sin(t)

-- integrate df along C
IC = defint(omega, t,-pi, +pi)
IC

```

EXERCISE 6.20



$$\Phi = \begin{bmatrix} 2 \cos(t) \\ 3 \sin(t) \end{bmatrix}$$

$$V = \begin{bmatrix} -2 \sin(t) \\ 3 \cos(t) \end{bmatrix}$$

$$\omega = 6 \cos(t)^2 - 4 \cos(t) \sin(t)$$

$$I_C = 6\pi$$

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2023

Preface

The book *A Geometric Approach to Differential Forms* from DAVID BACHMAN gives a wonderful concrete and explicit introduction to the theory and praxis of differential forms. His book focusses on comprehension and illuminates mathematical concepts by pictures and worked examples. Each example is followed by a well chosen set of exercises (problems) to gain a working knowledge of the learner. To control success, the pure results of most of the exercises are presented in an appendix. This makes the book a perfect candidate for an learning environment at an 'action and process level' in the sense of the APOS Theory, cf. the short summary [7] and [8].

Rounding off in the sense of the APOS theory this solution manual shows accompanying activities on the PC. The underlying didactical concept is that of CAS microworlds in the setting of a constructivist learning. To give an action-process-playground for the material presented, I have done the examples and exercises of BACHMAN's book using the language of the CAS EIGENMATH [7]. The solutions are presented in an intermediate precise CAS-language, that lives between the slang, in which mathematical concepts are vivid and roughly explained and the high precision of the formal mathematical language, which is not so easy to grasp at a first attempt. The EIGENMATH language allows to make the mathematical concepts of differential forms executable, to coin math processes (e.g. the concept of a directional or covariant derivative) into 'run'able procedures and therefore to allow own experiments. The results of one's thinking in the CAS language is immediately returned to the screen and helps to check the right understanding. Also, intermediate calculations may be outsourced to the CAS EIGENMATH. But, the presented structure of the EIGENMATH solutions of the exercises of the book admits the possibility of mathematically unfolding or reconstructing the hidden intermediate steps - while the overall plan resp. process of the solution is to be seen. This should help the users understanding.

This collection of small solution scripts not only want to help the reader to dive into the theory of differential forms along BACHMAN's book, but also to be comfortable with the use of the CAS EIGENMATH. The list of solutions is not yet complete, some exercises still need to be enhanced or corrected. Therefore, comments from the readers are very welcome. May this script be a source of help and inspiration for another round of thoughtful activity with respect to differential forms - supplemental to a previous or subsequent paper & pencil calculation, now from a different perspective.

For the inspection of a solution in the EIGENMATH language, no installation of any software is necessary, *everything runs directly online*: a click on a link in this text is enough to invoke the solution script - and by a click on the RUN button the calculation is made, allowing further free inputs from the user. If you own a Mac or Linux PC, there is the option to install the app EIGENMATH free of charge and run the scripts by *mark-copy-paste* into the EIGENMATH window.

Why EIGENMATH?

EIGENMATH is a small but well designed and powerful computer algebra system (CAS), that can be used to solve problems in mathematics and the natural and engineering sciences. It is a personal resource for students, teachers and scientists. EIGENMATH is compact, capable and free.

EIGENMATH ...

- focuses straight to the point, no frills, no gimmicks,
- makes learning easy: approximately 50 pages manual [12], only 100 commands
- but you will use only a good dozen of them for your work - that's all!
- has an intuitive user playground (IDE) with a two window frame,
- has an online version EIGENMATH^{online}[11], which runs in your browser,
- has mathematical language output in professional looking L^AT_EX printing,
- allows distraction-free experiences in Mathematics,
- is ideally suited for rapid prototyping.

Any feedback from the user is very welcome.

I want to thank George WEIGT for the development of the free CAS EIGENMATH and EIGENMATH^{online} over nearly 20 years and for his friendly support with tips and hints while writing these notes.

Wolfgang Lindner
Leichlingen, Germany
December 2023

The order of the following suite of examples and exercises (problems) follows the presentation in BACHMAN's book.

To unfold an example/exercise, please click the corresponding link

e.g. \triangleright *Click like here to run a script.*

Sometimes you have to enlarge the Eigenmath window or scroll down inside it in order to view the whole calculation.

It is recommended to look up a semi-automated CAS EIGENMATH solution only after one's own solution steps. At a minimum, the learner should, at least in advance, create a plan of the solution in his mind. Then you can look at the EIGENMATH calculations and, if necessary, expand them into a detailed traditional mathematical computation.

1 Introduction

LEXICON	<i>Bachman's book</i>	<i>this script</i>
	<i>Example 24.</i>	<i>Example 24.</i>
6.12. (numbered exercise)		P.6.12.

\triangleright *Example 00*

\triangleright **P.1.1.**

2 Prerequisites

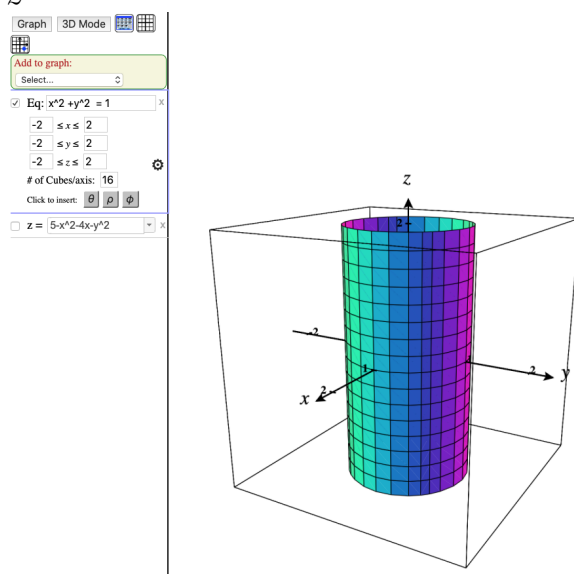
\triangleright **P.2.1.**

\triangleright *Example 1*

\triangleright **P.2.3.** \triangleright **P.2.4.** \triangleright **P.2.5.** \triangleright **P.2.7.** \triangleright **P.2.8.**

\triangleright **P.2.9.** \triangleright **P.2.10.** \triangleright **P.2.11.**

\triangleright *Example 2*



- ▷ **P.2.14.** ▷ **P.2.16.**
- ▷ *Example 4*
- ▷ **P.2.17.** ▷ **P.2.18.**
- ▷ *Example 5*
- ▷ **P.2.19.** ▷ **P.2.20.** ▷ **P.2.21.** ▷ **P.2.22.**
- ▷ *Example 7*
- ▷ **P.2.25.** ▷ **P.2.26.** ▷ **P.2.27.**
- ▷ *Example 8*
- ▷ *Example 9*
- ▷ **P.2.28.** ▷ **P.2.30.** ▷ **P.2.31.** ▷ **P.2.32.** ▷ **P.2.33.**
- ▷ **P.2.34.** ▷ **P.2.35.**
- ▷ *Example 11*
- ▷ **P.2.36.** ▷ **P.2.38.** ▷ **P.2.40.**

3 Forms

- ▷ **P.3.1.** ▷ **P.3.2.** ▷ **P.3.3.**
- ▷ *Example 13*
- It follows a suite of theoretical exercises, P.3.6 ... P.3.14. No solution here.
- ▷ **P.3.4.** ▷ **P.3.5.** ▷ **P.3.14.** ▷ **P.3.15.** ▷ **P.3.16.**
- ▷ **P.3.24.** ▷ **P.3.27.** ▷ **P.3.28.**
- ▷ *Example 13*
- ▷ **P.3.32.** ▷ **P.3.33.** ▷ **P.3.34.**
- ▷ *Example 16*
- ▷ **P.3.36.**

4 Differential Forms

- ▷ *Example 17*
- ▷ *Example 18*
- ▷ **P.4.1.** ▷ **P.4.2.**
- ▷ *Example 19*
- ▷ **P.4.3.a** ▷ **P.4.3.b** ▷ **P.4.3.c** ▷ **P.4.3.d** ▷ **P.4.4.**
- ▷ **P.4.5.** ▷ **P.4.6.** ▷ **P.4.7.** ▷ **P.4.8.** ▷ **P.4.9.**
- ▷ *Example 20*
- ▷ **P.4.10.** ▷ **P.4.11.** ▷ **P.4.12.**
- ▷ *Example 21*
- ▷ **P.4.13.** ▷ **P.4.14.** ▷ **P.4.15.** ▷ **P.4.16.** ▷ **P.4.17.**
- ▷ *Example 22*
- ▷ **P.4.18.**
- ▷ *Example 23*
- ▷ *Example 24*

- ▷ *Example 25*
- ▷ *Example 26*
- ▷ *Example 27*
- ▷ **P.4.19.** ▷ **P.4.20.** ▷ **P.4.21.** ▷ **P.4.22.** ▷ **P.4.23.**
- ▷ **P.4.24.** ▷ **P.4.25.**
- ▷ *Example 28*
- ▷ **P.4.26.** ▷ **P.4.27.** ▷ **P.4.28.** ▷ **P.4.29.** ▷ **P.4.30.** ▷ **P.4.31.**
- ▷ *Example 29*
- ▷ *Example 30*
- ▷ **P.4.32.**

5 Differentiation of Differential Forms

- ▷ *Example at p.70*
- ▷ **P.5.1.**
- ▷ *Example at p.71*
- ▷ **P.5.2.** ▷ **P.5.3.** ▷ **P.5.6.** ▷ **P.5.7.**
- ▷ *Example 31*
- ▷ *Example 32*
- ▷ **P.5.11.** ▷ **P.5.12.** ▷ **P.5.13.**

6 Stokes' Theorem

- ▷ **P.6.1.**
- ▷ *Example 35*
- ▷ *Example 37*
- ▷ *Example 38*
- ▷ *Example 39*
- ▷ **P.6.12.** ▷ **P.6.13.** ▷ **P.6.14.**
- ▷ *Example 40*
- ▷ **P.6.15.** ▷ **P.6.16.** ▷ **P.6.17.** ▷ **P.6.18.** ▷ **P.6.19.**
- ▷ **P.6.20.** ▷ **P.6.21.** ▷ **P.6.23.** ▷ **P.6.24.** ▷ **P.6.25.**
- ▷ *Example 41*
- ▷ *Example 42*
- ▷ *Example 43*
- ▷ *Example 44*
- ▷ **P.6.28.** ▷ **P.6.29.** ▷ **P.6.30.** ▷ **P.6.31.**

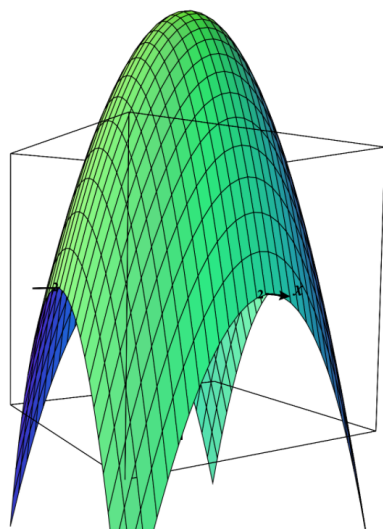


Fig. 6.31

7 Manifolds

- ▷ *Example 45*
- ▷ **P.7.1.**
- ▷ *Example 46*
- ▷ **P.7.5.** ▷ **P.7.6.**
- ▷ *Example 50*

8 Differential Geometry via Differential Forms

- ▷ *Example 53*
- ▷ *Example 54*
- ▷ *Example 55*
- ▷ *Example 56*
- ▷ **P.8.5.** ▷ **P.8.6.** ▷ **P.8.10.**

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