Week 7 Programming Assignment

Monday, February 20, 2023 2:00 PM

1. Multiclass Perceptron

(1) MultiClass Perceptron

(a) load in the data set data0.txt

In [96]:
```
data = np.loadtxt('data0.txt')
n, d = data.shape
# Create training set x and labels y
x_data = data[:, 0:d-1]
y_data = data[:, d-1]
```

(b) run the multiclass perceptron

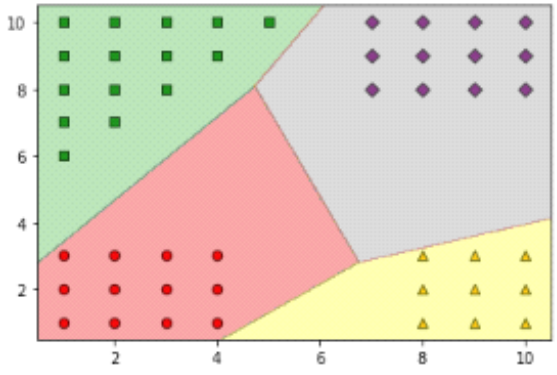
In [24]:
```
import numpy as np
import matplotlib.pyplot as plt

def evaluate_classifier(W, b, x):
    scores = np.dot(x, W) + b
    predicted_class = np.argmax(scores)
    return predicted_class
```

In [25]:
```
def train_perceptron(x, y, k, n_iters=100):
    n, d = x.shape
    W = np.zeros((d, k))
    b = np.zeros((k,))
    done = False
    converged = True
    iters = 0
    np.random.seed(None)
    while not(done):
        done = True
        I = np.random.permutation(n)
        for i in range(n):
            j = I[i]
            predicted_class = evaluate_classifier(W, b, x[j])
            if predicted_class != y[j]:
                W[:, predicted_class] -= x[j]
                b[predicted_class] -= 1
                W[:, y[j]] += x[j]
                b[y[j]] += 1
                done = False
        iters += 1
        if iters > n_iters:
            done = True
            converged = False
    if converged:
        print("Perceptron algorithm: iterations until convergence:", iters)
    else:
        print("Perceptron algorithm: did not converge within the specified number of iterations")
    return W, b, converged
```

In [26]:
```
def run_perceptron(datafile):
    data = np.loadtxt(datafile)
    n, d = data.shape
    # Create training set x and labels y
    x = data[:, 0:d-1]
    y = data[:, d-1].astype(int)
    k = len(np.unique(y))
    # Run the Perceptron algorithm for at most 100 iterations
    W, b, converged = train_perceptron(x, y, k, n_iters=100)
    # Determine the x1- and x2- limits of the plot
    x1min, x1max = np.min(x[:, 0]), np.max(x[:, 0])
    x2min, x2max = np.min(x[:, 1]), np.max(x[:, 1])
    x1_range, x2_range = np.meshgrid(np.linspace(x1min-1, x1max+1, num=1000), np.linspace(x2min-1, x2max+1, num=1000))
    grid = np.c_[x1_range.ravel(), x2_range.ravel()]
    Z = np.array([evaluate_classifier(W, b, pt) for pt in grid])
    Z = Z.reshape(x1_range.shape)
    # Plot the data points
    markers = ['o', 's', '^', 'D']
    colors = ["red", "green", "orange", "purple"]

    for i in range(k):
        plt.scatter(x[y==i, 0], x[y==i, 1], cmap=plt.cm.Set1, edgecolor='k', c=colors[i], marker=markers[i], s=40)


    # Plot the decision boundary
    plt.contourf(x1_range, x2_range, Z, alpha=0.4, cmap=plt.cm.Set1)
    plt.xlim(x1min-0.5, x1max+0.5)
    plt.ylim(x2min-0.5, x2max+0.5)
    plt.show()
```

In [27]:
```
run_perceptron('data0.txt')
```

```
Perceptron algorithm: iterations until convergence: 28
```



2. Multiclass SVM

(2) Multiclass SVM

(a) Load in the MNIST data

In [6]:
```
%matplotlib inline
import matplotlib.pyplot as plt
import gzip, os
import numpy as np
from scipy.stats import multivariate_normal
import sys
import pandas as pd

if sys.version_info[0] == 2:
    from urllib import urlretrieve
else:
    from urllib.request import urlretrieve

from sklearn.svm import SVC
from sklearn.svm import LinearSVC
from sklearn.metrics import accuracy_score
from sklearn.model_selection import cross_val_score
```

In [7]:
```
# Function that downloads a specified MNIST data file from Yann Le Cun's website
def download(filename, source='http://yann.lecun.com/exdb/mnist/'):
    print("Downloading %s" % filename)
    urlretrieve(source + filename, filename)

# Invokes download() if necessary, then reads in images
def load_mnist_images(filename):
    if not os.path.exists(filename):
        download(filename)
    with gzip.open(filename, 'rb') as f:
        data = np.frombuffer(f.read(), np.uint8, offset=16)
    data = data.reshape(-1,784)
    return data

def load_mnist_labels(filename):
    if not os.path.exists(filename):
        download(filename)
    with gzip.open(filename, 'rb') as f:
        data = np.frombuffer(f.read(), np.uint8, offset=8)
    return data
## Load the training set
train_data = load_mnist_images('train-images-idx3-ubyte.gz')
train_labels = load_mnist_labels('train-labels-idx1-ubyte.gz')

## Load the testing set
test_data = load_mnist_images('t10k-images-idx3-ubyte.gz')
test_labels = load_mnist_labels('t10k-labels-idx1-ubyte.gz')
```

(b) SVM classifier

In [8]:
```
def fit_classifier(C_value=1.0):
    clf = LinearSVC(loss='hinge', C=C_value, max_iter=10)
    clf.fit(train_data, train_labels)

    train_pred = clf.predict(train_data)
    train_error = float(np.sum((train_pred > 0.0) != (train_labels > 0.0)))/len(train_labels)

    test_pred = clf.predict(test_data)
    test_error = float(np.sum((test_pred > 0.0) != (test_labels > 0.0)))/len(test_labels)

    return train_error, test_error
```

Strategy for choosing optimal C value

- Test large range of potential c values, return training and test error values
- Test smaller range of potential c values around optimal c value from previous step, use 5-fold cross validation to determine lowest cv error and test error
- use argmin(error) to return the c value with the lowest cv and test error rates.

In [9]:
```
C_values = [1e-10, 1e-8, 1e-6, 1e-4, 1e-2, 1, 1e2, 1e4]
train_errors = []
test_errors = []

for c in C_values:
    train_error, test_error = fit_classifier(c)
    train_errors.append(train_error)
    test_errors.append(test_error)
```

In [10]:
```
d = {'C':C_values,'Training Error':train_errors, 'Test Error':test_errors}
df = pd.DataFrame(data=d).set_index('C')
df
```

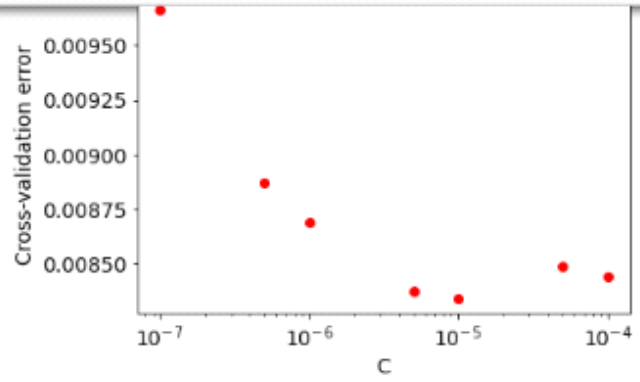
Out[10]:

C	Training Error	Test Error
1.000000e-10	0.061333	0.0607
1.000000e-08	0.011367	0.0105
1.000000e-06	0.007133	0.0066
1.000000e-04	0.023000	0.0231
1.000000e-02	0.008950	0.0081
1.000000e+00	0.009883	0.0103
1.000000e+02	0.012233	0.0124
1.000000e+04	0.008367	0.0099

In [11]:
```
def cross_validation_error(x,y,C_value,k):
    n = len(y)
    ## Randomly shuffle indices
    indices = np.random.permutation(n)

    ## Initialize error
    err = 0.0

    ## Iterate over partitions
    for i in range(k):
        ## Partition indices
        test_indices = indices[int(i*(n/k)):int((i+1)*(n/k) - 1)]
        train_indices = np.setdiff1d(indices, test_indices)

        ## Train classifier with parameter c
        clf = LinearSVC(C=C_value, loss='hinge')
        clf.fit(x[train_indices], y[train_indices])

        ## Get predictions on test partition
        preds = clf.predict(x[test_indices])

        ## Compute error
        err += float(np.sum((preds > 0.0) != (y[test_indices] > 0.0)))/len(test_indices)

    return err/k
```

In [18]:
```
def choose_parameter(x,y,k):
    ## Iterate over potential values of C
    c_vals = [.0000001, .0000005, .000001, .000005, .00001, .00005, .0001]
    errs = np.array([cross_validation_error(x, y, c, k) for c in c_vals])
    # Plot the cross-validation errors
    plt.plot(c_vals, errs, 'ro')
    plt.xscale('log')
    plt.xlabel('C', fontsize=14)
    plt.ylabel('Cross-validation error', fontsize=14)
    plt.show()
    ## Return best value of C and associated error estimate
    return c_vals[np.argmin(errs)], errs[np.argmin(errs)]
```

In [37]:
```
c, err = choose_parameter(train_data, train_labels, 5)
print("Choice of C: ", c)
print("Cross-validation error estimate: ", err)
## Train it and test it
clf = LinearSVC(C=c, loss='hinge')
clf.fit(train_data, train_labels)

preds = clf.predict(test_data)
error = float(np.sum((preds > 0.0) != (test_labels > 0.0)))/len(test_labels)

print("Test error: ", error)
```




```
Choice of C:  1e-05
Cross-validation error estimate:  0.008334027835652971
Test error:  0.0077
```

(c) Test error and linear separability

In [38]:
```
# train model at optimal c over 60,000 possible iterations to determine if it converges (training error = 0)
clf = LinearSVC(loss='hinge', C=5e-5, max_iter=60000)
clf.fit(train_data, train_labels)

y_pred = clf.predict(test_data)
accuracy = accuracy_score(test_labels, y_pred)

test_error = float(np.sum((y_pred > 0.0) != (test_labels > 0.0)))/len(test_labels)

train_pred = clf.predict(train_data)
train_error = float(np.sum((train_pred > 0.0) != (train_labels > 0.0)))/len(train_labels)

print("Test Error: ", test_error)
print("Training Error: ", train_error)
print("Optimal C: ", c)
```

```
Test Error:  0.0072
Training Error:  0.006266666666666667
Optimal C:  1e-05
```

```
C:\Users\Linds\anaconda3\lib\site-packages\sklearn\svm\_base.py:1206: ConvergenceWarning: Liblinear failed to converge, increas
e the number of iterations.
  warnings.warn(
```

For a dataset to be linearly separable, the value of C (ie slack) should be very large and the training error should be zero. In the case of the MNIST data set, the value of C is very close to zero (0.00001) and while the training error is close to zero, it is ultimately > 0. (Additionally, the model does not appear to converge after 60,000 iterations).

Therefore, the data is not linearly separable.

4. Multiclass kernel SVM

(4) Multiclass kernel SVM

(a) Load in the MNIST data

In [2]:
```
%matplotlib inline
import matplotlib.pyplot as plt
import gzip, os
import numpy as np
from scipy.stats import multivariate_normal
import sys
import pandas as pd

if sys.version_info[0] == 2:
    from urllib import urlretrieve
else:
    from urllib.request import urlretrieve

from sklearn.svm import SVC
from sklearn.svm import LinearSVC
from sklearn.metrics import accuracy_score
from sklearn.model_selection import cross_val_score

# Function that downloads a specified MNIST data file from Yann Le Cun's website
def download(filename, source='http://yann.lecun.com/exdb/mnist/'):
    print("Downloading %s" % filename)
    urlretrieve(source + filename, filename)

# Invokes download() if necessary, then reads in images
def load_mnist_images(filename):
    if not os.path.exists(filename):
        download(filename)
    with gzip.open(filename, 'rb') as f:
        data = np.frombuffer(f.read(), np.uint8, offset=16)
    data = data.reshape(-1,784)
    return data

def load_mnist_labels(filename):
    if not os.path.exists(filename):
        download(filename)
    with gzip.open(filename, 'rb') as f:
        data = np.frombuffer(f.read(), np.uint8, offset=8)
    return data
## Load the training set
train_data = load_mnist_images('train-images-idx3-ubyte.gz')
train_labels = load_mnist_labels('train-labels-idx1-ubyte.gz')

## Load the testing set
test_data = load_mnist_images('t10k-images-idx3-ubyte.gz')
test_labels = load_mnist_labels('t10k-labels-idx1-ubyte.gz')
```

(b) SVM classifier

In [3]:
```
def fit_classifier(C_value=1.0):
    clf = LinearSVC(loss='hinge', C=C_value, max_iter=100)
    clf.fit(train_data, train_labels)

    train_pred = clf.predict(train_data)
    train_error = float(np.sum((train_pred > 0.0) != (train_labels > 0.0)))/len(train_labels)

    test_pred = clf.predict(test_data)
    test_error = float(np.sum((test_pred > 0.0) != (test_labels > 0.0)))/len(test_labels)

    return train_error, test_error
```

In [4]:
```
C_values = [0.01, 0.1, 1.0, 10.0, 100.0]
train_errors = []
test_errors = []

for c in C_values:
    train_error, test_error = fit_classifier(c)
    train_errors.append(train_error)
    test_errors.append(test_error)
```

```
C:\Users\Linds\anaconda3\lib\site-packages\sklearn\svm\_base.py:1206: ConvergenceWarning: Liblinear failed to converge, increas
e the number of iterations.
  warnings.warn(
C:\Users\Linds\anaconda3\lib\site-packages\sklearn\svm\_base.py:1206: ConvergenceWarning: Liblinear failed to converge, increas
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  warnings.warn(
C:\Users\Linds\anaconda3\lib\site-packages\sklearn\svm\_base.py:1206: ConvergenceWarning: Liblinear failed to converge, increas
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  warnings.warn(
C:\Users\Linds\anaconda3\lib\site-packages\sklearn\svm\_base.py:1206: ConvergenceWarning: Liblinear failed to converge, increas
e the number of iterations.
  warnings.warn(
C:\Users\Linds\anaconda3\lib\site-packages\sklearn\svm\_base.py:1206: ConvergenceWarning: Liblinear failed to converge, increas
e the number of iterations.
  warnings.warn(
```

In [5]:
```
d = {'C':C_values,'Training Error':train_errors, 'Test Error':test_errors}
df = pd.DataFrame(data=d).set_index('C')
df
```

Out[5]:

C	Training Error	Test Error
0.01	0.010450	0.0128
0.10	0.011083	0.0106
1.00	0.011183	0.0129
10.00	0.008233	0.0092
100.00	0.011700	0.0132

No, the data does not appear to be linearly separable. Training error does not equal zero.

(c) quadratic kernel SVM

In [19]:
```
def quadratic_fit_classifier(C_value=1.0):
    clf = SVC(kernel='poly', degree=2, C=C_value, max_iter=1000)
    clf.fit(train_data, train_labels)

    train_pred = clf.predict(train_data)
    train_error = float(np.sum((train_pred > 0.0) != (train_labels > 0.0)))/len(train_labels)

    test_pred = clf.predict(test_data)
    test_error = float(np.sum((test_pred > 0.0) != (test_labels > 0.0)))/len(test_labels)

    n_supp_vect = sum(clf.n_support_)

    return train_error, test_error, n_supp_vect
```

In [21]:
```
quad_train_error, quad_test_error, n_supp_vect = quadratic_fit_classifier()
```

```
C:\Users\Linds\anaconda3\lib\site-packages\sklearn\svm\_base.py:284: ConvergenceWarning: Solver terminated early (max_iter=100
0).  Consider pre-processing your data with StandardScaler or MinMaxScaler.
  warnings.warn(
```

In [22]:
```
d = {'Training Error':quad_train_error, 'Test Error':quad_test_error, 'Number of Support Vectors':n_supp_vect}
d
```

Out[22]:
```
{'Training Error': 0.0012,
 'Test Error': 0.0029,
 'Number of Support Vectors': 10015}
```