Multilevel Linear Models

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1. Introduction

Multilevel model is an useful modeling tool for estimating variance components using the observational data having a hierarchical or clustered structure and make prediction based on these estimations.

Multilevel model compromises between the two extremes of excluding a categorical predictor from a model (**complete pooling**), or estimating separate models within each level of the categorical predictor (**no pooling**) (Gelman and Hill, 2007). Multilevel models allow a technique called "**shrinkage**", also know as "**partial pooling**" to partially pools the group-level parameters α_j toward their mean level μ_α.

In specific, there is more pooling when the group-level standard deviation σ_α is small, and more smoothing for groups with fewer observations. In general, the multilevel-modeling estimate of the group-level parameters α_j can be expressed as a weighted average of the no-pooling for its group $(\bar{y_j} - \beta\bar{x_j})$ and the mean μ_α (this expression is adopted from Gelman and Hill, 2007):

$$\alpha_j \approx \frac{n_j/\sigma_y^2}{n_j/\sigma_y^2 + 1/\sigma_\alpha^2}(\bar{y_j} - \beta\bar{x_j}) + \frac{1/\sigma_\alpha^2}{n_j/\sigma_y^2 + 1/\sigma_\alpha^2}\mu_\alpha$$

2. Goals

The aim of this tool is to illustrate random intercepts as well as fixed and random slopes in multilevel modelling. I first discussed the problems with the no-pooling and complete-pooling analyses. Then, I gradually build on the model and introduce the complexity of multilevel linear regression step by step with a figures of predicted model values. Specifically, I am going to explain the properties of the multilevel models by visualizing how the predicted values change under different model specifications. In addition, I discuss and compare model fit.

Overall, the goals of this tool are to:
- give a general idea of limitations of no-pooling and complete pooling estimations for clustered data
- give a general understanding of random intercepts across clusters in hierarchical data
- give a general understanding of random slope model, which allow the explanatory variable to have a different effect for each group

- Be able to interpret parameter estimates of fixed and random effects in multilevel models
- Be able to assess the model fit of multilevel models

3. Data

I used a simulated data for 2000 pupils in 100 schools. The main outcome variable is the pupil popularity, a popularity rating on a scale of 1-10 derived by a sociometric procedure. Typically, a sociometric procedure asks all pupils in a class to rate all the other pupils, and then assigns the average received popularity rating to each pupil. Because of the sociometric procedure, group effects as apparent from higher-level variance components are rather strong. The explanatory variables on individual level are pupil gender (boy = 0, girl = 1) and pupil extraversion (10-point scale); the explanatory variables on group level (class level) is teacher's experience (the data was adopted from: https://multilevel-analysis.sites.uu.nl/datasets/).

3.1 Level 1 observational unites (i)

Therefore, in this case, the level one is individual pupil

3.2 Level 2 observational units (j)

category of courses

```
## # A tibble: 6 x 6
##   pupil class extrav       sex  texp popular
##   <dbl> <dbl>  <dbl> <dbl+lbl> <dbl>   <dbl>
## 1     1     1      5  1 [girl]    24     6.3
## 2     2     1      7  0 [boy]     24     4.9
## 3     3     1      4  1 [girl]    24     5.3
## 4     4     1      3  1 [girl]    24     4.7
## 5     5     1      5  1 [girl]    24     6
## 6     6     1      4  0 [boy]     24     4.7
```

4. Analysis

```
#random intercept
m3<- lmer(formula = popular ~ 1 + (1|class), data=data,REML = TRUE)
m4<- lmer(formula = popular ~ 1 + sex + extrav+ (1|class), data=data,REML = TRUE)
m5 <- lmer(formula = popular ~ 1 + sex + extrav + texp + (1 + extrav| class),data=data,REML
```

4.1 problems with the no-pooling and complete-pooling analyses

The pooled method complete ignores any nesting structure in the data, which does not account for the variability in the response among the nesting groups. The unpooled method overstates the variability among the nesting groups by fitting separate estimates for each nesting group without taking into account information from other groups. Hierarchical models serve as a balance between these two methods. It accounts for the variability in the nesting groups while fitting group estimates by using information across all the groups.

4.2 Varying intercept

4.3 varying slope

$$y_i = \alpha_{j[i]} + \beta_{j[i]}x_{i1} + \beta_2 Z_{ij} + \epsilon_i, i = 1, ..., n$$
$$\begin{pmatrix}\alpha_j \\ \beta_j\end{pmatrix} \sim N(\begin{pmatrix}\mu_\alpha \\ \mu_\beta\end{pmatrix}, \begin{pmatrix}\alpha_\alpha^2 & \rho\sigma_\alpha\sigma_\beta \\ \rho\sigma_\alpha\sigma_\beta & \sigma_\beta^2\end{pmatrix}), j = 1, ..., J$$

4.5 Comparing models

```
anova(m3,m4,m5)
```

```
## refitting model(s) with ML (instead of REML)
```

```
## Data: data
## Models:
## m3: popular ~ 1 + (1 | class)
## m4: popular ~ 1 + sex + extrav + (1 | class)
## m5: popular ~ 1 + sex + extrav + texp + (1 + extrav | class)
##    npar    AIC    BIC  logLik deviance   Chisq Df Pr(>Chisq)
## m3    3 6333.5 6350.3 -3163.7   6327.5
## m4    5 4944.0 4972.0 -2467.0   4934.0 1393.52  2  < 2.2e-16 ***
## m5    8 4828.8 4873.6 -2406.4   4812.8  121.15  3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
class(m3) <- "lmerMod"
class(m4) <- "lmerMod"
class(m5) <- "lmerMod"
```

```
stargazer(m3,m4,m5,title = "OLS and linear mixed-effects model",column.labels = c("M1", "M2
```

Table 1: OLS and linear mixed-effects model

	Dependent variable:		
	popular		
	M1	M2	M3
sex		1.253*** (0.037)	1.252*** (0.037)
extrav		0.442*** (0.016)	0.453*** (0.025)
texp			0.091*** (0.009)
Constant	5.078*** (0.087)	2.141*** (0.117)	0.736*** (0.197)
Observations	2,000	2,000	2,000
Log Likelihood	−3,165.255	−2,474.149	−2,417.384
Akaike Inf. Crit.	6,336.510	4,958.299	4,850.768
Bayesian Inf. Crit.	6,353.312	4,986.303	4,895.576
Note:			*p<0.1; **p<0.05; ***p<0.01

“‘ # 5. Conclusion Instructors who are viewed as better looking receive higher instructional ratings, with the impact of a move from the 10th to the 90th percentile of beauty being substantial. This impact exists within university departments and even within particular courses, and is larger for male than for female instructors. Disentangling whether this outcome represents productivity or discrimination is, as with the issue generally, probably impossible.

Reference