Retail Demand Estimation using Public Spatial Data

An Equilibrium Configuration Approach

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Market Structure

- Multi-product retailers
- Chain level differentiation
  - Vertical
  - Horizontal
- Geographic differentiation
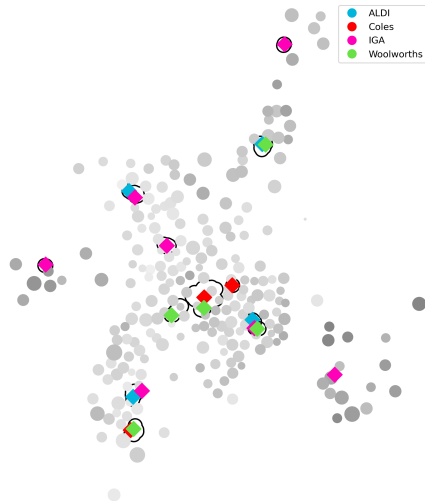 - Customer proximity
  - Local retail agglomeration

Retail Merger Counterfactuals

- Price effects
  - Unilateral effects
  - Distribution efficiencies
- Store rebranding & closures

Typical Public Data

- Stores (location, brand, owner)
- Households (location, income, size)
- Spatial demand & cost shifters

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ALDI
Coles
IGA
Woolworths

Demand Estimation

- **Model**: Household demand, equilibrium prices ✓
  - **Counterfactuals**: Mergers ✓
- **Data**: Store revenues/quantities/prices ✗
  - Davis (2000 EER; 2006 RJE)
  - Smith (2004 RES)
  - Thomadsen (2005 RJE; 2007 MS)
  - Manuszak (2010 IJIO)
  - Ho & Ishii (2011 IJIO)
  - Houde (2012 AER)
  - Seim & Waldfogel (2013 AER)
  - Aguirregabiria & Vicentini (2016 JIE)
  - Ellickson, Grieco & Khvastunov (2020 RJE)

Entry Games

- **Model**: Firm profits, equilibrium entry ✗
  - **Counterfactuals**: Entry ✗
- **Data**: Store locations ✓
  - Seim (2006 RJE)
  - Zhu & Singh (2009 QME)
  - Orhun (2013 QME)
  - Datta & Sudhir (2013 QME)

Present Paper

- **Model**: Household demand, equilibrium prices ✓
  - **Counterfactuals**: Mergers ✓
- **Data**: Store locations ✓

Data Requirements

- Markets: $\mathcal{M}$
  - Location set: $\mathcal{L}_m$
- Retail shopping precincts: $\mathcal{R}$
  - Location set: $\mathcal{L}_r$
  - Agglomeration: a_r
- Households types: $\mathcal{H}$
  - Location: l_h
  - Income: i_h
  - Count: n_h
  - Size: z_h (# Residents)
- Stores: $\mathcal{S}$
  - Location: l_s
  - Brand: b_s (Chain/Format)
  - Owner: o_s
- Wholesale distribution centres: $\mathcal{W}$
  - Location: l_w
  - Brand(s) supplied: $\mathcal{B}_w$

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Legend:
- ALDI
- Coles
- IGA
- Woolworths
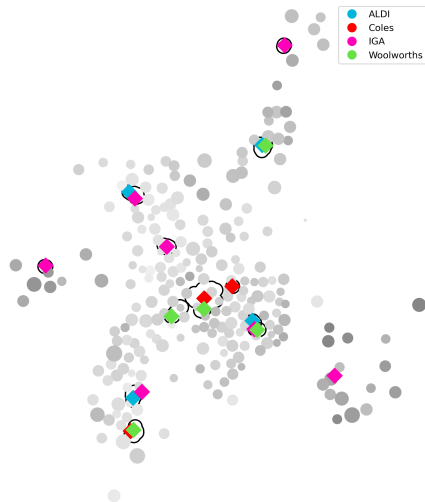
Model
Price Equilibrium

Household Store Choice

- Brand differentiation
  - Horizontal: ρ
  - Vertical: β_b
- Location differentiation
  - Distance: δ
  - Agglomeration: α
- Price disutility
  - $f(\text{income})$: η_0, η_i
- Demand
  - $f(\text{household size})$: ζ

$$u_{hs} = \underbrace{\alpha a_r + \beta_b + \delta d_{hs} + \eta_h p_s}_{\overline{u}_{hs}} + \underbrace{\epsilon_{hb} + (1-\rho)\epsilon_{hs}}_{\nu_{hs}}$$

$$\eta_h = \eta_0 + \eta_i i_h \text{ and } \mathrm{Corr}[\nu_{hs}, \nu_{hs'}] = \rho \text{ for } s, s' \in \mathcal{S}_b$$

$$q_s = \sum_h n_h \underbrace{z_h{}^\zeta}_{q_h} \cdot \underbrace{\frac{\exp(\frac{\overline{u}_{hs}}{1-\rho})}{\Sigma_{bh}}}_{\Pr[s_h=s|b_h=b]} \cdot \underbrace{\frac{\Sigma_{bh}{}^{1-\rho}}{1 + \sum_b \Sigma_{bh}{}^{1-\rho}}}_{\Pr[b_h=b]} \text{ with } \Sigma_{bh} = \sum_{s \in \mathcal{S}_b} \exp(\frac{\overline{u}_{hs}}{1-\rho})$$

Stores

- Fixed costs
  - $f(\text{brand})$: F_b
- Marginal costs
  - $f(\text{brand, distance})$: μ_b, μ_w

$$\pi_o = \sum_{s \in \mathcal{S}_o} \pi_s \text{ with } \pi_s = (p_s - c_s)q_s - f_s \text{ and } c_s = \mu_b + \mu_w d_{sw}, \ \Pr[f_s < x] = F_b(x)$$

Price Equilibrium

- Nash-Bertrand

$$p_o^* = \arg\max_{p_o} \pi_o(p_o, p_{-o}) \quad \forall o \in \mathcal{O}$$

Spatial Equilibrium
Sutton (1997 RJE; 1998; 2000; 2007):
Equilibrium configuration

- Mutual best response in store networks
  - Testable marginal profit inequalities:

    $\Delta \pi_s := \pi_o[s \in \mathcal{S} | \mathcal{S}_{-s}] - \pi_o[s \notin \mathcal{S} | \mathcal{S}_{-s}]$

    - Observed stores: marginally profitable
      $\Delta \pi_s \geq 0 \quad \forall s \in \mathcal{S},$
    - Unobserved stores: marginally unprofitable
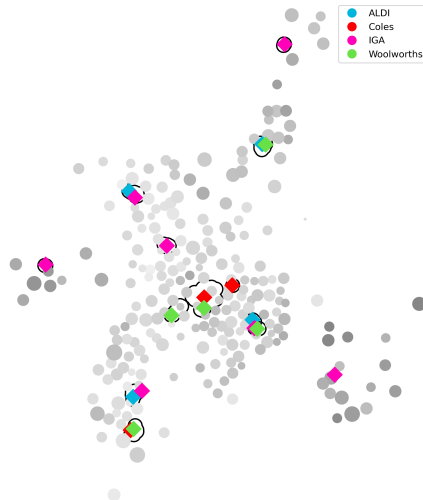 $\Delta \pi_{s'} \leq 0 \quad \forall s' \notin \mathcal{S}$

- Agnostic to order of entry
  - Robust to first mover advantages

- Agnostic to equilibrium selection
  - Robust to multiple equilibria

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- Vectorized Nash-Bertrand first order conditions:

$$\boldsymbol{p} = \boldsymbol{c} + \left(\boldsymbol{O} \odot \left(-\boldsymbol{\Delta}(\boldsymbol{p}) \right) \right)^{-1} \boldsymbol{q}(\boldsymbol{p}) \text{ with } \Delta_{ss'} = \frac{\partial q_s}{\partial p_{s'}}$$

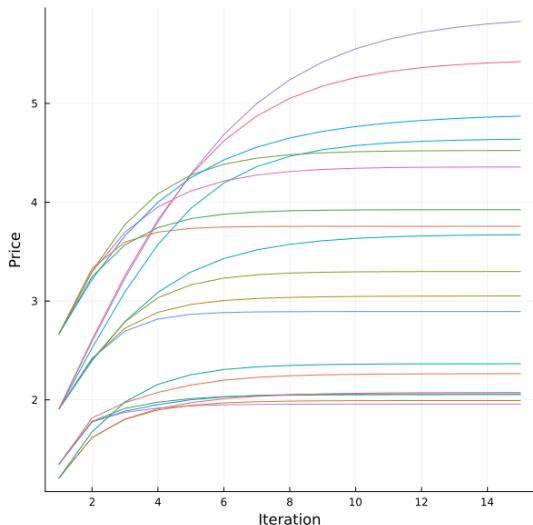$$\text{and } O_{ss'} = 1[o_s{=}o_{s'}]$$

Fixed Point Iteration
Morrow & Skerlos (2011 OR):

- Analytic demand derivative matrix decomposition:

$$\boldsymbol{\Delta} = \boldsymbol{\Lambda} - \boldsymbol{\Gamma} \text{ with } \Lambda_{ss} = \sum_h z_h^\zeta \, \eta_h \, \psi_{hs}$$

$$\text{and } \Gamma_{ss'} = \sum_h z_h^\zeta \, \eta_h \, \psi_{hs} \, \psi_{hs'}$$

- Fixed point iteration:

$$\boldsymbol{p} \leftarrow \boldsymbol{c} + \boldsymbol{\Lambda}(\boldsymbol{p})^{-1} \left(\left(\boldsymbol{O} \odot \boldsymbol{\Gamma}(\boldsymbol{p}) \right) \left(\boldsymbol{p} - \boldsymbol{c} \right) - \boldsymbol{q}(\boldsymbol{p}) \right)$$

Weighted Maximum Likelihood

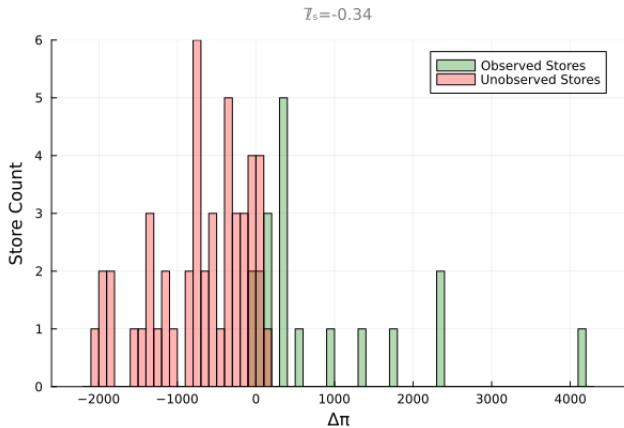- Maximise weighted log-likelihood of store existence & non-existence probabilities

$$\ell(\boldsymbol{\theta}) := |\mathcal{S}|^{-1} \sum_{s \in \mathcal{S}} \log\left[\underbrace{F_b\big(\Delta\pi_s(\boldsymbol{\theta}, \boldsymbol{\mathcal{X}}_s)\big)}_{\Pr[s \in \mathcal{S}]}\right] + |\mathcal{B} \times \mathcal{R}|^{-1} \sum_{s' \in \mathcal{B} \times \mathcal{R}} \log\left[\underbrace{1 - F_b\big(\Delta\pi_{s'}(\boldsymbol{\theta}, \boldsymbol{\mathcal{X}}_{s'})\big)}_{\Pr[s' \notin \mathcal{S}]}\right]$$

- Outcome-based sampling
  - $|\mathcal{S}|$ observed stores
  - $|\mathcal{B} \times \mathcal{R}|$ unobserved stores
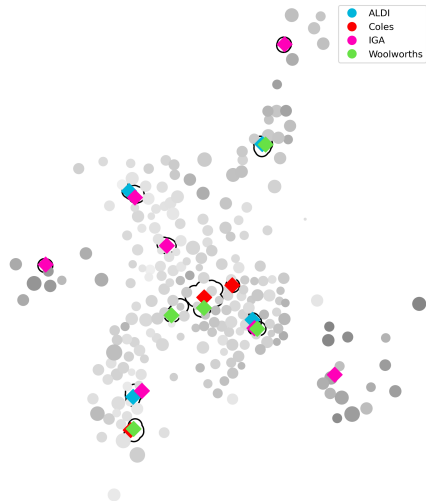   - Extra store of each brand at each retail precinct

$\bar{\ell}_s = -0.34$

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Results

Store	$\hat{\rho}_s$	$\hat{q}_s$	$\hat{\pi}_s$	$\widehat{\Delta\pi_s}$
ALDI EagleHawk	2.06	1269	28	24
ALDI Kangaroo Flat	2.05	1401	111	110
ALDI Epsom	2.26	1349	360	357
ALDI Strathdale	1.96	1439	0	-2
Coles Kangaroo Flat	1.99	1146	-11	-12
Coles McIvor Rd	2.37	1273	563	127
Coles Bendigo	2.07	2080	890	382
IGA EagleHawk	3.30	2608	1426	365
IGA Kangaroo Flat	3.05	2225	345	310
IGA Huntly	4.64	870	182	182
IGA Maiden Gully	5.87	1023	1852	1762
IGA Strathfieldsaye	5.44	1847	4326	4188
IGA Stonemans Village	2.89	3788	1532	1376
IGA Long Gully	3.67	3060	3204	2360
Woolworths Lansell Plaza	3.92	1371	71	57
Woolworths Golden Square	4.89	1851	2461	933
Woolworths Kennington	3.76	2413	974	338
Woolworths Epsom	4.52	2154	2345	2340
Woolworths Bendigo	4.36	2207	2074	503

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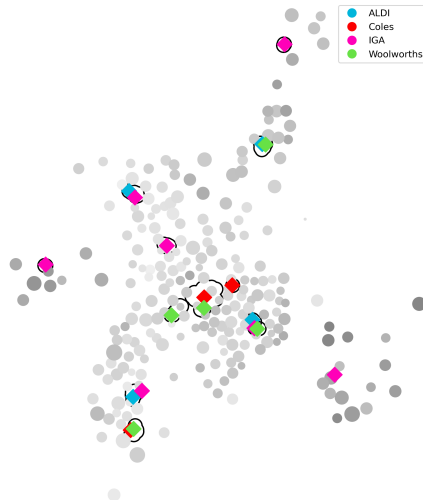
Retail Precinct r	Observed Brands $\mathcal{B}_r$	Profitable Entrants $b : \widehat{\Delta \pi_{br}} > 0$	Profit $\widehat{\Delta \pi_{br}}$
Golden Square	Woolworths	ALDI	164
Bendigo	Woolworths Coles	ALDI	77
Long Gully	IGA	ALDI Coles Woolworths	74 70 1

Demand

- Household Size: $\hat{\zeta} \approx 0.54 \implies q_h|i_h, \boldsymbol{p} \propto z_h^{0.5}$

Utility

- Agglomeration: $\hat{\alpha} \approx 0.19 \implies \hat{\alpha}a_s \in [0, 1]$
- Distance: $\hat{\delta} \approx -3.05$ (per Km) $\implies \hat{\delta}d_{hs} \in [-60, 0]$
- Taste Shocks:
  $\hat{\rho} \approx 0.67 \implies \text{Corr}[\varepsilon_{hs}, \varepsilon_{hs'}|s, s' \in \mathcal{S}_b] = 0.67$
- Price Sensitivity: $\hat{\eta}_h \approx -9.9 + 1.1i_h \implies \hat{\eta}_h p_s \in [-50, -5]$

Brand Vertical Differentiation & Cost Structure

Brand	$\hat{\phi}_b$	$\hat{\mu}_b$	$\hat{\beta}_b$
Woolworths	1648	2.67	12.49
IGA	2196	1.91	11.54
Coles	908	1.21	8.53
ALDI	873	1.35	8.54

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