ECE212 Homework 9

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Q1.

In the provided spaces, construct the straight-line approximations of the Bode plots for the following transfer functions. Clearly label all critical (corner) frequencies and the dB gains of "flat" regions.

a. $T(s) = \dfrac{70}{\left(1+\frac{s}{300}\right)\left(1+\frac{s}{70000}\right)}$

Write the transfer function in standard form:

$$T(j\omega) = \frac{70}{\left(1+\frac{j\omega}{300}\right)\left(1+\frac{j\omega}{70000}\right)}$$

$$= \frac{70}{\sqrt{1+\left(\frac{\omega}{300}\right)^2}\cdot e^{j\cdot\arctan\left(\frac{\omega}{300}\right)}\cdot\sqrt{1+\left(\frac{\omega}{70000}\right)^2}\cdot e^{j\cdot\arctan\left(\frac{\omega}{70000}\right)}}$$

$$= \frac{70}{\sqrt{1+\left(\frac{\omega}{300}\right)^2}\cdot\sqrt{1+\left(\frac{\omega}{70000}\right)^2}}\cdot e^{j\left(0^\circ-\arctan\left(\frac{\omega}{300}\right)-\arctan\left(\frac{\omega}{70000}\right)\right)}$$

*Two **critical frequencies** can be found at $\omega_1 = 300$, $\omega_2 = 70000$. The full derivations for gain and phase responses are:*

$$|T(j\omega)|_{dB} = 20\log_{10}\left(\frac{70}{\sqrt{1+\left(\frac{\omega}{300}\right)^2}\cdot\sqrt{1+\left(\frac{\omega}{70000}\right)^2}}\right)$$

$$= 20\left(\log_{10}(70) - \log_{10}\left(\sqrt{1+\left(\frac{\omega}{300}\right)^2}\right) - \log_{10}\left(\sqrt{1+\left(\frac{\omega}{70000}\right)^2}\right)\right)$$

$$\theta(\omega) = 0^\circ - \arctan\left(\frac{\omega}{300}\right) - \arctan\left(\frac{\omega}{70000}\right)$$

Approximated gain calculation based on above derivation:

$$\omega = 0:\ |T(0)| = 20\cdot\log_{10}(70) = 36.902\ dB \Rightarrow f = 0\ Hz$$

$$\omega = 300:\ f = \frac{300}{2\pi} = 47.75\ Hz$$

Approximated phase calculation based on above derivation:

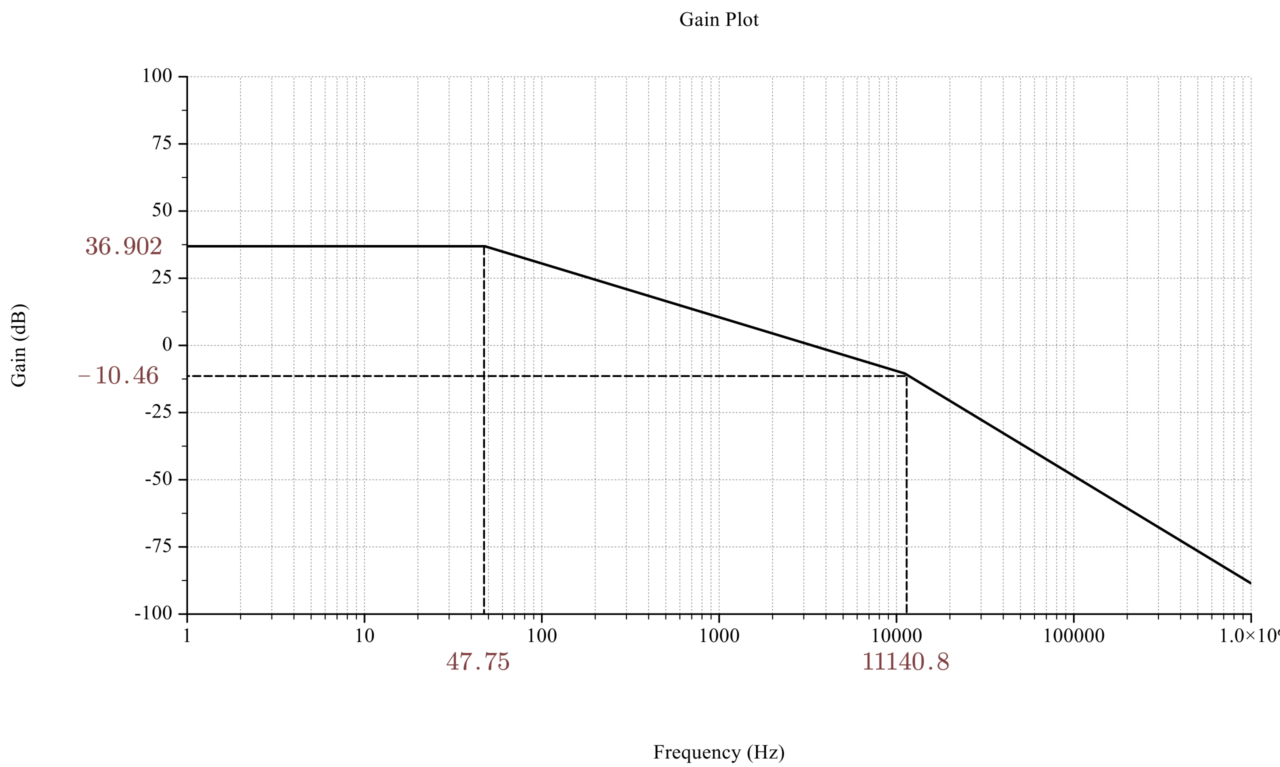
$$when\ \omega \ll 300\ \Rightarrow\ f \ll 47.75\ \Rightarrow f > \frac{1}{10} \cdot 47.75 = 4.77465\ Hz$$

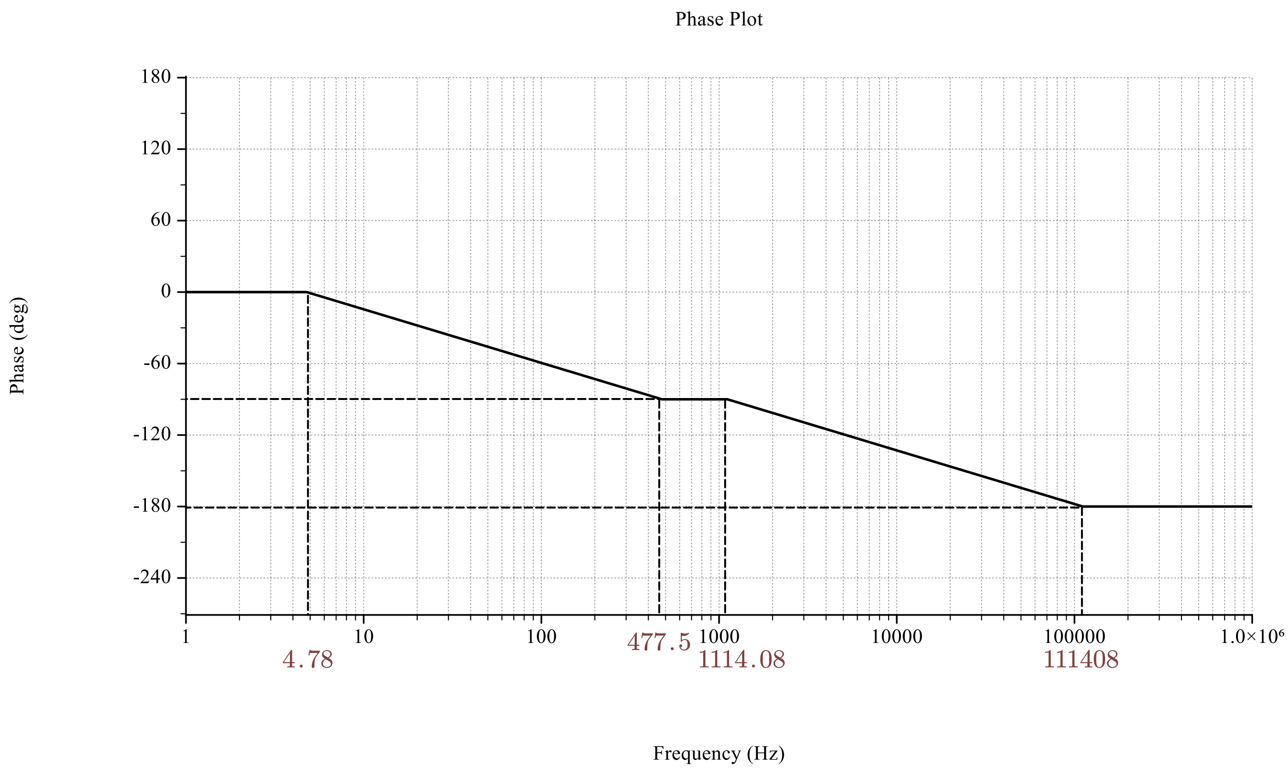
$$300 \gg \omega \Rightarrow\ f \gg 47.75\ \ \Rightarrow f < 10 \cdot 47.75 = 477.465\ Hz$$

$$\omega \ll 70000\ \Rightarrow\ f \ll 11140.8\ \Rightarrow f > \frac{1}{10} \cdot 11140.8 = 1114.08\ Hz$$

$$70000 \gg \omega \Rightarrow\ f \gg 11140.8\ \ \Rightarrow f < 10 \cdot 11140.8 = 111408\ Hz$$

Therefore, Bode plots for the stated transfer function can be drawn according the above calculations:





b. $T(s) = \dfrac{20}{s(s+0.01)}$

Write the transfer function in standard form:

$$T(j\omega) = \frac{2000}{j\omega(100j\omega+1)} = \frac{2000}{\omega \cdot e^{j\frac{\pi}{2}} \cdot \sqrt{1+\left(\frac{\omega}{0.01}\right)^2} \cdot e^{j \cdot \arctan\left(\frac{\omega}{0.01}\right)}} = \frac{2000}{\omega \cdot \sqrt{1+\left(\frac{\omega}{0.01}\right)^2}} \cdot e^{j\left(0° - 90° - \arctan\left(\frac{\omega}{0.01}\right)\right)}$$

*Two **critical frequencies** can be found at* $\omega_1 = 1,\ \omega_2 = 0.01$ *.The full derivations for gain/ phase responses are:*

$$|T(j\omega)|_{dB} = 20\log_{10}\left(\frac{2000}{\omega \cdot \sqrt{1+\left(\frac{\omega}{0.01}\right)^2}}\right) = 20\left(\log_{10}(2000) - \log_{10}(\omega) - \log_{10}\left(\sqrt{1+\left(\frac{\omega}{0.01}\right)^2}\right)\right)$$

$$\theta(\omega) = -90° - \arctan\left(\frac{\omega}{0.01}\right)$$

Approximated gain calculation based on above derivation:

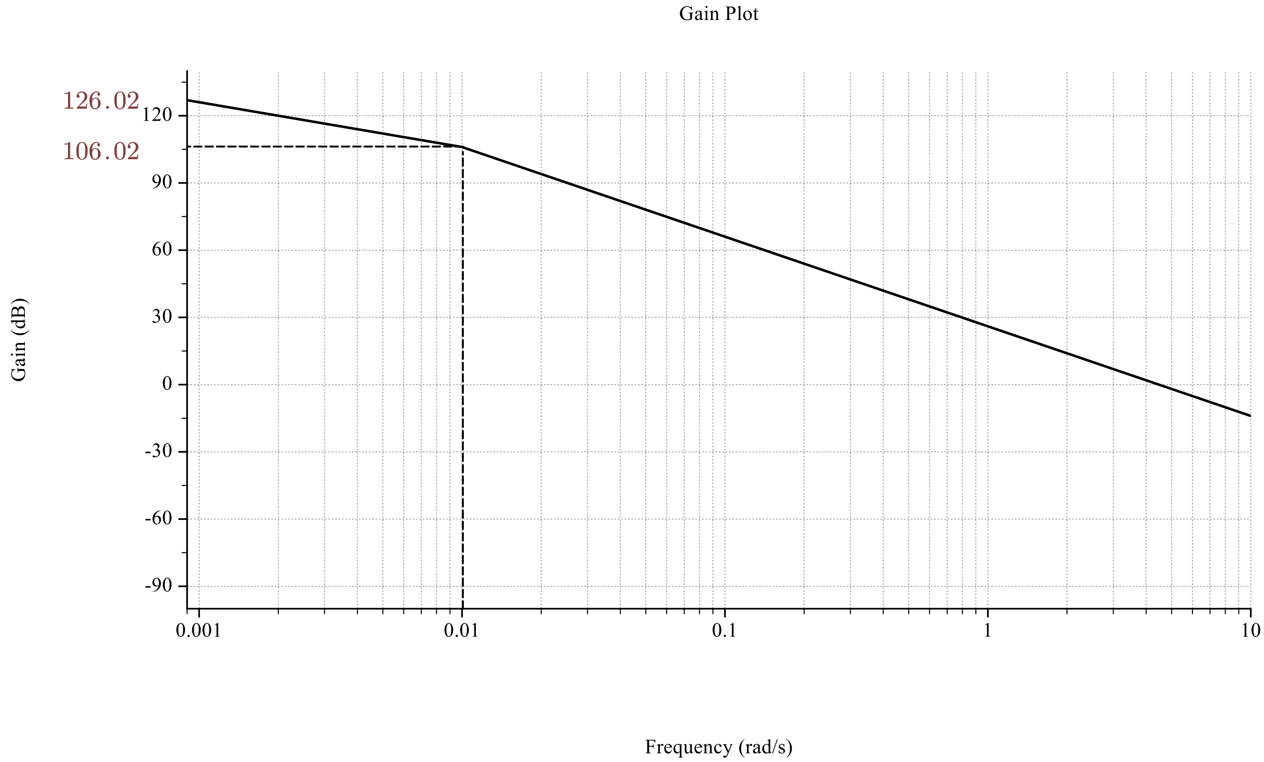
$$\omega = 0.001:\ |T(0.001)| = 20 \cdot \log_{10}(2000) - 20 \cdot \log_{10}(0.001) = 126.021\ dB$$

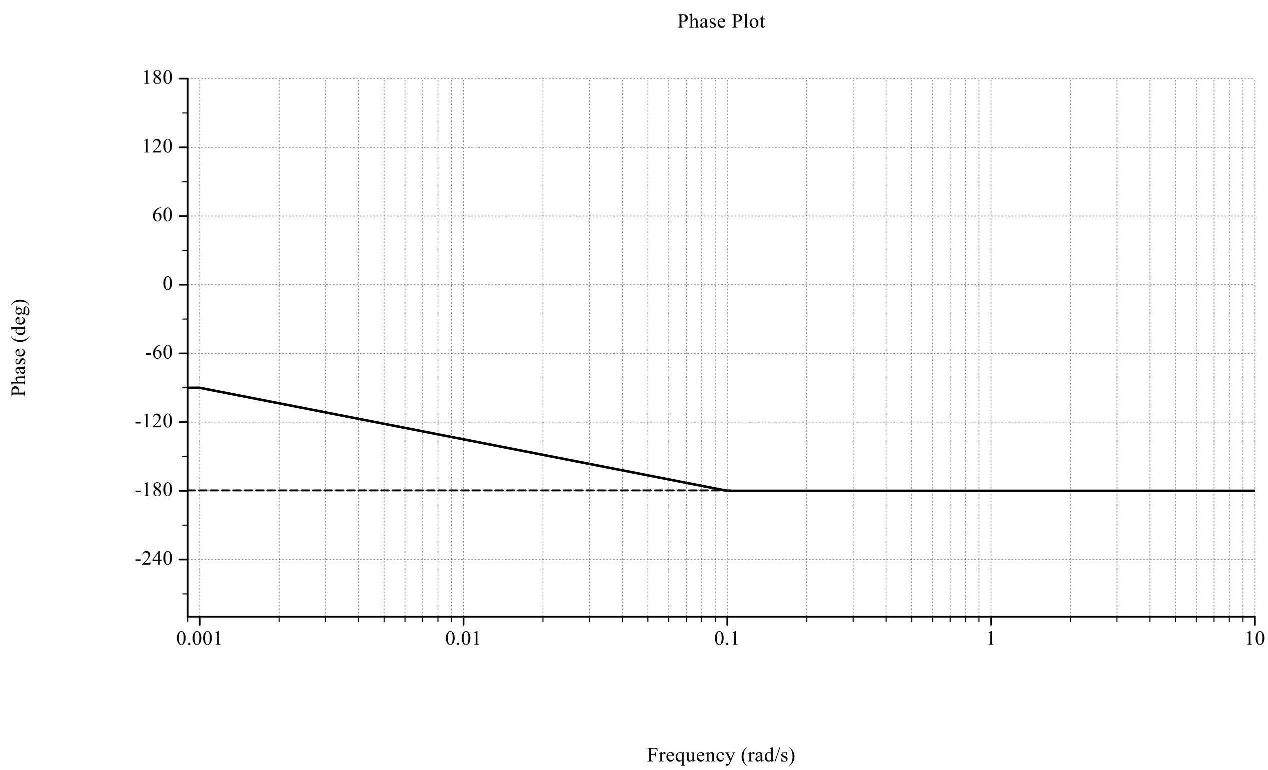
$$\omega = 0.01:\ |T(0.01)| = 20 \cdot \log_{10}(2000) - 20 \cdot \log_{10}(0.01) = 106.021\ dB$$

Approximated phase calculation based on above derivation:

$$when\ \omega \ll 0.01\ \Rightarrow \omega > \frac{1}{10} \cdot 0.01 = 0.001,\ when\ 0.01 \gg \omega\ \Rightarrow \omega < 10 \cdot 0.01 = 0.1$$

Therefore, Bode plots for the stated transfer function can be drawn according the above calculations:





c. $T(s) = \dfrac{200}{1 + \frac{s}{8000} + \frac{s^2}{16000}}$

Write the transfer function in standard form:

$$T(j\omega) = \frac{200}{1 + \frac{j\omega}{8000} - \frac{\omega^2}{16000}}$$

$$= \frac{200}{1 - \left(\frac{\omega}{40\sqrt{10}}\right)^2 + j \cdot 2 \cdot \frac{1}{40\sqrt{10}} \frac{\omega}{40\sqrt{10}}}$$

Two ***critical frequencies*** *can be found at* $\omega = 40\sqrt{10} = 126.491$ *. As the question is asking for straight-line approximation, only approximated calculations (damping factor is neglected) are provided as follows:*

$$\omega = 0 : \; |T(0)| = 20 \cdot \log_{10}(200) = 46.0206 \; dB$$

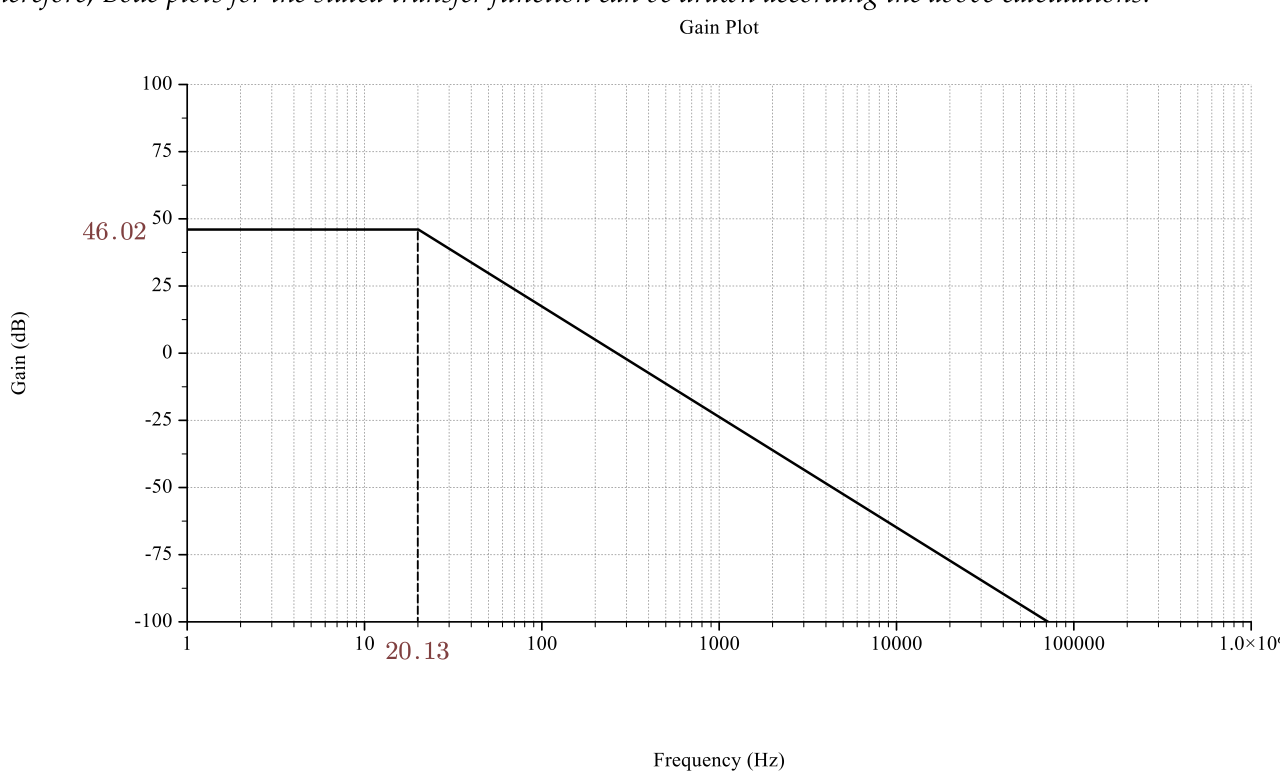
$$\omega = 40\sqrt{10} : \; f\left(40\sqrt{10}\right) = \frac{40\sqrt{10}}{2\pi} = 20.1317 \; Hz$$

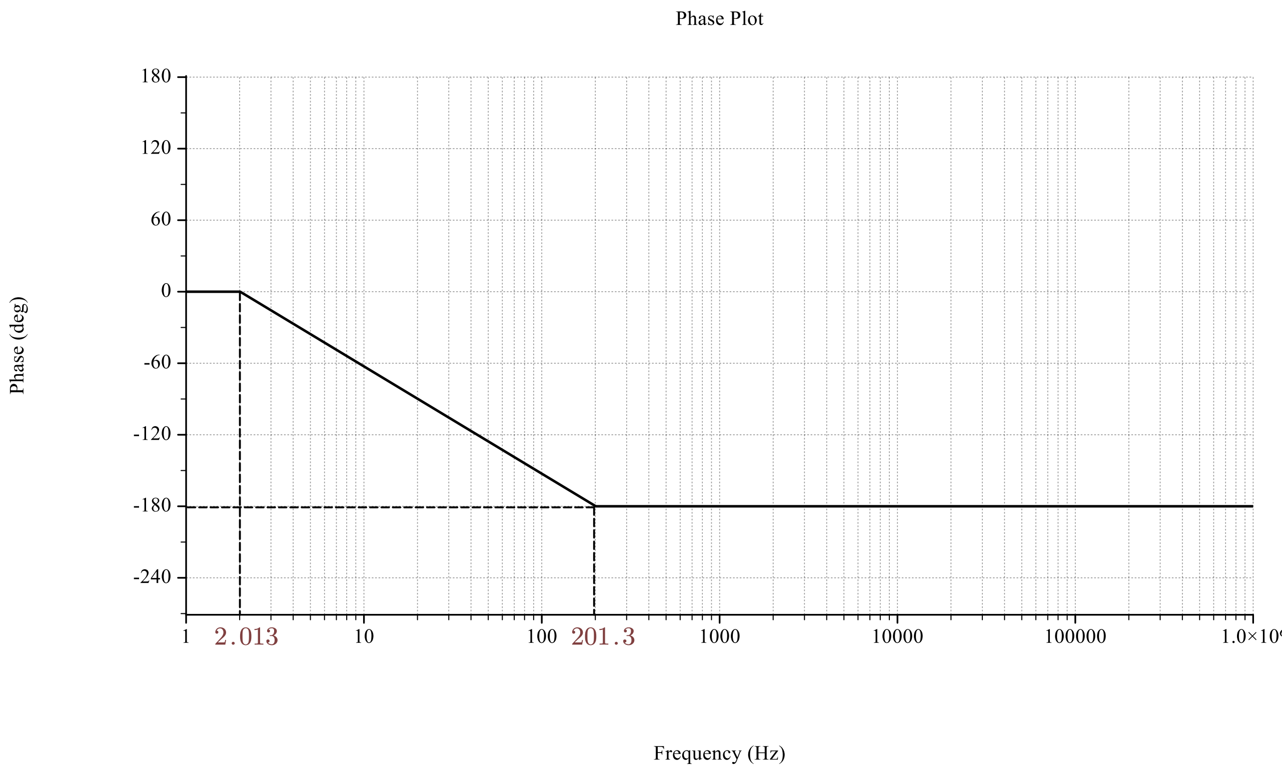
Approximated phase calculation based on above derivation:

$$when \; f \ll 20.13 \; \Rightarrow \omega > \frac{1}{10} \cdot 20.13 = 2.013,$$

$$20.13 \gg \omega \; \Rightarrow \omega < 10 \cdot 20.13 = 201.3$$

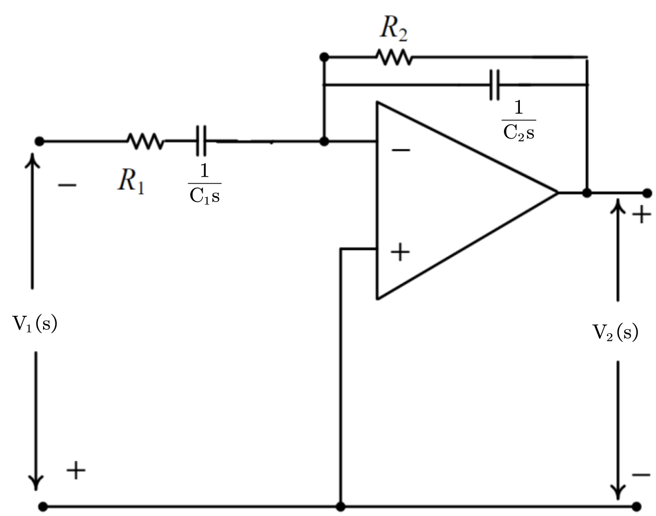
Therefore, Bode plots for the stated transfer function can be drawn according the above calculations:





Q2.

a. Transfer the circuit in s-domain



b. find the feedback and forward path impedance Z2 and Z1

Forward path impedance:

$$Z_1 = Z_{R1} + Z_{C1} = R_1 + \frac{1}{C_1 s}$$

Feedback path impedance:

$$Z_2 = Z_{R2} || Z_{C2} = \frac{R_2 \cdot \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = \frac{R_2}{R_2 C_2 s + 1}$$

c. find the Gain T(s) = v2(s)/v1(s)= Z2(s)/Z1(s)

$$T(s) = \frac{v_2(s)}{v_1(s)} = \frac{Z_2(s)}{Z_1(s)} = \frac{\frac{R_2}{R_2 C_2 s + 1}}{R_1 + \frac{1}{C_1 s}} = \frac{R_2 C_1 s}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

d. Select the values of Rs and Cs such that the critical (corner) frequencies and the gains of the OP.Amp. circuit match those shown with the Bode plots.

Write the transfer function in standard form:

$$T(s) = \frac{R_2 C_1 s}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)} = \frac{R_2 C_1 s}{\left(\frac{s}{\frac{1}{R_1 C_1}} + 1\right)\left(\frac{s}{\frac{1}{R_2 C_2}} + 1\right)}$$

Two critical points at $\omega_1 = 20\ Hz = 125.664\ rad/s$ *and* $\omega_1 = 10000\ Hz = 62831.9\ rad/s$ *can be examined from the given gain plot. Furthermore, the gain plot suggests that the gain at* $\omega_1 = 0\ rad/s$ *is approximately 14. This signifies that* $20 \cdot \log(R_2 C_1) \approx 14 \ \Rightarrow\ R_2 C_1 = 10^{14/20} = 5.01187$.

Therefore, a system of equations can be constructed as follows:

$$\begin{cases} \dfrac{1}{R_1 C_1} = 125.664 \\ \dfrac{1}{R_2 C_2} = 62831.9 \\ R_2 C_1 = 5.01187 \end{cases} \Rightarrow solve\ by\ setting\ R_2 = 1\Omega \Rightarrow \begin{cases} R_1 = 1.588\ m\Omega \\ C_2 = 15.92\ \mu F \\ C_1 = 5.01187\ F \end{cases}$$

Select $R_1 = 1.588\ m\Omega,\ R_2 = 1\Omega,\ C_1 = 5.01\ F,\ C_2 = 15.92\ \mu F$.