# ECE212 Homework 7

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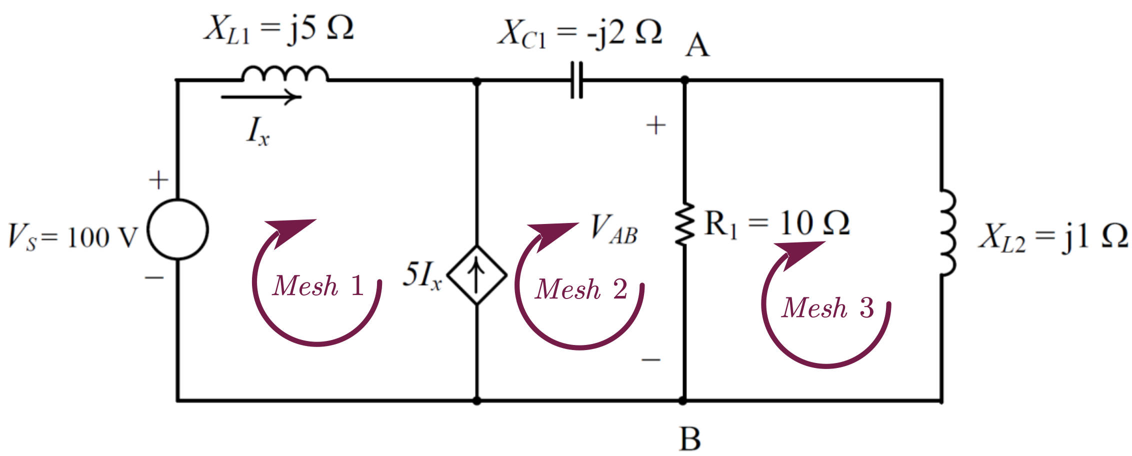
**Q1**.




1. **find the phasors $I_x$ and $V_{AB}$.**

   *Solve the question using mesh analysis:*

   *Finding equation on the supermesh consists of mesh 1 and mesh 2:*

$$V_s = I_1 \cdot X_{L1} + I_2 \cdot X_{C1} + (I_2 - I_3) \cdot R_1 \tag{1}$$

$$I_2 - I_1 = 5I_x \tag{2}$$

$$I_x = I_1 \tag{3}$$

*Finding equation on mesh 3:*

$$(I_3 - I_2) \cdot R_1 + I_3 \cdot X_{L2} = 0 \tag{4}$$

*After plugging in values, the above system of equations can be simplified to:*

$$\begin{cases} 100 = I_1 \cdot j5 + I_2 \cdot (-j2) + (I_2 - I_3)10 \\ (I_3 - I_2) \cdot 10 + I_3 \cdot j1 = 0 \\ I_2 = 6I_1 \end{cases}$$

*Solve the above system of equations:*

$$I_x = I_1 = 82.33\angle 60.7°A,\ I_2 = 493.99\angle 60.7°A, I_3 = 491.54\angle 55.0°A$$

$$V_{AB} = (I_2 - I_3)R_1 = (493.99\angle 60.7° - 491.54\angle 55.0°)10 = 491.54\angle 145.0°$$

2. **Based on your results from part a, find the Apparent, Complex, and Real Power:**

   **i) Delivered by the voltage source Vs**

   **ii) Consumed by the resistor R1**

   **iii) Consumed by the inductor L2, i.e. XL2.**

*i)*

$$I_{x\ rms} = \frac{I_x}{\sqrt{2}} = \frac{82.33}{\sqrt{2}} \angle 60.7° = 58.216 \angle 60.7°$$

$$V_{AB\ rms} = \frac{V_{AB}}{\sqrt{2}} = \frac{491.54}{\sqrt{2}} \angle 145.0° = 347.571 \angle 145.0°$$

$$\mathbf{S} = V_{rms} \cdot I_{rms}{}^{*} = \frac{100}{\sqrt{2}} \cdot 58.216 \angle -60.7° = 4116.5 \angle -60.7° W$$

$$S = |\mathbf{S}| = 4116.5\ W$$

$$P = \mathrm{Re}(\mathbf{S}) = S \cdot \cos(-60.7°) = 2014.54\ W$$

*ii)*

$$I_{R\ rms} = \frac{I_2 - I_3}{\sqrt{2}} = \frac{493.99\angle 60.7° - 491.54\angle 55.0°}{\sqrt{2}} = 34.69\angle 145.0°$$

$$\mathbf{S} = V_{rms} \cdot I_{rms}{}^{*} = 34.69\angle -145.0° \cdot 347.571 \angle 145.0° = 12057.2 \angle 0° W$$

$$S = |\mathbf{S}| = 12057.2\ W$$

$$P = \mathrm{Re}(\mathbf{S}) = S \cdot \cos(0°) = 12057.2\ W$$

*iii)*

$$I_{L\ rms} = \frac{I_3}{\sqrt{2}} = \frac{491.54\angle 55.0°}{\sqrt{2}} = 347.571 \angle 55.0°$$

$$\mathbf{S} = V_{rms} \cdot I_{rms}{}^{*} = 347.571 \angle -55.0° \cdot 347.571 \angle 145.0° = 120806 \angle 90° W$$

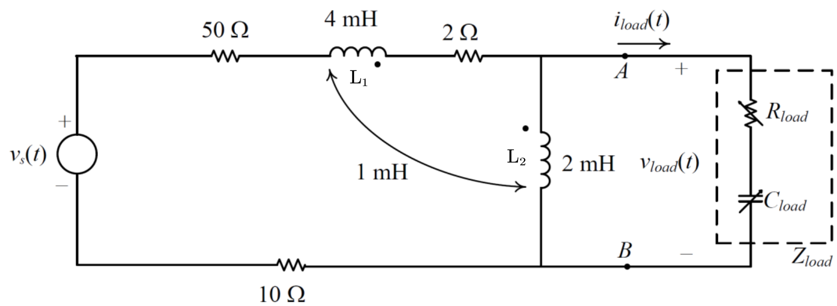
$$S = |\mathbf{S}| = 120806\ W$$

$$P = \mathrm{Re}(\mathbf{S}) = S \cdot \cos(90°) = 0\ W$$

3. **Are some parts of the complex power “missing” in parts ii) and iii)? Explain the results.**

   *In part ii), the complex power is missing an imaginary component, and in part iii) the complex power is missing a real component. This is because from Ohm’s Law,* $I = \frac{V}{R}$*, which means the voltage and the current of the same resistor will have the same phase (with different magnitudes), then by the definition of complex power, the phase of the complex power is the phase difference from the resistor voltage and the resistor current. Therefore, for* $R_1$*, the voltage phase cancels with the current phase, which result in a pure real complex power. On the other hand, for inductor,* $I = \frac{V}{Z_L} = \frac{V}{j\omega L} = -j\frac{V}{\omega L} = \frac{V}{\omega L}\angle \phi_V - 90°$*. Then, the phase difference* $(\phi_V - \phi_I)$ *for the inductor becomes* $\phi_V - (\phi_V - 90°) = 90°$*, which result in a pure imaginary complex power.*

**Q2.**



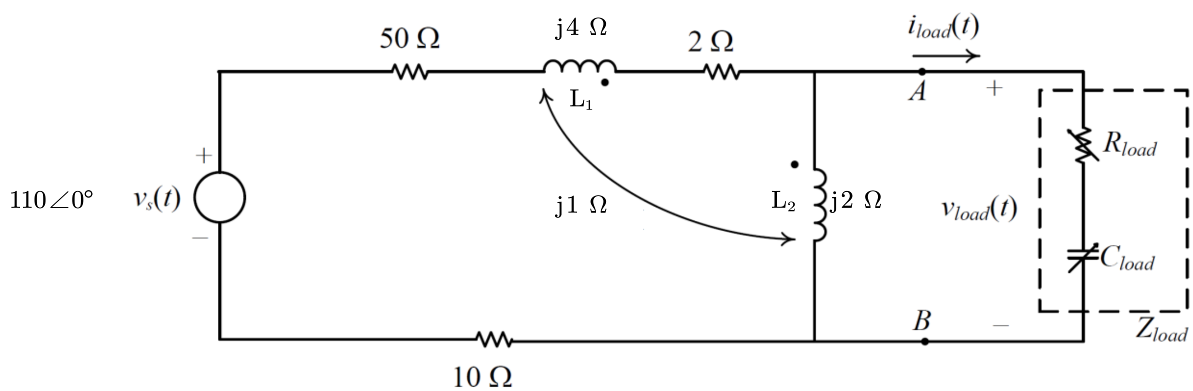

1. **Transform the circuit into phasor domain, i.e. draw its equivalent in the phasor domain, calculate and clearly label all impedances (you can express the equivalents of C<sub></sub>load and Rload in symbolic forms).**

$$V_s = 110\sqrt{2}\cos(1000t)V = 110\sqrt{2}\angle 0°$$

$$Z_{L1} = j\omega L_1 = j1000 \cdot 4 \cdot 10^{-3} = j4\ \Omega$$

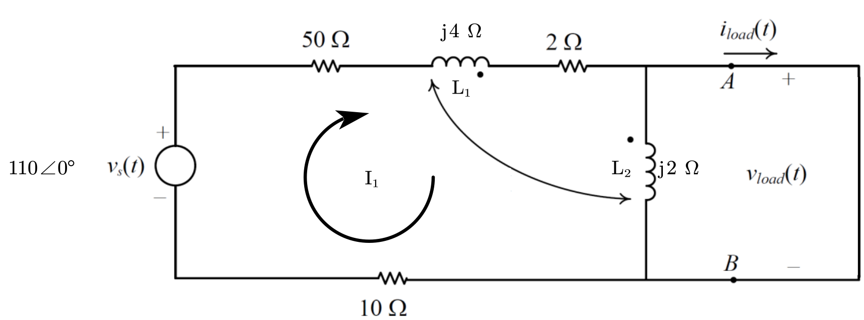
$$Z_{L2} = j\omega L_2 = j1000 \cdot 2 \cdot 10^{-3} = j2\ \Omega$$

*The phasor domain circuit configuration can be draw as follows (**in rms values**):*




2. **Calculate the values of the short circuit current and open circuit voltage between points A and B, i.e. the values of the current and the voltage when $Z_{load}$ is replaced with a short and an open circuit, respectively.**

   *Short circuit can be drawn as:*



$$V_{L1} = I_1 \cdot X_{L1} - M \cdot I_1 = I_1(j4 - j1) = I_1(j3)$$

$$V_{L2} = (I_1 - I_{load}) \cdot X_{L2} - M \cdot (I_1 - I_{load}) = (I_1 - I_{load})(j2 - j1) = (I_1 - I_{load})(j1) = 0V$$

$$\Rightarrow I_1 = I_{load},\ I_{L2} = 0V$$

$$\mathrm{R}_{eq} = 50 + 2 + 10 = 62\Omega$$

*Using KVL:*

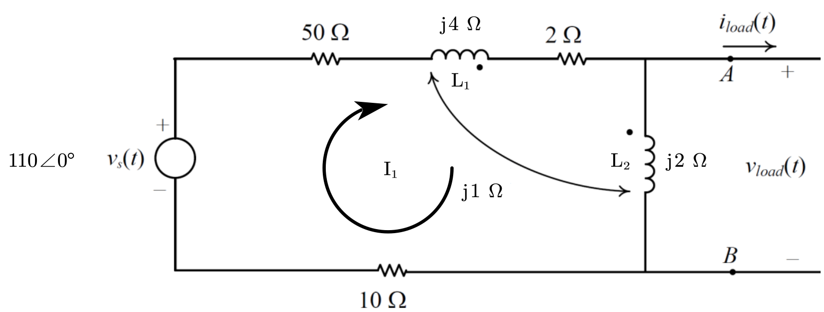
$$V_s = R_{eq} \cdot I_1 + V_{L1} + V_{L2}$$

$$\Rightarrow (62 + 3j) I_1 = 110$$

$$I_{load\ rms} = I_1 = \frac{110}{62 + 3j} A = 1.77\angle -2.77° A$$

$$I_{load} = 2.50\angle -2.77° A$$

*Open circuit can be drawn as:*




$$V_{L1} = I_1 \cdot X_{L1} - M \cdot I_1 = I_1(j4 - j1) = I_1(j3)$$

$$V_{L2} = I_1 \cdot X_{L2} - M \cdot I_1 = I_1(j2 - j1) = I_1(j1)$$

$$\mathrm{R}_{eq} = 50 + 2 + 10 = 62\Omega$$

*By KVL:*

$$V_s = R_{eq} \cdot I_1 + V_{L1} + V_{L2}$$

$$(62 + 4j) I_1 = 110$$

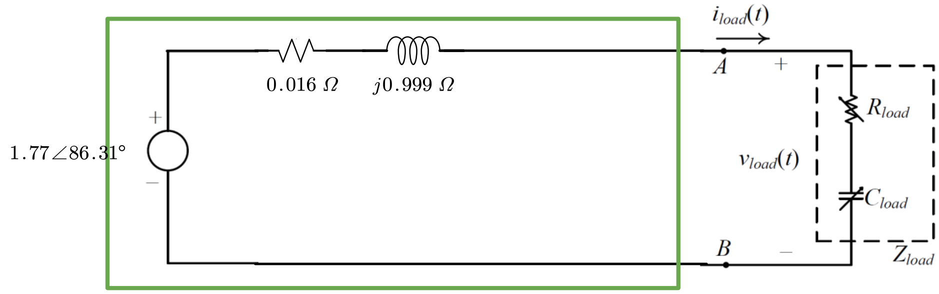
$$I_1 = \frac{110}{62 + 4j} A = 1.77\angle -3.69° A$$

$$V_{loadrms} = V_{L2} = I_1(j1) = 1.77\angle 86.31° V$$

$$V_{load} = 2.50\angle 86.31° V$$

**3.** Find the Thevenin equivalent circuit "seen" by ***Z**load*.

$$Z_{load} = \frac{V_{loadrms}}{I_{loadrms}} = \frac{1.77\angle 86.31°}{1.77\angle -2.77°} = 1.00\angle 89.08°\ \Omega = 0.016 + j0.999\ \Omega$$




**4. Find the values of $C_{load}$ and $R_{load}$ that will result in the maximum average power delivered to the load, $P_{load\_max}$, and calculate that power value.**

*To result in maximum average power for the load, the imaginary part of the Thevenin equivalent circuit impedance must be cancelled. This essentially means that the impedance of the inductor must be balanced with the impedance of the capacitor:*

$$Z_{load} = Z_{Th}^{*} = 1.00\angle -89.08°\ \Omega = 0.016 - j0.999\ \Omega$$

$$Z_{C\,load} = -j0.999\Omega = -j\frac{1}{\omega C}$$

$$0.999 = \frac{1}{1000C} \Rightarrow C_{load} = 1.00\ mF$$

$$R_{load} = 0.016\ \Omega$$

*Maximum power calculation:*

$$S_{load\max} = V_{rms} \cdot I_{rms}{*} = I_{rms} \cdot R_{load} \cdot I_{rms}{*}$$

$$= \frac{1.77\angle 86.31°}{0.032}\frac{1.77\angle -86.31°}{0.032} \cdot 1.00\angle 89.08° = 49.12 + j3059W$$

$$P_{load\max} = 49.12\ W$$