ECE212 Extra Credit Problems

Tiange Zhai

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Q1.

a.

Initial condition can be found as: $20 \cdot \log(50) = 33.9794$

The ω *of the observed zeroes and poles are converted as follows:*

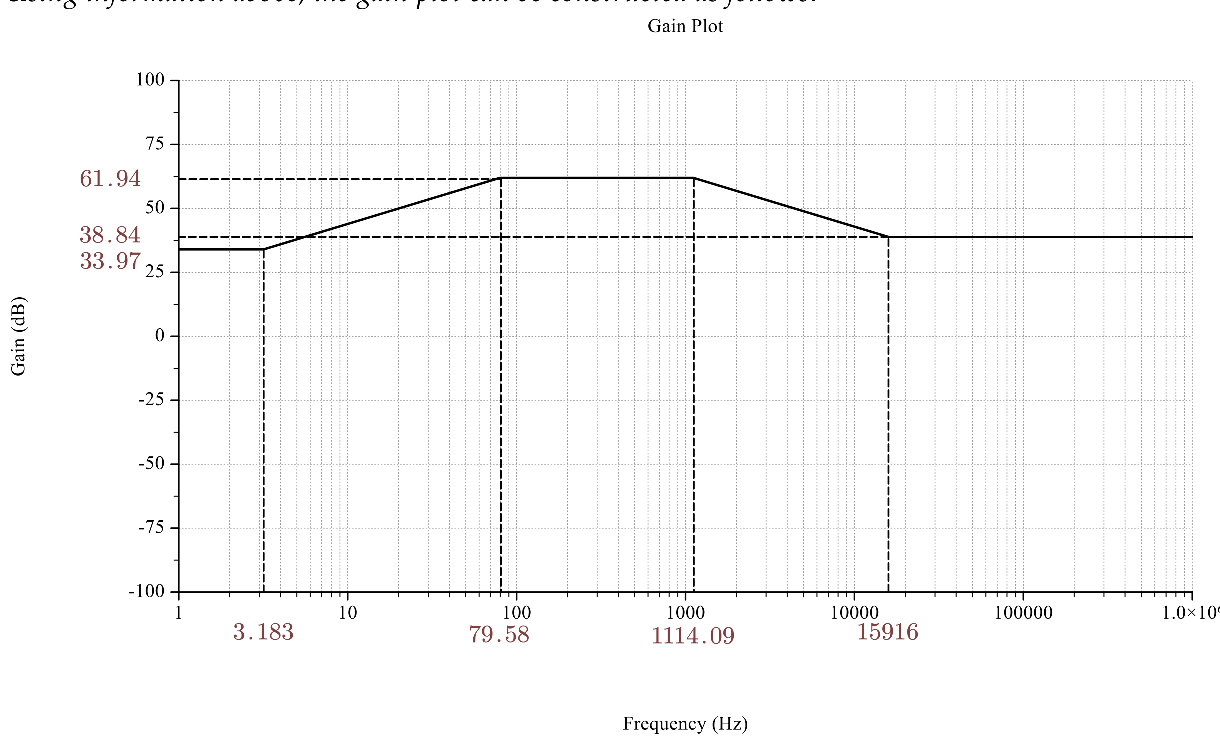
$$f(20) = \frac{20}{2\pi} = 3.1831\ Hz$$

$$f(500) = \frac{500}{2\pi} = 79.5775\ Hz$$

$$f(7000) = \frac{7000}{2\pi} = 1114.08\ Hz$$

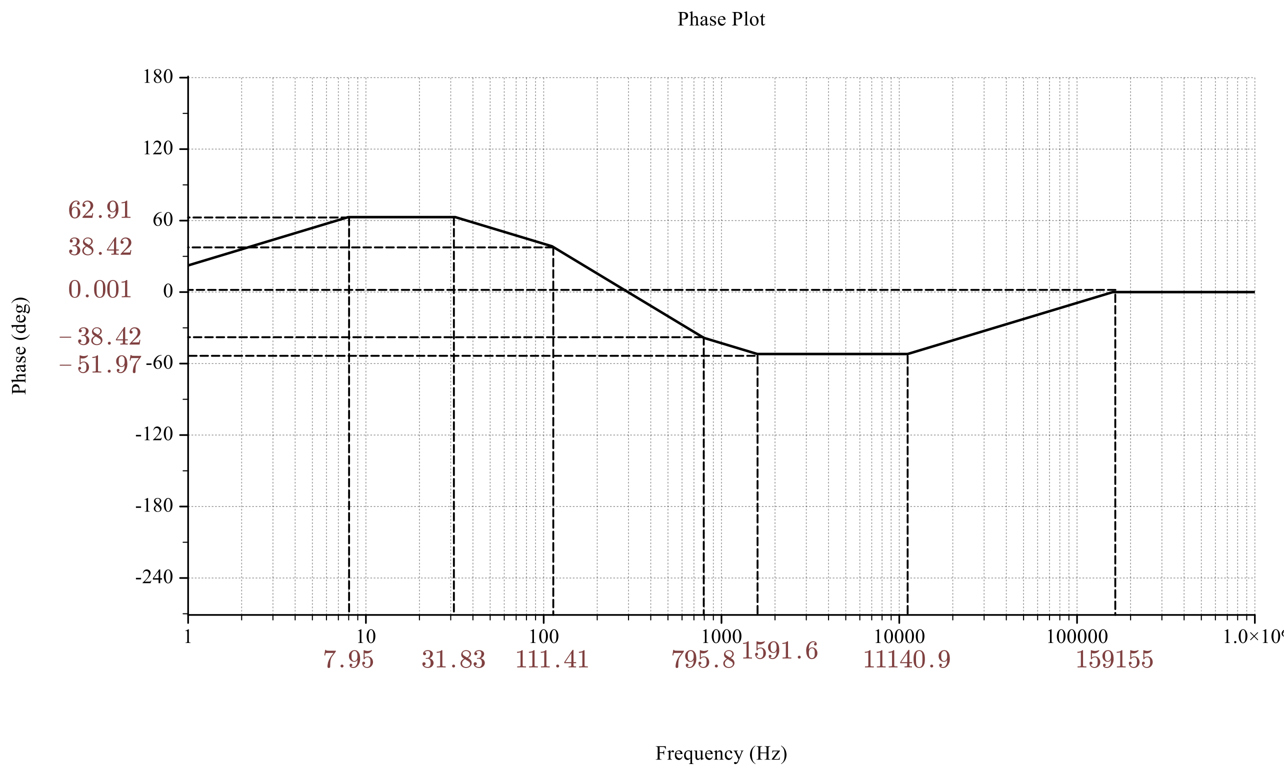
$$f(100000) = \frac{100000}{2\pi} = 15915.5\ Hz$$

Using information above, the gain plot can be constructed as follows:



The range of the phase plot can be considered as follows:

$$0.3183 \xrightarrow{+45°} 7.958 \xrightarrow{+0°} 31.83 \xrightarrow{-45°} 111.41 \xrightarrow{-90°} 795.8 \xrightarrow{-45°} 1591.6 \xrightarrow{0°} 11140.9 \xrightarrow{45°} 159155$$



b.

The ω *of the observed zeroes and poles are converted as follows:*

$$f(62.8) = \frac{62.8}{2\pi} = 9.99,\ f(251200) = \frac{251200}{2\pi} = 39979.7,\ f(3140) = \frac{3140}{2\pi} = 499.75$$

Write the transfer function in standard form:

$$T(j\omega) = 20\,\frac{(s+62.8)(s+251200)}{s(s+3140)}$$

$$= \frac{20\cdot 62.8\cdot 251200}{3140}\,\frac{\left(\frac{s}{62.8}+1\right)\left(\frac{s}{251200}+1\right)}{s\left(\frac{s}{3140}+1\right)}$$

$$= \frac{100480\cdot\sqrt{1+\left(\frac{\omega}{62.8}\right)^2}\cdot e^{j\cdot\arctan\left(\frac{\omega}{62.8}\right)}\cdot\sqrt{1+\left(\frac{\omega}{251200}\right)^2}\cdot e^{j\cdot\arctan\left(\frac{\omega}{251200}\right)}}{\sqrt{1+\left(\frac{\omega}{3140}\right)^2}\cdot e^{j\cdot\arctan\left(\frac{\omega}{3140}\right)}\cdot w\cdot e^{j\frac{\pi}{2}}}$$

$$= \frac{100480\cdot\sqrt{1+\left(\frac{\omega}{62.8}\right)^2}\cdot\sqrt{1+\left(\frac{\omega}{251200}\right)^2}}{\sqrt{1+\left(\frac{\omega}{3140}\right)^2}\cdot\omega}\cdot e^{j\left(0^\circ-90^\circ-\arctan\left(\frac{\omega}{3140}\right)+\arctan\left(\frac{\omega}{62.8}\right)+\arctan\left(\frac{\omega}{251200}\right)\right)}$$

Initial gain can be found at $\omega = 1\ rad/s$ *as:* $20\cdot\log(100480) = 100.042$

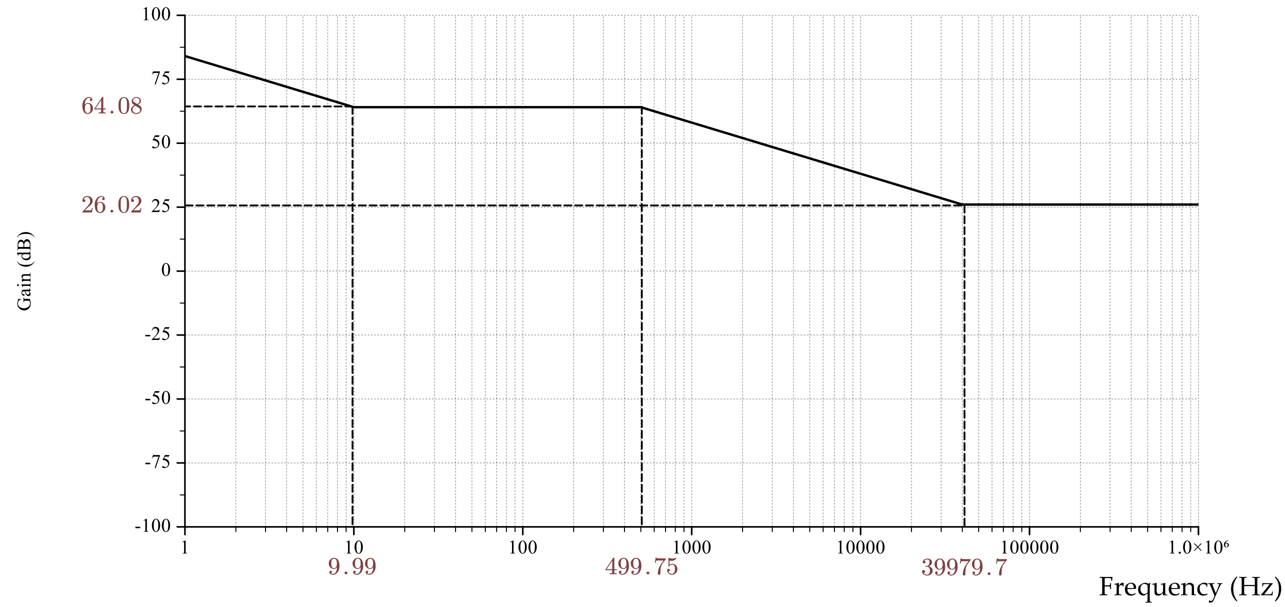
Two ***critical frequencies*** *can be found at* $\omega_1 = 62.8,\ \omega_2 = 3140,\ \omega_3 = 251200$*.The full derivations for gain and phase responses are:*

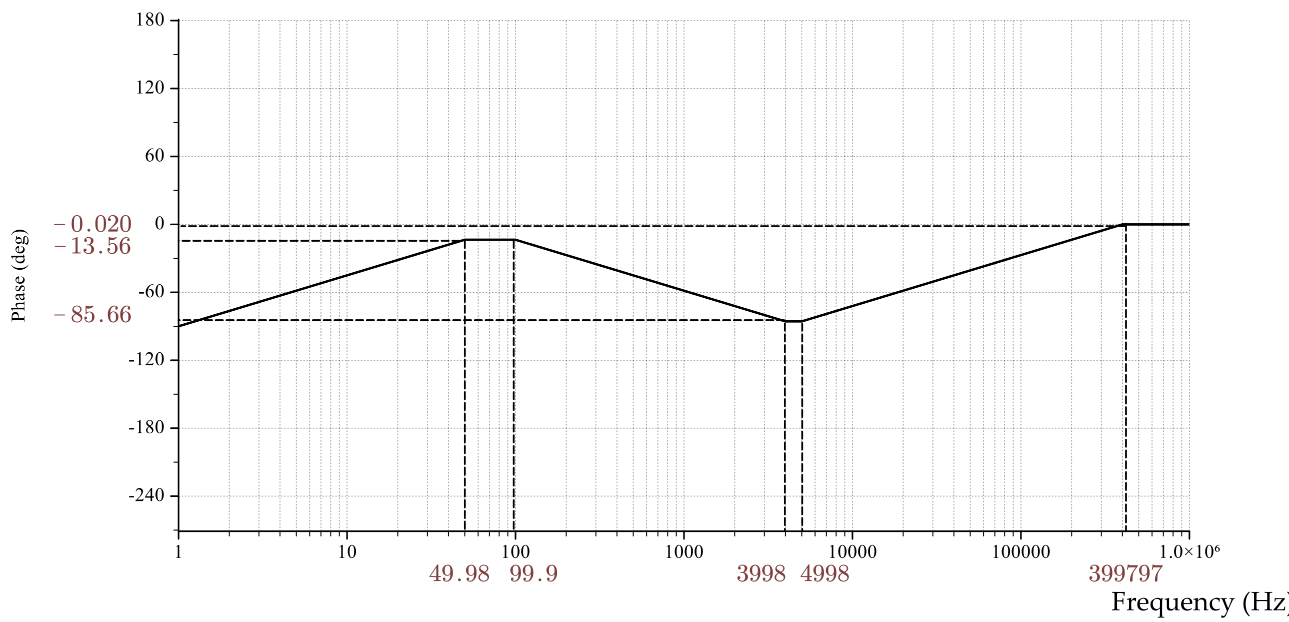
$$|T(j\omega)|_{dB}=20\log_{10}\left(\frac{100480\cdot\sqrt{1+\left(\frac{\omega}{62.8}\right)^2}\cdot\sqrt{1+\left(\frac{\omega}{251200}\right)^2}}{\sqrt{1+\left(\frac{\omega}{3140}\right)^2}\cdot\omega}\right)$$

$$=100.042+20\left(\log\left(\sqrt{1+\left(\frac{\omega}{62.8}\right)^2}\right)+\log\left(\sqrt{1+\left(\frac{\omega}{251200}\right)^2}\right)-\log\left(\sqrt{1+\left(\frac{\omega}{3140}\right)^2}\right)-\log(\omega)\right)$$

$$\theta(\omega)=-90°-\arctan\left(\frac{\omega}{3140}\right)+\arctan\left(\frac{\omega}{62.8}\right)+\arctan\left(\frac{\omega}{251200}\right)$$

Therefore, the gain/phase Bode plots can be constructed as follows based on above derivation model:





Q2.

a.

Trace the graph from left to right. The initial constant can be found as follows:

$20 \cdot \log(K) = 100 \Rightarrow \log(K) = 5 \Rightarrow K = 10^5$

The ω of the observed zeroes and poles are converted as follows:

$$\omega(10) = 10 \cdot 2\pi = 62.83$$

$$\omega(100) = 100 \cdot 2\pi = 628.32$$

$$\omega(3000) = 3000 \cdot 2\pi = 18849.6$$

$$\omega(30000) = 30000 \cdot 2\pi = 188496$$

$$\omega(100000) = 100000 \cdot 2\pi = 628319$$

The transfer function can be established as follows:

$$T(s) = 10^5 s^{\frac{20-100}{20}} \cdot \left(1 + \frac{s}{62.83}\right)^{\frac{-20-(-80)}{20}} \cdot \left(1 + \frac{s}{628.32}\right)^{\frac{0-(-20)}{20}} \cdot \left(1 + \frac{s}{18849.6}\right)^{\frac{-40}{20}} \cdot \left(1 + \frac{s}{188496}\right)^{\frac{0-(-40)}{20}}$$
$$\cdot \left(1 + \frac{s}{628319}\right)^{\frac{60}{20}}$$
$$= 10^5 \frac{\left(1 + \frac{s}{62.83}\right)^3 \left(1 + \frac{s}{628.32}\right) \left(1 + \frac{s}{188496}\right)^2 \left(1 + \frac{s}{628319}\right)^3}{s^4 \left(1 + \frac{s}{18849.6}\right)^2}$$

b.

The initial constant can be found as follows:

$20 \cdot \log(K) = 20 \Rightarrow \log(K) = 1 \Rightarrow K = 10$

Furthermore, inferring from the shape of the gain diagram as well as the phase shift of the phase diagram, the transfer function includes a pair of complex poles at $\omega = 3k$ (value retrieved by inspection). The transfer function can be established as follows:

$$T(j\omega) = \frac{10}{1 - \left(\frac{\omega}{3000 \cdot 2\pi}\right)^2 + j \cdot 2 \cdot \zeta \frac{\omega}{3000 \cdot 2\pi}}$$
$$= \frac{10}{1 - \left(\frac{\omega}{6000\pi}\right)^2 + j \cdot \zeta \frac{\omega}{3000\pi}}$$

The gain of this transfer function can be calculated as:

$$|T(j\omega)|_{dB} = 20\log\left(\frac{10}{\sqrt{\left(1-\left(\frac{\omega}{6000\pi}\right)^2\right)^2+\left(\zeta\frac{\omega}{3000\pi}\right)^2}}\right)$$

$$= 20\log 10 - 20\log\left(\sqrt{\left(1-\left(\frac{\omega}{6000\pi}\right)^2\right)^2+\left(\zeta\frac{\omega}{3000\pi}\right)^2}\right)$$

$$= 20\log 10 - 10\log\left(\left(1-\left(\frac{\omega}{6000\pi}\right)^2\right)^2+\left(\zeta\frac{\omega}{3000\pi}\right)^2\right)$$

Substitute $\omega = 3000\cdot 2\pi = 6000\pi$ *into gain calculation:*

$$|T(j6000\pi)|_{dB} = 20\log\left(\frac{10}{\sqrt{\left(1-\left(\frac{6000\pi}{6000\pi}\right)^2\right)^2+\left(\zeta\frac{6000\pi}{3000\pi}\right)^2}}\right)$$

$$= 20 - 20\log(2\zeta) = 40$$

$$\log(2\zeta) = -1$$

$$2\zeta = 0.1$$

$$\zeta = 0.05$$

Input $\zeta = 0.5$ *into the transfer function prototype:*

$$T(j\omega) = \frac{10}{1-\left(\frac{\omega}{3000\cdot 2\pi}\right)^2 + j\cdot 2\cdot 0.05\frac{\omega}{3000\cdot 2\pi}}$$

$$= \frac{10}{1+\left(\frac{j\omega}{6000\pi}\right)^2+\frac{j\omega}{60000\pi}}$$

$$= \frac{10}{1+\frac{s^2}{(6000\pi)^2}+\frac{s}{60000\pi}}$$