ECE212 Homework 6

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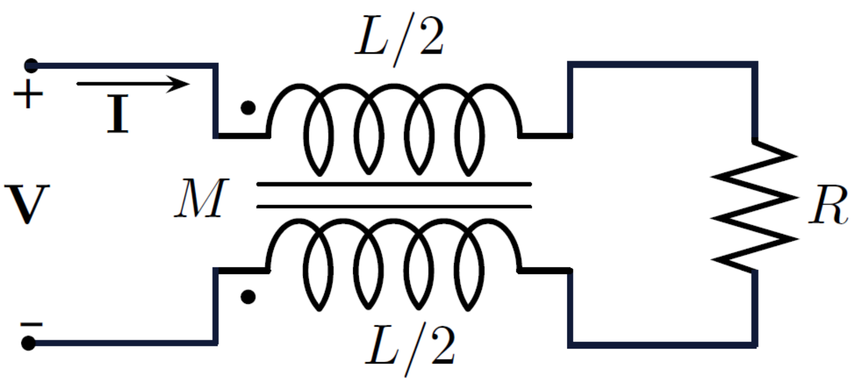
Q1.

The two coupled coils in this circuit have a mutual inductance M, a coupling coefficient k, and an inductance $\frac{L}{2}$ each. The resistor has a value R;.' and the input is a sinusoidal voltage source with frequency ω, represented with its phasor. Give an expression for the input impedance Z as seen by the voltage source. Simplify your answer so that it is in terms of only R, k, ω and L.



In phasor domain, let the voltage of the top inductor be V1, the voltage of the bottom inductor be V2 and mutual inductance be M:

$$V_1 = V_{11} + V_{12} = I \cdot \left(j\omega \frac{L}{2}\right) - I \cdot (j\omega M) = I \cdot \left(j\omega\left(\frac{L}{2} - M\right)\right)$$

$$V_2 = V_{22} + V_{21} = I \cdot \left(j\omega \frac{L}{2}\right) - I \cdot (j\omega M) = I \cdot \left(j\omega\left(\frac{L}{2} - M\right)\right)$$

$$\because k = \frac{M}{\sqrt{\frac{L}{2} \cdot \frac{L}{2}}} \Rightarrow M = k \cdot \frac{L}{2} = \frac{kL}{2}$$

$$\therefore V_1 = I \cdot \left(j\omega\left(\frac{L}{2} - \frac{kL}{2}\right)\right) = I \cdot \left(\frac{j\omega L(1-k)}{2}\right)$$

$$V_2 = I \cdot \left(j\omega\left(\frac{L}{2} - \frac{kL}{2}\right)\right) = I \cdot \left(\frac{j\omega L(1-k)}{2}\right)$$

By KVL around the loop:

$$V = V_1 + V_R + V_2$$

$$V = I \cdot \left(\frac{j\omega L(1-k)}{2}\right) + I \cdot R + I \cdot \left(\frac{j\omega L(1-k)}{2}\right)$$

$$V = I \cdot \left(\frac{j\omega L(1-k)}{2} + \frac{j\omega L(1-k)}{2} + R\right)$$

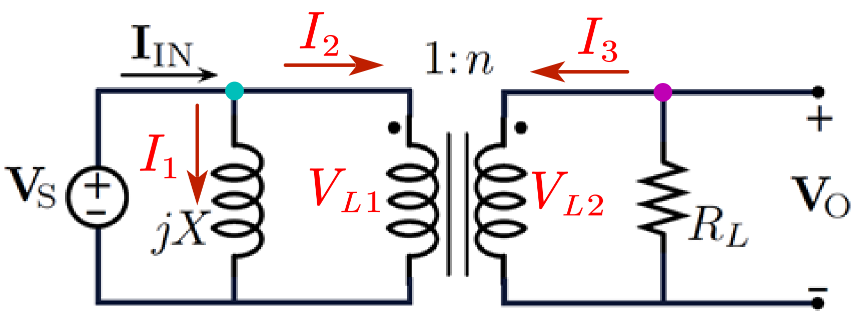
$$V = I \cdot \frac{2(1-k)j\omega L + 2R}{2} = I \cdot ((1-k)j\omega L + R)$$

Therefore, the input impedance can be calculated as $\frac{V}{I}$ *in phasor domain:*

$$Z_{eq} = \frac{V}{I} = \frac{I \cdot ((1-k)j\omega L + R)}{I} = (1-k)j\omega L + R \ \Omega$$

Q2.

For this circuit, the transformer is ideal with a turns ratio of n = 4. You are also given that X = 45 Ω and R_L= 720 Ω. Find I_{IN} and V_0 when Vs = $100\angle 0$ **V. Show your work in detail.**



Label the currents of the above circuit as shown (all currents entering the dots). As ideal transformer has the perfect coupling property of $k \approx 1$, *perfect coupling of ideal transformer implies* $\frac{V_{L2}}{V_{L1}} = -\frac{I_2}{I_3} = n = 4$, *which signifies that:*

$$V_O = V_{L2} = 4V_{L1} = 400\angle 0 \ V \tag{1}$$

$$I_2 = -4I_3 \tag{2}$$

As V_O is an open terminal, the L_2 inductor forms a close loop with R_L, by KVL around the close loop:

$$-I_3 R_L = V_{L2} = V_O$$

$$400 = -I_3 R_L \Rightarrow I_3 = \frac{-400}{R_L} = -\frac{400}{720} A = -0.556 A$$

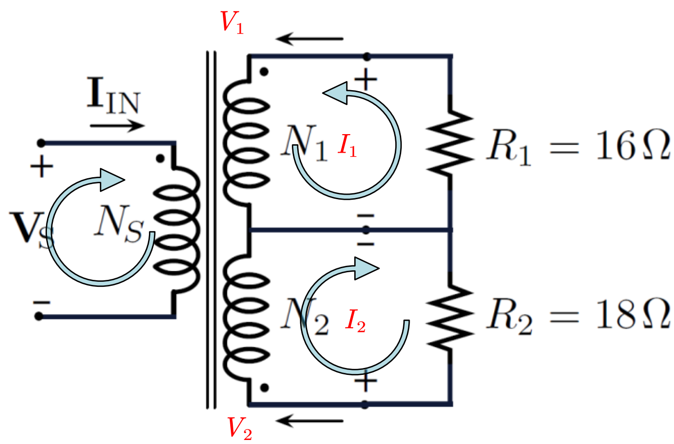
Plug in $I_3 = -0.556A$ to (2): $I_2 = -4I_3 = 2.222 \ A$

By KCL at node1 (where the node is highlighted with a blue dot), it can be inferred:

$$I_{IN} = I_1 + I_2 = \frac{V_S}{jX} + I_2 = \left(\frac{100}{j45} + 2.222\right) A = 3.143\angle -45^{\circ} A$$

Q3.

This ideal multi-winding transformer is driven by a sinusoidal voltage source $\mathbf{V}_S$, with a magnitude of 240 V (rms). The two output windings are connected to two resistive loads. If we want load 1 (R_1) to dissipate (i.e., absorb) 100 W and load 2 (R_2) to dissipate 200 W, find the required turns ratios in this transformer. Then, find the input impedance seen by the input source.



Using root mean square measure, the power of R_1 and R_2 can be analyzed as follows:

$$P_{R1} = \frac{V_1^{\,2}}{R_1} \Rightarrow V_1^{\,2} = P_{R1} \cdot R_1 = 100 \cdot 16 = 1600 \Rightarrow V_1 = 40V \ (rms)$$

$$I_1 = \frac{V_1}{R_1} = \frac{40V}{16\Omega} = 2.5A \ (rms)$$

$$P_{R2} = \frac{V_2^{\,2}}{R_2} \Rightarrow V_2^{\,2} = P_{R2} \cdot R_2 = 200 \cdot 18 = 3600 \Rightarrow V_2 = 60V \ (rms)$$

$$I_2 = \frac{V_2}{R_2} = \frac{60V}{18\Omega} = 3.333A \ (rms)$$

Let $n_1 = \frac{N_1}{N_S}$, $n_2 = \frac{N_2}{N_S}$. *As ideal transformer has the perfect coupling property of $k \approx 1$, perfect coupling of ideal transformer implies* $\frac{V_S}{V_1} = -\frac{I_1}{I_{IN}} = \frac{1}{n_1}$ *and* $\frac{V_S}{V_2} = -\frac{I_2}{I_{IN}} = \frac{1}{n_2}$, *which signifies that:*

$$n_1 = \frac{V_1}{V_S} = \frac{40}{240} = \frac{1}{6}$$

$$n_2 = \frac{V_2}{V_S} = \frac{60}{240} = \frac{1}{4}$$

Therefore, the required turn ratio in this transformer is $N_S{:}N_1{:}N_2 = 6:1:\frac{6}{4} = 12:2:3$.

Using the calculated turn ratio, the input current can be calculated as:

$$I_{IN1} = -I_1 n_1 = -\frac{2.5A}{6} = -0.417A\ (rms)$$

$$I_{IN2} = -I_2 n_2 = -\frac{3.33A}{4} = -0.833A\ (rms)$$

Therefore, as there does not exist a second independent power source in the shown circuit configuration, the input impedance can be directly calculated as:

$$Z_{IN} = \frac{-V_S}{I_{IN}} = \frac{-240V}{-0.417-0.833A} = 192\ \Omega$$

Q4.

In this circuit, we want to remove the impedance Z (replacing it by a short circuit) from the output side of the circuit and, instead, insert another impedance Z′ in the input side, in series with the primary winding, in such a way that the load Z_L has the same voltage and current as in the original circuit. Find the expression for Z′. Show your derivation in detail; simply giving the final answer will not be accepted, even if correct.

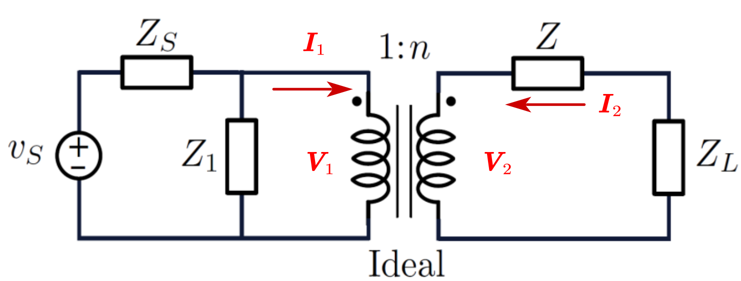


Fig 1. Original circuit

To reduce the complexity of circuit analysis, the above configuration can be transformed to the reduced circuit as follows, in which the impedance seen by the secondary winding ($Z_{L'}$) will be reduced to $\frac{Z_{L'}}{n^2}$

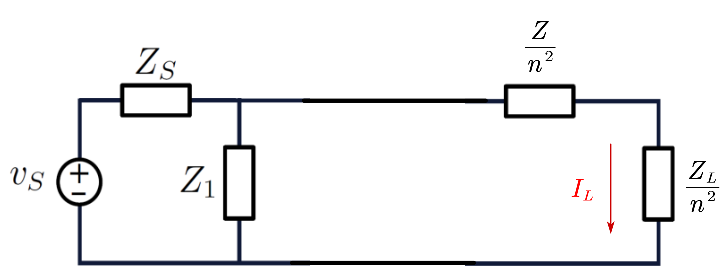


Fig 2. Reduced version of the original circuit

This transformation is valid for ideal transformer as the impedance seen by the original source (Z_{in}) can be expressed as $Z_{in} = \frac{V_1}{I_1} = \frac{\frac{V_2}{n}}{-I_2 \cdot n} = -\frac{V_2}{I_2} \cdot \frac{1}{n^2} = \frac{Z_{L'}}{n^2}$. *Noticed that both attached impedance (Z_L and Z) will need to be reduced:* $\frac{(Z+Z_L)}{n^2} = \frac{Z}{n^2} + \frac{Z_L}{n^2}$. *After transformation, the current flowing through Z_L is I_L.*

The modified circuit after removing the impedance Z and attaching another impedance Z′ is drawn as follows:

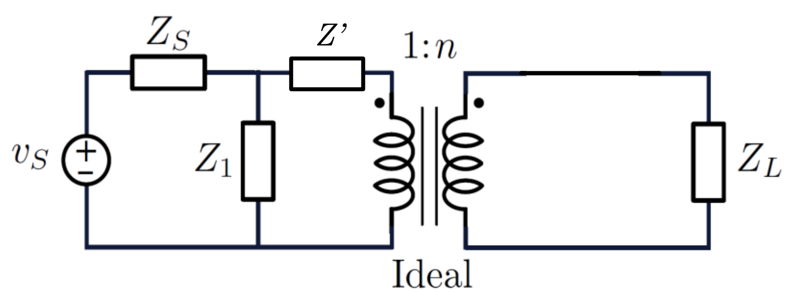


Fig 3. Modified circuit

This circuit can then be again transformed to the reduced circuit as follows:

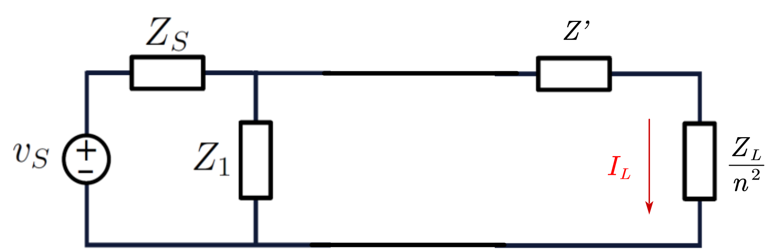


Fig 4. Reduced configuration of the modified circuit

Comparing Fig 4. with Fig 2., it can be observed that the only modified element of the reduced circuit schematics is the impedance connected in series with the Z_L. Therefore, to avoid changes of the properties of Z_L, it is necessary to set impedance of Z′ the same as $\frac{Z}{n^2}$.

Therefore, $Z' = \frac{Z}{n^2}$.