ECE216, Experiment 2
ECE, University of Toronto

Studying the Response of RLC Circuits to Sinusoidal Inputs Using Simulink

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IMPORTANT

Provide here the hash value you calculated for your group denoted by Θ (see lab instructions).

Hash value Θ = **1.12**

Include all other requested screenshots of the plots and answers as needed below.

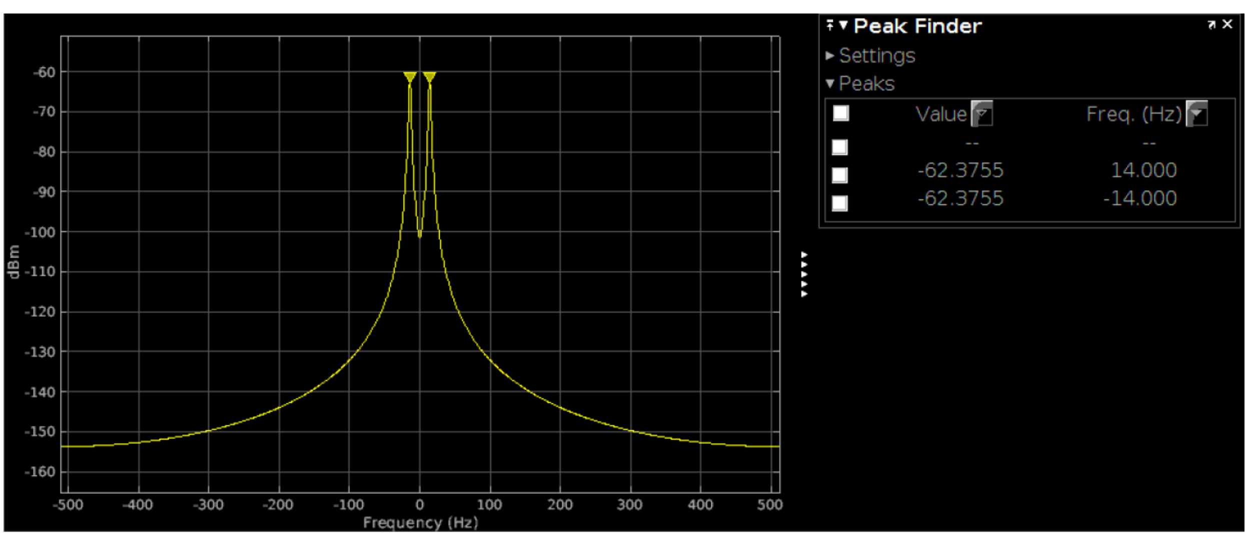
1. Natural frequency of an RLC Circuit

1.1 Exercise 1.1

Calculate the natural frequency for the following systems **(1 pt)**

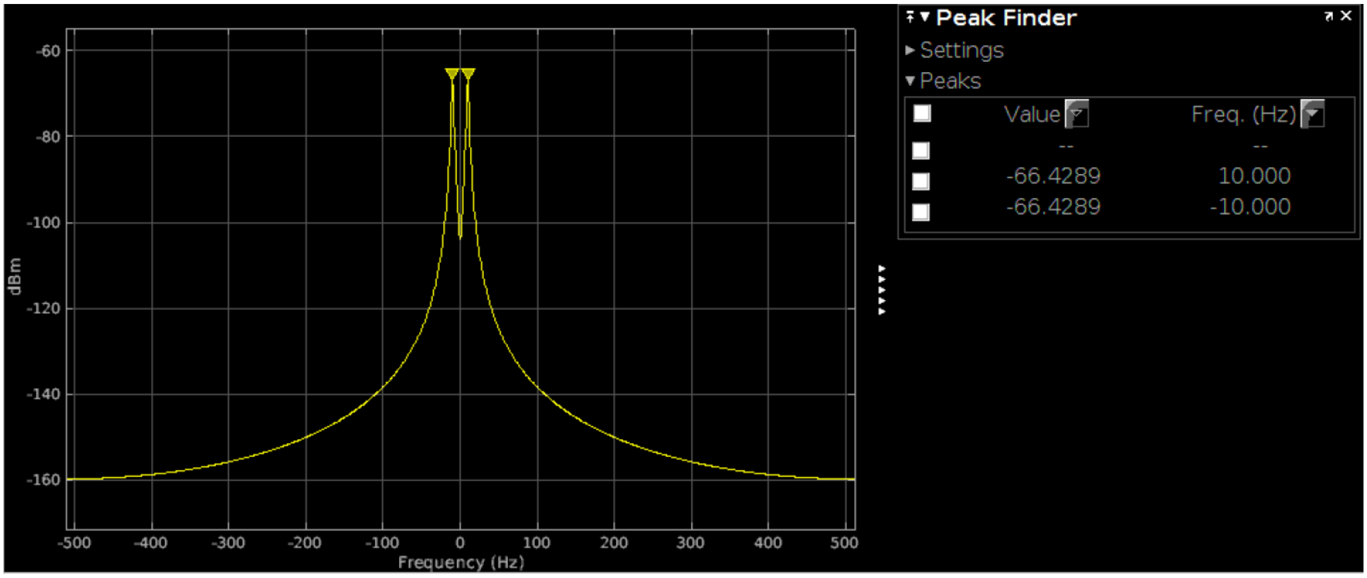
1. $C = 0.01 \times \Theta, L = 0.01 \times \Theta$

$$f_n = \pm \frac{1}{\sqrt{LC}} = \pm \frac{1}{2\pi \cdot \sqrt{(0.01 \cdot 1.12) \cdot (0.01 \cdot 1.12)}} = \pm 14.2103 \ Hz$$



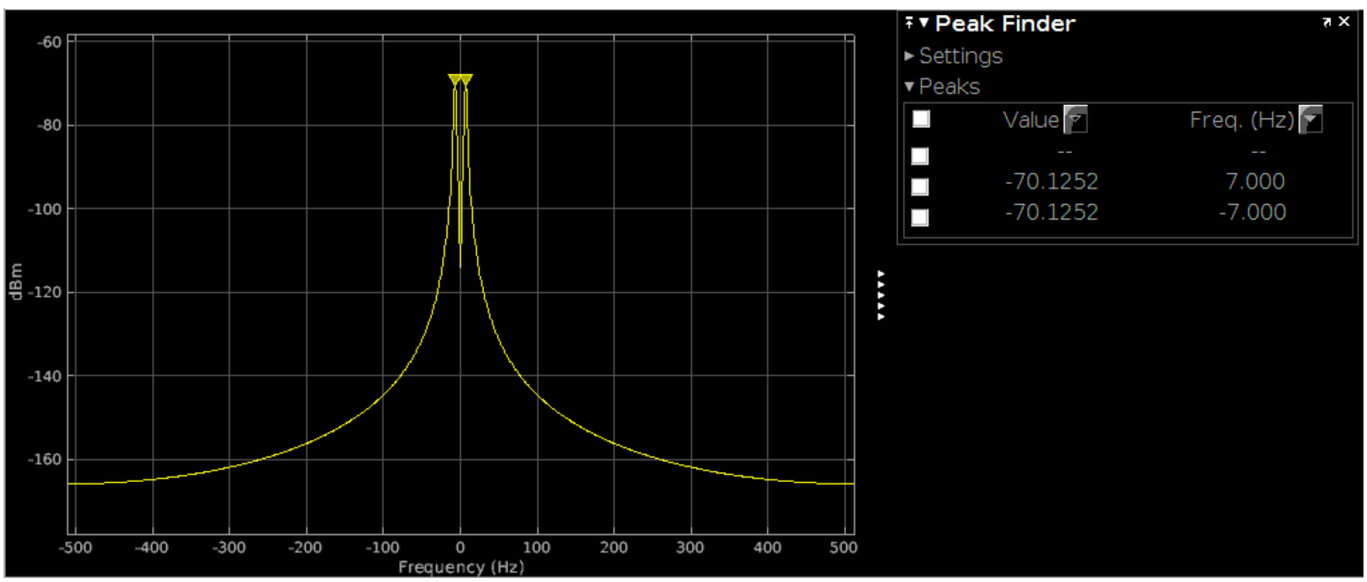
2. $C = 0.02 \times \Theta, L = 0.01 \times \Theta$

$$f_n = \pm \frac{1}{\sqrt{LC}} = \pm \frac{1}{2\pi \cdot \sqrt{(0.02 \cdot 1.12) \cdot (0.01 \cdot 1.12)}} = \pm 10.0482 \ Hz$$



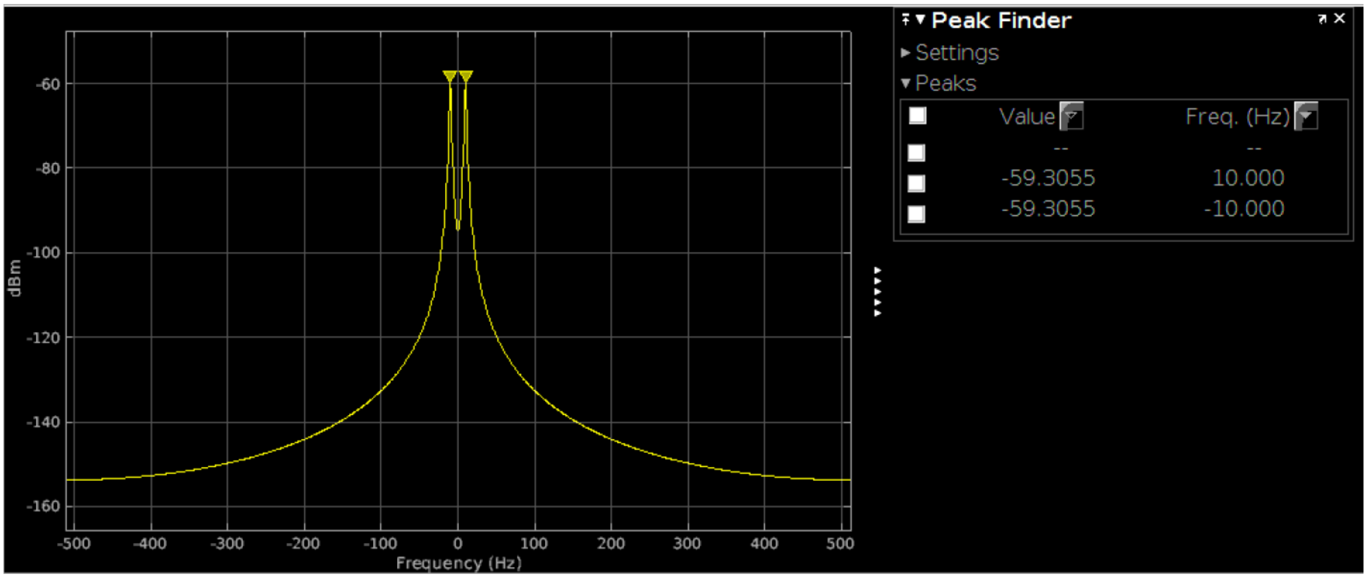
3. $C = 0.04 \times \Theta, L = 0.01 \times \Theta$

$$f_n = \pm \frac{1}{\sqrt{LC}} = \pm \frac{1}{2\pi \cdot \sqrt{(0.04 \cdot 1.12) \cdot (0.01 \cdot 1.12)}} = \pm 7.10513 \ Hz$$



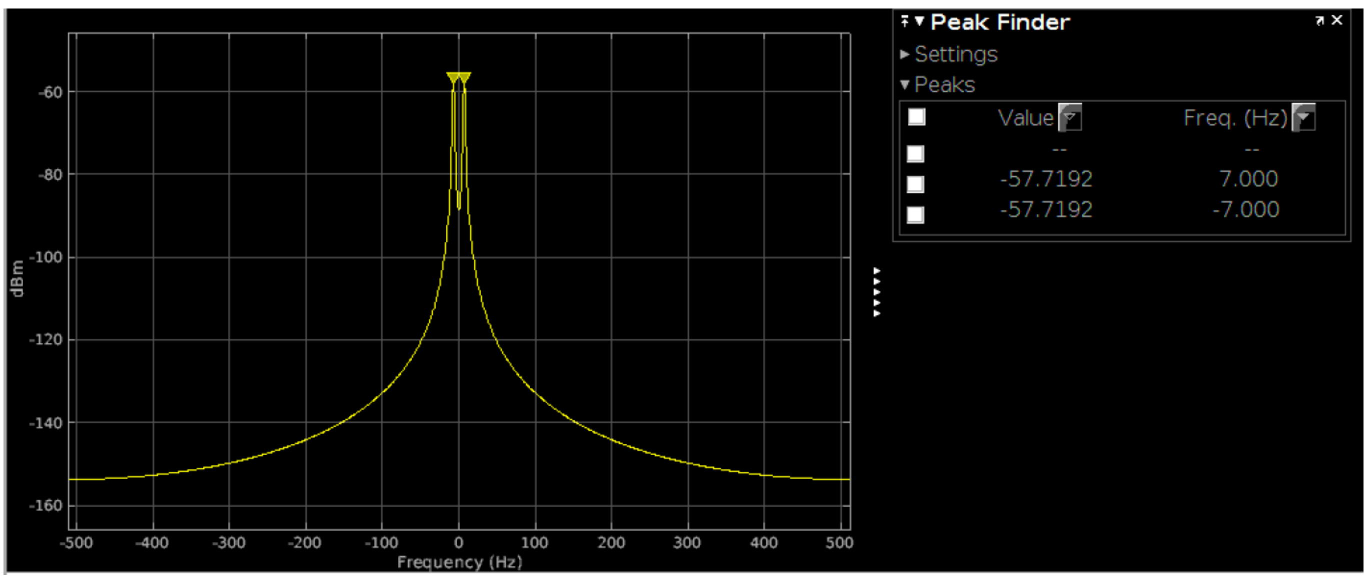
4. $C = 0.01 \times \Theta, L = 0.02 \times \Theta$

$$f_n = \pm \frac{1}{\sqrt{LC}} = \pm \frac{1}{2\pi \cdot \sqrt{(0.01 \cdot 1.12) \cdot (0.02 \cdot 1.12)}} = \pm 10.0482 \ Hz$$



5. $C = 0.01 \times \Theta, L = 0.04 \times \Theta$

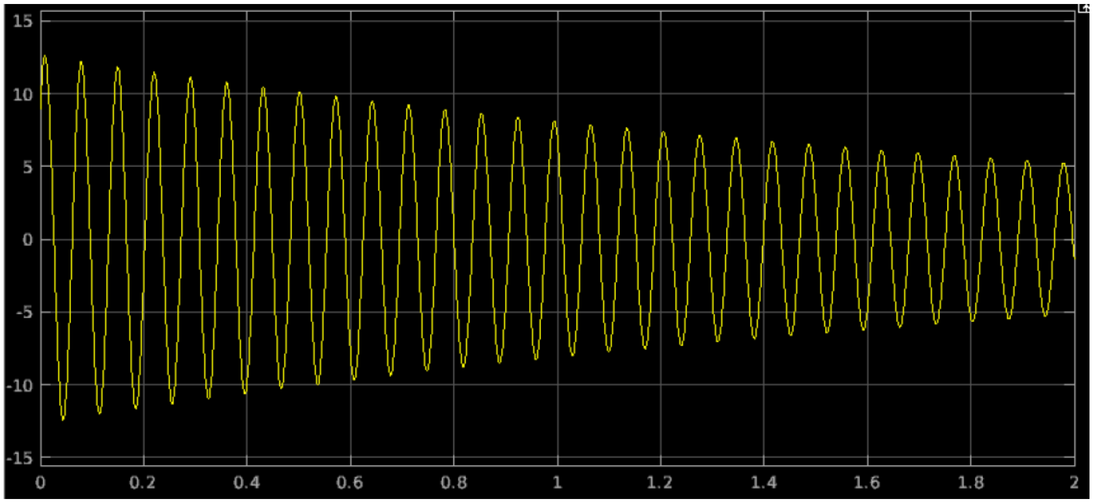
$$f_n = \pm \frac{1}{\sqrt{LC}} = \pm \frac{1}{2\pi \cdot \sqrt{(0.01 \cdot 1.12) \cdot (0.04 \cdot 1.12)}} = \pm 7.10513 \ Hz$$



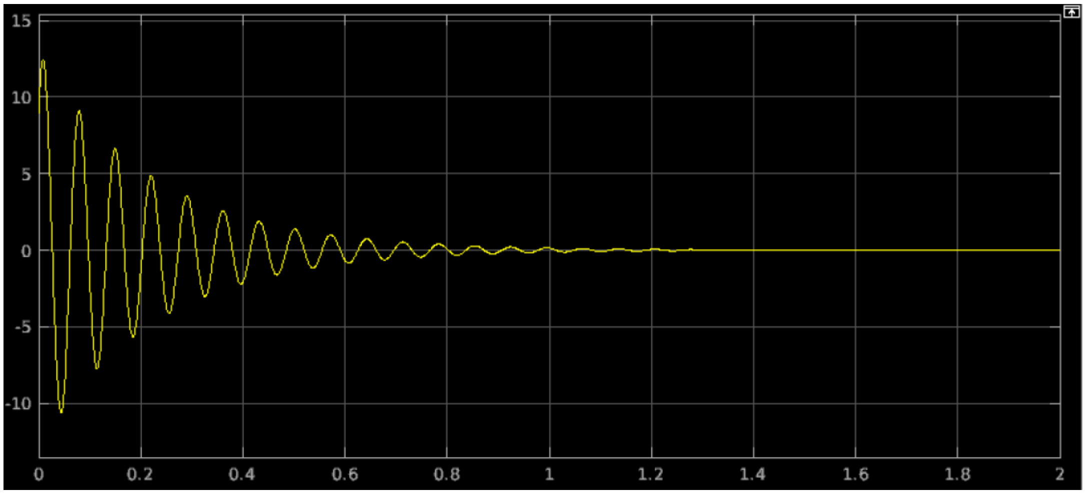
1.2 Exercise 1.2

a. *Vary the resistance values to show that damping factor increases as you increase the resistance (set* $C = 0.01 \times \Theta$ *and* $L = 0.01 \times \Theta$*).*
Save your screenshot of the capacitor voltage over time and attach to this document. **(1 pt)**

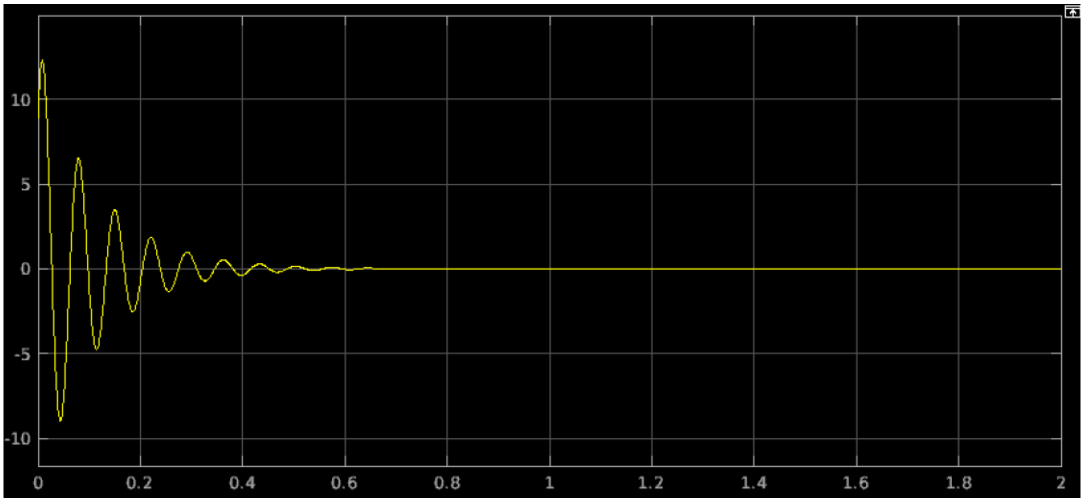
R = 0.01



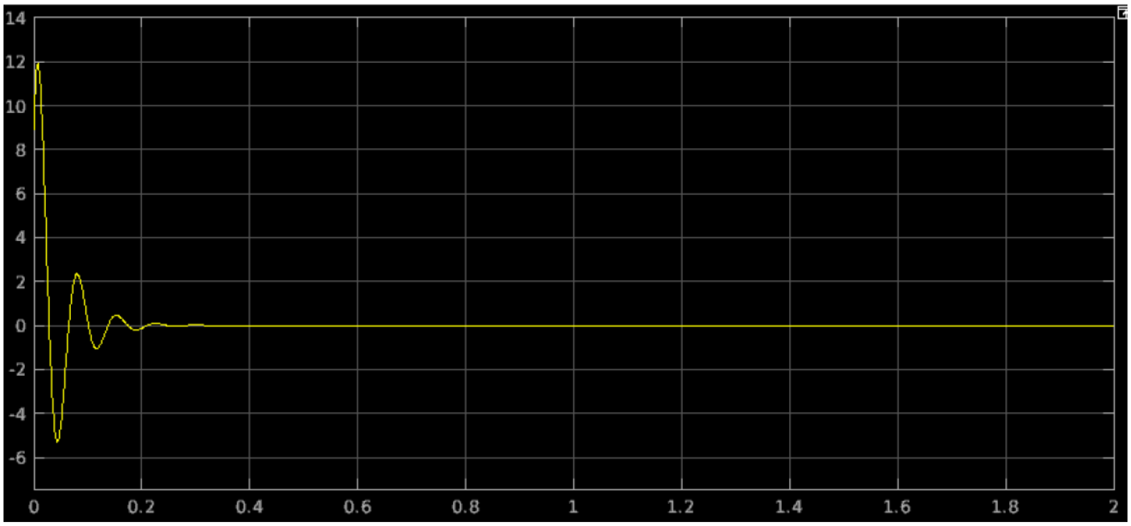
R = 0.1



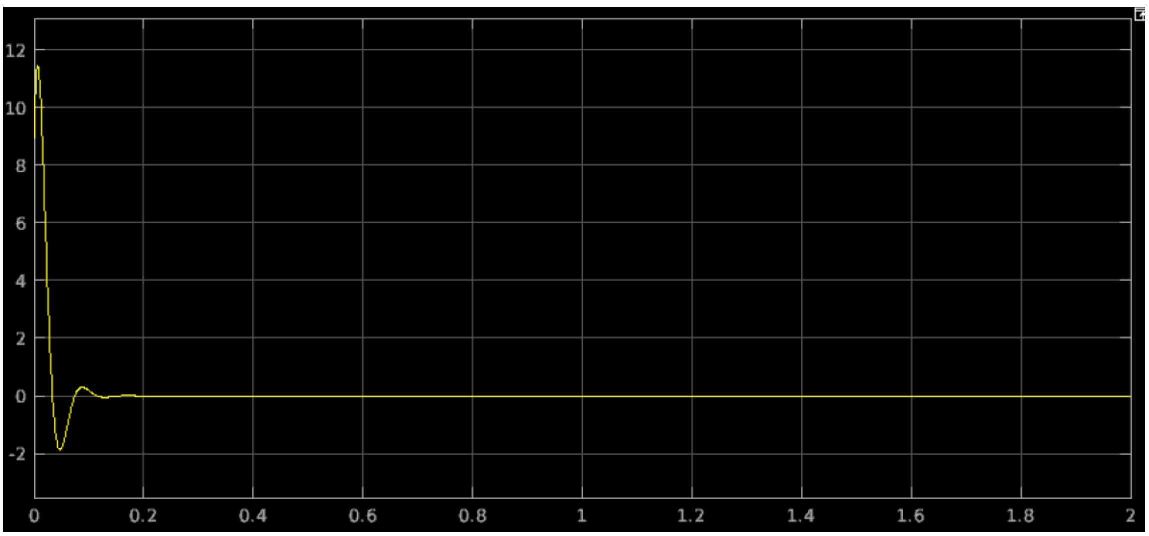
R = 0.2



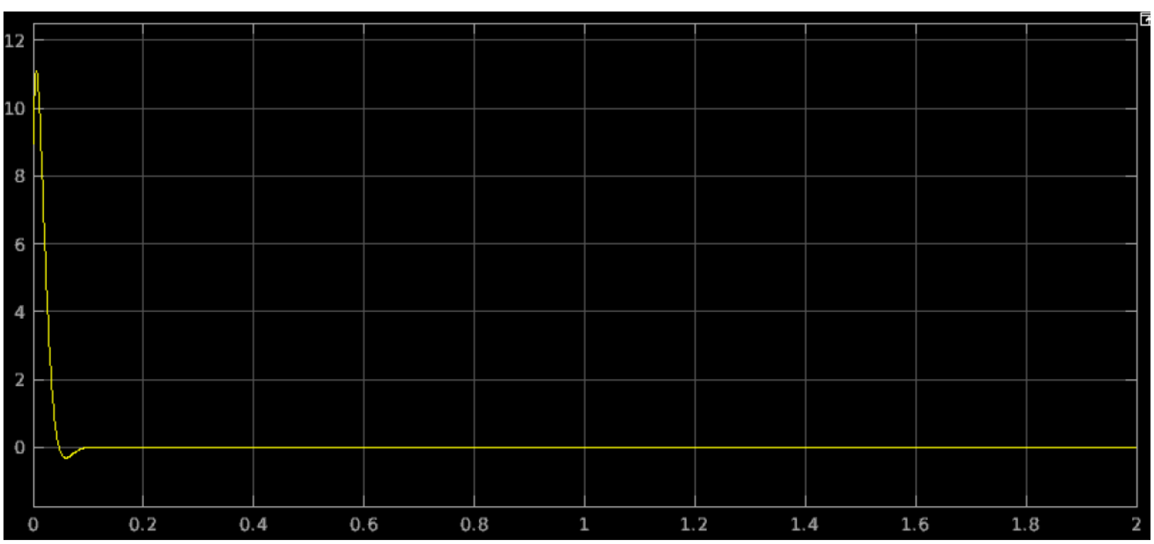
R =   0.5


14
12
10
8
6
4
2
0
-2
-4
-6
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

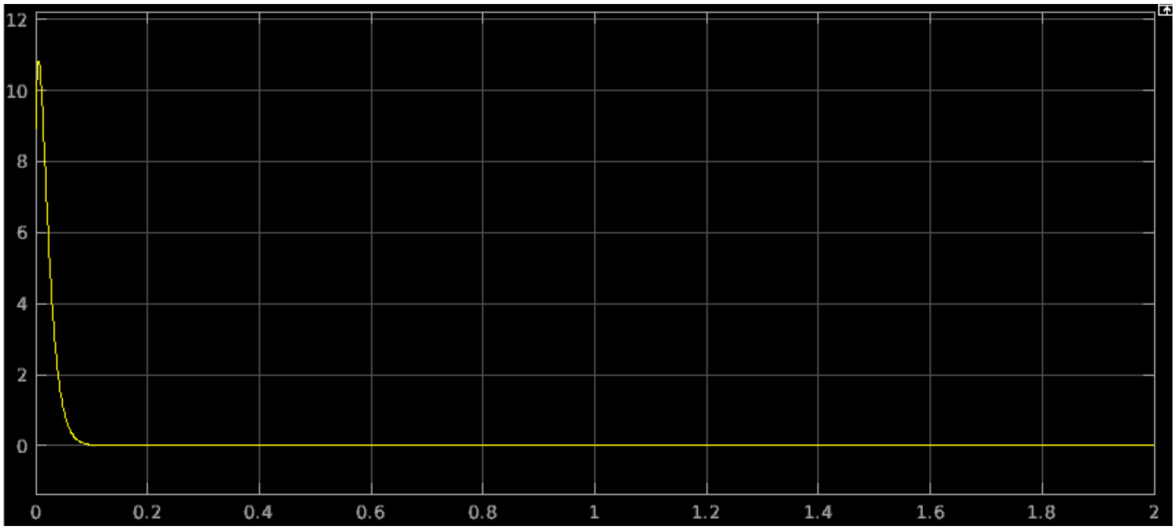
R =   1


12
10
8
6
4
2
0
-2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

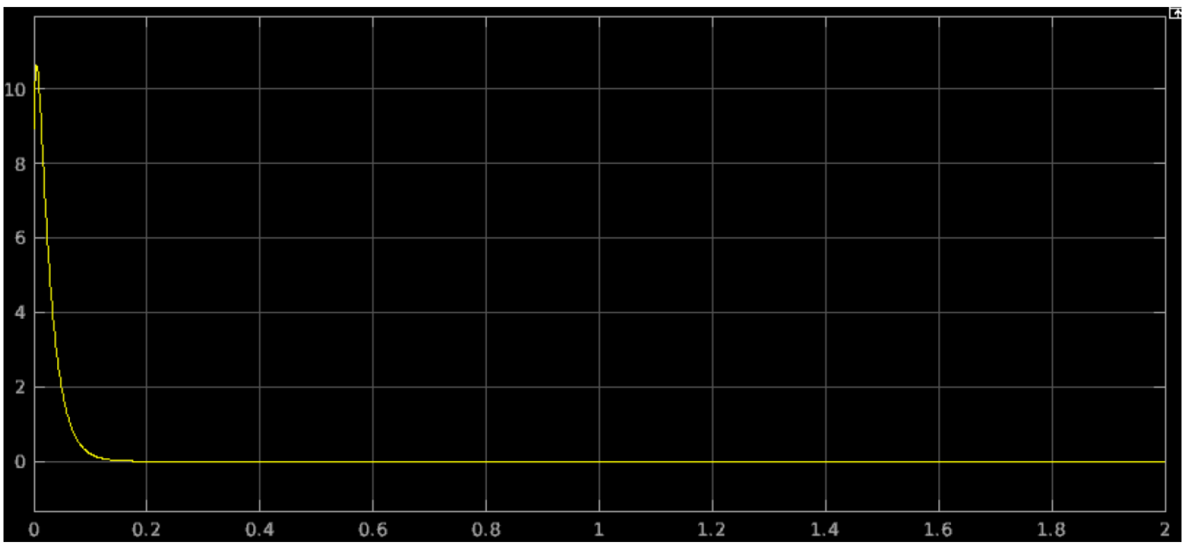
R =   1.5


12
10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

R = 2



R = 2.5



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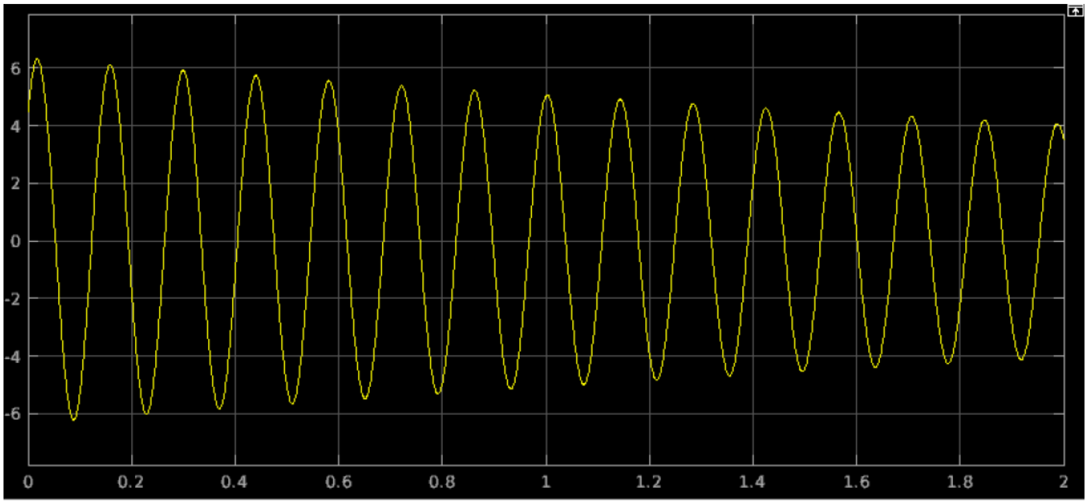
b. *At what resistance does the system transition from underdamped to overdamped? (Keep $L = 0.01$ Θ and $C = 0.01$ Θ)*
(0.5 pt)

Observing from the sequence of graphs as R vary, the pattern changes from underdamped to overdamped at $R = 2\Omega$***. This value can also be manually calculated as***

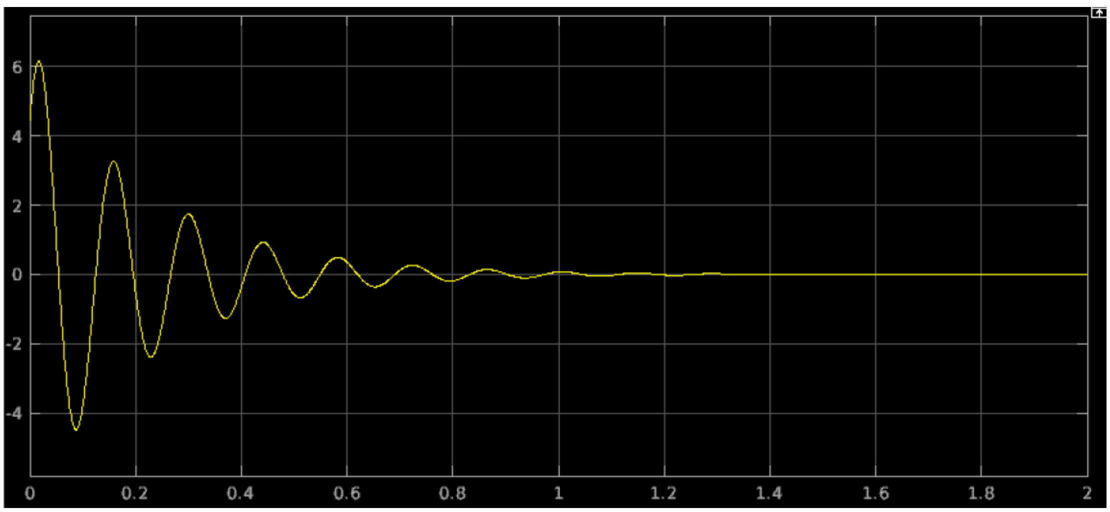
$$\zeta = 1 = \frac{R}{2L\omega_n} = \frac{R}{2 \cdot 0.0112 \cdot \frac{1}{0.0112}} \Rightarrow R = 2 \cdot 0.0112 \cdot \frac{1}{0.0112} = 2\Omega$$

c. *How would underdamped to overdamped transition change if you increase L to* $0.02\times\Theta$ *and C to* $0.02\times\Theta$*?* **(0.5 pt)**

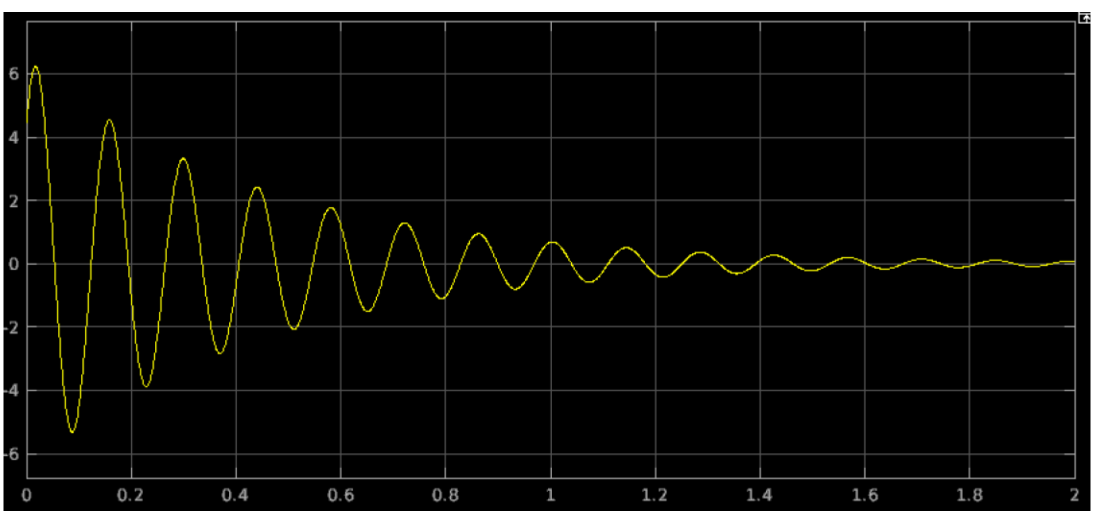
R = 0.01



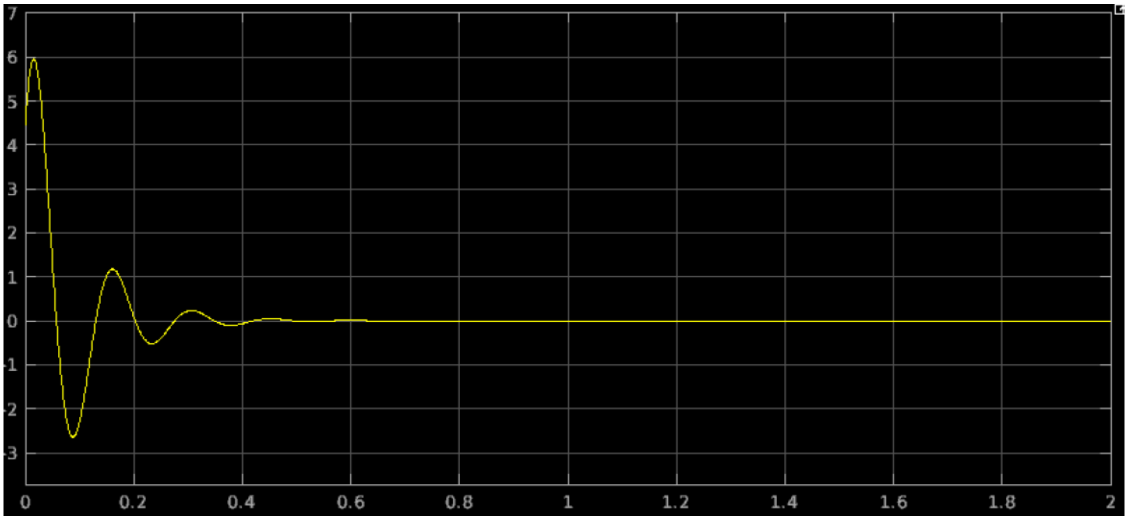
R = 0.1



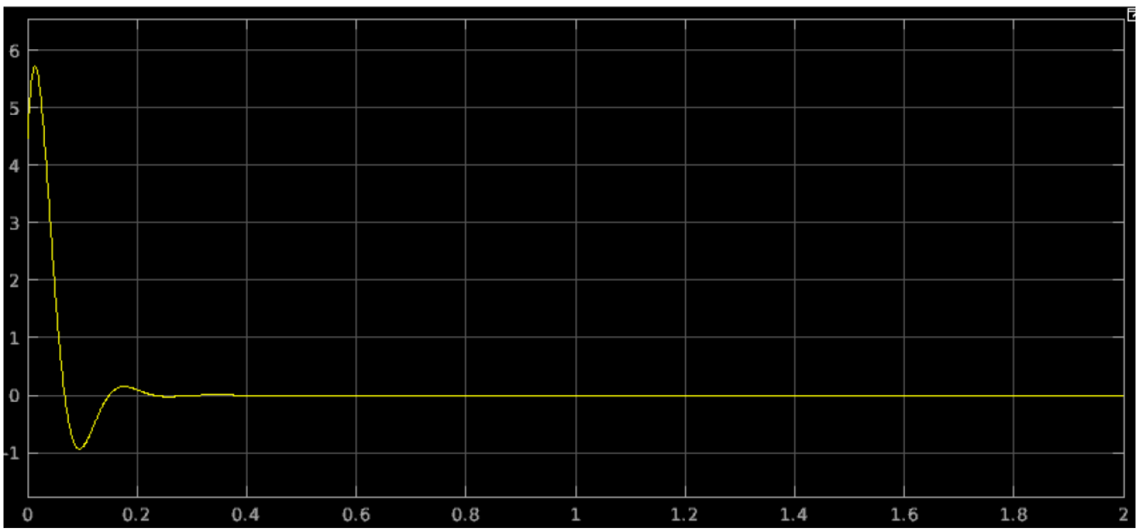
R = 0.2



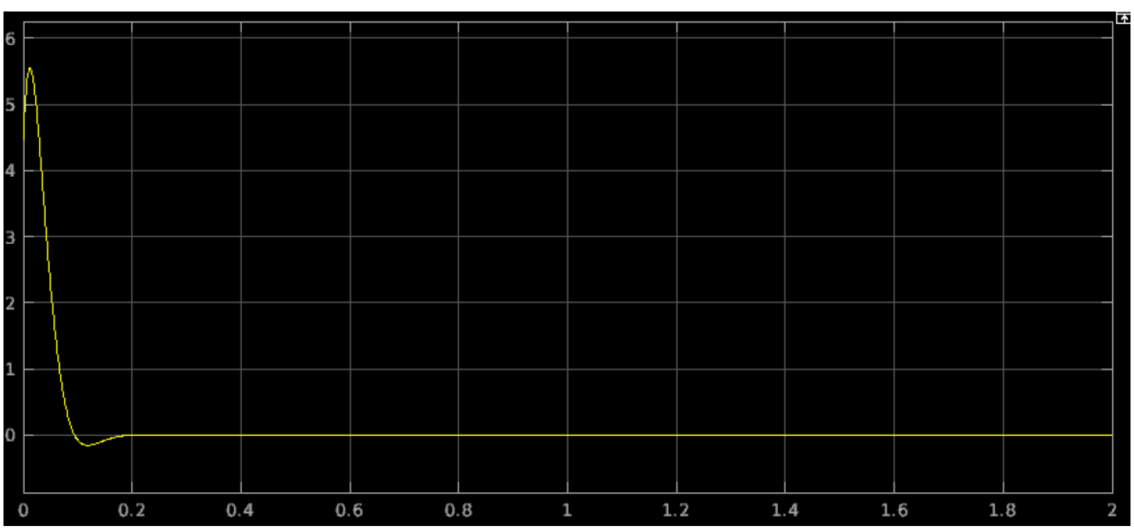
R = 0.5


7 6 5 4 3 2 1 0 -1 -2 -3
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

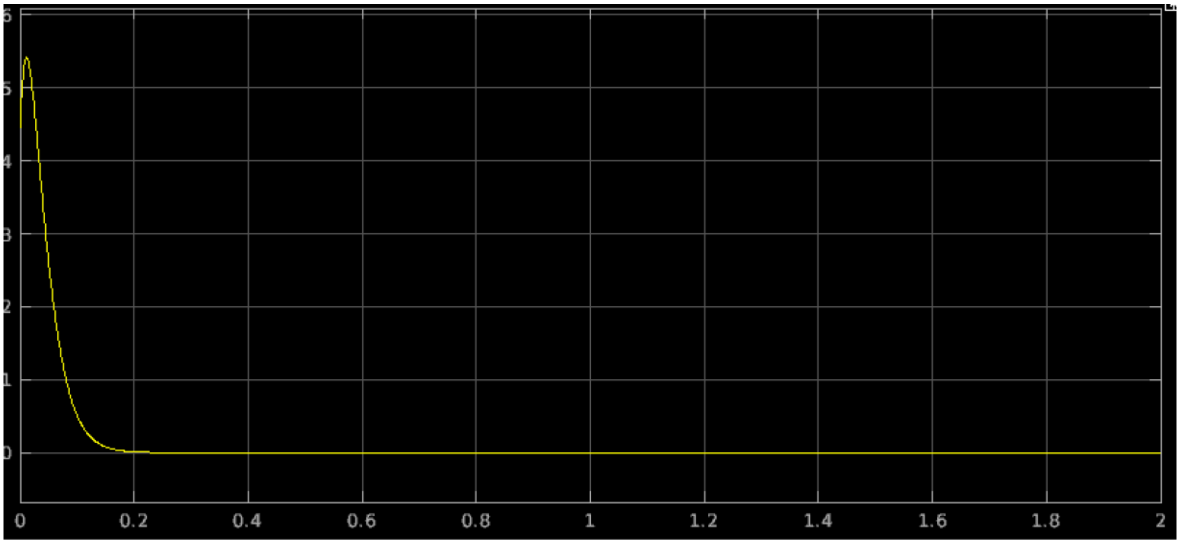
R=1


6 5 4 3 2 1 0 -1
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

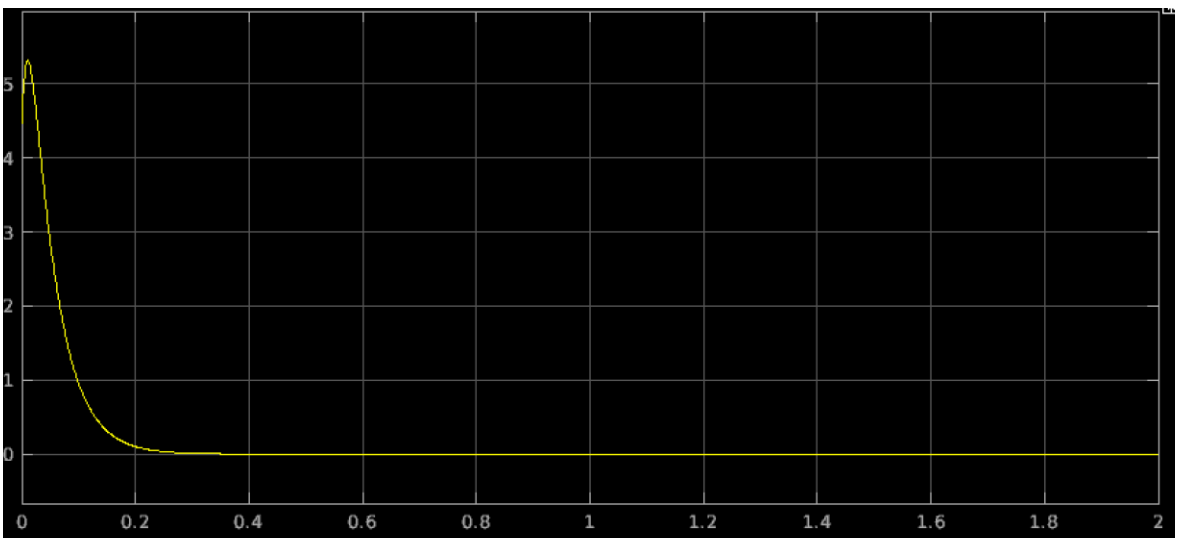
R = 1.5


6 5 4 3 2 1 0
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

R = 2



R = 2.5



Observing from the sequence of graphs as R vary, the pattern changes from underdamped to overdamped at $R = 2\Omega$***. This value can also be manually calculated as***

$$\zeta = 1 = \frac{R}{2L\omega_n} = \frac{R}{2 \cdot 0.0224 \cdot \frac{1}{0.0224}} \Rightarrow R = 2 \cdot 0.0224 \cdot \frac{1}{0.0224} = 2\Omega$$

2. RLC circuit response to an external voltage source

2.1 Exercise 2.1

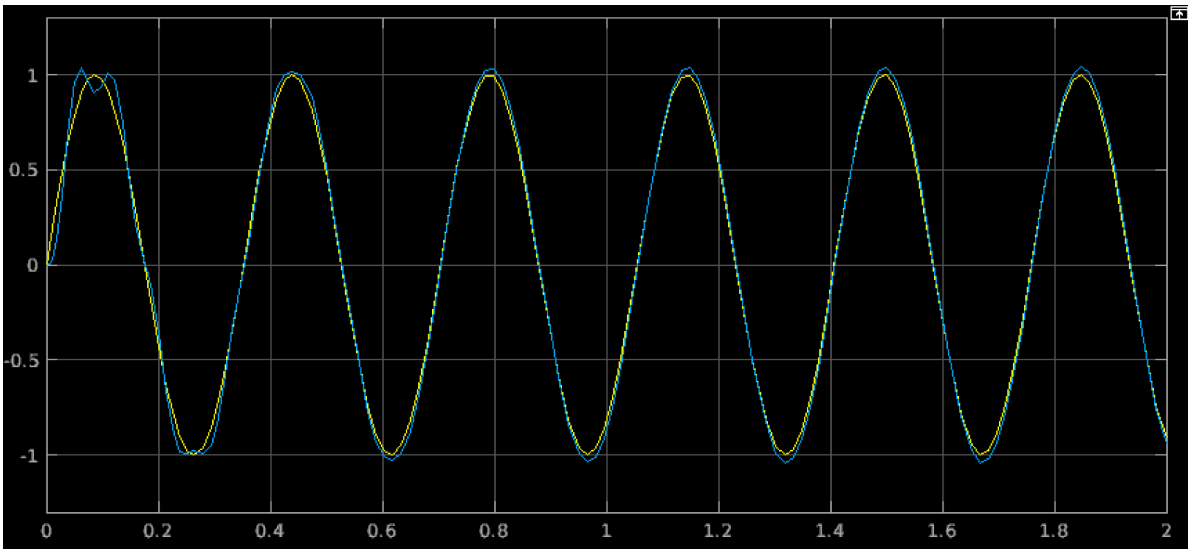
a. *Set the amplitude of your voltage source to 1 and measure the amplitude of the output response of the circuit for the case when the sine-wave input has the following frequencies.* **(1 pt)**

Natural frequency calculations:

$$\omega_n = \pm \frac{1}{\sqrt{LC}} = \pm \frac{1}{\sqrt{(0.01 \cdot 1.12) \cdot (0.01 \cdot 1.12)}} = \pm 89.2857\ rad/\text{sec}$$

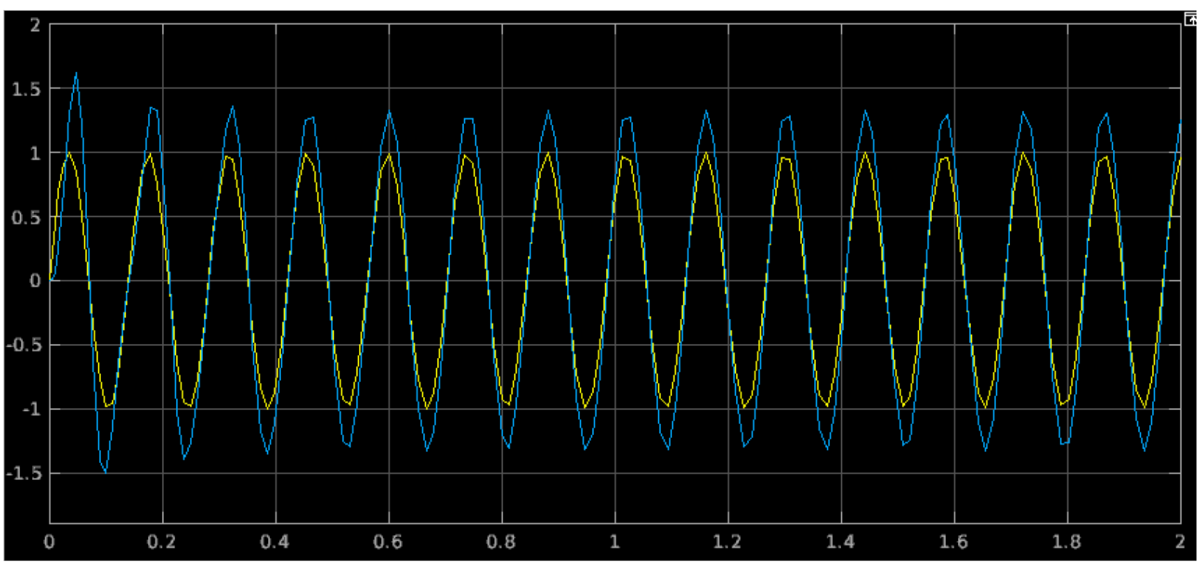
1. *Natural frequency / 5* ***(Steady-state Amp is 1.042V, Max Output is 1.042V, Min Output is -1.042V)***

$$\omega_n = \frac{89.2857}{5} = 17.8571\ rad/\text{sec}$$

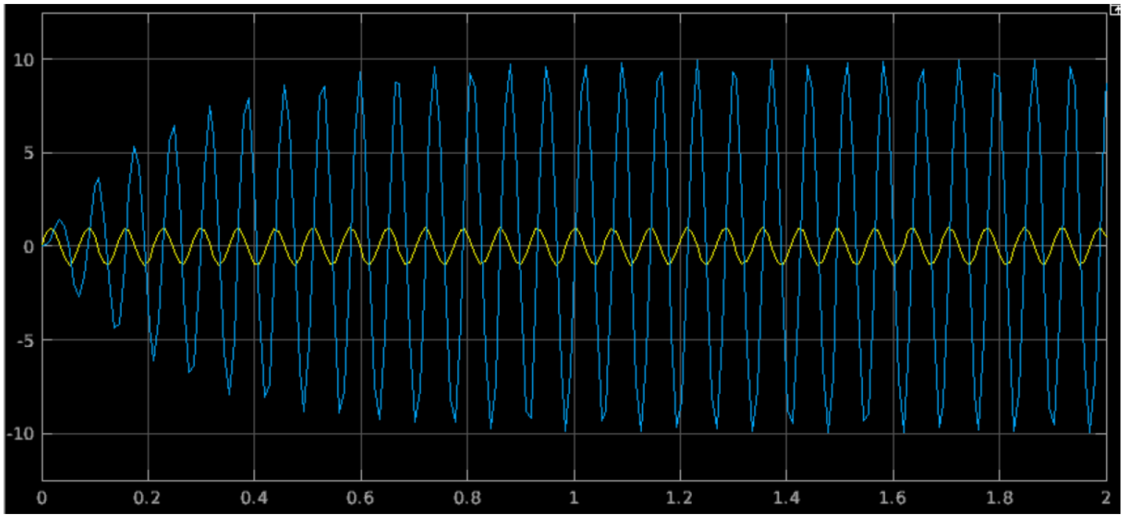


2. *Natural frequency / 2* ***(Steady-state Amp is 1.330V, Max Output is 1.620V, Min Output is -1.539V)***

$$\omega_n = \frac{89.2857}{2} = 44.6429\ rad/\text{sec}$$

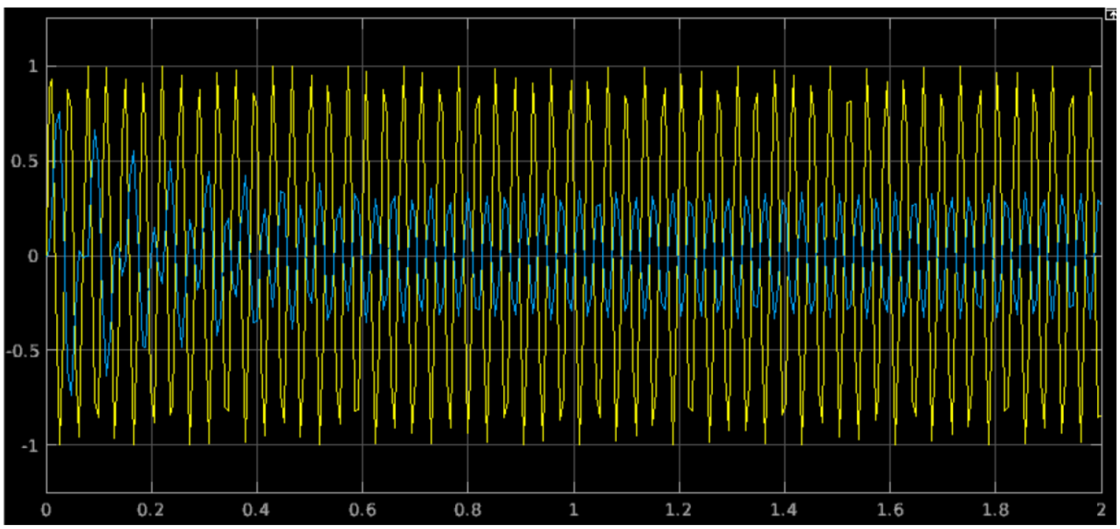


3. *Natural frequency* ***(Steady-state Amp is 9.995V, Max Output is 9.995V, Min Output is -9.996V)***



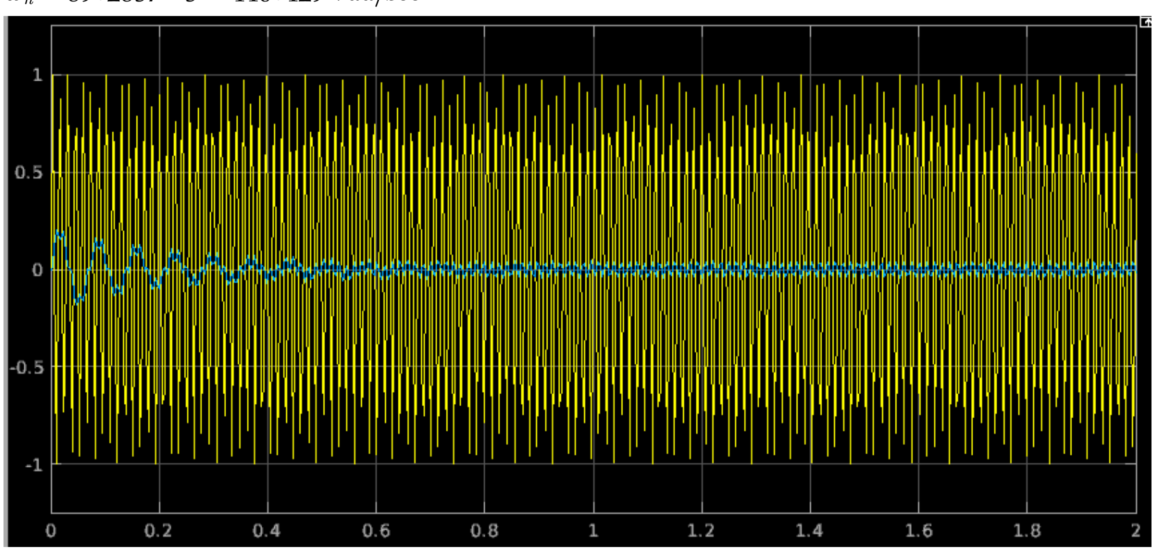
4. *Natural frequency* $*2$ ***(Steady-state Amp is 0.333V, Max Output is 0.809V, Min Output is -0.757V)***

$\omega_n = 89.2857 \cdot 2 = 178.571\ rad/\text{sec}$



5. *Natural frequency* $*5$ ***(Steady-state Amp is 0.042V, Max Output is 0.208V, Min Output is -0.182V)***

$\omega_n = 89.2857 \cdot 5 = 446.429\ rad/\text{sec}$



Explanation:

It can be observed that the amplitude of the output is the largest at natural frequency. As the circuit frequency drifts away from the natural frequency, the magnitudes of the maximum and minimum output also gradually reduces. The impedances of inductor and capacitor in the series RLC circuit configuration can be expressed as

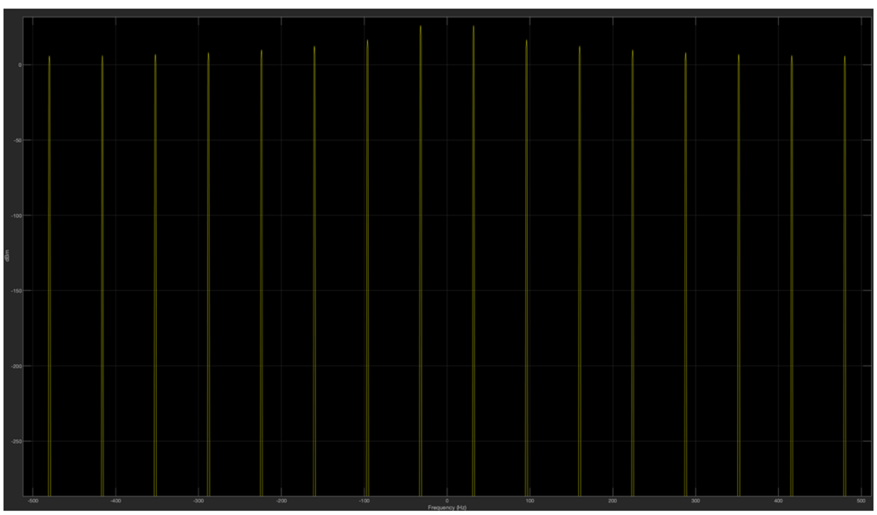
$X_L = \omega L,\ X_C = \frac{1}{\omega C}$***, respectively. At natural frequency, where*** $\omega_n = \frac{1}{\sqrt{LC}}$ ***, impedances can be calculated as***

$X_L = \frac{1}{\sqrt{LC}} L = \frac{1}{\frac{1}{\sqrt{LC}} C} = X_C$***. Therefore, the capacitive reactance cancelled with the inductive reactance, causing the circuit to act purely as a resistive circuit. Without any imaginary component, the current flow through the circuit will be maximized, thus causing the voltage to increase. A mathematical derivation of the capacitor voltage can be done to demonstrate that the max voltage across capacitor along occurs at*** $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$***, which is slightly lower than natural frequency. However, the natural frequency best approximates the max capacitor voltage frequency among the 5 trials with different frequencies, thus outputs the highest capacitor voltage.***

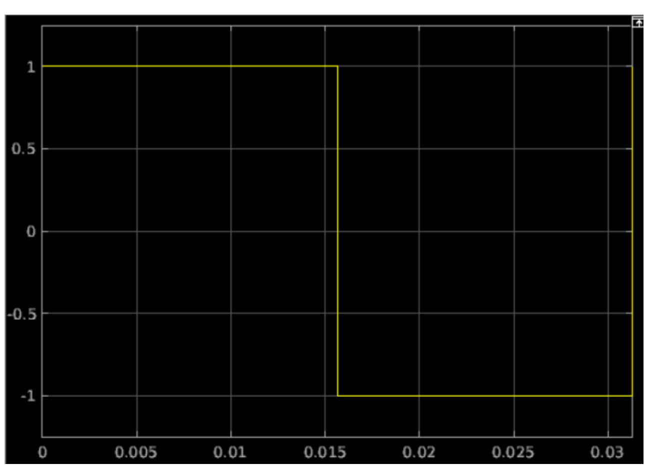
2.2 Exercise 2.2

Include a screenshot demonstrating the square-wave Simulink model (the time-domain plot and its frequency-domain spectrum) and provide an explanation of the peaks in the spectrum analyzer. **(1 pt)**

Square wave frequency domain plot:



Square wave time domain plot:



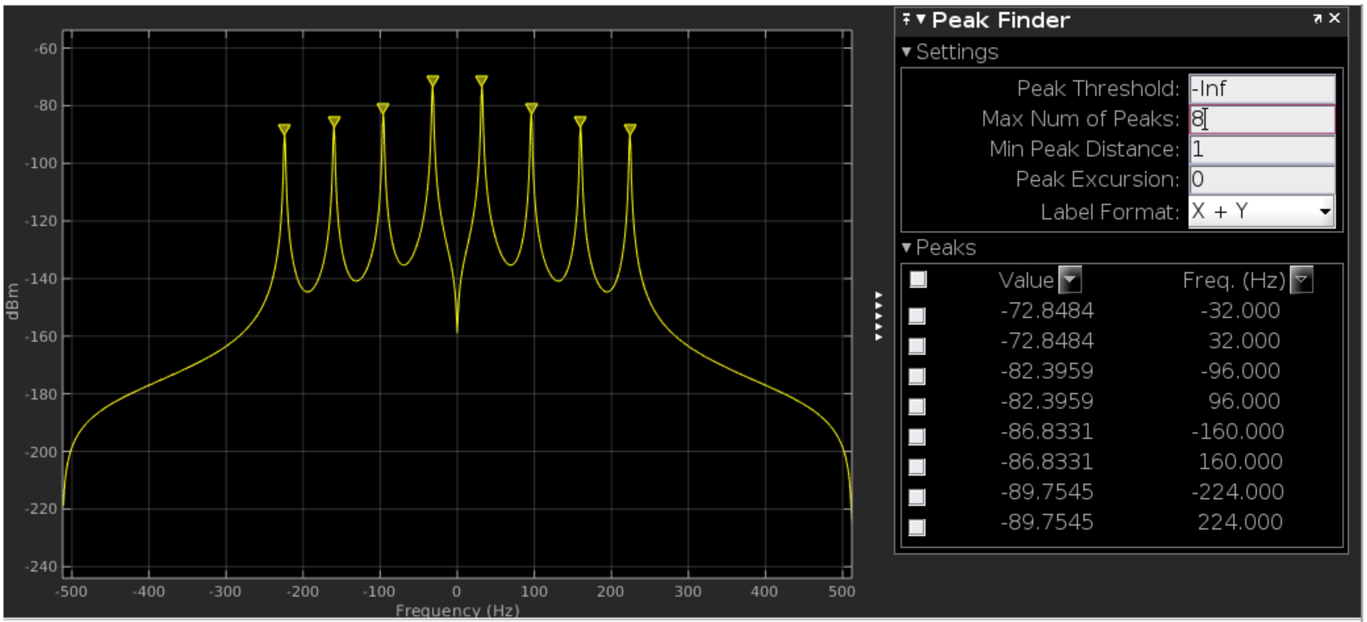
Explanation:

The peaks in the spectrum analyzer are resulted from the generation of the square wave. Essentially, a square wave is composed of multiple sine waves that are at the frequencies shown in the spectrum diagram. The detailed composition can be described by the Fourier expansion of $Square(t) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin(64\pi n t)$***, using*** $\frac{1}{32}$ ***second as period.***

3. Applying Fourier Series in circuit analysis

3.1 Exercise 3.1

a. *Use a square wave with 1/32 sec period. Read the frequency of the first 4 peaks on the frequency-domain spectrum, and record the results below. Include a screenshot of your plots.* **(0.5 pt)**



b. *Calculate the first 4 terms of the Fourier series for the square wave using the equation provided, and write down the frequency and amplitude of each term from the Fourier approximation below.* **(0.5 pt)**

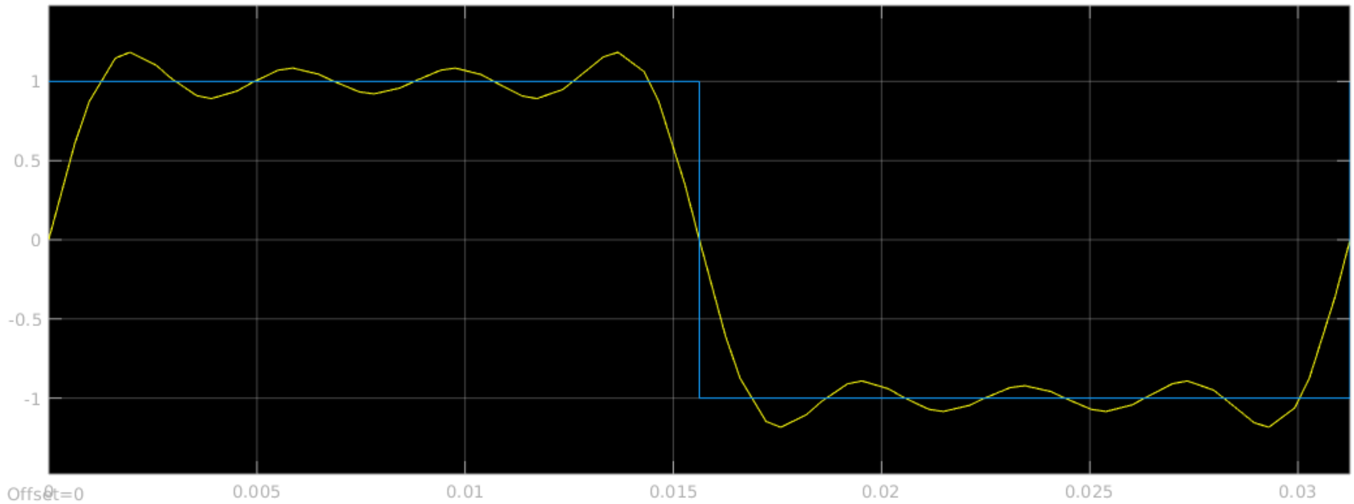
n=1: $\frac{4}{\pi} \cdot \sin(64\pi \cdot t)$ **(**amplitude is $\frac{4}{\pi}$**,** frequency is 32Hz**)**

n=2: $\frac{4}{3\pi} \cdot \sin(192\pi \cdot t)$ **(**amplitude is $\frac{4}{3\pi}$**,** frequency is 96Hz**)**

n=3: $\frac{4}{5\pi} \cdot \sin(320\pi \cdot t)$ **(**amplitude is $\frac{4}{5\pi}$**,** frequency is 160Hz**)**

n=4: $\frac{4}{7\pi} \cdot \sin(448\pi \cdot t)$ **(**amplitude is $\frac{4}{7\pi}$**,** frequency is 224Hz**)**

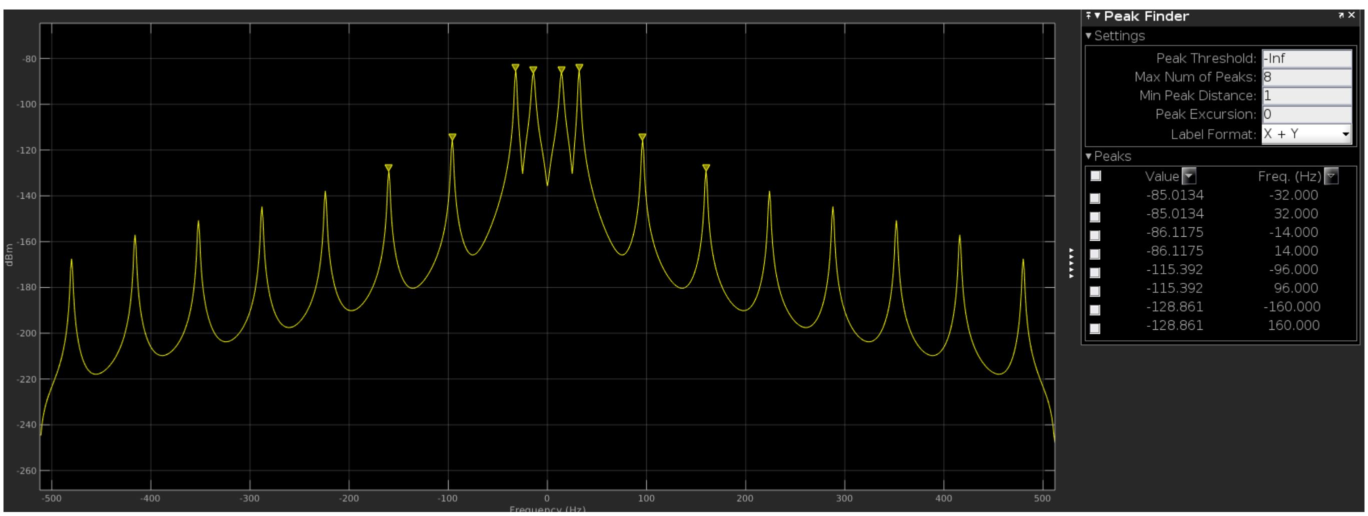
c. *Include a screenshot demonstrating how closely the 4-term Fourier series approximation matches the square wave.* **(0.5 pt)**



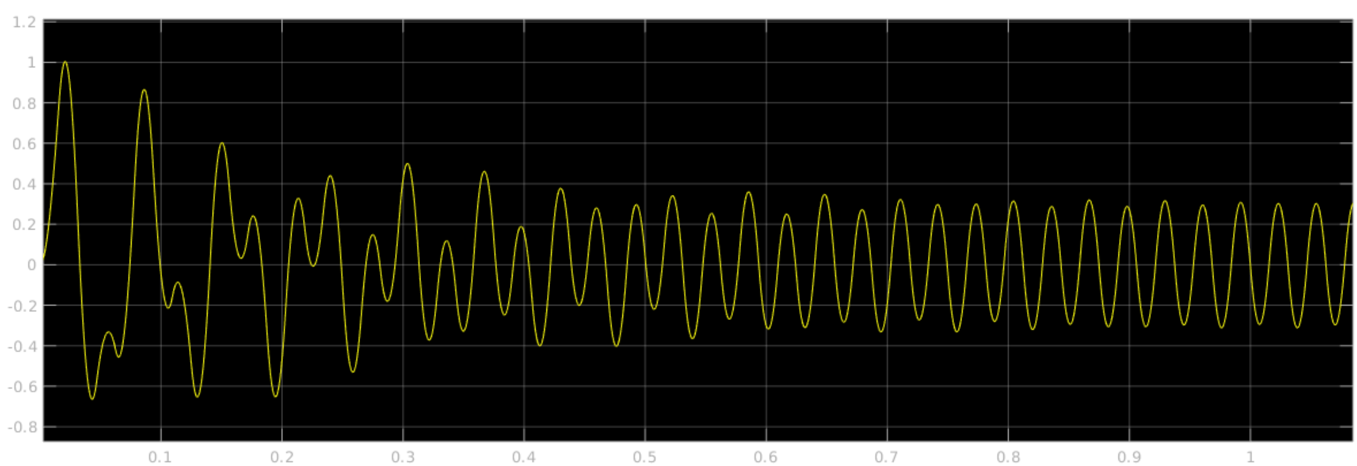
3.2 Exercise 3.2

Compare the output response of the RLC circuit to the 4-term Fourier series approximation input and the response to the square-wave input, respectively. Include a screenshot of the output response in the two cases, respectively. **(1 pt)**

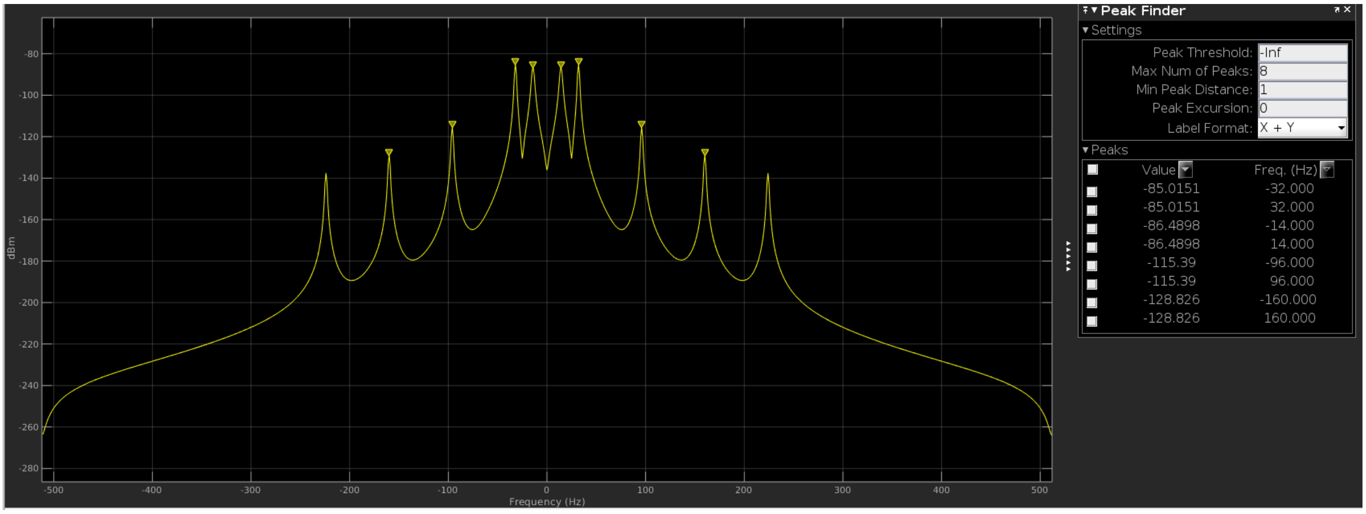
Square-wave input frequency domain plot:



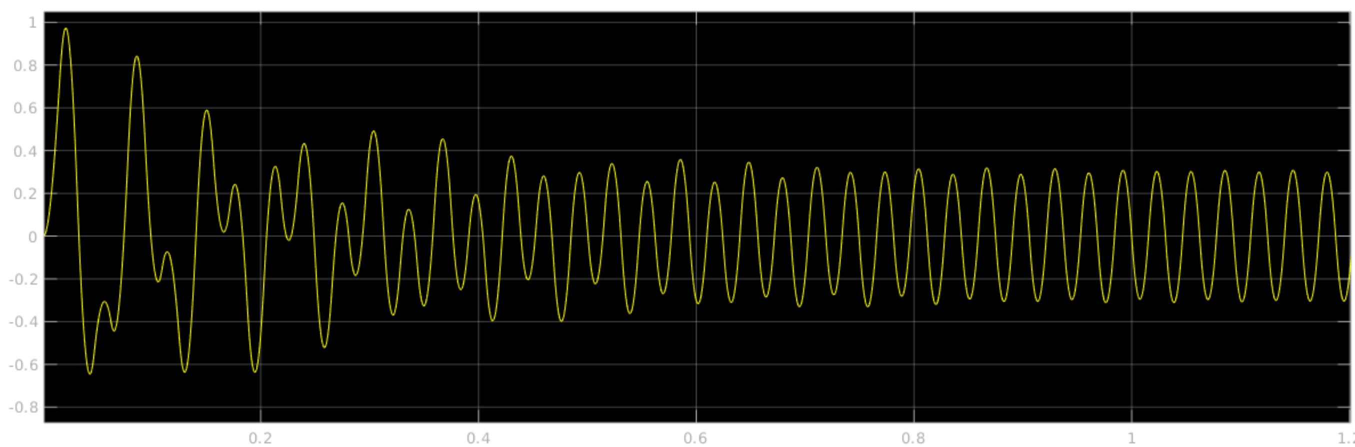
Square-wave input time domain plot:



4-term Fourier series approximation frequency domain plot:

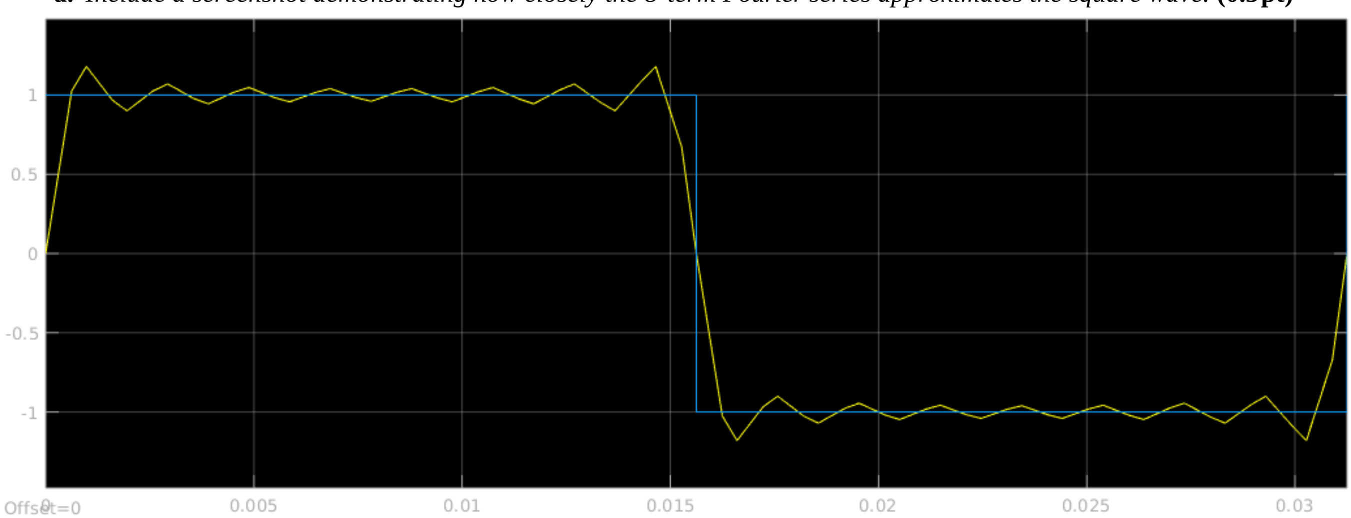


4-term Fourier series approximation time domain plot:



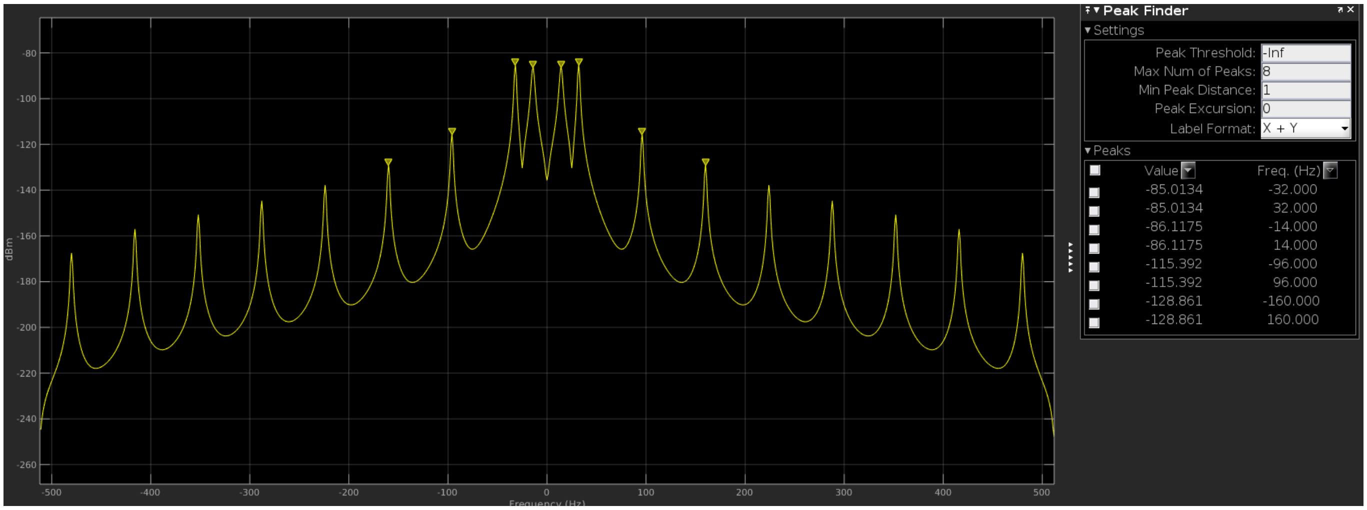
3.3 Exercise 3.3

a. *Include a screenshot demonstrating how closely the 8-term Fourier series approximates the square wave.* **(0.5pt)**

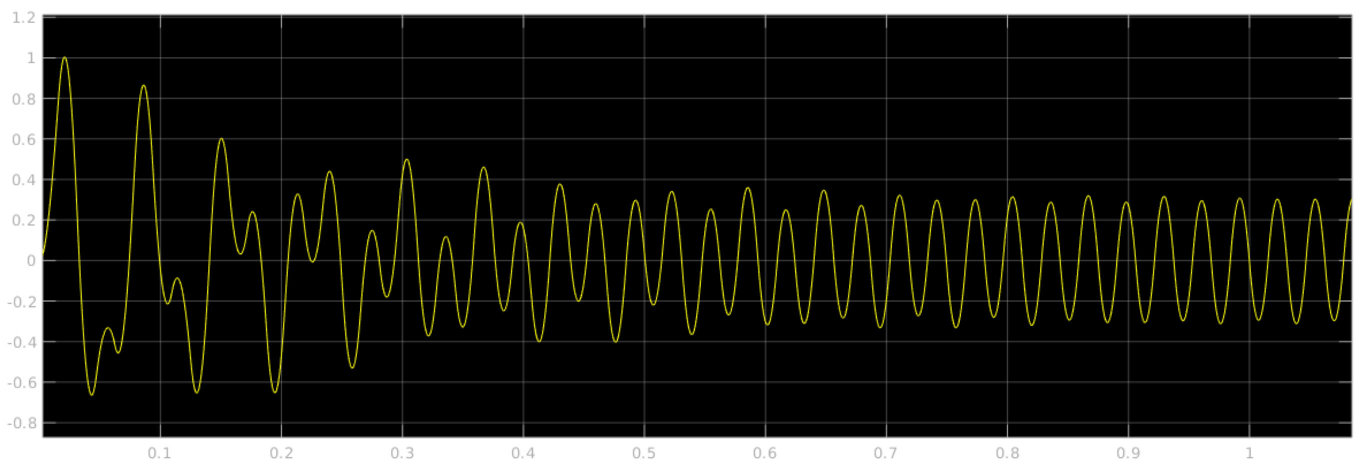


b. *Compare the output response of the RLC circuit to the 8-term Fourier series approximation input and the response to the square-wave input, respectively. Include a screenshot of the output response in the two cases, respectively.* **(1pt)**

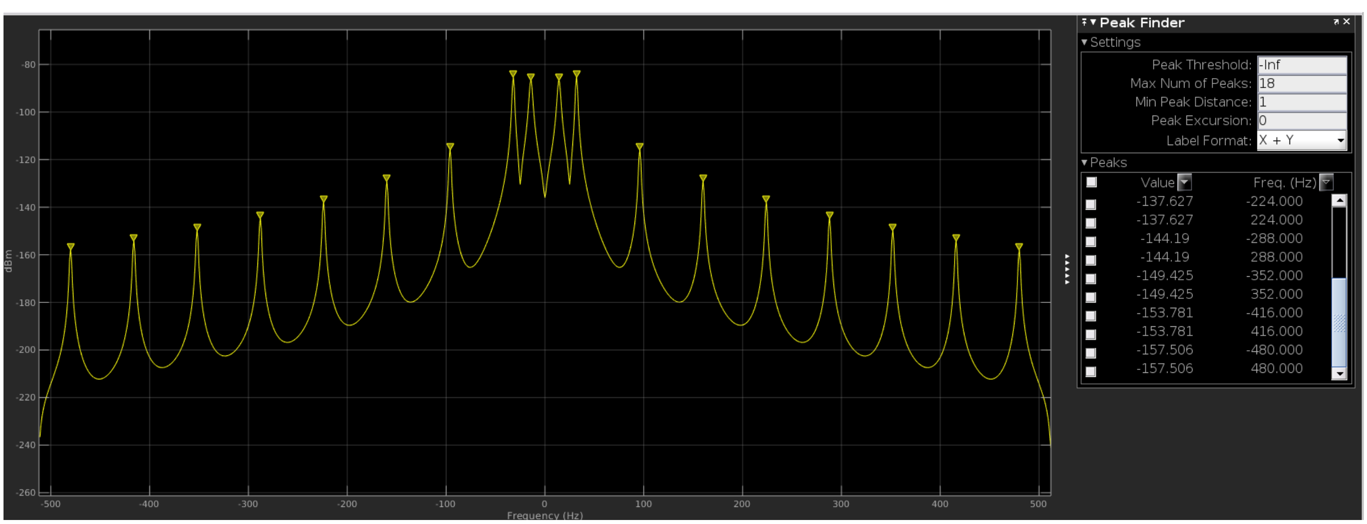
Square-wave input frequency domain plot (same as 3.2):



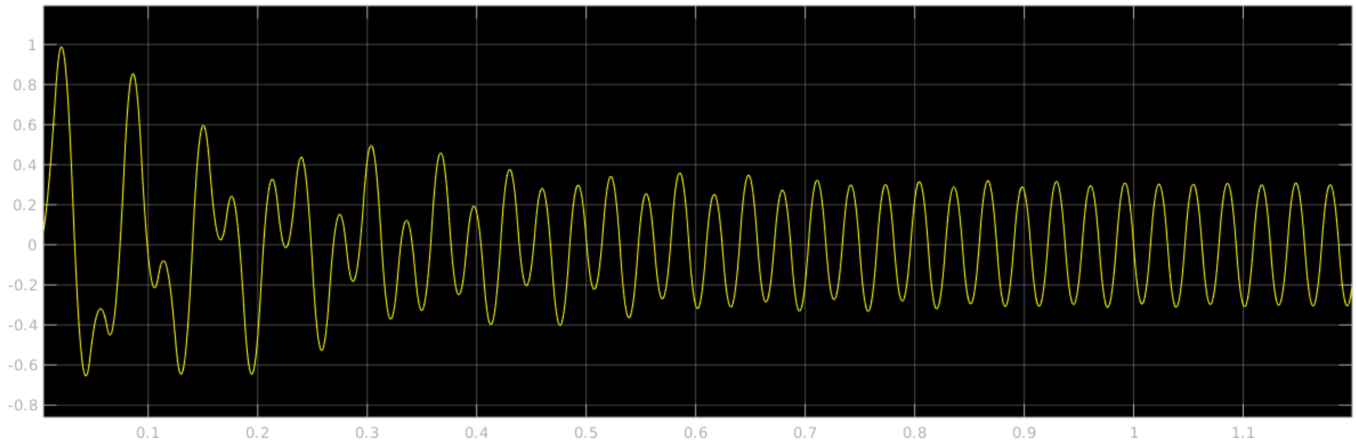
Square-wave input time domain plot (same as 3.2):



8-term Fourier series approximation frequency domain plot:



8-term Fourier series approximation time domain plot:



c. *Does the 8-term Fourier series approximate the square wave better than the 4-term Fourier series? Does the output response of the RLC circuit to the 8-term Fourier series input approximate the output response to the square-wave input better than when the 4-term Fourier series was used as input? Include an answer and justify your answer.* **(1pt)**

Comparing the time-domain simulation results, it is obvious that the 8-term Fourier series approximate the square wave better than the 4-term Fourier series. On the other hand, the output approximation of the RLC circuit to the 8-term Fourier series is only slightly better than the 4-term approximation.

Generally, more terms will result in a more accurate Fourier series approximation, as the boundaries of Fourier series sigma summation ranges from negative infinity to positive infinity. This means that a Fourier series signal with more composition terms will provide a better approximation.

Due to Gibbs phenomenon, where overshoot of Fourier series occurs at simple discontinuities, it is never possible to have a perfect approximation of the square wave functions using sine waves. However, for the RLC circuit output, where the original output is a continuous, smooth function, the Fourier transform approximation performs better, which means less terms could also return similar result.