ECE212 Homework 5

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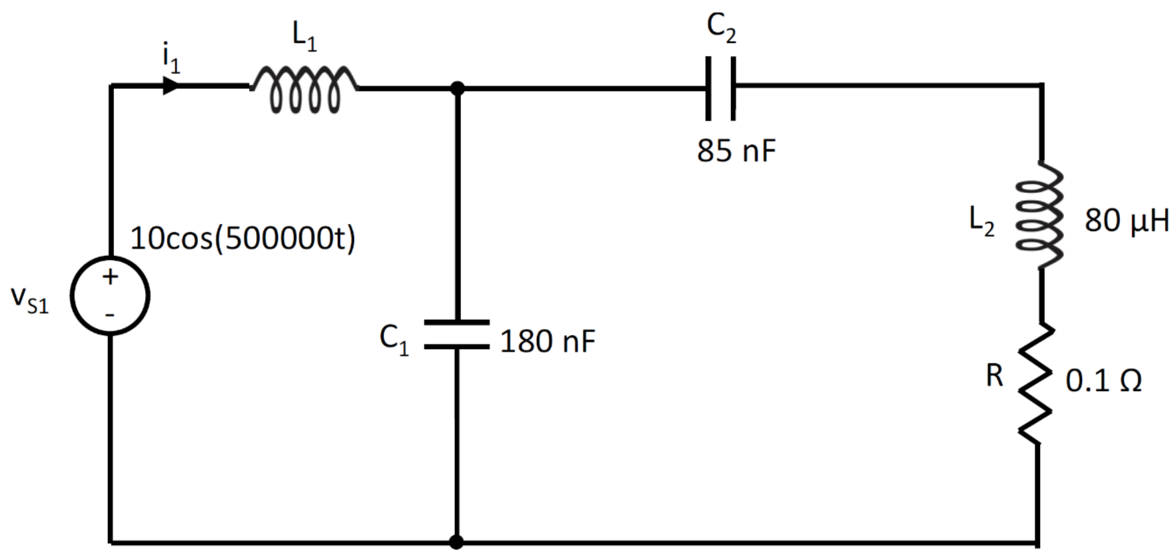
Q1.



1. **Calculate the equivalent impedance Z_{eq1} formed by the combination of C_2, L_2 and R.**

$$Z_{C2} = \frac{1}{j\omega C} = \frac{1}{j \cdot 5 \times 10^5 \cdot 85 \times 10^{-9}} = -j\frac{1}{425} \times 10^4 \Omega = -j23.529\Omega$$

$$Z_{L2} = j\omega L = j \cdot 5 \times 10^5 \cdot 80 \times 10^{-6} = j40\Omega$$

$$Z_R = R = 0.1\Omega$$

As C_2, L_2 and R are connected in series according to above circuit configuration, the equivalent impedance Z_{eq1} can be calculated by simple addition as follows:

$$Z_{eq1} = Z_{C2} + Z_{L2} + Z_R = -j23.529\Omega + j40\Omega + 0.1\Omega = 0.1 + j16.471\Omega = 16.471\angle 89.652°\Omega$$

2. **Calculate the equivalent impedance Z_{eq2} formed by the combination of Z_{eq1} and C_1.**

$$Z_{C2} = \frac{1}{j\omega C} = \frac{1}{j \cdot 5 \times 10^5 \cdot 180 \times 10^{-9}} = -j\frac{1}{9} \times 10^2 \Omega = -j11.11\Omega$$

As Z_{eq1} and C_1 are connected in parallel, Z_{eq2} can be calculated as follows:

$$\frac{1}{Z_{eq2}} = \frac{1}{Z_{eq1}} + \frac{1}{Z_{C1}} = \frac{1}{16.471\angle 89.652°} + \frac{1}{11.11\angle -90°} = \frac{1}{16.471}\angle -89.652° + \frac{1}{11.11}\angle 90°$$

$$= 0.000368752 - j0.029309 = 0.0293113\angle -78.759$$

$$Z_{eq2} = \frac{1}{0.029\angle-89.28}\Omega = 34.12\angle-89.28°\Omega = 0.429 - j34.13\Omega$$

3. Design a value of L₁ which gives rise to a reactance with the same magnitude as the complex part of Zeq2 but with the opposite sign.

From Part 2 it can be found that $Z_{eq2} = 0.429 - j34.13\Omega$, *which means* $X_{eq2} = -j34.13\Omega$.

$$X_{L1} = -X_{eq2} = j34.13\Omega$$

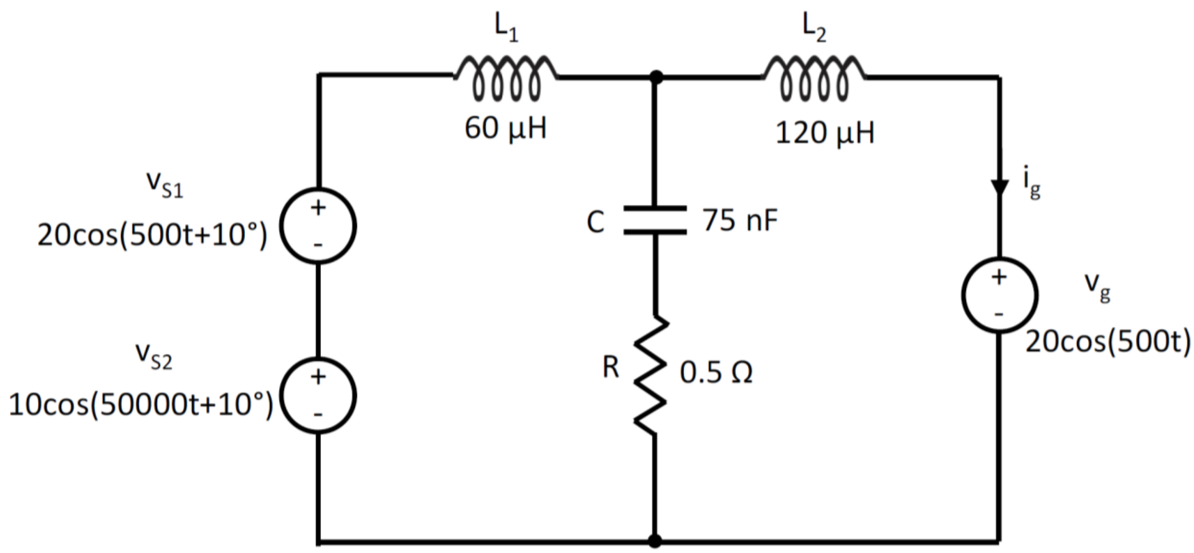
$$\Rightarrow\ j\omega L = \mathrm{Im}\{R_{L1}\} = X_{L1}$$

$$\Rightarrow\ L_1 = \frac{X_{L1}}{j\omega} = \frac{j34.13}{j500000} = 68.26\mu H$$

4. Derive an expression for the supply current i₁(t) (in the time domain, at sinusoidal steady state) if the value of L₁ calculated in part 3 is used.

$$i_1(t) = \frac{V_{S1}(t)}{\mathrm{R}_{eq2} + R_{L1}} = \frac{10\cos(500000t)}{0.429} = 23.31\cos(500000t)A$$

Q2.



1. **Calculate the impedance of all the dynamic elements in the circuit at ω_1 = 500 rad/s.**

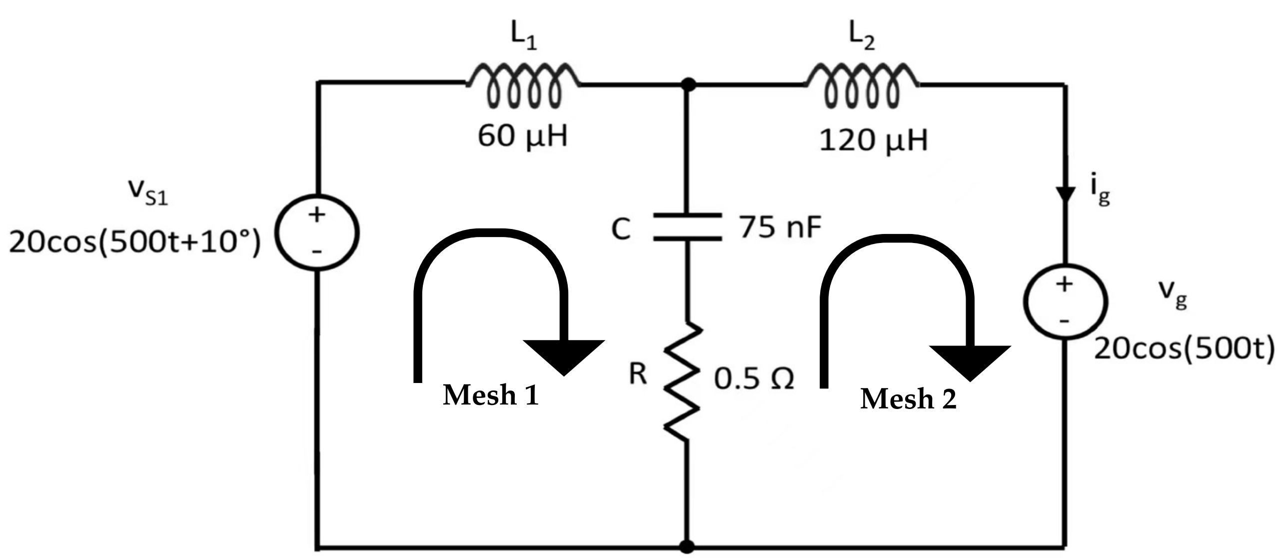
$$Z_{L1} = j\omega L_1 = j \cdot 5 \times 10^2 \cdot 60 \times 10^{-6} = j0.03\Omega = 0.03\angle 90°\ \Omega$$

$$Z_{L2} = j\omega L_2 = j \cdot 5 \times 10^2 \cdot 120 \times 10^{-6} = j0.06\Omega = 0.06\angle 90°\ \Omega$$

$$Z_{C2} = \frac{1}{j\omega C} = \frac{1}{j \cdot 5 \times 10^2 \cdot 75 \times 10^{-9}} = -j\frac{1}{375} \times 10^7 \Omega = -j26.6667k\Omega$$

$$Z_R = R = 0.5\Omega$$

2. **Draw the phasor domain circuit diagram at ω_1.**



3. Calculate the phasor current $I_g^{(1)}$ at ω_1.

First, transform the existing two voltage source into their phasor forms.

$$V_{S1} = 20\cos(500t + 10°) \;\Rightarrow\; \mathbf{V_{S1}} = 20\angle 10° V$$
$$V_g = 20\cos(500t) \;\Rightarrow\; \mathbf{V_g} = 20\angle 0° V$$

Then, combine C and R to reduce complexity of the above configuration.

$$Z_{eq} = Z_R + Z_{C2} = 0.5 - j26666.7\Omega = 26666.7\angle -89.999°$$

Using mesh analysis, at Mesh 1 (Mesh 1 has a mesh current of I_1), the following relationship can be found using KVL.

$$V_{L1} + V_C + V_R = V_{s1} \Rightarrow Z_{L1} \cdot I_1 + Z_{eq} \cdot (I_1 - I_2) = V_{s1}$$

$$0.03\angle 90° \cdot I_1 + 26666.7\angle -89.999° \cdot (I_1 - I_2) = 20\angle 10° \tag{1}$$

Using mesh analysis, at Mesh 2 (Mesh 2 has a mesh current of I_2), the following relationship can be found using KVL.

$$V_{L2} + V_C + V_R = -V_g \Rightarrow Z_{L2} \cdot I_2 + Z_{eq} \cdot (I_2 - I_1) = -V_g$$

$$0.06\angle 90° \cdot I_2 + 26666.7\angle -89.999° \cdot (I_2 - I_1) = -20\angle 0° \tag{2}$$

Solve the system of equation of (1)(2) by combining the two equations as follows.

$$0.03\angle 90° \cdot I_1 + 0.06\angle 90° \cdot I_2 = 20\angle 10° - 20\angle 0°$$
$$0.03\angle 90° \cdot I_1 + 0.06\angle 90° \cdot I_2 = -0.303845 + j3.47296 = 3.48623\angle 95.00°$$
$$\Rightarrow I_1 + 2I_2 = 116.208\angle 5.00°$$

$$\Rightarrow I_1 = 116.208\angle 5.00° - 2I_2 \tag{3}$$

Substituting (3) into (2),

$$0.06\angle 90° \cdot I_2 + 26666.7\angle -89.999° \cdot (3 \cdot I_2 - 116.208\angle 5.00°) = -20$$
$$I_2 = 38.736\angle 5.00° A \Rightarrow I_1 = 38.736\angle 5.00° A$$
$$i_g^{(1)} = I_2 = 38.736\angle 5.00° A = 38.736\cos(500t + 5.00°) A$$

4. **Calculate the impedance of all the dynamic elements in the circuit at ω_2 = 50000 rad/s.**

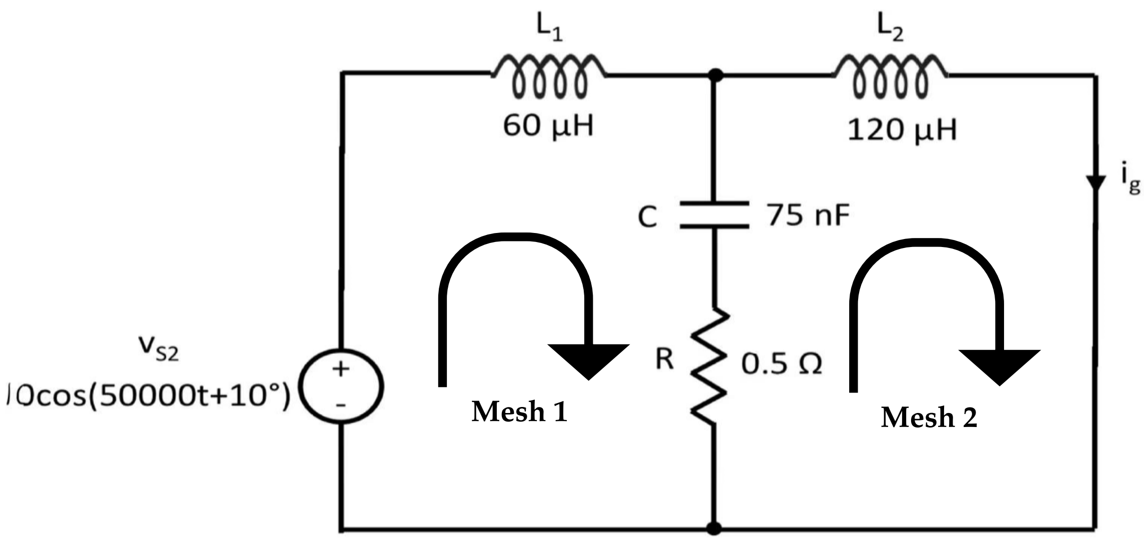
$$Z_{L1} = j\omega L_1 = j \cdot 5\times10^4 \cdot 60\times10^{-6} = j3\Omega = 3\angle 90°\ \Omega$$

$$Z_{L2} = j\omega L_2 = j \cdot 5\times10^4 \cdot 120\times10^{-6} = j6\Omega = 6\angle 90°\ \Omega$$

$$Z_{C2} = \frac{1}{j\omega C} = \frac{1}{j\cdot 5\times10^4 \cdot 75\times10^{-9}} = -j\frac{1}{375}\times10^5\Omega = -j266.667\Omega$$

$$Z_R = R = 0.5\Omega$$

5. **Draw the phasor domain circuit diagram at ω_2.**



6. **Calculate the phasor current Ig(1) at ω_1.**

First, transform the existing two voltage source into their phasor forms.

$$V_{S1} = 10\cos(50000t + 10°)\ \Rightarrow\ \mathbf{V_{S1}} = 10\angle 10° V$$

Then, combine C and R to reduce complexity of the above configuration.

$$Z_{eq} = Z_R + Z_{C2} = 0.5 - j266.667\Omega = 266.667\angle -89.893°\Omega$$

Using mesh analysis, at Mesh 1 (Mesh 1 has a mesh current of I_1), the following relationship can be found using KVL.

$$V_{L1} + V_C + V_R = V_{s2} \Rightarrow Z_{L1}\cdot I_1 + Z_{eq}\cdot (I_1 - I_2) = V_{s2}$$

$$3\angle 90°\cdot I_1 + 266.667\angle -89.893°\cdot (I_1 - I_2) = 10\angle 10° \quad (4)$$

Using mesh analysis, at Mesh 2 (Mesh 2 has a mesh current of I_2), the following relationship can be found using KVL.

$$V_{L2} + V_C + V_R = 0 \Rightarrow Z_{L2} \cdot I_2 + Z_{eq} \cdot (I_2 - I_1) = 0$$

$$6\angle 90° \cdot I_2 + 266.667\angle -89.893° \cdot (I_2 - I_1) = 0 \tag{5}$$

Solve the system of equation of (4)(5) *by combining the two equations as follows.*

$$3\angle 90° \cdot I_1 + 6\angle 90° \cdot I_2 = 10\angle 10°$$
$$\Rightarrow I_1 + 2I_2 = 3.33\angle -80°$$
$$\Rightarrow I_1 = 3.33\angle -80° - 2I_2 \tag{6}$$

Substituting (3) into (2),

$$6\angle 90° \cdot I_2 + 266.667\angle -89.893° \cdot (3 \cdot I_2 - 3.33\angle -80°) = 0$$
$$I_2 = 1.12\angle -80.00°\, A \Rightarrow I_1 = 1.09\angle -80.00°\, A$$
$$i_g^{(2)} = I_2 = 1.12\angle -80.00°\, A = 1.12\cos(50000t - 80.00°)\, A$$

7. **Using the solutions for parts 3 and 6, calculate the time domain current $i_g(t)$ at sinusoidal steady state.**

   By the principle of superposition, the time domain current $i_g(t)$ can be calculated as the sum of the solution from parts 3 and 6 as follows:

$$i_g(t) = i_g^{(1)}(t) + i_g^{(2)}(t) = 38.736\cos(500t + 5.00°) + 1.12\cos(50000t - 80.00°)\, A$$