Catalogue of 2D sensitivity patterns for ERT cross-borehole

HydroGeophysics Group, Aarhus University, Marts 2021

Introduction

We present a catalogue of 2D sensitivity patterns for electric resistivity tomography (ERT) cross-borehole setups. The sensitivity is computed for different dipole-dipole configurations, where A and B are the current electrodes and M and N are the potential electrodes (combined referred to as a quadrupole). The electrodes may be placed on the surface or buried in the ground.

Implementation

The 2D sensitivity distribution of a pole-pole configuration is given as

$$S = E_c E_p = E_c^x E_p^x + E_c^z E_p^z \quad (1)$$

where E_c and E_p is the electric field vector from the current electrode and the potential electrode acting as a current electrode (the adjoint source), respectively. Given a homogenous half-space, the electric field from a point source in the point $P(x, z)$ is

$$\boldsymbol{E} = \nabla V, \quad (2)$$

where V is the potential given by

$$V^{2D} = \frac{1}{2\pi}(\ln(R) + \ln(R_{im})) \, . \quad (3)$$

R and R_{im} are the distances from the point $P(x, y)$ to the current source and the mirrored (image) current source with respect to the surface at $z = 0$, respectively.

For an electrode placed in $P_s(r,s)$ and with z being negative in the ground:

$$R = \sqrt{|r-x|^2 + |s-z|^2} \text{ and } R_{im} = \sqrt{|r-x|^2 + |s+z|^2}. \quad (4)$$

Inserting equation 3 in equation 2, we get

$$\boldsymbol{E} = \nabla V = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right)\frac{1}{2\pi}[\ln(R) + \ln(R_{im})] = \frac{1}{2\pi}\left(\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right)\ln(R) + \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right)\ln(R_{im})\right), \quad (5)$$

where

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right)\ln(R) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right)\ln(\sqrt{|r-x|^2 + |s-z|^2}) = \frac{((r-x),(s-z))}{(r-x)^2+(s-z)^2} \quad (6)$$

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right)\ln(R_{im}) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial z}\right)\ln(\sqrt{|r-x|^2 + |s+z|^2}) = \frac{((r-x),(s+z))}{(r-x)^2+(s+z)^2}. \quad (7)$$

Dividing the electric field into a x and z component we get

$$E^x = \frac{1}{2\pi}\left(\frac{(r-x)}{(r-x)^2+(s-z)^2} + \frac{(r-x)}{(r-x)^2+(s+z)^2}\right)$$

$$E^y = \frac{1}{2\pi}\left(\frac{(s-z)}{(r-x)^2+(s-z)^2} + \frac{(r+z)}{(r-x)^2+(s+z)^2}\right).$$

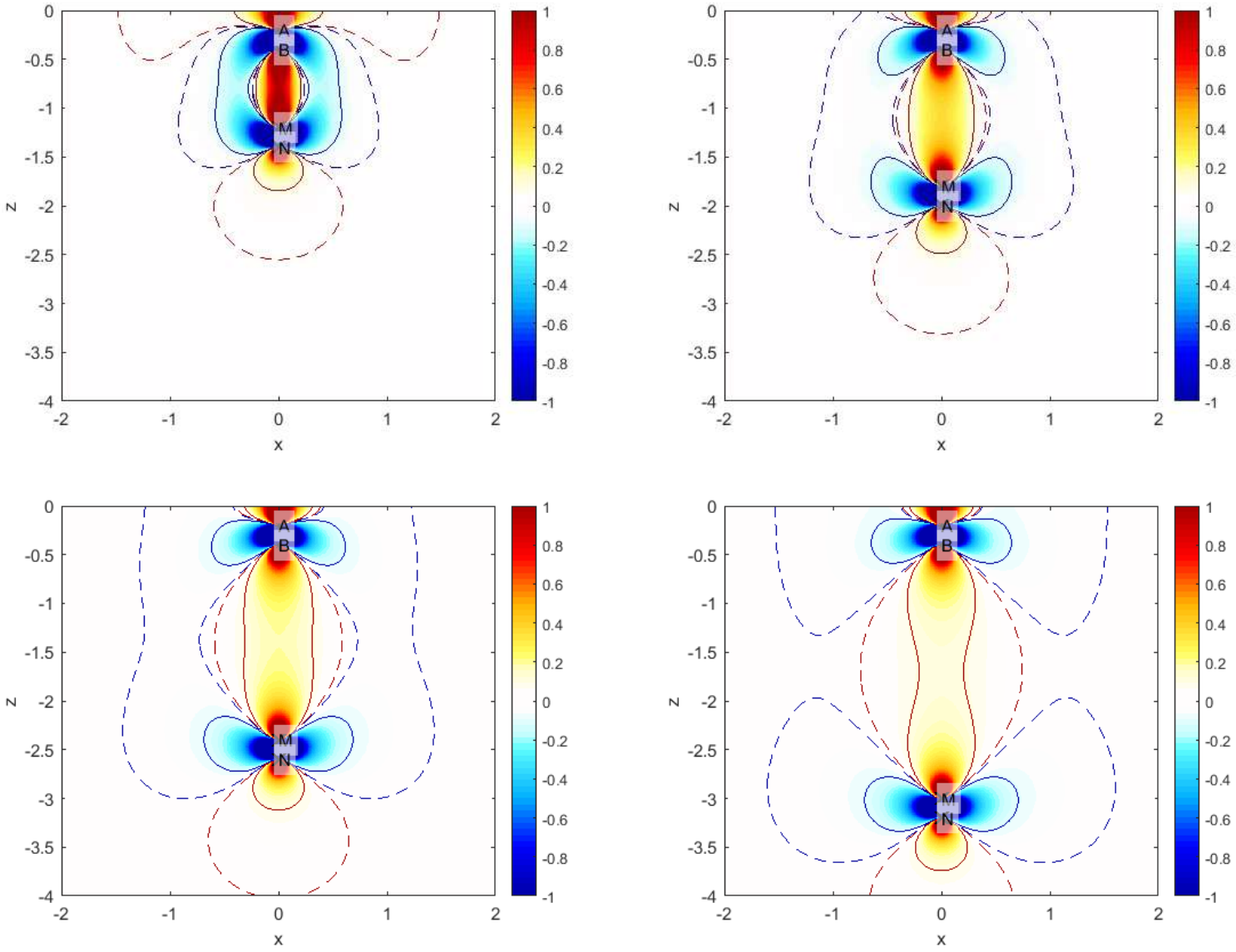
The sensitivity of the quadrupole can now be found from superposition of the pole-pole responses

$$S = S_{AM} - S_{AN} - S_{SM} + S_{BN}$$

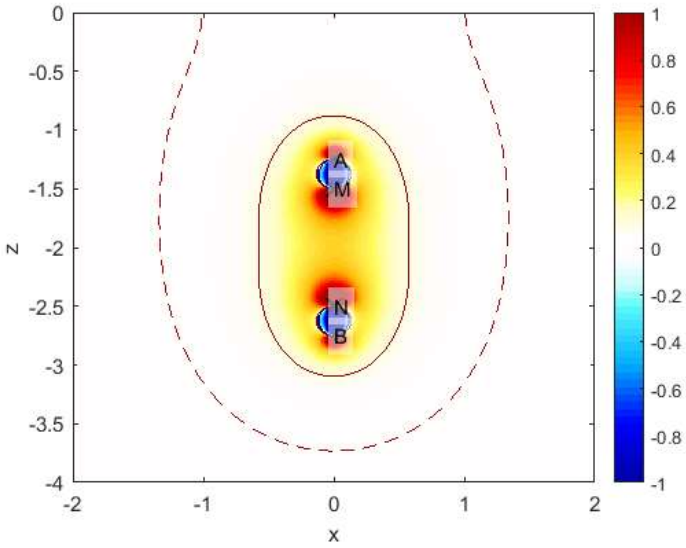
Visualization

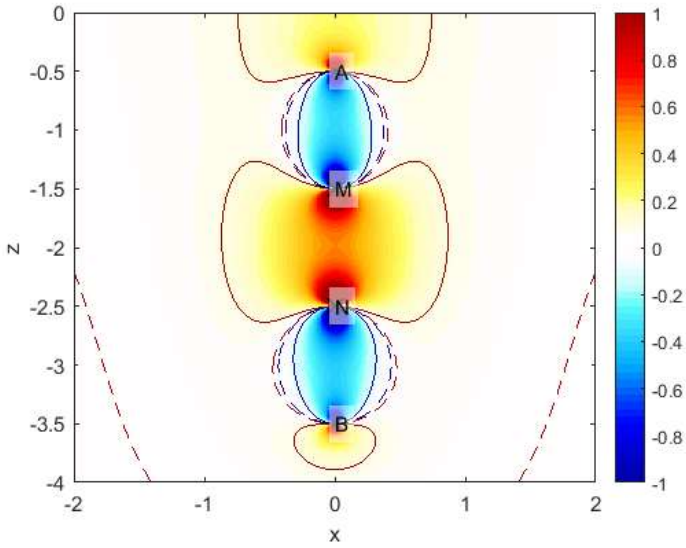
The sensitivity distributions are plotted as a normalised sensitivity between -1 and 1. The solid lines marks the $\pm$ 0.1 contour and the dashed lines the 0.01 contour. In the following pages, we show examples of different singe borehole configurations, where the four electrodes are placed in the same borehole, as well as cross-borehole configurations, where two boreholes are in use.

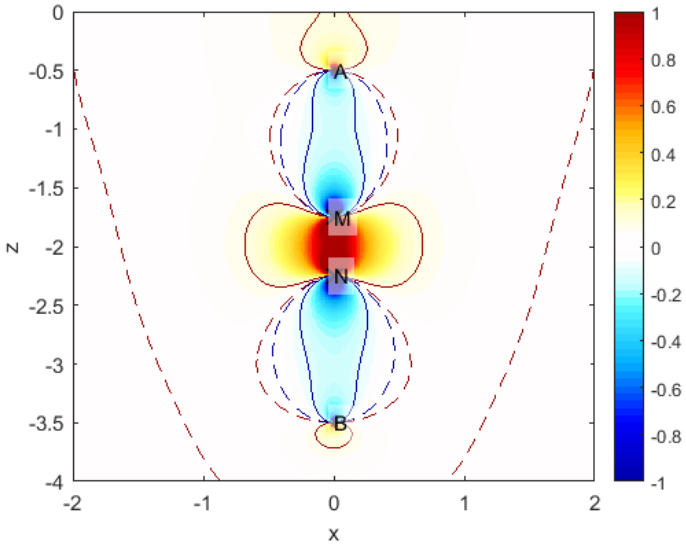
Single borehole: dipole-dipole

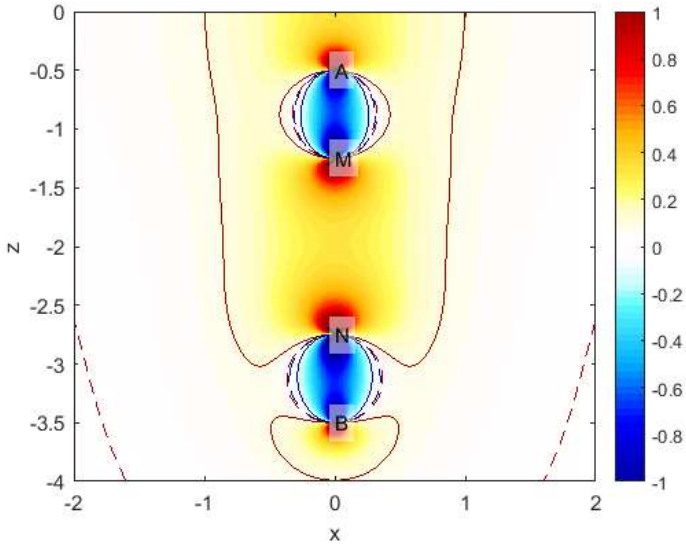


Single borehole: Schlumberger

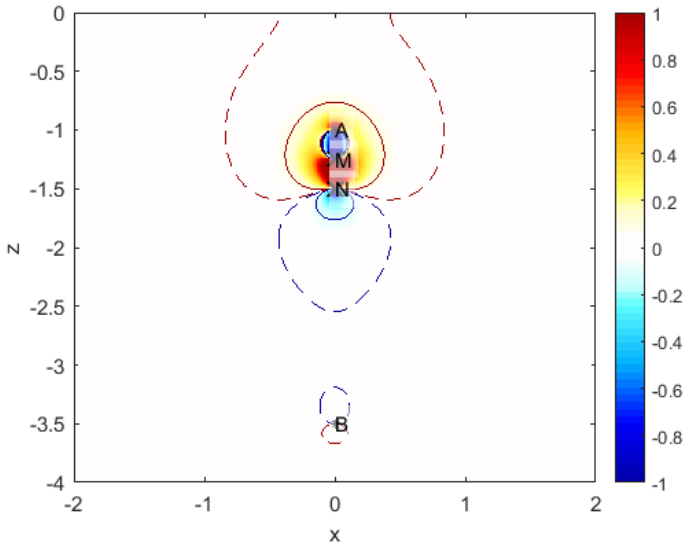


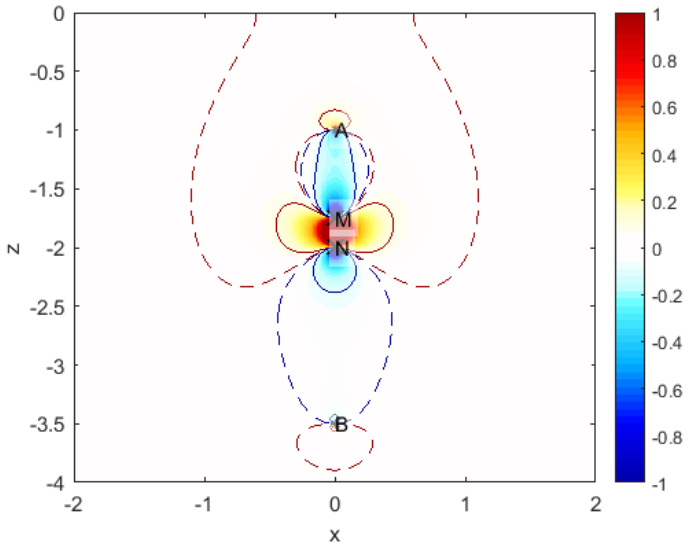


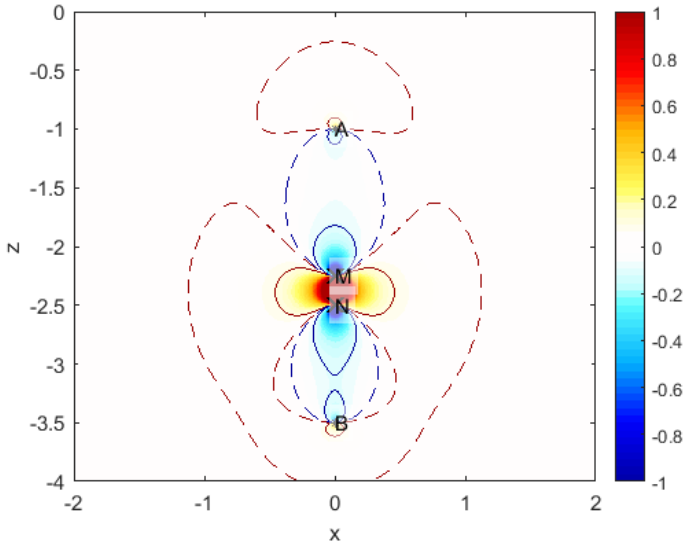


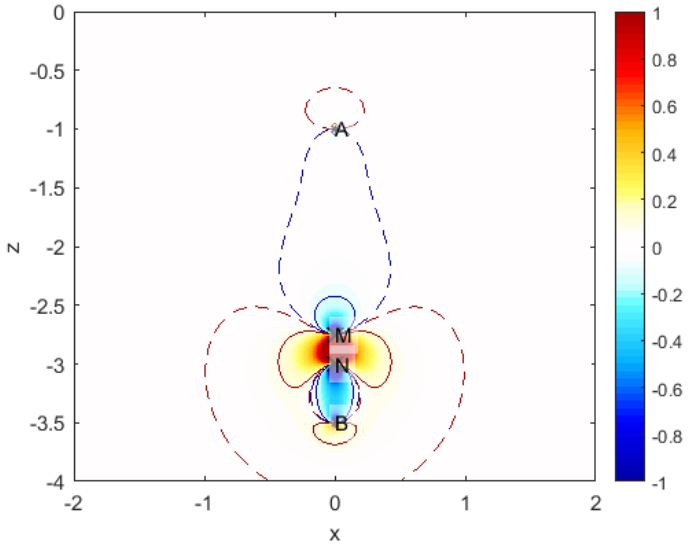


Single borehole: gradient

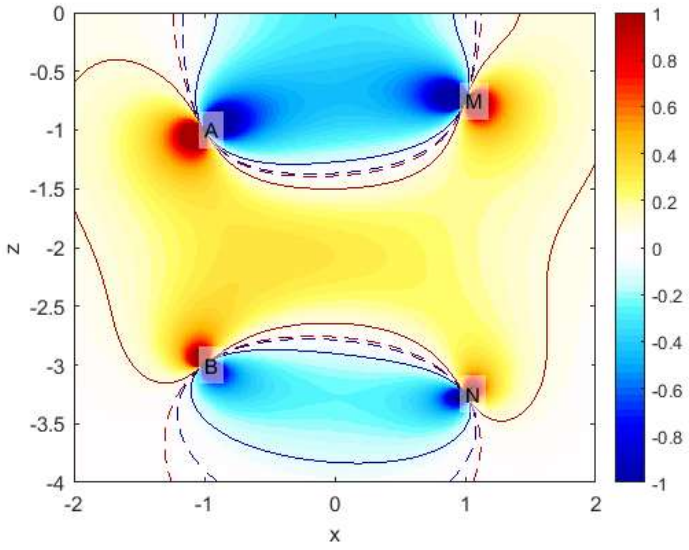


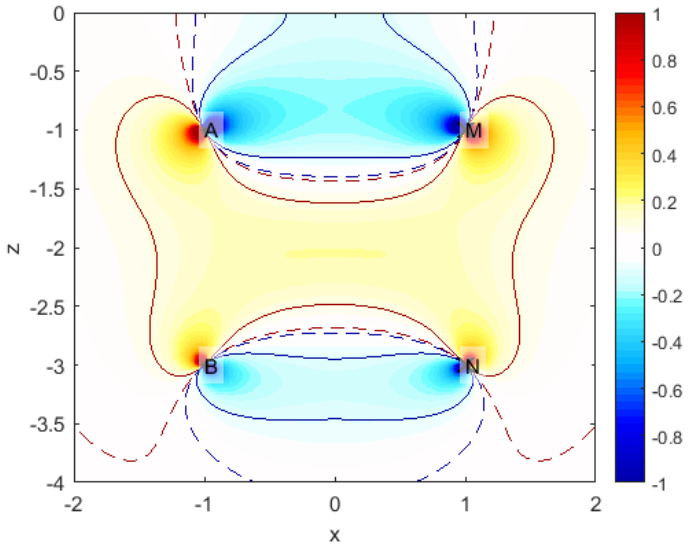


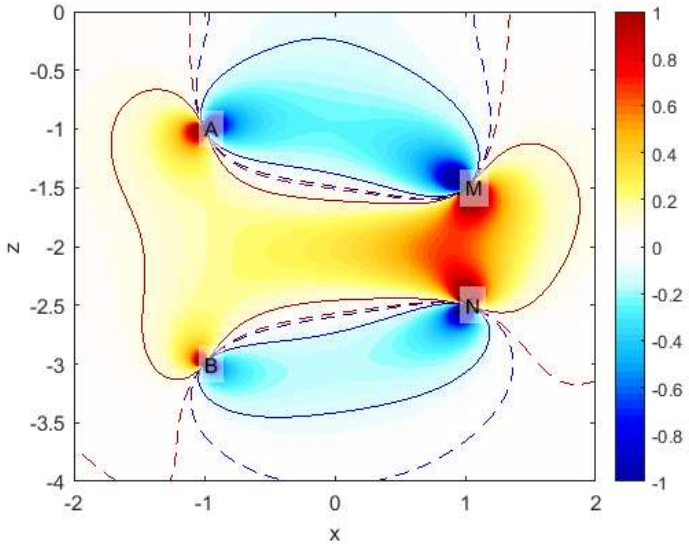


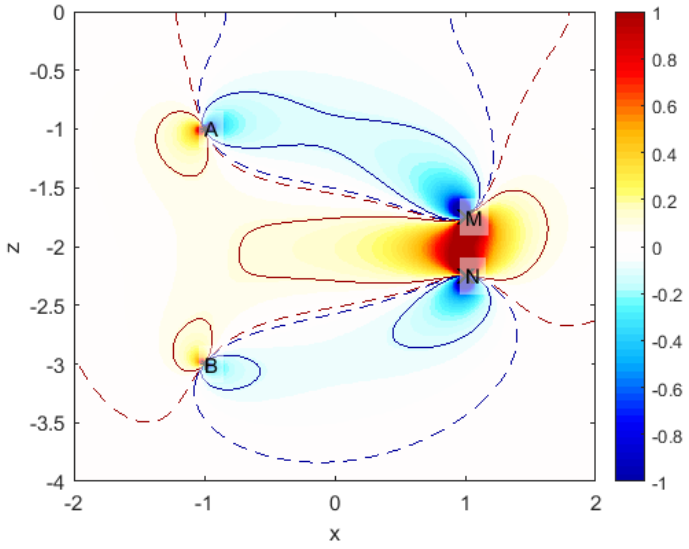


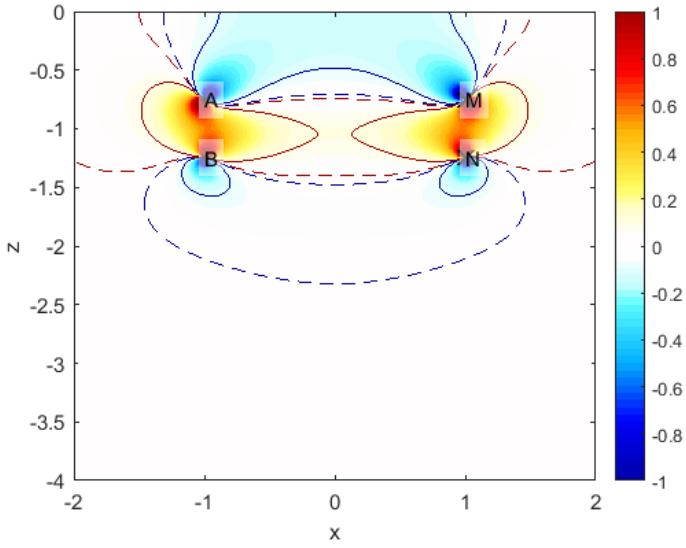
Cross-borehole: AB-MN







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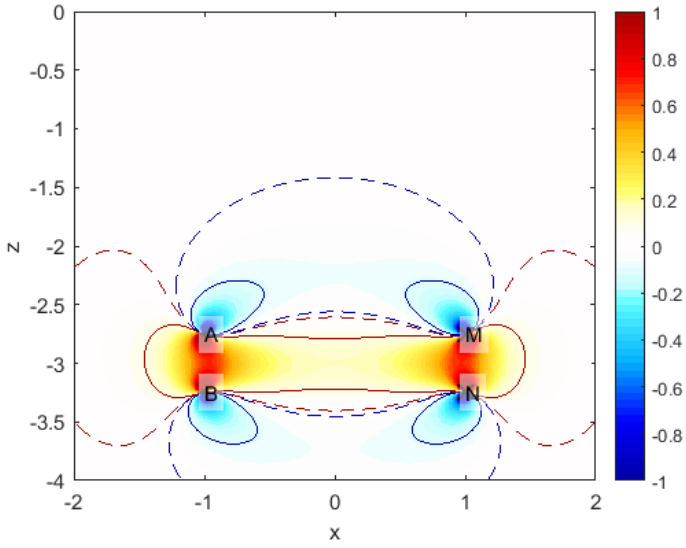
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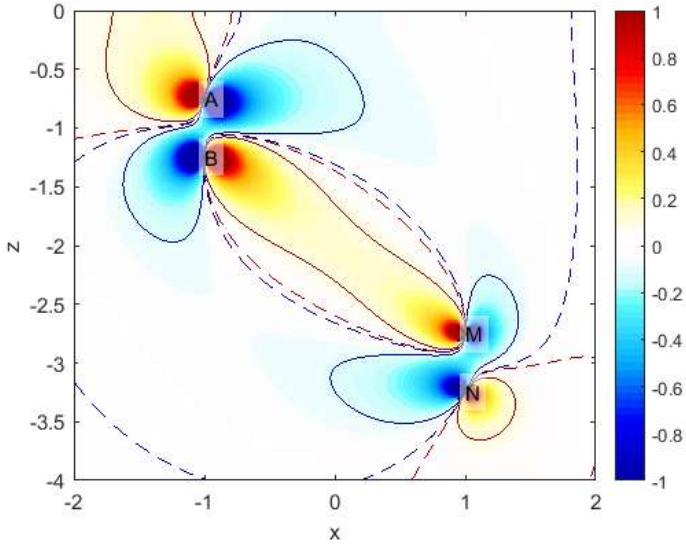
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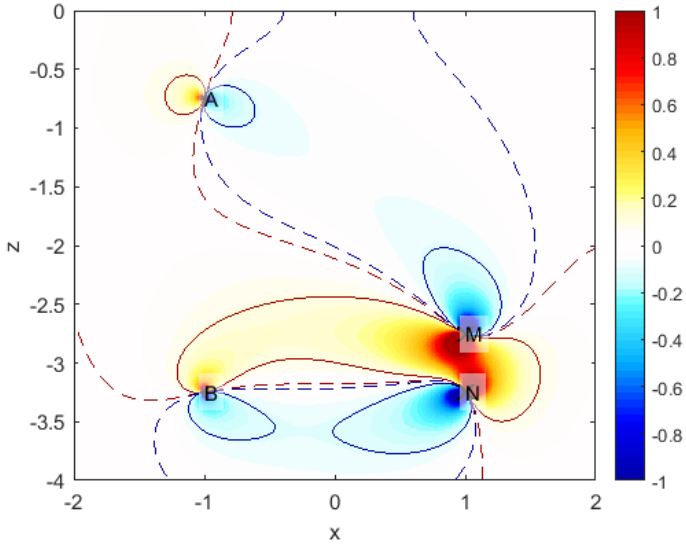
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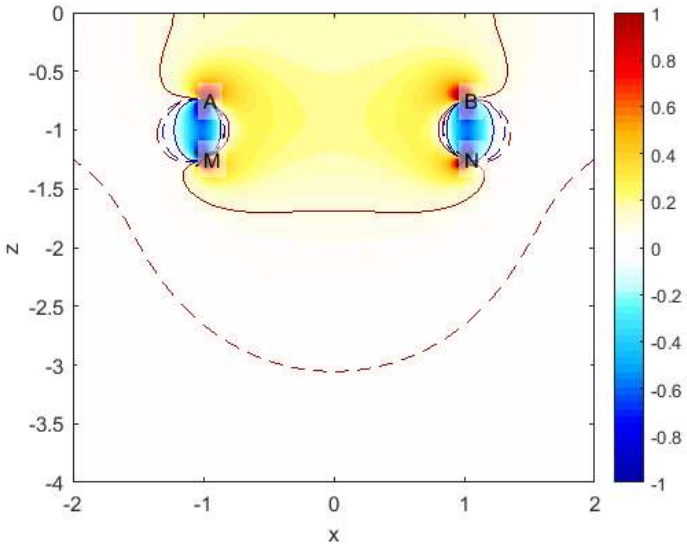
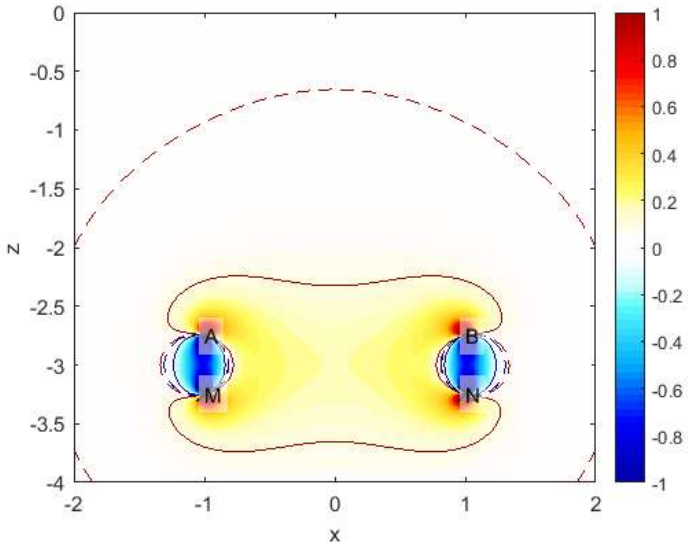
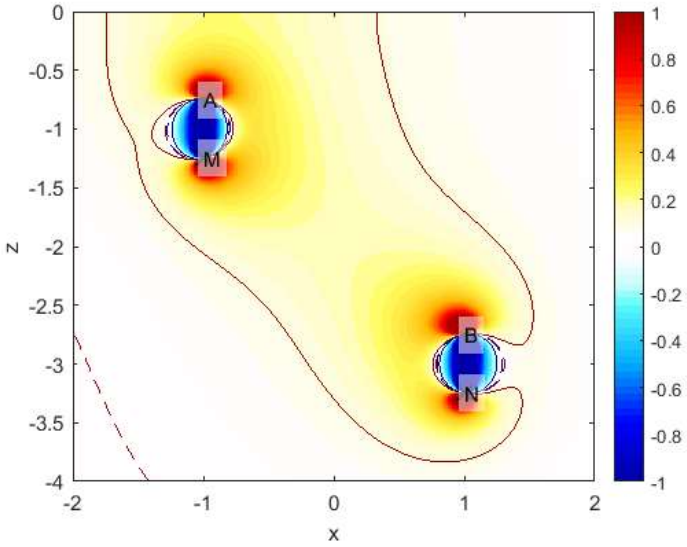
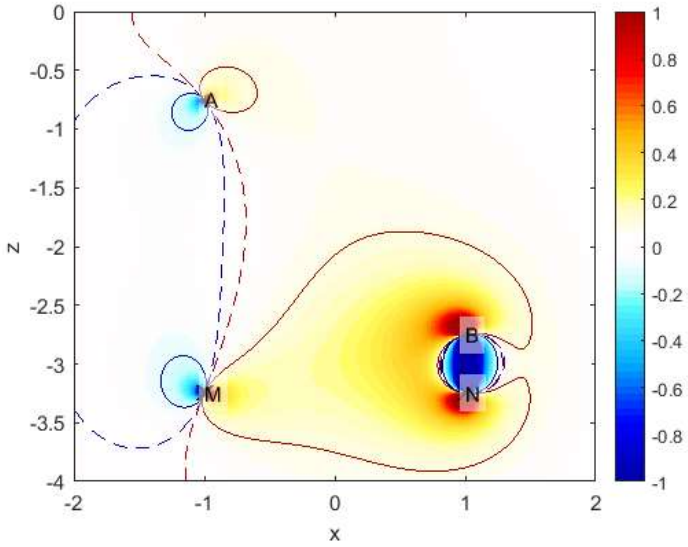
Cross-borehole: AM-BN

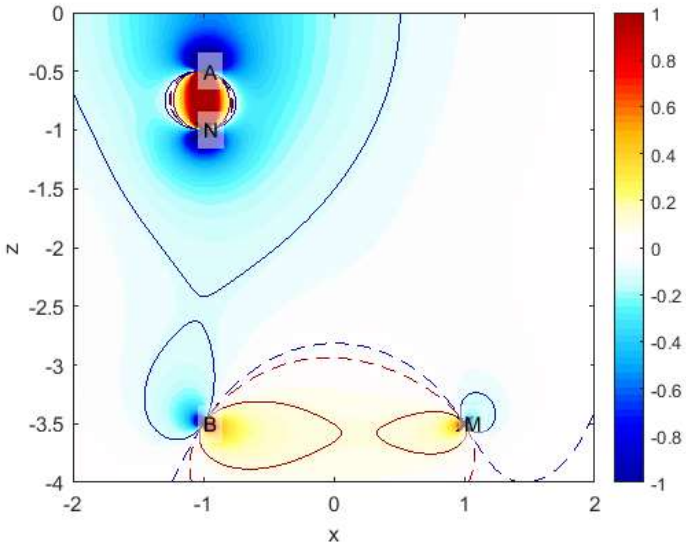


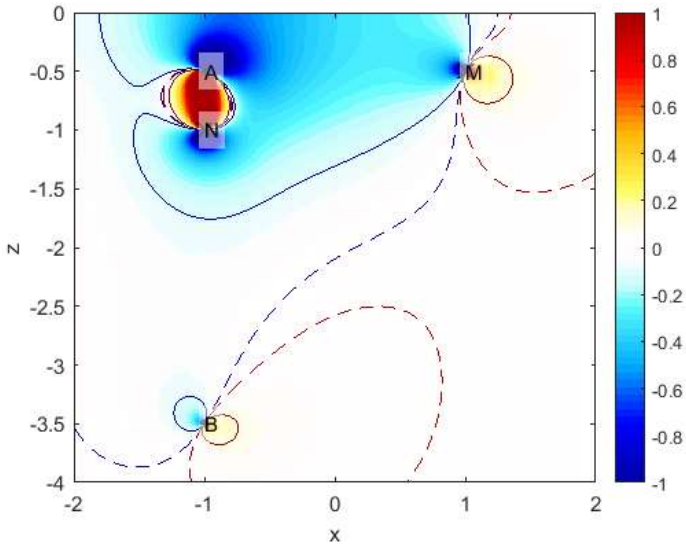


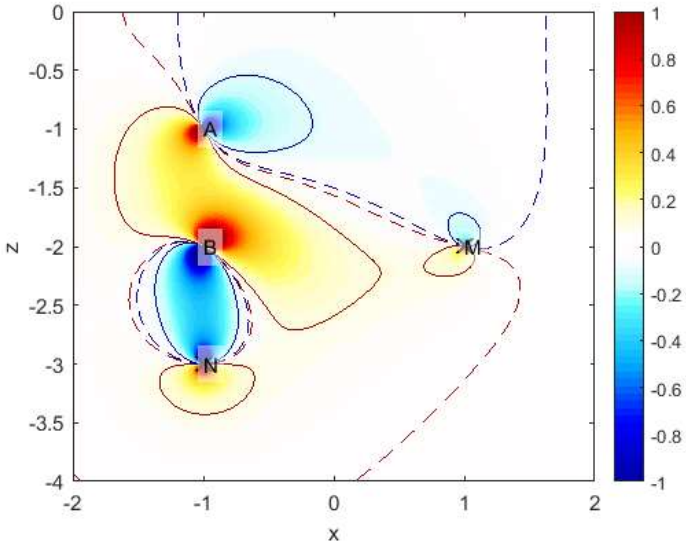


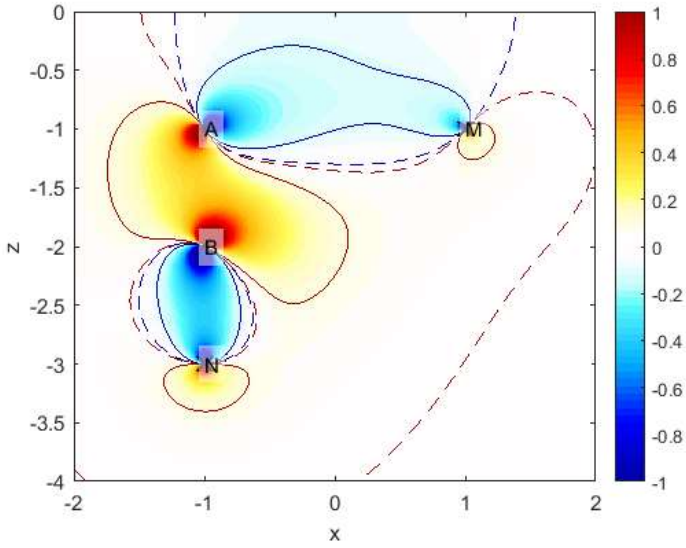


Cross-borehole: AMN-B









Surface-borehole