Schedule

	06/02 Monday	06/03 Tuesday	06/04 Wednesday	06/05 Thursday	06/06 Friday
8:20-8:30	Opening	Picture			
8:30-9:20	Richard Moeckel	Susanna Terracini	Tere Seara	Zhihong Xia	Alain Albouy
9:30-10:20	Kuo-chang Chen	Xijun Hu	Pau Martin	Gabriella Pinzari	Vivina Barutello
10:30-10:50	Tea Break				
10:50-11:40	Ezequiel Maderna	Hui Liu	Mitsuru Shibayama	Lei Zhao	Jian Wang
12：00	Lunch				
14:00-14:50	David Sauzin	Andrea Venturelli	Free Afternoon	Urs Frauenfelder	Otto van Koert
15：00-15：50	Pedro Salomao	Alessandro Portaluri		Inma Baldoma	Huagui Duan
16：00-16：20	Tea Break			Tea Break	
16：20-17：30	Zhifu Xie	Xiang Yu		Poster Session	Poster Session

Titles and Abstracts

The inverse problem for central configurations

Alain Albouy (Paris Observatory)

I will present the following result: If a 5-body planar configuration is central for two nonproportional sets of five masses $(m_1,\dots,m_5)$, one of them being of five positive masses, then the center of mass is the same for both sets of masses. We proved this theorem a few years ago with A.C. Fernandes (see [1]). I will present recent simplifications of the two main arguments, inspired by Williams' paper of 1938 [2]. Williams' equations, once simplified, reduce the question to the existence of ``flat Dziobek configurations''. That such configurations do not exist is proved in [3]. It was reproved by proving Williams' inequality [4]. I will present new general facts obtained with Fernandes about the inverse problem of central configurations in general.

[1] Multimass central configurations (with Antonio Carlos Fernandes), 15/6/2021, videoconference
https://sites.google.com/view/matemairacorana/schedule/records-of-previous-lectures
https://www.youtube.com/watch?v=LguGtxi3M04\&t=6s (starts at 23'00'')

[2] W.L. Williams, Permanent configurations in the problem of five bodies, Transactions of the American Mathematical Society, 44 (1938), pp. 563-579

[3] A limit of nonplanar 5-body central configurations is nonplanar, with A.C. Fernandes, Archive for Rational Mechanics and Analysis, 248 (2024), 12, 21 pp.

[4] Jiexin Sun, poster in this conference.

Chaotic phenomena beyond the Routh mass ratio

Immaculada Baldoma (Universitat Politècnica de Catalunya)

The equilibrium Lagrangian points L4 and L5 in the RPC3BP undergo a bifurcation at the mass ratio known as Routh mass-ratio. Indeed, they are elliptic for values of mass ration smaller than Routh mass-ratio, and a complex saddle afterwards. This is an example of a Hamiltonian-Hopf bifurcation. In this Hamiltonian setting, it is well known that the existence of transversal homoclinics associated to L4 (and L5) and they invariant manifolds, it give rise to rich dynamics: a plethora of horseshoes and what is known as the bluesky catastrophe.

We prove that, for any value of the mass ratio close enough and bigger than the Routh mass ratio, the invariant manifolds of L4 indeed intersect along a transverse homoclinic orbit, provided some coefficient is different from zero.

This is a join work with P. Martín and D. Scarcella

On Birkhoff conjecture for Kepler billiards

Vivina Barutello (University of Turin)

Keplerian billiards are classical billiards with a Keplerian potential inside. We consider the reflective case, where the particle reflects elastically on the boundary, as well as the refractive one, where the particle can cross the billiard's boundary entering a region with a harmonic potential.

In both cases we prove the presence of a symbolic dynamics, at least for high energies, under some assumptions on the boundary of the billiard table and the position of the Keplerian mass inside it.

Coautors: Stefano Baranzini, Irene De Blasi, Anna Maria Cherubini and Susanna Terracini

Geometric and mass constraints for some central configurations of the planar 5-body problem

Kuo-Chang Chen (National Tsing Hua University)

In this talk we will demonstrate some nontrivial criteria for central configurations of the planar 5-body problem. These criteria are combinations of geometric and mass constraints. We will focus on convex configurations, and concave rhomboidal configurations. For rhomboidal configurations, for any ordering of mutual distances, we either have existence and uniqueness, non-existence, or some necessary conditions on masses.

Maslov-type index theory and closed orbits

Huagui Duan (Nankai University)

In this talk, I will introduce two kinds of closed orbit problems, i.e., closed orbits on hypersurfaces with the fixed energy and closed geodesics on manifolds. Then I will introduce some recent progress in this field, and explain how to deal with these problems by using Maslov-type index theory.

Hamiltonian delay equations

Urs Frauenfelder (Augsburg University)

Different from an ODE a delay equation also depends on the past of the trajectory. In the talk I start historically with a variational approach of Carl Neumann taking into account retardation of the Coulomb potential and explain how this leads to Weber's electrodynamics. I than discuss how a recent new regularization technique due to Barutello, Ortega, and Verzini leads to delay equations and how in a joint work with Cieliebak and Volkov we applied this to the problem of existence of frozen planet orbits in the Helium atom. I finally plan to address the question how Hamiltonian delay equations could help to find suitable positions for a gateway to the moon.

References: https://arxiv.org/pdf/2103.15485
https://arxiv.org/pdf/1902.09612

TBD

Xijun Hu (Shandong University)

Proof of Hofer-Wysocki-Zehnder's two or infinity conjecture

Hui Liu (Wuhan University)

In this talk, we give a proof of the conjecture of Hofer-Wysocki-Zehnder published in 2003 asserting that a smooth and autonomous Hamiltonian flow on ${\bf R}^4$ has either two or infinitely many simple periodic orbits on any regular compact connected energy level that is transverse to the radial vector field. As its application, we also prove a longstanding conjecture of Bangert-Long that every irreversible Finsler metric on S^2 has either two or infinitely many prime closed geodesics. This talk is based on my recent joint work with Cristofaro-Gardiner, Hryniewicz and Hutchings.

Two birds, one stone

Ezequiel Maderna (Universidad de la Repúblicay)

Given three positive masses, the hyperbolic Lagrange homographic motion has two equilateral triangles T^+ and T^- as expansion limit shapes, for the future and for the past respectively. The constant introduced by Gascheau in 1843 \[\mu_G= \frac{(m_1+m_2+m_3)^2}{m_1m_2+m_2m_3+m_3m_1} \] measures the disparity of the masses and discriminates the cases in which the circular homographic orbit is linearly stable: a necessary and sufficient condition is that $\mu_G>27$. Note that for three equal masses $\mu_G=3$. We will show the following: if $\mu_G<27/8$ then \begin{itemize} \item The scattering problem -- see the Richard's new book [Mo24] -- has a local solution around any

Lagrange hyperbolic homographic motion. That is, given two configurations S^- and S^+ with the same size, and respectively close to T^- and T^+, there is an entire hyperbolic motion with these limit shapes for the past and the future.
The homographic elliptic Lagrange motions are linearly unstable, in particular orbital unstable, independently of the eccentricity of the orbits.
Joint work with Renato Iturriaga (CIMAT, México).

[Mo24] Montgomery, RFour Open questions for the N-body problem, Cambridge University Press (2024).

Parabolic saddles and Newhouse domains in Celestial Mechanics

Pau Martian (Universitat Politècnica de Catalunya)

In the 70s McGehee introduced a compactification of the phase space of the restricted 3-body problem by gluing a manifold of periodic orbits "at infinity". Although from the dynamical point of view these periodic orbits are parabolic (the linearization of the Poincaré map is the identity matrix), one of them, denoted here by O, possesses stable and unstable manifolds which, moreover, separate the regions of bounded and unbounded motion.
This observation prompted the investigation of the homoclinic picture associated to O, starting with the work of Alekseev and Moser. We continue this research and extend, to this degenerate setting, some classical results in the theory of homoclinic bifurcations. More concretely, we prove that there exist Newhouse domains N in parameter space (the ratio of masses of the bodies) and residual subsets R⊂N for which the homoclinic class of O has maximal Hausdorff dimension and is accumulated by generic elliptic periodic orbits.
One of the main consequences of our work is the fact that, for a (locally) topologically large set of parameters of the restricted 3-body problem the union of its elliptic islands forms an unbounded subset of the phase space and, moreover, the closure of the set of generic elliptic periodic orbits contains hyperbolic sets with Hausdorff dimension arbitrarily close to maximal. Other instances of the restricted n-body problem such as the Sitnikov problem and the case n=4 are also considered.

This a joint work with M. Garrido and J. Paradela

The corresponding preprint can be found in https://arxiv.org/abs/2411.02761, although it has already been accepted for publication in CIMP.

Partial results about partially rigid motions of the n-body problem

Richard Moeckel (University of Minnesota)

A solution of the n-body problem in dimension d is a relative equilibrium if all of the

mutual distance between the bodies are constant. In other words, the bodies undergo a rigid motion. One can ask about the possibility of partially rigid motions, where some but not all of the distances are constant. In particular, a hinged solution is one such that exactly one mutual distance varies. In the preprint found at arXiv:2407.17812 [math.DS], I show that hinged solutions don't exist when n=3 or n=4. For n=3 this means that if 2 of the 3 distances are constant so is the third and for n=4, if 5 of the 6 distances are constant, so is the sixth. These results hold independent of the dimension d of the ambient space. Conjecturally, partially rigid motions of any type are impossible, that is, assuming even one fixed distance should imply that the motion is a relative equilibrium. My talk will also discuss some recent progress about this question for the three-body problem.

An application of non-quasi-periodic normal form theory to celestial mechanics

Gabriella Pinzari (University of Padova)

Normal form theory is a tool in hamiltonian mechanics, going back to N. N.Nekhorossev, that has been refined and clarified by many authors. In this talk we shall show that an extension to it to damped oscillations can be successfully applied to a spin-orbit model with friction. As a result, we shall prove that certain dumped oscillations do persist in the model for exponentially-long times. The physical menaning of this result allows to provide an interpretation of resonance trapping through the occurrence of friction. This is joint work with B. Scoppola and M. Veglianti.

References: G. Pinzari, B. Scoppola, M. Veglianti, Two-Layer model via nonquasiperiodic Normal Form Theory. arXiv: 2505.03436, https://arxiv.org/abs/2505.03436

On the Morse Index in Singular Variational Problems

Alessandro Portaluri (University of Turin)

This talk explores how the intersection theory of curves of Lagrangian subspaces in a symplectic space serves as a powerful and flexible tool for studying the Morse index in singular variational problems.

We begin by revisiting the classical interpretation of the Morse index as the number of negative eigenvalues of a Sturm–Liouville boundary value problem. This viewpoint can be reframed in terms of intersection numbers between Lagrangian curves and hypersurfaces determined by boundary conditions, all within the Gelfand–Robbin quotient framework.

We then present a general spectral flow formula expressed as the Maslov index of a path of Cauchy data spaces. Building on this, we extend the theory to cover one- and two-sided singular Sturm–Liouville operators by employing symplectic reduction techniques. This

generalization captures and unifies existing abstract results underlying the index theory for asymptotic and doubly asymptotic motions—such as collisions or parabolic trajectories—in celestial mechanics.

Our results synthesize analytic and symplectic approaches, providing robust tools for computing spectral flows in singular variational settings. Applications span a range of fields, including celestial mechanics (e.g., perturbations in planetary systems), time-dependent oscillatory systems, wave propagation in inhomogeneous media, and quantum mechanics—particularly for Schrödinger operators with periodic or singular potentials.

Finite energy foliations for classical Hamiltonian systems

Pedro Salomao (Southern University of Science and Technology)

In this talk, I will explain the construction and use of finite energy foliations to study the dynamics of energy surfaces of Hamiltonian systems in dimension 4. I will mostly focus on mechanical Hamiltonians such as the spatial isosceles three-body problem and the Henon-Heiles potential for energies slightly above the critical value. I will also discuss how these foliations can be used to find homoclinic orbits to the Lyapunov orbit near the first Lagrange value in the circular planar restricted three-body problem.

Formal normalization and formal invariant foliation for an elliptic fixed point in the plane

David Sauzin (Paris Observatory/Capital Normal University)

Classically, for a local analytic diffeomorphism F of $(R^2,0)$ with a non-resonant elliptic fixed point (eigenvalues $\exp(\pm 2\pi i\omega)$ with ω real irrational), one can find formal normalizations, i.e. formal conjugacies to a formal diffeomorphism invariant under the group of rotations.

Less demanding is the notion of a "geometric normalization" that we introduce: this is a formal conjugacy to a formal diffeomorphism which maps any circle centered at 0 to a circle centered at 0. Geometric normalizations are not unique, but they correspond in a natural way to a unique formal invariant foliation (any formal leaf is mapped to a formal leaf by F).

We show that, generically, all geometric normalizations are divergent, so there is no analytic invariant foliation. This is a sequel—or rather a prequel—to [A. Chenciner, D. Sauzin, S. Sun & Q. Wei: Elliptic fixed points with an invariant foliation: Some facts and more questions, RCD 2022, Vol. 27], which shows that, in the exceptional situation where F leaves invariant an analytic foliation, formal normalizations are still generically divergent.

(joint work with Alain Chenciner (Paris), Shanzhong Sun and Qiaoling Wei (CNU))

A DEGENERATE ARNOLD DIFFUSION MECHANISM IN THE RESTRICTED 3-BODY PROBLEM

Tere M-Seara (Universitat Politècnica de Catalunya)

A major question in dynamical systems is to understand the mechanisms driving global instability in the 3-body problem (3BP), which models the motion of three bodies under Newtonian gravitational interaction. The 3BP is called restricted if one of the bodies has zero mass and the other two, the primaries,have strictly positive masses. We consider the Restricted Planar Elliptic 3-body problem (RPE3BP)where the primaries revolve in Keplerian ellipses and we prove that the RPE3BP exhibits topological instability:for any values of the masses, if they are different, we build orbits along which the angular momentumof the massless body experiences an arbitrarily large variation provided the eccentricity of the orbit of theprimaries is positive but small enough.In order to prove this result we show that a degenerate Arnold diffusion mechanism, which moreover involves exponentially small phenomena, takes place in the RPE3BP, improving a previous result Delshams, Kaloshin, de la Rosa, and Seara (2019) where an exponentially small ratio between the two masses was required.

This is a joint work with M. Guardia and J. Paradela.

Variational construction of heteroclinic orbits in the planar Sitnikov problem

Mitsuru Shibayama (Kyoto University)

Using the variational method, Chenciner and Montgomery (2000) proved the existence of a figure-eight orbit in the planar three-body problem with equal masses. Since then, a number of solutions to the N-body problem have been discovered. The Sitnikov problem is a special case of the three-body problem. This system is known to be chaotic and has been studied using symbolic dynamics (J. Moser, 1973). In this talk, we study the limiting case of the Sitnikov problem. Using the variational method, we show the existence of various kinds of solutions in the planar Sitnikov problem. For a given symbolic sequence, we demonstrate the existence of orbits realizing it. We also prove the existence of periodic orbits and heteroclinic orbits connecting them.

This is joint work with Yuika Kajihara and Guowei Yu.

Expanding solutions to the N-body problem

Susanna Terracini (University of Turin)

We deal, for the classical N-body problem, with the existence of action minimizing half entire expansive solutions with prescribed asymptotic direction and initial configuration of the bodies. We tackle the cases of hyperbolic, hyperbolic-parabolic and parabolic arcs in a unified manner. Our approach is based on the minimization of a renormalized Lagrangian action, on a suitable functional space. With this new strategy, we are able to confirm the already-known results of the existence of both hyperbolic and parabolic solutions, and we prove for the first time the existence of hyperbolic-parabolic solutions for any prescribed asymptotic expansion in a suitable class. Associated with each element of this class we find a viscosity solution of the Hamilton-Jacobi equation as a linear correction of the value function. Besides, we also manage to give a precise description of the growth of parabolic and hyperbolic-parabolic solutions.

These are joint works with Davide Polimeni and Diego Berti.

TBD

Otto van Koert (Soeul National University)

TBD

Andrea Venturelli (University of Avignon)

The Positive Fundamental Group of Sp(2n)

Jian Wang (Nankai University)

In this talk, we examine the homotopy classes of positive loops in Sp(2n). We show that two positive loops are homotopic if and only if they are homotopic through positive loops. This provides a positive answer to a conjecture raised by McDuff. As a consequence, we extend several results of McDuff and Chance to higher-dimensional symplectic manifolds, without any dimension restrictions. This is joint work with Qinglong Zhou.

Reference: http://arxiv.org/pdf/2405.07398

TBD

Zhihong Xia (Northwestern University/Great Bay University)

Implicit Function Theorem, Interval Computation, and the Uniqueness of Convex Four-Body Central Configurations

Zhifu Xie (The University of Southern Mississippi)

In this talk, we introduce an extension of the implicit function theorem that provides an estimate for the size of the domain where the implicit function exists. We then discuss interval computation, the Krawczyk operator, and their applications to central configurations in the N-body problem. A long-standing conjecture proposes that for any four positive masses arranged in a fixed order, there exists a unique convex central configuration. Many previous studies have addressed this conjecture by assuming some equal masses or by imposing geometric constraints such as trapezoidal or co-circular shapes. In this presentation, we review how the Krawczyk operator and the implicit function theorem can be combined to prove the uniqueness of a convex central configuration within a neighborhood of a given mass point. We will also discuss the challenges posed by small masses near zero.

This is a joint work with Shanzhong Sun and Peng You.

Reference https://arxiv.org/abs/2303.00201

On Periodic Orbits of the Planar N-body Problem

Xiang Yu (Tianjin University)

In this talk, we will discuss that there are abundant new periodic orbits near relative equilibria of the planar N-body problem. All of these periodic orbits lie on a $2d$-dimensional central manifold of the planar N-body problem, and generically the relative measure of the closure of the set of periodic orbits near relative equilibria on the central manifold is close to 1.

A common first integral from three-body secular theory and Kepler billiards

Lei Zhao (Augsburg University/Dalian University of Technology)

The study of G. Pinzari on the integrability of partially-averaged three-body problem and the study of G. Gallavotti-I. Jauslin and following works on Kepler billiards leads to the same first integral appearing as a linear combination of squared angular momentum and a component of the Laplace(-Runge-Lenz) vector. In this talk I shall explain this coincidence from the projective dynamics of the two-center problem and discuss some direct consequences. Joint work with G. Pinzari.

Reference: arXiv: 2504.17645