Accurate College of Engineering & Technology, Greater Noida

B. Tech. I Year Even Sem. 2023-24
Assignment - III
Mathematics - II (BAS203)

(2 Marks Questions for Section-A)

Q.1 What are the applications Fourier series?

Q.2Find the Fourier coefficienta_0 for $f(x) = x\cos x \quad , -\pi \le x \le \pi$.

Q.3Find the Fourier coefficient for the function $f(x) = x^2 \quad ,0 < x < 2\pi$

Q.4Find value ofa_0 in Fourier series expansion of $f(x) = x\sin x$, $0 < x < 2\pi$.

Q.5Find the Fourier coefficient a_1 for $f(x) = x^2 \quad , -\pi \le x \le \pi$.

Q.6Find the value of Fourier coefficient a_0 for the function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$.

Q.7Find the Fourier coefficient a_n for $f(x) = x\cos x$in the interval $(-\pi , \pi)$.

Q.8 If $f(x) = 1$, $0 < x < \pi$ is expanded in half range sine series then find the value of b_n.

Q.9Find the constant term when is $f(x) = 1 + |x|$ is expanded in Fourier series in the interval $(-3 ,3)$.

Q.10 Find the constant term if $f(x) = x + x^2$ expanded in Fourier series defined in $(-1 ,1)$.

Q.11 Find the constant term when is $f(x) = |x|$ expanded in Fourier series in the interval $(-2 ,2)$.

Q.12 Expand $f(x) = x$ as a half-range Sine series in $0 < x < 2$.

Q.13 Find the value of a_0 in the expansion of Fourier series of $f(x) = x^2 - 2$ when $-2 \le x \le 2$.

(7 Marks Questions for Section-B&C)

Q.14 Find the Fourier series to represent $x - x^2$ from $x = -\pi \; to \; x = \pi$ .Hence show that

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{12}$.

Q.15 Obtain the Fourier series expansion of $f(x) = e^{ax} \quad ,0 < x < 2\pi$.

Q.16Obtain the Fourier series for the function $f(x) = x \; \sin x \quad ,0 < x < 2\pi$.

Q.17 If $f(x) = \left[\frac{\pi - x}{2}\right]^2 \quad ,0 < x < 2\pi$ then show that $f(x) = \frac{\pi^2}{12} - \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$ .Hence deduce that

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{6}$.

Q.18 Obtain the Fourier series of the function $f(t) = \begin{cases} t, -\pi < x < 0 \\ -t, \; 0 < x < \pi \end{cases}$. Hence, deduce that

$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{8}$.

Q.19 Find the Fourier series to represent the function $f(x)$ given by $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ (2\pi - x), & \pi \le x \le 2\pi \end{cases}$.

Q.20 Obtain Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 < x < \pi \end{cases}$.

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{8}$.

Q.21 Obtain the Fourier expansion of $f(x) = x \sin x$ $, -\pi \le x \le \pi$ as cosine series in and hence show that $\frac{1}{1\times3} - \frac{1}{3\times5} + \frac{1}{5\times7} - \frac{1}{7\times9} + \cdots \ldots \ldots \ldots \ldots, = \left(\frac{\pi - 2}{4}\right)$.

Q.22 Find the Fourier series of $f(x) = x \sin x$ $, -\pi \le x \le \pi$.

Q.23 Find the Fourier series expansion of the periodic function $f(x) = x \cos x$ $, -\pi < x < \pi$.

Q.24Obtain the Fourier series for the function $f(x) = x^2$ $, -\pi < x < \pi$.

Q.25 Find the Fourier series for the function $f(x) = x^3$ $, -\pi < x < \pi$.

Q.26 Find half range Fourier sine series for $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$

Q.27 Find the Fourier half range cosine series for the function $f(x) = \begin{cases} 0, & 0 < x < \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} < x < \pi \end{cases}$.

Q.28 A periodic function of period 4 is defined as $f(x) = |x|, -2 < x < 2$,find its Fourier series expansion.

. Q.29 Obtain the Fourier series expansion of $f(x) = \frac{\pi - x}{2}$ $, 0 < x < 2$.

Q.30Expand $f(x) = 2x - 1$ as a cosine series in $0 < x < 2$.

Q.31Obtain a half range cosine series for $f(x) = \begin{cases} k\,x, & 0 < x < \frac{l}{2} \\ k\,(l - x), & \frac{l}{2} < x < l \end{cases}$.

Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \ldots \ldots \ldots \ldots$

Q.32 Obtain a half range cosine series for the function $f(t) = \begin{cases} 2\,t, & 0 < x < 1 \\ 2\,(2 - t), & 1 < x < 2 \end{cases}$.

Q.33 Find half range sine series of $f(x) = \begin{cases} x, & 0 < x < 2 \\ (4 - x), & 2 < x < 4 \end{cases}$.

Q.34 Obtain Fourier series for $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$.

Q.35Find the half range cosine series for the function $f(x) = (x - 1)^2$ in the interval $(0, 1)$.

Hence, prove that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \ldots \ldots \ldots \ldots = \frac{\pi^2}{8}$

(2 Marks Questions for Section-A)

Q.1 Discuss the convergence of sequence $\{U_n\}$,where $U_n = \sin\frac{1}{n}$.

Q.2 Discuss the convergence of the sequence $\{a_n\}$,where $a_n = \frac{n+1}{n}$.

Q.3 Test the series $\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{1}{n}$.

Q.4 Discuss the convergence of sequence $(1, 2^1, 2^2, 2^3, 2^4 \ldots\ldots..)$.

Q.6 Discuss the convergence of sequence $a_n = \frac{2n}{n^2+1}$.

(7 Marks Questions for Section-B&C)

Q.7 State D Alembert's test. Test the series $1 + \frac{x^1}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \frac{x^4}{17} + \cdots \ldots\ldots.. + \frac{x^n}{n^2+1} + \cdots \ldots\ldots\ldots\ldots$

Q.8 Examine the series for convergence or divergence $1 + \frac{1!}{2}x + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \frac{4!}{5^4}x^4 + \cdots \ldots\ldots\ldots$

Q.9 Test the series: $\frac{x^1}{1\,.\,2} + \frac{x^2}{3\,.\,4} + \frac{x^3}{5\,.\,6} + \frac{x^4}{7\,.\,8} + \cdots \ldots\ldots\ldots$

Q.10 Test for convergence of the series: $\frac{(a+x)^1}{1!} + \frac{(a+2x)^2}{2!} + \frac{(a+3x)^3}{3!} + \frac{(a+4x)^4}{4!} + \cdots \ldots\ldots\ldots$

Q11 Discuss the convergence of series:$\sum \sqrt{n-1} - \sqrt{n}$.

Q.12 Interpret for convergence of series: $1 + \frac{1}{2}x + \frac{1\,.\,3}{2\,.\,4}x^2 + \frac{1\,.\,3\,.5}{2\,.\,4\,.6}x^3 + \frac{1\,.\,3\,.\,5\,.\,7}{2\,.\,4\,.\,6\,.\,8}x^4 + \cdots \ldots\ldots.., x > 0$

Q.13 Test for convergence of following series: $\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{x^2}{7.8.9} + \cdots \ldots..$, where x is a real number.

Q.14 Test the convergence of the series:$\frac{1}{1} + \frac{1\,.\,3}{1\,.\,4} + \frac{1\,.\,3\,.5}{1\,.\,4\,.7} + \frac{1\,.\,3\,.\,5\,.\,7}{1\,.\,4\,.\,7\,.\,10} + \cdots \infty$