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Model Predictive Control of Residential Demand in Low Voltage Network using Ice Storage

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***Abstract*—Smart buildings and communities with residential storage systems can shift their electricity demand from high-priced peak periods to low-priced off-peak periods. Most of the literature in this field focuses on chemical batteries or hot water tanks as storage devices. Although residential ice storage systems are becoming increasingly popular, there is little research on modeling and evaluating their performance on low voltage networks. In this paper, a model of an ice storage system is presented taking into account its thermodynamic and heat transfer constraints. The model is linearized to take advantage of fast linear program solvers. The linearized model is then used in the framework of model predictive control (MPC) to determine the optimum demand of the buildings to achieve minimum electricity cost. The results of MPC shows that ice storage systems can be effective in shifting the cooling demand to the off-peak periods and improve the voltage profile of the low voltage electricity grid.**

***Index Terms*—Joint electrical and thermodynamic model, model predictive control, demand response, thermostatically controlled loads, community energy.**

Nomenclature

ϵ_i	Heat transfer effectiveness of ice storage.
C_i	Thermal inertia of the building element i.
COP	Coefficient of performance.
h_{fg}	Latent heat of ice fusion.
m_{ice}	Mass of ice in the ice tank.
m_{tank}	Maximum mass of ice in the ice tank.
NTU	Number of Transfer Units method.
$P_{HVAC}^{h,k}$	HVAC demand of the building h at time k.
$P_{nonflex}^{h,k}$	Non-flexible demand of the building h at time k.
$P_i^{k,D}$	Active power consumption at node i at time k.
$P_i^{k,G}$	Active power generation at node i at time k.
p_t	Time-of-use signal.
$P_{L,i}$	Active power flow through lines from node i to $i+1$.
pf^h	Power factor for house h.
$Q_i^{k,D}$	Reactive power consumption at node i at time k.
$Q_i^{k,G}$	Reactive power generation at node i at time k.
$Q_{L,i}$	Reactive power flow through lines from node i to $i+1$.
R_i	Thermal resistance of the element i in the building.
$R_{L,i}$	Line resistance from node i to $i+1$.
t_{cha}	Amount of time to charge the ice tank.
t_{dis}	Amount of time to discharge the ice tank.
T_i^k	Temperature of the building element i at time k.
T_{max}^k	Maximum temperature for the building at time k.
T_{min}^k	Minimum temperature for the building at time k.
T_{tank}	Temperature of the ice storage tank.
u_{tank}	Energy stored in the ice tank.
V_i	Voltage magnitude at node i.
$X_{L,i}$	Line reactance from node i to $i+1$.
U_{Cool}^{max}	Maximum cooling of the HVAC system.
U_{Heat}^{max}	Maximum heating of the HVAC system.
u_{ch}	Rate of charging of the ice tank.
u_{dch}	Rate of discharging of the ice tank.
U_{HVAC}	HVAC thermal demand.
u_{loss}	Rate of energy loss from the ice tank.

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I. Introduction

The increasing number of smart appliances and residential storage devices enables smart buildings and communities to shift some of their flexible loads such as heating, ventilation, and air-conditioning (HVAC) from peak periods, when the price of electricity is high, to off-peak periods, when the price of electricity is low [1]. HVAC demand is the main cause for peak electricity demand in the residential sector in many countries, such as Australia [2].

Conventional building energy management mostly relies on feedback and rule-based control, where an error signal i.e. difference between the desired and the actual output, is used along with a set of heuristic rules to decide on cooling/heating or charging/discharging actions. Model predictive control (MPC) uses a model of the system along with a forecast of the input parameters to determine an optimal set of actions while satisfying the constraints of the system [3]. MPC can be used to minimize the building electricity cost when time-of-use electricity prices are implemented. The building MPC charges the building storage devices during the off-peak times, when the cost of electricity is low, and discharge them in peak periods, when the price of electricity is high [4]. This shift of demand from peak times to off-peak times may have an effect on the low voltage network. However, the effect of ice storage, as the building energy storage system, on low voltage network is not well-studied in the literature.

Rismanchi et al. [5] use heuristic controls to model an ice storage device for an office building in Malaysia. Their results show the ice storage can reduce the size of HVAC system and the air handling pumps and overall reduce the energy demand by about 4%. Drees and Braun [6] developed a non-linear MPC model of a building with ice storage and show 6-20% reduction in monthly electricity cost is achievable. Candanedo et al. [7] compares the cost of electricity of a single office building using MPC model and a heuristic control model that uses chiller or ice storage

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according to some simple control rules. The results show that MPC can save between 5-30% depending on the heuristic control rules.

In this paper, the effect of the ice storage systems on the low voltage network is studied by using a linearized joint electrical-thermodynamic model of low voltage network, ice storage systems and residential buildings. The main contributions of this paper are:

- Linearizing the performance of ice storage systems, which are used in the residential buildings in a low voltage network.
- Developing and implementing an MPC framework to optimize the operation of a smart community equipped with ice storage systems.

The rest of the paper is organized as follows. In Section II, the linearized models of the low voltage network, ice storage and building models are presented. In Section III, the MPC model is presented. The case study is introduced in Section IV, followed by the results in Section V. The paper concludes with Section VI.

II. Joint Electrical and Thermodynamic Model

In this section, the joint electrical-thermodynamic model of low voltage network, ice storage and buildings is presented.

1) ***Low Voltage Network :*** The complex power flow at each node i can be described by the DistFlow equations [8]:

$$P^k_{L,i+1} = P^k_{L,i} - R_{L,i}\frac{{P^k_{L,i}}^2 + {Q^k_{L,i}}^2}{{V^k_i}^2} - P^k_{i+1} \tag{1}$$

$$Q^k_{L,i+1} = Q^k_{L,i} - X_{L,i}\frac{{P^k_{L,i}}^2 + {Q^k_{L,i}}^2}{{V^k_i}^2} - Q^k_{i+1} \tag{2}$$

$$\begin{aligned} {V^k_{i+1}}^2 &= {V^k_i}^2 - 2(R_{L,i}P^k_i + X_{L,i}Q^k_i) + ... \\ &(R^2_{L,i} + X^2_{L,i})\frac{{P^k_{L,i}}^2 + {Q^k_{L,i}}^2}{{V^k_i}^2} \end{aligned} \tag{3}$$

$$P^k_i = P^{k,D}_i - P^{k,G}_i, \quad Q^k_i = Q^{k,D}_i - Q^{k,G}_i \tag{4}$$

To obtain the linear DistFlow equations, the non-linear terms representing losses in the network are assumed to be much smaller than the branch power terms P_i and Q_i and the voltage at node i remains in within 10% of the nominal voltage [9]. This simplification is justified since in practice, the power lines in distribution network are usually over sized. Therefore the linearized DistFlow equations are:

$$P^k_{L,i+1} = P^k_{L,i} - P^k_{i+1} \tag{5}$$

$$Q^k_{L,i+1} = Q^k_{L,i} - Q^k_{i+1} \tag{6}$$

$$V^k_{i+1} = V^k_i - \frac{R_{L,i}P^k_{L,i} + X_{L,i}Q^k_{L,i}}{V_0} \tag{7}$$

along with (4). Furthermore, for a residential network, where loads have power factor is close to unity, DC approximation can be used by only considering active power.

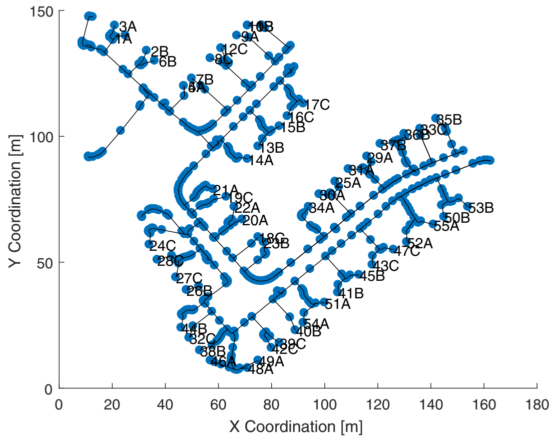


Fig. 1: The comparison between linear approximation and the non-linear LVN models.

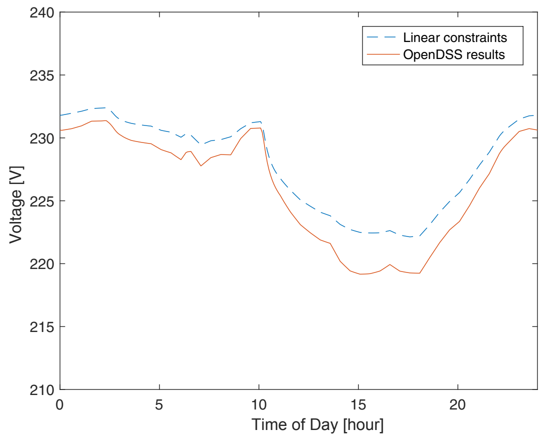


Fig. 2: The comparison between linear approximation and the non-linear LVN models for the base case without ice storage.

These simplifications are justified by comparing the linear model with a non-linear LVN model simulated with OpenDSS software. An IEEE radial LVN, presented in Figure 1, is selected. The results in Figure 2 shows the voltage from the linear model is within 3% of the actual voltage value obtained from OpenDSS software.

2) ***Ice Storage:*** In general case, the mass and energy balance of an ice storage system is [10]:

$$\begin{aligned} \alpha(m_{tank} - m_{ice})c_w\frac{dT_{tank}}{dt} + \beta(m_{ice})c_w\frac{dT_{tank}}{dt} + ... \\ h_{fg}\frac{dm_{ice}}{dt} = u_{loss} + u_{ch} - u_{dch} \end{aligned} \tag{8}$$

with the following conditions:

$$\begin{aligned} m_{ice} = m_{tank}, \alpha = 1, \beta = 1 \quad & T_{tank} < 0°C \\ m_{ice} = 0, \alpha = 1, \beta = 1 \quad & T_{tank} > 0°C \\ \alpha = 0, \beta = 0 \quad & T_{tank} = 0°C \end{aligned} \tag{9}$$

We can simplify (8) by assuming, the heat loss is small ($u_{loss} = 0$), full charge is when all the water is turned into

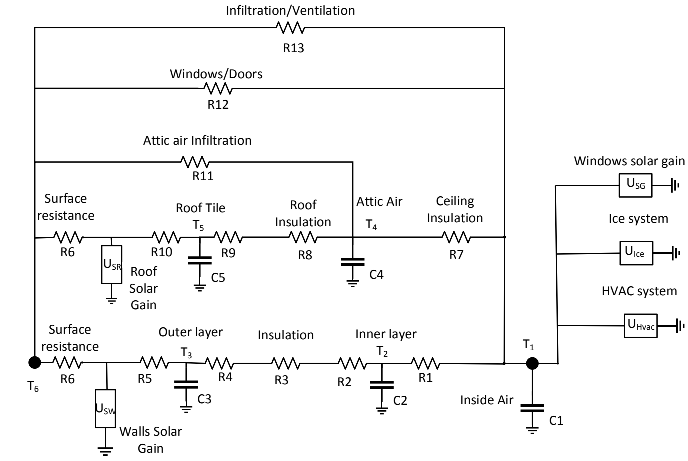


Fig. 3: The schematic of the building thermodynamic model with the HVAC and the ice storage systems.

ice at zero degree, and full discharge is when all the ice is turned into water at zero degree. Hence, (8) becomes:

$$h_{fg}\frac{dm_{ice}}{dt} = u_{ch} - u_{dch} \tag{10}$$

Furthermore, (10) is discretized to a set of difference equations [11]:

$$m_{ice}^{k+1} = m_{ice}^{k} + \frac{\Delta t(u_{ch}^{k} - u_{dch}^{k})}{h_{fg}} \tag{11}$$

The charging and discharging rate of the ice storage system is [12]:

$$-\epsilon_i^k \frac{m_{tank}\cdot h_{fg}}{t_{dis}} \leq u_i^k \leq \epsilon_i^k \frac{m_{tank}\cdot h_{fg}}{t_{cha}} \tag{12}$$

where $i \in \{charge, discharge\}$. The ϵ_i^k term is non-linear in nature as presented in $\epsilon_i^k = 1 - exp(-NTU^k) = 1 - exp(1/(R_{ice,t}^k \dot{m}^k c_h))$, where $R_{ice,t}^k$ is the thermal resistance of the ice around the coils which changes over time. The ϵ_i is linear when the state of charge is kept between 10-90% in which $\epsilon_i^k \approx 1$ [6].

3) ***Building****:* The thermodynamic model of a building is developed using the first law of thermodynamics and the heat transfer equations. For a single node in Figure 3, the energy stored in the node is equal to the input energy minus output energy:

$$C_1\frac{dT_1^k}{dt} = \frac{T_2^k - T_1^k}{R_1} + \frac{T_4^k - T_1^k}{R_7} + \frac{T_6^k - T_1^k}{R_{eq}} + U_1^k \tag{13}$$

where U_1 is the external effects on the node one, i.e. HVAC, ice, and window solar gain and $R_{eq} = \frac{R_{12}\cdot R_{13}}{R_{12}+R_{13}}$ is the equivalent thermal resistance between T_1 and T_6. To present the equation in a form of a set of discretized equations that

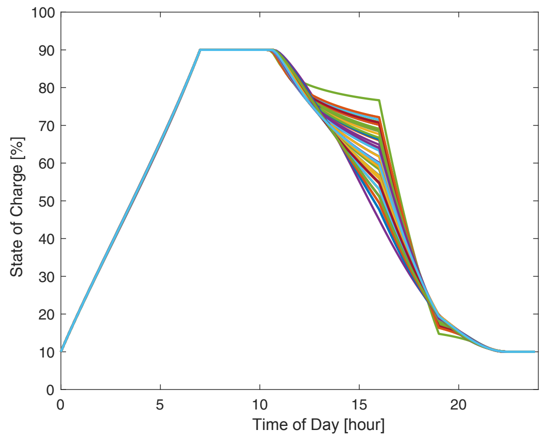


Fig. 4: State of charge of the ice storage system for all the 55 houses in the LVN.

are solved iteratively, the equation is rearranged to obtain T_1^{k+1}:

$$\begin{aligned} T_1^{k+1} =& (-\frac{\Delta t}{C_1\cdot R_1} - \frac{\Delta t}{C_1\cdot R_7} - \frac{\Delta t}{C_1\cdot R_{eq}} + 1)T_1^k + ... \\ & (\frac{\Delta t}{C_1\cdot R_1})T_2^k + (\frac{\Delta t}{C_1\cdot R_7})T_4^k + ... \\ & (\frac{\Delta t}{C_1\cdot R_{eq}})T_6^k + (\frac{\Delta t}{C_1})U_1^k \end{aligned} \tag{14}$$

For a more complex building model with several temperature nodes, the thermodynamic behavior in (14) for each of the building nodes are stacked to provide a state-space representation of the building model [13]:

$$T^{k+1} = A\cdot T^k + B\cdot D^k \tag{15}$$

where, T is the column vector of the temperatures $[T1; T2; T3; T4; T5]$ and D is the column vector of external weather inputs $[U_{HVAC}; U_{ICE}; U_{SW}; U_{SG}; U_{SR}; T_6]$. The heating and cooling can be within the rating power of the HVAC system of $-U_{Cool}^{max} \leq U_{HVAC}^{k,h} \leq U_{Heat}^{max}$. The electric active and reactive power of the HVAC system is calculated from:

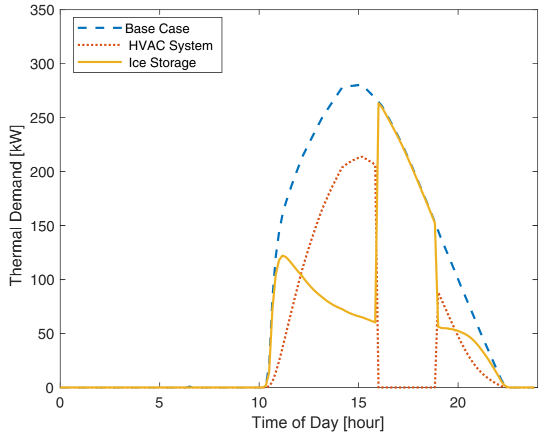
$$\begin{aligned} P_{HVAC}^{k,h} =& \frac{U_{HVAC}^{k,h}}{COP_{HVAC}} + \frac{u_{Ice,ch}^{k,h}}{COP_{Ice,ch}} + \frac{u_{Ice,dch}^{k,h}}{COP_{Ice,dch}} \\ Q_{HVAC}^{k,h} =& P_{HVAC}^{k,h}\sqrt{\frac{1}{(pf^h)^2} - 1} \end{aligned} \tag{16}$$

The occupied areas of the building needs to be within a temperature range to provide thermal comfort for the occupants:

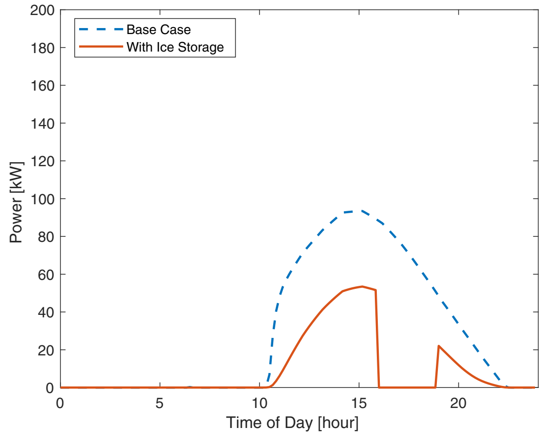
$$T_{h,min} \leq T_{1,h} \leq T_{h,max} \tag{17}$$

III. Model Predictive Control

The aim of the optimization is to minimize the electricity cost of the residential buildings over 24 hours by controlling the air conditioning and ice storage systems. We consider linear objective functions due to several nice properties of linear programming, such as fast convergence, convex



(a) Aggregated thermal demand for cooling supplies by Ice or HVAC systems



(b) Aggregated electricity demand required for cooling the buildings

Fig. 5: The thermal and electric demand used for cooling of the buildings in the LVN.

properties, and low computational demand. The objective function for this problem is :

$$\begin{aligned} \underset{P_{HVAC}^{k,h}}{\text{minimize}} \; & \sum_{k=0}^{K-1} p_t \cdot (P_{nonflex}^{k,h} + P_{HVAC}^{k,h}) \\ s.t. \quad & (4)-(7), \\ & (11)-(12), \\ & (15)-(17). \end{aligned} \tag{18}$$

where $h \in \{1, ...H\}$. Furthermore, a sliding horizon of 24 hours is used. At each time step, (18) is performed, then, the first action (charge, discharge) is implemented and the horizon is slide one time step in which again optimization is conducted. This process is repeated until the end of the first day (24 hours).

IV. Case Study

As a case study, we have considered a radial low voltage network with 55 residential buildings connected to a radial network [14]. The non-HVAC demand of the customers are estimated from the real smart meter data of 55 customers from 2015-2016 with 30-minutes resolution. To obtain the non-HVAC demand the smart meter data of the days that may require cooling (max temperature above 26°C) or heating (min temperature below 15°C) are discarded. The non-HVAC demand for each 30-minute is the simple average of the remaining demands in that time slot.

The HVAC demand is calculated using the state-space equations for the buildings with brick veneer walls with specifications presented in [15]. The ice tank capacity is 28 kWh of thermal energy and 7 hours charging and 2 hours discharging time. The price of electricity is 0.05 per kWh between midnight and 7am, 1.4 per kWh between 4pm and 7pm and 0.2 per kWh in other times.

V. Results

In this section the aggregated demand of the community and the voltage profile of the low voltage network are presented. The community is simulated in two cases: with and without ice storage. The community without ice storage, represents a typical community without any intelligent building energy management system in which the HVAC devices are turned on when the temperature is above the comfort limit and turned off when it is within the comfort limit. The community with ice storage also takes

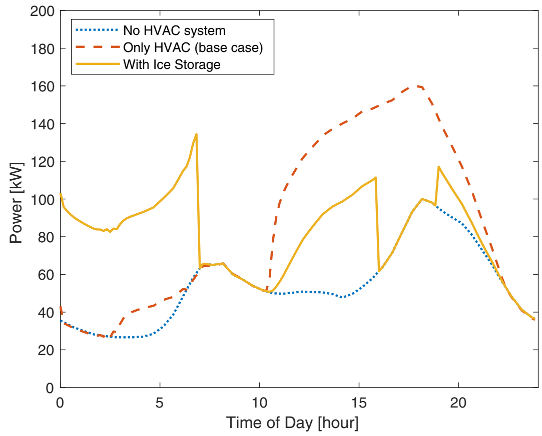


Fig. 6: Comparison of total electricity demand of the radial network with different cooling methods.

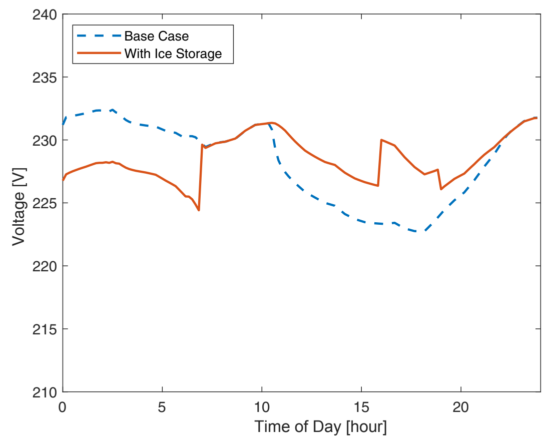


Fig. 7: The minimum voltage in the radial network with and without ice storage.

advantage of MPC frameworks which dictates the charging and discharging behavior of the ice storage systems.

The state of charge for the ice storage systems are presented in Figure 4. The ice storage devices are charged between 10% and 90% of their nominal capacity. This is to ensure that the assumptions for the linear model are valid. The charging period is during the off-peak period where the price of electricity is cheap. The ice storage systems start discharging only when there is cooling demand. During the peak period the ice storage is discharged at a rate that enabled the HVAC systems to be turned off.

The thermal demand of the HVAC systems and the ice storage systems for the community with ice storage are presented in Figure 5(a). The electricity demand required for providing the cooling thermal demand is presented in Figure 5(b), in which the electricity demand for ice discharge is negligible compared to the electricity demand of the HVAC system. The figure shows during the peak period, ice storage systems are supplying the entire thermal demand of the community. As the ice storage capacity is larger than the aggregated demand during the peak period, some of the ice is used to minimize the operation of HVAC systems during the off-peak periods.

The aggregated demand of the customers on the LVN is presented in Figure 6. The demand of the community without the ice storage presents a typical demand profile with a small peak in the morning, around 8am, and a larger peak (160 kW) in the afternoon, around 5pm. The ice storage systems, in the community with such storage, are charged over night, when the cost of electricity is low. This leads to an increase of the demand from 50 kW to about 130 kW. The ice storage systems are then discharged during the high-priced periods in the afternoon, reducing the demand at peak hour from 160 kW to 80 kW.

The effect of the ice storage systems on the voltage profile of the network is presented in Figure 7. This figure presents the minimum voltage in the network. The ice storage systems reduce the voltage at night time to $225V$ compared to the community without ice storage ($230V$). However, the ice storage systems have a positive effect on the voltage during the day, where, the voltage is improved by about $8V$ during the peak demand.

VI. Conclusion

This paper investigated the effects of ice storage systems on the low voltage network using an MPC framework. The MPC uses a linear joint electrical-thermodynamic model to minimize the cost of the electricity for the community by optimum operation of the HVAC systems and ice storage systems under full information. The general model of low voltage network, building thermodynamic model, and ice storage system are presented. Then, these models are linearized to take advantage of fast convergence rate and other convex properties of linear programming. The results show ice storage operation using MPC improves the voltage deviation from the nominal voltage by about 60% during the peak electricity demand. The cooling demand is successfully shifted to the off-peak periods at night time which results in demand reduction of about 50% during the peak electricity time. In the future, the performance of residential ice storage systems will be compared with residential chemical battery systems. In particular, the effect of different wall constructions will be modeled.

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