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Senior Thesis FRAB (FIBO Robotics and Automation: Bachelor)

2-WHEELS SELF-BALANCING ROBOT

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บทคัดย่อ

This project simulates the kinematics and dynamics of a two-wheels self-balancing robot, using dynamics and PID to maintain balance and reach the set target position. The system uses robot dynamics to feed forward gravitational term, and PID to creating additional torque to maintain equilibrium for others term. Using position and velocity PID controllers generate torque to drive the robot to the target.

คำสำคัญ: 2-wheels robot, Self-balancing, PID Control, Inverted pendulum

1. บทนำ (Introduction)

1.1 จุดประสงค์โครงการ

1. To simulate 2-wheels robot self – balancing.
2. To control a 2-wheels robot to target position in x-y plane.
3. To control a linear velocity of 2-wheel robot.
4. To control a rotation velocity around z-axis of 2-wheel robot.
5. To learn dynamic of 2-wheels self – balancing robot.

1.2 ขอบเขต

1) Kinematic of Robot
   - *a.* Dynamics in 2-wheels self – balancing robot.

2) Control Part
   - *a.* Self-balancing control using feed forward to counter gravitational term and Using PID in other terms.
   - *b.* Using PID to control position of robot in X-Y Plane.
   - *c.* Using PID to control linear velocity in X-axis of 2-wheel robot.
   - *d.* Using PID to control Rotation velocity around z-Axis of 2-wheel robot.

3) Constrain of Robot
   - *a.* Robot must have only 2 joint (2 Wheel).

b. Input Velocities and Input Position will be separate.

c. The robot should move forward, backward, and turn effectively.

d. The robot has no movement normal to the wheel plane.

e. The robot wheel movement is rolling without slipping.

f. The robot will conduct on flat, not slanted floor and even surfaces only.

g. Target result of 2-wheels robot have error +-5%. (velocity and position)

h. No external force impact to our robot.

4) Input to calculation

a. Position target in X-Y Plane in range -3 to 3

b. Linear speed in range -1 to 1

c. Rotation speed in range -0.5 to 0.5

5) Tools & Simulation

a. Using Python.

b. Using PyBullet Library to simulation.

c. Using Pybullet Library Physics Engine to calculate physics in simulation.

2. Literature Review

2.1 Kinematics of 2-wheels self-balancing robot

2.1.1 Dynamics Model of Two wheeled inverted pendulum

The dynamics equation of two wheeled inverted pendulum creates for simulation how robot move when conduct on flat, not slanted floor and even surfaces when generalize coordinate is

$$q = [x \;\; \theta \; \psi]^T \tag{1}$$

When in each term is

x = Linear Position

θ = Pendulum Position

ψ = Yaw position (around Z-axis)

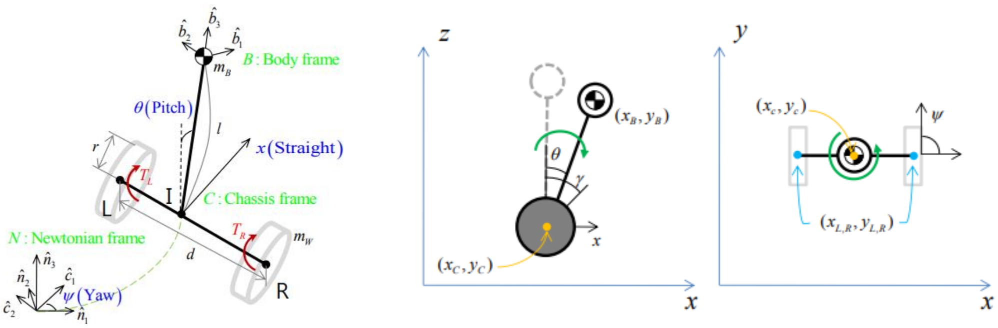

Figure [1] Configuration coordinate of 2-wheels self-balancing robot

Then we apply Lagrangian method to find the dynamics equation of two wheeled inverted pendulum

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau \tag{2}$$

From the lagrange equation we get the torque left and right and can arrange it and rewrite in the exact dynamic equation form

$$M(q,\dot{q})\ddot{q} + V(q,\dot{q}) + G(q) = B(q)\tau \tag{3}$$

When in each term is

$M(q,\dot{q})$	is Term of Mass	=	$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}$
$V(q,\dot{q})$	is Term of Coriolis	=	$[v_1 \quad v_2 \quad v_3]^T$
$G(q)$	is Term of Gravitational	=	$[0 \quad G_2 \quad 0]^T$
$B(q)$	is Term of Frame	=	$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$
τ	is Term of Output Torque	=	$[T_L \quad T_R]^T$

And in each variables is

Variables in Mass term

m_{11} = $m_B + 2m_W + 2J/r$

m_{12} = $m_B l cos\theta$

m_{21} = $m_B l cos\theta$

m_{22} = $I_2 + m_B l^2$

m_{33} = $I_3 + 2K + \frac{m_w d^2}{2}$

Variable in Coriolis term

v_1 = $-m_b l sin\theta(\dot{\psi}^2 + \dot{\theta}^2)$

v_2 = $(I_3 - I_1 - m_B l^2) sin\theta cos\theta \psi$

v_3 = $\{2\dot{\theta}(I_1 - I_3) cos\theta + m_B l(\dot{x} + 2l\dot{\theta} cos\theta)\}\dot{\psi} sin\theta$

Variables in Gravitational term

G_2 = $-m_B l g sin\theta$

Variable in Damping term

b_{11} = $1/r$

b_{12} = $1/r$

b_{21} = -1

b_{22} = -1

b_{31} = $-d/(2r)$

b_{32} = $d/(2r)$

2.1.2 Kinematic model of motion

The differential kinematic model of the two-wheel robot can be described in the context of a differential drive. The differential drive mechanism allows the two wheels to rotate at independent speeds, enabling both linear and angular motion for the robot.

First, the robot's location relative to its environment is defined by establishing two reference frames: the global frame and the robot frame. The origin of the robot frame is located at the robot's center of mass, which is positioned equidistantly between the two wheels. The positive x-axis of the robot frame points is in the direction of the robot's front.

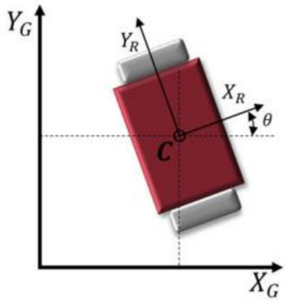

Figure [2]: Differential Drive Robot Reference Frame in a Global Reference Frame.

The robot's position and orientation are defined by three values: $x, y,$ and θ. The coordinates $x,$ and y indicate the robot's location, while θ represents its rotation in relation to the global frame. These three values are encapsulated in a vector as

$$\xi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{4}$$

The rotational matrix from robot frame to global frame is

$${}^R_G R(\theta) = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The robot's motion (velocity) in global frame can describe by derivative of position vector that equal to

$$\dot{\xi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \tag{6}$$

To understand the robot's movement, we looked at how wheels create motion. As the wheels turn, the point where each wheel touches the ground moves along the surface of the wheel. The distance this point travels matches the distance the center of the wheel moves, calculated by multiplying the wheel's radius by the angle it turned as the equation

$$x(t) = r\gamma(t) \tag{7}$$

For

r is the wheel radius

$\gamma(t)$ is the wheel's angular position

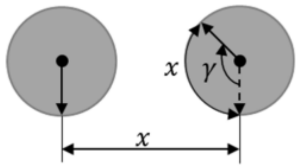

Figure [3]: Surface arc on the wheel as it turns is equal to the distance traveled by the wheel

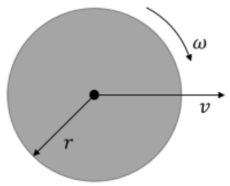

Figure [4]: Relationship between tangential velocity and angular momentum.

As represented in Figure [9], the tangential velocity of the center of rotation is equal to the radius of the wheel times the angular velocity of the wheel.

$$v = \dot{x} = r\dot{\gamma} = r\omega \tag{8}$$

Velocity of robot in X and Y axis can describe by, $\dot{x}$ is equal to the sum of the velocity of each wheel as experienced at the center and $\dot{y}$ is equal to zero since the robot's motion is constrained to prevent movement normal to the wheel.

$$\dot{x} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}r(\omega_1 + \omega_2) \tag{9}$$

$$\dot{y} = 0 \tag{10}$$

To verify the equation $\dot{x} = \frac{1}{2}r(\omega_1 + \omega_2)$ if robot wheel has same angular velocity in difference direction. The robot should spin around center of mass and $\dot{x}$ should be zero as equation

$$\dot{x} = \frac{1}{2}r(\omega - \omega) = 0 \tag{11}$$

To find the equation of $\dot{\theta}$ can describe by give $\omega_1 = 0$ the robot will rotate around the stationary wheel as in Figure [5].

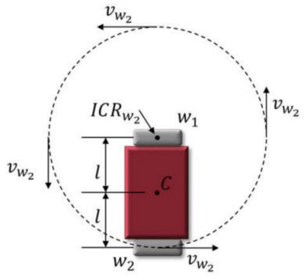

Figure [5]: Instantiation Center of Rotation (ICR) for wheel 2 when wheel 1 is held stationary.

The angular velocity of disk can describe by

$$\omega_{disk} = \dot{\gamma} = \frac{\dot{x}}{r_{disk}} = \frac{v_{disk}}{r_{disk}} \tag{12}$$

The tangential velocity of the center of the disk was the tangential velocity of ω_2 and for ω_2 that in counterclockwise direction of rotation given the equation

$$v_{disk} = v_{\omega_2} = (r\omega)_{\omega_2} \tag{13}$$

$$\omega_{disk} = -\frac{(r\omega)_{\omega_2}}{2l} \tag{14}$$

And in the same way but give $\omega_2 = 0$ will get equation

$$\omega_{disk} = \frac{(r\omega)_{\omega_1}}{2l} \tag{15}$$

The angular velocity of the robot $(\dot{\theta})$, combines contributions from each wheel's rotation as

$$\dot{\theta} = \frac{r}{2l}(\omega_1 - \omega_2) \tag{16}$$

In conclusion robot's motion can describe by

$$\dot{\xi}_R = \begin{bmatrix} \frac{1}{2} r(\omega_1 + \omega_2) \\ 0 \\ \frac{r}{2l}(\omega_1 - \omega_2) \end{bmatrix} \tag{17}$$

3. เนื้อหาในรายวิชาที่เกี่ยวข้อง

1) Transformation Of Coordinate Frame
2) Dynamics
3) Closed-loop Control
4) PID Control

4. System Diagram / System Overview (Function and Argument)

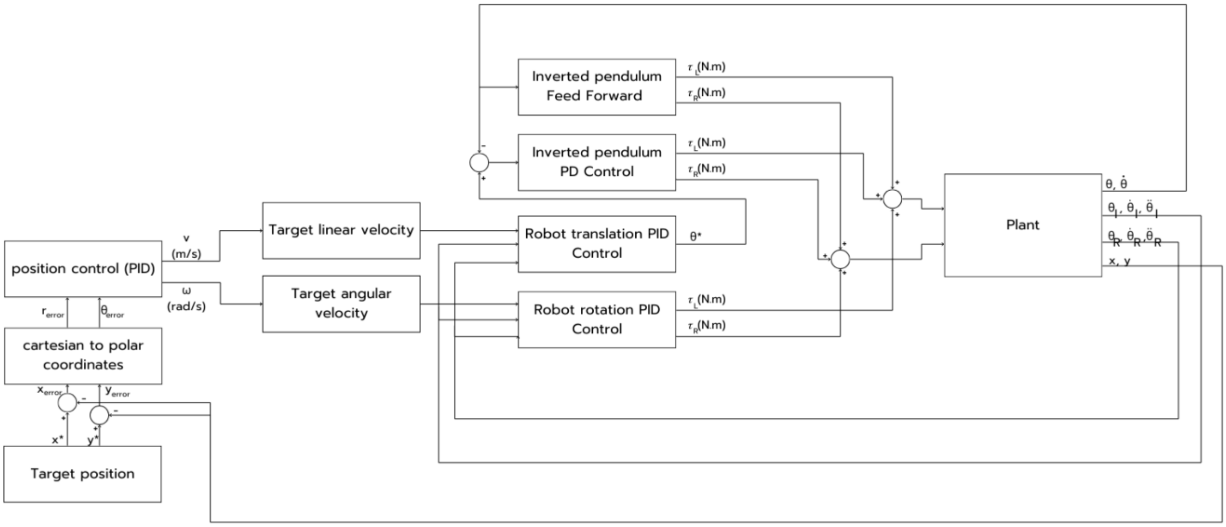

Fig [6]: System Diagram

- **Target position:** provides the target position in X-Y axis for the robot.
- **Target linear velocity:** provides the target linear velocity for the robot.
- **Target angular velocity:** provides the target angular velocity for the robot.
- **Cartesian to polar coordinates:** Calculate polar coordinates between robot and target from cartesian coordinates.
- **Position PID control:** Using PID control to generates angular velocity and linear velocity based on distance between robot and target.
- **Inverted pendulum Feed Forward:** get input that is current body rotation around y-axis and calculate torque to counter gravitational terms.
- **Inverted pendulum PD control:** Using PD control to generates torque for each wheel based on body rotation around y-axis.
- **Robot rotation PID control:** using diff-drive kinematics to calculate current angular velocity of robot, compare to target velocity and using PID control to generate wheel torque for the left and right wheels.
- **Robot translation PID control:** using diff-drive kinematics to calculate current linear velocity of robot, compare to target velocity and using PID control to generate robot pitch angle (cascade control).
- **Plant:** Receive a torque, which is a combine torques from the Inverted pendulum Feed Forward, Inverted pendulum PD control, Robot rotation PID control and Robot translation PID control, of each wheel and return output as rotation of each wheel, tilt angle and position of robot in X-Y plane.

5. ผลการศึกษาที่คาดหวัง

1. Function that can calculate dynamics of 2-wheel self-balance robot.
2. Simulation of 2-wheel robot that can self – balancing itself.
3. Simulation of 2-wheel robot that can input velocities of 2-wheel self-balance robot and move
4. Simulation of 2-wheel robot that can input position in X-Y Plane and 2-wheel self-balance robot move

6. รายละเอียดโครงการ

ลำดับ	การดำเนินงาน	สัปดาห์ที่ 1	สัปดาห์ที่ 2	สัปดาห์ที่ 3	สัปดาห์ที่ 4	สัปดาห์ที่ 5	สัปดาห์ที่ 6
1	Understanding about stabilize system and Kinematics of 2-wheel robot						
2	Make a Proposal						
3	Develop function of Inverse differential kinematics of 2-wheel robot and inverted pendulum dynamics.						
4	Develop simulation of 2-wheel robot.						
5	Testing and refinement.						

7. เอกสารอ้างอิง (References)

[1] Kathryn Remell. “Mathematical Modeling of a Two Wheeled Robotic Base” Uark ScholarWorks, 2021

[2] Nguyen Cao Cuong., Hoang Dinh Co. “Balancing and Trajectory Tracking Control for Two Wheeled Self-Balancing Robot”, 2024

[3] Sangtae Kim., SangJoo Kwon. “On the Dynamic Model of a Two-Wheeled Inverted Pendulum Robot”, 2014