

FE 535: Introduction to Financial Risk Management

Project 2

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Instructions

1. This is a teamwork project - please check your assigned team in the course website.
2. There are **6 main independent questions** in this project.
3. Each member should contribute equally.
4. Feel free to use the handouts and the published codes to do your project.
5. You are welcome to use any programming language or statistical software. Also, you are welcome to use any library/package unless stated otherwise.
6. You will need to download data on your own - unless provided otherwise.
7. The final report should be written using a special document editor, e.g. Word, Latex, Markdown, etc. **Any form of document with a handwriting would not be accepted.**
8. Please submit a pdf copy of your final report, please include your code as an appendix or as part of the your Markdown output. Note that Canvas will not accept any other formats than pdf.
9. Not including the code appendix, the maximum report's length **should be no more than 15 pages** - with font 11 point size, 1.5 line space, and 1in margin (just like this document).
10. Please **avoid taking picture snapshots**. You should report your results in an organized table. The same applies to plots and other visualizations - do not paste any low resolution figures.
11. Please use a special equation editor to write any math, in case needed.

1 Interest Rate Risk: Empirical Investigation (20 Points)

Download adjusted prices for three **ETFs: SPY, IEF, and SHY** from Yahoo Finance. The first one tracks the S&P 500 index, whereas the other two track bond indices. In particular, IEF tracks a bond index that represents the prices of Treasury bonds with maturity ranging between 7 and 10 years, whereas the SHY ETF tracks the prices of government bonds with lower maturity ranging between 1 and 3 years. After merging altogether, the data should date **between August 2002 and October 2020**.

1. Your first task is to consider the return on each ETF. In particular, compute the mean return for each ETF and report in annual basis. How does the SPY compare with the bond ETFs? In terms of risk-return reward trade-off how do you explain the heterogeneity in the mean returns of the ETFs? (5 Points)
2. For each ETF compute a time series of monthly returns. In other words, for a given ETF, consider the sum of daily returns over each month in the time series to represent the monthly return. Report the Pearson correlation coefficients for the three ETF monthly returns (note that in total you should have three unique coefficients). In addition, plot the cumulative monthly return of each ETF in one figure. Looking at the plot, how can you justify the sign and magnitude of each correlation coefficient? Provide some economic rationale. (5 Points)
3. As a final perspective, let's investigate how the interest rates affect the returns of the three ETFs. To do so, consider the Federal Funds rate, which corresponds to the target interest rate set by the Federal Reserve to monitor economic activity:
 - (a) Go to Federal Reserve Economic Data (FRED) at St. Louis Fed website (see [link](#)) and download a time series data for the effective Federal Funds rate.¹ The Fed Funds rate is reported on a monthly basis. To merge with the ETF data, refer to the monthly returns from question 2 above. Note that to merge the Yahoo Finance and FRED data, you need to find a common key index. One suggestion is to create a year-month time index. After merging, this should result in 4 monthly time series. Similar to the figure from question 2 above, plot the cumulative returns over time for each ETF along with the corresponding Fed Funds rate. Ideally, you should use two y-axes, where the left hand side refers to the ETF cumulative return and the right-hand side to the Fed Funds Rate. Provide a number of insights. (5 Points)
 - (b) Consider the monthly change in the Fed Funds rate denoted by Δr_t , which is the difference between the current month rate and the previous one. As a final exercise, regress the monthly return of each ETF against Δr_t using a linear regression. Report the beta of each regression. How do explain the magnitudes and signs of each beta? (5 Points)

¹You may also download this using the R quantmod package, the symbol for which is "FEDFUNDS".

Note on the Fed Funds Rate: when it comes to the yield curve, the Fed Funds rate is considered the interest charged for loans with the shortest maturity. These loans, mostly, take place overnight between banks and financial institutions. The Federal Reserve can influence the amount of cash in the economy by setting the Fed Funds rate, which, as a result, would influence how much money is being circulated in the economy. Hence, changes in the Fed Fund rates should, eventually, be reflected in the yield curve, i.e. affect the yield on bonds with higher maturity as well as reflect economic activity.

2 Bond Portfolio Management (30 Points)

Consider the following Treasury bond data collected from different dates over the last three years:

Number	Coupon	Price	Yield	Maturity
Panel (a) Oct 5, 2018				
1	2.75	99.74	2.88	2
2	2.88	99.11	3.07	5
3	2.88	97.00	3.23	10
4	3.00	92.47	3.40	30
Panel (b) Oct 7, 2019				
5	1.50	100.06	1.47	2
6	1.50	100.50	1.40	5
7	1.63	100.55	1.56	10
8	2.25	104.28	2.06	30
Panel (c) March 20, 2020				
9	1.13	101.57	0.31	2
10	1.13	103.25	0.46	5
11	1.50	106.20	0.85	10
12	2.00	114.17	1.42	30
Panel (d) Nov 6, 2020				
13	0.13	99.95	0.15	2
14	0.25	99.45	0.36	5
15	0.63	98.19	0.82	10
16	1.38	94.69	1.60	30

Given the above table, address the following questions:

1. Use the pricing equation of a fixed-coupon bond to price each of the above bonds. I recommend writing a function that takes yield, coupon, face value, and maturity as its main arguments. The resulting prices should correspond to the ones reported above. Hence, you should plot the computed prices against the reported ones. To confirm, you should observe a 45-degree line. (6 Points)
2. Prices should reflect investors' perception of future interest rates. Rather than computing the prices using yields as the case in the previous question, in practice, it is the other way around. We try to deduce yields from market prices. Hence, given a pricing function, you need to find the yield that matches the market price. For each bond, find the implied yield and plot it against the corresponding yield reported in the table above. Again, this should result in a 45-degree line. (6 Points)

Hint: This relies on a numerical solution. Recall that the solution for function f is the x^* that satisfies $f(x^*) = 0$. Since the price of the bond is a function of yield, i.e. $f(y) = P$, design a function $g(y) = f(y) - P_0$, where P_0 is fixed using the values from the above table.

As a result, the implied yield is the solution y^* that satisfies $g(y^*) = 0$, i.e. $f(y^*) = P$. In R, you may refer to the `uniroot` function. In Excel, this can be attained using “goalseek”.

3. Compute and report the Macaulay duration for each of the 16 US T. bonds. Ideally, you should report this in four 2×4 tables, where the first row corresponds to the bond number and the second to the Macaulay duration. The first table refers to the data from 2018, the second to 2019, the third to March 2020, and the fourth to Nov 2020. Given these tables, how do they compare? **Provide some rationale.** (6 Points)
4. Using first order Taylor expansion, calculate the change in the Treasury bond prices, if the yield curve in the US shifts **down** by 25 bps. Focus only on the recent bond data to answer this part, i.e. bonds numbered 13, 14, 15, and 16. To summarize, plot both the original and new prices against maturity. How do you justify this observation? (6 Points)

Note: since you have a pricing function for a fixed coupon bond, you should confirm whether the new price is correct. For instance, if the price P is a function of yield y , then we know that price is $P = f(y)$. To check whether your answer is correct, you should compare your Taylor expansion results with the exact price, which would be $P_1 = f(y + \Delta y)$.

5. Assume that the prices in the above table reflect the dollar price of each bond, e.g. the price of bond 9 is \$101.57. As a portfolio manager, you need to allocate \$100,000 between bonds 13 and 14 from the above table. If you believe that the Federal Reserve will increase interest rates in the near future, you need to limit your portfolio duration to 3 years. As a result, how many units of each bond you need to purchase to satisfy this? How would your answer change if you target a duration of 6 years instead? Explain why these numbers make sense. (6 Points)
6. **Bonus Question** Consider the details from the previous question. However, in this case, you need to allocate \$100,000 among the four Treasury bonds numbered 13, 14, 15, and 16. If you are targeting a portfolio duration of 6 years, how many units of each bond you need to buy? The position in each one of the four bonds should not be zero. (6 Points)

Hint: In this case, you need to satisfy two conditions by choosing four unknowns. This results in an under-determined linear system of equations. To solve this, you need to think in terms of a generalized solution. A possible suggestion is to look into a generalized matrix inverse - for instance, see Moore-Penrose pseudoinverse (Wiki [page](#)). As a confirmation, check whether the proposed solution satisfies the two requirements.

3 The Yield Curve (20 Points)

This question contains two main parts

Part I

The yield curve is plot of the yield on bonds with differing terms to maturity but the same credit risk, liquidity, and tax considerations. Over time, we have witnessed different shapes of the yield curve:

- Upward-sloping: long-term rates are above short-term rates
- Flat: short- and long-term rates are the same
- Inverted: long-term rates are below short-term rates

Your first task is to provide empirical evidence for each of the above shapes using real-data. To do so, you need to download data for Treasury yields of different maturities using the FRED database. In particular, you need to download data for the following codes DGS1MO, DGS3MO, DGS1, DGS2, DGS5, DGS7, and DGS10. After merging and dropping missing values, the final dataset is daily and dates between late July 31st, 2001 and Oct 31, 2020.

1. Given the data, provide three plots of the yield curve from different dates in which we witnessed one of the above three shapes, i.e. upward-sloping, flat, and inverted. (3 Points)
2. As a summary, you need to report a number of statistics for each maturity: mean, standard deviation, skewness, and kurtosis. You should summarize your results in a 7×4 table, where rows refer to maturities and columns to statistics. (4 Points)
3. Given the previous summary statistics how do you justify the magnitude of the standard deviation and kurtosis with respect to maturity? **Elaborate in no more than two sentences.** (3 Points)
4. Let's control for interest rate regimes, where you round the 1-month yields to the closest 0.25 unit. For instance, if the 1-month yield is 1.71, it becomes 1.75. For 1.11, it becomes 1.00. After rounding the 1-month yields, the data now corresponds to 22 interest rate regimes in which the 1-month yield takes values in the range of $\{0, 0.25, 0.50, 0.75, \dots, 5.00, 5.25\}$. Conditional on each regime, compute the standard deviation of yields for both the 10 years and 3 months maturities. As a summary, plot the ratio between the former and the latter versus the rounded 1-month yields. Note that the plot should correspond to 22 points. How do you justify this ratio? Recall the implications of interest rate risk and convexity. (4 Points)

Part II

The theory of the term structure of interest rates tries to explain the following facts about the yield curve

- Interest rates on bonds of different maturities move together over time.
- When short-term interest rates are low, yield curves are more likely to have an upward slope. Alternatively, when short-term rates are high, yield curves are more likely to slope downward and be inverted.
- Yield curves almost always slope upward.

Your second task is to empirically validate the above facts. My recommendation is to provide a statistical evidence using the full data to check each one of the above facts. You may consider performing a statistical test with significance levels, but basic descriptive statistics combined with qualitative description should suffice. (6 Points)

Note: The insights made in Part I could be used to further validate the above facts.

4 Forward Contracts and No-Arbitrage Pricing (20 Points)

Under no-arbitrage pricing it follows that future price of a stock index corresponds to the following geometric Brownian motion (GBM):

$$S_T = S_t \times \exp \left(\left(r - d - \frac{\sigma^2}{2} \right) \tau + \sigma Z_\tau \right) \quad (1)$$

with r is the risk-free rate, d is the continuous annual dividend yield, and Z_τ is a standard Brownian motion.

To address the following questions, assume that $r = 1.75\%$ and $\sigma = 0.2$, while $d = 0$, i.e. the underlying stocks of the index pay no dividends. Additionally, suppose that the spot price is \$100. Given this information, address the following questions:

1. Under no-arbitrage pricing, what is the fair value of a k -years forward contract on the above stock index? Report your answer for $k = 1, 2, 3, 4, 5$. As a summary, plot the forward price versus k . What does the graph say? (4 Points)
2. Repeat the previous part but using Monte Carlo simulation. In particular, you will need to simulate the future price of the index for $k = 1, 2, 3, 4, 5$ years. Using a boxplot, plot the distribution of the simulated price for each year and highlight the forward price. How does your answer compare with the previous part? (4 Points)

Hint: Remember the economic implications of the forward contract.

3. Suppose you have a long position in the above stock index and you are planning to liquidate your position exactly one year from now.
 - Using the 1-year forward contract, explain how would you hedge your position? Elaborate (2 Points)
 - Given the previous answer, you need to evaluate the profit and loss (P&L) of the final payoff of the hedged position. As a summary, report the Value-at-Risk of the hedged position? Elaborate (2 Points)
4. Suppose that you are an arbitrageur and that the market price of the 1-year forward contract is trading \$0.25 lower than the price you computed in the first part of this question. Describe a trading strategy that would exploit this mispricing. As a summary, you need to simulate and plot two price paths in which the future stock index either increases or decreases. For each scenario, how does your arbitrage strategy perform? Elaborate (4 Points)
5. An assumption of the risk-neutral and, hence, the no-arbitrage pricing is that r is constant over time. Suppose after executing your arbitrage strategy the Fed cuts the interest rate by 50 basis points exactly six months after. How does this affect your arbitrage strategy? Elaborate (4 Points)

5 Managing Linear Risk (30 Points)

Case: Today's date is **Nov 16th, 2020**, and a U.S. exporter has been promised a payment of **1,250,000 euros** in **Oct 1st, 2021**. The **spot exchange rate** on Nov 16th, 2020 is **\$1.1836**. According to the Chicago Mercantile Exchange (CME Group), the exporter can trade EUR/USD contracts with face amount of 125,000 euros that will expire in December 2021. The CME Group states that futures contracts are terminated on the second business day immediately preceding the third Wednesday of the contract month.

Calibrating the Exchange Rate Process (10 Points)

Table 1: On Nov 16th, 2020, according to [investing.com](https://www.investing.com), the spot price of the exchange rate is \$1.1836, while the quotes for the EUR/USD forward contracts are given in the table below.

Name	Bid	Ask	High	Low
EURUSD 1M FWD	7.64	7.79	7.87	7.6
EURUSD 2M FWD	19.36	19.6	19.95	18.14
EURUSD 3M FWD	27.08	27.28	28.21	26.29
EURUSD 4M FWD	34.25	34.55	34.63	34.2
EURUSD 5M FWD	42.94	43.32	43.51	42.79
EURUSD 6M FWD	50.38	51.2	51.34	49.66
EURUSD 7M FWD	58.81	59.31	59.58	59.12
EURUSD 8M FWD	67.1	68.1	68.59	66.79
EURUSD 9M FWD	75.07	76.32	77.32	74.76
EURUSD 10M FWD	84.12	85.02	84.92	83.95
EURUSD 11M FWD	91.88	93.46	94.56	92.1
EURUSD 1Y FWD	100.61	101.58	101.6	100.46
EURUSD 15M FWD	128.61	129.98	129.44	128.27
EURUSD 21M FWD	183.96	185.89	184.66	184.22
EURUSD 2Y FWD	213.08	215.3	213.71	213.47
EURUSD 3Y FWD	334.16	339.8	338.93	338.88

1. Let S_t denote the EUR/USD exchange rate, i.e. the amount of dollars needed to purchase a single Euro at time t . Under no-arbitrage pricing (risk-neutral valuation), S_t follows a Geometric Brownian Motion (GBM), such that the future spot price is given by

$$S_T = S_t \times \exp\left(\left(\theta - \frac{\sigma^2}{2}\right)\tau + \sigma B_\tau\right) \quad (2)$$

where

- $\theta = r - r_f$ denotes the difference between the US risk-free rate and the EU risk-free rate.
- σ is the annual volatility of the exchange rate
- $B_\tau \sim N(0, \tau)$ is a standard Brownian motion

Your first task is to calibrate the GBM:

- (a) For θ , you need to refer to the interest rate parity and estimate θ using the forward quotes from Table 1. Note that this a “forward-looking” approach(5 Points)
- (b) For σ , you need to download data for the daily EUR/USD exchange rate using the “EURUSD=X” symbol from Yahoo Finance. Your data should be daily and range between 2016-01-01 and 2020-04-03. Given the adjusted prices, you need to calibrate σ using the historical returns. Note that this calibration is backward-looking, which is in line with what you did in Project 1. (5 Points)

Hint: For foreign exchange rates, it is common to relate to the interbank lending rate in terms risk-free rate. For instance, see the USD and EUR LIBOR rates according to the market data from Wall Street Journal available [here](#). According to that week, the LIBOR for USD is 0.34188 bps, for instance.

VaR for the Unhedged (5 Points)

- 2. Assume that the exporter does not hedge the exchange rate risk. In this case, the exporter exchanges the euros in the spot market upon receiving the payment in the future. Let V_T denote the profit/loss (P&L) of the exporter at delivery time, which is given by $S_T - S_t$. What is the 99% VaR of the exporter’s P&L in \$? (5 Points)

Recall that if V_T denotes the P&L at time T , then the $1 - \alpha$ VaR is

$$VaR_t(V_T, \alpha) = \mathbb{E}_t[V_T] - Q_t(V_T, \alpha) \quad (3)$$

Hint: The answer should be between \$100K and \$200K. If the euro weakens relative to the dollar, then the exporter gets paid less dollars in the future. Given that the exporter expects 1.25M euros, the future P&L depends on the future spot rate. You need to solve this question using a MC simulation.

Unitary Hedge (10 Points)

- 3. Consider a unitary hedge, in which the exporter shorts 10 futures contracts today and closes the position when the euro payment is received (Oct 1st, 2020). If the risk-free rates are fixed and there is no arbitrage, the price of the futures contract should obey to the interest rate parity. In other words, the futures contract price at time t is given by

$$F_t = S_t \times e^{(r-r_f) \times (t_d-t)} \quad (4)$$

with t_d denoting the maturity time of the futures contract, which is mid-December 2021. Assume that there is no transactions cost, i.e. you are able to buy and sell futures contract with respect to the price implied by the interest rate parity. Using a MC simulation, address the following:

- (a) What is the 99% VaR of the P&L with unitary hedging? (5 Points)
- (b) Suppose instead you use the futures contract expiring in September 2021 (before delivery). What is the 99% VaR of the P&L now? **Elaborate** in terms of basis risk (5 Points)

Hedging using ETFs (5 Points)

4. Suppose for some reason the exporter decides to use ETFs (or ETNs) to hedge currency exposure instead of using futures or forward contracts. Your task is to screen 5 different ETFs. For each ETF, provide an economic rationale behind of which to serve as a EUR/USD hedge. Justify your reasoning by reporting the hedge effectiveness of each.

Note this is an open question without a unique answer. However, your reasoning should make sense in terms of economic mechanisms behind the EUR/USD exchange rate movement.

6 Stochastic Interest Rate Models

To be completed...