Crosstabs: Counts, Proportions, and More

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It took me two years to figure out how to do cross-tabs in *R* the way that *Goldvarb* does cross-tabs. Below I show you how to build cross-tabs from scratch.

Token Counts

A good starting point is the function `table()`. This function returns token numbers.[1]

```
# Get the number of tokens by level of Dep.Var
table(td$Dep.Var)
```

```
Deletion Realized
     386       803
```

This tells you that there are 386 `Deletion` tokens and 803 not deleted, or `Realized` tokens. If you add another factor group like `Age.Group`, you get the number of tokens for each level of `Dep.Var` for each level of that additional factor group. These two factor groups are returned as the rows and then columns in the table.

```
# Get the number of tokens by level of Dep.Var and Age.Group
table(td$Dep.Var, td$Age.Group)
```

	Old	Middle	Young
Deletion	67	125	194
Realized	134	235	434

---

[1]If you don't have the `td` dataset loaded in *R*, go back to Doing it all again, but `tidy` and run the code.

If you add one more factor group, Sex, it divides the data in what R calls "pages". The first page is the number of tokens for each level of Dep.Var by each level of Age.Group for female data (Sex = F), and then the same for the male data (Sex = M).

```
# Get the number of tokens by Dep.Var, Sex, and Age.Group
table(td$Dep.Var, td$Age.Group, td$Sex)
```

```
, ,  = F


          Old Middle Young
  Deletion  43     73    72
  Realized 107    165   199

, ,  = M


          Old Middle Young
  Deletion  24     52   122
  Realized  27     70   235
```

You can add the option deparse.level = 2 to include the names of the columns in the table.

```
# Get the number of tokens by Dep.Var, Sex, and Age.Group
table(td$Dep.Var, td$Age.Group, td$Sex, deparse.level = 2)
```

```
, , td$Sex = F

            td$Age.Group
td$Dep.Var Old Middle Young
  Deletion  43     73    72
  Realized 107    165   199

, , td$Sex = M

            td$Age.Group
td$Dep.Var Old Middle Young
  Deletion  24     52   122
  Realized  27     70   235
```

If you wrap the `table()` function in the `addmargins()` function you get the sums of each row and column, and another page for both the male and the female data together.

```
# Get the number of tokens by Dep.Var, Sex, and Age.Group, with column, row and page total
addmargins(table(td$Dep.Var, td$Age.Group, td$Sex, deparse.level = 2))
```

```
, , td$Sex = F

            td$Age.Group
td$Dep.Var  Old Middle Young  Sum
  Deletion   43     73    72  188
  Realized  107    165   199  471
  Sum       150    238   271  659

, , td$Sex = M

            td$Age.Group
td$Dep.Var  Old Middle Young  Sum
  Deletion   24     52   122  198
  Realized   27     70   235  332
  Sum        51    122   357  530

, , td$Sex = Sum

            td$Age.Group
td$Dep.Var  Old Middle Young  Sum
  Deletion   67    125   194  386
  Realized  134    235   434  803
  Sum       201    360   628 1189
```

If you change the order of factor groups you include in the `table()` function you can change which factors are rows, which are columns, and which are pages. You can also keep adding factors as additional pages. The order is always: rows, columns, page 1, page 2, etc.

```
# Get the number of tokens by Age.Group, Education, Sex, and Dep.Var, with row, column, an
addmargins(table(td$Age.Group, td$Education, td$Sex, td$Dep.Var, deparse.level = 2))
```

```
, , td$Sex = F, td$Dep.Var = Deletion

              td$Education
```

```
td$Age.Group Educated Not Educated Student  Sum
     Old            2           41        0   43
     Middle        68            5        0   73
     Young         20            0       52   72
     Sum           90           46       52  188

, , td$Sex = M, td$Dep.Var = Deletion

          td$Education
td$Age.Group Educated Not Educated Student  Sum
     Old            0           24        0   24
     Middle        16           36        0   52
     Young         48           24       50  122
     Sum           64           84       50  198

, , td$Sex = Sum, td$Dep.Var = Deletion

          td$Education
td$Age.Group Educated Not Educated Student  Sum
     Old            2           65        0   67
     Middle        84           41        0  125
     Young         68           24      102  194
     Sum          154          130      102  386

, , td$Sex = F, td$Dep.Var = Realized

          td$Education
td$Age.Group Educated Not Educated Student  Sum
     Old           30           77        0  107
     Middle       153           12        0  165
     Young         52            0      147  199
     Sum          235           89      147  471

, , td$Sex = M, td$Dep.Var = Realized

          td$Education
td$Age.Group Educated Not Educated Student  Sum
     Old            0           27        0   27
     Middle        30           40        0   70
     Young         77           31      127  235
     Sum          107           98      127  332

, , td$Sex = Sum, td$Dep.Var = Realized
```

```
          td$Education
td$Age.Group Educated Not Educated Student  Sum
      Old             30          104        0  134
      Middle         183           52        0  235
      Young          129           31      274  434
      Sum            342          187      274  803

, , td$Sex = F, td$Dep.Var = Sum

          td$Education
td$Age.Group Educated Not Educated Student  Sum
      Old             32          118        0  150
      Middle         221           17        0  238
      Young           72            0      199  271
      Sum            325          135      199  659

, , td$Sex = M, td$Dep.Var = Sum

          td$Education
td$Age.Group Educated Not Educated Student  Sum
      Old              0           51        0   51
      Middle          46           76        0  122
      Young          125           55      177  357
      Sum            171          182      177  530

, , td$Sex = Sum, td$Dep.Var = Sum

          td$Education
td$Age.Group Educated Not Educated Student   Sum
      Old             32          169        0   201
      Middle         267           93        0   360
      Young          197           55      376   628
      Sum            496          317      376  1189
```

The above function produces 9 "pages", one for each combination of `Sex` (two levels) and `Dep.Var` (two levels), plus the sum of each (one additional level each), and the sum for both. With more than three factor groups like this it is very useful to have the column names included in the output. Scroll to the sixth page, for example (the one that begins `, , td$Sex = Sum, td$Dep.Var = Realized`). It shows the number of tokens by `Age.Group` and `Education` (the first two factor groups in the function), when `Sex` equals `Sum` (e.g., M and F combined) and `Dep.Var` equals `Realized`.

One advantage of doing cross-tabs in *R*, rather than *Goldvarb*, is that you can simultaneously cross more than two factor groups at once. But, the presentation of these factors in pages may not be the most useful. The function `ftable()` in the package `vcd` presents the cross-tab in a more condensed format. The last factor group in the `table()` function will be the variable for the columns in `ftable()`, so you always want to make that the dependent variable. Below is the `ftable()` for the cross-tab of `Age.Group`, `Education`, `Sex`, and `Dep.Var`. You can see, for example, that there are 52 `Deletion` tokens from young, student, female speakers and that there are no tokens from old, educated men.

```
# Get the number of tokens by Age.Group, Education, Sex, and Dep.Var, with row,
# column and page totals, presented in a flattened table
library(vcd)
ftable(table(td$Age.Group, td$Education, td$Sex, td$Dep.Var))
```

			Deletion	Realized
Old	Educated	F	2	30
		M	0	0
	Not Educated	F	41	77
		M	24	27
	Student	F	0	0
		M	0	0
Middle	Educated	F	68	153
		M	16	30
	Not Educated	F	5	12
		M	36	40
	Student	F	0	0
		M	0	0
Young	Educated	F	20	52
		M	48	77
	Not Educated	F	0	0
		M	24	31
	Student	F	52	147
		M	50	127

```
# Do the same but include the margin values
ftable(addmargins(table(td$Age.Group, td$Education, td$Sex, td$Dep.Var)))
```

Deletion Realized Sum

Old	Educated	F	2	30	32
		M	0	0	0
		Sum	2	30	32
	Not Educated	F	41	77	118
		M	24	27	51
		Sum	65	104	169
	Student	F	0	0	0
		M	0	0	0
		Sum	0	0	0
	Sum	F	43	107	150
		M	24	27	51
		Sum	67	134	201
Middle	Educated	F	68	153	221
		M	16	30	46
		Sum	84	183	267
	Not Educated	F	5	12	17
		M	36	40	76
		Sum	41	52	93
	Student	F	0	0	0
		M	0	0	0
		Sum	0	0	0
	Sum	F	73	165	238
		M	52	70	122
		Sum	125	235	360
Young	Educated	F	20	52	72
		M	48	77	125
		Sum	68	129	197
	Not Educated	F	0	0	0
		M	24	31	55
		Sum	24	31	55
	Student	F	52	147	199
		M	50	127	177
		Sum	102	274	376
	Sum	F	72	199	271
		M	122	235	357
		Sum	194	434	628
Sum	Educated	F	90	235	325
		M	64	107	171
		Sum	154	342	496
	Not Educated	F	46	89	135
		M	84	98	182
		Sum	130	187	317
	Student	F	52	147	199

```
            M           50      127  177
            Sum         102     274  376
    Sum     F           188     471  659
            M           198     332  530
            Sum         386     803 1189
```

Of course we can use the pipe **%>%** to make things a bit easier

```
# Get the number of tokens by Age.Group, Education, Sex, and Dep.Var, with row,
# column and page totals, presented in a flattened table
table(td$Age.Group, td$Education, td$Sex, td$Dep.Var) %>%
    addmargins() %>%
    ftable()
```

```
                              Deletion Realized  Sum

Old     Educated     F            2        30    32
                     M            0         0     0
                     Sum          2        30    32
        Not Educated F           41        77   118
                     M           24        27    51
                     Sum         65       104   169
        Student      F            0         0     0
                     M            0         0     0
                     Sum          0         0     0
        Sum          F           43       107   150
                     M           24        27    51
                     Sum         67       134   201
Middle  Educated     F           68       153   221
                     M           16        30    46
                     Sum         84       183   267
        Not Educated F            5        12    17
                     M           36        40    76
                     Sum         41        52    93
        Student      F            0         0     0
                     M            0         0     0
                     Sum          0         0     0
        Sum          F           73       165   238
                     M           52        70   122
                     Sum        125       235   360
Young   Educated     F           20        52    72
```

```
           M             48       77  125
           Sum           68      129  197
    Not Educated F        0        0    0
           M             24       31   55
           Sum           24       31   55
    Student     F        52      147  199
           M             50      127  177
           Sum          102      274  376
    Sum         F        72      199  271
           M            122      235  357
           Sum          194      434  628
Sum Educated    F        90      235  325
           M             64      107  171
           Sum          154      342  496
    Not Educated F       46       89  135
           M             84       98  182
           Sum          130      187  317
    Student     F        52      147  199
           M             50      127  177
           Sum          102      274  376
    Sum         F       188      471  659
           M            198      332  530
           Sum          386      803 1189
```

Another `tidy` way to find out the number of tokens by the different levels of a factor group is using the `group_by()` and `tally()` functions. First, we specify how to group the data, i.e., what combination of factors we want to investigate. In this case, we want the number of tokens for every combination of `Age.Group`, `Education`, `Sex` and `Dep.Var`. Next we use the `tally()` function to provide the token counts for each of those combinations. The results are very similar to those produced by `ftable(table())`.

```
# Group data by Age, Education, and Sex then tally each group
td %>%
    group_by(Age.Group, Education, Sex, Dep.Var) %>%
    tally()
```

```
# A tibble: 24 x 5
# Groups:   Age.Group, Education, Sex [12]
  Age.Group Education    Sex   Dep.Var      n
  <fct>     <fct>        <fct> <fct>    <int>
 1 Old       Educated     F     Deletion     2
```

```
 2 Old        Educated     F     Realized    30
 3 Old        Not Educated F     Deletion    41
 4 Old        Not Educated F     Realized    77
 5 Old        Not Educated M     Deletion    24
 6 Old        Not Educated M     Realized    27
 7 Middle     Educated     F     Deletion    68
 8 Middle     Educated     F     Realized   153
 9 Middle     Educated     M     Deletion    16
10 Middle     Educated     M     Realized    30
# ... with 14 more rows
# i Use `print(n = ...)` to see more rows
```

As the results of `tally()` is a *tibble*, only the first 10 rows will be printed. To print all the rows add `print(n=Inf)` at the end.

```
# Group data by Age, Education, and Sex, tally each group, then print all rows
td %>%
    group_by(Age.Group, Education, Sex, Dep.Var) %>%
    tally() %>%
    print(n = Inf)
```

```
# A tibble: 24 x 5
# Groups:   Age.Group, Education, Sex [12]
   Age.Group Education    Sex   Dep.Var       n
   <fct>     <fct>        <fct> <fct>     <int>
 1 Old       Educated     F     Deletion      2
 2 Old       Educated     F     Realized     30
 3 Old       Not Educated F     Deletion     41
 4 Old       Not Educated F     Realized     77
 5 Old       Not Educated M     Deletion     24
 6 Old       Not Educated M     Realized     27
 7 Middle    Educated     F     Deletion     68
 8 Middle    Educated     F     Realized    153
 9 Middle    Educated     M     Deletion     16
10 Middle    Educated     M     Realized     30
11 Middle    Not Educated F     Deletion      5
12 Middle    Not Educated F     Realized     12
13 Middle    Not Educated M     Deletion     36
14 Middle    Not Educated M     Realized     40
15 Young     Educated     F     Deletion     20
16 Young     Educated     F     Realized     52
```

```
17 Young      Educated      M      Deletion      48
18 Young      Educated      M      Realized      77
19 Young      Not Educated M       Deletion      24
20 Young      Not Educated M       Realized      31
21 Young      Student       F      Deletion      52
22 Young      Student       F      Realized      147
23 Young      Student       M      Deletion      50
24 Young      Student       M      Realized      127
```

The above code gives us the number of `Realized` and `Deletion` tokens for each combination of `Age.Group`, `Education`, and `Sex`. What if we want the total number of tokens for each combination, rather than the number of each level of `Dep.Var`. In this case, you can just drop `Dep.Var` from the `group_by()` function.

```
# Get total number of tokens per group by removing Dep.Var
td %>%
    group_by(Age.Group, Education, Sex) %>%
    tally() %>%
    print(n = Inf)
```

```
# A tibble: 12 x 4
# Groups:   Age.Group, Education [7]
   Age.Group Education    Sex       n
   <fct>     <fct>        <fct> <int>
 1 Old       Educated     F        32
 2 Old       Not Educated F       118
 3 Old       Not Educated M        51
 4 Middle    Educated     F       221
 5 Middle    Educated     M        46
 6 Middle    Not Educated F        17
 7 Middle    Not Educated M        76
 8 Young     Educated     F        72
 9 Young     Educated     M       125
10 Young     Not Educated M        55
11 Young     Student      F       199
12 Young     Student      M       177
```

We know now that there are 32 tokens from `Old`, `Educated`, `F` (female) speakers. The previous `tally()` shows us that 2 of the tokens are `Deletion` and 30 are of `Realized`.

An alternative to `tally()` is the much more flexible `summarize()` function.[2] With this function you can apply a summary statistic function to each combination of the grouping variables. If no summary statistic function is created, the a tibble of the combination of the groups is produced.

```
# Create a tibble of all combinations of Age.Group, Education, and Sex (for
# which there are rows of data)
td %>%
    group_by(Age.Group, Education, Sex) %>%
    summarize()
```

```
# A tibble: 12 x 3
# Groups:   Age.Group, Education [7]
   Age.Group Education     Sex
   <fct>     <fct>         <fct>
 1 Old       Educated      F
 2 Old       Not Educated  F
 3 Old       Not Educated  M
 4 Middle    Educated      F
 5 Middle    Educated      M
 6 Middle    Not Educated  F
 7 Middle    Not Educated  M
 8 Young     Educated      F
 9 Young     Educated      M
10 Young     Not Educated  M
11 Young     Student       F
12 Young     Student       M
```

To get the count, or number of rows, of each combination, we create a new column in the tibble that is the output of `summarize()` and assign to it the value of the count function `n()`

```
# Create a tibble of grouping variables, then add a new column 'Tokens' with
# the value of the count function
td %>%
    group_by(Age.Group, Education, Sex, Dep.Var) %>%
    summarize(Tokens = n()) %>%
    print(n = Inf)
```

```
# A tibble: 24 x 5
```

---

[2] `summarise()` and `summarize()` are synonyms.

```
# Groups:   Age.Group, Education, Sex [12]
   Age.Group Education    Sex  Dep.Var  Tokens
   <fct>     <fct>        <fct> <fct>    <int>
 1 Old       Educated     F    Deletion      2
 2 Old       Educated     F    Realized     30
 3 Old       Not Educated F    Deletion     41
 4 Old       Not Educated F    Realized     77
 5 Old       Not Educated M    Deletion     24
 6 Old       Not Educated M    Realized     27
 7 Middle    Educated     F    Deletion     68
 8 Middle    Educated     F    Realized    153
 9 Middle    Educated     M    Deletion     16
10 Middle    Educated     M    Realized     30
11 Middle    Not Educated F    Deletion      5
12 Middle    Not Educated F    Realized     12
13 Middle    Not Educated M    Deletion     36
14 Middle    Not Educated M    Realized     40
15 Young     Educated     F    Deletion     20
16 Young     Educated     F    Realized     52
17 Young     Educated     M    Deletion     48
18 Young     Educated     M    Realized     77
19 Young     Not Educated M    Deletion     24
20 Young     Not Educated M    Realized     31
21 Young     Student      F    Deletion     52
22 Young     Student      F    Realized    147
23 Young     Student      M    Deletion     50
24 Young     Student      M    Realized    127
```

The `summarize()` function can be used with a number of summary statistic functions, including, but not limited to. the following:

Type	Some Useful Functions
Center	`mean()`, `median()`
Spread	`sd()`, `IQR()`
Range	`min()`, `max()`
Position	`first()`, `last()`, `nth()`
Count	`n()`, `n_distinct()`
Logical	`any()`, `all()`

Summary Statistics for Continous Variables

This seems like an appropriate place to describe how to summarize values that are continous, like `YOB`. Normally in variationist sociolinguistics we are very concerned with frequency and proportion of usage, and we will explore how to generate those statistics in the following section. Here, however, let's explore the functions available to use inside `summarize()`. These functions can be used on their own, also. For example, the first two, `mean()` and `median()` provide the arithmetic mean (basically the average) of a set of numbers while the `median()` provides the exact middle number of a set of values organized from smallest to largest (if there are an even number of values, `median()` returns the halfway point between the two middle numbers).

```
# Get mean year of birth
mean(td$YOB)
```

```
[1] 1969.447
```

```
# Get median year of birth
median(td$YOB)
```

```
[1] 1984
```

We already know that the mean year of birth for the `td` data set is 1969.447. You can also see that the middle number of all years of birth organized from oldest to youngest is 1984. If we wanted to find the mean or median year of birth for either just male or just female speakers, we have two options. We can use the base filter technique, or we can use the `tidy` method to group the data and summarize it.

```
# Get mean year of birth of just female speakers
mean(td$YOB[td$Sex == "F"])
```

```
[1] 1963.487
```

```
# Get mean year of birth of just male speaker
mean(td$YOB[td$Sex == "M"])
```

```
[1] 1976.857
```

```r
# Get mean year of birth for each level of Sex
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB))
```

```
# A tibble: 2 x 2
  Sex    Mean.YOB
  <fct>     <dbl>
1 F         1963.
2 M         1977.
```

Dealing with Decimals

Tibbles are intended to be succinct and concise, so they provide very few values after the decimal place by default. If you require more decimal values, the easiest (trust me) thing to do is to convert the tibble into a *data frame*.

```r
# Get mean year of birth by Sex, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB)) %>%
    as.data.frame()
```

```
  Sex Mean.YOB
1   F 1963.487
2   M 1976.857
```

data frames will display whole numbers, and numbers with decimals up to the total number of digits set by `options()` function. Keep in mind, though, that changing this value changes the global options for *R*. An alternative is to use the `format()` function.

```r
# Change number of significant digits displayed to 10
options(digits = 6)
# Get mean year of birth by sex, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB)) %>%
    as.data.frame()
```

```
  Sex Mean.YOB
1   F  1963.49
2   M  1976.86
```

```r
# Change number of significant digits displayed to 10
options(digits = 10)
# Get mean year of birth by sex, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB)) %>%
    as.data.frame()
```

```
  Sex    Mean.YOB
1   F 1963.487102
2   M 1976.856604
```

```r
# Change number of significant digits displayed to 3
options(digits = 3)
# Get mean year of birth by sex, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB)) %>%
    as.data.frame() %>%
    format(digits = 10)
```

```
  Sex    Mean.YOB
1   F 1963.487102
2   M 1976.856604
```

For very large numbers R will often display values in exponential notation. We can alter this by setting the value of `scipen` inside the `option()` function. Gain, though, remember that this is a global change for your whole R session. For `scipen` positive values increase the likelihood of using real numbers, negative values increase the likelihood of using exponential notation. To ensure printouts are always real numbers, set `scipen` to 9999 (this is the default). To ensure printouts are always exponential notation, set `scipen` to -9999. To demonstrate, below we multiply mean `YOB` by 10000.

```r
# Change number of significant digits displayed to 6, alter the likelihood of
# use of real number rather than scientific notation by 0
```

```r
options(digits = 6, scipen = 0)
# Get mean year of birth by sex multiplied by 100000, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB) * 1e+05) %>%
    as.data.frame()
```

```
  Sex  Mean.YOB
1   F 196348710
2   M 197685660
```

With `scipen` set to 0, we still get real numbers as the values `Mean.YOB` are not too big. To ensure we have real numbers, though, we change the `scipen` value.

```r
# Change number of significant digits displayed to 6, alter the likelihood of
# use of real number rather than scientific notation by 9999
options(digits = 6, scipen = 9999)
# Get mean year of birth by sex multiplied by 100000, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB) * 10000) %>%
    as.data.frame()
```

```
  Sex Mean.YOB
1   F 19634871
2   M 19768566
```

If, instead we prefer exponential notation, we use the maximum negative `scipen` value, -9999/

```r
# Change number of significant digits displayed to 6, alter the likelihood of
# use of real number rather than scientific notation by -9999
options(digits = 6, scipen = -9999)
# Get mean year of birth by sex multiplied by 100000, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB) * 10000) %>%
    as.data.frame()
```

```
   Sex    Mean.YOB
1    F 1.96349e+07
2    M 1.97686e+07
```

Above, the value `1.96349e+07` means 1.96349×10^7. The easiest way to calculate this is to simply move the decimal places 7 spaces to the right (as the exponent is positive), which gives `19639000`. Notice some precision is lost because our number of `digits` is only 6.

```r
# Change number of significant digits displayed to 10, alter the likelihood of
# use of real number rather than scientific notation by -9999
options(digits = 1e+01, scipen = -9.999e+03)
# Get mean year of birth by sex multiplied by 100000, converted to data frame
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB) * 1e+04) %>%
    as.data.frame()
```

```
   Sex        Mean.YOB
1    F 1.963487102e+07
2    M 1.976856604e+07
```

Now, with more `digits` we have more precision; $1.963487102 \times 10^7 = 19634671.02$. If the exponential values are negative, move the decimal place to the left. For example, $1.963487102 \times 10^{-7} = 0.0000001963467102$.

Similarly, we can set whether or not we want scientific notation using the `format()` function. The `scientific` option can be either `TRUE` or `FALSE`, or a value like `scipen`.

```r
# Change number of significant digits displayed to 3, alter the likelihood of
# use of real number rather than scientific notation by 9999
options(digits = 3e+00, scipen = 9.999e+03)
# Get mean year of birth by sex multiplied by 100000, converted to data frame,
# digits formatted to 10 significant digits, and exponential notation
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB) * 1e+04) %>%
    as.data.frame() %>%
    format(digits = 1e+01, scientific = TRUE)
```

```
   Sex        Mean.YOB
1    F 1.963487102e+07
2    M 1.976856604e+07
```

More Summary Statistics for Continous Variables

The other summary statistics for continuous variables include spread functions and the range functions. Some spread functions are `sd()`, which returns the standard deviation; and `IQR()` which returns the interquartile range.[3] Some range functions include: `min()`, which returns the lowest value; `max()`, which returns the highest value. To find the maximum spread (from highest to lowest), we can either subtract the `min()` value from the `max()` value, or employ the `diff()` function plus the `range()` function(which produces a vector containing the minimum and maximum values).

We can include these functions inside the same `summarize()` function as we used above.

```
# Get mean, standard deviation, interquartile range, minimum value, maximum
# value, and range of values (twice) for year of birth
td %>%
    group_by(Sex) %>%
    summarize(Mean.YOB = mean(YOB), SD.YOB = sd(YOB), IQR.YOB = IQR(YOB), Min.YOB = min(YO
        Max.YOB = max(YOB), Range = max(YOB) - min(YOB), Range2 = diff(range(YOB)))
```

```
# A tibble: 2 x 8
  Sex   Mean.YOB SD.YOB IQR.YOB Min.YOB Max.YOB Range Range2
  <fct>    <dbl>  <dbl>   <dbl>   <int>   <int> <int>  <int>
1 F        1963.   26.5      45    1915    1999    84     84
2 M        1977.   19.6      33    1921    1994    73     73
```

Based on these values, we can make the following statements:

- Among females in the (t, d) data, the average or mean year of birth is 1963 ± 26.5 years.

- The oldest female speakers was born in 1915, and the youngest female speaker was born in 1999.

- Fifty-percent of women were born in the 45 years centered around 1963.

- The female data represents 84 years of apparent time.

---

[3]If we order the data from lowest to highest values, 50% of the data will be less than the mean, and 50% of the data will be higher than the mean. The mean is also called the 2nd quartile. The first quartile is halfway between the mean and the lowest value in the data. The third quartile is halfway betwen the mean and the highest value in the data. The interquartile range is the difference between the 3rd quartile and the 1st quartile and represents the spread of the middle 50% of the data.

Position functions with `summarize()`

The position functions `first()`, `last()`, and `nth()` also work on the data created by `group_by()` and `summarize()`. `first()` returns the first value, `last()` returns the last value, and `nth()` returns the value after a specific number of rows.

```
# Get first six rows of just Sex and Dep.Var columns of td
td %>%
    select(Sex, Dep.Var) %>%
    head()
```

```
  Sex  Dep.Var
1   F Realized
2   F Deletion
3   F Deletion
4   F Deletion
5   M Realized
6   M Deletion
```

```
# Get last six rows of just Sex and Dep.Var columns of td
td %>%
    select(Sex, Dep.Var) %>%
    tail()
```

```
     Sex  Dep.Var
1184   F Realized
1185   F Realized
1186   F Realized
1187   M Realized
1188   M Deletion
1189   M Realized
```

Above we use the `select()` function to choose just the **Sex** and **Dep.Var** columns and run the `head()` and `tail()` functions in order to see the first and last six values for both in the data. We do this just for comparisons sake. Now, lets use the position functions an compare them to our results.

```
# Get first, last, second, and second to last value of Dep.Var by Sex
td %>%
    group_by(Sex) %>%
```

```
        summarize(First = first(Dep.Var), Last = last(Dep.Var), Second = nth(Dep.Var,
            2), Second.Last = nth(Dep.Var, -2))
```

```
# A tibble: 2 x 5
  Sex   First    Last      Second    Second.Last
  <fct> <fct>    <fct>     <fct>     <fct>
1 F     Realized Realized Deletion Realized
2 M     Realized Realized Deletion Deletion
```

Compare the male values with those from the `head()` and `tail()` functions above. The first (row 5) is `Realized`, the last (row 1198) is `Realized`. The second (row 6) is `Deletion`, and the second to last (row 1188) is also `Deletion`.

Count functions with `summarize()`

We've already looked at `n()` above, but there is also the `n_distinct()` function, which reports the number of distinct values. We can use this, for example, to find the number of speakers in each social category. To do this using base R filtering is a lot more complicated to code (so much so its not even worth doing). One example is shown below. It would need to be repeated for every combination of sex, education, and age group.

```
# Example using base R filtering, finding the number of unique speakers who are
# female, educated, and middle aged

n_distinct(td$Speaker[td$Sex == "F" & td$Education == "Educated" & td$Age.Group ==
    "Middle"])
```

```
[1] 12
```

```
# Much easier way to find number of unique speakers for every combination of
# Sex, Education, and Age. Group

td %>%
    group_by(Sex, Education, Age.Group) %>%
    summarize(Speaker.Count = n_distinct(Speaker)) %>%
    print(n = Inf)
```

```
# A tibble: 12 x 4
# Groups:   Sex, Education [6]
   Sex   Education     Age.Group Speaker.Count
   <fct> <fct>         <fct>             <int>
 1 F     Educated      Old                   1
 2 F     Educated      Middle               12
 3 F     Educated      Young                 3
 4 F     Not Educated  Old                   6
 5 F     Not Educated  Middle                1
 6 F     Student       Young                11
 7 M     Educated      Middle                3
 8 M     Educated      Young                 6
 9 M     Not Educated  Old                   5
10 M     Not Educated  Middle                7
11 M     Not Educated  Young                 3
12 M     Student       Young                 8
```

You'll notice that there are is no value for older educated males. This is because there are no speakers in the data from this group.

Logical functions

The two logical functions only work on data that is logical (i.e., is `TRUE` or `FALSE`). `any()` returns the answer to the question "Are any values `TRUE`?" and `all()` returns the answer to the question "Are all values `TRUE`?". There are no logical values in the `td` data set, so lets make some as an example.

```
# Create a new column in which all values are FALSE
td$Logical.Test <- FALSE
# Modify the new column so for any tokens from young female speakers are coded
# as TRUE instead of FALSE
td$Logical.Test[td$Sex == "F" & td$Age.Group == "Young"] <- TRUE

# Get logical value (TRUE or FALSE) of whether any tokens and all tokens of
# Logical.Test are TRUE, by Sex
td %>%
    group_by(Sex) %>%
    summarize(Any.True = any(Logical.Test), All.True = all(Logical.Test))
```

```
# A tibble: 2 x 3
  Sex   Any.True All.True
```

```
   <fct> <lgl>    <lgl>
1 F     TRUE     FALSE
2 M     FALSE    FALSE
```

Above we created a logical column in which only tokens from young females are set to TRUE. The `any()` function returns TRUE for F but not for M because there is at least one TRUE value in the female data. Conversely, the `all()` function returns FALSE for F because not all of the female values are TRUE.

Proportions

Finding out the proportion of a variant is just like finding out the number of tokens. Using the base R methods, you simply wrap the `table()` function in a `prop.table()` function.

```
# Proportion of each level of Dep.Var
prop.table(table(td$Dep.Var))
```

```
Deletion Realized
   0.325    0.675
```

Usually proportions are expressed as hundredths. To force R to express numbers in hundredths, you can use the `options()` function to set the number of significant digits displayed to two.

```
# Display values rounded to nearest hundredth.
options(digits = 2)

# Proportion of each level of Dep.Var
prop.table(table(td$Dep.Var))
```

```
Deletion Realized
    0.32     0.68
```

In the example above there is only one dimension: `Dep.Var`. The `prop.table()` outer function takes the `table()` inner function and divides the number of tokens in each cell by some total (e.g. denominator). The default denominator is the total number of tokens in the whole table. Because, in the example above, the total number of tokens in the one dimension table is the same as the total number of `Dep.Var` tokens, you don't need to specify anything further. In

the example below, however, there are two dimensions: `Dep.Var` and `Age.Group`. If you do not specify which total to use as a denominator, the proportions expressed use the total number of tokens in the table as the denominator.[4] If you want to know the percentage of deletion tokens that come from `Young`, `Middle` and `Old` speakers, you set `margin = 1`, meaning that you want the total (e.g., denominator) to be the sum of the tokens for the first variable in the function, (e.g., rows total). If instead you want to know the percentage of `Young` tokens (or `Middle` tokens, or `Old` tokens) that are `Deletion`, and the percentage that are `Realized`, you set `margin = 2`, or rather set the denominator to the sum of the second factor group in the function (e.g., column total). This follows R's global pattern of rows, columns, page 1, page 2, etc. You can verify this by adding up the proportions in each table below. In the first table all of the proportions add up to 1. In the second table, on the other hand, the proportions add up to 1 going across the rows. In the third table they add up to 1 going down the columns.

```
# Proportion of each level of Dep.Var and Age.Group (all values sum to 1)
prop.table(table(td$Dep.Var, td$Age.Group))
```

```
            Old Middle Young
 Deletion 0.056  0.105 0.163
 Realized 0.113  0.198 0.365
```

```
# Proportion of each level of Age.Group for each level of Dep.Var (each row
# sums to 1)
prop.table(table(td$Dep.Var, td$Age.Group), margin = 1)
```

```
            Old Middle Young
 Deletion 0.17   0.32  0.50
 Realized 0.17   0.29  0.54
```

```
# Proportion of each level of Dep.Var for each level of Age.Group (each column
# sums to 1)
prop.table(table(td$Dep.Var, td$Age.Group), margin = 2)
```

```
            Old Middle Young
```

---

[4]You'll notice that the values in this table are expressed in thousandths instead of hundredths. This is because the proportion for `Deletion` and `Old` tokens requires three decimal places to have two meaningful digits.

```
Deletion 0.33    0.35  0.31
Realized 0.67    0.65  0.69
```

In order to achieve the three-dimension cross-tabs you get from *Goldvarb*, with one dependent variable and two independent variables, you must set up the `prop.table(table())` function with your variables in the following order: *independent variable 1*, *independent variable 2*, *dependent variable*. You must also specify a particular `margin`, e.g., denominator. In a *Goldvarb*-style cross-tab each cell is the number of tokens for one level of the dependent variable (e.g., the application or non-application value) divided by the total number of tokens for that cell. In an *R* proportion table the total number of tokens per cell is the number of tokens for the value of the row and the column at the same time — not the row total, or the column total. To specify that you want the denominator to be the cell total you set *margin = c(1,2)*, where the `c()` concatenating function specifies both row (1) and column (2). The result is a separate page for proportions of each level of `Dep.Var`. The proportions for the corresponding cells in each page add up to 1.

```
# Proportion of each level of Dep.Var for each level of Age.Group and Sex (all
# corresponding cells sum to 1)
prop.table(table(td$Age.Group, td$Sex, td$Dep.Var), margin = c(1, 2))
```

```
, ,  = Deletion


          F    M
 Old    0.29 0.47
 Middle 0.31 0.43
 Young  0.27 0.34

, ,  = Realized


          F    M
 Old    0.71 0.53
 Middle 0.69 0.57
 Young  0.73 0.66
```

You can keep adding factor groups to your proportion table, but you must do two things. You must keep the dependent variable, `Dep.Var`, as the rightmost variable in the function, and you must include all the other variables in the margin specification. For example, below you add `Education` as the third variable, and add 3 to the margin specification. There will be a separate page for each combination of the levels of `Education` and `Dep.Var`.

```
# Proportion of each level of Dep.Var for each level of Age.Group, Sex and
# Education
prop.table(table(td$Age.Group, td$Sex, td$Education, td$Dep.Var), margin = c(1, 2,
    3))
```

, , = Educated, = Deletion

           F   M
  Old   0.062
  Middle 0.308 0.348
  Young 0.278 0.384

, , = Not Educated, = Deletion

           F   M
  Old   0.347 0.471
  Middle 0.294 0.474
  Young   0.436

, , = Student, = Deletion

           F   M
  Old
  Middle
  Young 0.261 0.282

, , = Educated, = Realized

           F   M
  Old   0.938
  Middle 0.692 0.652
  Young 0.722 0.616

, , = Not Educated, = Realized

           F   M

```
  Old     0.653 0.529
  Middle  0.706 0.526
  Young         0.564


, ,   = Student,   = Realized


            F     M
  Old
  Middle
  Young   0.739 0.718
```

Again, you can make these larger tables easier to read by flattening the pages using `ftable()`. Here the `NaN` means there is no data in the cell.

```
# Proportion of each level of Dep.Var for each level of Age.Group, Sex and
# Education, presented as a flattened table. Here the `NaN' just means there is
# no data in the cell.
library(vcd)
ftable(prop.table(table(td$Age.Group, td$Sex, td$Education, td$Dep.Var), margin = c(1,
    2, 3)))
```

```
                         Deletion Realized

Old     F Educated          0.062    0.938
          Not Educated      0.347    0.653
          Student             NaN      NaN
        M Educated            NaN      NaN
          Not Educated      0.471    0.529
          Student             NaN      NaN
Middle  F Educated          0.308    0.692
          Not Educated      0.294    0.706
          Student             NaN      NaN
        M Educated          0.348    0.652
          Not Educated      0.474    0.526
          Student             NaN      NaN
Young   F Educated          0.278    0.722
          Not Educated        NaN      NaN
          Student           0.261    0.739
        M Educated          0.384    0.616
          Not Educated      0.436    0.564
          Student           0.282    0.718
```

There are a number of functions specifically designed to create cross-tables that are somewhat easier to use, but can be somewhat less flexible. Generally, they are most useful for one independent variable and one dependent variable. I tend to use the `CrossTable()` function from the `gmodels` package frequently.

```
# Load gmodels
library(gmodels)

# Generate cross tab of Sex and Dep.Var in which the row proportions are
# displayed, but table proportions, column proportions, and contribution to
# chi-square are suppressed, with 0 decimal values displayed, and missing
# combinations included.
CrossTable(td$Sex, td$Dep.Var, prop.r = TRUE, prop.c = FALSE, prop.t = FALSE, prop.chisq =
    format = "SPSS", digits = 0, missing.include = TRUE)
```

```
   Cell Contents
|-------------------------|
|                  Count  |
|            Row Percent  |
|-------------------------|

Total Observations in Table:  1189

             | td$Dep.Var
     td$Sex  | Deletion  | Realized  | Row Total |
-------------|-----------|-----------|-----------|
          F  |      188  |      471  |      659  |
             |      29%  |      71%  |      55%  |
-------------|-----------|-----------|-----------|
          M  |      198  |      332  |      530  |
             |      37%  |      63%  |      45%  |
-------------|-----------|-----------|-----------|
Column Total |      386  |      803  |     1189  |
-------------|-----------|-----------|-----------|
```

For the `CrossTable()` function you can set the denominator to row total with the option `prop.r=TRUE`. If instead you wanted to the proportion by column, you set `prop.c = TRUE`, and if you want the proportion across the entire table you can set `prop.t = TRUE`. You can

actually set all of these to `TRUE` to get all three. There are other values that can be generated, including values for calculating chi-square (see the `CrossTable()` documentation here). The above code includes the minimal number of options needed to generate the type of cross-table we generally want.

To produce proportions using the `tidy` method, we combine the `group_by()` and `summarize()` functions with the `mutate()` discussed in an earlier section.

```
# Generate tibble of combination of Sex and Dep.Var with token counts and
# proportion of each level of Dep.Var by Sex
td %>%
    group_by(Sex, Dep.Var) %>%
    summarize(Count = n()) %>%
    mutate(Prop = Count/sum(Count))
```

```
# A tibble: 4 x 4
# Groups:   Sex [2]
  Sex   Dep.Var  Count  Prop
  <fct> <fct>    <int> <dbl>
1 F     Deletion   188 0.285
2 F     Realized   471 0.715
3 M     Deletion   198 0.374
4 M     Realized   332 0.626
```

After grouping the data by `Sex` and `Dep.Var`, we create a new column `Count` with values equal to the number of tokens for the particular combination, then we create a new column using `mutate()` and the a math equation to generate proportions. It is important here that your dependent variable `Dep.Var` is the last grouping variable. If we change the order, instead of generating the proportion of `Realized` and `Deletion` tokens, it will instead return the percentage of `Realized` tokens that are `M` and the percentage that are `F`, which is the incorrect denominator for our purposes.

```
# Generate tibble of combination of Dep.Var and Sex with token counts and
# proportion of each level of Sex by Dep.Var
td %>%
    group_by(Dep.Var, Sex) %>%
    summarize(Count = n()) %>%
    mutate(Prop = Count/sum(Count))
```

```
# A tibble: 4 x 4
# Groups:   Dep.Var [2]
```

```
  Dep.Var  Sex    Count  Prop
  <fct>    <fct> <int> <dbl>
1 Deletion F        188 0.487
2 Deletion M        198 0.513
3 Realized F        471 0.587
4 Realized M        332 0.413
```

Unlike the `CrossTable()` function, we can include multiple independent variables. To include every combination (including those for which there are no tokens), we can add `.drop = FALSE` to the `group_by()` function.

```
# Generate tibble of combination of Sex, Edcuation, Age.Group, and Dep.Var with
# all combinations included, with token counts and proportion of each level of
# Dep.Var by each combination of other variables
td %>%
    group_by(Sex, Education, Age.Group, Dep.Var, .drop = FALSE) %>%
    summarize(Count = n()) %>%
    mutate(Prop = Count/sum(Count)) %>%
    print(n = Inf)
```

```
# A tibble: 36 x 6
# Groups:   Sex, Education, Age.Group [18]
   Sex   Education     Age.Group Dep.Var   Count     Prop
   <fct> <fct>         <fct>     <fct>     <int>    <dbl>
 1 F     Educated      Old       Deletion      2   0.0625
 2 F     Educated      Old       Realized     30   0.938
 3 F     Educated      Middle    Deletion     68   0.308
 4 F     Educated      Middle    Realized    153   0.692
 5 F     Educated      Young     Deletion     20   0.278
 6 F     Educated      Young     Realized     52   0.722
 7 F     Not Educated  Old       Deletion     41   0.347
 8 F     Not Educated  Old       Realized     77   0.653
 9 F     Not Educated  Middle    Deletion      5   0.294
10 F     Not Educated  Middle    Realized     12   0.706
11 F     Not Educated  Young     Deletion      0  NaN
12 F     Not Educated  Young     Realized      0  NaN
13 F     Student       Old       Deletion      0  NaN
14 F     Student       Old       Realized      0  NaN
15 F     Student       Middle    Deletion      0  NaN
16 F     Student       Middle    Realized      0  NaN
17 F     Student       Young     Deletion     52   0.261
```

```
18 F    Student      Young    Realized  147   0.739
19 M    Educated     Old      Deletion    0 NaN
20 M    Educated     Old      Realized    0 NaN
21 M    Educated     Middle   Deletion   16   0.348
22 M    Educated     Middle   Realized   30   0.652
23 M    Educated     Young    Deletion   48   0.384
24 M    Educated     Young    Realized   77   0.616
25 M    Not Educated Old      Deletion   24   0.471
26 M    Not Educated Old      Realized   27   0.529
27 M    Not Educated Middle   Deletion   36   0.474
28 M    Not Educated Middle   Realized   40   0.526
29 M    Not Educated Young    Deletion   24   0.436
30 M    Not Educated Young    Realized   31   0.564
31 M    Student      Old      Deletion    0 NaN
32 M    Student      Old      Realized    0 NaN
33 M    Student      Middle   Deletion    0 NaN
34 M    Student      Middle   Realized    0 NaN
35 M    Student      Young    Deletion   50   0.282
36 M    Student      Young    Realized  127   0.718
```

Notice that for the missing combinations the `count()` is 0, and the percentage is `NaN`, which stands for "not a number", the result of trying to divide 0 by something. `NaN` is similar to `NA`, but `NA` stands for "no data", and is used for empty cells.

```r
# Assign the tibble generated in the previous code to an object called results
results <- td %>%
    group_by(Sex, Education, Age.Group, Dep.Var, .drop = FALSE) %>%
    summarize(Count = n()) %>%
    mutate(Prop = Count/sum(Count))

# Recode all NaN in results to 0
results$Prop[is.nan(results$Prop)] <- 0
# Print results
print(results, n = Inf)
```

```
# A tibble: 36 x 6
# Groups:   Sex, Education, Age.Group [18]
   Sex    Education    Age.Group Dep.Var   Count   Prop
   <fct>  <fct>        <fct>     <fct>     <int>  <dbl>
 1 F      Educated     Old       Deletion      2 0.0625
 2 F      Educated     Old       Realized     30 0.938
```

```
 3 F       Educated     Middle    Deletion    68 0.308
 4 F       Educated     Middle    Realized   153 0.692
 5 F       Educated     Young     Deletion    20 0.278
 6 F       Educated     Young     Realized    52 0.722
 7 F       Not Educated Old       Deletion    41 0.347
 8 F       Not Educated Old       Realized    77 0.653
 9 F       Not Educated Middle    Deletion     5 0.294
10 F       Not Educated Middle    Realized    12 0.706
11 F       Not Educated Young     Deletion     0 0
12 F       Not Educated Young     Realized     0 0
13 F       Student      Old       Deletion     0 0
14 F       Student      Old       Realized     0 0
15 F       Student      Middle    Deletion     0 0
16 F       Student      Middle    Realized     0 0
17 F       Student      Young     Deletion    52 0.261
18 F       Student      Young     Realized   147 0.739
19 M       Educated     Old       Deletion     0 0
20 M       Educated     Old       Realized     0 0
21 M       Educated     Middle    Deletion    16 0.348
22 M       Educated     Middle    Realized    30 0.652
23 M       Educated     Young     Deletion    48 0.384
24 M       Educated     Young     Realized    77 0.616
25 M       Not Educated Old       Deletion    24 0.471
26 M       Not Educated Old       Realized    27 0.529
27 M       Not Educated Middle    Deletion    36 0.474
28 M       Not Educated Middle    Realized    40 0.526
29 M       Not Educated Young     Deletion    24 0.436
30 M       Not Educated Young     Realized    31 0.564
31 M       Student      Old       Deletion     0 0
32 M       Student      Old       Realized     0 0
33 M       Student      Middle    Deletion     0 0
34 M       Student      Middle    Realized     0 0
35 M       Student      Young     Deletion    50 0.282
36 M       Student      Young     Realized   127 0.718
```

The easiest way to convert NaN (or Na) to 0 is to assign the above to a variable, then replace NaN with 0 using the function is.nan(). IF there were NA values, you can do the same thing as above, but replace is.nan() with is.na()

When we report proportions in sociolinguistics manuscripts, we often only report the proportion of one level of the dependent variable (called the application value). Do only display one of the two levels of Dep.Var — for instance, if we just want to show the rates of Deletion, which we might decide is our application value — we can use the subset() function.

```
# Create the results object, but subsetted to include only Deletion tokens
results <- td %>%
    group_by(Sex, Education, Age.Group, Dep.Var, .drop = FALSE) %>%
    summarize(Count = n()) %>%
    mutate(Prop = Count/sum(Count)) %>%
    subset(Dep.Var == "Deletion")

# Recode NaN to 0
results$Prop[is.nan(results$Prop)] <- 0
# Print results
print(results, n = Inf)
```

```
# A tibble: 18 x 6
# Groups:   Sex, Education, Age.Group [18]
   Sex   Education     Age.Group Dep.Var   Count   Prop
   <fct> <fct>         <fct>     <fct>     <int>  <dbl>
 1 F     Educated      Old       Deletion      2 0.0625
 2 F     Educated      Middle    Deletion     68 0.308
 3 F     Educated      Young     Deletion     20 0.278
 4 F     Not Educated  Old       Deletion     41 0.347
 5 F     Not Educated  Middle    Deletion      5 0.294
 6 F     Not Educated  Young     Deletion      0 0
 7 F     Student       Old       Deletion      0 0
 8 F     Student       Middle    Deletion      0 0
 9 F     Student       Young     Deletion     52 0.261
10 M     Educated      Old       Deletion      0 0
11 M     Educated      Middle    Deletion     16 0.348
12 M     Educated      Young     Deletion     48 0.384
13 M     Not Educated  Old       Deletion     24 0.471
14 M     Not Educated  Middle    Deletion     36 0.474
15 M     Not Educated  Young     Deletion     24 0.436
16 M     Student       Old       Deletion      0 0
17 M     Student       Middle    Deletion      0 0
18 M     Student       Young     Deletion     50 0.282
```

Finally, if we also want to add the total number of tokens per category (something we usually report alongside the application value) we can add another column using `mutate()`. Also, if we want the percentage instead of proportion, we can add `100 *` to the proportion equation (as percentage is proportion $\times 100$)

```
# Generate results object with percentage instead of proportion and a column
# with total tokens per combination.
results <- td %>%
    group_by(Sex, Education, Age.Group, Dep.Var, .drop = FALSE) %>%
    summarize(Count = n()) %>%
    mutate(Percentage = 100 * Count/sum(Count), Total.N = sum(Count)) %>%
    subset(Dep.Var == "Deletion")

# Recode NaN to 0
results$Percentage[is.nan(results$Percentage)] <- 0
# Print results
print(results, n = Inf)
```

```
# A tibble: 18 x 7
# Groups:   Sex, Education, Age.Group [18]
   Sex   Education     Age.Group Dep.Var  Count Percentage Total.N
   <fct> <fct>         <fct>     <fct>    <int>      <dbl>   <int>
 1 F     Educated      Old       Deletion     2       6.25      32
 2 F     Educated      Middle    Deletion    68      30.8      221
 3 F     Educated      Young     Deletion    20      27.8       72
 4 F     Not Educated  Old       Deletion    41      34.7      118
 5 F     Not Educated  Middle    Deletion     5      29.4       17
 6 F     Not Educated  Young     Deletion     0       0          0
 7 F     Student       Old       Deletion     0       0          0
 8 F     Student       Middle    Deletion     0       0          0
 9 F     Student       Young     Deletion    52      26.1      199
10 M     Educated      Old       Deletion     0       0          0
11 M     Educated      Middle    Deletion    16      34.8       46
12 M     Educated      Young     Deletion    48      38.4      125
13 M     Not Educated  Old       Deletion    24      47.1       51
14 M     Not Educated  Middle    Deletion    36      47.4       76
15 M     Not Educated  Young     Deletion    24      43.6       55
16 M     Student       Old       Deletion     0       0          0
17 M     Student       Middle    Deletion     0       0          0
18 M     Student       Young     Deletion    50      28.2      177
```

The above results show that there are 32 tokens from old, educated females, 2 of which (or 6.25%) are `Deletion`.