NAME: ____

CSE585/EE555: Digital Image Processing II
Spring 2020
Exam #2

given: 1:30PM — 3:15PM, Thursday, 30 April 2020. On-line: CANVAS/Zoom.

Two $8\frac{1}{2}$" $\times$11" pages of notes, both sides, allowed.

	1.	22 pts
	2.	18 pts
	3.	25 pts
	4.	17 pts
	5.	18 pts
	TOTAL	100 pts

1. (22 pts) Answer each of the following T (true) or F (False). (2 pts each.)

____ **a.** Suppose $p(i, j, \theta)$ is the $N_g \times m$ run length matrix ($\theta = 0^o$) for a digital image having N_g distinct gray levels and run lengths up to length m. Then

$$\sum_i \sum_j p(i, j, \theta) = 1$$

____ **b.** The opening filter is a very good filter to apply for reducing pepper noise.

____ **c.** The median filter has the property of edge preservation.

____ **d.** Suppose we use the following conduction coefficient

$$c(x, y) = \begin{cases} 1, & \text{if } ||\nabla I(x, y)|| < 20 \\ 0, & \text{if } ||\nabla I(x, y)|| \geq 20 \end{cases}$$

in anisotropic diffusion to filter an 8-bit gray-scale image where $\nabla I(x, y)$ signifies the gradient of the image at (x, y). Then, this could be an effective $c(x, y)$ to use for noise reduction and edge preservation.

Consider two random fractals defined by functions, $I_1(x, y)$ and $I_2(x, y)$. We known that their respective Hurst parameters are $H_1 = 0.01$ and $H_2 = 0.99$.

____ **e.** The fractal dimension of $I_1(x, y) = 1.99$.

____ **f.** Fractal I_1 is rougher than fractal I_2.

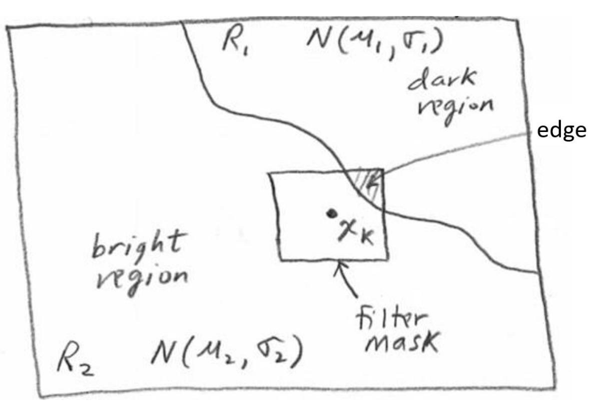
____ **g.** Fractal I_1 has a higher persistence than I_2.

____ **h.** The function

$$g(x) = 2I_1(x) \ - 3 \ + \ \frac{1}{2} I_1 \left(x - \frac{1}{2} \right)$$

is also fractal Brownian and characterized by the Hurst parameter $H = 0.01$.

Gray-scale image I below has two regions, R_1 and R_2, separated by an edge, as discussed in L12-9. Assume dark region R_1 and bright region R_2 have $\sigma_1 = \sigma_2$. Also, at pixel location x_k, a 7×7 filter is applied as indicated by the filter mask; assume when the filter is applied, it is used to filter the whole image. Answer the questions below.



____ **i.** After a sigma filter ($\sigma = \sigma_1$) is applied I, σ_1 for R_1 is likely to get smaller.

____ **j.** After applying a median filter to original image I, the bright region becomes brighter.

____ **k.** After an alpha-trimmed mean filter($\alpha = 0.49$) is applied to original image I, the edge is likely to become blurred.

2. (18 pts) Consider the two 5×5 textures, I_1 and I_2, below. The pixels in these two textures can take on gray-level values 0, 1, 2, or 3.

0	1	2	3	3
0	1	2	3	3
0	1	2	3	3
0	1	2	3	3
0	1	2	3	3

I_1

1	1	1	0	0
1	1	1	0	0
1	1	1	0	0
1	1	1	0	0
1	1	1	0	0

I_2

Consider the gray-level difference method (GLDM), where

$$p'_j(i, \vec{d}) = \text{probability of gray-level difference } i \text{ between pixels } (x, y) \text{ and } (x, y) + \vec{d}$$

for image I_j, where $\vec{d} = (\Delta x, \Delta y)$ (x is horizontal and y is vertical).

(a) (12 pts) For $\vec{d} = (2, 0)$, compute $p'_1(i, \vec{d})$ and $p'_2(i, \vec{d})$, for I_1 and I_2, respectively. Give your answers in the table below. Note: you only have 15 pairs of pixels to consider for each texture.

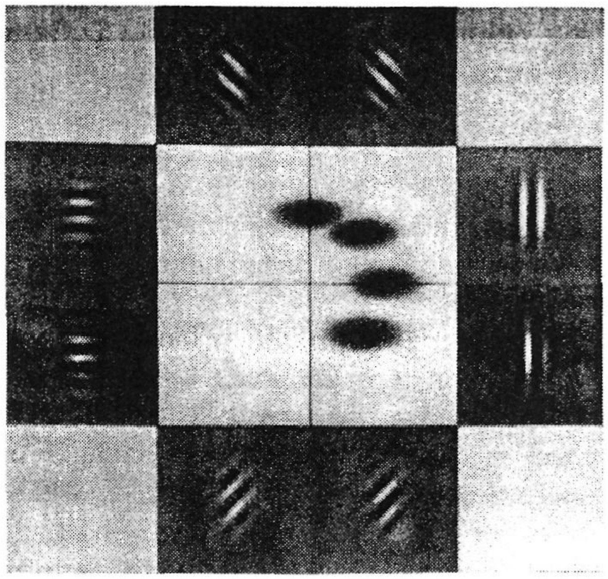
i	0	1	2	3
I_1 : $p'_1(i, \vec{d})$				
I_2 : $p'_2(i, \vec{d})$				

(b) (6 pts) Compute the *Contrast (CON)* for I_1 and I_2. Based on your results, which texture has larger overall gray-level changes? Note: the *CON* of texture I_j is given by

$$CON(I_j) = \sum_i \left[i^2 p'_j(i, \vec{d}) \right]$$

3. (25 pts) Consider the family of four GEFs plotted in the figure below (L17-18 of class notes, Bovik's paper). These GEFs share the following parameters (**changed** slightly from L17!):

- $\lambda = 1/3$, $F = 16$ cycles/image, $\sigma_x = 30$
- The Gaussian part of the GEF is not rotated.



From L17, a GEF is given by the following functions in the space- and frequency-domains:

space: $h(x,y) = g(x',y') \exp\left[j2\pi(Ux + Vy)\right]$, where

$$g(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2\right]\right\}$$

$$(x',y') = (x\cos\phi + y\sin\phi \ , \ -x\sin\phi + y\cos\phi)$$

$$U = F\cos\theta^o, \quad V = F\sin\theta^o$$

frequency: $H(u,v) = \exp\left\{-2\pi^2\left[(\sigma_x[u-U]')^2 + (\sigma_y[v-V]')^2\right]\right\}$,

$$([u-U]',[v-V]') = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} u-U \\ v-V \end{bmatrix}$$

Consider the GEF for $\theta = 90^o$. Assume that the GEF is applied to a 512×512 image $i(x,y)$.

(a) (8 pts) Give the parameters σ_y, U, V, and ϕ defining $h(x,y)$ and $H(u,v)$.

(b) (3 pts) What is F if the units are changed to cycles/pixel?

Problem 3 continued.

(c) (8 pts) Consider the following bipartite-textured image:

$$i(x,y) = \begin{cases} \text{texture } t_1 : & \exp\{j2\pi(15x \;+\; y)\}, \quad x < 0 \;\text{ and }\; y \text{ any} \\ \text{texture } t_2 : & \exp\{j2\pi(x \;+\; 15y)\}, \quad \text{otherwise} \end{cases}$$

Using the GEF above, which texture will the Gabor filter below pass and why?

$$m(x,y) \;=\; |i(x,y) \;*\; h(x,y)|$$

(d) (6 pts) The GEF $h(x,y)$ is *separable* in x and y; i.e., it can be written as

$$h(x,y) = h_1(x) \cdot h_2(y).$$

Thus, from L18, the discrete implementation of the Gabor filter can be computed efficiently with successive 1-D operations. In particular, we can perform the initial calculation for each y [rows] using

$$i_1(x,y) = \sum_{x'=-2\sigma_x}^{+2\sigma_x} i(x-x',y)\widehat{h}_1(x') \tag{1}$$

where $\widehat{h}_1(x)$ is a truncated version of $h_1(x)$ consisting of $4\sigma_x + 1$ values. Following our procedures of Project #4 and assuming that the indices of the 512×512 image $i(x,y)$ range as $0 \le x \le 511$ and $0 \le y \le 511$, what are the minimum and maximum values of x for which we can compute valid values of $i_1(x,y)$?

4. (17 pts) Consider a 1-D image x_k whose pixels vary via the following pdf $f(x)$ and cdf $F(x)$:

$$f(x) \;=\; \begin{cases} 2x, & \text{if } \; 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases} \qquad , \qquad F(x) \;=\; \begin{cases} 0, & \text{if } \; x < 0 \\ x^2, & \text{if } \; 0 \le x \le 1 \\ 1, & \text{if } \; x > 1 \end{cases}$$

Assume all image pixels are iid random variables. We are going to apply a 3-point minimum filter to this image to give filtered image y_k:

$$y_k = X_{(1)} = \min(x_{k-1}, x_k, x_{k+1})$$

For this situation, we know that the cdf of this 3-point minimum filter is given by (L13-2):

$$F_{(1)}(x) = 1 \;-\; [1 - F(x)]^3$$

(a) (8 pts) What is Prob$\{x > 1/2\}$ and Prob$\{X_{(1)} > 1/2\}$? (Hint: This is NOT hard!)

(b) (3 pts) We are told that $x_0 > 1/2$. What does this tell us about the probability that $x_1 > 1/2$?

(c) (6 pts) Give an expression for the pdf $f_{(1)}(x)$ in terms of $f(x)$ and $F(x)$.

5. (18 pts) A deterministic fractal $f(x)$ has fractal dimension

$$D = \frac{\log 6}{\log 5} \quad .$$

(a) (6 pts) Consider a replacement rule for constructing $f(x)$ with this value of D.
(i) How many segments are needed for the replacement rule?
(ii) What is the length of each segment relative to the current scale?
(iii) Is this replacement rule unique for this particular D?

Now, consider the affine linear mapping $\quad w \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

defined on complete metric space $(\mathcal{R}^2, d)$, where $\quad d(p_1, p_2) = |x_1 - x_2| + |y_1 - y_2|$

and $p_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ and $p_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ are two points in $\mathcal{R}^2$. (The metric d is commonly referred to as the "city block" distance or l_1 norm.) We now apply mapping $w \begin{bmatrix} x \\ y \end{bmatrix}$ to the line segment delineated by the points $\quad p_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 16 \\ 16 \end{bmatrix}$ and $p_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 32 \\ 32 \end{bmatrix}$.

(b) (8 pts) Apply mapping $w \begin{bmatrix} x \\ y \end{bmatrix}$ two successive times to these two points; i.e., compute $w{\circ}1 \begin{bmatrix} x \\ y \end{bmatrix}$ and $w{\circ}2 \begin{bmatrix} x \\ y \end{bmatrix}$ for p_1 and p_2.

(c) (4 pts) From your results of (b), does $w \begin{bmatrix} x \\ y \end{bmatrix}$ appear to be a contractive transformation? If so, give a plausible contractivity factor c.