NAME: ____

CSE585/EE555: Digital Image Processing II
Spring 2020
Exam #1: Exam Questions

given: 6:15PM — 9:00PM, Wednesday, 18 March 2020.

One $8\frac{1}{2}$"$\times$11" page of notes, both sides, "allowed." (Effectively open notes.)

Work SEPARATELY! It is not permitted to work with others on this exam.

Give all answers on the ANSWER SHEET.

	1.	24 pts
	2.	10 pts
	3.	12 pts
	4.	18 pts
	5.	20 pts
	6.	16 pts
	TOTAL	100 pts

Dr. Higgins on call at 814-865-0186 during the exam.

During the exam, stay on CANVAS (Exam #1 tab) and email to receive possible updates.

1. (24 pts) Answer each of the following T (true) or F (False). (2 pts each.)

____ **a.** Suppose f is a root of operator $\Psi(f)$; i.e., $\Psi(f) = f$. Then, $\Psi(\cdot)$ is idempotent.

____ **b.** Consider 3-point filter $y_k = f(x_{k-1}, x_k, x_{k+1})$, where f is the exclusive-or function. Then, this filter can be defined as a stack filter and associated positive Boolean function.

____ **c.** Suppose $f(x)$ and $g(x)$ are 1-D gray-scale images satisfying $f(x) \geq g(x)$ for all x. Then, $f(x) \geq g^G(x)$ for all x, where g^G is the closing of g by structuring element G.

____ **d.** Suppose for a function f, $\mathrm{med}(f; A) = f$. Then, $\mathrm{med}^{\infty}(f; A) = f$.

____ **e.** We know that $x \in X \ominus B = \bigcap_{b \in B^s} X_b$. Then, $\forall b \in B^s, x_{-b} \in B$.

____ **f.** Consider the following statement

$$(X \ominus B_1^s) \oplus B_2^s = B_2 \oplus (X \ominus B_1)$$

where $X \neq \emptyset$, $B_1 \neq \emptyset$, $B_2 \neq \emptyset$, and B^s signifies the symmetrical set of B. The above statement is true if $B_1 = B_1^s$ and $B_2 = B_2^s$.

Consider sets $X \neq \emptyset$ and $Y \neq \emptyset$ and structuring elements $B \neq \emptyset$ and $C \neq \emptyset$.

____ **g.** For all sets $X, Y, B \subset \mathcal{R}^n$ and structuring element $B \neq \emptyset$, operator $\Psi_B(X)$ satisfies

$$X \subset Y \Longrightarrow \Psi_B(X) \subset \Psi_B(Y).$$

Then, $Y \subset \Psi(Y)$.

____ **h.** $(X \ominus B) \oplus C = (X \oplus C) \ \ominus B$.

____ **i.** $X \ominus (C \oplus B) = (X \ominus B) \ominus C$.

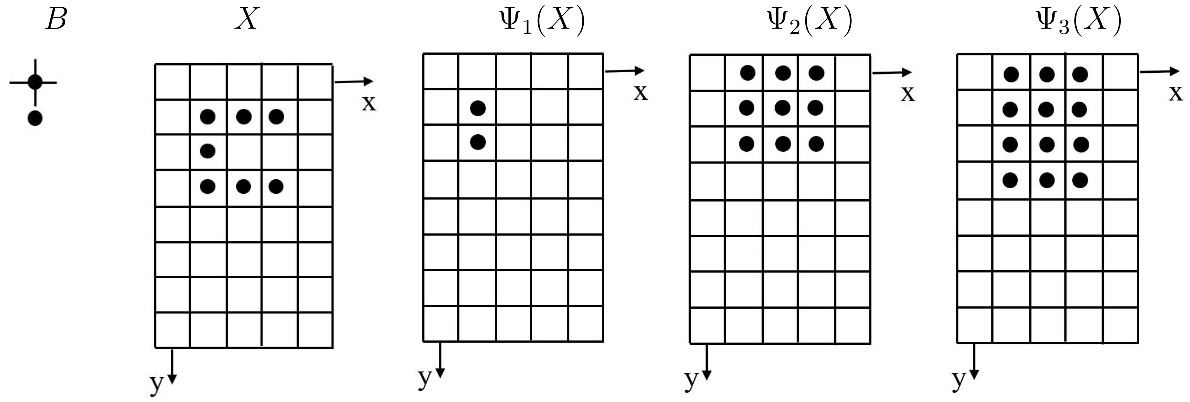
Now, consider gray-scale images $f(x)$ and $g(x)$ defined (i.e., $\neq -\infty$) for $-\infty \leq x \leq \infty$.

____ **j.** $\mathrm{U}[T(f)] = f$.

____ **k.** $f \vee g = \mathrm{T}(U(f \vee g))$.

____ **l.** $\mathrm{U}(f \wedge g) = \mathrm{U}[f]) \cap \mathrm{U}[g]$.

2. (10 pts) Binary Morphology. Consider the 2-point structuring element B and 8×5 image X depicted below. The "•" symbol denotes "1" pixels, "+" denotes the origin of B, and the upper left-hand corner of X denotes X's origin. You may disregard border pixels; i.e., assume pixels outside of X are "0" pixels.



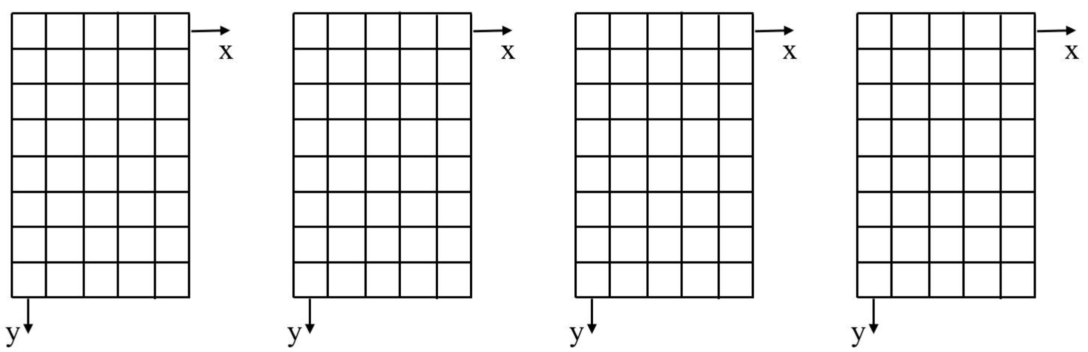
The figures $\Psi_i(X)$, $i = 1, 2, 3$, represent X after an operation $\Psi_i(\cdot)$ is performed on X. These operations correspond to one of the choices listed below:

(a) X dilated by B (b) $X \oplus B$ (c) X closed by B

(d) X eroded by B (c) $X \ominus B$ (e) X opened by B

Specify the choice (a-e) that corresponds to each output $\Psi_i(X)$, $i = 1, 2, 3$.

You can use the blank figures below to help you arrive at your answers — these will NOT be considered in your answers to the questions above.



3. (12 pts) Skeletonization. Consider the morphological skeleton $sk(X)$ of a 2-D digital object X, based on structuring element B. Some helpful general relations related to $sk(X)$ appear below:

$$sk(X) = \bigcup_{0 \le n \le N} S_n(X) \qquad \text{where:}$$

$$S_n(X) = (X \ominus nB) - (X \ominus nB)_B \qquad n^{th} \text{ skeletal component}$$

$$nB = \underbrace{(B \oplus B \oplus B \oplus \ldots \oplus B)}_{n\ B's} \qquad n^{th}\text{-order homothetic of } B$$

$$\text{where } n = 0, 1, 2, \ldots, N, \qquad N = \max\{k : \ X \ominus kB \neq \emptyset\}$$

$$skf(x,y) = \begin{cases} n, & (x,y) \in S_n(X) \\ \text{undefined}, & \text{otherwise} \end{cases}$$

$$X_{kB} = \bigcup_{n=k}^{N} (S_n(X) \oplus nb)$$

Let $B = 3 \times 3$ square centered about (0,0). We are told the following about a specific $X \neq \emptyset$:
i.) $X_B = X$; ii.) $X_{2B} \subset X_B$; iii.) $X_{2B} \neq \emptyset$, but $X_{3B} = \emptyset$.
Answer the following questions.

(a) (4 pts) Which skeletal components $S_n(X)$ do not equal the empty set?

(b) (3 pts) What is $skf(x,y)$ when $(x,y) \in S_2(X)$?

(c) (5 pts) X_B can be written as the union of select components $S_n(X) \oplus nB$. Precisely define this union by only including components $\neq \emptyset$.

4. (18 pts) Size distribution and pattern spectrum. $U(n) = \text{area}\,[X_{nB}]$ and $f(n) = \frac{U(n)-U(n+1)}{\text{area}(X)}$ denote the size distribution and pattern spectrum of an object X relative to 3×3 square structuring element B. Also, we know

a.) $$U(n) = \begin{cases} 50, & \text{if } n = 0 \\ \text{unknown}, & \text{if } n = 1 \\ 30, & \text{if } n = 2 \\ 0, & \text{if } n > 2 \end{cases}$$

b.) $f(1) = 0.36$

Answer the following questions.

(a) (4 pts) What is area(X)?

(b) (14 pts) Give the complete size distribution and pattern spectrum of X in the table below.

n	0	1	2	3
$U(n)$				
$f(n)$				

5. (20 pts) Order-statistics filtering. A 1-D gray-scale image x_k passes through a 3-point minimum filter given by

   $y_k = f(x_{k-1}, x_k, x_{k+1})$ to produce output y_k, where $x_k \in Q = \{0, 1, 2, 3\}$.

   For the situation shown below, we know the following:

   1. The input x_k is known for $1 \le k \le 6$, while x_k is unknown at $k = 0$ and $k = 7$.
   2. The output y_k is known at $k = 1$ and $k = 6$.
   3. Per our class convention, $T_i(\cdot)$ denotes the threshold set at value i, while

      $f(T_i(\cdot))$ denotes the filtered version of $T_i(\cdot)$.
   4. "X" values in the output signify points NOT to process.
   5. "minimum" and "binary min" denote that minimum filter applied to gray-scale and

      binary data, respectively.

INPUT

	0	1	2	3	4	5	6	7
k:	0	1	2	3	4	5	6	7
x_k:	a	2	3	0	3	1	3	b
$T_3(\cdot)$:								
$T_2(\cdot)$:								
$T_1(\cdot)$:								

Input		Filter		Output
x_k	$\rightarrow$	minimum	$\rightarrow$	y_k
$T_3(\cdot)$	$\rightarrow$	binary min	$\rightarrow$	$f(T_3(\cdot))$
$T_2(\cdot)$	$\rightarrow$	binary min	$\rightarrow$	$f(T_2(\cdot))$
$T_1(\cdot)$	$\rightarrow$	binary min	$\rightarrow$	$f(T_1(\cdot))$

OUTPUT

k:	0	1	2	3	4	5	6	7	
	X	1					1	X	y_k
	X							X	$f(T_3(\cdot))$
	X							X	$f(T_2(\cdot))$
	X							X	$f(T_1(\cdot))$

(a) (8 pts) What are the possible values of "a" and "b"?

(b) (14 pts) Fill in the table above. In particular, be sure to define

output y_k, threshold sets $T_i(\cdot)$, and filtered threshold sets $f(T_i(\cdot))$.

If "a" or "b" can take on more than one value, pick a value for your results.

6. (16 pts) Stack filter. We know that a stack filter

$$y_k = S_f(x_{k-1}, x_k, x_{k+1})$$

is defined by positive Boolean function B. $B = 1$ is when both (a.) and (b.) below are true:

a.) $x_k = 1$

b.) $x_{k-1} = 1$ or $x_{k+1} = 1$.

Derive the positive Boolean function for this filter. In particular, use the answer sheet to give a truth table for this filter and to give your final positive Boolean function.

Note: feel free to write terms such as x_{k-1} as $x(k-1)$ when giving your answer.