Ling Zhang HW4

1. (a) $K = \begin{bmatrix} \phi_x & \gamma & \delta_x \\ 0 & \phi_y & \delta_y \\ 0 & 0 & 1 \end{bmatrix}$

$$[K\ I] = \begin{bmatrix} \phi_x & \gamma & \delta_x & 1 & 0 & 0 \\ 0 & \phi_y & \delta_y & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} \phi_x & \gamma & 0 & 1 & 0 & -\delta_x \\ 0 & \phi_y & 0 & 0 & 1 & -\delta_y \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \phi_x & 0 & 0 & 1 & -\frac{\gamma}{\phi_y} & \frac{-\delta_x\phi_y + \gamma\delta_y}{\phi_y} \\ 0 & 1 & 0 & 0 & \frac{1}{\phi_y} & -\frac{\delta_y}{\phi_y} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{1}{\phi_x} & -\frac{\gamma}{\phi_x\phi_y} & \frac{-\delta_x\phi_y + \gamma\delta_y}{\phi_x\phi_y} \\ 0 & 1 & 0 & 0 & \frac{1}{\phi_y} & -\frac{\delta_y}{\phi_y} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad K^{-1} = \begin{bmatrix} \frac{1}{\phi_x} & -\frac{\gamma}{\phi_x\phi_y} & \frac{-\delta_x\phi_y + \gamma\delta_y}{\phi_x\phi_y} \\ 0 & \frac{1}{\phi_y} & -\frac{\delta_y}{\phi_y} \\ 0 & 0 & 1 \end{bmatrix}$$

K^{-1} is also an upper triangular matrix

(b) $$x' = \frac{x}{\phi_x} - \frac{\gamma y}{\phi_x\phi_y} + \frac{-\delta_x\phi_y + \gamma\delta_y}{\phi_x\phi_y}$$

$$y' = \frac{y}{\phi_y} - \frac{\delta_y}{\phi_y}$$

2. $$P_{c_1} = R_1 P_w + t_1$$
$$P_w = R_1^{-1}(P_{c_1} - t_1)$$

$$P_{c_2} = R_2 R_1^{-1}(P_{c_1} - t_1) + t_2$$
$$= R_2 R_1^{-1} P_{c_1} - R_2 R_1^{-1} t_1 + t_2$$

$$\therefore \quad \Omega = R_2 R_1^{-1}$$
$$\tau = t_2 - R_2 R_1^{-1} t_1$$

3. $\lambda_i = P_{31} u_i + P_{32} v_i + P_{33} w_i + P_{34}$

$\lambda x_i = P_{11} u_i + P_{12} v_i + P_{13} w_i + P_{14}$

$\lambda y_i = P_{21} u_i + P_{22} v_i + P_{23} w_i + P_{24}$

$$P_{11} u_i + P_{12} v_i + P_{13} w_i + P_{14} - (P_{31} u_i + P_{32} v_i + P_{33} w_i + P_{34}) x_i = 0$$

$$P_{21} u_i + P_{22} v_i + P_{23} w_i + P_{24} - (P_{31} u_i + P_{32} v_i + P_{33} w_i + P_{34}) y_i = 0$$

$X = [P_{11} \ P_{12} \ P_{13} \ P_{14} \ P_{21} \ P_{22} \ P_{23} \ P_{24} \ P_{31} \ P_{32} \ P_{34} \ P_{44}]^T$

$$A: \begin{bmatrix} u_1 & v_1 & w_1 & 1 & 0 & 0 & 0 & 0 & -u_1x_1 & -v_1x_1 & -w_1x_1 & -x_1 \\ 0 & 0 & 0 & 0 & u_1 & v_1 & w_1 & 1 & -u_1y_1 & -v_1y_1 & -w_1y_1 & -y_1 \\ & & & & & \vdots & & & & & & \\ u_N & v_N & w_N & 1 & 0 & 0 & 0 & 0 & -u_Nx_N & -v_Nx_N & -w_Nx_N & -x_N \\ 0 & 0 & 0 & 0 & u_N & v_N & w_N & 1 & -u_Ny_N & -v_Ny_N & -w_Ny_N & -y_N \end{bmatrix}$$

Columns : 2N Rows : 12

4. (a). RMS for rubik1 is 3.572.

for rubik2 is 3.163

(b) Yes. they are reasonable. δx and δy are close.

Yes. $\frac{\phi x}{\phi y} \approx 1$ Yes. skew γ is small.

Here is the screenshot of K1 and K2

Pworldpts | Pimagepts1 | K2 | K1

K1: 3x3 double

	1	2	3
1	0.0526	-1.4509e-04	0.0213
2	0	0.0525	0.0152
3	0	0	2.1659e-05

Pworldpts | Pimagepts1 | K2 | K1

K2: 3x3 double

	1	2	3
1	0.0562	-2.5104e-04	0.0199
2	0	0.0565	0.0182
3	0	0	2.3462e-05

(c).

```
field of view of camera1 horizontal is 0.522
field of view of camera1 vertical is 0.759
field of view of camera2 horizontal is 0.528
field of view of camera2 vertical is 0.764
>>
```

(d). We only use 37 data point. No enough data points Pictures taken from real world to digital. Some information lost.

(e) norm (inv(K1)* K2) = 1.1121

norm (inv (K2) * K1) = 0.9585

Both of them are close to 1.

Screenshot for inv(K1)*K2 and inv(K2)*K1

```
>> inv(K1)*K2

ans =

    1.0687   -0.0018   -0.0586
         0    1.0758    0.0331
         0         0    1.0833
```

```
>> inv(K2)*K1

ans =

    0.9358    0.0016    0.0506
         0    0.9295   -0.0284
         0         0    0.9231
```

5.(a). RMS is 0.111