Problem Set 9

1. A non-relativistic particle of mass m and charge q is in the ground state of a one dimensional harmonic oscillator potential $V(x) = m\omega^2 x^2/2$. At $t = 0$ it is subjected to an electric field pulse of duration τ. The perturbation is equivalent to a potential of the form

$$V(x,t) \;=\; \begin{cases} -q\mathcal{E}x & 0 \le t \le \tau \\ 0 & t < 0 \text{ and } t > \tau \end{cases} ,$$

   where x is the position of the particle and $\mathcal{E}$ is the electric field. Compute the amplitude of finding the particle in the first excited state at the end of the pulse as a power series in $\mathcal{E}$ up to second order by using the fact that the additional potential simply translates the harmonic oscillator potential by $X_0 = q\mathcal{E}/m\omega^2$. Compare this to the expression you would obtain using time dependent perturbation theory up to second order.

2. Repeat the previous problem for the amplitude of finding the particle in the second excited state at the end of the pulse.

3. A spin-1/2 particle is subjected to a time dependent magnetic field such that the Hamiltonian for the problem is given by

$$H \;=\; \omega_0 S_z + \lambda \exp(-t^2/\tau^2) S_x .$$

   If the particle is in the eigenstate of S_z with eigenvalue $-\hbar/2$ at $t = -\infty$, using first order perturbation theory compute the probability to find the particle in the eigenstate of S_z with eigenvalue $+\hbar/2$ at $t = +\infty$.

4. Consider a spin-1 atom subjected to a time dependent electromagnetic field such that the Hamiltonian is given by

$$H \;=\; \omega\Big(J_x \cos(\omega t) + J_y \sin(\omega t)\Big)$$

   If the atom is in the eigenstate of J_z with eigenvalue $-\hbar$ at $t = 0$, find the probability to find it in the eigenstate of J_z with eigenvalue $\hbar$ at a later time t without approximations.

5. Consider the physics of a resonant level decay. Begin with an $N+1$ dimensional state space spanned by basis vectors $|n\rangle, n = 0, 1, 2..., N-1$ and the renonant level $|b\rangle$. The Hamiltonian for the system is given by

$$H \;=\; \hbar\omega_0 |b\rangle\langle b| + \hbar\omega \sum_n (2|n\rangle\langle n| - |n\rangle\langle n+1| - |n+1\rangle\langle n|) + \hbar\delta(|b\rangle\langle 0| + |0\rangle\langle b|)$$

   where we assume periodic boundary conditions $|n = N\rangle \equiv |n = 0\rangle$. The particle is given to be in the state $|b\rangle$ at $t = 0$. For $N = 100$ use mathematica (or a similar software) plot the probability of finding the particle in the resonant level at time $0 \le \omega t \le 200$ assuming $\omega_0/\omega = 0.5$ and $\delta/\omega = 0.1$. Repeat this calculation with $\delta/\omega = 0.5$.

6. In class we learned how to deal with the previous problem using Fermi's Golden rule to first order. Plot this simple analytic prediction on top of the exact plots for the two cases from the previous problem. Find the rough time t when the exact answer begins to deviate from first order perturbation theory by about 10%.