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Tilt and azimuth angles in solar energy applications – A review

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ABSTRACT

This paper presents a review of tilt angle and azimuth angles in solar energy applications. The paper involves an overview of design parameter, applications, simulations and mathematical techniques covering different usage application. The number of references analysing the tilt angle deployment in the context of the research papers of the different countries currently having operations in solar systems is much more significant. Different kinds of models and test methods of optimum tilt angle in different solar systems have been developed since 1956 which can be distinguished by their particular mathematical models or tracking techniques as shown in the latest researches. The mathematical models allows the calculation of different parameters of the solar radiation, the angle of inclination, and the optimum tilt angle of the collecting surface and the effects acting on the system.

1. Introduction

Concentrated solar power generation is considered one of the most promising renewable sources as the technologies are very close, in concept to conventional and traditional forms of power generation based on fossil-fuel combustion. Solar concentration is carried out in most of the solar systems by tracking the sun direction to focus the incident rays on a receiver, where a thermal process and generator unit is located to convert the solar energy into electric energy. This technology has many applications in relatively small, medium and large capacities. However, larger capacities are achieved by integrating small units to achieve solar collectors' farm. Currently, there are four main technologies that utilize the concentrated solar thermal energy which are (a) the parabolic trough systems, (b) the solar tower systems, (c) the Stirling solar dish systems and (d) the linear Fresnel systems. There are other applications to utilize the solar radiation in cooking or solar water heating. The researches discussed the best performance, design, simulation for the solar energy systems using optimum tilt angles [1–202]. There are number of studies and researches that were carried out in order to find the best performance of solar system areas around the world, and others in a comparison between different locations. There are numerous applications regarding the optimum tilt angle for a specific geographic location, as for example Photovoltaic systems [1–47], Solar Water Heater [48–55], solar cooker [56,57], solar still [58–63], solar powered thermoacoustic engines [64–67], building-integrated photovoltaic system (BIPV) [28,68–72], solar cooling [73], solar updraft tower power plant [74], and solar collectors [75–112], all of which are discussed in detail in this study.

A number of studies were carried out to find the optimum tilt angle and orientation (azimuth) of PV systems, solar collectors, or any other application in certain areas around the world, such as (Brisbane, Australia [3], Abu Dhabi, UAE [5], eight provinces of Turkey [8], Turkey [40], Izmir in Turkey [92,175,192], Athens, Greece [13], Greece [197], Madinah, Saudi Arabia [17], United States of America (USA) [21,188], Carbondale, Illinois, USA [189], North America [107], Canada [23], Taiwan [26], Sanliurfa, New Delhi, India [34], Egypt [39,104], Helwan, Egypt [141], Jordan [55], Iran [85,87], Basra, Iraq [103], Syria [89], 30 cities in China [93], Changsha, China [202], Malaysian territory [96], Perlis, Northern Malaysia [199], Kuala Lumpur, Johor Bharu, Ipoh, Kuching, Alor Setar in Malaysia [200], Kuala Lumpur, Malaysia [201], India [126], Dhaka, Bangladesh [127], South Africa [177,185], Japan [187], Cyprus [190], Burgos, Spain [191], Italy [193], Romania [194], Brunei Darussalam [195], Ghana [196], Singapore [198], the examples could continue and many more).

Abbreviation: ADHDEOA, AntDirection Hybrid Differential Evolution Algorithm; AI, Artificial Intelligence; ANN, Artificial Neural Network; BIPV, Building Integrated Photovoltaic System; CC, Compensation Chamber; CO_2, Carbon Dioxide; DFR, Diffuse Flat Reflector; DG, Diesel Generator; GA, Genetic Algorithm; GRNN, Generalized Regression Neural Networks; HDKR, Hay, Davies, Klucher, Reindl; LATITS, Large Angle Tilt Implantation of dopant Through Gate Sidewall Spacer; MAPV, Mirror Augmented Photovoltaic System;; PSO, Particle-Swarm Optimization; PSO—NTVE, Particle-Swarm Optimization Method with Nonlinear Time-Varying Evolution; PV, Photovoltaic; PVSYST, Photovoltaic Systems; RBFNN, A Radial Basis Function Neural Network; SNAOA, Sequential Neural-Network Approximation and Orthogonal Arrays; SPVEGS, Stand alone Photovoltaic Electricity Generation Systems; SUPP, Solar Updraft Tower power plant; SWH, Solar Water Heaters; TFT, Thin Film Transistor; TRNSYS, Transient System; UAE, United Arab Emirates; US, United States

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Nomenclature

G_T	The global irradiance, W/m^2
G_{B_t}	Total Beam solar radiation on a tilted surface, W/m^2
G_{D_t}	Total Diffuse solar radiation on a tilted surface, W/m^2
G_{R_t}	Total Ground-reflected solar radiation on a tilted surface, W/m^2
G_B	Beam radiation on a horizontal surface, W/m^2
G_{B_n}	Beam radiation in the direction of the rays, W/m^2
R_B	The beam radiation tilt factor
R_D	The diffuse radiation tilt factor
R_R	The ground-reflected radiation tilt factor

Greek symbols

β_{opt}	Optimum tilt angle, °
φ	Geographical latitude, °
δ	Declination angle, °
ω_{ss}	Sunset hour angle, °
β	Surface slope from the horizontal, °
ρ	Ground albedo

The techniques to optimize the tilt angle are shown in details where the most effective methods of maximizing solar radiation or energy collected on the surfaces, ANN, GA, SA and PSO techniques and all these techniques are shown by different optimum tilt angles equations such as Stanciu C. and Stanciu D. [78], $\beta_{opt} = \varphi - \delta$, Bakirci [8], $\beta_{opt} = 34.783 - 1.4317\delta - 0.0081\delta^2 + 0.0002\delta^3$, Rowlands et al. [23], $\beta_{opt} = \varphi$, Lave and Kleissl [21], $\beta_{opt} = \varphi - (1^o - 10^o)$, Moghadam et al. [87],$\beta_{opt} = 0.917\varphi \pm 0.321^o$, Benghanem [17], $\beta_{opt} = \varphi$, Ahmad and Tiwari [34] India in Summer is 13°, $\beta_{opt} = \varphi - 60^o$, And Winter is 47.5°, $\beta_{opt} = \varphi + 90^o$, Calabròa [31], $\beta_{opt} = \varphi - (26^o, 27^o, 28^o)$, Gunerhan and Hepbasli [92], In summer $\beta_{opt} = \varphi + 15^o$, In winter $\beta_{opt} = \varphi - 15^o$, Ulgen [175], for summer months is $\beta_{opt} = \varphi - 34^o$, and that for winter months is $\beta_{opt} = \varphi + 19^o$, Elminir et al. [141], $\beta_{opt} = \varphi \pm 15^o$, Duffie and Beckman [176], $\beta_{opt} = \varphi \pm 15^o$, Rusheng Tang and Tong [93], $\beta_{opt} = \varphi + (4^o \rightarrow -10^o)$, Gopinathan [177],$\beta_{opt} = \varphi$, El-Kassaby [104], $\beta_{opt} = \varphi + 3.5^o$, Lewis [178], $\beta_{opt} = \varphi \pm 8^o$, Garg [179], $\beta_{opt} = \varphi \pm 15^o$ or $\beta_{opt} = 0.9\varphi$, Duffie and Beckman [180], $\beta_{opt} = (\varphi + 15^o) \pm 15^o$, Lunde [181], $\beta_{opt} = \varphi \pm 15^o$ Kern and Harris [165], $\beta_{opt} = \varphi + 10^o$, Löf and Tybout [182], $\beta_{opt} = \varphi + (10^o \rightarrow 30^o)$, Yellott [183], $\beta_{opt} = \varphi + 20^o$, Heywood [184], $\beta_{opt} = \varphi - 10^o$, Chinnery [185], $\beta_{opt} = \varphi + 10^o$, Hottel [186], $\beta_{opt} = \varphi + 20^o$.

The main aim of this study is to review the current optimum tilt angles calculation methods to optimize the best design for the solar systems. The study takes into consideration the available solar potential, different techniques of solar tilt calculations, as well as all available analyses for different parts of the system. In Section 2, the use of solar tilt angles in different solar energy systems applications are described. The techniques of the solar radiation calculation are discussed in Section 3. In Section 4, the techniques to optimize the tilt angle and related equations are described. Finally, the conclusion is summarized in Section 5.

2. Tilt angles applications

This section presents an overview on most effective technologies and methods applied, in the latest researches, on the design parameters, applications, simulations and mathematical techniques of a tilt angle in different applications. The optimum design for any solar system will be achieved by the selection of the optimum components and materials, best analysis using simulation programs, mathematical techniques. The present research is focused on the effect of tilt angle on design. Many researchers discussed the optimum tilt angle, as showed in Tables (1–4) to achieve more efficient solar system designs using different techniques. This section will describe briefly these techniques in order to reach the most appropriate application for the system.

Most of the solar energy systems track the direction of the sun to focus the heat on the receiver, where a thermal process and generator unit is located or to collect the maximum solar radiation from the sun. This technology has many applications in relatively small, medium and large capacities. However, larger capacities are achieved by integrating small units to achieve solar collectors' farm. There are numerous applications regarding the optimum tilt angle for a specific geographic location, as for example Photovoltaic systems [1–47], Solar Water Heater [48–55], solar cooker [56,57], solar still [58–63], solar powered thermoacoustic engines [64–67], building-integrated photovoltaic system (BIPV) [28,68–72], solar cooling [73], solar updraft tower power plant [74], and solar collectors [75–112] these applications are discussed in details in this section.

Table (1) shows the current researches in the tilt angle covering different technologies used in Photovoltaic systems. Table (2) shows the current researches in the tilt angle covering the technologies used in solar water heaters, solar cookers, and solar still systems. Table (3) shows the current researches in the tilt angle used in solar powered thermoacoustic engines, building-integrated photovoltaic system (BIPV), solar cooling, and solar updraft tower power plant systems. Table (4) shows the current researches in the tilt angle in solar collectors systems.

2.1. Photovoltaic cells, modules, panels, and power plants

Photovoltaic system is one on the most promising applications in the solar energy field. Optimization of tilt angle is can to achieve the annual optimum tilt angle of the solar panels surface at a certain site or to achieve optimum tilt angle for PV modules to obtain maximum output power.

Hartner et al. [1] analyzed the wholesale market value of PV, potential fuel and CO_2 cost reductions through PV deployment under different tilt angles and orientations in 23 regions of Austria and Germany. Jeyaprabha and Selvakumar [2] described the optimal sizing of PV/battery/DG based hybrid system using optimal tilting of PV panel for remote locations using artificial intelligence (AI) techniques and without metrological data. Yan et al. [3] showed a theoretical model to estimate PV system performance with different tilt angles and orientations calculated in Brisbane, Australia, 26° N facing true North approximately. Ismail et al. [4] showed a hybrid power system from photovoltaic and microturbine at Palestine where optimization of PV tilt angle performed, which varies from 0° to 90° to maximize the annual energy production. Jafarkazemi and Saadabadi [5] obtained optimum tilt angle and orientation of solar cells panels and solar collectors in Abu Dhabi, United Arab Emirates (UAE). Based on the calculation results, the optimum is to change the tilt angle, at least twice a year. Yadav and Chandel [6] reviewed different optimization techniques and methods for determining optimum solar panel tilt angle at any site. Kaldellis et al. [7] investigated the optimum tilt angle in Athens, Greece. One of the PV panels at a fixed angle equals to the theoretical optimum angle and the other panel set to vary under standard angle step at a 15°, from 0° to 90°. Bakirci [8] optimized the tilt angles for the solar panels using solar radiation data measured to eight provinces in Turkey where the optimum tilt angle varies from 0° to 65° throughout the year. Lucio et al. [9] evaluated the optimum tilt angle using an algorithm for obtaining load minimum loss probability and optimum design of stand-alone photovoltaic systems in Europe. Asowata et al. [10] determined and validated the optimum tilt angles

Table 1
The current researches in the tilt angle for covering different technologies used in Photovoltaic systems.

Research Application	Authors	Year	Research Data	Ref.
Photovoltaic Systems	Hartner et al.	(2015)	Analyzed the wholesale market value of PV, potential fuel and CO_2 cost reductions through PV deployment under different tilt angles and orientations in various 23 regions of Austria and Germany.	[1]
	Jeyaprabha and Selvakumar	(2015)	Described the optimal sizing of PV/battery/DG based hybrid system including using optimal tilting of PV panel for different remote locations using artificial intelligence (AI) techniques and without metrological data.	[2]
	Yan et al.	(2013)	Showed a theoretical model to estimate PV system performance with different tilt angles and orientations where calculated it in Brisbane, Australia, 26° N facing true North approximately.	[3]
	Ismail et al.	(2013)	Showed a hybrid power system from photovoltaic and microturbine at Palestine where optimization of PV tilt angle performed, which varies from 0° to 90° to maximize the annual energy production.	[4]
	Jafarkazemi and Saadabadi	(2013)	Obtained optimum tilt angle and orientation of solar cells panels and solar collectors in Abu Dhabi, United Arab Emirates (UAE). Based on the calculation results, the optimum is to change the tilt angle, at least twice a year.	[5]
	Yadav and Chandel	(2013)	Reviewed different optimization techniques and methods for determining optimum solar panel tilt angle at any site.	[6]
	Kaldellis et al.	(2012)	Investigated the optimum tilt angle for PV panels at Athens, Greece. One of the PV panels at a fixed angle equal to the theoretical optimum angle and the other panel set to vary under standard angle step at a 15°, from 0° to 90°.	[7]
	Bakirci	(2012)	Optimized the tilt angles for the solar panels using solar radiation data measured to eight provinces in Turkey where the optimum tilt angle varies from 0° to 65° throughout the year.	[8]
	Lucio et al.	(2012)	Evaluated the optimum tilt angle using an algorithm for obtaining load minimum loss probability and optimum design of stand-alone photovoltaic systems in Europe.	[9]
	Asowata et al.	(2012)	Determined and validated the optimum tilt angles 16°, 26° and 36° analytically using photovoltaic experimental setup for winters at Vaal Triangle, South Africa (Lat. 29° 00′ S and Long 24° 00′ E).	[10]
	Bojić et al.	(2012)	Determined the optimum tilt angles for PV systems that are located in four different towns (Les Avirons, Petite-France, Saint-Benoit, and Piton Saint-Leu) in Reunion Island, France.	[11]
	Siraki and Pillay	(2012)	Calculated the optimum angle of PV array needs at any site using a modified HDKR model (Hay, Davies, Klucher, Reindl) anisotropic sky model that also considers the effects of the adjacent buildings in urban locations.	[12]
	Kaldellis and Zafirakis	(2012)	Evaluated the performance of different PV panel tilt angles during the summer at Athens, Greece and found that the optimum angle during the summer is 15° (+−2.5°).	[13]
	Liu et al.	(2012)	Calculated the optimal tilt angles of PV panels depending on techno-economic optimization of PV system connected to a grid and electricity tariffs.	[14]
	Beringer et al.	(2011)	Investigated the difference between the performances of different eight multicrystalline solar cells at various tilt angles from 0° to 70° in steps of 10° to achieve monthly optimum tilt angles at Hannover, Germany. The maximum power is found to beat 50–70° tilt angles in winter and 0–30° in summer months.	[15]
	Wada et al.	(2011)	Analyzed the annual generated energy of each array in a 100 kW PV system at Gobo city, Wakayama prefecture, Japan, and found that the tilt angle for arrary maximum annual energy is not the same tilt angle to achieve the maximum anuual irradiation.	[16]
	Benghanem	(2011)	Performed a study to achieve the annual optimum tilt angle of the solar panels surface for receiving a maximum solar radiation and found that is approximately equal to the latitude of the site. At Madinah, Saudi Arabia, annual optimum tilt angle is 23.5° with respect to the latitude of Madinah site 24.5°; and for the winter months is 37° and for the summer months is 12°.	[17]
	Talebizadeha et al.	(2011)	Calculated hourly, daily, monthly, seasonally and yearly optimum tilt angle of PV panels and solar collectors using GA at Iran and showed that optimum hourly azimuth angle is not zero.	[18]
	Sunderan et al.	(2011)	Presented a way to enhance the power generation from a stand alone Photovoltaic Electricity Generation Systems (SPVEGS) using the optimum tilt angle and orientation in Ipoh, Malaysia where the optimum PV orientation is North from April to August and south from September to March. The gains for monthly and yearly optimum tilt angles are 6.4% and 6.1% respectively.	[19]
	Maatallah et al.	(2011)	Investigated the effects of a zimuth and tilt angles on the output power of a photovoltaic module.	[20]
	Lave and Kleissl	(2011)	Determined the optimum tilt angle and azimuth angle of solar PV panels at US and achieved to the fixed tilted panel and to two axis tracking 10–25%, 25–45% higher irradiation respectively.	[21]
	Lubitz	(2011)	Showed the effect of various manual tilt angels on fixed and tracking PV panels. The optimum tilt angle for an azimuth tracking was 19° closer to vertical than for a fixed, south-facing panel.	[22]
	Rowlands et al.	(2011)	Evaluated the optimal tilt angle and azimuth of a PV panel in Canada hourly for two locations and they find only slight deviations from output that are maximizing angle combinations.	[23]
	Mehleri et al.	(2010)	Presented a method to calculate the optimal tilt angle and orientation of PV panels in order to maximize the solar irradiance on the array.	[24]
	Chang et al.	(2010)	Showed the calculation of the optimum tilt angle of PV modules to maximize the power output in Taipei area using orthogonal array experiment technique and an antdirection hybrid differential evolution algorithm (ADHDEOA). The annual optimal angle for Taipei area is determined (Taichung 17.3°, Tainan 16.15°, Kaosiung 15.79°, Hengchung 15.17°, Hualian1 7.16°, Taitung 15.94°).	[25]
	Chang	(2010)	Showed a mathematical model to achieve optimal tilt angle on PV panels at Taiwan in order to maximize electrical energy output from PV modules and found that the optimal annual tilt angle is equal to the latitude.	[26]
	Chang	(2010)	Determined the optimal tilt angles of PV modules fixed south facing for maximum output electrical energy of the modules in Taiwan using a particle-swarm optimization method with nonlinear time-varying evolution (PSO—NTVE) and found the annual optimal angle for the Taipei area is 18.16° and 17.3°, 16.15°, 15.79°, 15.17°, 17.16°, 15.94° for Taichung, Tainan, Kaosiung, Hengchung, Hualian, Taitung respectively.	[27]
	Cheng et al.	(2009)	Investigated the correlation between the latitude of 20 locations on the northern hemisphere and the tilt angle of a fixed solar collector using PVSYST simulation software.	[28]
	Chang	(2009)	Determined the optimum tilt angle for PV modules to achieve the maximum output power energy in seven areas of Taiwan using sequential neural-network approximation and	[29]

(continued on next page)

Table 1 *(continued)*

Research Application	Authors	Year	Research Data	Ref.
			orthogonal arrays (SNAOA) and found the annual optimal angle for the Taipei area is 23.25°; for Taichung, 22.25°; for Tainan, 21.25°; for Kaosiung, 20.75°; for Hengchung, 20.25°; for Hualian, 22.25°; and for Taitung, 21°.	
	Juang	(2009)	Proposed a formation of poly-Si thin-film transistor (TFT) by large-angle-tilt-implantation of dopant through gate sidewall spacer (LATITS).	[30]
	Calabrò	(2009)	Proposed a relationship between the optimum tilt angles of PV panels and the latitude outside tropics from 36° to 46° and showed the optimum tilt angles for winter months are very different from the summer months.	[31]
	Chang	(2009)	Compered between single axis tracking and fixed of PV panel, In addation, analyzed the gain in extraterrestrial radiation and found the incident angle of sunlight upon the tracked panel is smaller than the fixed panel, except at solar noon.	[32]
	Chang	(2009)	Presented theoretically the different tilt and azimuth angles to calculate the electric energy of PV panels in Taiwan and found substantial gains of 51.4%, 28.5% and 18.7% from the extraterrestrial, predicted and observed radiations respectively using a single axis tracking system.	[33]
	Ahmad et al.	(2009)	Showed different tilt angle optimatiztion methods of PV panels as monthly, seasonal, yearly-based optimization to improve the efficiency and the reliability of the system.	[34]
	Ko et al.	(2007)	Determined the optimal combination of inertiaweights and acceleration coefficients by orthogonal array experiments as proposed using PSO–NTVE method and then evaluated and compared this method and three other PSO methods.	[35]
	Burger and Rüther	(2006)	Showed that annual PV generation power have small effects on it from azimuthal deviations at lower latitude site (Florianópolis, Brazil, 27° S) than on a higher latitude site (Freiburg, Germany, 48° N) and the vertical facades at higher latitude sites led to lower relative energy generation.	[36]
	Chen et al.	(2005)	Calculated the optimum installation angle of fixed solar panels using a genetic algorithm (GA) and a simulated annealing method.	[37]
	El et al.	(2005)	Showed that at low solar radiation periods conditions lead to efficient operations of PV modules depending on the maximum energy yield.	[38]
	Hussein et al.	(2004)	Calculated the optimum tilt angle and montly solar radiation at Cairo, Egypt. Then compared these data and the output power of solar cells using TRNSYS simulation software and found the yearly optimum tilt angle to be $\varphi \rightarrow \varphi - 10^o$.	[39]
	Kacira et al.	(2004)	Determined the optimal monthly tilt angle and orientation of a PV panel in Sanliurfa, Turkey depend on the local latitude, the climate and the load consumption temporal profile.and found the minimum value as 13° in June and maximum value as 61° in December.	[40]
	Ahmad et al.	(2003)	Showed the performance of PV modules using Fortran subprogram and connected it to the TRNSYS simulation program. The Fortran subprogram verified experimentally under different parameters as tilt angle and under the actual meteorological conditions of Cairo, Egypt.	[41]
	Nakamura et al.	(2001)	Used three tilt angles in a period of 6 months (September–February) to investigate the output power of solar cells in Hamamatsu, Japan and found the optimum tilt angle is 30° facing south, i.e. ($\beta_{opt} \sim \phi$).	[42]
	Asl-Soleimani et al.	(2001)	Determined the optimum tilt angle analytically and experimentally of PV modules (five, each one 11 W mono-crystalline) by maximize the energy output at Tehran.	[43]
	Fordham	(2001)	Showed the optimal tilt angle of PV is equal to the latitude of the site minus 20° ($\beta_{opt} = \varphi - 20^o$) at Eskdalemuir, Scotland.	[44]
	Neocleous and Schizas	(2000)	Proposed method for solving the tilt angle of PV modules problem using sequential feed forward multilayer neural network and ANN to estimate of the 4-h-ahead electric load in a Power Plant.	[45]
	Fordham et al.	(1999)	Showed different tilt angles and orientations at London, UK to compare the output of a PV array mono-crystalline silicon, 50 m^2 to achieve the maximum output energy at zero surface azimuth angle and 30° tilt angle.	[46]
	Duffie and Beckman	(1991)	Showed the yearly optimal tilt angle of PV modules to achieve the maximum yearly incident solar energy which is equal to the local latitude as $\beta_{opt} = (\varphi + 15) \pm 15^o$.	[47]

16°, 26° and 36° analytically using photovoltaic experimental setup for winters at Vaal Triangle, South Africa (Lat. 29° 00′ S and Long 24° 00′ E).

Bojić et al. [11] determined the optimum tilt angles for PV systems that are located in four different towns (Les Avirons, Petite-France, Saint-Benoit, and Piton Saint-Leu) in Reunion Island, France. Siraki and Pillay [12] calculated the optimum angle of PV array needs at any site using a modified HDKR model (Hay, Davies, Klucher, Reindl) anisotropic sky model that also considers the effects of the adjacent buildings in urban locations. Kaldellis and Zafirakis [13] evaluated the performance of different PV panel tilt angles during the summer at Athens, Greece and found that the optimum angle during the summer is 15° (± 2.5°). Liu et al. [14] calculated the optimal tilt angles of PV panels depending on techno-economic optimization of PV system connected to a grid and electricity tariffs. Beringer et al. [15] investigated the difference between the performances of different eight multicrystalline solar cells at various tilt angles from 0° to 70° in steps of 10° to achieve monthly optimum tilt angles at Hannover, Germany. The maximum power is found to be at 50–70° tilt angles in winter and 0–30° in summer months. Wada et al. [16] analyzed the annual generated energy of each array in a 100 kW PV system at Gobo city, Wakayama prefecture, Japan, and found that the tilt angle for array maximum annual energy is not the same tilt angle to achieve the maximum anuual irradiation. Benghanem [17] performed a study to achieve the annual optimum tilt angle of the solar panels surface for receiving a maximum solar radiation and found that it is approximately equal to the latitude of the site. At Madinah, Saudi Arabia, annual optimum tilt angle is 23.5° with respect to the latitude of Madinah site 24.5°; and for the winter months is 37° and for the summer months is 12°. Talebizadeha et al. [18] calculated hourly, daily, monthly, seasonally and yearly optimum tilt angle of PV panels and solar collectors using GA at Iran and showed that optimum hourly azimuth angle is not zero. Sunderan et al. [19] presented a way to enhance the power generation from a stand alone Photovoltaic Electricity Generation Systems (SPVEGS) using the optimum tilt angle and orientation in Ipoh, Malaysia where the optimum PV orientation is North from April to August and south from September to March. The gains for monthly and yearly optimum tilt angles are 6.4% and 6.1% respectively. Maatallah et al. [20] investigated the effects of azimuth and tilt angles on the output power of a photovoltaic module.

Lave and Kleissl [21] determined the optimum tilt angle and azimuth angle of solar PV panels at US and achieved to the fixed tilted panel and to two axis tracking 10–25%, 25–45% higher irradiation respectively. Lubitz [22] showed the effect of various manual tilt angels on fixed and tracking PV panels. The optimum tilt angle for an azimuth tracking was 19° closer to vertical than for a fixed, south-facing panel. Rowlands et al. [23] evaluated the optimal tilt angle and azimuth of a PV panel in Canada hourly for two locations and they found only slight deviations from output that are maximizing angle combinations. Mehleri et al. [24] presented a method to calculate the optimal tilt angle and orientation of PV panels in order to maximize the solar irradiance on the array. Chang et al. [25] showed the calculation of the optimum tilt angle of PV modules to maximize the power output in Taipei area using orthogonal array experiment technique and an antdirection hybrid differential evolution algorithm (ADHDEOA). The annual optimal angle for Taipei area is determined (Taichung 17.3°, Tainan 16.15°, Kaosiung 15.79°, Hengchung 15.17°, Hualian1 7.16°, Taitung 15.94°). Chang [26] showed a mathematical model to achieve optimal tilt angle on PV panels at Taiwan in order to maximize electrical energy output from PV modules and found that the optimal annual tilt angle is equal to the latitude. Chang [27] determined the optimal tilt angles of PV modules fixed south facing for maximum output electrical energy of the modules in Taiwan using a particle-swarm optimization method with nonlinear time-varying evolution (PSO—NTVE) and found the annual optimal angle for the Taipei area is 18.16° and 17.3°, 16.15°, 15.79°, 15.17°, 17.16°, 15.94° for Taichung, Tainan, Kaosiung, Hengchung, Hualian, Taitung respectively. Cheng et al. [28] investigated the correlation between the latitude of 20

Table 2
The current researches in the tilt angle for covering different technologies used in solar water heaters, solar cooker, and solar still systems.

Research Application	Authors	Year	Research Data	Ref.
Solar Water Heater	Bracamonte et al.	(2015)	Showed effect of the tilt angle (10°, 27° and 45°) on thermal efficiency and stratification of water in glass evacuated tube passive solar water heater.	[48]
	Manouchehri et al.	(2015)	Presented impact of small tilt angles of 0°, 2°, 5°, 10° and 15° with respect to the vertical on the performance of falling film drain water heat recovery systems.	[49]
	Tang et al.	(2014)	Investigated the thermal performance of the water in glass evacuated tube in the solar water heaters (SWH) at nights where the larger the tilt-angle of the collector lead to increase the reverse flow rate.	[50]
	Zhang et al.	(2014)	Showed that the tilt angle did not affect on the performance of the solar water heaters at china.	[51]
	Skerlić et al.	(2013)	Investigated various tilt angles for the collectors at 2°, 4° and 12°, placed in north–south direction, yearly optimized for heating of domestic hot water in Belgrade, Serbia.	[52]
	Tang et al.	(2011)	Showed the performance for two identical solar water heaters in the design under various collector tilt-angle from the horizon by 22° and the other one at 46°.	[53]
	Tang et al.	(2009)	Developed a mathematical model to evacuated glass tube collectors at SWH for determining the optimum tilt angles to maximize the annual solar radiation in China and found for the sites with latitudes greater than 30°, the optimum tilt angles are about 10° less than the latitude for sites.	[54]
	Shariah et al.	(2002)	Presented the optimum tilt angles for a thermosyphoning solar water heater using the annual solar fraction as an indicator at Jordan and found at high solar fraction system; the optimum tilt angle varies from ϕ to ϕ7201.	[55]
Solar Cooker	Sethi et al.	(2014)	Presented the optimum tilt angle of an inclined box type solar cooker with single reflector mirror where the tilt angle will affect in maximizing the reflected solar radiation during all months at selected latitudes of 10°, 20°, 30°, 40° and 50° N.	[56]
	Al-Soud et al.	(2010)	Designed and tested a parabolic solar cooker and showed effects of the tilt angle with automatic two axes sun tracking, and used a programmable logic controller to control the motion of the solar cooker.	[57]
Solar Still	Tanaka	(2016)	Proposed and theoretically analyzed a combination of vertical multiple-effect diffusion solar still and tilted wick still.	[58]
	Khalifa	(2011)	Described a relation between the cover tilt angle and productivity of simple solar still in various seasons, with a relation between the optimum tilt angle and the latitude angle.	[59]
	Aybar and Assefi	(2009)	Examined the cover tilt angles for treated glass as high as 85°.	[60]
	El-Bahi and Inan	(1999)	Examined the cover tilt angles for ordinary glass as low as 4°.	[61]
	Porta et al.	(1997)	Showed the cover tilt angles of solar still to reach the optimum design for ordinary glass as low as 4°.	[62]
	Bahadori and Edlin	(1973)	Showed that the cover tilt angles of the solar still for treated glass as low as 1.5°.	[63]

Table 3
The current researches in the tilt angle for covering different technologies used in solar powered thermoacoustic engines, building-integrated photovoltaic system (BIPV), solar cooling, and solar updraft tower power plant systems.

Research Application	Authors	Year	Research Data	Ref.
Solar Powered Thermoacoustic Engines	Pan et al.	(2014)	Investigated the heat transfer of the solar powered thermoacoustic engine under five tilt angles including −45°, −90°, 0°, 45° and 90°.	[64]
	Pan et al.	(2013, 2012)	Showed the flow and heat transfer characteristic of thermoacoustic engine at horizontal case (0°).	[65,66]
	Shen et al.	(2009)	Investigated effect of tilt angles on the onset temperature and system pressure of thermoacoustic engine.	[67]
BIPV	Fortunato et al.	(2014)	Presented a mathematical model of 2 kWp mirror augmented photovoltaic system (MAPV) with mirrors inclined at 60° is comparable with a 3 kWp building-integrated photovoltaic system (BIPV) with a tilt angle equal to 30°.	[68]
	Sun et al.	(2012)	Determined the optimum tilt angles for the BIPV, shading-type, claddings at different orientations at Hong Kong [lat 22.25° N Long 114.1667° S]. The maximum electricity generation per unit PV area is found when the PV modules are installed on south façades at the tilt angle of 10°, thus increasing the energy gain significantly.	[69]
	Santos and Rüther	(2012)	Showed the different tilt angles effects on the building integrated photovoltaic systems (BIPV) at the top pf all of the available roof areas.	[70]
	Hachem et al.	(2012)	Considered the optimal sizing to avoid the building revamping where there is a significant increase in the total electricity generation through the BIPV of housing units with certain shape site configurations.	[71]
	El hassan et al.	(2011)	Determined the optimal tilt angles for building integrated photovoltaic (BIPV) design and applications of four PV modules experimental setup inclined in the East, West, North, and South directions to determine the optimum tilt angles for these directions at Kuala Lumpur, Malaysia (Lat. 2° 30′ N, Long. 112° 30′ E) and found the optimum tilt angle for this location is nearly equal to the latitude of the location.	[72]
	Cheng et al.	(2009)	Proposed to increase the realabity of the system and reduce the calculations for optimal tilted angles using $\beta = \phi$. Calculations were made for a BIPV south orientated tilted roof at 20 different locations in 14 countries.	[28]
SC	Corrada et al.	(2014)	Showed the solar cooling system where Compensated the cost of implementing by optimization of panels and the tilt angles of it to achieve the required cooling in summer, and use it in the winter for for heating requirement with the same number of panels.	[73]
SUPP	Gitan et al.	(2015)	Showed the results of the SUPP (Solar Updraft Tower power plant) of slope collector angle of 10°, which can produce higher power generation over the whole year under Malaysia climate conditions.	[74]

locations on the northern hemisphere and the tilt angle of a fixed solar collector using PVSYST simulation software. Chang [29] determined the optimum tilt angle for PV modules to achieve the maximum output power energy in seven areas of Taiwan using sequential neural-network approximation and orthogonal arrays (SNAOA) and found the annual optimal angle for the Taipei area is 23.25°; for Taichung, 22.25°; for Tainan, 21.25°; for Kaosiung, 20.75°; for Hengchung, 20.25°; for Hualian, 22.25°; and for Taitung, 21°. Juang [30] proposed a formation of poly-Si thin-film transistor (TFT) by large-angle-tilt-implantation of dopant through gate sidewall spacer (LATITS).

Calabrò [31] proposed a relationship between the optimum tilt angles of PV panels and the latitude outside tropics from 36° to 46° and showed that the optimum tilt angles for winter months are different from the summer months. Chang [32] compered between single axis tracking and fixed PV panel. In addition, he analyzed the gain in extraterrestrial radiation and found the incident angle of sunlight upon the tracked panel is smaller than the fixed panel, except at solar noon. Chang [33] presented theoretically the different tilt and azimuth angles to calculate the electric energy of PV panels in Taiwan and found substantial gains of 51.4%, 28.5% and 18.7% from the extraterrestrial, predicted and observed radiations respectively using a single axis tracking system. Ahmad et al. [34] showed different tilt angle optimatiztion methods of PV panels as monthly, seasonal, yearly-based optimization to improve the efficiency and the reliability of the system. Ko et al. [35] determined the optimal combination of inertiaweights and acceleration coefficients by orthogonal array experiments as proposed using PSO–NTVE method and then evaluated and compared this method and three other PSO methods. Burger and Rüther [36] showed that annual PV generation power have small effects on it from azimuthal deviations at lower latitude site (Florianópolis, Brazil, 27° S) than on a higher latitude site (Freiburg, Germany, 48o N) and the vertical facades at higher latitude sites led to lower relative energy generation. Chen et al. [37] calculated the optimum installation angle of fixed solar panels using a genetic algorithm (GA) and a simulated annealing method. El et al. [38] showed that at low solar radiation periods conditions lead to efficient operations of PV modules depending on the maximum energy yield. Hussein et al. [39] calculated the optimum tilt angle and montly solar radiation in Cairo, Egypt. Then compared these data and the output power of solar cells using TRNSYS simulation software and found the yearly optimum tilt angle to be $\varphi \rightarrow \varphi - 10^{o}$. Kacira et al. [40] determined the optimal monthly tilt angle and orientation of a PV panel in Sanliurfa, Turkey depends on the local latitude, the climate and the load consumption temporal profile and found the minimum value to be 13° in June and maximum value to be 61° in December.

Ahmad et al. [41] showed the performance of PV modules using Fortran subprogram and connected it to the TRNSYS simulation program. The Fortran subprogram verified tilt angle under the actual meteorological conditions of Cairo, Egypt. Nakamura et al. [42] used three tilt angles in a period of 6 months (September–February) to investigate the output power of solar cells in Hamamatsu, Japan and found the optimum tilt angle to be 30° facing south, i.e. ($\beta_{opt} \sim \phi$). Asl-Soleimani et al. [43] determined the optimum tilt angle analytically and experimentally of PV modules (five, each one 11 W mono-crystalline) by maximizing the energy output at Tehran. Fordham [44] showed the optimal tilt angle of PV is equal to the latitude of the site minus 20° ($\beta_{opt} = \varphi - 20^{o}$) at Eskdalemuir, Scotland. Neocleous and Schizas [45] proposed a method for solving the tilt angle of PV modules problem using sequential feed forward multilayer neural network and ANN to estimate of the 4-h-ahead electric load in a Power Plant. Fordham et al. [46] showed different tilt angles and orientations at London, UK to compare the output of a PV array mono-crystalline silicon, 50 m^2 to achieve the maximum output energy at zero surface azimuth angle and 30° tilt angle. Duffie and Beckman [47] showed the yearly optimal tilt angle of PV modules to achieve the maximum yearly incident solar

Table 4
The current researches in the tilt angle for covering different technologies used in solar collectors systems.

Research Application	Authors	Year	Research Data	Ref.
Solar Collector	Stanciu et al.	(2016)	Presented a mathematical model for selecting the optimum tilt angle for solar collectors in isotropic sky.	[75]
	Soulayman and Sabbagh	(2015)	Proposed an algorithm to determine the optimum tilt angle (β_{opt}) at any latitude (ϕ) and for any direction (surface azimuth angle).	[76]
	Despotovic and Nedic	(2015)	Showed the optimum tilt angles of solar collectors at yearly, biannual, seasonal, monthly, fortnightly and daily level where four different seasonal scenarios and two different biannual scenarios considered at Belgrade, Serbia.	[77]
	Gitan et al.	(2015)	Developed a mathematical model to estimate the performance of (SUPP) based on tracking solar collector consideration and optimize the slope angle of tilted tracking solar collector in Malaysia.	[74]
	Stanciu, C., and Stanciu, D.	(2014)	Proposed a numerical simulation of a flat plate collector for the optimum tilt angle at different latitudes from 0° to 80° as $\beta_{opt} = \varphi - \delta$ function on the latitude (ϕ) and solar declination (δ).	[78]
	Moghadam and Deymeh	(2014)	Determined the optimum location and optimum tilt angle of solar collectors on the roof, with respect to the shadow of adjacent buildings.	[79]
	Bai et al.	(2014)	Investigated the effect of three evaporator tilt angles experimentally and theoretically on the operating temperature of a loop heat pipe: where the evaporator (a) horizontal with the compensation chamber (CC), (b) vertically below the CC, and (c) above the CC with a tilt angle of 1.80°.	[80]
	Rahman et al.	(2014)	Investigated a range of tilt angles from 0 to 60° on a solar collector, which is an important factor and the heat transfer, can be maximized for solid volume fraction of the nanofluid and a specific tilt angle.	[81]
	Khorasanizadeh et al.	(2014)	Calculated the optimum tilt angle of south-facing solar surfaces in Tabass, Iran, for the monthly, seasonal, semi-yearly and yearly where the monthly optimum tilt varies from 0° in June and July up to 64° in December and the yearly optimum tilt is around 32°, which is very close to latitude of Tabass (33.36°). The semi-yearly tilt adjustment of 0° for (April–September) and 55° for (October–March) was preferred.	[82]
	Patkó et al.	(2013)	Determined theoretically the optimal tilt angle of sun collectors for the four seasons (autumn, winter, spring, summer).	[83]
	Darhmaoui and Lahjouji	(2013)	Developed a mathematical model for determining the optimal tilt angle at 35 sites in the Mediterranean region selected for the study.	[84]
	Jafarkazemi and Saadabadi	(2013)	Investigated the effect of the optimum tilt angle and orientation of PV panels and solar collectors by change the tilt angle, at least twice a year in Abu Dhabi, UAE.	[5]
	Handoyo and Ichsani	(2013)	Obtained the optimal tilt angle of a solar collector to maximize the solar radiation received at Surabaya – Indonesia and found the optimal tilt angle during March 12 – September 30 is varied between 0 and 40° (face to the North) and during October 1 – March 11 is between 0 and 30° (face to the South).	[30]
	Jafarkazemi et al.	(2012)	Proposed the optimum tilt angle's maps by an anisotropic method to estimate solar radiation on tilted surface using twenty years average cloud factor for all regions in Iran.	[85]
	Fahl and Ganapathisubbu	(2011)	Computed optimum tilt angles of south facing collectors at Bangalore (Lat 12.97° N, Long 77.56° E) under various tracking conditions and found he optimum tilt angles are between 15° and 17° for.	[86]
	Moghadam et al.	(2011)	Performed an optimization of solar flat collector inclination to determine monthly, seasonal, semi-annual and annual optimum tilt angles.	[87]
	Armstrong and Hurley	(2010)	Presented a review for optimizations of flat plate collectors taking into account cloudiness, deduced the optimum tilt particularly for climates susceptible to frequently over cast skies and validated for other climate types as well.	[88]
	Tang et al.	(2009)	Developed a mathematical method to calculate daily collectible radiation on a single tube of collectors with any structural and geometrical parameters in China. In case of the site latitude larger than 30°, T-type collectors with a tilt-angle about 10° less than the site latitude, whereas for H-type collectors without DFR, the reasonable tilt-angle should be about 20° less than the site latitude.	[54]
	Skeiker	(2009)	Presented a mathematical model to compute the optimum tilt angle and orientation (surface azimuth angle) of solar collector Syrian zones and recommend that by changing the tilt angle 12 times in a year and found the solar radiation approximately is the maximum data.	[89]
	Cheng et al.	(2009)	Investigated the correlation between the tilt angle of a fixed solar collector and the latitude using the simulation software PVSYST at 20 locations in 14 countries on the northern hemisphere and found there are increase of 98.5% for a solar power plant from its full capacity by using the latitude angle for the tilted panel.	[28]
	Chang	(2008)	Calculated the optimal tilt angles using three different radiation types for six locations in Taiwan, i.e. the extraterrestrial, predicted global radiation model and ten-year data from 1990 to 1999 for measured solar radiation.	[90]
	Ertekin et al.	(2008)	Showed the optimum tilt angles of the solar collector surfaces to maximize solar radiation in Turkey and found that during the summer (March to August) low tilt angles are the best, and during the autumn (September to November) and winter (December to February) high tilt angles are the optimum. Monthly optimum tilt angles were depend on the latitude and day of the year.	[91]
	Gunerhan and Hepbasli	(2007)	Determined the monthly optimum orientation and tilt angles of solar collectors β_{opt} equal to latitude φ throughout the year at Izmir, Turkey and suggest, while for summer $\beta_{opt} = \varphi - 15^o$ and for winter $\beta_{opt} = \varphi + 15^o$.	[92]
	Tang and Wu	(2004)	Developed a mathematical procedure to determine the optimal tilt angle of a collector using the monthly global and diffuse radiation on horizontal radiation for 30 cities in China.	[93]
	Bari	(2001)	Suggested an algorithm to determine the optimum tilt of solar collector at latitudes of odd and even values of interval [4–20°] in the Philippines territory.	[94]
	Bari et al.	(2001)	Proposed a polynomial equation of six order for calculating the optimum tilt angle (β_{opt}) for different latitudes in the Thailand territory.	[95]
	Bari	(2000)	Showed a method to determine the optimum tilt angle and orientation of solar collectors to utilize both the direct and diffuse components of solar radiation for different periods of operation at latitudes 1°, 3°, 5° and 7° in the Malaysian territory.	[96]
	Hussein et al.	(2000)	Developed theoretically, analysis of the instantaneous, daily, and yearly solar energy collection of a tilted flat-plate solar collector augmented by a plane reflector.	[97]
	Tiris, M., & Tiris, C.	(1998)	Discussed the optimal tilt angle and orientation of south-facing solar collector in different regions, and found the optimum tilt depends upon the latitude and the day of the year.	[98]
	Morcos	(1994)	Derived with Liu and Jordan model by an analytical expression of the optimum tilt to maximize total solar energy at Assiut, Egypt where the tilt angle is depend on azimuth angle, declination, latitude, ground reflectance, zenith and hour angles.	[99]

(continued on next page)

Table 4 (*continued*)

Research Application	Authors	Year	Research Data	Ref.
	Soulayman	(1991)	Proposed a general algorithm to calculate optimum tilt angle (β_{opt}) of south facing solar absorber plate collector and showed the optimum tilt angle is almost equal to the latitude.	[100]
	Gopinathan	(1991)	Showed different tilt angles and orientations at three inclinations for six different azimuth angles for summer, winter and annual collection, and showed the optimum tilt angle is equal to the latitude.	[101]
	Wenxian	(1989)	Suggested an empirical correlation to estimate optimal tilt angle of a solar collector and calculate of monthly diffuse solar radiation data of 30 cities around the country.	[102]
	Saraf and Hamad	(1988)	Found that the yearly optimum tilt angle by a flat plate collector was higher than the latitude by about 8° in Basra, Iraq.	[103]
	El-Kassaby	(1988)	Determined the optimum tilt and azimuth angles for an absorber plate covered by one or two glass covers of solar collector and suggested that using two glass covers instead of one did not affect the value of β_{opt}.	[104]
	Chau	(1982)	Showed the optimum tilt angles of solar collectors with a cylindrically curved transparent cover in clear sky conditions at various latitudes for all 12 months of the year.	[105]
	Chinnery	(1981)	Suggested that the optimal tilt angle of collectors was latitude plus 10° in South Africa.	[106]
	Moon et al.	(1981)	Determined the optimum tilt angles of the solar collectors using regression analysis for several different periods of the year in 171 locations at North America.	[107]
	Iqbal	(1979)	Investigated the optimum collectors lope for residential heating in adverse climates.	[108]
	Beekley and Mather	(1978)	Developed a mathematical model for calculating collectible radiation on south-facing single tube collectors.	[109]
	Heywood	(1971)	Suggested the optimal tilt angle of a yearly collector equal to latitude minus 10°.	[110]
	Page	(1961)	Calculated the monthly diffuse radiation and estimate the optimal tilt angle of a solar collector in the provinces considered using a simple mathematical procedure.	[111]
	Hottel	(1954)	Showed the optimal tilt angle of collectors used for space heating should be latitude plus 20° in the US.	[112]

energy which is equal to the local latitude as $\beta_{opt} = (\varphi + 15) \pm 15^{o}$.

2.2. Solar Water Heaters

The tilt angle calculations are applied in many designs in solar water heater applications and especially in the collectors design and tubes in the collectors. Bracamonte et al. [48] showed effect of the tilt angle (10°, 27° and 45°) on thermal efficiency and stratification of water in glass evacuated tube passive solar water heater. Manouchehri et al. [49] presented impact of small tilt angles of 0°, 2°, 5°, 10° and 15° with respect to the vertical on the performance of falling film drain water heat recovery systems. Tang et al. [50] investigated the thermal performance of the water in glass-evacuated tube in the solar water heaters (SWH) at nights where the larger the tilt-angle of the collector lead to increase the reverse flow rate. Zhang et al. [51] showed that the tilt angle did not affect on the performance of the solar water heaters at china. Skerlić et al. [52] investigated various tilt angles for the collectors at 2°, 4° and 12°, placed in north–south direction, yearly optimized for heating of domestic hot water in Belgrade, Serbia. Tang et al. [53] showed the performance for two identical solar water heaters in the design under various collector tilt-angle from the horizon by 22° and the other one at 46°. Tang et al. [54] developed a mathematical model to evacuated glass tube collectors at SWH for determining the optimum tilt angles to maximize the annual solar radiation in China and found for the sites with latitudes greater than 30°, the optimum tilt angles are about 10° less than the latitude for sites. Shariah et al. [55] presented the optimum tilt angles for a thermosyphoning solar water heater using the annual solar fraction as an indicator at Jordan and found at high solar fraction system; the optimum tilt angle varies from ϕ to ϕ7201.

2.3. Solar cookers

Calculation of optimum tilt angle can also be applied to stand-alone cooking systems. Sethi et al. [56] presented the optimum tilt angle of an inclined box type solar cooker with single reflector mirror where the tilt angle will affect in maximizing the reflected solar radiation during all months at selected latitudes of 10°, 20°, 30°, 40° and 50° N. Al-Soud et al. [57] designed and tested a parabolic solar cooker and showed the effects of the tilt angle with automatic two axes sun tracking, and used a programmable logic controller to control the motion of the solar cooker.

2.4. Solar still

Some researchers discussed some models for tilt angle calculations in solar still. Tanaka [58] proposed and theoretically analyzed a combination of vertical multiple-effect diffusion solar still and tilted wick still. Khalifa [59] described a relation between the cover tilt angle and productivity of simple solar still in various seasons, with a relation between the optimum tilt angle and the latitude angle. Aybar and Assefi [60] examined the cover tilt angles for treated glass as high as 85°. El-Bahi and Inan [61] examined the cover tilt angles for ordinary glass as low as 4°. Porta et al. [62] showed the cover tilt angles of solar still to reach the optimum design for ordinary glass as low as 4°. Bahadori and Edlin [63] showed that the cover tilt angles of the solar still for treated glass as low as 1.5°.

2.5. Solar powered thermoacoustic engines

Pan et al. [64] investigated the heat transfer of the solar powered thermoacoustic engine under five tilt angles including −45°, −90°, 0°, 45° and 90°. Pan et al. [65,66] showed the flow and heat transfer characteristic of thermoacoustic engine in the horizontal case (0°). Shen et al. [67] investigated the effect of tilt angles on the onset temperature and system pressure of thermoacoustic engine.

2.6. Building-integrated photovoltaic system (BIPV)

Fortunato et al. [68] presented a mathematical model comparing 2 kWp mirror augmented photovoltaic system (MAPV) with mirrors inclined at 60° with a 3 kWp building-integrated photovoltaic system (BIPV) with a tilt angle equal to 30°. Sun et al. [69] determined the optimum tilt angles for the BIPV, shading-type, claddings at different orientations in Hong Kong [lat 22.25° N Long 114.1667° S]. The maximum electricity generation per unit PV area is found when the PV modules are installed on south façades at a tilt angle of 10°, thus increasing the energy gain significantly. Santos and Rüther [70] showed the different tilt angles effects on the building integrated photovoltaic systems (BIPV) at the top of all of the available roof areas. Hachem et al. [71] considered the optimal sizing to avoid the building revamping where there is a significant increase in the total electricity generation through the BIPV of housing units with certain shape site configurations. El hassan et al. [72] determined the optimal tilt angles for building integrated photovoltaic (BIPV) design and applications of four PV modules experimental setup inclined to the East, West, North, and South directions to determine the optimum tilt angles for these directions at Kuala Lumpur, Malaysia (Lat. 2° 30′ N, Long. 112° 30′ E) and found the optimum tilt angle for this location is nearly equal to the latitude of the location. Cheng et al. [28] proposed to increase the reliability of the system and reduce the calculations for optimal tilted angles using $\beta=\phi$. Calculations were made for a BIPV south-orientated tilted roof at 20 different locations in 14 countries.

2.7. Solar cooling

Corrada et al. [73] discussed a solar cooling system where the cost of implementing was compensated by the optimization of panels and the tilt angles of it to achieve the required cooling in summer, and use it in the winter for heating requirement with the same number of panels.

2.8. Solar updraft tower power plant

Gitan et al. [74] showed the results of the SUPP (Solar Updraft Tower power plant) of slope collector angle of 10°, which can produce higher power generation over the whole year under Malaysia climate conditions.

2.9. Solar collectors

Stanciu et al. [75] presented a mathematical model for selecting the optimum tilt angle for solar collectors in isotropic sky. Soulayman and Sabbagh [76] proposed an algorithm to determine the optimum tilt angle (β_{opt}) at any latitude (ϕ) and for any direction (surface azimuth angle). Despotovic and Nedic [77] showed the optimum tilt angles of solar collectors at yearly, biannual, seasonal, monthly, fortnightly and daily level where four different seasonal scenarios and two different biannual scenarios considered at Belgrade, Serbia. Gitan et al. [74] developed a mathematical model to estimate the performance of (SUPP) based on tracking solar collector consideration and optimize the slope angle of tilted tracking solar collector in Malaysia. Stanciu, C., and Stanciu, D. [78] proposed a numerical simulation of a flat plate collector optimum tilt angle at different latitudes from 0° to 80° as a function of the latitude (ϕ) and solar declination (δ). Moghadam and Deymeh [79] determined the optimum location and optimum tilt angle of solar collectors on the roof, with respect to the shadow of adjacent buildings. Bai et al. [80] investigated the effect of three evaporator tilt angles experimentally and theoretically at the operating temperature of a loop heat pipe. Rahman et al. [81] investigated a range of tilt angles from 0 to 60° on a solar collector can be maximized for solid volume fraction of the nanofluid and a specific tilt angle. Khorasanizadeh et al. [82] calculated the optimum tilt angle of south-facing solar surfaces in Tabass, Iran, for the monthly, seasonal, semi-yearly and yearly where the monthly optimum tilt varies from 0° in June and July up to 64° in December and the yearly optimum tilt is around 32°, which is very close to the latitude of Tabass (33.36°). The semi-yearly tilt adjustment of 0° for (April–September) and 55° for (October–March) was preferred. Patkó et al. [83] determined theoretically the optimal tilt angle of sun collectors for the four seasons (autumn, winter, spring and summer). Darhmaoui and Lahjouji [84] developed a mathematical model for determining the optimal tilt angle at 35 sites in the Mediterranean region selected for the study. Jafarkazemi and Saadabadi [5] investigated the effect of the optimum tilt angle and orientation of PV panels and solar collectors by changing the tilt angle, at least twice a year in Abu Dhabi, UAE. Handoyo and Ichsani [30] obtained the optimal tilt angle of a solar collector to maximize the solar radiation received at Surabaya – Indonesia and found the optimal tilt angle during March 12 – September 30 is varied between 0 and 40° (face to the North) and during October 1 – March 11 is between 0 and 30° (face to the South).

Jafarkazemi et al. [85] proposed the optimum tilt angle's maps by an anisotropic method to estimate solar radiation on tilted surface using twenty years average cloud factor for all regions in Iran. Fahl and Ganapathisubbu [86] computed optimum tilt angles of south facing collectors at Bangalore (Lat 12.97° N, Long 77.56° E) under various tracking conditions and found that the optimum tilt angles are between 15° and 17°. Moghadam et al. [87] performed an optimization of solar flat collector inclination to determine monthly, seasonal, semi-annual and annual optimum tilt angles. Armstrong and Hurley [88] presented a review for optimizations of flat plate collectors taking into account cloudiness, deduced the optimum tilt particularly for climates susceptible to frequently over cast skies and validated for other climate types as well. Tang et al. [54] developed a mathematical method to calculate daily collectible radiation on a single tube of collectors with any structural and geometrical parameters in China. In case of the site latitude larger than 30°, T-type collectors with a tilt-angle about 10° less than the site latitude, whereas for H-type collectors without DFR, the reasonable tilt-angle should be about 20° less than the site latitude. Skeiker [89] presented a mathematical model to compute the optimum tilt angle and orientation (surface azimuth angle) of solar collector in Syrian zones and recommend that by changing the tilt angle 12 times in a year and the solar radiation will be approximately the maximum. Cheng et al. [28] investigated the correlation between the tilt angle of a fixed solar collector and the latitude using the simulation software PVSYST at 20 locations in 14 countries on the northern hemisphere and found that there are power increase of 98.5% for a solar power plant from its full capacity by using the latitude angle for the tilted panel. Chang [90] calculated the optimal tilt angles using three different radiation types for six locations in Taiwan. Ertekin et al. [91] showed the optimum tilt angles of the solar collector surfaces to maximize solar radiation in Turkey and found that during the summer (March to August) low tilt angles are the best, and during the autumn (September to November) and winter (December to February) high tilt angles are optimum. Monthly optimum tilt angles were dependent on the latitude and day of the year. Gunerhan and Hepbasli [92] determined the monthly optimum orientation and tilt angles of solar collectors β_{opt} equal to latitude φ throughout the year at Izmir, Turkey and suggested, for summer, $\beta_{opt} = \varphi - 15^o$, and for winter, $\beta_{opt} = \varphi + 15^o$.

Tang and Wu [93] developed a mathematical procedure to determine the optimal tilt angle of a collector using the monthly global and diffuse radiation on horizontal radiation for 30 cities in China. Bari [94] suggested an algorithm to determine the optimum tilt of solar collectors at latitudes of odd and even values of interval [4–20°] in the Philippines territory. Bari et al. [95] proposed a polynomial equation of six order for calculating the optimum tilt angle (β_{opt}) for different latitudes in the Thailand territory. Bari [96] showed a method to determine the optimum tilt angle and orientation of solar collectors to utilize both the direct and diffuse components of solar radiation for different periods of operation at latitudes 1°, 3°, 5° and 7° in the Malaysian territory. Hussein et al. [97] developed a theoretical analysis of the instantaneous, daily, and yearly solar energy collection of a tilted flat-plate solar collector augmented by a plane reflector. Tiris, M., & Tiris, C. [98] discussed the optimal tilt angle and orientation of south-facing solar collectors in different regions, and found the optimum tilt depends upon the latitude and the day of the year. Morcos [99] derived with Liu and Jordan a model by an analytical expression of the optimum tilt to maximize total solar energy at Assiut, Egypt where the tilt angle is dependent on azimuth angle, declination, latitude, ground reflectance, zenith and hour angles. Soulayman [100] proposed a general algorithm to calculate optimum tilt angle (β_{opt}) of south facing solar absorber plate collector and showed the optimum tilt angle is almost equal to the latitude. Gopinathan [101] showed different tilt angles and orientations at three inclinations for six different azimuth angles for summer, winter and annual collection, and showed the optimum tilt angle is equal to the latitude. Wenxian [102] suggested an empirical correlation to estimate optimal tilt angle of a solar collector and calculated the monthly diffuse solar radiation of 30 cities around the country. Saraf and Hamad [103] found that the yearly optimum tilt angle by a flat plate collector was higher than the latitude by about 8° in Basra, Iraq. El-Kassaby [104] determined the optimum tilt and azimuth angles for an absorber plate covered by one or two glass covers of solar collector and suggested that using two glass covers instead of one did not affect the value of β_{opt}. Chau [105] showed the optimum tilt angles of solar collectors with a cylindrically curved transparent cover in clear sky conditions at various latitudes for all 12 months of the year.

Chinnery [106] suggested that the optimal tilt angle of collectors to be equal to the latitude plus 10° in South Africa. Moon et al. [107] determined the optimum tilt angles of the solar collectors using regression analysis for several different periods of the year in 171 locations at North America. Iqbal [108] investigated the optimum collectors lope for residential heating in adverse climates. Beekley and Mather [109] developed a mathematical model for calculating collectible radiation on south-facing single tube collectors. Heywood [110] suggested the optimal tilt angle of a yearly collector equal to latitude minus 10°. Page [111] calculated the monthly diffuse radiation and estimated the optimal tilt angle of a solar collector in the provinces using a simple mathematical procedure. Hottel [112] showed the optimal tilt angle of collectors used for space heating should be equal to the latitude plus 20° in the US.

3. Solar radiation models

Maximizing solar radiation is the main target to change the tilt angle in any solar energy system. Therefore, accurate calculation of the solar radiation is very important to achieve the optimum tilt angle in the solar energy system.

3.1. Global, direct, diffuse solar radiation

The average global radiation may be estimated using the calculated solar radiation data available for solar energy system in certain location. The solar radiation may be calculated by the set of the following equations:

$$G_T = G_{B_t} + G_{\mathbf{D}_t} + G_{R_t} \quad (1)$$

$$G_{B_t} = (G_{R_t} - G_{\mathbf{D}_t})\ R_B \quad (2)$$

$$G_{R_t} = G_{B_t}\rho\left(\frac{1-\cos\beta}{2}\right) \quad (3)$$

where β is the tilt angle of the collector, ρ is the ground albedo and R_B is the ratio of monthly mean daily beam radiation on a tilted surface to that on a horizontal surface. R_B for fixed slope surfaces faced towards the equation equator in the northern hemisphere outlined by [1] is

$$R_B = \frac{\cos(\varphi-\beta)\cos\delta\sin\omega_{ss} + \omega_{ss}\sin(\varphi-\beta)\sin\delta}{\cos\varphi\cos\delta\sin\omega_{ss} + \omega_{ss}\sin\varphi\sin\delta} \quad (4)$$

Where ω_{ss} is the sunset hour angle for the tilted surface for the mean day of the month, which is given by

$$\omega_{ss} = \min\begin{bmatrix}\cos^{-1}(-\tan\varphi\tan\delta)\\ \cos^{-1}(-\tan(\varphi-\beta)\tan\delta)\end{bmatrix} \quad (5)$$

where 'min' means the smaller of the two terms in the bracket.

For surfaces in the southern hemisphere sloped towards the equator, the equations are [167]:

$$R_B = \frac{\cos(\varphi+\beta)\cos\delta\sin\omega_{ss} + \omega_{ss}\sin(\varphi+\beta)\sin\delta}{\cos\varphi\cos\delta\sin\omega_{ss} + \omega_{ss}\sin\varphi\sin\delta} \quad (6)$$

$$\omega_{ss} = \min\begin{bmatrix}\cos^{-1}(-\tan\varphi\tan\delta)\\ \cos^{-1}(-\tan(\varphi+\beta)\tan\delta)\end{bmatrix} \quad (7)$$

$$G_{\mathbf{D}_t} = G_D\, R_D \quad (8)$$

3.1.1. Global solar radiation

Chang [90] calculated the optimal tilt angle by three different radiation models for six locations in Taiwan, using ten-year data from 1990 to 1999 for measured solar radiation. Tang and Wu [93] developed a mathematical procedure to determine the optimal tilt angle of a collector using the monthly global and diffuse radiation on horizontal radiation for 30 cities in China. Shaddel et al. [114] used Artificial Neural Network (ANN) of Matlab software to estimate hourly global solar irradiation on tilted surfaces at Mashhad, Iran. Shamim et al. [119] proposed a methodology to estimate clear sky global solar radiation, analytical tools for determining optical transmittance and surface albedo. Notton et al. [123] developed three ANN models to estimate hourly global irradiation on the tilted plane at Mediterranean site of Ajaccio, France. Mehleri et al. [128] estimated global solar irradiance on a horizontal surface, extraterrestrial radiation, solar zenith angle and solar incidence angle on a tilted plane as inputs to RBFNN for estimating global solar irradiance on inclined surfaces at Athens, Greece. El-Sebaii et al. [129] showed the optimal tilt angle equals to the latitude of the place and estimated the amount of global solar radiation of horizontal surfaces using various climatic parameters, such as sunshine duration, cloud cover, humidity, maximum and minimum ambient temperatures, and wind speed at Jeddah, Saudi Arabia. Bakirci [132] developed the correlations to estimate of daily global solar radiation with hours of bright sunshine in Turkey. Janjai et al. [133] proposed a model to calculate the monthly average hourly global radiation in the tropics with high aerosol load using satellite data at Thailand from 1994 to 2005 to predict daily global solar radiation from sunshine hours, air temperature, total precipitation and dew point at Nanchang station (China). Bulut and Buyukalaca [139] proposed a model to estimate monthly average daily global solar radiation on horizontal surface in 68 provinces in Turkey. Sen [140] proposed a nonlinear model to estimate global solar radiation through sunshine hour data. El-Sebaii [144] showed global solar radiation to achieve the optimal solar energy conversion system design and prediction of its performance. Ulgen and Hepbasli [147] presented the diffuse fraction of daily and monthly global radiation at Izmir, Turkey. Sabbagh et al. [164] determined the daily global radiation at different locations in Lebanon, Kuwait, Sudan, Egypt and Saudi Arabia.

3.1.2. Direct solar radiation

Bari [96] showed a method to determine the optimum tilt angle and orientation of solar collectors to utilize both the direct and diffuse components of solar radiation for different periods of operation at latitudes 1°, 3°, 5° and 7° in the Malaysian territory. Gorantla and Setty [118] observed the direct solar radiation through glazing at different latitudes with different glass materials. Kern and Harris [165] calculated the optimum tilt angle by maximizing the direct solar radiation incident on the collector.

3.1.3. Diffuse solar radiation

Tang and Wu [93] developed a mathematical procedure to determine the optimal tilt angle of a collector using the monthly global and diffuse radiation on horizontal radiation for 30 cities in China. Bari [96] showed a method to determine the optimum tilt angle and orientation of solar collectors to utilize both the direct and diffuse components of solar radiation for different periods of operation at latitudes 1°, 3°, 5° and 7° in the Malaysian territory. Wenxian [102] suggested an empirical correlation to estimate optimal tilt angle of a solar collector and calculated the monthly diffuse solar radiation data of 30 cities around China. Page [111] calculated the monthly diffuse radiation and estimated the optimal tilt angle of a solar collector in the provinces of China considered using a simple mathematical procedure. Simón-Martín et al. [117] presented a new device to measure diffuse solar radiation simultaneous under different azimuth and tilt angles. Vignola et al. [125] presented a prototype that can simultaneously measure diffuse irradiance on planes tilted from 60° to 90° oriented towards the four cardinal directions: North, South, East and West. Pandey and Katiyar [130] presented at different tilt angles (15°, 30°, 45° and 60°), the different diffuse components of the solar radiation at Lucknow (Latitude 26.75°, Longitude 80.85°), India. Noorian et al. [135] showed hourly diffuse solar radiation on tilted surfaces to 12 isotropic and anisotropic models in Iran. Soares et al. [146] showed an estimation of hourly horizontal diffuse radiations using neural-networks technique. Ulgen and Hepbasli [147] presented the diffuse fraction of daily and monthly global radiation at Izmir, Turkey. El-Sebaii and Trabea [148] proposed the correlations to estimate the horizontal diffuse radiation in Egypt by correlating (H_d/H_g) and (H_d/H_o) with K_T and (S/S_{max}). Tiris et al. [152] calculated the monthly average daily global, diffuse and beam radiations with hours of bright sunshine at Gebze, Turkey. Khalil and Alnajjar [153] proposed annual optimum tilt angle of solar collector is 24° using the global, beam, and diffuse solar radiation on inclined surface by clear sky model at Al Ain, UAE. Reindl et al. [155] obtained hourly diffuse solar radiation on horizontal surface by using hourly total solar radiation, an independent data from United States and Europe. Erbs et al. [158] developed a model to estimate the monthly average diffuse radiation on a horizontal surface using the data from United States and Australian metrological stations. Hay and Davies [160] developed an isotropic model by adding circumsolar radiation to diffuse radiation. Klucher [161] proposed to add factor-representing effect of cloudiness on diffuse solar radiation, called an anisotropic model to estimate solar radiation on the south-facing inclined surface. Collares-Pereira and Rabl [162] developed a model to estimate daily diffuse radiation on horizontal surface to obtain monthly average diffuse radiation based on summation. Liu and Jordan [166] estimated the solar radiation; beam, diffuse and ground-reflected radiations on an inclined surface using anisotropic model.

3.1.4. Ground albedo

Psiloglou and Kambezidis [131] reported several methods to calculate ground albedo underlining its important influence on the estimation of solar radiation incident on tilted surfaces.

3.2. Isotropic and anisotropic models

Agarwal et al. [126] compared the tilt angles numerically derived by 4 isotropic and 4 anisotropic models and maximized terrestrial solar energy in India. Noorian et al. [135] showed hourly diffuse solar radiation on tilted surfaces to 12 isotropic and anisotropic models in Iran. Temps and Coulson [163] calculated the solar radiation on an inclined surface by adding horizontal brightening item to isotropic model.

3.2.1. Isotropic models

Stanciu et al. [75] presented a mathematical model for selecting the optimum tilt angle for solar collectors in isotropic sky. Ghosh et al. [127] determined hourly and seasonal optimum tilt angles and solar radiations of single axis and double axis tracking surfaces using three mathematical models.

The isotropic model is given by Badescu model [168] as follow:

$$R_D = \frac{3 + \cos(2\beta)}{4} \tag{9}$$

Tian et al. model [169] is given by the relation:

$$R_D = 1 - \frac{\beta}{180} \tag{10}$$

Koronakis model [170] is given as follow:

$$R_D = \frac{2 + \cos(\beta)}{3} \tag{11}$$

Liu and Jordan model [167] is given as follow:

$$R_D = \frac{1 + \cos(\beta)}{2} \tag{12}$$

3.2.2. Anisotropic models

Siraki and Pillay [12] calculated the optimum angle of PV, at any site, using a modified HDKR model (Hay, Davies, Klucher, Reindl) anisotropic sky model that also considers the effects of the adjacent buildings in urban locations. Jafarkazemi et al. [85] proposed the optimum tilt angle's maps by an anisotropic method to estimate solar radiation on tilted surface using twenty years average cloud factor for all regions in Iran. Elminir et al. [141] compared three anisotropic models to estimate solar radiation and determined the optimum tilt angle of solar collector to maximize solar energy availability in Egypt. Hay and Davies [160] developed an isotropic model by adding circumsolar radiation to diffuse radiation. Klucher [161] proposed to add a factor-representing effect of cloudiness on diffuse solar radiation to estimate solar radiation on the south-facing inclined surface. Liu and Jordan [166] estimated the solar radiation; beam, diffuse and ground-reflected radiations on an inclined surface using anisotropic model (Tables 5, 6).

Reindl et al. model [171] is given by the relation:

$$R_D = \frac{G_B}{G_{B_n}} R_B + \left(1 - \frac{G_B}{G_{B_n}}\right)\left(1 + \frac{\cos\beta}{2}\right)\left(1 + \sqrt{\frac{G_B}{G_{B_t}}}\sin^3(\frac{\beta}{2})\right) \tag{13}$$

Skartveit and Olseth model [172] is given as follow:

$$R_D = \frac{G_B}{G_{B_n}} R_B + \Omega\cos(\beta) + \left(1 - \frac{G_B}{G_{B_n}}\right)\left(1 + \frac{\cos\beta}{2}\right)\left(1 + \sqrt{\frac{G_B}{G_{B_t}}}\sin^3(\frac{\beta}{2})\right) \tag{14}$$

$$\Omega = \left\{Max\left[0, \left(0.3 - 2\frac{G_B}{G_{B_n}}\right)\right]\right\} \tag{15}$$

Steven and Unsworth model [173] is given as follow:

$$R_D = 0.51 R_B + \frac{1 + \cos\beta}{2} - \frac{1.74}{1.26\pi}\left[\sin\beta - \left(\beta\frac{\pi}{180}\right)\cos\beta - \pi\sin^2\left(\frac{\beta}{2}\right)\right] \tag{16}$$

Table 5
The solar radiation diffuse factor with respect to tilt angle in the previous researches.

Model	Authors	Optimum Tilt Angle with respect to altitude	Ref.
Isotropic	Badescu (2002)	$R_D = \frac{3+\cos(2\beta)}{4}$	[167]
Isotropic	Tian et al. (2001)	$R_D = 1 - \frac{\beta}{180}$	[168]
Isotropic	Koronakis (1986)	$R_D = \frac{2+\cos(\beta)}{3}$	[169]
Isotropic	Liu and Jordan (1961)	$R_D = \frac{1+\cos(\beta)}{2}$	[170]
Anisotropic	Reindl et al. (1990)	$R_D = \frac{G_B}{G_{B_n}} R_B + \left(1 - \frac{G_B}{G_{B_n}}\right)\left(1 + \frac{\cos\beta}{2}\right)\left(1 + \sqrt{\frac{G_B}{G_{B_t}}}\sin^3(\frac{\beta}{2})\right)$	[171]
Anisotropic	Skartveit and Olseth (1986)	$R_D = \frac{G_B}{G_{B_n}} R_B + \Omega\cos(\beta) + \left(1 - \frac{G_B}{G_{B_n}}\right)\left(1 + \frac{\cos\beta}{2}\right)\left(1 + \sqrt{\frac{G_B}{G_{B_t}}}\sin^3(\frac{\beta}{2})\right)$ $\Omega = \left\{Max\left[0, \left(0.3 - 2\frac{G_B}{G_{B_n}}\right)\right]\right\}$	[172]
Anisotropic	Steven and Unsworth (1980)	$R_D = 0.51 R_B + \frac{1+\cos\beta}{2} - \frac{1.74}{1.26\pi}\left[\sin\beta - \left(\beta\frac{\pi}{180}\right)\cos\beta - \pi\sin^2\left(\frac{\beta}{2}\right)\right]$	[173]
Anisotropic	Hay (1979)	$R_D = \frac{G_B}{G_{B_n}} R_B + \left(1 - \frac{G_B}{G_{B_n}}\right)\left(1 + \frac{\cos\beta}{2}\right)$	[174]
Anisotropic	Klucher (1979)	$R_D = \frac{1}{2}(1 + \cos\beta)[1 + F\sin^3(\beta/2)]\,(1 + F\cos^2\theta\sin^3\theta_z)$ $F = 1 - (G_{D_t}/G_B)^2$ -	[161]
		$\cos\theta = \sin\delta\sin(\varphi-\beta) + \cos\delta\cos(\varphi-\beta)\cos\omega\sin\theta_z = \sqrt{1 - (\sin\delta\sin\varphi + \cos\delta\cos\varphi\cos\omega)^2}$ -	
		$\omega = \cos^{-1}(-\tan\varphi\tan\delta)$	

Hay model [174] is given by the relation:

$$R_D = \frac{G_B}{G_{B_n}} R_B + \left(1 - \frac{G_B}{G_{B_n}}\right)\left(1 + \frac{\cos\beta}{2}\right) \tag{17}$$

Klucher [161]

$$R_D = \frac{1}{2}(1 + \cos\beta)[1 + F\sin^3(\beta/2)]\,(1 + F\cos^2\theta\sin^3\theta_z) \tag{18}$$

$$F = 1 - (G_{D_t}/G_B)^2 \tag{19}$$

$$\cos\theta = \sin\delta\sin(\varphi - \beta) + \cos\delta\cos(\varphi - \beta)\cos\omega \tag{20}$$

$$\sin\theta_z = \sqrt{1 - (\sin\delta\sin\varphi + \cos\delta\cos\varphi\cos\omega)^2} \tag{21}$$

$$\omega = \cos^{-1}(-\tan\varphi\tan\delta) \tag{22}$$

4. Tilt angle optimization techniques

Many techniques to optimize the tilt angle have been developed. The most effective methods are by maximizing solar radiation or energy collected on the surface, ANN, GA, SA and PSO techniques. The optimum tilt angle calculated by the following equations, Stanciu C. and Stanciu D. [78], $\beta_{opt} = \varphi - \delta$, Bakirci [8], $\beta_{opt} = 34.783 - 1.4317\delta - 0.0081\delta^2 + 0.0002\delta^3$, Rowlands et al. [23], $\beta_{opt} = \varphi$, Lave and Kleissl [21], $\beta_{opt} = \varphi - (1^o - 10^o)$, Moghadam et al. [87], $\beta_{opt} = 0.917\varphi \pm 0.321^o$, Benghanem [17], $\beta_{opt} = \varphi$, Ahmad and Tiwari [34] India in Summer is 13°, $\beta_{opt} = \varphi - 60^o$, And Winter is 47.5°, $\beta_{opt} = \varphi + 90^o$, Calabròa [31], $\beta_{opt} = \varphi - (26^o, 27^o, 28^o)$, Gunerhan and Hepbasli [92], In summer $\beta_{opt} = \varphi + 15^o$, In winter $\beta_{opt} = \varphi - 15^o$, Ulgen [175], for summer months is $\beta_{opt} = \varphi - 34^o$, and that for winter months is $\beta_{opt} = \varphi + 19^o$, Elminir et al. [141], $\beta_{opt} = \varphi \pm 15^o$, Duffie and Beckman [176], $\beta_{opt} = \varphi \pm 15^o$, Rusheng Tang and Tong [93], $\beta_{opt} = \varphi + (4^{\,o} \to -10^o)$, Gopinathan [177], $\beta_{opt} = \varphi$, El-Kassaby [104], $\beta_{opt} = \varphi + 3.5^o$, Lewis [178], $\beta_{opt} = \varphi \pm 8^o$, Garg [179], $\beta_{opt} = \varphi \pm 15^o$ or $\beta_{opt} = 0.9\varphi$, Duffie and Beckman [180], $\beta_{opt} = (\varphi + 15^o) \pm 15^o$, Lunde [181], $\beta_{opt} = \varphi \pm 15^o$ Kern and Harris [165], $\beta_{opt} = \varphi + 10^o$, Löf and Tybout [182], $\beta_{opt} = \varphi + (10^o \to 30^o)$, Yellott [183], $\beta_{opt} = \varphi + 20^o$, Heywood [184], $\beta_{opt} = \varphi - 10^o$, Chinnery [185], $\beta_{opt} = \varphi + 10^o$, Hottel [186], $\beta_{opt} = \varphi + 20^o$.

4.1. Tilt angle optimization using Artificial Neural Network (ANN) techniques

Chang [29] determined the optimum tilt angle for PV modules to achieve the maximum output power energy in seven areas of Taiwan using sequential neural-network approximation and orthogonal arrays (SNAOA) and found the annual optimal angle for the Taipei area is 23.25°; for Taichung, 22.25°; for Tainan, 21.25°; for Kaosiung, 20.75°; for Hengchung, 20.25°; for Hualian, 22.25°; and for Taitung, 21°. Neocleous and Schizas [45] proposed a method for solving the tilt angle of PV modules problem using sequential feed forward multilayer neural network and ANN to estimate of the 4-h-ahead electric load in a Power Plant. Shaddel et al. [114] used Artificial Neural Network (ANN) of Matlab to estimate hourly global solar irradiation on tilted surfaces at Mashhad, Iran. Celik and Muneer [121] predicted solar radiation on the tilted surface using generalized regression neural networks (GRNN) at Iskenderun, Turkey. Chatterjee and Keyhani [124] estimated the optimum tilt and total irradiance on tilted surface by ANN using 14 inputs (latitude, ground reflectivity and 12 months irradiance value). Soares et al. [146] showed an estimation of hourly horizontal diffuse radiations using neural-networks technique.

Table 6
The solar radiation calculation techniques and effects in the previous researches.

Research Application	Authors	Year	Research Data	Ref.
Solar Radiation Data	Smith et al.	(2016)	Determined tilted irradiance, horizontal irradiance and optimal tilt angle using a computational method by a radiative transfer for calculating the all-sky irradiance.	[113]
	Shaddel et al.	(2016)	Using Artificial Neural Network (ANN) of Matlab to estimate hourly global solar irradiation on tilted surfaces at Mashhad, Iran.	[114]
	Shukla et al.	(2015)	Showed comparative study of 6 different models for estimation of solar radiation at a tilt angle of 23.26° (latitude of Bhopal, India).	[115]
	Khahro et al.	(2015)	Determined the optimum tilt angle for monthly, seasonally, half-yearly and yearly adjustment which varies from 0° in May, June and July to 49° in December. The yearly optimum tilt angle was found as 23°, which is close to latitude of investigated location (25° 07′N) in southern region of Sindh, Pakistan	[116]
	Simón-Martín et al.	(2015)	Presented a new device to measure diffuse solar radiation simultaneous under different azimuth and tilt angles.	[117]
	Gorantla and Setty	(2015)	Observed the direct solar radiation through glazing at different latitudes with different glass materials. When solar radiation passing through the glass in the south direction is least in North, South-East and South-West directions, when compared to other directions.	[118]
	Shamim et al.	(2015)	Proposed a methodology to estimate clear sky global solar radiation, analytical tools for determining optical transmittance and surface albedo.	[119]
	David et al.	(2013)	Showed models to estimate diffuse radiation on tilted surface using four (Hay, Skartveit and Olseth, Gueymard, Perez) at Reunion Island and found the Perez model shows the best performance and albedo constant i.e. 0.2 provides accurate results.	[120]
	Celik and Muneer	(2013)	Predicted solar radiation on the tilted surface using generalized regression neural networks (GRNN) at Iskenderun, Turkey.	[121]
	Yadav and Chandel	(2013)	Showed a review of the tilt angle to maximize the solar irradiation, which varies depend on both time and location, as solar irradiation is different on different locations and months.	[6]
	Čongradac et al.	(2012)	Presented a method to find out optimum blind tilt angle using GA and fuzzy logic process for maintaining accurate brightness of the room.	[122]
	Notton et al.	(2012)	Developed three ANN models to estimate hourly global irradiation on the tilted plane at Mediterranean site of Ajaccio, France.	[123]
	Chatterjee and Keyhani	(2012)	Estimated the optimum tilt and total irradiance on tilted surface by ANN using 14 inputs (latitude, ground reflectivity and 12 months irradiance value).	[124]
	Vignola et al.	(2012)	Presented a prototype that is can simultaneously measure diffuse irradiance on planes tilted from 60° to 90° oriented towards the four cardinal directions: North, South, East and West.	[125]
	Agarwal et al.	(2012)	Compared the tilt angles numerically derived by 4 isotropic and 4 anisotropic models and maximize terrestrial solar energy and at India.	[126]
	Ghosh et al.	(2010)	Determined hourly and seasonal optimum tilt angles and solar radiations of single axis and double axis tracking surfaces using three mathematical models are Isotropic, Klucher and Perez model for at Dhaka.	[127]
	Mehleri et al.	(2010)	Estimated global solar irradiance on a horizontal surface, extraterrestrial radiation, solar zenith angle and solar incidence angle on a tilted plane as inputs to RBFNN for estimating global solar irradiance on inclined surfaces at Athens, Greece.	[128]
	El-Sebaii et al.	(2010)	Showed the optimal tilt angle equals to the latitude of the place and estimate the amount of global solar radiation of horizontal surfaces using various climatic parameters, such as sunshine duration, cloud cover, humidity, maximum and minimum ambient temperatures, and wind speed at Jeddah.	[129]
	Pandey and Katiyar	(2009)	Presented at different tilt angles (15°, 30°, 45° and 60°), the different diffuse components of the solar radiation at Lucknow (Latitude 26.75°, Longitude 80.85°), India.	[130]
	Chang	(2009)	Compered between single axis tracking and fixed of PV panel, In addation, analyzed the gain in extraterrestrial radiation and found the incident angle of sunlight upon the tracked panel is smaller than the fixed panel, except at solar noon.	[32]
	Psiloglou and Kambezidis	(2009)	Reported several methods to calculate ground albedo under lining its important influence on the estimation of solar radiation incident on tilted surfaces.	[131]
	Bakirci	(2009)	Developed the correlations to estimate of daily global solar radiation with hours of bright sunshine in Turkey.	[132]
	Janjai et al.	(2009)	Proposed a model to calculate the monthly average hourly global radiation in the tropics with high aerosol load using satellite data at Thailand.	[133]
	Li et al.	(2008)	Calculated solar radiation and the optimum tilt angle on the tilted surface with different orientations based on sunshine hour data model where the maximum error in this model is found to be less than 5.2%.	[134]
	Noorian et al.	(2008)	Showed hourly diffuse solar radiation on tilted surfaces to 12 isotropic and anisotropic models in Iran.	[135]
	Mondol et al.	(2008)	Studied the variance of daily and seasonal solar radiation and PV output and the effect on potential savings in residential electricity bills at Northern Ireland and found that slightly westerly (190°) angles is the best.	[136]
	Li et al.	(2007)	Found the optimum tilt angle of a solar collector to be around 20° due south to receive the annual solar yield over 1598 kWh/m^2 in Hong Kong and found that the tilt angle approximately equal to latitude of the place could receive maximum annual solar radiation.	[137]
	Wu et al.	(2007)	Used the metrological data from 1994 to 2005 to predict daily global solar radiation from sunshine hours, air temperature, total precipitation and dew point at Nanchang station (China).	[138]
	Bulut and Buyukalaca	(2007)	Proposed a model to estimate monthly average daily global solar radiation on horizontal surface in 68 provinces at Turkey.	[139]
	Sen	(2007)	Proposed a nonlinear model to estimate global solar radiation through sunshine hour data.	[140]
	Elminir et al.	(2006)	Compared three anisotropic models to estimate solar radiation and determine the optimum tilt angle of solar collector to maximize solar energy availability in Egypt.	[141]
	Gunerhan	(2005)	Calculated the optimum tilt angle by maximize the extraterrestrial solar radiation.	[142]
	Chandel et al.	(2005)	Developed a model to determine hourly solar radiation using sunshine hours and temperature data.	[143]
	El-Sebaii	(2005)	Showed global solar radiation to achieve the optimal solar energy conversion system design and prediction of its performance.	[144]
	Sugden	(2004)	Review the calculations of the view factors that is of interest and proposed for application to solar radiation for e-cast.	[145]
	Soares et al.	(2004)	Estimation of hourly horizontal diffuse radiations using neural-networks technique.	[146]
	Ulgen and Hepbasli	(2003)	Presented the diffuse fraction of daily and monthly global radiation at Izmir, Turkey.	[147]
	El-Sebaii and Trabea	(2003)	Proposed the correlations to estimate the horizontal diffuse radiation in Egypt by correlating (H_d/H_g) and (H_d/H_o) with KT and (S/S_{max}).	[148]

(continued on next page)

Table 6 *(continued)*

Research Application	Authors	Year	Research Data	Ref.
	Gueymard	(2000)	Calculated the hourly solar radiation from daily solar radiation data using daily integration approach.	[149]
	Hartely et al.	(1999)	Compared three models (Temps–Coulson, Klucher and Hay) to estimate total radiation on vertical planes facing north, south, east and west at Valencia Spain (Lat 39.5° N, Long 0.67° W).	[150]
	Hartley and Martinez-Lozano	(1999)	Used Liu and Jordan model to maximize monthly average solar irradiance on a horizontal plane but without considering the view factor of the collector to the ground.	[151]
	Tiris et al.	(1996)	Calculated the monthly average daily global, diffuse and beam radiations with hours of bright sunshine at Gebze, Turkey.	[152]
	Khalil and Alnajjar	(1995)	Proposed annual optimum tilt angle of solar collector is 24° using the global, beam, and diffuse solar radiation on inclined surface by clear sky model at Al Ain, UAE.	[153]
	Zuhairy and Sayigh	(1995)	Carried out simulation and modeling of solar radiation in Saudi Arabia.	[154]
	Reindl et al.	(1990)	Obtained hourly diffuse solar radiation on horizontal surface by using hourly total solar radiation, an independent data from United States and Europe.	[155]
	Elsayed	(1989)	Showed a correlation of the optimum tilt angle as a function of clearness index, latitude and day number and presented an analytical model depend on long-term of solar data at Iran.	[156]
	Abdalla and Feregh	(1988)	Showed optimum tilt and orientation angles of solar surfaces and solar energy potential and resources at United Arab Emirates.	[157]
	Erbs et al.	(1982)	Developed a model to estimate the monthly average diffuse radiation on a horizontal surface using the data from United States and Australian metrological stations.	[158]
	Klein and Theilacker	(1981)	Showed a model to estimate the solar radiation on an inclined surface at different orientations to calculate sunrise and sunset for different surface orientations and solar radiations between tevery 2 h.	[159]
	Hay and Davies	(1980)	Developed an isotropic model by adding circumsolar radiation to diffuse radiation.	[160]
	Klucher	(1979)	Proposed to add factor-representing effect of cloudiness on diffuse solar radiation and is called as anisotropic model to estimate solar radiation on the south-facing inclined surface.	[161]
	Collares-Pereira and Rabl	(1979)	Developed a model to estimate daily diffuse radiation on horizontal surface to obtain monthly average diffuse radiation based on summation.	[162]
	Temps and Coulson	(1977)	Calculated the solar radiation on an inclined surface by adding horizontal brightening item to isotropic model.	[163]
	Sabbagh et al.	(1977)	Determined the daily global radiation at different locations in Lebanon, Kuwait, Sudan, Egypt and Saudi Arabia.	[164]
	Kern and Harris	(1975)	Calculated the optimum tilt angle by maximize the direct solar radiation incident on the collector.	[165]
	Liu and Jordan	(1960)	Estimated the solar radiation; beam, diffuse and ground-reflected radiations on an inclined surface using an isotropic model.	[166]

4.2. Tilt angle optimization using GA, SA and PSO techniques

Talebizadeha et al. [18] calculated hourly, daily, monthly, seasonally and yearly optimum tilt angle of PV panels and solar collectors using GA at Iran and showed that optimum hourly azimuth angle is not zero. Chang [27] determined the optimal tilt angles of PV modules fixed south facing for maximum output electrical energy of the modules in Taiwan using a particle-swarm optimization method with nonlinear time-varying evolution (PSO—NTVE) and found the annual optimal angle for the Taipei area is 18.16° and 17.3°, 16.15°, 15.79°, 15.17°, 17.16°, 15.94° for Taichung, Tainan, Kaosiung, Hengchung, Hualian, Taitung respectively. Ko et al. [35] determined the optimal combination of inertiaweights and acceleration coefficients by orthogonal array experiments as proposed using PSO–NTVE method and compared this method and three other PSO methods. Čongradac et al. [122] presented a method to find out optimum blind tilt angle using GA and fuzzy logic process for maintaining accurate brightness of the room.

4.3. Tilt angle optimization using other techniques

Shukla et al. [115] showed comparative study of six different models for estimation of solar radiation at a tilt angle of 23.26° (latitude of Bhopal, India). Hartely et al. [150] compared three models (Temps–Coulson, Klucher and Hay) to estimate total radiation on vertical planes facing north, south, east and west at Valencia Spain (Lat 39.5° N, Long 0.67° W). Hartley and Martinez-Lozano [151] used Liu and Jordan model to maximize monthly average solar irradiance on a horizontal plane without considering the view factor of the collector to the ground.

Smith et al. [113] determined tilted irradiance, horizontal irradiance and optimal tilt angle using a computational method by a radiative transfer for calculating the all-sky irradiance. Khahro et al. [116] determined the optimum tilt angle for monthly, seasonally, half-yearly and yearly adjustment which varies from 0° in May, June and July to 49° in December. The yearly optimum tilt angle was found as 23°, which is close to latitude of investigated location (25° 07′N) in southern region of Sindh, Pakistan. Mondol et al. [136] studied the variance of daily and seasonal solar radiation and PV output and the effect on potential savings in residential electricity bills at Northern Ireland and found that slightly westerly (190°) angles is the optimum. Gueymard [149] calculated the hourly solar radiation from daily solar radiation data using daily integration approach.

Li et al. [134] calculated solar radiation and the optimum tilt angle on the tilted surface with different orientations based on sunshine hour data model where the maximum error in this model is found to be less than 5.2%. Chandel et al. [143] developed a model to determine hourly solar radiation using sunshine hours and temperature data. Klein and Theilacker [159] showed a model to estimate the solar radiation on an inclined surface at different orientations to calculate sunrise and sunset.

Yadav and Chandel [6] showed a review of the tilt angle to maximize the solar irradiation, which varies depending on both time and location. Chang [32] compared between single axis tracking and fixed PV panels. In addition, he analyzed the gain in extraterrestrial radiation and found the incident angle of sunlight upon the tracked panel is smaller than the fixed panel, except at solar noon. Li et al. [137] found the optimum tilt angle of a solar collector to be around 20° due south to receive the annual solar yield over 1598 kWh/m^2 in Hong Kong and found that the tilt angle is approximately equal to the latitude of the place to receive maximum annual solar radiation. Wu et al. [138] used the metrological data. Gunerhan [142] calculated the optimum tilt angle by maximizing the extraterrestrial solar radiation. Sugden [145] reviewed the calculations of the view factors and it is effected on solar radiation. Zuhairy and Sayigh [154] carried out simulation and modeling of solar radiation in Saudi Arabia. Elsayed [156] showed a correlation of the optimum tilt angle as a function of clearness index, latitude and day number and presented an analytical model which depends on long-term solar data at Iran. Abdalla and Feregh [157] showed optimum tilt and orientation angles of solar surfaces and solar energy potential and resources at United Arab Emirates.

Table (7) shows the optimum tilt angle relations with respect to altitude in the previous researches. Table (8) shows optimum tilt angle data (Monthly, Seasonally, Yearly) in the previous researches at many locations. Other suggestions for fixed optimum tilt angle were found as follows:

Stanciu, C., and Stanciu, D. [78] proposed a numerical simulation of a flat plate collector for the optimum tilt angle at different latitudes from 0° to 80° as function of the latitude (ϕ) and solar declination (δ).

$$\beta_{opt} = \varphi - \delta \tag{23}$$

Bakirci [8] optimized the tilt angles for solar panels using solar radiation data measured using a polynomial correlation for the optimum tilt angle as a function of solar declination to eight provinces in Turkey where the optimum tilt angle varies from 0° to 65° throughout the year.

$$\beta_{opt} = 34.783 - 1.4317\delta - 0.0081\delta^2 + 0.0002\delta^3 \tag{24}$$

Rowlands et al. [23] evaluated the optimal tilt angle and azimuth of a PV panel in Canada for two locations and they found only slight deviations from output that are maximizing angle combinations.

$$\beta_{opt} = \varphi \tag{25}$$

Lave and Kleissl [21] determined the optimum tilt angle and azimuth angle of solar PV panels in US and achieved the fixed tilted panel and two axis tracking 10–25%, 25–45% higher irradiation respectively.

$$\beta_{opt} = \varphi - (1^o - 10^o) \tag{26}$$

Moghadam et al. [87] performed an optimization of solar flat collector inclination to determine monthly, seasonal, semi-annual and annual optimum tilt angles.

$$\beta_{opt} = 0.917\varphi \pm 0.321^o \tag{27}$$

Benghanem [17] performed a study to achieve the annual optimum tilt angle of the solar panels surface for receiving a maximum solar radiation and found that it is approximately equal to the latitude of the site at Madinah, Saudi Arabia. The annual optimum tilt angle was found to be 23.5° with respect to the latitude of Madinah site 24.5°, and for the winter months is 37° and for the summer months is 12°.

$$\beta_{opt} = \varphi \tag{28}$$

Ahmad et al. [34] showed different tilt angle optimatiztion methods of PV panels as monthly, seasonal, yearly-based optimization to improve the efficiency and the reliability of the system at New Delhi, India where the optimum tilt angle in summer months is 13°.

$$\beta_{opt} = \varphi - 60^o \tag{29}$$

And winter is 47.5°

$$\beta_{opt} = \varphi + 90^o \tag{30}$$

Calabrò [31] proposed a relationship between the optimum tilt angles of PV panels and the latitude outside tropics from 36° to 46° at USA and Europe and showed the optimum tilt angles for winter months are very different from the summer months.

$$\beta_{opt} = \varphi - (26^o, 27^o, 28^o) \tag{31}$$

Gunerhan and Hepbasli [92] determined the monthly optimum orientation and tilt angles of solar collectors β_{opt} equal to latitude φ throughout the year at Izmir, Turkey and suggested for summer

Table 7
Optimum Tilt Angle relations with respect to altitude in the previous researches.

Authors	Year	Location	Optimum Tilt Angle with respect to altitude Math. Model[a]	Ref.
Stanciu C. and Stanciu D.	(2014)	Romania	$\beta_{opt} = \varphi - \delta$	[78]
Uba and Sarsah	(2013)	Ghana	$\beta_{opt} = \varphi + 17^o$	[196]
Bakirci	(2012)	Eight provinces of Turkey	$\beta_{opt} = 34.783 - 1.4317\delta - 0.0081\delta^2 + 0.0002\delta^3$	[8]
Rowlands et al.	(2011)	Ontario, Canada	Yearly, $\beta_{opt} = \varphi$	[23]
		Ottawa & Toronto, Canada	Yearly, $\beta_{opt} = \varphi - (7^{\,o} \rightarrow 12^o)$	[23]
Lave and Kleissl	(2011)	United States	$\beta_{opt} = \varphi - (1^o - 10^o)$	[21]
Moghadam et al.	(2011)	Iran	$\beta_{opt} = 0.917\varphi \pm 0.321^o$	[87]
Benghanem	(2011)	Madinah, Saudi Arabia	Yearly, $\beta_{opt} = \varphi$	[17]
Ahmad and Tiwari	(2009)	New Delhi, India	Summer; $\beta_{opt} = \varphi - 60^o$; Winter; $\beta_{opt} = \varphi + 90^o$	[34]
Calabròa	(2009)	USA and Europe	$\beta_{opt} = \varphi - (26^o, 27^o, 28^o)$; Where φ varies from 36° to 46°	[31]
Gunerhan and Hepbasli	(2007)	Izmir, Turkey	Summer or Winter $\beta_{opt} = \varphi \pm 15^o$ March and September; $\beta_{opt} = \varphi$	[92]
Ulgen	(2006)	Izmir, Turkey	Summer; $\beta_{opt} = \varphi - 34^o$; Winter; $\beta_{opt} = \varphi + 19^{\,o}$	[175]
Elminir et al.	(2006)	Helwan, Egypt	$\beta_{opt} = \varphi \pm 15^o$	[141]
Duffie and Beckman	(2006)		$\beta_{opt} = (\varphi + 15^o) \pm 15^o$	[176]
Tang and Wu	(2004)	China	$\beta_{opt} = \varphi + (4^{\,o} \rightarrow -10^o)$	[93]
Shariah et al.	(2002)	Amman, Jordan	$\beta_{opt} = \varphi - 3^o$	[55]
		Aqaba, Jordan	$\beta_{opt} = \varphi$	[55]
Ibrahim	(1995)	Cyprus	Summer; $\beta_{opt} = \varphi - 21^o$; Winter; $\beta_{opt} = \varphi + 13^{\,o}$	[190]
Gopinathan	(1991)	South Africa	$\beta_{opt} = \varphi$	[177]
El-Kassaby	(1988)	Egypt	$\beta_{opt} = \varphi + 3.5^o$	[104]
Lewis	(1987)		$\beta_{opt} = \varphi \pm 8^o$	[178]
Buresch	(1983)		$\beta_{opt} = \varphi \pm 11^o$	[203]
Garg	(1982)		$\beta_{opt} = \varphi \pm 15^o$ or $\beta_{opt} = 0.9\varphi$	[179]
Duffie and Beckman	(1980)		$\beta_{opt} = (\varphi + 15^o) \pm 15^o$	[180]
Lunde	(1980)		$\beta_{opt} = \varphi \pm 15^o$	[181]
Iqbal	(1979)		$\beta_{opt} = \varphi + (-10^o \rightarrow 15^o)$	[108]
Garg and Gupta	(1978)		$\beta_{opt} = \varphi \pm 5^o$	[204]
Kern and Harris	(1975)		$\beta_{opt} = \varphi + 10^o$	[165]
Löf and Tybout	(1973)		$\beta_{opt} = \varphi + (10^o \rightarrow 30^o)$	[182]
Yellott	(1973)		$\beta_{opt} = \varphi \pm 20^o$	[183]
Heywood	(1971)		$\beta_{opt} = \varphi - 10^o$	[184]
Chinnery	(1967)	South Africa	$\beta_{opt} = \varphi + 10^o$	[185]
Hottel	(1954)	USA	$\beta_{opt} = \varphi + 20^o$	[186]

[a] ±The minus sign refers to the summer season and the plus sign is for the winter.

$$\beta_{opt} = \varphi + 15^o \tag{32}$$

And for winter

$$\beta_{opt} = \varphi - 15^o \tag{33}$$

Ulgen [175] showed that optimum tilt angle, in Izmir, Turkey, for summer season is

$$\beta_{opt} = \varphi - 34^o \tag{34}$$

And that for winter season is

$$\beta_{opt} = \varphi + 19^o \tag{35}$$

Elminir et al. [141] compared three anisotropic models to estimate solar radiation and determined the optimum tilt angle of solar collector to maximize solar energy availability in Egypt.

$$\beta_{opt} = \varphi \pm 15^o \tag{36}$$

Duffie and Beckman [176] showed optimal tilt angle as

$$\beta_{opt} = (\varphi + 15^o) \pm 15^o \tag{37}$$

Tang and Wu [93] developed a mathematical procedure to determine the optimal tilt angle of a collector using the monthly global and diffuse radiation on horizontal radiation for 30 cities in China.

$$\beta_{opt} = \varphi + (4^{\,o} \rightarrow -10^o) \tag{38}$$

Gopinathan [177] obtained the optimum tilt angle in South Africa as

$$\beta_{opt} = \varphi \tag{39}$$

El-Kassaby [104] determined the optimum tilt and azimuth angles for an absorber plate covered by one or two glass covers of solar collector and suggested that using two glass covers instead of one did not affect the value of β_{opt} in Egypt.

Table 8
Optimum tilt angle (monthly, seasonally, yearly) in the previous researches.

Authors	Year	Location	Optimum Tilt Angle Monthly/ Seasonally/ Yearly	Seasonally	Ref.
Yan et al.	(2013)	Brisbane, Australia	Yearly	26°N	[3]
Jafarkazemi and Saadabadi	(2013)	Abu Dhabi, UAE	Monthly	June −9°, December 52°	[5]
			Yearly	22°	
Uba and Sarsah	(2013)	Ghana	Monthly	40° (January); 28.7° (February);10.6° (March);	[196]
			Yearly	26.8°	
Colli and Zaaiman	(2012)	Italy	Yearly	30°	[193]
Elhab et al.	(2012)	Kuala Lumpur, Malaysia	Yearly	10°	[201]
Kaldellis and Zafirakis	(2012)	Athens, Greece	Monthly	15° with (± 2.5°) deviation for summer	[13]
Khatib et al.	(2012)	Kuala Lumpur, Johor Bharu, Ipoh, Kuching, Alor Setar in Malaysia	Seasonally	Maximum in summer and minimum in winter	[200]
Xianping	(2012)	Changsha, China	Yearly	19.22° (i.e. 10° less than latitude)	[202]
Agarwal et al.	(2012)	Nandha Haryana, India	Seasonally	(December) 62°; Liu and Jorden, 63.5°; Reindel, 63°; Hay, 62.5°; Badescu	[126]
			Yearly	30.61°, Liu and Jorden; 31.66°, Reindel; 31.20°, Hay; 29.58°, Badescu	
Agarwal et al.	(2012)	Delhi, Haryana, India	Monthly	(January) 60°; Liu and Jorden, 61.5°; Reindel, 61°; Hay, 60.5°; Badescu (June) 0° to all models (Liu and Jorden, Reindel, Hay, Badescu)	[126]
			Seasonally	55° (Winter); 8° (Summer)	
			Yearly	28.29°; Liu and Jorden, 29.41°; Reindel, 29.08°; Hay, 27.58°; Badescu	
Bakirci	(2012)	Turkey	Monthly	0° (June) and 65° (December)	[8]
			Seasonally	21.17° (spring), 5.67° (summer), 46.48° (autumn), 57.29° (winter)	
Bakirci	(2012)	Adana, Turkey	Monthly	0° (June and July); 60° (December)	[8]
			Yearly	31.2°	
		Ankara, Turkey	Monthly	0° (June); 60° (December)	
			Yearly	32.7°	
		Diyarbakir, Turkey	Monthly	0° (June and July); 61° (December)	
			Yearly	32.6°	
		Izmir, Trabzon, Turkey	Monthly	0° (June); 61° (December)	
			Yearly	32.8°	
		Erzurum, Turkey	Monthly	0° (June); 65° (December)	
			Yearly	34.3°	
		Istanbul, Turkey	Monthly	0° (June); 59° (December)	
			Yearly	32.6°	
		Samsun, Turkey	Monthly	0° (June); 62° (December)	
			Yearly	33.2°	
Rowlands et al.	(2011)	Ontario, Canada	Yearly	Equal to latitude, $\beta_{opt} = \varphi$	[23]
Rowlands et al.	(2011)	Ottawa, Canada	Yearly	$\beta_{opt} = 36^{o} \rightarrow 38^{o}$;$\varphi$= 45°	[23]
Rowlands et al.	(2011)	Toronto, Canada	Yearly	$\beta_{opt} = 32^{o} \rightarrow 35^{o}$;$\varphi$= 44°	[23]
Daut et al.	(2011)	Perlis, Northern Malaysia	Monthly	−17.160° to 29.740° (± means PV facing is north, south)	[199]
Lave and Kleissl	(2011)	United States of America (USA)	Yearly	Optimum orientation (Tilt/Azimuth) Orlando, FL, 29.1°/7.6°E; Dallas, TX, 30.5°/5.9°E; Phoenix, AZ, 33.4°/0.3°W; Los Angeles, CA, 32.4°/3.8°W; St. Louis, MO, 34.8°/1.0°W	[21]
Moghadam et al.	(2011)	Zahedan, Iran	Bi-Annual	5°, 50°	[87]
			Yearly	28°	
Benghanem	(2011)	Madinah, Saudi Arabia	Monthly	40° (January); 10° (June); 40° (December)	[17]
			Seasonally	37° (Winter); 12° (Summer); 17° (Spring); 28° (Autumn)	
			Yearly	23.5°, Equal to latitude, $\beta_{opt} = \varphi$	
Chang	(2010)	Taipei, Taiwan	Yearly	18.16°	[26]
		Taichung, Taiwan		17.3°	
		Tainan, Taiwan		16.15°	
		Kaosiung, Taiwan		15.79°	
		Hengchung, Taiwan		15.17°	
		Hualian, Taiwan		17.16°	
		Taitung, Taiwan		15.94°	
Zhao et al.	(2010)	Singapore	Monthly	Daily average varies from 0.1° to 25.9°; Monthly average varies from 0.1° to 28.7°	[198]
Ghosh et al.	(2010)	Dhaka, Bangladesh	Seasonally	10° (March–September), 40° (October–February)	[127]
			Yearly	28°	

(continued on next page)

Table 8 *(continued)*

Authors	Year	Location	Optimum Tilt Angle Monthly/ Seasonally/ Yearly	Seasonally	Ref.
Skeiker	(2009)	Syria	Monthly Yearly	0° (June, July); 63° (December) 30.56°	[89]
Ahmad and Tiwari	(2009)	New Delhi, India	Seasonally	Summer 13° (φ− 60°), Winter 47.5° (φ+ 90°)	[34]
			Yearly	Equal to latitude,$\beta_{opt} = \varphi$	
Gunerhan and Hepbasli	(2007)	Izmir in Turkey	Monthly	38.46° (March and September)	[92]
			Seasonally Yearly	53.46° (Summer); 23.46° (Winter) Equal to latitude,$\beta_{opt} = \varphi$	
Ulgen	(2006)	Izmir in Turkey	Monthly Yearly	0° (June); 61° (December) 30.3°	[175]
Elminir et al.	(2006)	Helwan, Egypt	Monthly	55° (January); 45° (February); 30° (March); 15° (April); 5° (May); 5° (June); 5° (July); 15° (August); 25° (September); 40° (October); 50° (November); 55° (December)	[141]
			Seasonally Yearly	43.33° (Winter); 15° (Summer) 28.75°	
Gong and Kulkarni	(2005)	Carbondale, Illinois, USA	Yearly	$\beta_{opt} = 30^{o}$;φ= 37° 46′	[189]
Hussein	(2004)	Egypt	Yearly	$\beta_{opt} = 20^{o} \rightarrow 30^{o}$	[39]
Kacira et al.	(2004)	Sanliurfa, Turkey	Monthly	13° (June); 61° (December)	[40]
Hiraoka et al.	(2003)	Japan	Yearly	26.5°	[187]
Shariah et al.	(2002)	Amman, Jordan	Yearly	8°	[55]
		Aqaba, Jordan	Yearly	5°	
Yakup and Malik	(2001)	Brunei Darussalam	Monthly Yearly	1.6° (September), 32.3° (December) 3.3°	[195]
De Miguel et al.	(1995)	Burgos, Spain	Monthly	70° (January); 2° (June); 80° (December)	[191]
Ibrahim	(1995)	Cyprus	Seasonally Yearly	22° (Spring), 14° (summer), 40° (autumn), 48° (winter) 31°	[190]
El-Kassaby	(1988)	Egypt	Monthly	58° (November, December, January); 46° (February, October); 30° (March, September); 10° (April, August); 0° (May, June, July)	[104]
Saraf and Hamad	(1988)	Basra, Iraq	Monthly Yearly	15° (June); 63° (December); 38.13°	[103]
Tsalides and Thanailakis	(1985)	Greece	Yearly	± 60° (Greater than latitude)	[197]

$$\beta_{opt} = \varphi + 3.5^{o} \tag{40}$$

Lewis [178] presented optimum tilt angle as

$$\beta_{opt} = \varphi \pm 8^{o} \tag{41}$$

Garg [179] suggested optimal tilt angle as

$$\beta_{opt} = \varphi \pm 15^{o} \tag{42}$$

or

$$\beta_{opt} = 0.9\varphi \tag{43}$$

Duffie and Beckman [180] obtained as

$$\beta_{opt} = (\varphi + 15^{o}) \pm 15^{o} \tag{44}$$

The minus sign refers to the summer season and the plus sign is for the winter.

Lunde [181] showed that optimum tilt angle as

$$\beta_{opt} = \varphi \pm 15^{o} \tag{45}$$

Kern and Harris [165] calculated the optimum tilt angle by maximizing the direct solar radiation incident on the collector.

$$\beta_{opt} = \varphi + 10^{o} \tag{46}$$

Löf and Tybout [182] showed different tilt angle optimatiztion methods to be

$$\beta_{opt} = \varphi + (10^{o} \rightarrow 30^{o}) \tag{47}$$

Yellott [183] presented optimal tilt angle as

$$\beta_{opt} = \varphi + 20^{o} \tag{48}$$

Heywood [184] performed a study to achieve the annual optimum tilt angle as

$$\beta_{opt} = \varphi - 10^{o} \tag{49}$$

Chinnery [185] proposed a method to calculate the optimum tilt angle as

$$\beta_{opt} = \varphi + 10^{o} \tag{50}$$

Hottel [186] showed tilt angle as

$$\beta_{opt} = \varphi + 20^{o} \tag{51}$$

5. Conclusion

This paper contributes an in-depth description of most tilt angle design criteria regarding solar energy technologies. Likewise, the review has allowed analysis of several measures consecutively applied to achieve the best output on the electricity system of the solar system or the reach of the best thermal heat collected from the solar radiation. The optimum tilt angle is low in the summer and spring, high in the winter, and autumn; where the lowest optimum tilt angle during the months (May, June, and July) is 0° and the highest values during the months (November, December, and January) is more than 30° in different countries. The tracking system is one of the most promising systems where it used in different solar energy applications and the main application of it in solar thermal application (point concentration systems) that used dual axes of the tracking systems as a mandatory system such as solar tower systems. The photovoltaic systems are showed a great gain in the systems that used optimum yearly tilt angle where the tracking system is not the preferred design because in the developing countries there are a leakage in maintenance. The main objective of the current research is to review different techniques for optimizing the tilt angle and study the effects of different parts of the solar radiation (beam, diffuse, and ground reflected) on the design performance. The theoretical concept and calculations were carried out in many solar systems with different factors in various locations. The above solar radiation models are applied using different models and specially Liu and Jordan model to calculate the average daily solar radiation on tilted surface to achieve the optimum tilt angle. The most researchers uses a basic equations and simulate it using models to make appropriate site selections in order to achieve sustainable development by effective solar radiation site.

References

[1] Hartner M, Ortner A, Hiesl A, Haas R. East to west–The optimal tilt angle and orientation of photovoltaic panels from an electricity system perspective. Appl Energy 2015;160:94–107.
[2] Jeyaprabha SB, Selvakumar AI. Optimal sizing of photovoltaic/battery/diesel based hybrid system and optimal tilting of solar array using the artificial intelligence for remote houses in India. Energy Build 2015;96:40–52.
[3] Yan R, Saha TK, Meredith P, Goodwin S. Analysis of yearlong performance of differently tilted photovoltaic systems in Brisbane, Australia. Energy Convers Manag 2013;74:102–8.
[4] Ismail MS, Moghavvemi M, Mahlia TMI. Design of an optimized photovoltaic and microturbine hybrid power system for a remote small community: case study of Palestine. Energy Convers Manag 2013;75:271–81.
[5] Jafarkazemi F, Saadabadi SA. Optimum tilt angle and orientation of solar surfaces in Abu Dhabi, UAE. Renew Energy 2013;56:44–9.
[6] Yadav AK, Chandel SS. Tilt angle optimization to maximize incident solar radiation: a review. Renew Sustain Energy Rev 2013;23:503–13.
[7] Kaldellis J, Kavadias K, Zafirakis D. Experimental validation of the optimum photovoltaic panels' tilt angle for remote consumers. Renew Energy 2012;46:179–91.
[8] Bakirci K. General models for optimum tilt angles of solar panels: turkey case study. Renew Sustain Energy Rev 2012;16(8):6149–59.
[9] Lucio JH, Valdés R, Rodr´guez LR. Loss-of-load probability model for stand-alone photovoltaic systems in Europe. Sol Energy 2012;86(9):2515–35.
[10] Asowata O, Swart J, Pienaar C. Optimum Tilt Angles for Photovoltaic Panels during Winter Months in the Vaal Triangle, South Africa. Smart Grid Renew Energy SGRE 2012;03(02):119–25.
[11] Bojić M, Bigot D, Miranville F, Parvedy-Patou A, Radulović J. Optimizing performances of photovoltaics in Reunion Island-tilt angle. Prog Photovolt: Res Appl 2012;20(8):923–35.
[12] Gharakhani Siraki A, Pillay P. Study of optimum tilt angles for solar panels in different latitudes for urban applications. Sol Energy 2012;86(6):1920–8.
[13] Kaldellis J, Zafirakis D. Experimental investigation of the optimum photovoltaic panels' tilt angle during the summer period. Energy 2012;38(1):305–14.
[14] Liu G, Rasul MG, Amanullah MTO, Khan MMK. Techno-economic simulation and optimization of residential grid-connected PV system for the Queensland climate. Renew Energy 2012;45:146–55.
[15] Beringer S, Schilke H, Lohse I, Seckmeyer G. Case study showing that the tilt angle of photovoltaic plants is nearly irrelevant. Sol Energy 2011;85(3):470–6.
[16] Wada H, Yamamoto F, Ueta K, Yamaguchi T. Generation characteristics of 100 kW PV system with various tilt angle and direction arrays. Sol Energy Mater Sol Cells 2011;95(1):382–5.
[17] Benghanem M. Optimization of tilt angle for solar panel: case study for Madinah, Saudi Arabia. Appl Energy 2011;88(4):1427–33.
[18] Talebizadeh P, Mehrabian MA, Abdolzadeh M. Prediction of the optimum slope and surface azimuth angles using the Genetic Algorithm. Energy Build 2011;43(11):2998–3005.
[19] Sunderan P, Ismail AM, Singh B, Mohamed NM. Optimum tilt angle and orientation of stand-alone photovoltaic electricity generation systems for rural electrification. J Appl Sci 2011;11(7):1219–24.
[20] Maatallah T, El Alimi S, Nassrallah SB. Performance modeling and investigation of fixed, single and dual-axis tracking photovoltaic panel in Monastir city, Tunisia. Renew Sustain Energy Rev 2011;15(8):4053–66.
[21] Lave M, Kleissl J. Optimum fixed orientations and benefits of tracking for capturing solar radiation in the continental United States. Renew Energy 2011;36(3):1145–52.
[22] Lubitz WD. Effect of manual tilt adjustments on incident irradiance on fixed and tracking solar panels. Appl Energy 2011;88(5):1710–9.
[23] Rowlands IH, Kemery BP, Beausoleil-Morrison I. Optimal solar-PV tilt angle and azimuth: an Ontario (Canada) case-study. Energy Policy 2011;39(3):1397–409.
[24] Mehleri ED, Zervas PL, Sarimveis H, Palyvos JA, Markatos NC. Determination of the optimal tilt angle and orientation for solar photovoltaic arrays. Renew Energy 2010;35(11):2468–75.
[25] Chang YP. An ant direction hybrid differential evolution algorithm in determining the tilt angle for photovoltaic modules. Expert Syst Appl 2010;37(7):5415–22.
[26] Chang YP. Optimal the tilt angles for photovoltaic modules in Taiwan. Int J Electr Power Energy Syst 2010;32(9):956–64.
[27] Chang YP. Optimal the tilt angles for photovoltaic modules using PSO method with nonlinear time-varying evolution. Energy 2010;35(5):1954–63.
[28] Cheng CL, Jimenez CSS, Lee MC. Research of BIPV optimal tilted angle, use of latitude concept for south orientated plans. Renew Energy 2009;34(6):1644–50.
[29] Chang YP. Optimal design of discrete-value tilt angle of PV using sequential neural-network approximation and orthogonal array. Expert Syst Appl 2009;36(3):6010–8.
[30] Handoyo EA, Ichsani D. The optimal tilt angle of a solar collector. Energy Procedia 2013;32:166–75.
[31] Calabrò E. Determining optimum tilt angles of photovoltaic panels at typical north-tropical latitudes. J Renew Sustain Energy 2009;1(3):033104.
[32] Chang TP. The gain of single-axis tracked panel according to extraterrestrial radiation. Appl Energy 2009;86(7–8):1074–9.
[33] Chang TP. Output energy of a photovoltaic module mounted on a single-axis tracking system. Appl Energy 2009;86(10):2071–8.
[34] Jamil Ahmad M, N Tiwari G. Optimization of tilt angle for solar collector to receive maximum radiation. The open renewable. Energy J 2009;2(1):19–24.
[35] Ko CN, Chang YP, Wu CJ. An orthogonal-array-based particle swarm optimizer with nonlinear time-varying evolution. Appl Math Comput 2007;191(1):272–9.
[36] Burger B, Rüther R. Inverter sizing of grid-connected photovoltaic systems in the light of local solar resource distribution characteristics and temperature. Sol Energy 2006;80(1):32–45.
[37] Chen YM, Lee CH, Wu HC. Calculation of the optimum installation angle for fixed solar-cell panels based on the genetic algorithm and the simulated-annealing method. IEEE Trans Energy Convers 2005;20(2):467–73.
[38] El K, Kavadias K, Kaldellis JK. Techno-economic evaluation of autonomous building integrated photovoltaic systems in Greece. In International Conference on" Integration of RES into Buildings, Patras, Greece; 2005.
[39] Hussein HMS, Ahmad GE, El-Ghetany HH. Performance evaluation of photo-voltaic modules at different tilt angles and orientations. Energy Convers Manag 2004;45(15):2441–52.
[40] Kacira M, Simsek M, Babur Y, Demirkol S. Determining optimum tilt angles and orientations of photovoltaic panels in Sanliurfa, Turkey. Renew Energy 2004;29(8):1265–75.
[41] Ahmad GE, Hussein HMS, El-Ghetany HH. Theoretical analysis and experimental verification of PV modules. Renew Energy 2003;28(8):1159–68.
[42] Nakamura H, Yamada T, Sugiura T, Sakuta K, Kurokawa K. Data analysis on solar irradiance and performance characteristics of solar modules with a test facility of various tilted angles and directions. Sol Energy Mater Sol Cells 2001;67(1):591–600.
[43] Asl-Soleimani E, Farhangi S, Zabihi M. The effect of tilt angle, air pollution on performance of photovoltaic systems in Tehran. Renew Energy 2001;24(3–4):459–68.
[44] Fordham M. Thomas R, editor. Photovoltaics and architecture. London: Spon Press; 2001. p. 17–32.
[45] Neocleous C, Schizas CN. A study on the effects of environmental factors for the forecasting of electric power load. In Proceedings 2000 electrotechnical conference:1185–1188; 2000.
[46] Fordham M. Partners In Association with Feilden Clegg Architects. Photovoltaics in Buildings-A Design Guide. Report No. ETSU S/P2/00282/REP, sl: Department of Trae and Industry:ETSU, UK; 1999.
[47] Duffie JA, Beckman WA. Solar engineering of thermal processes. New York: John Wiley & Sons; 1991.
[48] Bracamonte J, Parada J, Dimas J, Baritto M. Effect of the collector tilt angle on thermal efficiency and stratification of passive water in glass evacuated tube solar water heater. Appl Energy 2015;155:648–59.
[49] Manouchehri R, Banister CJ, Collins MR. Impact of small tilt angles on the performance of falling film drain water heat recovery systems. Energy Build 2015;102:181–6.
[50] Tang R, Yang Y. Nocturnal reverse flow in water-in-glass evacuated tube solar water heaters. Energy Convers Manag 2014;80:173–7.
[51] Zhang X, You S, Xu W, Wang M, He T, Zheng X. Experimental investigation of the higher coefficient of thermal performance for water-in-glass evacuated tube solar water heaters in China. Energy Convers Manag 2014;78:386–92.
[52] Skerlić J, Radulović J, Nikolić D, Bojić M. Maximizing performances of variable tilt flat-plate solar collectors for Belgrade (Serbia). J Renew Sustain Energy

2013;5(4):041820.
[53] Tang R, Yang Y, Gao W. Comparative studies on thermal performance of water-in-glass evacuated tube solar water heaters with different collector tilt-angles. Sol Energy 2011;85(7):1381–9.
[54] Tang R, Gao W, Yu Y, Chen H. Optimal tilt-angles of all-glass evacuated tube solar collectors. Energy 2009;34(9):1387–95.
[55] Shariah A, Al-Akhras MA, Al-Omari IA. Optimizing the tilt angle of solar collectors. Renew Energy 2002;26(4):587–98.
[56] Sethi VP, Pal DS, Sumathy K. Performance evaluation and solar radiation capture of optimally inclined box type solar cooker with parallelepiped cooking vessel design. Energy Convers Manag 2014;81:231–41.
[57] Al-Soud MS, Abdallah E, Akayleh A, Abdallah S, Hrayshat ES. A parabolic solar cooker with automatic two axes sun tracking system. Appl Energy 2010;87(2):463–70.
[58] Tanaka H. Theoretical analysis of a vertical multiple-effect diffusion solar still coupled with a tilted wick still. Desalination 2016;377:65–72.
[59] Khalifa AJN. On the effect of cover tilt angle of the simple solar still on its productivity in different seasons and latitudes. Energy Convers Manag 2011;52(1):431–6.
[60] Aybar HŞ, Assefi H. Simulation of a solar still to investigate water depth and glass angle. Desalin Water Treat 2009;7(1–3):35–40.
[61] El-Bahi A, Inan D. A solar still with minimum inclination, coupled to an outside condenser. Desalination 1999;123(1):79–83.
[62] Porta MA, Chargoy N, Fernández JL. Extreme operating conditions in shallow solar stills. Sol Energy 1997;61(4):279–86.
[63] Bahadori MN, Edlin FE. Improvement of solar stills by the surface treatment of the glass. Sol Energy 1973;14(3):339–52.
[64] Pan N, Wang S, Shen C. A fundamental study on characteristic of thermoacoustic engine with different tilt angles. Int J Heat Mass Transf 2014;74:228–37.
[65] Pan N, Shen C, Wang S. Experimental study on the flow and heat transfer characteristics of thermoacoustic core. Exp Therm Fluid Sci 2013;44:219–26.
[66] Pan N, Wang S, Shen C. Visualization investigation of the flow and heat transfer in thermoacoustic engine driven by loudspeaker. Int J Heat Mass Transf 2012;55(25):7737–46.
[67] Shen C, He Y, Li Y, Ke H, Zhang D, Liu Y. Performance of solar powered thermoacoustic engine at different tilted angles. Appl Therm Eng 2009;29(13):2745–56.
[68] Fortunato B, Torresi M, Deramo A. Modeling, performance analysis and economic feasibility of a mirror-augmented photovoltaic system. Energy Convers Manag 2014;80:276–86.
[69] Sun L, Lu L, Yang H. Optimum design of shading-type building-integrated photovoltaic claddings with different surface azimuth angles. Appl Energy 2012;90(1):233–40.
[70] Santos ÍPdos, Rüther R. The potential of building-integrated (BIPV) and building-applied photovoltaics (BAPV) in single-family, urban residences at low latitudes in Brazil. Energy Build 2012;50:290–7.
[71] Hachem C, Athienitis A, Fazio P. Evaluation of energy supply and demand in solar neighborhood. Energy Build 2012;49:335–47.
[72] Elhassan ZAM, Zain MFM, Sopian K, Awadalla A. Output energy of photovoltaic module directed at optimum slope angle in Kuala Lumpur, Malaysia. Res J Appl Sci 2011;6(2):104–9.
[73] Corrada P, Bell J, Guan L, Motta N. Optimizing solar collector tilt angle to improve energy harvesting in a solar cooling system. Energy Procedia 2014;48:806–12.
[74] Gitan AA, Abdulmalek SH, Dihrab SS. Tracking collector consideration of tilted collector solar updraft tower power plant under Malaysia climate conditions. Energy 2015;93:1467–77.
[75] Stanciu D, Stanciu C, Paraschiv I. Mathematical links between optimum solar collector tilts in isotropic sky for intercepting maximum solar irradiance. J Atmos Sol-Terr Phys 2016;137:58–65.
[76] Soulayman S, Sabbagh W. Comment on 'Optimum tilt angle and orientation for solar collectors in Syria' by Skeiker, K. Energy Convers Manag 2015;89:1001–2.
[77] Despotovic M, Nedic V. Comparison of optimum tilt angles of solar collectors determined at yearly, seasonal and monthly levels. Energy Convers Manag 2015;97:121–31.
[78] Stanciu C, Stanciu D. Optimum tilt angle for flat plate collectors all over the World–A declination dependence formula and comparisons of three solar radiation models. Energy Convers Manag 2014;81:133–43.
[79] Moghadam H, Deymeh SM. Determination of optimum location and tilt angle of solar collector on the roof of buildings with regard to shadow of adjacent neighbors. Sustain Cities Soc 2015;14:215–22.
[80] Bai L, Lin G, Zhang H, Wen D. Effect of evaporator tilt on the operating temperature of a loop heat pipe without a secondary wick. Int J Heat Mass Transf 2014;77:600–3.
[81] Rahman MM, Mojumder S, Saha S, Mekhilef S, Saidur R. Effect of solid volume fraction and tilt angle in a quarter circular solar thermal collectors filled with CNT–water nanofluid. Int Commun Heat Mass Transf 2014;57:79–90.
[82] Khorasanizadeh H, Mohammadi K, Mostafaeipour A. Establishing a diffuse solar radiation model for determining the optimum tilt angle of solar surfaces in Tabass, Iran. Energy Convers Manag 2014;78:805–14.
[83] Patkó I, Szeder A, Patkó C. Evaluation the impact tilt angle on the sun collectors. Energy Procedia 2013;32:222–31.
[84] Darhmaoui H, Lahjouji D. Latitude based model for tilt angle optimization for solar collectors in the mediterranean region. Energy Procedia 2013;42:426–35.
[85] Jafarkazemi F, Saadabadi SA, Pasdarshahri H. The optimum tilt angle for flat-plate solar collectors in Iran. J Renew Sustain Energy 2012;4(1):013118.
[86] Fahl P, Ganapathisubbu S. Tracking benefits for solar collectors installed in Bangalore. J Renew Sustain Energy 2011;3(2):023103.
[87] Moghadam H, Tabrizi FF, Sharak AZ. Optimization of solar flat collector inclination. Desalination 2011;265(1–3):107–11.
[88] Armstrong S, Hurley WG. A new methodology to optimise solar energy extraction under cloudy conditions. Renew Energy 2010;35(4):780–7.
[89] Skeiker K. Optimum tilt angle and orientation for solar collectors in Syria. Energy Convers Manag 2009;50(9):2439–48.
[90] Chang TP. Study on the optimal tilt angle of solar collector according to different radiation types. Int J Appl Sci Eng 2008;6(2):151–61.
[91] Ertekin C, Evrendilek F, Kulcu R. Modeling spatio-temporal dynamics of optimum tilt angles for solar collectors in Turkey. Sensors 2008;8(5):2913–31.
[92] Gunerhan H, Hepbasli A. Determination of the optimum tilt angle of solar collectors for building applications. Build Environ 2007;42(2):779–83.
[93] Tang R, Wu T. Optimal tilt-angles for solar collectors used in China. Appl Energy 2004;79(3):239–48.
[94] Bari S. Seasonal orientation of solar collectors for the Philippines. ASEAN J Sci Technol Dev 2001;18(1):45–54.
[95] Bari S, Lim TH, Yu CW. Slope angle for seasonal applications of solar collectors in Thailand. Int Energy J 2001;2(1):43–51.
[96] Bari S. Optimum slope angle and orientation of solar collectors for different periods of possible utilization. Energy Convers Manag 2000;41(8):855–60.
[97] Hussein HMS, Ahmad GE, Mohamad MA. Optimization of operational and design parameters of plane reflector-tilted flat plate solar collector systems. Energy 2000;25(6):529–42.
[98] Tiris M, Tiris C. Optimum collector slope and model evaluation: case study for Gebze, Turkey. Energy Convers Manag 1998;39(3):167–72.
[99] Morcos VH. Optimum tilt angle and orientation for solar collectors in Assiut Egypt. Renew Energy 1994;4(3):291–8.
[100] Soulayman SS. On the optimum tilt of solar absorber plates. Renew Energy 1991;1(3):551–4.
[101] Gopinathan KK. Solar radiation on variously oriented sloping surfaces. Sol Energy 1991;47(3):173–9.
[102] Wenxian L. Optimum inclinations for the entire year of south-facing solar collectors in China. Energy 1989;14(12):863–6.
[103] Saraf GR, Hamad FAW. Optimum tilt angle for a flat plate solar collector. Energy Convers Manag 1988;28(2):185–91.
[104] EL-Kassaby MM. Monthly and daily optimum tilt angle for south facing solar collectors; theoretical model, experimental and empirical correlations. Sol Wind Technol 1988;5:589–96.
[105] Chau KV. Optimum tilt angles for solar collectors in clear sky conditions. J Agric Eng Res 1982;27(4):321–8.
[106] Chinnery DNW. Solar heating in South Africa. CSIR-Research Report 248, Pretoria; 1981.
[107] Moon SH, Felton KE, Johnson AT. Optimum tilt angles of a solar collector. Energy 1981;6(9):895–9.
[108] Iqbal M. Optimum collector slope for residential heating in adverse climates. Sol Energy 1979;22(1):77–9.
[109] Beekley DC, Mather GRJ. Analysis and experimental tests of a high-performance evacuated tubular collector. ISES Congress, Los Angeles, California, USA, 220; 1978.
[110] Heywood H. Operating experience with solar heating. JIHVE 1971;39(6):61–9.
[111] Page JK. The estimate of monthly mean values of daily total short-wave radiation on vertical and inclined surface from sunshine records for latitude 40°N–40°S. Proceedings of the UNConference on New Sources of Energy;4:378–390; 1961.
[112] Hottel HC. Performance of flat-plate solar energy collectors for space heating with solar energy. in: Proceeding of a Course-Symposium. Cambridge, MA: MIT Press; 1954.
[113] Smith CJ, Forster PM, Crook R. An all-sky radiative transfer method to predict optimal tilt and azimuth angle of a solar collector. Sol Energy 2016;123:88–101.
[114] Shaddel M, Javan DS, Baghernia P. Estimation of hourly global solar irradiation on tilted absorbers from horizontal one using Artificial Neural Network for case study of Mashhad. Renew Sustain Energy Rev 2016;53:59–67.
[115] Shukla KN, Rangnekar S, Sudhakar K. Comparative study of isotropic and anisotropic sky models to estimate solar radiation incident on tilted surface: a case study for Bhopal, India. Energy Rep 2015;1:96–103.
[116] Khahro SF, Tabbassum K, Talpur S, Alvi MB, Liao X, Dong L. Evaluation of solar energy resources by establishing empirical models for diffuse solar radiation on tilted surface and analysis for optimum tilt angle for a prospective location in southern region of Sindh, Pakistan. Int J Electr Power Energy Syst 2015;64:1073–80.
[117] de Simón-Martín M, Alonso-Tristán C, González-Peña D, Díez-Mediavilla M. New device for the simultaneous measurement of diffuse solar irradiance on several azimuth and tilting angles. Sol Energy 2015;119:370–82.
[118] Gorantla K, Setty ABTP. Study of optimum inward glass tilt angle for window glass in different indian latitudes to gain minimum heat into buildings. Energy Procedia 2015;79:1039–45.
[119] Shamim MA, Remesan R, Bray M, Han D. An improved technique for global solar radiation estimation using numerical weather prediction. J Atmos Sol Terr Phys 2015;129:13–22.
[120] David M, Lauret P, Boland J. Evaluating tilted plane models for solar radiation using comprehensive testing procedures, at a southern hemisphere location. Renew Energy 2013;51:124–31.
[121] Celik AN, Muneer T. Neural network based method for conversion of solar radiation data. Energy Convers Manag 2013;67:117–24.
[122] Čongradac V, Prica M, Paspalj M, Bojanić D, Čapko D. Algorithm for blinds control based on the optimization of blind tilt angle using a genetic algorithm and

fuzzy logic. Sol Energy 2012;86(9):2762–70.
[123] Notton G, Paoli C, Vasileva S, Nivet ML, Canaletti JL, Cristofari C. Estimation of hourly global solar irradiation on tilted planes from horizontal one using artificial neural networks. Energy 2012;39(1):166–79.
[124] Chatterjee A, Keyhani A. Neural network estimation of micro grid Maximum solar power. IEEE Trans Smart Grid 2012;3(4):1860–6.
[125] Vignola F, Michalsky J, Stoffel T. Solar and infrared radiation measurements. Energy and the Environment, 1. Boca Raton: CRC Press; 2012.
[126] Agarwal A, Vashishtha VK, Mishra SN. Comparative approach for the optimization of tilt angle. Int J Eng Res Technol 2012;1(5).
[127] Ghosh HR, Bhowmik NC, Hussain M. Determining seasonal optimum tilt angles, solar radiations on variously oriented, single and double axis tracking surfaces at Dhaka. Renew Energy 2010;35(6):1292–7.
[128] Mehleri ED, Zervas PL, Sarimveis H, Palyvos JA, Markatos NC. A new neural network model for evaluating the performance of various hourly slope irradiation models: implementation for the region of Athens. Renew Energy 2010;35(7):1357–62.
[129] El-Sebaii AA, Al-Hazmi FS, Al-Ghamdi AA, Yaghmour SJ. Global, direct and diffuse solar radiation on horizontal and tilted surfaces in Jeddah, Saudi Arabia. Appl Energy 2010;87(2):568–76.
[130] Pandey CK, Katiyar AK. A note on diffuse solar radiation on a tilted surface. Energy 2009;34(11):1764–9.
[131] Psiloglou BE, Kambezidis HD. Estimation of the ground albedo for the Athens area, Greece. J Atmos Sol Terr Phys 2009;71(8–9):943–54.
[132] Bakirci K. Correlations for estimation of daily global solar radiation with hours of bright sunshine in Turkey. Energy 2009;34(4):485–501.
[133] Janjai S, Pankaew P, Laksanaboonsong J. A model for calculating hourly global solar radiation from satellite data in the tropics. Appl Energy 2009;86(9):1450–7.
[134] Li DHW, Lam TNT, Chu VWC. Relationship between the total solar radiation on tilted surfaces and the sunshine hours in Hong Kong. Sol Energy 2008;82(12):1220–8.
[135] Noorian AM, Moradi I, Kamali GA. Evaluation of 12 models to estimate hourly diffuse irradiation on inclined surfaces. Renew Energy 2008;33(6):1406–12.
[136] Mondol JD, Yohanis YG, Norton B. Solar radiation modelling for the simulation of photovoltaic systems. Renew Energy 2008;33(5):1109–20.
[137] Li DHW, Lam TNT. Determining the optimum tilt angle and orientation for solar energy collection based on measured solar radiance data. Int J Photo 2007;2007:1–9.
[138] Wu G, Liu Y, Wang T. Methods and strategy for modeling daily global solar radiation with measured meteorological data – A case study in Nanchang station, China. Energy Convers Manag 2007;48(9):2447–52.
[139] Bulut H, Büyükalaca O. Simple model for the generation of daily global solar-radiation data in Turkey. Appl Energy 2007;84(5):477–91.
[140] Şen Z. Simple nonlinear solar irradiation estimation model. Renew Energy 2007;32(2):342–50.
[141] Elminir HK, Ghitas AE, El-Hussainy F, Hamid R, Beheary MM, Abdel-Moneim KM. Optimum solar flat-plate collector slope: case study for Helwan, Egypt. Energy Convers Manag 2006;47(5):624–37.
[142] Gunerhan H. Determination of the optimum tilt of solar collectors for building applications. Build Environ 2005;42(2):779–83.
[143] Chandel SS, Aggarwal RK, Pandey AN. New correlation to estimate global solar radiation on horizontal surfaces using sunshine hour and temperature data for indian sites. J Sol Energy Eng 2005;127(3):417.
[144] El-Sebaii AA, Trabea AA. Estimation of global solar radiation on horizontal surfaces over Egypt. Egypt J Solids 2005;28(1):163–75.
[145] Sugden S. View factor for inclined plane with Gaussian source. Appl Math Model 2004;28:1063–82.
[146] Soares J, Oliveira AP, Božnar MZ, Mlakar P, Escobedo JF, Machado AJ. Modeling hourly diffuse solar-radiation in the city of São Paulo using a neural-network technique. Appl Energy 2004;79(2):201–14.
[147] Ulgen K, Hepbasli A. Comparison of the diffuse fraction of daily and monthly global radiation for Izmir, Turkey. Energy Sources 2003;25(7):637–49.
[148] El-Sebaii AA, Trabea AA. Estimation of horizontal diffuse solar radiation in Egypt. Energy Convers Manag 2003;44(15):2471–82.
[149] Gueymard C. Prediction and performance assessment of mean hourly global radiation. Sol Energy 2000;68(3):285–303.
[150] Hartley LE, Martinez-Lozano JA, Utrillas MP, Tena F, Pedros R. The optimisation of the angle of inclination of a solar collector to maximise the incident solar radiation. Renew Energy 1999;17(3):291–309.
[151] Hartley LE, Martinez-Lozano JA. The optimization of the angle of inclination of a solar collector to maximize the incident solar radiation. Renew Energy 1999;17:291–309.
[152] Tiris M, Tiris Ç, Türe İE. Correlations of monthly-average daily global, diffuse and beam radiations with hours of bright sunshine in Gebze, Turkey. Energy Convers Manag 1996;37(9):1417–21.
[153] Khalil A, Alnajjar A. Experimental and theoretical investigation of global and diffuse solar radiation in the United Arab Emirates. Renew Energy 1995;6(5):537–43.
[154] Zuhairy AA, Sayigh AAM. Simulation and modeling of solar radiation in Saudi Arabia. Renew Energy 1995;6(2):107–18.
[155] Reindl DT, Beckman WA, Duffie JA. Diffuse fraction correlations. Sol Energy 1990;45(1):1–7.
[156] Elsayed MM. Optimum orientation of absorber plates. Sol Energy 1989;42(2):89–102.
[157] Abdalla YAG, Feregh GM. Contribution to the study of solar radiation in Abu Dhabi. Energy Convers Manag 1988;28(1):63–7.
[158] Erbs DG, Klein SA, Duffie JA. Estimation of the diffuse radiation fraction for hourly, daily and monthly-average global radiation. Sol Energy 1982;28(4):293–302.
[159] Klein SA, Theilacker JC. An algorithm for calculating monthly-average radiation on inclined surfaces. J Sol Energy Eng 1981;103(1):29–33.
[160] Hay JE, Davies JA. Calculation of the solar radiation incident on an inclined surface. In: Proceedings of the First Canadian Solar Radiation Data Workshop (Eds: JE Hay and TK Won), Ministry of Supply and Services Canada:(Vol. 59); 1980.
[161] Klucher TM. Evaluation of models to predict insolation on tilted surfaces. Sol Energy 1979;23(2):111–4.
[162] Collares-Pereira M, Rabl A. The average distribution of solar radiation-correlations between diffuse and hemispherical and between daily and hourly insolation values. Sol Energy 1979;22(2):155–64.
[163] Temps RC, Coulson KL. Solar radiation incident upon slopes of different orientations. Sol Energy 1977;19(2):179–84.
[164] Sabbagh JA, Sayigh AAM, El-Salam EMA. Estimation of the total solar radiation from meteorological data. Sol Energy 1977;19(3):307–11.
[165] Kern J, Harris I. On the optimum tilt of a solar collector. Sol Energy 1975;17(2):97–102.
[166] Liu BY, Jordan RC. The interrelationship and characteristic distribution of direct, diffuse and total solar radiation. Sol Energy 1960;4(3):1–19.
[167] Badescu V. A new kind of cloudy sky model to compute instantaneous values of diffuse and global solar irradiance. Theor Appl Climatol 2002;72(1–2):127–36.
[168] Tian YQ, Davies-Colley RJ, Gong P, Thorrold BW. Estimating solar radiation on slopes of arbitrary aspect. Agric For Meteorol 2001;109(1):67–74.
[169] Koronakis PS. On the choice of the angle of tilt for south facing solar collectors in the Athens basin area. Sol Energy 1986;36(3):217–25.
[170] Liu B, Jordan R. Daily insolation on surfaces tilted towards equator. ASHRAE J 1961:10.
[171] Reindl DT, Beckman WA, Duffie JA. Evaluation of hourly tilted surface radiation models. Sol Energy 1990;45(1):9–17.
[172] Skartveit A, Olseth JA. Modelling slope irradiance at high latitudes. Sol Energy 1986;36(4):333–44.
[173] Steven MD, Unsworth MH. The angular distribution and interception of diffuse solar radiation below overcast skies. Q J R Meteorol Soc 1980;106(447):57–61.
[174] Hay JE. Calculation of monthly mean solar radiation for horizontal and inclined surfaces. Sol Energy 1979;23(4):301–7.
[175] Ulgen K. Optimum tilt angle for solar collectors. Energy Sources Part A: Recov Util Environ Eff 2006;28(13):1171–80.
[176] Duffie J, Beckman W. Solar engineering of thermal processes. John Wiley & Sons Inc; 2006.
[177] Gopinathan KK. Optimization of tilt angle of solar collectors for maximum irradiation on sloping surfaces. Int J Sol Energy 1991;10(1–2):51–61.
[178] Lewis G. Optimum tilt of a solar collector. Sol Wind Technol 1987;4(3):407–10.
[179] Garg HP. Treatise on solar energy, Fundamentals of Solar Energy, 1. New York: John Wiley & Sons; 1982.
[180] Duffie JA, Beckman WA. Solar engineering of thermal processes. New York: John Wiley & Sons; 1980.
[181] Lunde PJ. Solar thermal engineering: space heating and hot water systems. New York: John Wiley & Sons; 1980.
[182] Löf GO, Tybout RA. Cost of house heating with solar energy. Sol Energy 1973;14(3):253–78.
[183] Yellott J. Utilization of sun and sky radiation for heating and cooling of buildings. ASHRAE J 1973;15:31–42.
[184] Heywood H. Operating experiences with solar water heating. IHVE J 1971;39:63–9.
[185] Chinnery DNW. Solar water heating in South Africa. CSIR Report;248:44; 1967.
[186] Hottel HC. Performance of flat-plate energy collectors. in: Space Heating with Solar Energy, Proceeding sofa course symposium. Cambridge: MIT Press; 1954.
[187] Hiraoka S, Fujii T, Takakura H, Hamakawa Y. Tilt angle dependence of output power in an 80kWp hybrid PV system installed at Shiga in Japan. Sol Energy Mater Sol Cells 2003;75(3):781–6.
[188] Best SR, Rodiek JA, Brandhorst Jr HW. Comparison of solar modeling data to actual pv installations: power predictions and optimal tilt angles. In Photovoltaic Specialists Conference (PVSC) 37th IEEE pp. 001994–001999. IEEE; 2011.
[189] Gong X, Kulkarni M. Design optimization of a large scale rooftop photovoltaic system. Sol Energy 2005;78(3):362–74.
[190] Ibrahim D. Optimum tilt angle for solar collectors used in Cyprus. Renew Energy 1995;6(7):813–9.
[191] De Miguel A, Bilbao J, Diez M. Solar radiation incident on tilted surfaces in Burgos, Spain: isotropic models. Energy Convers Manag 1995;36(10):945–51.
[192] Ulgen K, Hepbasli A. Prediction of solar radiation parameters through clearness index for Izmir, Turkey. Energy Sources 2002;24(8):773–85.
[193] Colli A, Zaaiman WJ. Maximum-power-based PV performance validation method: application to single-axis tracking and fixed-Tilt c-Si systems in the Italian Alpine region. IEEE J Photovolt 2012;2(4):555–63.
[194] Iacobescu F, Badescu V. The potential of the local administration as driving force for the implementation of the National PV systems strategy in Romania. Renew Energy 2012;38(1):117–25.
[195] Yakup MABHM, Malik AQ. Optimum tilt angle and orientation for solar collector in Brunei Darussalam. Renew Energy 2001;24(2):223–34.
[196] Uba FA, Sarsah EA. Optimization of tilt angle for solar collectors in WA, Ghana, Pelagia Research Library. Adv Appl Sci Res 2013;4(4):108–14.
[197] Tsalides P, Thanailakis A. Direct computation of the array optimum tilt angle in constant-tilt photovoltaic systems. Sol Cells 1985;14(1):83–94.

[198] Zhao Qian, Wang Peng, Goel Lalit. Optimal PV panel tilt angle based on solar radiation prediction IEEE In: Proceedings of the 11th International Conference on Probabilistic Methods Applied to Power Systems pp. 425–430. IEEE; 2010.
[199] Daut I, Irwanto M, Irwan YM, Gomesh N, Ahmad NS. Clear sky global solar irradiance on tilt angles of photovoltaic module in Perlis, Northern Malaysia. International Conference on Electrical, Control and Computer Engineering (INECCE) on pp. 445–450. IEEE; 2011.
[200] Khatib T, Mohamed A, Sopian K. On the monthly optimum tilt angle of solar panel for five sites in Malaysia IEEE International Power Engineering and Optimization Conferenc (PEDCO) Melaka, Malaysia, pp. 7–10. IEEE; 2012.
[201] Elhab BR, Sopian K, Mat S, Lim C, Sulaiman MY, Ruslan MH, Saadatian O. Optimizing tilt angles and orientations of solar panels for Kuala Lumpur, Malaysia. Sci Res Essays 2012;7(42):3758–65.
[202] Liu X. Calculation and Analysis of Optimal Tilt Angle for PV/T Hybrid Collector s International Conference on Intelligent System Design and Engineering Application, 791–795; 2012.
[203] Buresch M. Photovoltaic energy systems. McGraw-Hill Book Company; 1983, [ISBN 0-07-008952-3].
[204] Garg HP, Gupta GL. in: Proceedings of the International Solar Energy Society, Congress, New Delhi;1134; 1978.