Challenge: Static Discrete Choice Game–Model and Simulation

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Abstract

In this paper, we first introduce the original model of Ellickson and Misra (2011)'s static discrete game of complete information. Then we discuss the estimation of such game with an interesting simultaneous move situation where two college students choose whether go to work or continue studying. And we use R to simulate data and estimate the parameters.

Keywords: Discrete choice game   static game   game theory

1 The Original Model

Ellickson and Misra (2011) focus on static discrete game of complete information, using the Wal-Mart and Kmart Entry Game as an example. Suppose Kmart and Wal-Mart compete in a collection of well-defined local markets, which means each firm operates at most one store. The existence of Target is ignored since they are focusing on small, rural markets. While their stores were actually sited over a 40 or 50 year period, their strategic choice of entry can be well-approximated by a static discrete game.

Their dataset was collected by Panle Jia (2008). Conditional on each firm operates at most one store in a local market, this leaves 2,065 relatively small and isolated markets, assumed to be independent replications of this simple 2*2 discrete game (two firms choosing either “enter” or “don't enter”).

The estimation of discrete games relies on the same revealed preference logic as discrete choice: the choice the firm actually made must have yielded higher profits than the alternatives that it did not choose, conditional on the equilibrium choices of its rivals.

Elickson and Misra (2011) followed the structure of Berry(1992) to establish their model. The profit function of firm I = K,W in local market m be given by $\pi_{im}(\theta; y_{im})$ where y_{im} is the action (enter or do not enter) of firm i, $y{-}im$ is the action of its rivals (just one rival firm, in their example), and θ is a finite-dimensional parameter vector. The function π_{im} will typically contain covariates specific to both the market

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and the firms, so let X_m be a vector of market characteristics common to both firms and $Z_m = (Z_{Km}, Z_{Wm})$ represent firm characteristics which enter only into the focal firm's profit function (e.g, cost variable) and do not impact the profit of its rivals. After those simplified assumptions, the profit function of firm i in market m be given by

$$\pi_{im} = \alpha_i^{'} X_m + \beta_i^{'} Z_{im} + \delta_i y_{-im} + \varepsilon_{im} \tag{1}$$

Where ε_{im} is a component of profits that is unobservable to the econometrician. Thus, expected profits (net of ε_{im}) are a function of only the common market characteristics, the firm's own characteristics, and its rival's chosen action. The ε are perfectly observed by both players makes this a game of complete information. Assuming that the firms make choices simultaneously, the complete information Nash equilibrium can be characterized by the following system of inequalities which, in this case, represents the non-negative profit conditions for Kmart and Wal-Mart respectively.

$$y_{Km} = 1[\alpha_K^{'} X_m + \beta_K^{'} Z_{Km} + \delta_K y_{Wm} + \varepsilon_{Km} \geq 0] \tag{2}$$

$$y_{Wm} = 1[\alpha_W^{'} X_m + \beta_W^{'} Z_{Wm} + \delta_W y_{Km} + \varepsilon_{Wm} \geq 0] \tag{3}$$

The presence of a rival's choice variables on the right hand side of each firm's profit function are what distinguished discrete games from discrete choice problems, however this independent structure raises problems for estimation and identification. The major problem is coherency problem (visualized in Figure

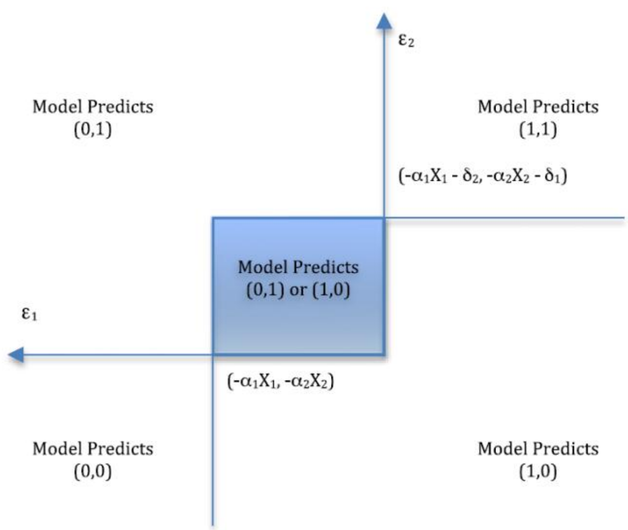

Figure 1: Coherency problem

1) in which the mapping from parameters to outcomes is non-unique. For example, if the δ's are assumed to be negative (facing competition reduces your profits), multiple equilibria arise in the region of ε space for which

$$-(\alpha_i^{'} X + \beta_i^{'} Z_i) \leq \varepsilon_i \leq (\alpha_i^{'} X + \beta_i^{'} Z_i) - \delta_{3-i} \quad for i = 1, 2$$

. Intuitively, this represents the settings in which a local market can only "fit" one firm and neither firm's monopoly profits are large enough to preempt entry by the other, i.e each firm's monopoly profits are only slightly greater than zero, so neither one wants to be there if the other is). From a practical standpoint, in the 2*2 game considered above, the likelihood for the individual firm's choice probabilities will sum to more than 1, violating the law of total probability.

Here we will only introduce the aggregating method which was first proposed by Bresnahan and Reiss (1991), as we will use it in our own example. Based on Bresnahan and Reiss (1991). We assume that firms are exchangeable and profits depend only on market level factors

$$\pi_{im} = \alpha' X_m - \delta y_{-im} + \varepsilon_{im} \tag{4}$$

Assuming ε_{im} are i.i.d. standard normal deviates, the likelihood of observing n_m firms in a given market m can be computed in closed form. For example, the probability of seeing a duopoly is

$$Pr(n_m = 2) = \prod_i Pr(\alpha' X_m - \delta y_{-im} + \varepsilon_{im} \geq 0) \tag{5}$$

The sample log-likelihood is then

$$ln\zeta = \sum_{m=1}^{M} \sum_{l=0}^{2} 1(n_m = l) ln Pr(n_m = l) \tag{6}$$

Estimation is carried out using full-information maximum likelihood (FIML). Results are presented in Figure 2.

Table 1: Estimation Results from Complete Information Games

Variable	B&R (Homogeneous)	Berry (Profit)	Berry (Wal-Mart)	Berry (Kmart)
Common Effects				
Population	1.32*	1.69	1.67	1.69
Retail Sales per capita	1.13	1.54	1.52	1.54
Urban	1.03	1.20	1.19	1.20
δ	0.65	0.39	0.40	0.38
Wal-Mart Specific Effects				
Intercept (Wal-Mart)	-14.03**	-11.87	-11.76	-11.90
Distance to Bentonville, AK		-1.07	-1.06	-1.07
South		0.72	0.72	0.71
Kmart specific Effects				
Intercept (Kmart)	-14.03**	-19.76	-19.56	-19.57
MidWest		0.37	0.37	0.37

*All coefficients are significant at the 5% level.

**Intercepts are common across both firms in this specification.

Figure 2: Result of Ellickson and Misra (2011)'s paper

2 Our Model–Further development for college students

To apply the discrete choice model, we take a game of further development for college students as an example.

Suppose a college undergraduate student named Diligent and another one called Average are roommate of each other. The name fits the person. Diligent is more diligent while Average just as normal people do. When they graduate, they will entry labor market with various kinds of jobs or continue their further study in college. As the set of Ellicken and Misra (2011), the strategic entry to job market is a static discrete game. Diligent and Average will choose between entry the job market or stay in college. And we assume if they choose to entry, they will go to the same company since it is their common dream generated when they saw a TV show together. Nevertheless, the company is too hot to hire many students from the same college in the same year. As a result, if two or above two students of the same college apply the position at the same time, it will choose the ones with more skills besides their major, such as software, operating Wechat official account, creating posters and so on, as its candidates and then allow the students decide whether they will come or not. In that the more diligent the more skills one may study, Diligent has overwhelming superiority comparing with Average. In other words, Diligent will have larger possibility to get an offer if they both apply. In this case, we use the first option in Ellicken and Misra (2011)' s paper, aggregating to a prediction which is unique, to address the problem of multiple equilibrium. In our example, two students will make

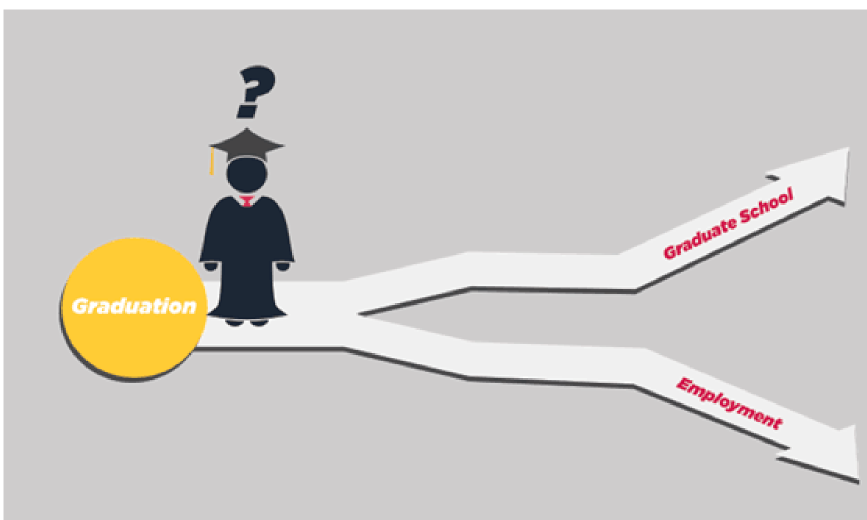

Figure 3: Which Road to go?

their choice according to their utility simultaneously. the π_{im} is the utility of student $i = D, A$ at the time they make the choice (not containing long term). We let the utility of staying college equals to 0 to represent the base line making it easy to compare. y_{im} is the action of student I equaling 1 if he chooses to go to the company while y_{-im} is the action of his counterpart. And for going to a company, X_m is a vector of college student characteristics, such as the holistic preference between going to a company as soon as graduate and staying in college to continue further study, social value system and so on. These social characteristics are the common effect for one' s choice. $Z_{im} = (Z_{Dm}, Z_{Am})$ contains students personal characteristics, for example, family influence, experience effect and so on. yibuxilong is a component of utility each student observes but we do not. For instance, they both worry about that if they go to a company they will only get a scholar degree but it may be not enough for the future society as well as if they choose to stay at college they will not have certain income to afford a family so that it will be late to have a wife and children. Also, peer pressure. The measurements of these things are in the students' mine and randomly change.

$$\pi_{Dm} = \alpha'_D X_m + \beta'_D Z_{Dm} + \gamma_D y_{Am} + \varepsilon_{Dm} \tag{7}$$

$$\pi_{Am} = \alpha'_A X_m + \beta'_A Z_{Am} + \gamma_A y_{Dm} + \varepsilon_{Am} \tag{8}$$

The complete information Nash Equilibrium can be expressed as

$$Payoff_{Dm} = \pi_{Dm} 1[\alpha'_D X_m + \beta'_D Z_{Dm} + \gamma_D y_{Am} + \varepsilon_{Dm} \geq 0] \tag{9}$$

$$Payoff_{Wm} = \pi_{Dm} 1[\alpha'_A X_m + \beta'_A Z_{Am} + \gamma_A y_{Dm} + \varepsilon_{Am} \geq 0] \tag{10}$$

Shown in a table, it will be:

N.E.	$\pi_A < 0$	$\pi_A < 0\&\pi_{AD} < 0$	$\pi_A > 0$
$\pi_D < 0$	(0,0)	(0,π_A)	(0,π_A)
$\pi_D < 0\&\pi_{DA} < 0$	(π_D,0)	(π_D,0)	(π_{DA},π_{AD})
$\pi_D > 0$	(π_D,0)	(π_D,0)	(π_{DA},π_{AD})

Table 1: Nash Equilibrim under each condition

Each cell represents an equilibrium on the condition given by corresponding row and column. For example, when $\pi_{DA} > 0$ and $\pi_A > 0$ but $\pi_{AD} > 0$, which means if they both choose to go to the company Diligent will have higher utility than staying in college but Average will have lower utility than staying in college or only himself going to the company, Diligent will choose to go to the company while Average will not.

Then we simulate our data. First, $X_m = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ where $X_i \sim N(2, 1)$ and they are independent so that $X_m \sim \left(\begin{pmatrix} 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$. Similarly, let $Z_{Dm} \sim \left(\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$ and $Z_{Am} \sim \left(\begin{pmatrix} 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$. We assume the true value of parameters $\{\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_D, \gamma_A, \sigma\} = \{1, -1, 1, -1, -1, -1, 1\}$. sigmaD and sigmaA can be interpreted as that if one of them go to the company the other one will be worried. Also, for epoxilon we assume it follows the normal distribution, i.e. $\varepsilon \sim N(0, 1)$. Assume there are 1000 possible companies that may be their common dream company.

```r
library(MASS)
set.seed(111)
n = 3000
alpha <- c(1,-1)
belta <- c(1,-1)
sigma <- 1
gammaD <- -1
gammaA <- -1
eD <- rnorm(n,mean = 0,sd = 1)
eA <- rnorm(n,mean = 0,sd = 1)
cors <- matrix(c(1,0,0,1),nrow = 2)
x <- mvrnorm(n, c(2,2), cors)
zd <- mvrnorm(n, c(3.5,2), cors)
za <- mvrnorm(n, c(2,2), cors)
parameter <- c(alpha, belta, sigma, gammaD)
```

```r
game <- function(parameter){
comp <- matrix(1, nrow = n, ncol = 2)
alpha = parameter[1:2]
belta = parameter[3:4]
sigma = parameter[5]
gammaD = parameter[6]
gammaA = parameter[6]
utilityD <- x  %*% alpha + zd %*% belta + eD
utilityDA <- x  %*% alpha + zd %*% belta + gammaD + eD
utilityA <- x  %*% alpha + za %*% belta + eA
utilityAD <- x  %*% alpha + za %*% belta + gammaA + eA
for (i in 1:n) {
if (utilityD[i] < 0 & utilityA[i] < 0){
comp[i,] <- c(0,0)
}
else if (utilityDA[i] >= 0 & utilityAD[i] >= 0){
comp[i,] <- c(utilityDA[i],utilityAD[i])
}
else if (utilityD[i] >= 0 & utilityAD[i] < 0){
comp[i,] <- c(utilityD[i],0)
}
else {
comp[i,] <- c(0,utilityA[i])
}
}
return(comp)
}
```

```
utility <- game(parameter)
result <- matrix(utility != 0, nrow = n)
table(result[,1],result[,2])
      FALSE  TRUE
FALSE   549   338
TRUE   1345   768
```

As shown in the result, among 3000 possible results the situation Diligent will go to the company and Average stay in college has the most weight, which is consistent with our assumption that if they compete Diligent is more likely to win the position.

3 Simulation

As we say before, we take advantage of the first option in Ellicken and Misra (2011)' s paper to aggregate to a unique prediction for the purpose of addressing the problem of multiple equilibrium. We predict how many students will go to the company.

The likelihood of observing n_c students in a given company c can be computed in closed form

$$\begin{aligned} Pr(n_m = 2) &= \prod_i Pr(\alpha X_m + \beta Z_{im} - \gamma_D y_{-im} + \varepsilon_{im} \geq 0) \\ Pr(n_m = 0) &= \prod_i Pr(\alpha X_m + \beta Z_{im} - \gamma_A y_{-im} + \varepsilon_{im} < 0) \\ Pr(n_m = 1) &= 1 - Pr(n_m = 2) - Pr(n_m = 0) \end{aligned} \tag{11}$$

The sample log-likelihood is then

$$ln\zeta = \sum_{m=1}^{M} \sum_{l=0}^{2} 1(n_m = l) lnPr(n_m = l) \tag{12}$$

We use normal probability function to estimate the probability of each student to go to the company. Then we simulate it in R and estimate the parameters by maximizing MLE. Our estimated parameters are shown in Table 2

α_1	α_2	β_1	β_2	σ	γ_i
1.0954216	-1.0178220	1.0781046	-0.9827813	1.0120998	-1.4980074

Table 2: Estimated Parameters

```
Pr <- function(parahat = c(1,-1,1,-1,1,-1)){
prob <- rep(0,n)
alphah <- parahat[1:2]
beltah <- parahat[3:4]
gammaDh <- parahat[6]
```

```
gammaAh <- parahat[6]
sigmah <- parahat[5]
for (i in 1:n) {
if (result[i,1] == 1 & result[i,2] == 1){
prob[i] <- (1-pnorm(-(x[i,]%*%alphah + zd[i,]%*%beltah + gammaDh)/sigmah))*(1-
    pnorm(-(x[i,]%*%alphah + za[i,]%*%beltah + gammaAh)/sigmah))
}
else if (result[i,1] == 1 & result[i,2] == 0){
prob[i] <- (1-pnorm(-(x[i,]%*%alphah + zd[i,]%*%beltah)/sigmah))*(pnorm(-(x[i,]%*%
    alphah + za[i,]%*%beltah + gammaAh)/sigmah))
}
else if (result[i,1] == 0 & result[i,2] == 0){
prob[i] <- (pnorm(-(x[i,]%*%alphah + zd[i,]%*%beltah + gammaDh)/sigmah))*(pnorm(-(
    x[i,]%*%alphah + za[i,]%*%beltah)/sigmah))
}
else {
prob[i] <- ((pnorm(-(x[i,]%*%alphah+zd[i,]%*%beltah)/sigmah))*(1-pnorm(-(x[i,]%*%
    alphah+za[i,]%*%beltah)/sigmah))+(pnorm(-(x[i,]%*%alphah+zd[i,]%*%beltah+
    gammaDh)/sigmah)-pnorm(-(x[i,]%*%alphah+zd[i,]%*%beltah)/sigmah))*(1-pnorm(-(x[
    i,]%*%alphah+za[i,]%*%beltah+gammaAh)/sigmah)))
}
}
return(prob)
}

likelihood <- function(parahat){
-sum(log(Pr(parahat)))
}

maxlikelihood <- optim(par = c(1,-1,1,-1,1,-1),likelihood)
maxlikelihood
$par
[1]  1.0954216 -1.0178220  1.0781046 -0.9827813  1.0120998 -1.4980074

$value
[1] 1569.204

$counts
function gradient
373       NA

$convergence
[1] 0
```

```
$message
NULL
```

```
est <- maxlikelihood$par
eutility <- game(est)
eresult <- matrix(eutility != 0, nrow = n)
table(eresult[,1],eresult[,2])

    FALSE  TRUE
FALSE    411   337
TRUE    1526   726
```

Like the table we show before, our estimation still holds that Diligent has dominate strength.

we would like to clarify that it' s not a general result and just based on our assumption and generated data. Different number of players and different power can make significant difference. It' s also important that the choice of the payoff specification is key. If we allow the players to have different coefficients (like γ_i), it will have significant impact on estimates.

```
table(eresult,result)

    result
eresult FALSE TRUE
FALSE    2599   86
TRUE     182    3133
```

As shown in the table, the accuracy is

$$accuracy = \frac{True\ Positive + Ture\ Negative}{TP + TN + FP + FN} = \frac{2599 + 3133}{2599 + 86 + 182 + 3133} = 95.5\%$$

and the precision is

$$precision = \frac{True\ Positive}{True\ Positive + False\ Positive} = \frac{3133}{182 + 3133} = 94.5\%$$

Then we can get the conclusion that MLE is feasible for our design.

4 References

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