Methods in Psycholinguistics — Mixed effects linear regression —

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Generalized Linear Mixed Models

Experiments typically have more than one **participant** and more than one **item**
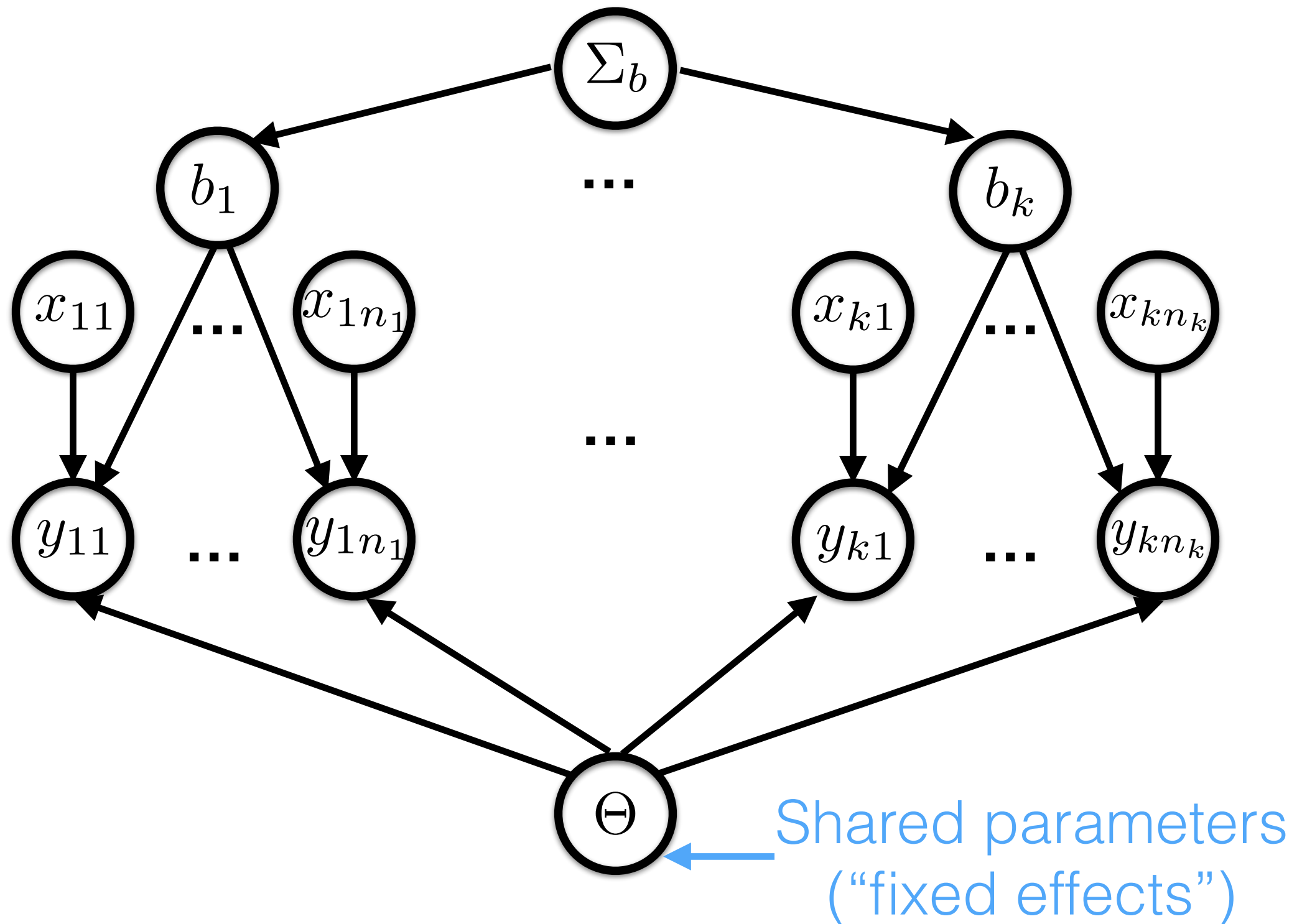—> violation of the assumption of independence!

We would like to take these idiosyncrasies into account, and maybe even investigate them.

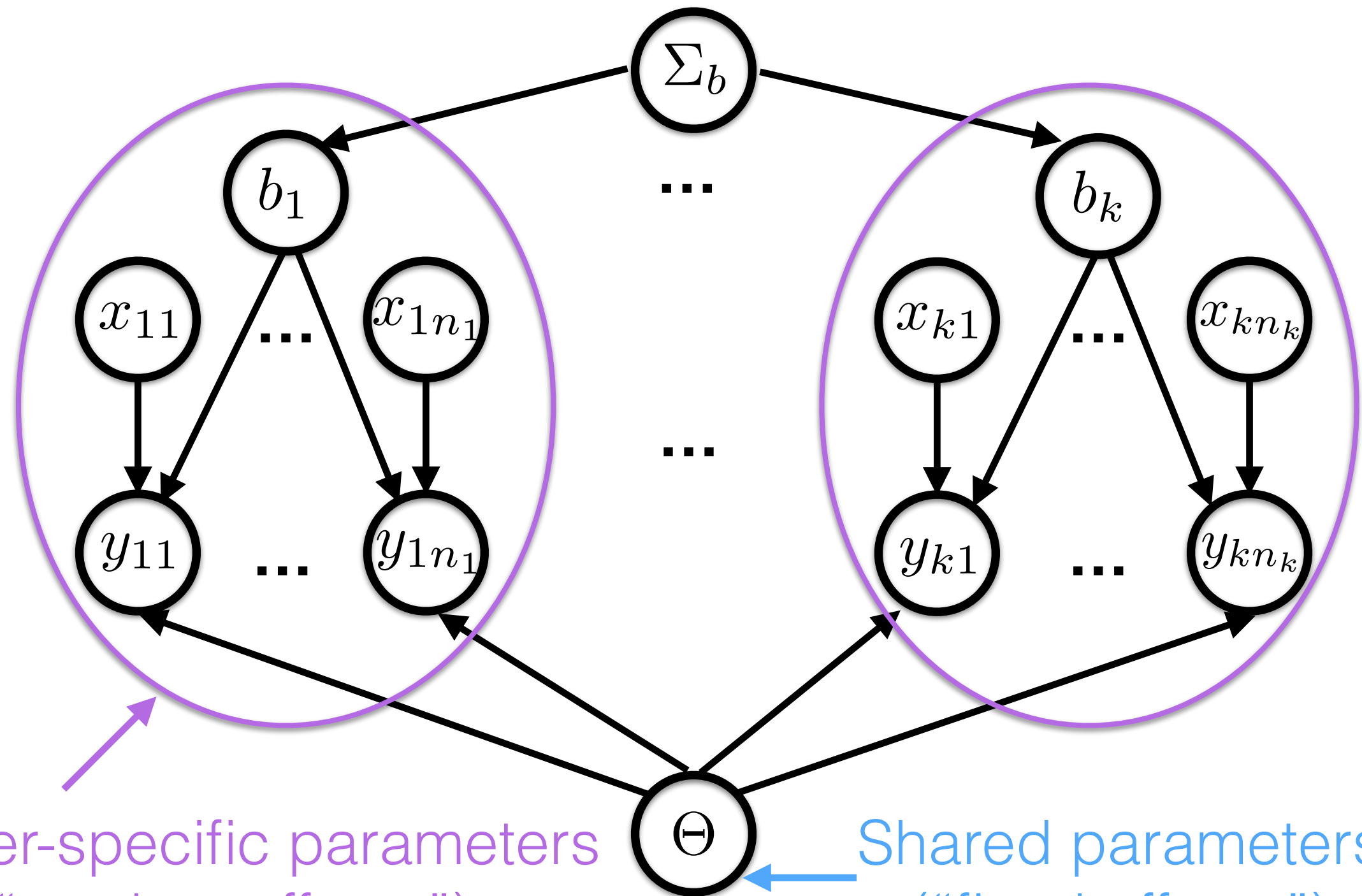Solution: Generalized Linear Mixed or Multilevel Models (a.k.a. hierarchical, mixed-effects)

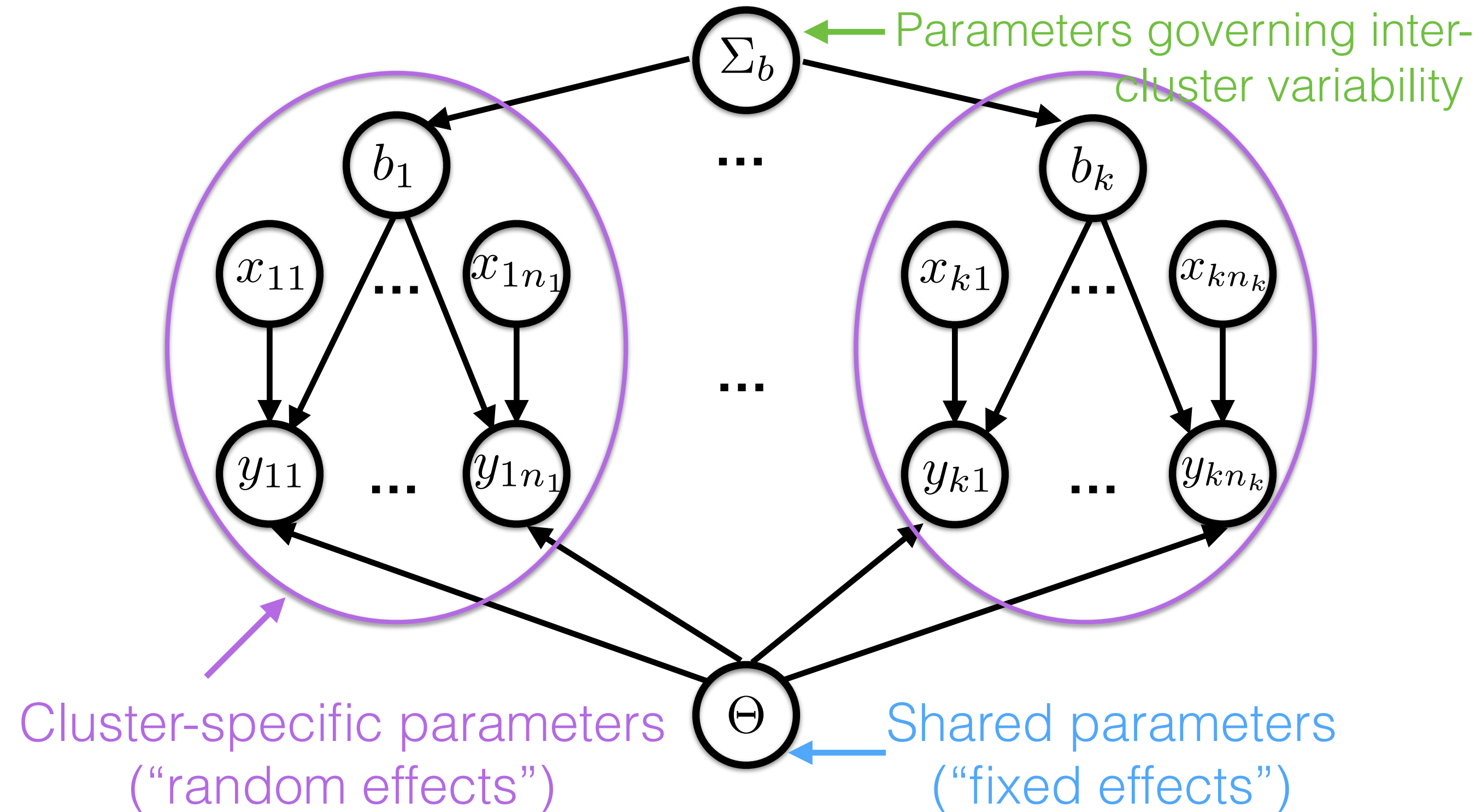# Recall: Generalized Linear Model
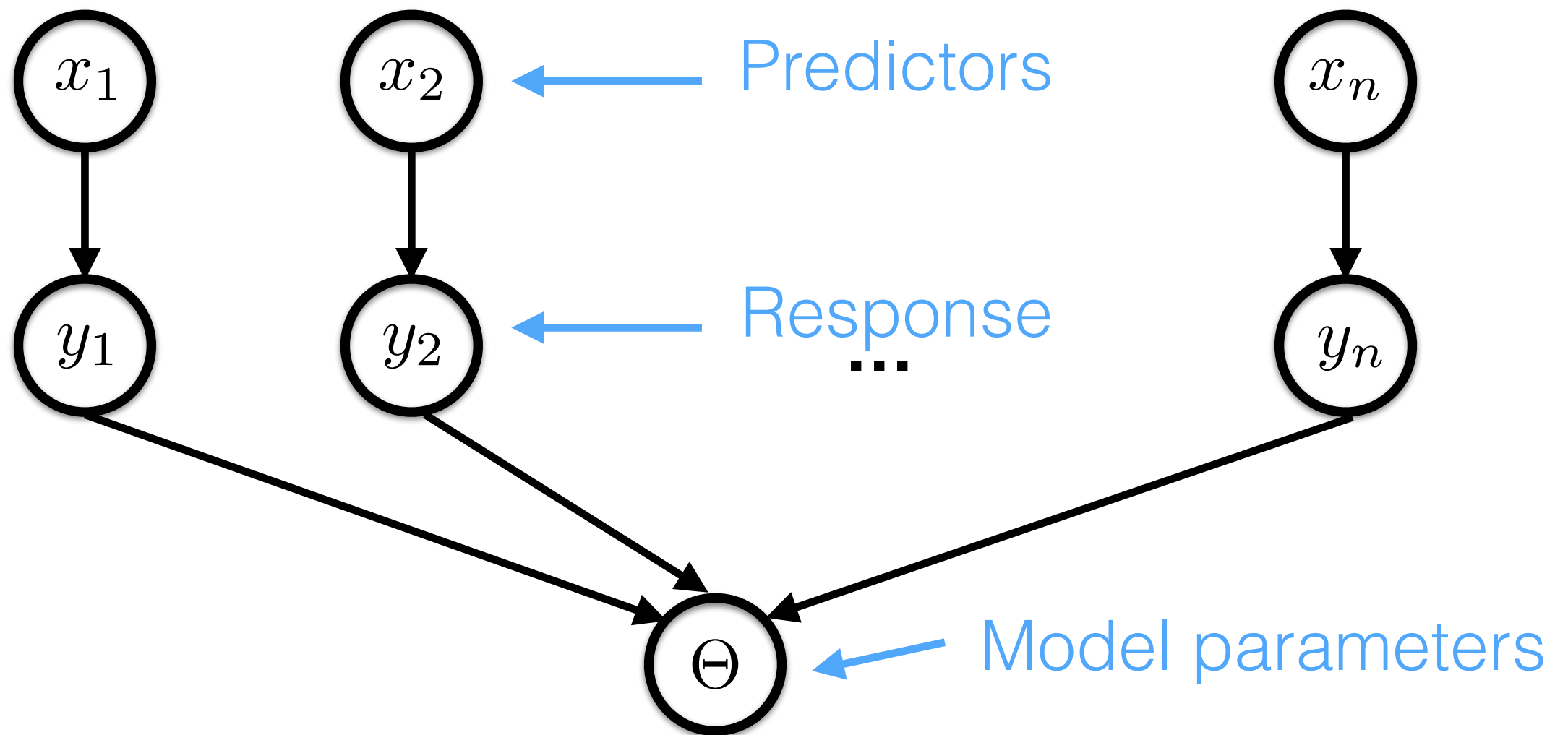
Generalized Linear Mixed Model

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Mixed linear model

…back to our lexical decision experiment.

A number of predictors seem to affect RTs:
- frequency
- family size
- native language
- interactions

Additionally, different participants may have
- different overall decision speeds
- differing sensitivity to predictors

We want to draw inferences about all of this at the same time!

Mixed linear model

Let's allow each participant i to have idiosyncratic differences in response times:

$$RT_{ij} = \alpha + \beta x_{ij} + \overbrace{b_i}^{\sim N(0,\sigma_b)} + \epsilon_{ij}$$

Mixed linear model

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- Idea: model distribution of participant differences as deviation from grand mean
- Mixed models approximate deviation by fitting a normal distribution
  - with mean 0
  - only additional parameter to fit is variance

Mixed linear model

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Let's translate this into R!

Interpretation of the output

Random
effects
estimates

→

```
Random effects:
  Groups    Name           Variance Std.Dev.
  Subject  (Intercept) 0.02373  0.1541
  Residual              0.03274  0.1809
Number of obs: 1659, groups:  Subject, 21
```

Fixed
effects
estimates
(same)

→

```
Fixed effects:
                Estimate Std. Error t value
(Intercept)  6.588778    0.037737    174.6
Frequency   -0.042872    0.003485    -12.3
```

2 notable differences:
1. random effects estimates
2. no more p-values!

Mixed models with one intercept

Let's look at the by-subject adjustments to the intercept: Best Unbiased Linear Predictors (BLUPs)

BLUPs are *not* fitted parameters. Only one degree of freedom added to the model. BLUPs are estimated a posteriori based on the fitted model.
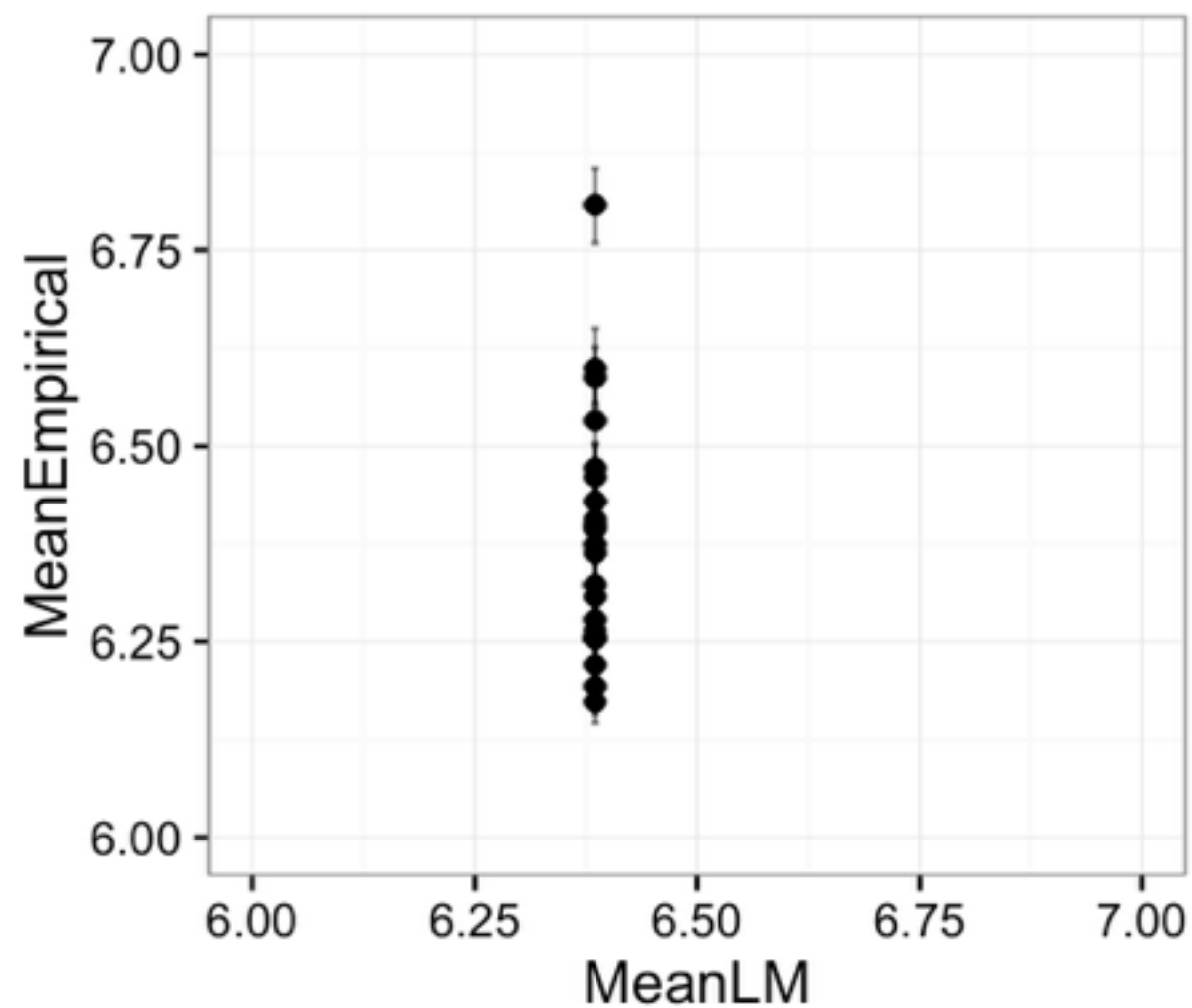
$$P(b_i | \hat{\alpha}, \hat{\beta}, \hat{\sigma}_b, \hat{\sigma}_\epsilon, X, Y)$$

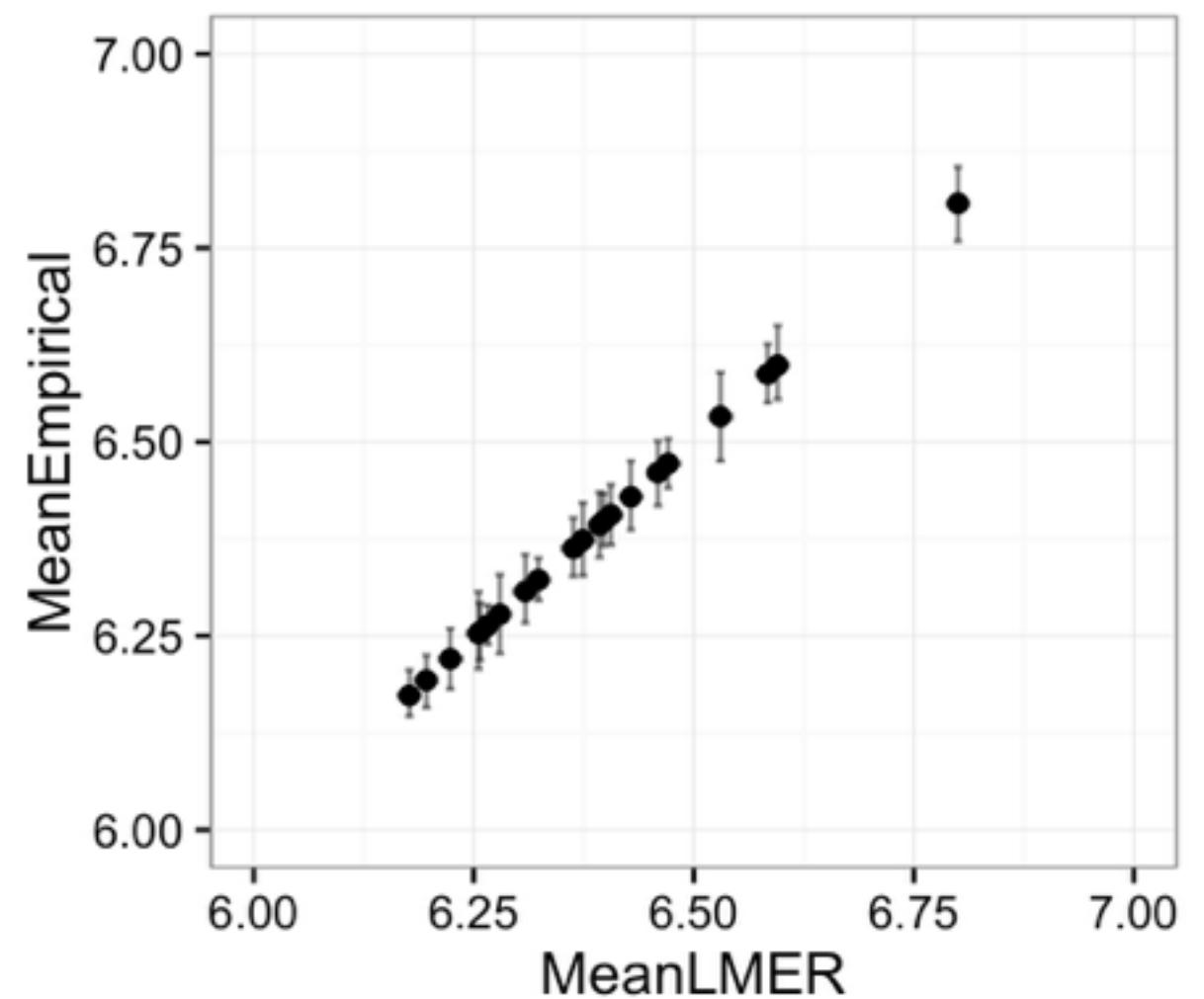The BLUPs are chosen to maximize this probability.
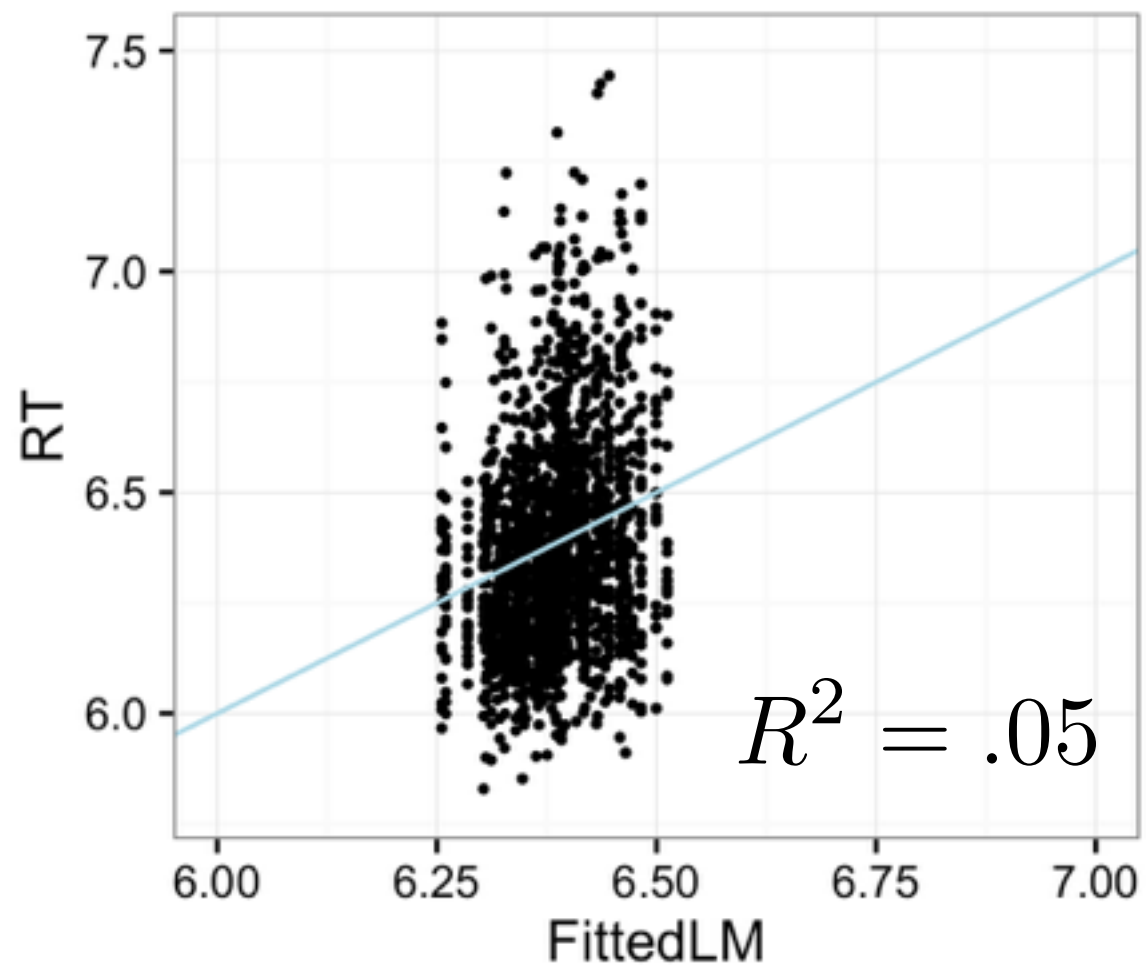
The effect

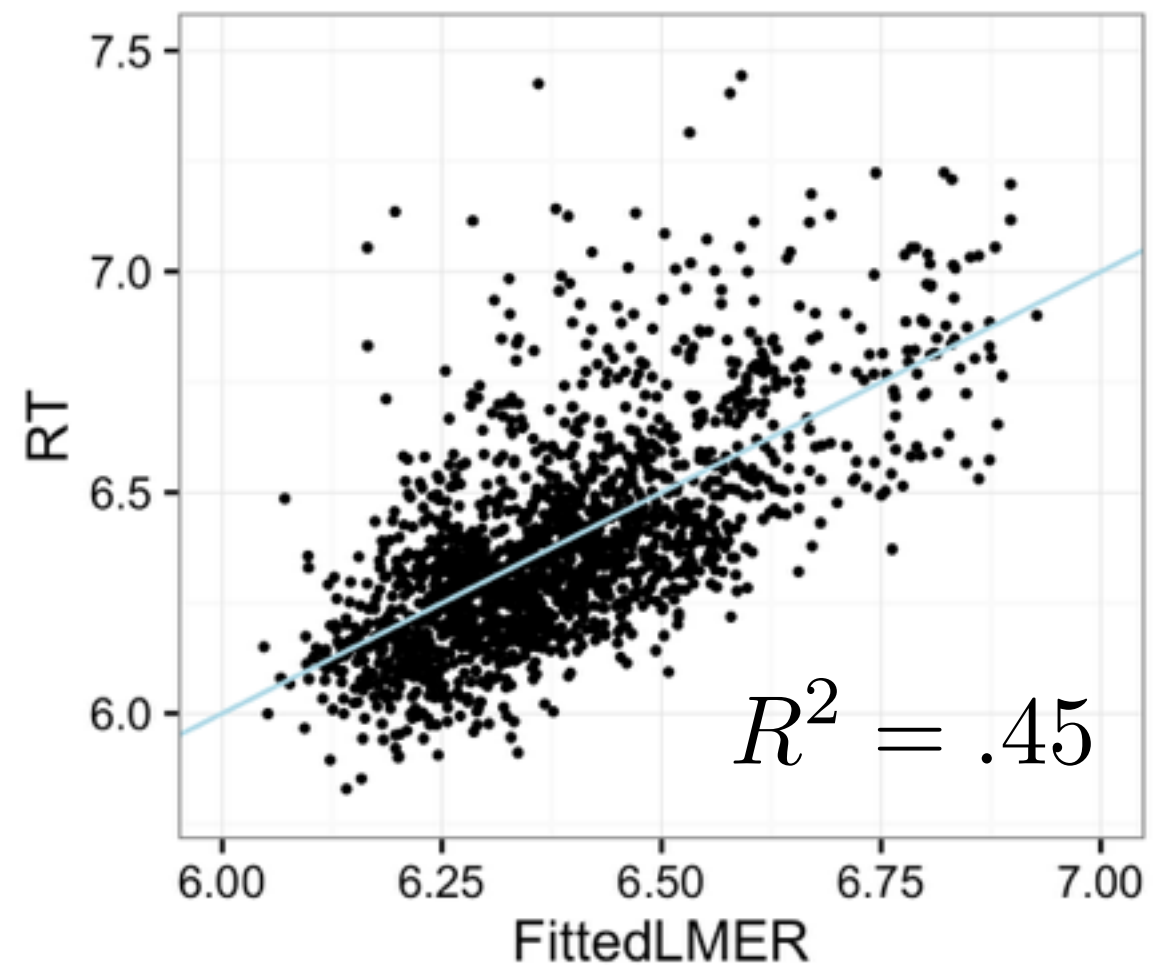## Predicted by-subject means

Linear model

Mixed linear model

The effect

Predicted RTs

Linear model

$R^2 = .05$

Mixed linear model
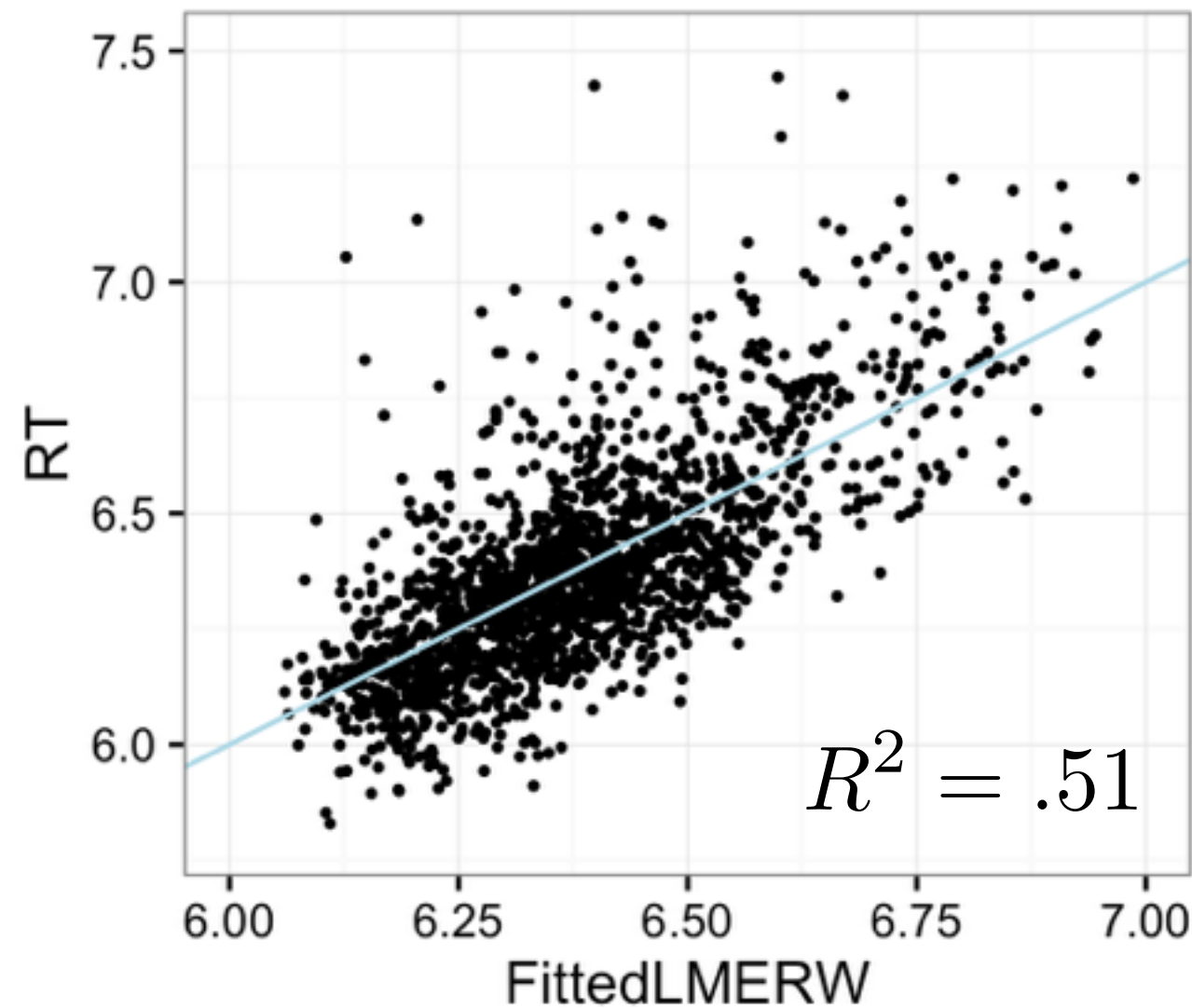

$R^2 = .45$

A second random intercept

A second random intercept

$R^2 = .51$
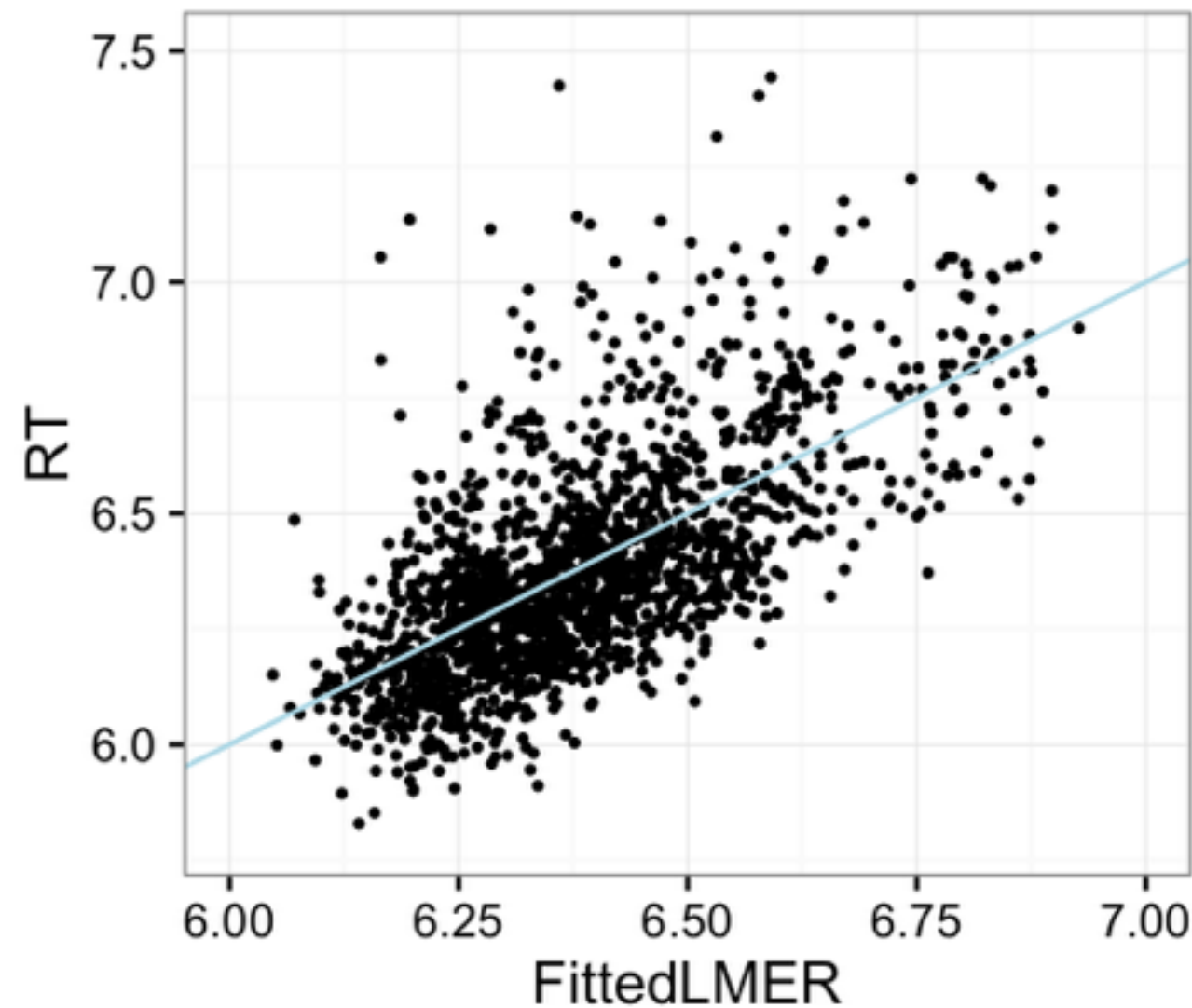
Mixed models with random slopes

Not just the intercept, but **any of the slopes** (of the predictors) may differ between individuals.

For example, subjects may show different sensitivity to frequency:

```
> head(ranef(m)$Subject)
      (Intercept)   Frequency
A1  -0.113081677  0.002001991
A2  -0.237501826  0.015898141
A3  -0.005238541  0.003482887
C   -0.132056269  0.014382981
D    0.001134412  0.003810049
I   -0.141643431  0.002989325
```
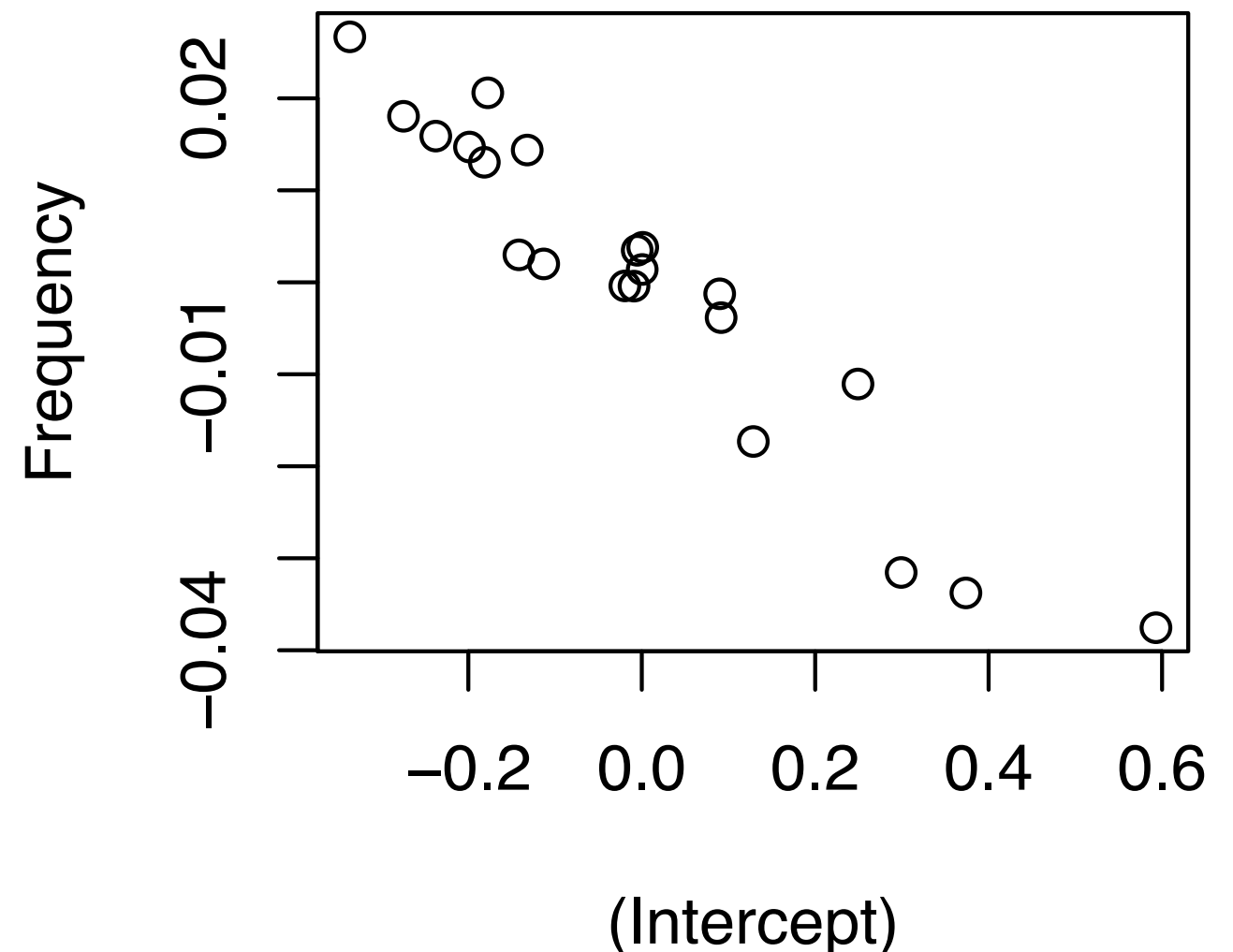
The BLUPs of the random slope reflect the by-subject adjustments to the overall frequency effect.

Random intercept, slope, and covariance

Random effects (e.g., intercepts and slopes) may be correlated.

By default, R fits these covariances, introducing additional degrees of freedom (parameters).

What do correlations between random effects mean?

Where did our p-values go?

- t-value anti-conservative, so no p-values!

- there are many different options for computing p-values if you really need them. You can check these in R directly by saying `?pvalues`

- some reasonably easy to use ones for **fixed effects**:

  - likelihood ratio tests via anova

  - lmerTest package (uses Satterthwaite or Kenward-Roger approximations)