1.

(P)
$$\begin{aligned} \min \quad & z = x_1 + 2x_2 - \theta(x_1 + x_2) \\ \text{s.t.} \quad & x_1 + x_2 \le \theta \\ & x_1 \ge \theta \\ & x_2 \le 4 \\ & x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

$\Leftrightarrow$
$$\begin{aligned} \min \quad & \underbrace{(1-\theta)}_{b_1}x_1 + \underbrace{(2-\theta)}_{b_2}x_2 \\ \text{s.t.} \quad & x_1 + x_2 \le \theta \\ & -x_1 \le -\theta \\ & x_2 \le 4 \\ & x_1 \ge 0, x_2 \ge 0. \end{aligned}$$

a) Find Dual

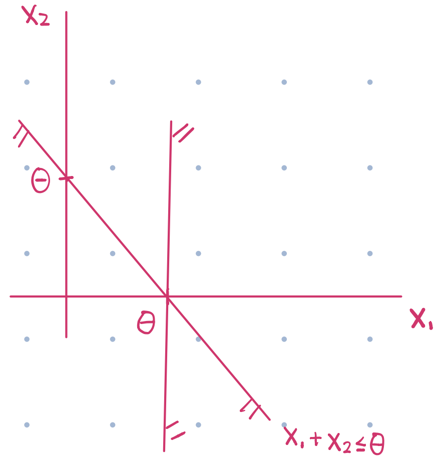
(D)
$$\begin{aligned} \max \quad & -\theta y_1 + \theta y_2 - 4y_3 \\ \text{s.t.} \quad & -y_1 + y_2 \le (1-\theta) \\ & -y_1 - y_3 \le (2-\theta). \\ & y \ge 0. \end{aligned}$$

b) **(P:10pts; S:10pts)** Give the optimal value of the objective function of this linear program, $z = z(\theta)$ and the optimal solution $x(\theta)$ as a function of θ for all nonnegative real values of θ. (Hints: it may be helpful to graph this problem).

The only feasible solution is $(\theta, 0)$.

$$x(\theta) = (\theta, 0)$$

$$z(\theta) = (1-\theta)\theta.$$



c) For θ nonnegative, show that or explain why the optimal dual solution is not unique.

$$\begin{aligned} & -\theta y_1 + \theta y_2 - 4y_3 = (1-\theta)\theta \\ & -y_1 + y_2 \le (1-\theta) \\ & -y_1 - y_3 \le (2-\theta). \\ & y \ge 0. \end{aligned}$$

$\Longrightarrow$ we can fix y_2 and y_3

$$\begin{aligned} & -\theta y_1 + \theta y_2 - 4y_3 = (1-\theta)\theta \\ & -y_1 + y_2 = (1-\theta) \\ & y_3 = 0 \\ & y_1 \ge (\theta - 2). \end{aligned}$$

free.