Benchmark Methods for Solving the SFPTMP

This document provides the modelling and algorithmic details of the benchmark methods used in the paper entailed "Optimizing Freight Procurement for Transportation-Inventory Systems Under Supply and Demand Uncertainty" by Lingxiao Wu, Wenxuan Shan, Yossiri Adulyasak, and Jean-Franois Cordeau. We first provide a summery of notation used for describing the methods in Section 1, and then introduce the methods in Section 2.

1.   Notation Summery

Table 1 summarizes the commonly used notation used for describing the approaches.

Table 1: Notation Used in the Document.

	Sets
$\mathcal{T}$	set of periods; $\mathcal{T} = \{1, ..., \bar{t}\}$.
$\mathcal{P}$	set of shipping stages; $\mathcal{P} = \{1, ..., \bar{p}\}$.
$\mathcal{N}$	set of nodes in the space-time network.
$\mathcal{N}^S$	set of nodes associated with supply sites.
$\mathcal{N}^D$	set of nodes associated with demand sites.
$\mathcal{N}_p$	set of nodes associated with stage $p \in \mathcal{P}$.
$\mathcal{A}$	set of arcs in the space-time network.
$\mathcal{A}^1$	set of arcs representing shipments in capacity contracts.
$\mathcal{A}^2$	set of arcs representing shipments via non-contractual shipments.
$\mathcal{A}_b^1$	set of arcs representing shipments in bid $b \in \mathcal{B}$.
$\mathcal{A}_p$	set of arcs associated with stage $p \in \mathcal{P}$.
$\mathcal{A}^+(n)$	set of outgoing arcs of node $n \in \mathcal{N}$.
$\mathcal{A}^-(n)$	set of incoming arcs of node $n \in \mathcal{N}$.
Ξ	set of scenarios.
Ω_p	set of stage scenarios associated with stage $p \in \mathcal{P}$.
	Parameters
$\underline{m}_b$	lower bound for the capacity of bid $b \in \mathcal{B}$.
$\overline{m}_b$	upper bound for the capacity of bid $b \in \mathcal{B}$.
q_n^0	initial inventory at node $n \in \mathcal{N}$.
$\bar{q}_n$	inventory capacity at node $n \in \mathcal{N}$.
d_n^ω	demand or supply generated at node $n \in \mathcal{N}$ under stage scenario $\omega \in \Omega_p$ at stage $p \in \mathcal{P}$.
$d_{i,t}^\xi$	demand or supply generated at node $n \in \mathcal{N}$ under scenario $\xi \in \Xi$.
h_n	unit inventory holding cost at node $n \in \mathcal{N}$.
e_n	unit penalty price for the backlogged supply or demand at node $n \in \mathcal{N}$.
c_a	unit variable transportation cost on arc $a \in \mathcal{A}$.

ϱ_ω	probability of stage scenario $\omega \in \Omega_p$, $p \in \mathcal{P}$.
ρ_ξ	probability of scenario $\xi \in \Xi$.
	Decision Variables
x_b	binary variable, which equals 1 if and only if the shipper accepts bid $b \in \mathcal{B}$.
y_b	continuous variable, which represents the capacity purchased by the shipper for each shipment in the for-hire contract associated with bid $b \in \mathcal{B}$.
$z_{\xi,a}$	continuous variable, which represents the volume of the commodity allocated on arc $a \in \mathcal{A}$ under scenario $\xi \in \Xi$.
$u_{\xi,n}$	continuous variable, which represents the inventory level at node $n \in \mathcal{N}$ under scenario $\xi \in \Xi$.
$v_{\xi,n}$	continuous variable, which represents the volume of the supply or demand backlogged at node $n \in \mathcal{N}$ under scenario $\xi \in \Xi$.

2. Benchmark Methods for the SFPTMP

Section 2.1 describes the details of NC, where the for-hire contracts are not considered. Section 2.2 introduces the myopic decision policy (MD). We then describe the heuristic SDDP method (HS) in Section 2.3. Finally, the two-stage stochastic optimization approach (TS) is introduced in Sections 2.4.

2.1. NC

Analogous to the SDDP approached proposed in the paper, the approach NC solves the SFPTMP through multi-stage stochastic optimization. However, in NC, we assume that the shipper cannot select any bid and the resulting problem ($\mathbf{P}_{\mathrm{NC}}$) can be formulated as the following linear programming model.

$$\mathbf{P}_{\mathrm{NC}} = \min \sum_{\xi \in \Xi} \rho_\xi \left(\sum_{n \in \mathcal{N}} (h_n u_{\xi,n} + e_n v_{\xi,n}) + \sum_{a \in \mathcal{A}} c_a z_{\xi,a} \right) \tag{1}$$

$$\text{s.t.} \quad z_{\xi,a} \le 0 \ \forall a \in \mathcal{A}_b^1, \forall b \in \mathcal{B}, \forall \xi \in \Xi \tag{2}$$

$$u_{\xi,n_1} + v_{\xi,n_1} = d_{\xi,n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a \in A^+(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^S, \forall \xi \in \Xi \tag{3}$$

$$u_{\xi,n} + v_{\xi,n} = d_{\xi,n} + q_n^0 - \sum_{a \in A^+(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^S, \forall \xi \in \Xi \tag{4}$$

$$u_{\xi,n_1} - v_{\xi,n_1} = d_{\xi,n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a \in A^-(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^D, \forall \xi \in \Xi \tag{5}$$

$$u_{\xi,n} - v_{\xi,n} = d_{\xi,n} + q_n^0 + \sum_{a \in A^-(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^D, \forall \xi \in \Xi \tag{6}$$

$$u_{\xi,n} \le \bar{q}_n \ \forall n \in \mathcal{N}, \forall \xi \in \Xi \tag{7}$$

$$z_{\xi_1,a} = z_{\xi_2,a} \ \forall a \in \mathcal{A}_p, \forall (\xi_1, \xi_2) \in \Lambda_p, \forall p \in \mathcal{P} \tag{8}$$

$$u_{\xi,n}, v_{\xi,n}, z_{\xi,a} \geq 0 \ \forall n \in \mathcal{N}, \forall a \in \mathcal{A}, \forall \xi \in \Xi. \tag{9}$$

We solve the $\mathbf{P}_{\mathrm{NC}}$ using the SDDP approach described in Section 5 of the paper with the following additional constraints in problem $\mathbf{P}_0$:

$$x_b = 0, \ \forall b \in \mathcal{B}.$$

15 Based on the cost-to-go functions returned by NC, we then derive the upper bound obtained

16 by NC for each instance using the same approach described in Section 6.1 of the paper. Note that

17 when deriving the upper bound for any instance, we set $\boldsymbol{y} = \mathbf{0}$.

18 ## 2.2. MD

19 Approach MD solves the SFPTMP in a multi-stage manner as our SDDP. It first solves a two-stage

20 approximation of the SFPTMP to determine the for-hire contracts and the capacity purchased for

21 each shipment in the contracts (i.e., decisions in $\boldsymbol{x}$ and $\boldsymbol{y}$). We then solve stage-wise problems,

22 from stage 1 to $|\mathcal{P}|$, based on the determined capacity of each shipment associated with the for-hire

23 contracts. The stage-wise problems are solved myopically, without cost-to-go functions. Below, we

24 provide more details of the approach.

25 Let $\mathbf{P}_{\mathrm{MD}}$ denote the two-stage approximation to the SFPTMP. $\mathbf{P}_{\mathrm{MD}}$ is formulated based on a set

26 of Ξ' scenarios, where $|\Xi'| = |\Omega_1|$. Each $\xi \in \Xi'$ is associated with a stage scenario $\omega \in \Omega_1$, denoted

27 by $\omega(\xi)$. Each $\xi \in \Xi'$ is associated with a probability $\rho'_\xi = \frac{1}{|\Xi'|}$. For each $\xi \in \Xi'$, and $n \in \mathcal{N}$ we define

28 the supply and demand parameters (denoted by $d'_{\xi,n}$) as follows:

$$d'_{\xi,n} = \begin{cases} d'_{\omega(\xi),n}, & \text{if } n \in \mathcal{N}_1, \\ \sum_{\omega \in \Omega_p} \varrho_\omega d_{\omega,n}, & \text{if } n \in \mathcal{N}_p, p \in \mathcal{P} \setminus \{1\}. \end{cases}$$

Given Ξ' and $d'_{\xi,n}$, $\mathbf{P}_{\mathrm{MD}}$ can be formulated as the following mixed-integer linear programming (MILP) model:

$$\mathbf{P}_{\mathrm{MD}} = \min \sum_{b \in \mathcal{B}} F_b y_b + \sum_{\xi \in \Xi'} \rho'_\xi \left(\sum_{n \in \mathcal{N}} (h_n u_{\xi,n} + e_n v_{\xi,n}) + \sum_{a \in \mathcal{A}} c_a z_{\xi,a} \right) \tag{10}$$

$$\text{s.t.} \ \ y_b \geq \underline{m}_b x_b \ \forall b \in \mathcal{B} \tag{11}$$

$$y_b \leq \overline{m}_b x_b \ \forall b \in \mathcal{B} \tag{12}$$

$$z_{\xi,a} \leq y_b \ \forall a \in \mathcal{A}_b^1, \forall b \in \mathcal{B}, \forall \xi \in \Xi' \tag{13}$$

$$u_{\xi,n_1} + v_{\xi,n_1} = d'_{\xi,n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a \in A^+(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^S, \forall \xi \in \Xi' \tag{14}$$

$$u_{\xi,n} + v_{\xi,n} = d'_{\xi,n} + q_n^0 - \sum_{a \in A^+(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^S, \forall \xi \in \Xi' \tag{15}$$

$$u_{\xi,n_1} - v_{\xi,n_1} = d'_{\xi,n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a \in A^-(n_1)} z_{\xi,a}$$

$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^D, \forall \xi \in \Xi' \tag{16}$$

$$u_{\xi,n} - v_{\xi,n} = d'_{\xi,n} + q_n^0 + \sum_{a \in A^-(n)} z_{\xi,a} \quad \forall n = (i,1) \in \mathcal{N}^D, \forall \xi \in \Xi' \tag{17}$$

$$u_{\xi,n} \leq \bar{q}_n \quad \forall n \in \mathcal{N}, \forall \xi \in \Xi' \tag{18}$$

$$x_b \in \{0,1\} \quad \forall b \in \mathcal{B} \tag{19}$$

$$y_b \geq 0 \quad \forall b \in \mathcal{B} \tag{20}$$

$$u_{\xi,n}, v_{\xi,n}, z_{\xi,a} \geq 0 \quad \forall n \in \mathcal{N}, \forall a \in \mathcal{A}, \forall \xi \in \Xi'. \tag{21}$$

29   Since the number of scenarios in Ξ' is relatively small, $\mathbf{P}_{\text{MD}}$ can be efficiently solved directly

30   with commercial solvers (e.g., CPLEX). Let $\bar{\boldsymbol{y}}$ denote the solution to the $\boldsymbol{y}$ variables of $\mathbf{P}_{\text{MD}}$. Based

31   on $\bar{\boldsymbol{y}}$, we then derive the upper bound obtained by MD for any instance using the same approach

32   described in Section 6.1 of the paper. Note that when deriving the upper bound of an instance, we

33   solve problem $\mathbf{P}_{\xi,p}$ for any sampled scenario ξ and any stage $p \in \mathcal{P}$ without cost-to-go functions,

34   i.e., we have $\mathcal{K}_p = \varnothing, \forall p \in \mathcal{P}$.

35   **2.3.   HS**

In this approach, we first solve a linear programming relaxation of problem $\mathbf{P}$ defined in the paper,

denoted by $\mathbf{P}_{\text{HS}}$, where the following constraints in $\mathbf{P}$:

$$x_b \in \{0,1\} \quad \forall b \in \mathcal{B}$$

are relaxed to:

$$x_b \in [0,1] \quad \forall b \in \mathcal{B}.$$

36   We solve the problem $\mathbf{P}_{\text{HS}}$ through the SDDP proposed in the paper. Let $\bar{\boldsymbol{x}} \in \mathbb{R}^{|\mathcal{B}|}$ and $\bar{\boldsymbol{y}} \in \mathbb{R}^{|\mathcal{B}|}$

37   denote the solution of the variables in $\boldsymbol{x}$ and $\boldsymbol{y}$ returned by the SDDP to the problem, respectively.

38   We then construct a feasible solution, denoted by $(\boldsymbol{x}', \boldsymbol{y}')$ to the problem $\mathbf{P}$ based on $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$. In

39   particular, we let

$$x'_b = \begin{cases} 1, \text{ if } \bar{x}_b \geq 0.5, \\ 0, \text{ if } \bar{x}_b < 0.5, \end{cases}$$

$$y'_b = \begin{cases} \bar{y}_b, \text{ if } \bar{x}_b \geq 0.5 \wedge \underline{m}_b \leq \bar{y}_b \leq \overline{m}_b, \\ \underline{m}_b, \text{ if } \bar{x}_b \geq 0.5 \wedge \bar{y}_b < \underline{m}_b, \\ \overline{m}_b, \text{ if } \bar{x}_b \geq 0.5 \wedge \bar{y}_b > \overline{m}_b, \\ 0, \quad \text{ if } \bar{x}_b < 0.5. \end{cases}$$

40   Based on $\boldsymbol{y}'$, we then derive the upper bound obtained by HS for each instance using the same

41   approach described in Section 6.1 of the paper.

2.4.   TS

Unlike the approaches introduced previously, the approach TS constructs solutions to the SFPTMP in a two-stage manner. In the first decision stage, we solve a two-stage approximation of problem $\mathbf{P}$, denoted by $\mathbf{P}_{\mathrm{TS}}$ to determine the for-hire contracts and the capacities of the shipments associated with the contracts. In the second decision stage, given the first-stage decisions and the observed supplies and demands at the shipping stage $p = 1$, the shipper determine the volumes of all shipments in all the shipping stages from $p = 1$ to $p = \bar{p}$.

In $\mathbf{P}_{\mathrm{TS}}$, we consider a subset $\Xi' \subseteq \Xi$ of scenarios. In our implementation, if $|\Xi| \leq 1,000$, we let $\Xi' = \Xi$. Otherwise, we generate Ξ' by randomly sample $1,000$ scenarios from Ξ. For each scenario $\xi \in \Xi'$, let ρ'_ξ denote its probability. Furthermore, we use $d'_{\xi,n}$ denote the demand or supply of node $n \in \mathcal{N}$ under scenario $\xi \in \Xi'$, which is set as:

$$\hat{d}_{\xi,n} = \begin{cases} d'_{\xi,n}, & \text{if } p = 1, \\ \min_{\omega \in \Omega_q} d_{\omega,n}, & \text{if } n \in \mathcal{N}^S, p \geq 2 \\ \max_{\omega \in \Omega_q} d_{\omega,n}, & \text{if } n \in \mathcal{N}^D, p \geq 2. \end{cases} \tag{22}$$

The $\mathbf{P}_{\mathrm{TS}}$ is formulated as follows:

$$\mathbf{P}_{\mathrm{MD}} = \min \sum_{b \in \mathcal{B}} F_b y_b + \sum_{\xi \in \Xi'} \rho'_\xi \left(\sum_{n \in \mathcal{N}} (h_n u_{\xi,n} + e_n v_{\xi,n}) + \sum_{a \in \mathcal{A}} c_a z_{\xi,a} \right) \tag{23}$$

$$\text{s.t.} \ \ y_b \geq \underline{m}_b x_b \ \forall b \in \mathcal{B} \tag{24}$$

$$y_b \leq \overline{m}_b x_b \ \forall b \in \mathcal{B} \tag{25}$$

$$z_{\xi,a} \leq y_b \ \forall a \in \mathcal{A}_b^1, \forall b \in \mathcal{B}, \forall \xi \in \Xi' \tag{26}$$

$$u_{\xi,n_1} + v_{\xi,n_1} = d'_{\xi,n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a \in A^+(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i, t-1) \in \mathcal{N}^S, \forall \xi \in \Xi' \tag{27}$$

$$u_{\xi,n} + v_{\xi,n} = d'_{\xi,n} + q_n^0 - \sum_{a \in A^+(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^S, \forall \xi \in \Xi' \tag{28}$$

$$u_{\xi,n_1} - v_{\xi,n_1} = d'_{\xi,n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a \in A^-(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i, t-1) \in \mathcal{N}^D, \forall \xi \in \Xi' \tag{29}$$

$$u_{\xi,n} - v_{\xi,n} = d'_n + q_n^0 + \sum_{a \in A^-(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^D, \forall \xi \in \Xi' \tag{30}$$

$$u_{\xi,n} \leq \bar{q}_n \ \forall n \in \mathcal{N}, \forall \xi \in \Xi' \tag{31}$$

$$\hat{u}_{\xi,n_1} + \hat{v}_{\xi,n_1} = \hat{d}_{\xi,n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a \in A^+(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i, t-1) \in \mathcal{N}^S, \forall \xi \in \Xi' \tag{32}$$

$$\hat{u}_{\xi,n} + \hat{v}_{\xi,n} = \hat{d}_{\xi,n} + q_n^0 - \sum_{a \in A^+(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^S, \forall \xi \in \Xi' \tag{33}$$

$$\hat{u}_{\xi,n_1} - \hat{v}_{\xi,n_1} = \hat{d}_{\xi,n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a \in A^-(n_1)} z_{\xi,a}$$

$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^D, \forall \xi \in \Xi' \tag{34}$$

$$\hat{u}_{\xi,n} - \hat{v}_{\xi,n} = \hat{d}_{\xi,n} + q_n^0 + \sum_{a \in A^-(n)} z_{\xi,a} \ \ \forall n = (i,1) \in \mathcal{N}^D, \forall \xi \in \Xi' \tag{35}$$

$$\hat{u}_{\xi,n} \leq \bar{q}_n \ \ \forall n \in \mathcal{N}, \forall \xi \in \Xi' \tag{36}$$

$$x_b \in \{0,1\} \ \ \forall b \in \mathcal{B} \tag{37}$$

$$y_b \geq 0 \ \ \forall b \in \mathcal{B} \tag{38}$$

$$u_{\xi,n}, v_{\xi,n}, \hat{u}_{\xi,n}, \hat{v}_{\xi,n}, z_{\xi,a} \geq 0 \ \ \forall n \in \mathcal{N}, \forall a \in \mathcal{A}, \forall \xi \in \Xi'. \tag{39}$$

Note that at the beginning of the second stage, only the supplies and demands associated with nodes in $\mathcal{N}_1$ are fully known to the shipper, while the shipper has to make volume decisions for the shipments during all stages in the shipping phase (i.e., from $p = 1$ to $p = \bar{p}$). Therefore, we use constraints (32)–(36) and (39) to ensure the feasibility of the solution of variables in $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ obtained by the two-stage decision framework when implemented in a multi-stage framework.

2.4.1. Benders Reformulation Observe that there is no non-anticipativity constraints in the formulation of $\mathbf{P}_{\text{MD}}$. This enables the application of Benders decomposition to efficiently solve the problem. The master problem of the decomposition (**MP**) is formulated as follows:

$$\mathbf{MP} = \min \sum_{b \in \mathcal{B}} F_b y_b + \sum_{\xi \in \Xi'} \rho'_\xi \eta_\xi \tag{40}$$

$$\text{s.t.} \ \ (24),(25),(37),(38)$$

$$\eta_\xi \geq 0 \ \ \forall \xi \in \Xi' \tag{41}$$

Benders optimality cuts,

where η_ξ represents the second-stage cost under scenario $\xi \in \Xi'$. Furthermore, given any first-stage solution $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ to the master problem, there is always a feasible solution to the second-stage problem under any scenario $\xi \in \Xi$. Hence, only optimality cuts are needed for the master problem.

Let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\eta}})$ denote the solution to the master problem. Given $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\eta}})$, we solve a subproblem for each scenario $\xi \in \Xi$. Let $\mathbf{SP}_\xi$ denote the subproblem associated with $\xi \in \Xi'$, which can be formulated as follows:

$$\mathbf{SP}_\xi = \min \sum_{n \in \mathcal{N}} (h_n u_{\xi,n} + e_n v_{\xi,n}) + \sum_{a \in \mathcal{A}} c_a z_{\xi,a} \tag{42}$$

$$\text{s.t.} \ \ y_b = \bar{y}_b \ \ \forall b \in \mathcal{B} \tag{43}$$

$$z_{\xi,a} \leq y_b \ \forall a \in \mathcal{A}_b^1, \forall b \in \mathcal{B} \tag{44}$$

$$u_{\xi,n_1} + v_{\xi,n_1} = d'_{\xi,n_1} + u_{\xi,n_2} + v_{\xi,n_2} - \sum_{a \in A^+(n_1)} z_{\xi,a} \ \forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^S \tag{45}$$

$$u_{\xi,n} + v_{\xi,n} = d'_{\xi,n} + q_n^0 - \sum_{a \in A^+(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^S \tag{46}$$

$$u_{\xi,n_1} - v_{\xi,n_1} = d'_{\xi,n_1} + u_{\xi,n_2} - v_{\xi,n_2} + \sum_{a \in A^-(n_1)} z_{\xi,a} \ \forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^D \tag{47}$$

$$u_{\xi,n} - v_{\xi,n} = d'_{\xi,n} + q_n^0 + \sum_{a \in A^-(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^D \tag{48}$$

$$u_{\xi,n} \leq \bar{q}_n \ \forall n \in \mathcal{N} \tag{49}$$

$$\hat{u}_{\xi,n_1} + \hat{v}_{\xi,n_1} = \hat{d}_{\xi,n_1} + \hat{u}_{\xi,n_2} + \hat{v}_{\xi,n_2} - \sum_{a \in A^+(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^S \tag{50}$$

$$\hat{u}_{\xi,n} + \hat{v}_{\xi,n} = \hat{d}_{\xi,n} + q_n^0 - \sum_{a \in A^+(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^S, \forall \xi \in \Xi' \tag{51}$$

$$\hat{u}_{\xi,n_1} - \hat{v}_{\xi,n_1} = \hat{d}_{\xi,n_1} + \hat{u}_{\xi,n_2} - \hat{v}_{\xi,n_2} + \sum_{a \in A^-(n_1)} z_{\xi,a}$$
$$\forall n_1 = (i,t), n_2 = (i,t-1) \in \mathcal{N}^D \tag{52}$$

$$\hat{u}_{\xi,n} - \hat{v}_{\xi,n} = \hat{d}_{\xi,n} + q_n^0 + \sum_{a \in A^-(n)} z_{\xi,a} \ \forall n = (i,1) \in \mathcal{N}^D \tag{53}$$

$$\hat{u}_{\xi,n} \leq \bar{q}_n \ \forall n \in \mathcal{N} \tag{54}$$

$$u_{\xi,n}, v_{\xi,n}, \hat{u}_{\xi,n}, \hat{v}_{\xi,n}, z_{\xi,a} \geq 0 \ \forall n \in \mathcal{N}, \forall a \in \mathcal{A}. \tag{55}$$

We solve the dual of $\mathbf{SP}_\xi$ to derive Benders optimality cuts. Let Z_ξ^{SP} denote the objective function value to the dual of $\mathbf{SP}_\xi$ and $\boldsymbol{\pi}$ be the optimal solution to dual variables associated with constraints (43). If $Z_\xi^{\text{SP}} > \bar{\eta}_\xi + \epsilon$, where $\epsilon > 0$ is a preset residual term, we add the following Benders cut into the master problem $\mathbf{MP}$, which will be solved again:

$$\eta_\xi \geq Z_\xi + \sum_{b \in \mathcal{B}} \pi_b (y_b - \bar{y}_b). \tag{56}$$

71 **2.4.2. Benders Decomposition Algorithm** The Benders decomposition algorithm solves
72 the problem $\mathbf{P}_{\mathrm{TS}}$ by iterating between the mater problem and subproblems. Algorithm 1 illustrates
73 the procedures of the algorithm.

Algorithm 1 Benders Decomposition.

1: Initialization: set the set of Benders cuts as $\varnothing$; set the upper bound as $UB \leftarrow +\infty$; set the lower bound
    as $LB \leftarrow -\infty$; set the stopping flag as $True$; set the optimal solution as $(\boldsymbol{x}^*, \boldsymbol{y}^*) \leftarrow (\mathbf{0}, \mathbf{0})$

2: **while** Stopping flag is $False$ **do**

3:    Solve **MP**

4:    Collect the solution $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\eta}})$ and the objective function value Z^{MP}

5:    Set $LB \leftarrow Z^{MP}$

6:    **for** $\xi \in \Xi'$ **do**

7:        Solve the dual of $\mathbf{SP}_\xi$

8:        Collect Z^{SP}_ξ and $\boldsymbol{\pi}$

9:        **if** $Z^{\mathrm{SP}}_\xi > \bar{\eta}_\xi + \epsilon$ **then**

10:           Expand the set of Benders optimality cuts by adding constraint (56).

11:        **end if**

12:    **end for**

13:    **if** $UB > Z^{MP} - \sum_{\xi \in \Xi'} \rho'_\xi \bar{\eta}_\xi + \sum_{\xi \in \Xi'} \rho'_\xi Z^{SP}_\xi$ **then**

14:        Set $UB \leftarrow Z^{MP} - \sum_{\xi \in \Xi'} \rho'_\xi \bar{\eta}_\xi + \sum_{\xi \in \Xi'} \rho'_\xi Z^{SP}_\xi$.

15:        Set the best solution as $(\boldsymbol{x}^*, \boldsymbol{y}^*) \leftarrow (\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$.

16:    **end if**

17:    **if** $UB - LB < \epsilon$ **then**

18:        Set the stopping flag as $True$

19:    **end if**

20: **end while**

21: Return the solution $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ as the first-stage solution to $\mathbf{P}_{\mathrm{TS}}$.

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74 **2.4.3. Deriving Upper Bounds** To derive the upper bounds obtained by TS for any
75 instance, we design a simulation method that mimics the *two-stage solution framework*. In this
76 framework, the for-hire carries and their carrying capacities in each shipment are determined in
77 the first stage, while in the second stage, the shipment volumes on all lines over the entire planning
78 horizon covering all stages $p \in \mathcal{P}$ are determined.

79 Let $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}})$ be the solution returned by algorithm 1. To obtain the upper bound, we first use the
80 same sampling approach as described in Section 6.1 of the paper. For each sampled scenario ξ, we
81 first recast the supply and demand parameters into the two-stage optimization logic. In particular,
82 for each $n \in \mathcal{N}_p$ with $p \in \mathcal{P}$ we set the supply and demand parameters as follows:

$$d'_{\xi,n} = \begin{cases} d'_{\omega(\xi),n}, & \text{if } p = 1, \\ \sum_{\omega \in \Omega_p} \varrho_\omega d_{\omega,n}, & \text{if } p \geq 2. \end{cases}$$

Then we solve the second-stage problem $(\mathbf{SP}_\xi)$ defined on $\bar{\boldsymbol{y}}$ and $\boldsymbol{d}'$ to determine the shipping volumes. Let $\bar{\boldsymbol{z}}$ be the solution to the z_a variables in the problem. Based on $\bar{\boldsymbol{z}}$, we solve stage-wise problems $\mathbf{P}_{\xi,p}$ with the following additional constraints to ensure that all shipment volume decisions are made at the second stage:

$$z_a = \bar{z}_a \ \ \forall a \in \mathcal{A}_p.$$

All the other components of the method for deriving upper bounds remain unchanged as described in Section 6.1.