Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
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FACULTY OF SCIENCE, TECHNOLOGY, ENGINEERING AND MATHEMATICS

SCHOOL OF ENGINEERING

Electronic and Electrical Engineering

Engineering MAI/MSc **Semester 1, 2022**

EEP55C25 Algorithms for Quantum Computing

16/12/2022 **RDS – SIMMONSCOURT** **09:30–11:30**

Prof. B. Basu

Instructions to Candidates:

Answer any four (4) questions.

Start the answer to each question on a new page.

Use of calculator is permitted.

Q.1 **[Total: 25 marks]**

(a) Draw a quantum circuit to transform the quantum state β_{00}, i.e. $\frac{1}{\sqrt{2}}(|00> + |11>)$ to the state $|0\rangle|0\rangle$, supported by mathematical calculations.

[8 marks]

(b) What is the significance of a quantum state which cannot be represented as a point on the surface of a Bloch sphere?

[8 marks]

(c) Find the quantum states which are the eigenstates of the NOT gate

$$X = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

and comment on them.

[9 marks]

Q.2 **[Total: 25 marks]**

(a) Explain with an example the difference between a classical probabilistic computing and quantum computing.

[7 marks]

(b) Draw a quantum circuit explaining with mathematical calculations to show how the phase $\omega = 0.0x_1x_2$ can be estimated.

[8 marks]

(c) Consider the following state in a 4-dimensional Hilbert space

$$H_A \otimes H_B, \quad |\varphi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|01\rangle.$$

Derive the Schmidt bases and the coefficients.

[10 marks]

Q.3 **[Total: 25 marks]**

(a) We have a blackbox U_f to compute an unknown function f: {0,1,2, …, 100} → {0,1}. Suppose there are 10 solutions to f(x) = 1. Show that amplitude amplification algorithm finds a solution with a probability of approximately 90%.

Aide memoir: After k iterations of applying Q = $U_{\psi}^{\perp} U_f$, the initial state becomes $Q^k \, |\psi\rangle = \cos((2k+1)\theta)|\psi_{bad}\rangle + \sin((2k+1)\theta)|\psi_{good}\rangle$. The notations have their usual meaning. **[9 marks]**

(b) Given any 1 qubit quantum gate denoted by U, a controlled U gate can be defined as a 2-qubit gate denoted by c-U which acts on the second qubit only if the first qubit is in state $|1\rangle$. Prove that c-U and c-($e^{i5\pi}$ U) are not equivalent.

[8 marks]

(c) Draw a circuit for {H, T} gates applied in a series and show its effect on a qubit (α_0 α_1).

[8 marks]

Q.4 **[Total: 25 marks]**

(a) Consider the following state:

$$|\psi\rangle = \sqrt{\frac{1}{11}}|0\rangle|0\rangle + \sqrt{\frac{5}{11}}|0\rangle|1\rangle + \sqrt{\frac{2}{11}}|1\rangle|0\rangle + \sqrt{\frac{3}{11}}|1\rangle|1\rangle.$$

Calculate the probability of measuring 1 in the second qubit if the measurement of first qubit gives a value of 0.

[6 marks]

(b) Consider the following matrix representation of a biased Hadamard quantum gate

$$H_b = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 \\ \sqrt{2} & -\sqrt{2} \end{bmatrix}.$$

Calculate the probability of measuring an output of 0 if the gate acts twice successively (without intermediate measurement) on the quantum state $|0\rangle$.

[9 marks]

(c) Compute the probabilities of accessing the states in a 1 step quantum random walk starting from origin $a_0 = (0,0)$. Compare and contrast the result with a 1 step random walk when starting from the quantum state $\frac{1}{\sqrt{2}}(a_0 + i a_1)$, where $a_1 = (0,1)$.

[10 marks]

Q.5 **[Total: 25 marks]**

(a) Prove that the controlled gate c-U for any given 1 qubit gate U corresponds to the operator

$|0\rangle\langle 0| \otimes I + \ |1\rangle\langle 1| \otimes U.$ **[10 marks]**

(b) Prove that the density operator ρ for an ensemble of pure states satisfies the conditions: Trace (ρ) = 1.

[8 marks]

(c) Is the state $\frac{1}{\sqrt{2}}\,(|01> + |10>)$ entangled? Justify your answer.

[7 marks]