Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
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FACULTY OF SCIENCE, TECHNOLOGY, ENGINEERING AND MATHEMATICS

SCHOOL OF ENGINEERING

Electronic and Electrical Engineering

Engineering **Semester 2, 2021**
MAI

Self-Organising Systems

17/12/2021 **RDS Simmonscourt** **09:30–11:30**

Prof. Nicola Marchetti

Instructions to Candidates:
Answer EXACTLY FIVE questions.

Materials permitted for this examination:
This is a closed book examination. Materials Permitted: Calculator; Mathematical Tables.

Q.1 **[Total: 20 marks]**

(a) Calculate the state probability distribution of the Markov process in Fig. Q1.

[14 marks]

(b) Calculate the Shannon entropy of the Markov process in Fig. Q1.

[6 marks]

- Note: Assume that when there is a choice for the transitions from a state, all such transitions are equiprobable.
- Note: Please show all your workings.

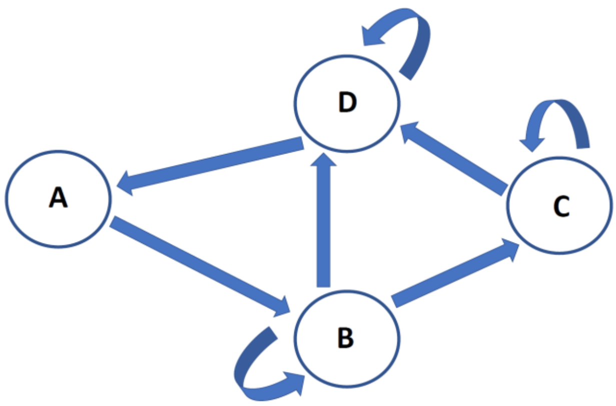

Fig. Q1

Q.2 **[Total: 20 marks]**

Consider the process defined by the finite automaton shown in Fig. Q.2. When two arcs leave a node, they have the same probability.

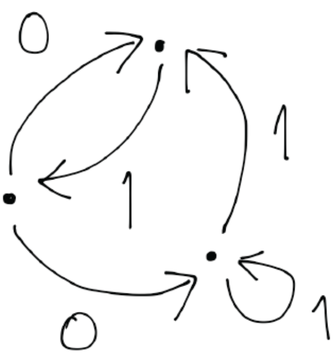

Fig. Q.2

(a) Determine the correlation length m and the probability mass function for the Markov model graph nodes.

[10 marks]

(b) Calculate the correlation complexity η.

[10 marks]

Q.3 **[Total: 20 marks]**

(a) The following network models are used to generate graphs which exhibit important properties observed in real-world networks.

- Erdos-Renyi model;
- Barabasi-Albert model.

(i) Outline the steps necessary to produce graphs using the two above models.

[5 marks]

(ii) Write the degree distributions of the two above models.

[5 marks]

(b) For the graph G shown in Fig Q.3, calculate:

(i) Calculate the degree k_i of each node in G.

[1.5 marks]

(ii) Calculate the degree degree distribution P(k) of G.

[1.5 marks]

(iii) Calculate the average path length <l> of G.

[3.5 marks]

(iv) Calculate the global clustering coefficient C of G.

[3.5 marks]

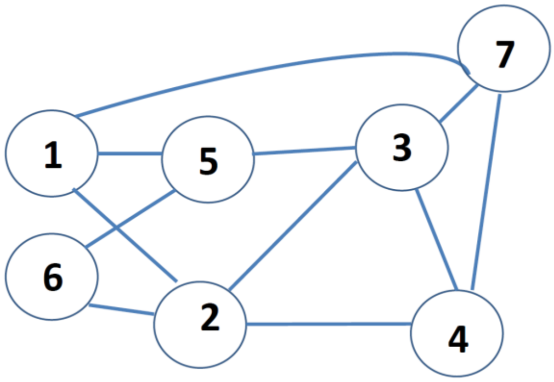

Fig. Q.3

Q.4 **[Total: 20 marks]**

For a random graph:

(a) Prove that the average degree is:

$$< k > = (N-1)p$$

where *N* is the number of nodes in the graph, and *p* is the probability of an edge being created.

[10 marks]

(b) Prove that the degree distribution is:

$$P(k) = \exp(-< k >)\frac{< k >^k}{k!}$$

[10 marks]

Q.5 **[Total: 20 marks]**

Consider the traveling salesman problem. We try to solve the problem using Simulated Annealing (SA) algorithm.

(a) What is a possible example of 'configuration' in SA algorithm with reference to travelling salesman problem and how could it be changed?

[2.5 marks]

(b) In reference to travelling salesman problem, what is analogous to 'metal' and 'energy state' in SA algorithm?

[1.5 marks]

(c) Please interpret 'temperature' in relation to SA algorithm. What are the main principles to design a temperature schedule for travelling salesman problem?

[4 marks]

(d) What is Metropolis criterion? Please write down pseudo code for metropolis function in relation to travelling salesman problem.

[4 marks]

(e) What is muting in SA and why is it used? How is this implemented in Metropolis criterion? Please explain clearly by referring to pseudocode steps.

[4 marks]

(f) What is the role of temperature (heating and cooling) in Metropolis criterion/muting frequency? Please explain clearly by referring to pseudocode.

[4 marks]

Q.6 **[Total: 20 marks]**

Suppose that the waiting time between packet arrivals in a communication system, is modelled by the following probability density function, $p(x) = Ax^{-\alpha}$, where $x \in [1, \infty[$ seconds.

(a) Using the fact that the probability must be normalized, solve for A as a function of α, where $\alpha > 1$.

[5 marks]

(b) For α=3, compute:

i. the probability that the waiting time is between 2 and 5 s;

ii. the probability that the waiting time is larger than 15 s.

[5 marks]

(c) Determine the mean waiting time for $\alpha=3$.

[5 marks]

(d) Show that the mean waiting time is infinite for $\alpha \leq 2$.

[5 marks]

Q.7 **[Total: 20 marks]**

Consider a network where packets must traverse multiple intermediate nodes to be delivered from source to destination, as in Fig. Q.7. In the figure, S denotes the source, D the destination, and A and B are intermediate routers that face the decision of whether to forward a packet.

Fig. Q.7

When each of the intermediate routers forwards a packet, it incurs a cost c = 3 units. Each time a packet successfully reaches the destination (i.e., both A and B decide to forward the packet), each of the routers receives a reward r = 5 units.

We wish to model the forwarding decision as a game.

(a) Set up the game in strategic form, namely:

(i) Define the set of players;

[1 mark]

(ii) Define the action set for each player;

[1 mark]

(iii) Provide the set of payoffs for each joint action, in matrix form.

[2 marks]

(b) Set up the game in extensive form, i.e., as a decision tree with each player's utilities shown in the leaves.

[4 marks]

(c) Find all Nash equilibria for this game.

[4 marks]

(d) For each Nash equilibrium, determine whether it is Pareto optimal.

[4 marks]

(e) Is this game equivalent to the Prisoner's Dilemma? Describe what fundamental concept about game theory the Prisoner's dilemma illustrates.

[4 marks]