Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF ENGINEERING

Electronic & Electrical Engineering

Engineering **Michaelmas Term, 2020-21**

SS / MAI / MSc

Annual Examinations

4C7/5C7 SELF ORGANISING SYSTEMS

19th January 2021 **Online** **12:00 - 02:00**

Prof Nicola Marchetti

Instructions to Candidates:

- Please answer EXACTLY FOUR questions for part A and EXACTLY EIGHT questions for part B.

Materials Permitted for this Examination:

- Mathematical Tables
- Calculator
- Lecture Notes
  - Clean slides without annotations
  - No tutorial slides allowed

PART A

Q.1

(a) Calculate the state probability distribution and the Shannon entropy of the below Markov process.

- Note: Assume that when there is a choice for the transitions from a state, all such transitions are equiprobable.
- Note: Please show all your workings.

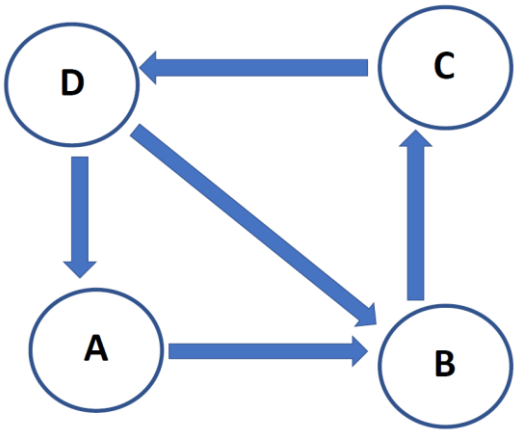


Fig. Q1

[7.5 marks]

(b) Assume the same Markov process as in Fig. Q1, with the addition of loops for every state, e.g. there is a transition from A to A, from B to B etc.

- Note: Assume that when there is a choice for the transitions from a state, all such transitions are equiprobable.
- Note: Please show all your workings.

[7.5 marks]

Q.2

Consider an object encoded in a sequence of bits of length 12. We aim to minimize the number of ones in the first third of the sequence. We apply a genetic algorithm to find the solution of the problem.

We initialize with a generation of 5 chromosomes which are encoded as follows:

A_1 = 0110 1011 0011

A_2 = 1000 0101 1010

A_3 = 1111 1101 1110

A_4 = 1010 0111 0001

A_5 = 0001 1011 0010

Please answer the following questions.

(a) Define an appropriate fitness function for the problem defined. Apply your fitness function to compute the fitness values of each chromosome (A_1-A_5).

[4 marks]

(b) Based on the results in (a), design a selection criterion. What is the probability of selection of each chromosome for the next generation?

[3 marks]

(c) Based on your results in (b), perform a selection for the next generation.

[4 marks]

(d) Perform one-point cross over operation to A_3 and A_4. Then, perform two-point cross over operation to A_1 and A_5. Define the cross over points of your choice and describe them clearly.

[2 marks]

(e) Perform mutation with probability 0.25 on A_1. Clearly mention the mutated bit(s).

[2 marks]

Q.3

The following directed graph shows the communication network topology for an ad-hoc network, where an edge is present if the receiving node can decode the signal from the transmitting node.

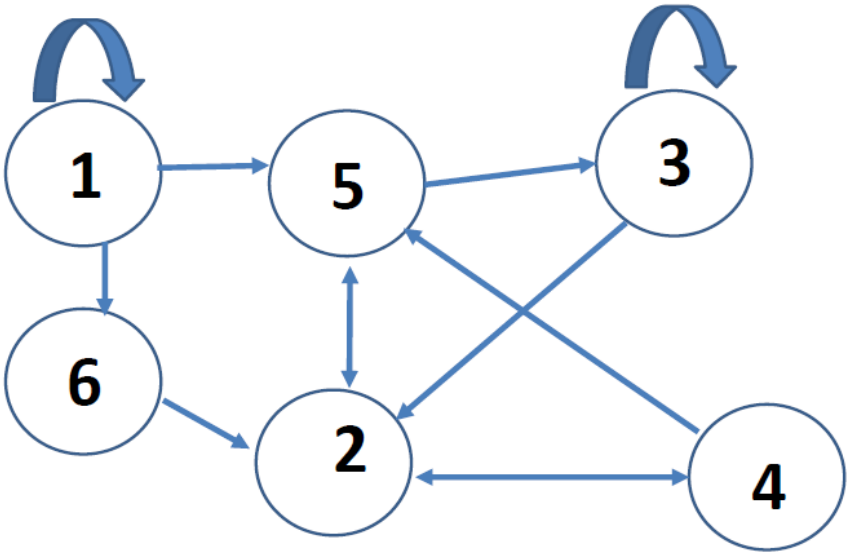


Fig. Q.3

Calculate the following quantities, providing an explanation for each answer.

(a) Calculate the in-degree of each vertex.

[3 marks]

(b) Calculate the out-degree of each vertex.

[3 marks]

(c) Calculate the in-degree distribution of the graph.

[3 marks]

(d) Calculate the out-degree distribution of the graph.

[3 marks]

(e) Calculate the adjacency matrix of the graph.

[3 marks]

Q.4

The graph in Fig. Q.4, shows the load balancing topology for a cellular network, where an edge is present if the two base stations exchange users based on their resource availability.

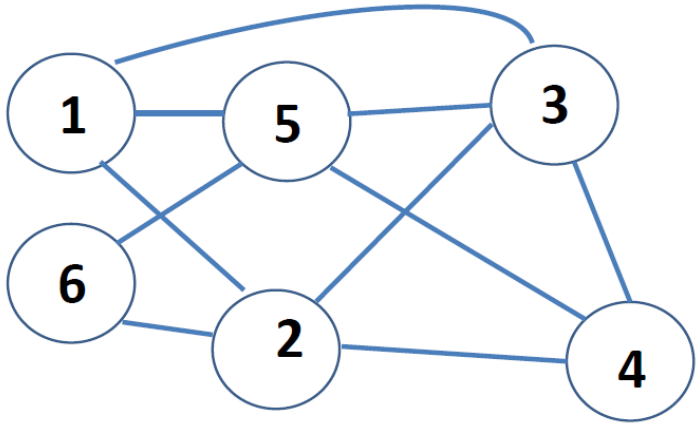


Fig. Q.4

(a) Compute the closeness centrality for each node.

[5 marks]

(b) Compute the betweenness centrality for each node, considering only the geodesic paths from node 1 to node 4.

[4 marks]

(c) List all the 3-cliques. Adding what link results in two additional 4-cliques?

[3 marks]

(d) Apply pruning to find the 3-cores.

[3 marks]

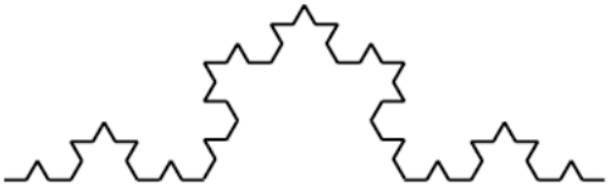
Provide an explanation for each answer.

Q.5

(a) For a fractal antenna modelled as a Cantor set (see figure below), calculate the fractal dimension and the total length at iteration n=3. Assume the initial side length to be 1.

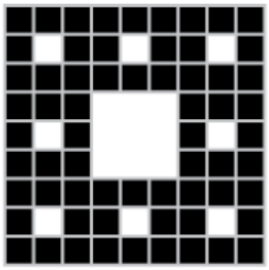
[3.75 marks]

(b) For a fractal antenna modelled as a Koch curve (see figure below), calculate the fractal dimension, and the total length at iteration n=3. Assume the initial side length to be 1.



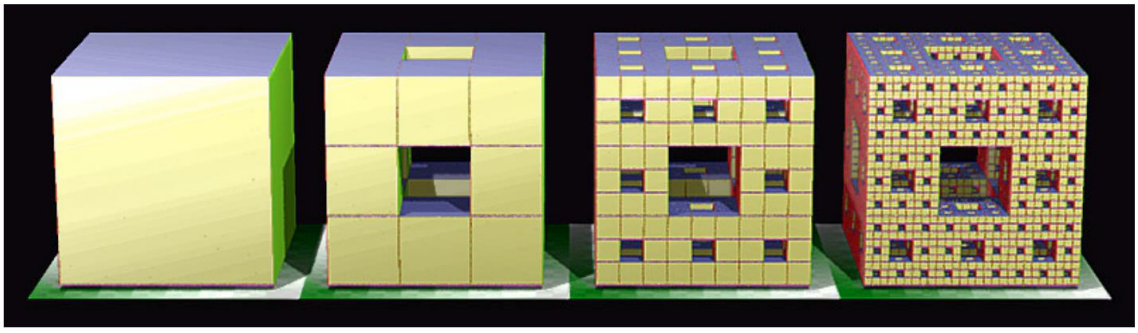
[3.75 marks]

(c) For a fractal antenna modelled as a Sierpinski carpet (see figure below), calculate the fractal dimension, and the total area at iteration n=3. Assume the initial side length to be 1.



[3.75 marks]

(d) For a fractal antenna modelled as a Sierpinski cube (see figure below), calculate the fractal dimension, and the total volume at iteration n=3. Assume the initial side length to be 1.



[3.75 marks]

Q.6

Consider the following situation, to be modeled using game theory.

Two firms, A and B, produce a product to serve the same market. They have constant costs of producing the item, equal to €6 per item. Each firm can choose to sell the item at a high price (€25) or a low price (€15). When both firms set a high price, the total demand for the product is 20,000 units, which is evenly split between the two firms. When both firms set a low price, the total demand is 34,000 units, again evenly split between both. If one firm sets a low price and the other a high price, the low-priced firm sells 30,000 units while the high-priced firm sells only 5,000 units.
The utility function for each firm is its expected profit (revenue minus costs).

a) Set up the problem as a non-cooperative game, in normal form (i.e., draw a matrix with the utilities achieved by each player under each action vector).

[4 marks]

b) Derive the Nash equilibrium (or equilibria) for this game. Assume that both firms set their prices simultaneously.

[4 marks]

c) In what sense is this problem equivalent to the prisoner's dilemma?

[4 marks]

d) Now assume that firm A sets its price first, and firm B follows. What is the Stackelberg equilibrium for this game?

[3 marks]

PART B

Q.7

Which one among the following statements about the Shannon information is incorrect?

(a) It can capture the meaning of messages
(b) It captures the amount of surprise in a message
(c) It depends on the probability distribution of the source's message alphabet
(d) It depends on the size of the source's message alphabet

[5 marks]

Q.8

Which one among the following statements about the field of network science is incorrect?

(a) It relies on graph theory
(b) At present its knowledge of network structure surpasses its knowledge about the network dynamics
(c) It is mainly focusing on representations of real networks
(d) The only model to capture randomness in a graph is Erdos-Renyi

[5 marks]

Q.9

Which of the following statements is incorrect about statistical mechanics entropy?

(a) It relates the microstates of a system to a macrostate
(b) The number of macrostates represents the system's disorder
(c) It is a generalisation of thermodynamics' entropy
(d) It is related to Shannon's entropy

[5 marks]

Q.10

Which of following statements is incorrect about a radio channel assignment problem?

(a) The random allocation is adapting to changes in a way that decreases interference
(b) The self-organised allocation has a higher complexity than the random allocation
(c) The periodic allocation shows a complexity of zero
(d) The random allocation shows a non-zero entropy

[5 marks]

Q.11

Which of following statements is incorrect about a statistical mechanics model of wireless network interference?

(a) The degeneracy of channel allocations reflects the interference cancellation hardware capabilities
(b) The degeneracy of channel allocations reflects the amount of RF bandwidth available
(c) The graph topology reflects the position of devices
(d) None of the above

[5 marks]

Q.12

Which one among the following methods can be applied to study a social network being designed but that is not yet operational?

(a) Survey
(b) Direct observation
(c) Archival data
(d) None of the above

[5 marks]

Q.13

For the channel gain $|h|^2$, which of the following statements is incorrect?

(a) h=0 is an attracting fixed point
(b) h=1 is a repelling fixed point
(c) h=2 is not a fixed point
(d) 1, 1.21, 1.46…, 2.14… is a possible orbit

[5 marks]

Q.14

For the Shannon capacity $\log_2(1+x^2)$, where x is the signal-to-noise ratio S/N, which of the following statements is incorrect?

(a) S=0 corresponds to a fixed point
(b) S=N corresponds to a fixed point
(c) S=1 corresponds to a fixed point
(d) S/N = 1 corresponds to a fixed point

[5 marks]

Q.15

Which one among the following examples cannot be considered a part of an action set according to game theory?

(a) Carrier frequency
(b) Energy efficiency
(c) Transmit antennas
(d) Modulation order

[5 marks]

Q.16

Give the strategic form payoff table $\begin{bmatrix}(a,a) & (b,0)\\ (0,0) & (0,0)\end{bmatrix}$, with a>0 and b<0, which of the following statements is incorrect?

(a) (a,a) is a Nash equilibrium
(b) (a,a) is Pareto optimal
(c) (b,0) is a Nash equilibrium
(d) (b,0) is not Pareto optimal

[5 marks]

Q.17

Which one among the following statements is incorrect?

(a) A k-clique is also a k-plex
(b) A k-plex is also a k-clique
(c) A k-clique cannot contain (k-1)-cliques
(d) A k-core in a graph might not exist

[5 marks]

Q.18

Which one among the following statements is incorrect for a directed interference graph?

(a) The adjacency matrix's transpose is not equal to the adjacency matrix
(b) The in-degree distribution is different from the out-degree distribution
(c) We cannot define the graph's clustering coefficient
(d) We cannot define the average path length

[5 marks]