



Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin

Ollscoil Átha Cliath | The University of Dublin

**FACULTY OF SCIENCE, TECHNOLOGY, ENGINEERING AND
MATHEMATICS**

SCHOOL OF ENGINEERING

Electronic and Electrical Engineering

Engineering
Senior Sophister

Semester 1, 2021

Self-Organising Systems

17/12/2021

RDS Simmonscourt

09:30–11:30

Prof. Nicola Marchetti

Instructions to Candidates:

Answer EXACTLY FIVE questions.

Materials permitted for this examination:

This is a closed book examination. Materials Permitted: Calculator;
Mathematical Tables.

Q.1

(a) Calculate the state probability distribution of the Markov process in Fig. Q1.

[14 marks]

(b) Calculate the Shannon entropy of the Markov process in Fig. Q1.

[6 marks]

- Note: Assume that when there is a choice for the transitions from a state, all such transitions are equiprobable.
- Note: Please show all your workings.

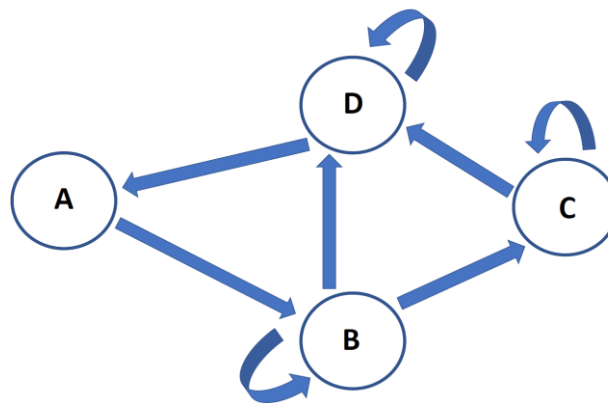


Fig. Q1

Q.2

[Total: 20 marks]

Consider the process defined by the finite automaton shown in Fig. Q.2. When two arcs leave a node, they have the same probability.

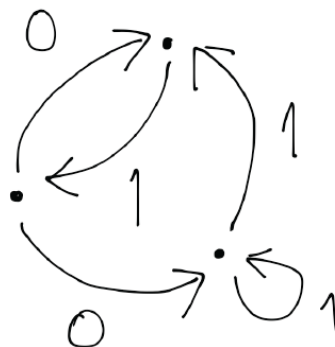


Fig. Q.2

- (a) Determine the correlation length m and the probability mass function for the Markov model graph nodes.

[10 marks]

- (b) Calculate the correlation complexity η .

[10 marks]

Q.3**[Total: 20 marks]**

- (a) The following network models are used to generate graphs which exhibit important properties observed in real-world networks.

- Erdos-Renyi model;
- Barabasi-Albert model.

- (i) Outline the steps necessary to produce graphs using the two above models.

[5 marks]

- (ii) Write the degree distributions of the two above models.

[5 marks]

- (b) For the graph G shown in Fig Q.3, calculate:

- (i) Calculate the degree k_i of each node in G .

[1.5 marks]

- (ii) Calculate the degree degree distribution $P(k)$ of G .

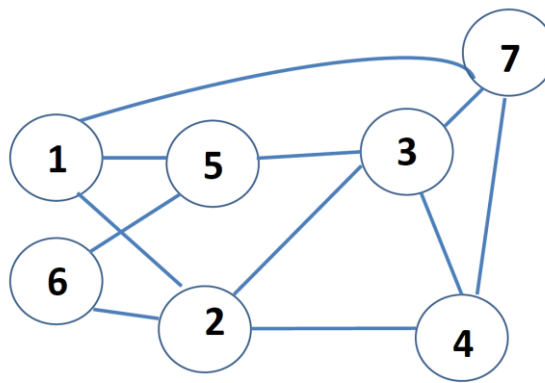
[1.5 marks]

- (iii) Calculate the average path length $\langle l \rangle$ of G .

[3.5 marks]

- (iv) Calculate the global clustering coefficient C of G .

[3.5 marks]

**Fig. Q.3****Q.4****[Total: 20 marks]**

For a random graph:

(a) Prove that the average degree is:

$$\langle k \rangle = (N - 1)p$$

where N is the number of nodes in the graph, and p is the probability of an edge being created.

[10 marks]

(b) Prove that the degree distribution is:

$$P(k) = \exp(-\langle k \rangle) \frac{\langle k \rangle^k}{k!}$$

[10 marks]**Q.5****[Total: 20 marks]**

Consider the traveling salesman problem. We try to solve the problem using Simulated Annealing (SA) algorithm.

(a) What is a possible example of 'configuration' in SA algorithm with reference to travelling salesman problem and how could it be changed?

[2.5 marks]

- (b) In reference to travelling salesman problem, what is analogous to 'metal' and 'energy state' in SA algorithm?

[1.5 marks]

- (c) Please interpret 'temperature' in relation to SA algorithm. What are the main principles to design a temperature schedule for travelling salesman problem?

[4 marks]

- (d) What is Metropolis criterion? Please write down pseudo code for metropolis function in relation to travelling salesman problem.

[4 marks]

- (e) What is muting in SA and why is it used? How is this implemented in Metropolis criterion? Please explain clearly by referring to pseudocode steps.

[4 marks]

- (f) What is the role of temperature (heating and cooling) in Metropolis criterion/muting frequency? Please explain clearly by referring to pseudocode.

[4 marks]

Q.6

[Total: 20 marks]

Suppose that the waiting time between packet arrivals in a communication system, is modelled by the following probability density function, $p(x) = Ax^{-\alpha}$, where $x \in [1, \infty[$ seconds.

- (a) Using the fact that the probability must be normalized, solve for A as a function of α , where $\alpha > 1$.

[5 marks]

- (b) For $\alpha=3$, compute:

- the probability that the waiting time is between 2 and 5 s;
- the probability that the waiting time is larger than 15 s.

[5 marks]

(c) Determine the mean waiting time for $\alpha=3$.

[5 marks]

(d) Show that the mean waiting time is infinite for $\alpha \leq 2$.

[5 marks]

Q.7

[Total: 20 marks]

Consider a network where packets must traverse multiple intermediate nodes to be delivered from source to destination, as in Fig. Q.7. In the figure, S denotes the source, D the destination, and A and B are intermediate routers that face the decision of whether to forward a packet.



Fig. Q.7

When each of the intermediate routers forwards a packet, it incurs a cost $c = 3$ units. Each time a packet successfully reaches the destination (i.e., both A and B decide to forward the packet), each of the routers receives a reward $r = 5$ units.

We wish to model the forwarding decision as a game.

(a) Set up the game in strategic form, namely:

(i) Define the set of players;

[1 mark]

(ii) Define the action set for each player;

[1 mark]

(iii) Provide the set of payoffs for each joint action, in matrix form.

[2 marks]

- (b)** Set up the game in extensive form, i.e., as a decision tree with each player's utilities shown in the leaves.

[4 marks]

- (c)** Find all Nash equilibria for this game.

[4 marks]

- (d)** For each Nash equilibrium, determine whether it is Pareto optimal.

[4 marks]

- (e)** Is this game equivalent to the Prisoner's Dilemma? Describe what fundamental concept about game theory the Prisoner's dilemma illustrates.

[4 marks]