1. Alice encodes the EPR pair

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

using the quantum one-~~time~~ pad. What is the state of the encoded quantum state as seen by Bob who knows the key bits k_1 and k_2?

We use k_1 k_2 represented the key bits and $\bar{k_1}$, $\bar{k_2}$ means the opposite state of k_1 k_2

$|\psi_{encrypted}\rangle = \frac{1}{\sqrt{2}}(|k_1 k_2\rangle + |\bar{k_1}\bar{k_2}\rangle)$

In one-time pad situation,

if $k_1 = 0$ && $k_2 = 0$, state stay still

$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

if $k_1 = 1$ && $k_2 = 0$, first state turn into opposite.

$|\psi\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$

if $k_1 = 0$ && $k_2 = 1$, second bit state turn opposite.

$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

if $k_1 = 1$ && $k_2 = 1$ so both of them turn over.

$|\psi\rangle = \frac{1}{\sqrt{2}}(|11\rangle + |00\rangle)$

2. Alice prepares the state $\rho_{AB} = |\phi\rangle\langle\phi|$ which is the singlet $|\phi\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$. Compute ρ_A.

2. $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

$\because \rho_{AB} = |\psi\rangle\langle\psi|$

$|01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\rho_{AB} = \frac{1}{2}(|01\rangle - |10\rangle)(\langle 01| - \langle 10|)$

$\rho_{AB} = \frac{1}{2}(|01\rangle\langle 01| - |01\rangle\langle 10| - |10\rangle\langle 01| + |10\rangle\langle 10|)$

$\because \rho_A = Tr_B(\rho_{AB}) \qquad Tr_B(\rho_{AB}) = \sum_{i=0}^{1} \langle i_B|\rho_{AB}|i_B\rangle$

$|i_B\rangle$ is the state of Bob's qubit, for singlet state only $|01\rangle$ and $|10\rangle$. when taking the trace, the crossing terms will cancel out.

$\therefore$

$\rho_A = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

'A.

3. Alice want to send a simple message consisting of a single bit $a \in \{0, 1\}$. She encodes by preparing a qubit in the standard basis:

$$0 \rightarrow |0\rangle$$
$$1 \rightarrow |1\rangle$$

Unfortunately, Alice's qubit preparation machine broke down and when asked to prepare the state in the from $|0\rangle$, it actually prepared

$$|\phi\rangle = \frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle .$$

Bob know this and measures the received state in the standard basis. What is the probability of obtaining the outcome 0?

3. $P(0) = |\langle 0|\psi\rangle|^2$

$P(0) = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = \frac{2}{3} \approx 66.67\%$

(not sure above this).