Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
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FACULTY OF SCIENCE, TECHNOLOGY, ENGINEERING AND MATHEMATICS

SCHOOL OF ENGINEERING

Electronic and Electrical Engineering

Engineering MAI/MSc **Semester 2, 2022**

EEP55C25 Algorithms for Quantum Computing

05/05/2022 **RDS – MAIN HALL** **09:30–11:30**

Prof. B. Basu

Instructions to Candidates:

Answer any four (4) questions.

Start the answer to each question on a new page.

Use of calculator is permitted.

Q.1 **[Total: 25 marks]**

(a) Describe the effect of the CNOT gate with respect to the following bases:

$$B_1 = \left\{|0\rangle\left(\frac{|0\rangle+|1\rangle}{2}\right), |0\rangle\left(\frac{|0\rangle-|1\rangle}{2}\right), |1\rangle\left(\frac{|0\rangle+|1\rangle}{2}\right), |1\rangle\left(\frac{|0\rangle-|1\rangle}{2}\right)\right\}$$

Express your answer using Dirac notation. **[3 marks]**

(b) Given any 1 qubit quantum gate denoted by U, a controlled U gate can be defined as a 2-qubit gate denoted by c-U which acts on the second qubit only if the first qubit is in state $|1\rangle$. Prove that c-U and c-($e^{i\theta}$ U) are not equivalent for θ not equal to an integer multiple of 2π. **[8 marks]**

(c) A photon is passed through two successive beam splitters along a straight line. Explain what happens with mathematical justification.

[7 marks]

(d) Find out the eigenvector of the X(NOT) gate with eigenvalue -1.

Show that this gate is useful for phase kickback algorithm.

[7 marks]

Q.2 **[Total: 25 marks]**

(a) Illustrate with a diagram how a state of a qubit can be represented on the surface of the Bloch sphere. Also show how a probabilistic bit can be represented. **[7 marks]**

(b) Verify that Hadamard bases in a 2D Hilbert space form a set of orthonormal bases. **[4 marks]**

(c) Consider the state $|\varphi\rangle = \left(\frac{|0\rangle+|1\rangle}{2}\right)$. Compute the quantum state $|\varphi\rangle|\varphi\rangle$.

Is it a pure state? Why/Why not? **[6 marks]**

(d) We have a blackbox U_f to compute an unknown function f: {0,1,2, …, N} → {0,1}. Suppose there are *t* solutions to f(x) = 1, with 0 < t < N and *t* known. Show that amplitude amplification algorithm finds a solution with probability at least 1 − O (t/N).

Aide memoir: After k iterations of applying Q = $U_{\psi}^{\perp}U_f$, the initial state becomes Q^k $|\psi\rangle = \cos((2k+1)\theta)|\psi_{bad}\rangle + \sin((2k+1)\theta)|\psi_{good}\rangle$. The notations have their usual meaning. **[8 marks]**

Q.3 **[Total: 25 marks]**

(a) Consider the following state in a 4-dimensional Hilbert space

$$H_A \otimes H_B, \quad |\varphi\rangle = \frac{1+\sqrt{6}}{2\sqrt{6}}\,|00\rangle + \frac{1-\sqrt{6}}{2\sqrt{6}}|01\rangle + \frac{\sqrt{2}-\sqrt{3}}{2\sqrt{6}}|10\rangle + \frac{\sqrt{2}+\sqrt{3}}{2\sqrt{6}}|11\rangle.$$

If one of the Schmidt bases is

$\left\{ \frac{1}{\sqrt{3}}\,|0\rangle + \frac{\sqrt{2}}{\sqrt{3}}\,|1\rangle, \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}\,|1\rangle \right\}$ with Schmidt coefficient ¼,

derive the other Schmidt basis and coefficient. **[14 marks]**

(b) Derive the density matrix of the following state:

$$\left\{\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, \frac{1}{2}\right), \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle, \frac{1}{2}\right)\right\}.$$

[4 marks]

(c) Verify if the following state on the 4-dimensional space

$$|\varphi\rangle = \frac{1}{2}\,|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

is entangled.

[7 marks]

Q.4 **[Total: 25 marks]**

(a) Explain with mathematical calculations how the phase ω = 0.0x can be estimated. **[8 marks]**

(b) What is a phase rotation gate? Express the gate R_4^{-1} in a matrix form. **[6+2 = 8 marks]**

(c) Compute the probabilities of accessing the states in a 2 step quantum random walk starting from origin (0,0). Compare and contrast the result with a classical 2 step random walk. What happens if the walk starts from (0,1)? **[9 marks]**

Q.5 **[Total: 25 marks]**

(a) What is the fundamental difference between quantum computing and classical computing? Explain the concept of superposition of a quantum state. **[4 marks]**

(b) A pure state can be decomposed as

$$|\psi\rangle = \sum_i \alpha_i \, |\psi_i\rangle$$

where $|\psi_i\rangle$ form a set of orthonormal bases and $|\alpha_i|^2 = p(i)$, with $p(i)$ as the probability of occurrence of state ψ_i. Show that

(i) $\alpha_i = \langle\psi_i|\psi\rangle$;

(ii) $p(i) = \langle\psi|P_i|\psi\rangle$, where P_i is an orthogonal projection operator such that $P_i P_j = 0$ for $i \neq j$

and

(iii) $|\psi_i\rangle = \frac{P_i|\psi\rangle}{\sqrt{p(i)}}$ (i.e., are equivalent states differing by a global phase).

[15 marks]

(c) Implement a circuit for basis change from the computational basis to the Bell basis

$$\beta = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \ .$$

[6 marks]