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Next Generation Networks

Traffic models

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Traffic models

- All *performance evaluation techniques must make some assumptions regarding traffic* or workload
  - Queuing models: arrival and service processes
  - Simulation: traffic generators
  - Experimental: workload generators
- **Assumptions are captured in traffic models**
  - Closed-form analytical solutions for queuing theory only available for certain (simple) traffic models

Poisson Process (1)

- Stochastic process in which events occur continuously and independently of one another
- Collection $\{N(t) : t \geq 0\}$ of random variables, where $N(t)$ is the number of events that have occurred up to time t (starting from time 0)
- The **number of events** between time a and time b is given as $N(b) - N(a)$ and has a Poisson distribution

Poisson Process (2)

- The **homogeneous** Poisson process is characterized by a rate parameter λ, such that the number of events in time interval $(t,t+\tau]$ follows a Poisson distribution with associated parameter $\lambda\tau$
- This relation is given as

$$P\left[\left(N(t+\tau)-N(t)\right)=k\right]=\frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!}, \mathrm{k}=0,1,\ldots$$

where $N(t+\tau)-N(t)=k$ is the number of events in time interval $(t,t+\tau]$

- Process is characterized by its rate parameter λ, which is the *expected* number of "events" or "arrivals" that occur per unit time

Poisson Process (3)

- In general, the rate parameter may change over time
- Such a process is called a **non-homogeneous** Poisson process
- In this case, the generalized rate function is given as $\lambda(t)$ and we can write

$$\lambda_{a,b} = \int_a^b \lambda(t)dt$$

- The number of arrivals in the time interval (a,b], given as N(b)-N(a), follows a Poisson distribution with associated parameter $\lambda_{a,b}$

$$P\left[\left(N(b)-N(a)\right)=k\right]=\frac{e^{-\lambda_{a,b}}\left(\lambda_{a,b}\right)^k}{k!}, \mathrm{k}=0,1,\ldots$$

Queuing Theory – Kendalls' Notation (revisited)

- G/G/n/k queue
  - G: General - can be any distribution
  - First letter: Arrival process
    - M: exponential (memoryless) interarrival times - Poisson arrival process

  Second letter: Service times distribution
    - M: exponential, D: Deterministic
  - Third letter: Number of servers
  - Fourth letter: Number of customers in system (including both number in queue and number being served)

Burstiness (1)

- A **bursty** source generates traffic in random clusters
- Deterministic traffic is not bursty
- Poisson process (in continuous time) and Bernoulli process (in discrete time) are bursty as single processes
- Burstiness of Poisson and Bernoulli processes removed for aggregated traffic sources
- Other traffic sources exhibit more burstiness

Burstiness (2)

A *self-similar* object is exactly or approximately similar to a part of itself (i.e., the whole has the same shape as one or more of the parts). Many objects in the real world, e.g., coastlines, are *statistically self-similar*: they show the same statistical properties at many scales.

- **Self-similar** traffic is not smoothed through aggregation
  - Maintains its burstiness at any time scale

- Burstiness is important
  - Peak traffic demands on buffer resources can lead to overflow and lost traffic
    - ✓ Peak demands may create quality of service (QoS) problems in a network
    - ✓ Need to characterize burstiness for traffic sources in a QoS environment

- Taking into account the "peak busy hours" allows to deal with long-term burstiness

Self-similar phenomena

"The unifying concept underlying fractals, chaos and power laws is self-similarity. Self-similarity, or **invariance against changes in scale or size**, is an attribute of many laws of nature and innumerable phenomena in the world around us. Self-similarity is, in fact, one of the decisive symmetries that shape our universe and our efforts to comprehend it"

-- Manfred Schroeder

Semester 2 module "Self-Organising Technological Networks"

Basic concepts

- A phenomenon that is self-similar looks or behaves the same when viewed at different degrees of "magnification" or different scales on a dimension (time or space)

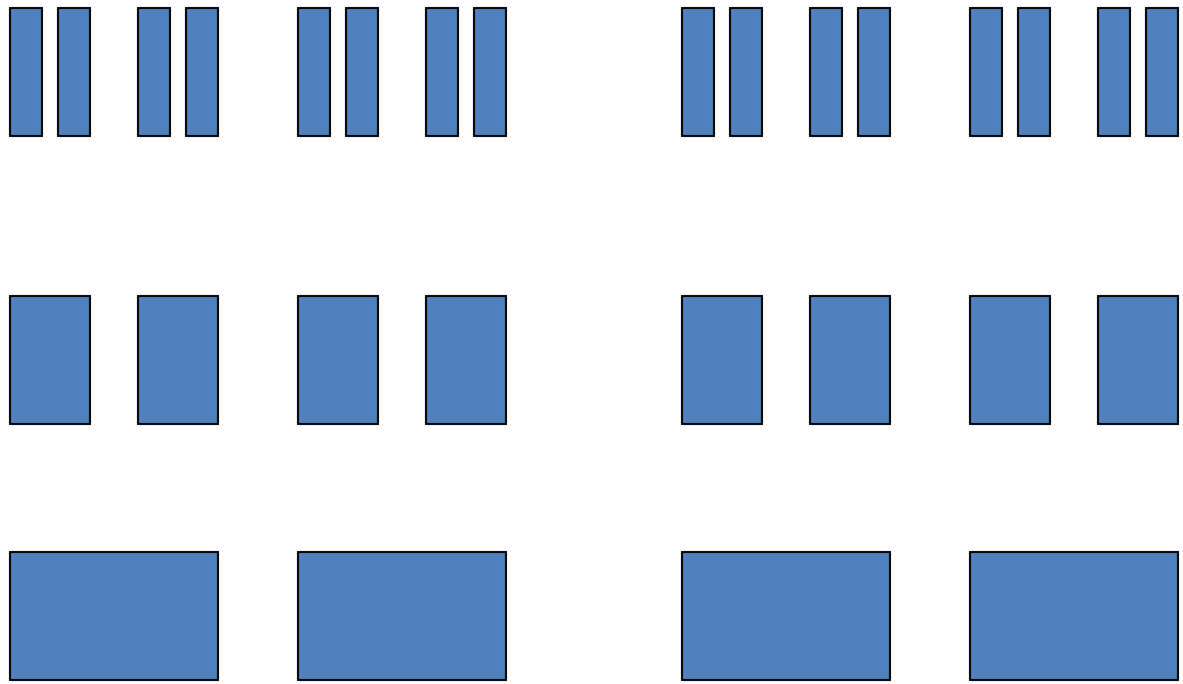
- We are concerned with time series and stochastic processes that exhibit **self-similarity with respect to time**

Visual example: aggregated time series

A time series

Aggregated time series

Aggregated again



Properties

1. Self-similar phenomena have structure at arbitrarily small scales

2. A self-similar structure contains smaller replicas of itself at all scales
   - for real phenomena, properties do not hold indefinitely; however, they hold over a large range of scales

Relevance

- If network traffic is self-similar, there is significant amount of clustering at all time scales, requiring more buffering

- Contrast with Poisson traffic, where clustering occurs in the short term but traffic smoothes out over the long term

Some Memories of Statistics (1)

- A random variable X has p.d.f. f_X if:
  - $$P[a \leq X \leq b] = \int_a^b f_X(x)dx$$
  - f_X is non-negative for all x
  - $$\int_{-\infty}^{\infty} f_X(x)dx = 1$$
- Cumulative distribution function

$$F_X(x) = \int_{-\infty}^{x} f_X(u)du = P[X \leq x]$$

$$f_X(x) = \frac{d}{dx} F_X(x)$$

Some Memories of Statistics (2)

- Expected value (mean; continuous case)

$$\mu = E[X] = \int_{-\infty}^{\infty} x f_X(x)dx$$

- Variance

$$\sigma^2 = Var[X] = E[(X-\mu)^2] = E[X^2] - (E[X])^2$$

- Auto-correlation

$$R_{XX}(t,s) = \frac{E[(X_t-\mu_t)(X_s-\mu_s)]}{\sigma_t \sigma_s}$$

Statistical self-similarity definition

- A stochastic process X(t) is statistically **self-similar** with parameter H ($0.5 \le H \le 1$) if for any real $a > 0$:

$$(i) \quad E[X(t)] = \frac{E[X(at)]}{a^H} \quad Mean$$

$$(ii) \; Var[X(t)] = \frac{Var[X(at)]}{a^{2H}} \quad Variance$$

$$(iii) \; R_{XX}(t,s) = \frac{R_{XX}(at,as)}{a^{2H}} \quad Autocorrelation$$

Hurst parameter

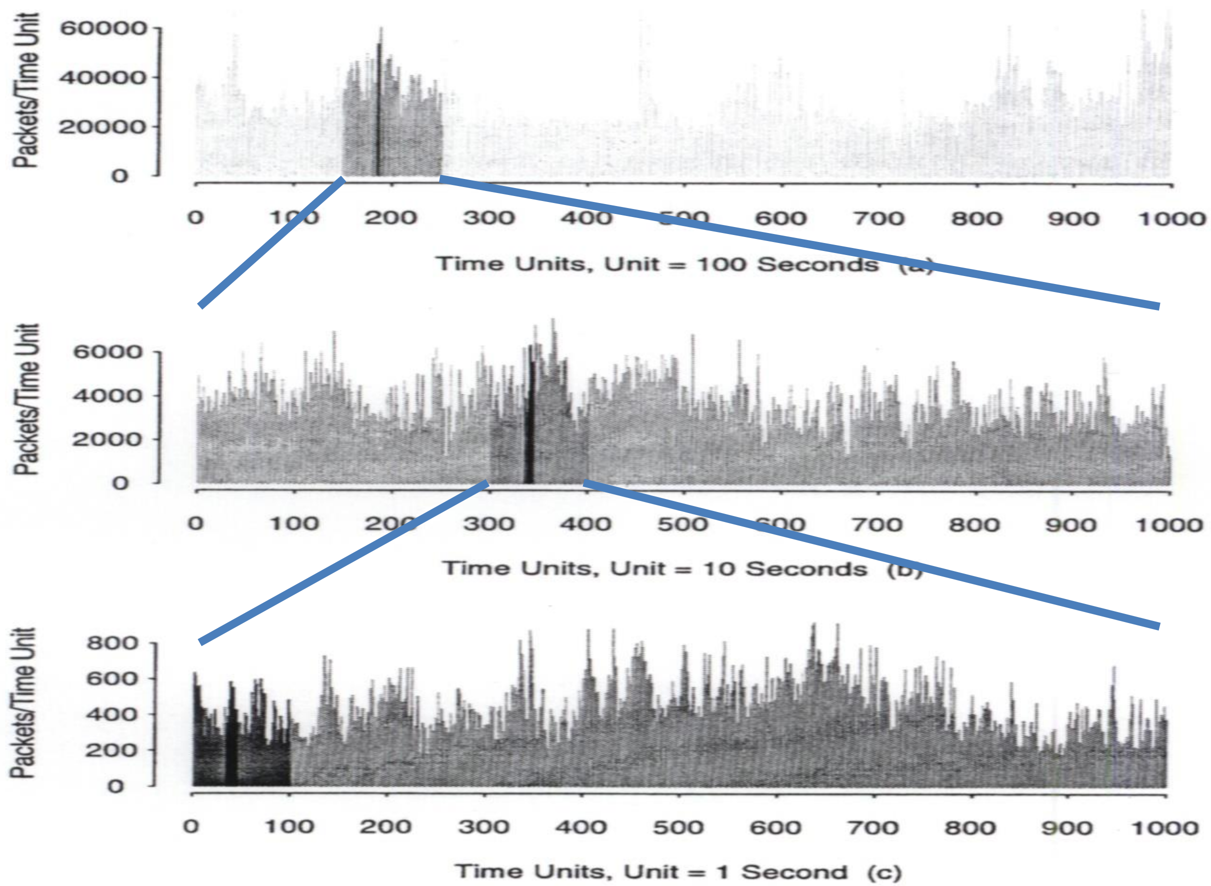
- H is the **Hurst Parameter**, a key measure of self-similarity
- H=0.5 indicates the absence of self-similarity
- H closer to 1 indicates a higher degree of persistence of long-range dependence (self-similarity)

Self-similarity and network traffic

- It turns out data traffic is well-modeled as a *self-similar* process in many practical networking situations, including:
    - Ethernet traffic
    - WWW traffic
    - TCP, FTP traffic
    - Variable bit rate (VBR) video

- Straightforward **queuing analysis** using Poisson traffic assumptions **inadequate** to model this type of traffic

Ethernet traffic (1)



Source: W. E. Leland et al., "On the Self-Similar Nature of Ethernet Traffic," ACM SIGComm'93.

Ethernet traffic (2)

- Authors estimated Hurst parameter of $H = 0.9$
  - The higher the load, the higher H

- Aggregating several streams does not remove self-similarity

- We know that Poisson is not a good model in this case. But what is?
  - Superposition of many Pareto-like ON/OFF sources

Pareto distribution

Source: W. Stallings, High Speed Networks: TCP/IP and ATM Design Principles, Prentice-Hall, 1998.

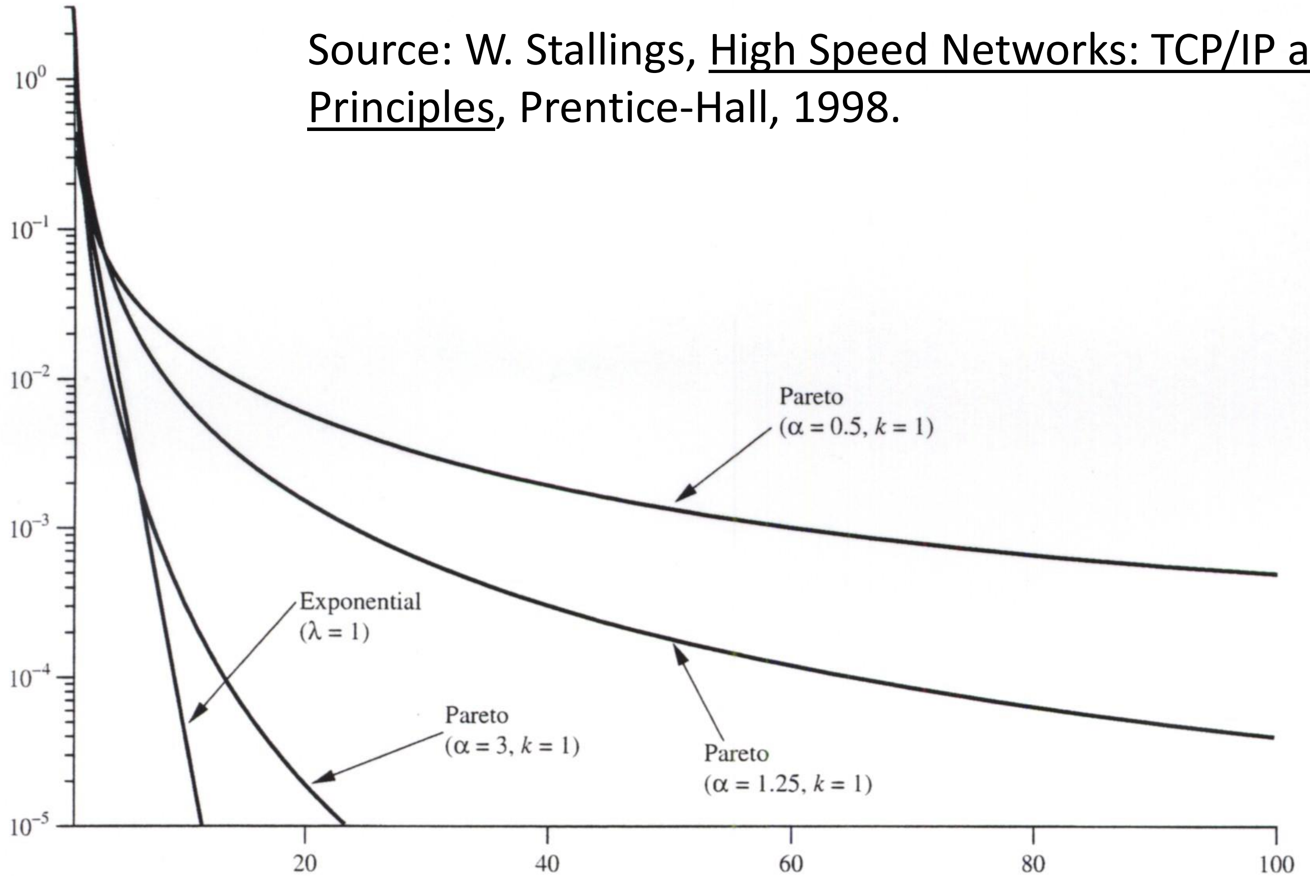


Figure 8.4 Pareto and exponential probability density functions.

Performance implications

- If actual data is more bursty than originally modeled, then the **original models underestimate average delay and blocking**
  - Self-similarity leads to higher delays and higher blocking probabilities (since burstiness increases the stress on buffers)

- In other words, self-similarity leads to a poor fit with traditional queuing theory results

Mean waiting time results

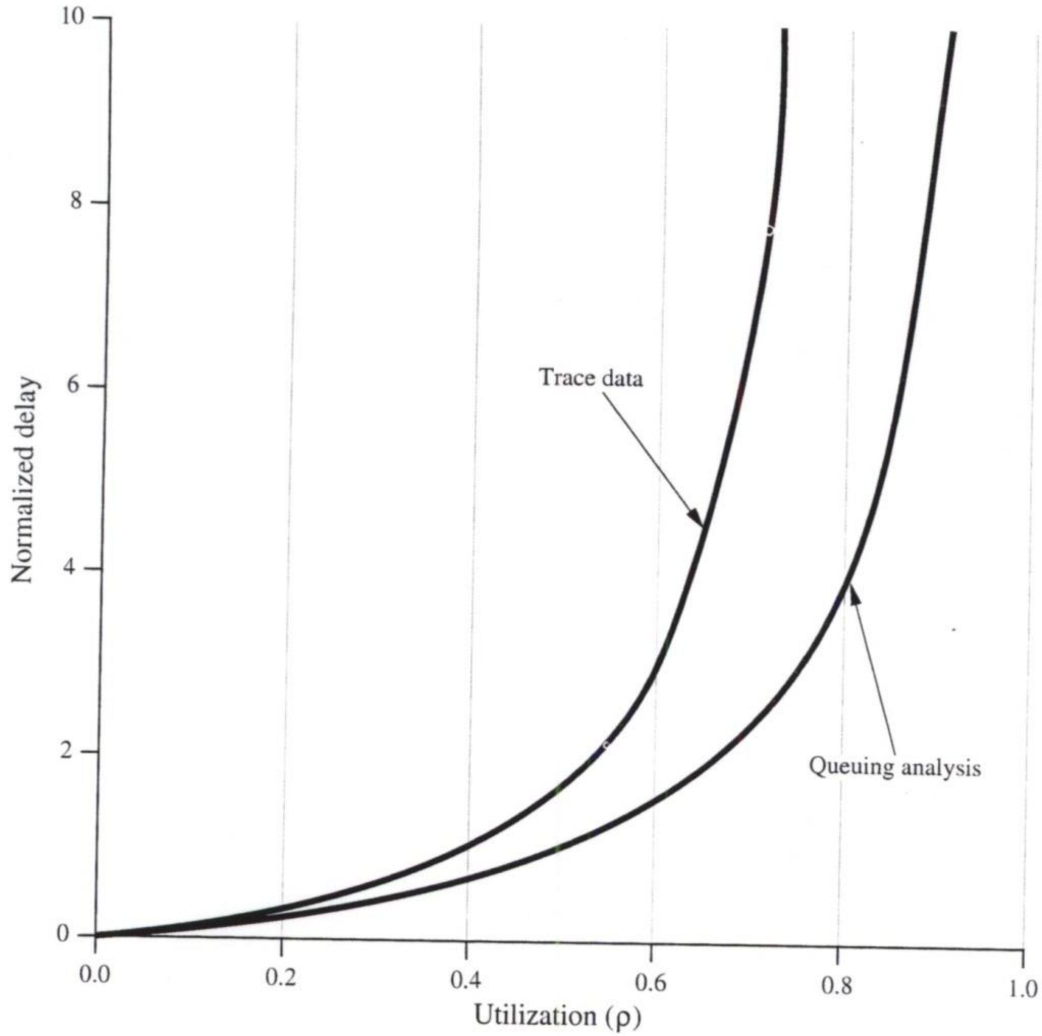


Figure 8.6 Mean waiting time results.

Source: W. Stallings, High Speed Networks: TCP/IP and ATM Design Principles, Prentice-Hall, 1998.

Queue occupancy results

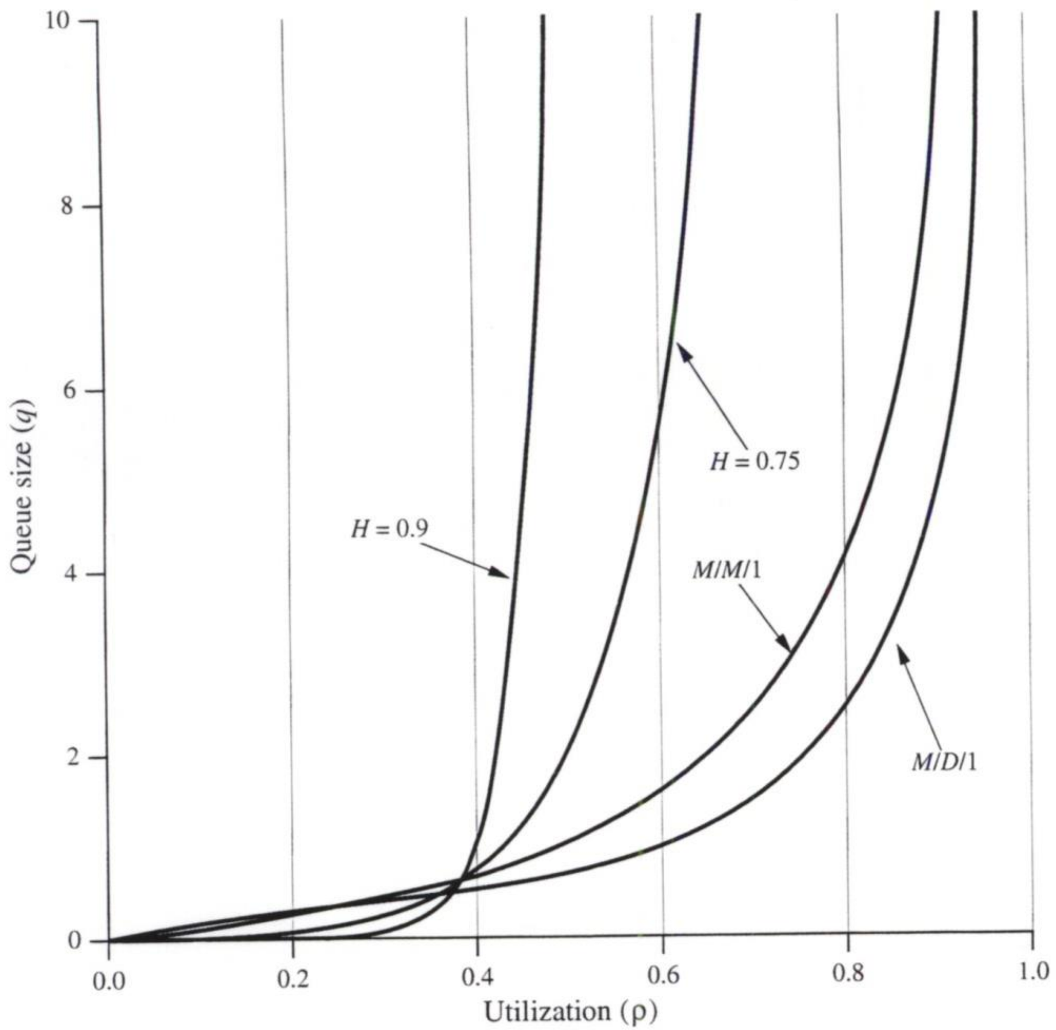


Figure 8.7 Self-similar storage model.

Source: W. Stallings, High Speed Networks: TCP/IP and ATM Design Principles, Prentice-Hall, 1998.

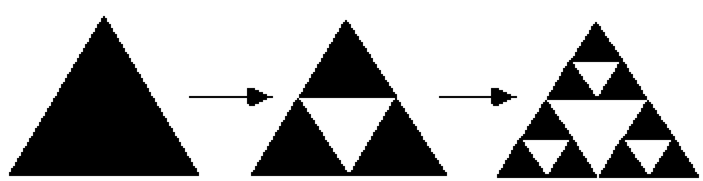
Problem

The *fractal dimension* of a self-similar set is defined as *log(C)/log(b)*, where the set is covered by *C* copies of itself, each scaled by a factor of 1/*b*.

Calculate the fractal dimension of a certain wireless traffic pattern, in case it can be modeled as:

a) the Koch's Snowflake

b) the Sierpinski's Triangle



c) a segment, a square, and a cube.

a) For the Koch's Snowflake, $C=4$, $b=3$, thus the fractal dimension is $\log(4)/\log(3) \approx 1.26$

b) For the Sierpinski's Triangle, $C=3$, $b=2$, thus the fractal dimension is $\log(3)/\log(2) \approx 1.59$

c) In general a segment can be divided into k copies of itself, each scaled by $1/k$, thus $\log(k)/\log(k)=1$.
A square can be divided into k^2 copies of itself, each scaled by $1/k$, thus $\log(k^2)/\log(k)=2$.
A cube can be divided into k^3 copies of itself, each scaled by $1/k$, thus $\log(k^3)/\log(k)=3$.

Problem

Let *X(t)* be a stochastic process representing the downlink throughput of a certain base Station. At time instants *t'=ct,* where *c>0* an arbitrary positive real number, we have that:

$$E[X(t)] = \frac{\sqrt{c}}{c} E[X(t')]$$

$$Var[X(t)] = \frac{Var[X(t')]}{c}$$

Furthermore, considering also another arbitrary time instant *s*, at time instants *t'=ct*, *s'=cs*, we have that:

$$R_{XX}(t,s) = \frac{R_{XX}(t',s')}{c}$$

(a) Is the base station throughput self-similar? Why?

Now assume that another base station throughput process Y(t) is such that:

$$E[Y(t)] = \frac{E[Y(t')]}{c}$$

$$Var[Y(t)] = \frac{Var[Y(t')]}{c^2}$$

$$R_{YY}(t,s) = \frac{R_{YY}(t',s')}{c^2}$$

(b) Is the process self-similar? Why?

(c) Explain what conclusions can be drawn in terms of self-similarity for a process *Z(t)* such that:

$$E[Z(t)] = \frac{E[Z(t')]}{c}$$

(a) The base station throughput has a Hurst parameter $H=0.5$, therefore is non self-similar.

(b) The base station throughput has a Hurst parameter $H=1$, therefore is completely self-similar.

(c) We cannot conclude anything for what concerns the self-similarity of $Z(t)$, as we only know that the self-similarity mean relation holds, but we don't know whether the relations in terms of variance and autocorrelation hold.