Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Information Theoretical Aspects of Complex Systems

Lecture 2.10

EEU45C09 / EEP55C09
Self Organising Technological Networks

CONNECT
Networks of the Future

Nicola Marchetti
nicola.marchetti@tcd.ie

01

Application of Unit 2 to Cellular Networks

Approach

- We take a simple approach of modeling a self-organizing network using *Cellular Automata* (CA), and explore the resulting behavior to determine whether it can be considered a complex entity

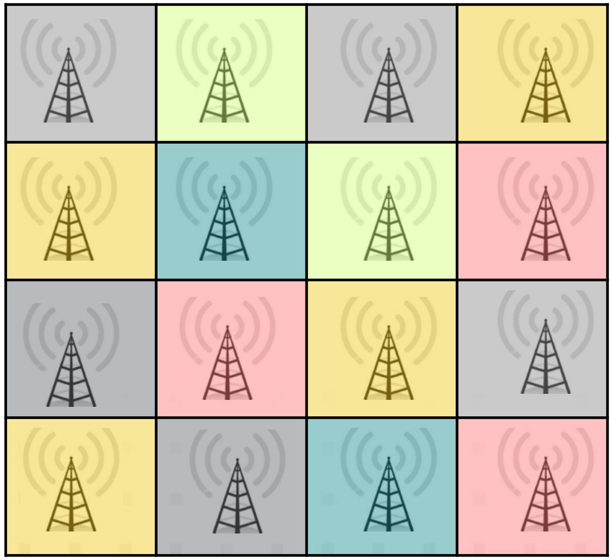
- We adopt a *measure of complexity related to organizational aspects*, i.e. the difficulty of describing organizational structure

- The complex behavior exhibited by a collection of self-organizing wireless networks has attractive properties in terms of **robustness to changes** in the environment

Channel Assignment and Cellular Automata

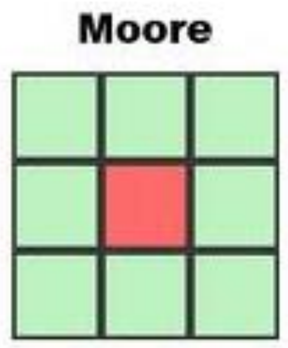
- A CA is a discrete model that consists of a finite dimensional lattice of cells, each in one of a finite number of states

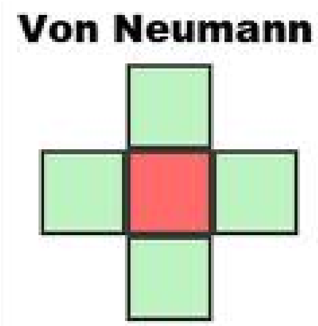
- Each cell represents a wireless node, and the state of a cell is the channel selected by the node



Channel Assignment and Cellular Automata

- Each cell interacts with a subset of cells, called *neighbourhood*





- In our model the neighbourhood of a cell is the set of nodes that interfere with that cell when using the same channel

Channel Assignment Algorithm

- Time is slotted and all cells can measure the interference caused by the transmission of adjacent nodes on all channels
- A cell is active at time t if the cell detected a variation on the measured interference on any channel in the previous time slot
- Only active cells at time t can update their state (channel)
- At the beginning of each time slot, each active cell stops transmitting and initialises a random timer (with period less than the duration of the time slot)
- When the timer expires, each active cell updates its state, i.e. it selects a channel and starts transmission

Moore Neighbourhood: Active cells evolution

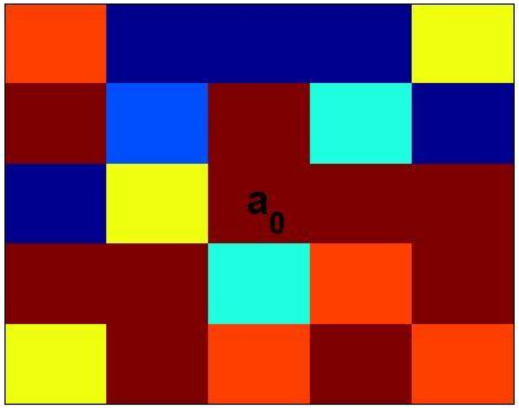
		a_3	a_3	a_3	a_3	a_3	a_3	a_3		
		a_3	a_2	a_2	a_2	a_2	a_2	a_3		
		a_3	a_2	a_1	a_1	a_1	a_2	a_3		
		a_3	a_2	a_1	a_0	a_1	a_2	a_3		
		a_3	a_2	a_1	a_1	a_1	a_2	a_3		
		a_3	a_2	a_2	a_2	a_2	a_2	a_3		
		a_3	a_3	a_3	a_3	a_3	a_3	a_3		

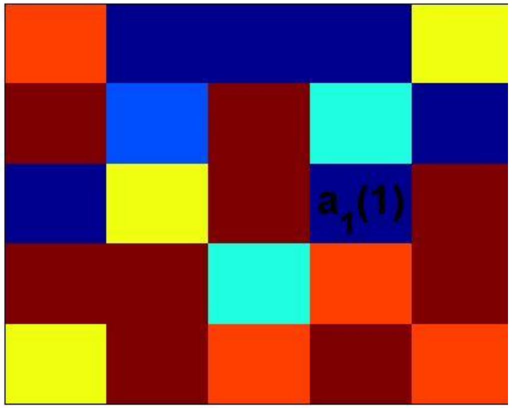
- An active cell selects a channel randomly among the channels which have not been selected by the subset S of its neighbours that changed state in the previous and current time slot
- After updating its state, an active cell will remain inactive for the two subsequent time slots

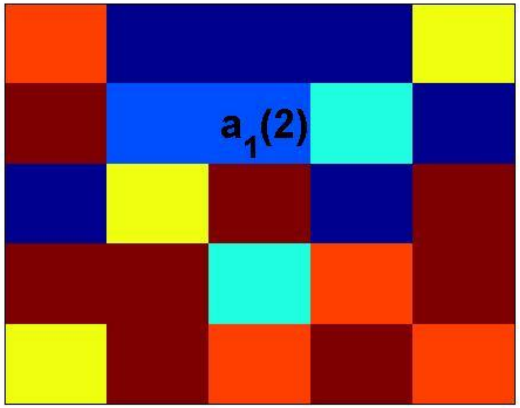
Example

t = 0

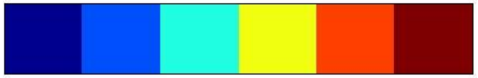
t = 1



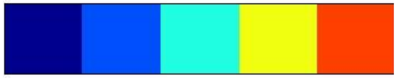




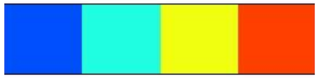
Channels available to cell $a_0(0)$



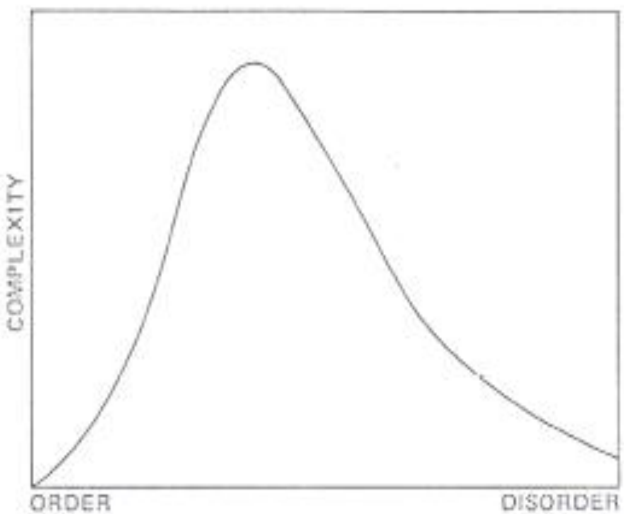
Channels available to cell $a_1(1)$



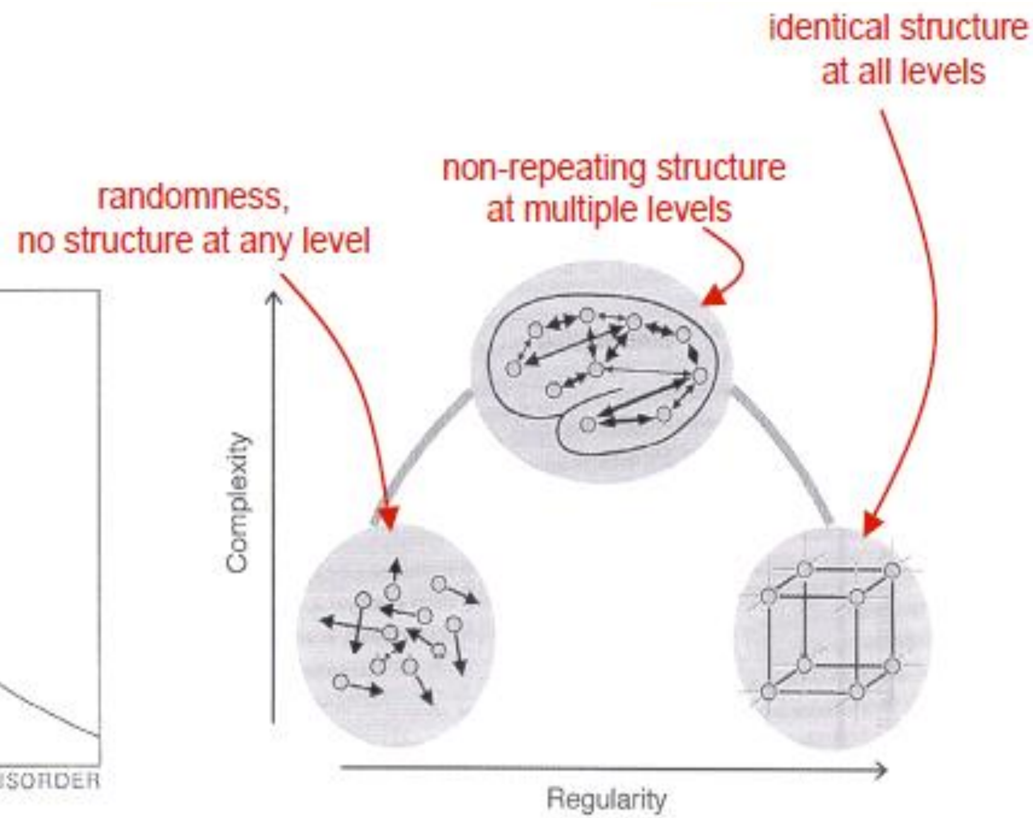
Channels available to cell $a_1(2)$



Order & Complexity



Reference:
B.A. Huberman and T.Hogg (1986) Physica 22D, 376.

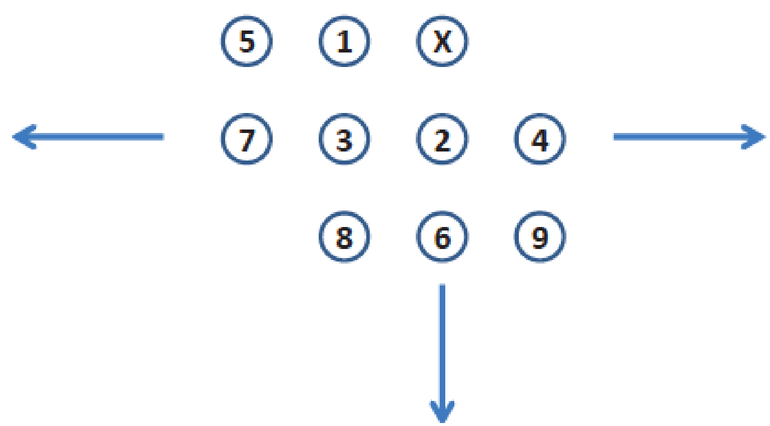


Reference:
G. Tononi, G.M. Edelman, O. Sporns (1998) TICS 2, 474.

Complexity

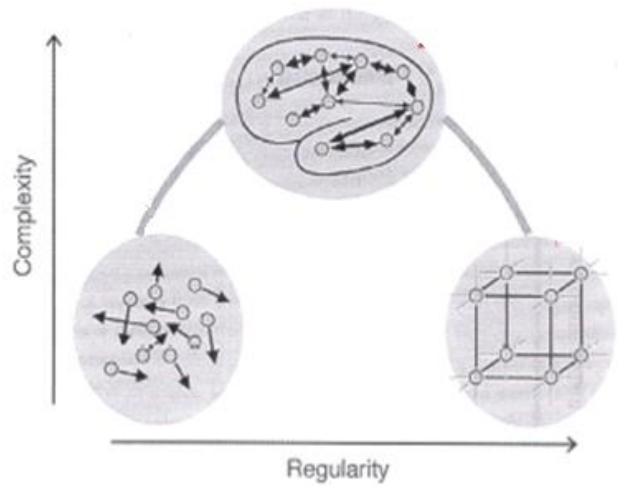
[1] D. P. Feldman and J. P. Crutchfield, “Structural information in two dimensional patterns: Entropy convergence and excess entropy,” Physical Review E, vol. 67, no. 5.

- We use convergence *excess entropy* E_C [1] to measure complexity
  - Obtained by considering how entropy density estimates converge to their asymptotic value h
- In two dimension the entropy density h is: $\lim_{M \to \infty} h(M)$
  - h(M) is the entropy of the target cell X conditioned on cells 1,2,…,M.
  - h(M’) ≤ h(M’’), for all M’>M’’
- Cell numbers show how cells are added to the template (Euclidean distance)

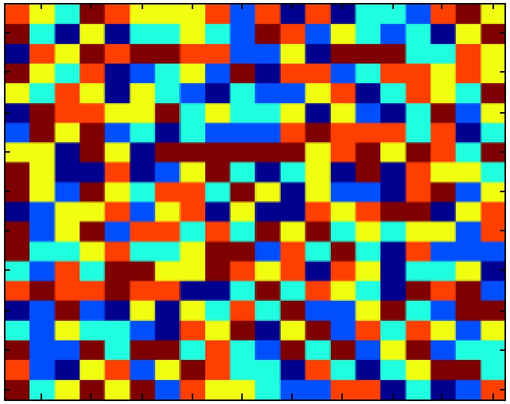


$$E_C = \sum_{M=1}^{\infty} (h(M) - h)$$

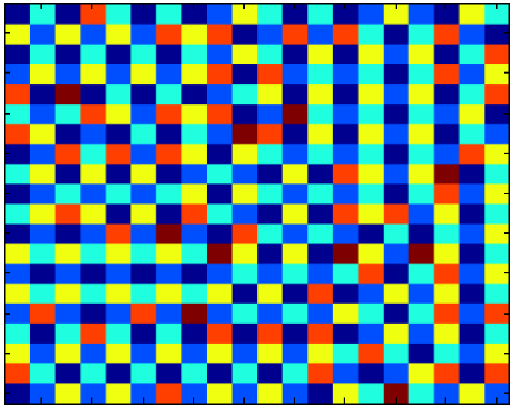
Complexity Analysis



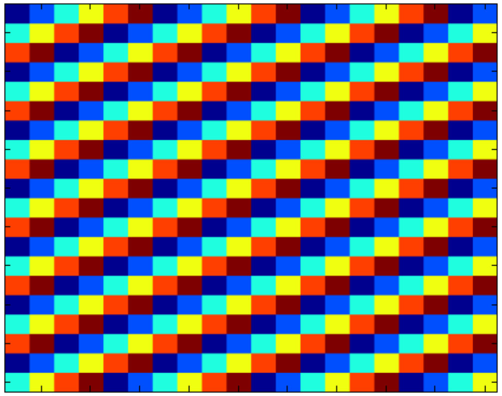
h=2.58, E_C=0



h=1.29, E_C=2.04

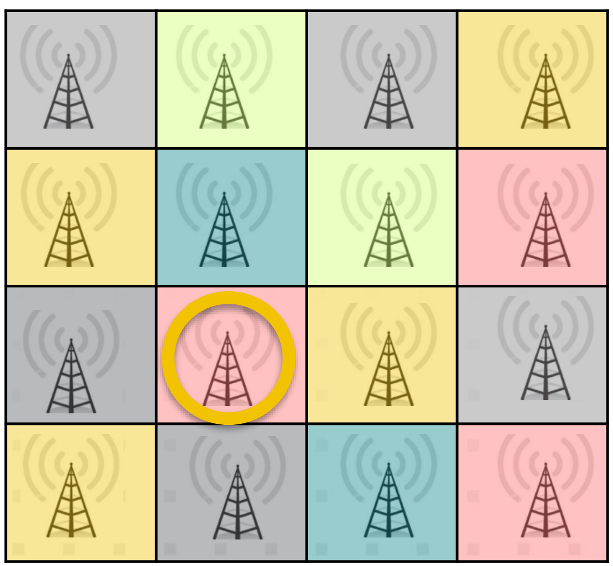


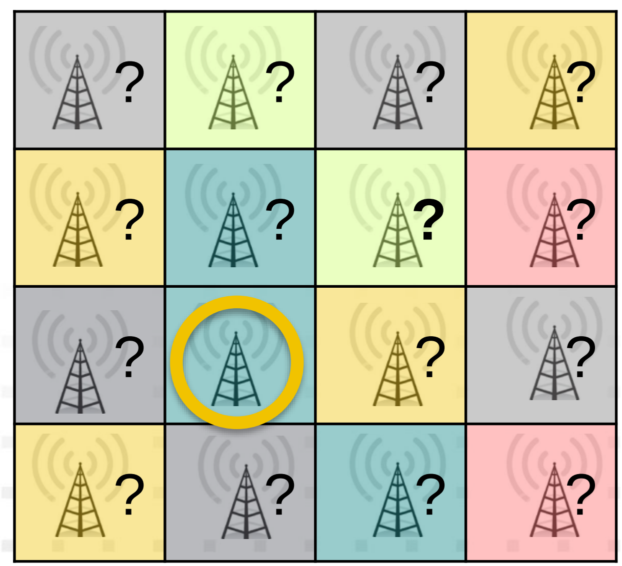
h=0, E_C=0



Complexity and robustness

- Minimum number of changes to return to an interference free allocation, after one cell in the lattice is required to change channel of operation





Complexity and robustness

- Only cells in a neighborhood $V_r(n)$ of radius r of the cell n are allowed to change channel
- 10^2 instances of the self-organising frequency allocation algorithm using $10^2 \times 10^2$ lattices
- A locally perturbed channel allocation resulting from a central frequency planner requires a larger number of changes to return to an interference-free allocation, than in the case of self-organised channel assignments

Channel Allocation	Neighbourhood radius	$Prob$ (no solution)	$Prob$ $(c = 0)$	$Prob$ $(1 \leq c \leq 4)$	$Prob$ $(c \geq 5)$
Self-organised $(\tilde{S}_{SOA})$	$r = 1$	0.21	0.06	0.67	0.06
	$r = 2$	0.0	0.06	0.86	0.08
Centralized planner $(\tilde{S}_{CA})$	$r = 1$	0.4	0.0	0.4	0.2
	$r = 2$	0.0	0.0	0.6	0.4

02

Application of Unit 2 to Wireless Sensor Networks

WSN

- Wireless Sensor Networks (WSN) represent large deployments of unattended sensors, which are disposable and expected to last until their energy drains

- Clustering partitions a network of nodes into a number of smaller groups
  – Clustering algorithms have a great influence on scalability, energy efficiency, load balancing, delay reduction, etc.

- Understanding the *organization and communication characteristics of different clustering algorithms* allows us to comprehend which aspects of a specific implementation lead to certain characteristics, e.g. scalability and energy efficiency

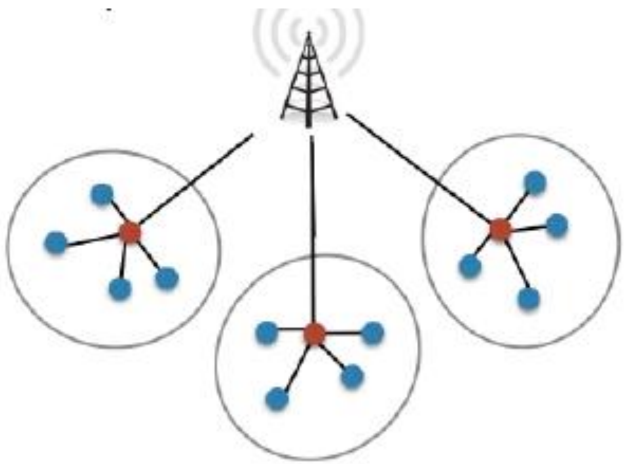
Approach

Quantify the amount of *uncertainty of interaction* that exists within smaller subparts of the system, as compared to the uncertainty in the whole system

Clustering algorithms

LEACH algorithm

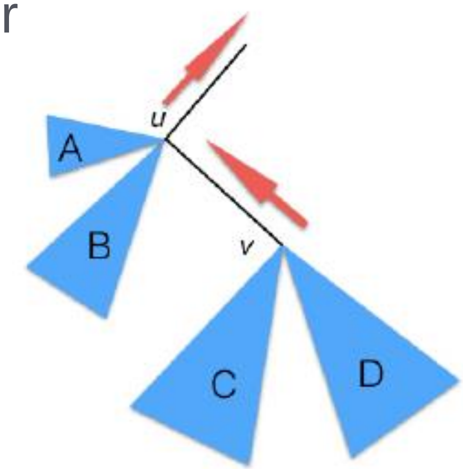
- Ordinary nodes transfer sensing information to the cluster-heads, which forward this information to the base station



Heinzelman et al, “An application-specific protocol architecture for wireless microsensor networks,” *IEEE Trans on Wireless Communications*, 2002.

HCC algorithm

- Each node discovers its subtree size and forwards this information upstream to its parent
- If the subtree size is big enough (predefined cluster size), the subtree becomes a cluster

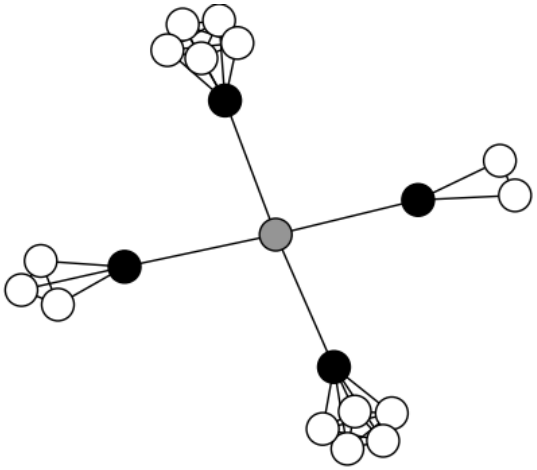


Banerjee et al, “A Clustering Scheme for Hierarchical Control in Wireless Networks,” *IEEE INFOCOM*, 2001.

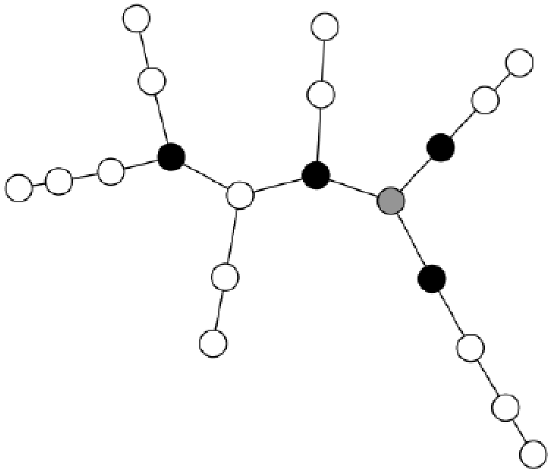
Functional topology

- A functional topology is a graph that shows how parts of the system interact with each other, in order to **execute a network function**

- Nodes in this graph represent functional entities involved in the implementation of the network function (in this case clustering in WSN), and links represent the interactions between them



LEACH



HCC

Functional complexity

- Based on Shannon entropy $H(x_n)$
- Λ_k^j is the k-th subgraph with j nodes
- $p_r\,(x_n=1)$ is the probability that any given interaction in the course of a function operation involves node n for a given scale r
- For the clustering functions, $p_r\,(x_n = 1) = i_r^n / j$, where i_r^n is the number of neighbors of node n for a given scale r, and j is the number of nodes for the given subgraph

$$C_F = \frac{1}{R-1}\sum_{r=1}^{R-1}\sum_{j=1+r}^{N}\left|\langle I_r(\Lambda^j)\rangle - \frac{r+1-j}{r+1-N} I_r(\Lambda^N)\right|$$

$$I_r(\Lambda_k^j) = \sum_{n\in\Lambda_k^j} H(x_n)$$

Functional complexity

- $I_r(\Lambda_k^j)$ is the amount of information of the k-th subgraph with j nodes

- The functional complexity compares the uncertainty of interactions for a smaller subset ($\langle I_r(\Lambda^j)\rangle$) to the uncertainty which is expected from the calculation performed on the whole system ($I_r(\Lambda^N)$)

- A non-complex model assumes that **every part always contributes the same amount of information**

$$C_F = \frac{1}{R-1}\sum_{r=1}^{R-1}\sum_{j=1+r}^{N}\left|\langle I_r(\Lambda^j)\rangle - \frac{r+1-j}{r+1-N}I_r(\Lambda^N)\right|$$

$$I_r(\Lambda_k^j) = \sum_{n\in\Lambda_k^j} H(x_n)$$

Symbol	Meaning
N	total number of nodes in the functional topology
j	subgraph size - number of nodes in the subgraph
r	scale size
R	maximum scale size, which is defined as the longest shortest path in the whole functional topology
$H(x_n)$	entropy of node n which indicates the uncertainty of interactions of node n in the operation of a network function
Λ_k^j	k[th] subgraph with j nodes
$I_r(\Lambda^N)$	the total amount of information of the subgraph with N nodes for scale r
$\langle I_r(\Lambda^j)\rangle$	the average amount of information over all sub-graphs with the size j

Functional complexity

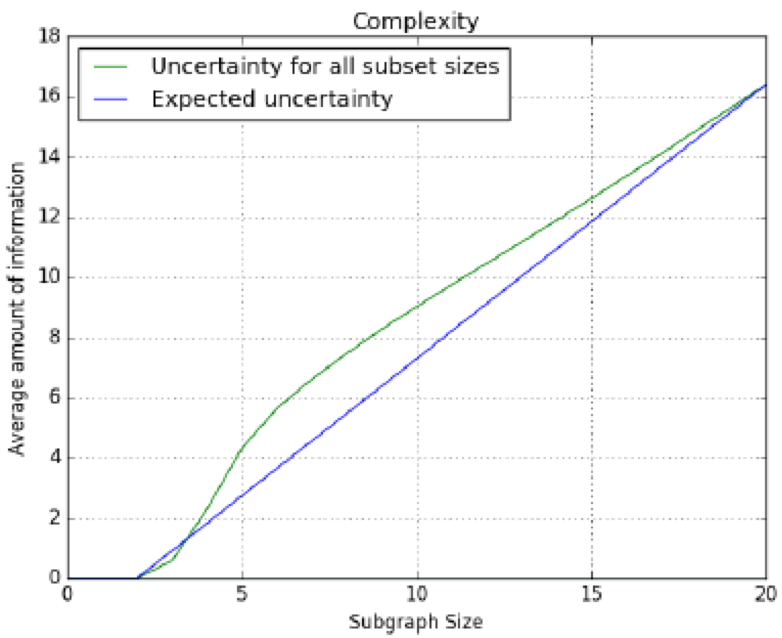
- $H(x_n)$ reaches its maximum if the probability of interaction with node n is $p_r(x_n = 1) = 0.5$
  - Zero entropy for a node indicates that this node functionally represents a hub or a disconnected node

- As the distribution of links among nodes for a sparse graph is almost uniform, a sparse graph results in high values of $H(x_n)$
  - High values of $H(x_n)$ result in high values of $<I_r(\Lambda_k^j)>$

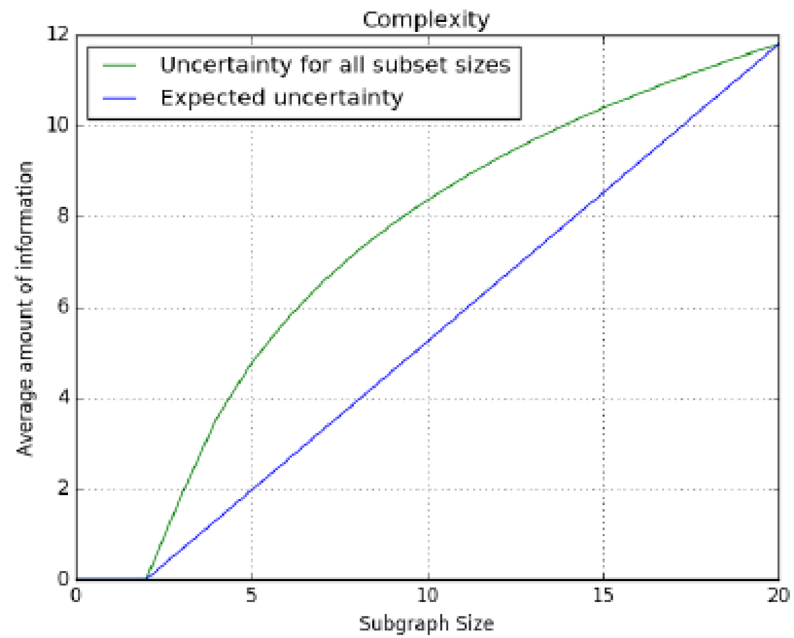
- Functional complexity is high for a sparse graph, with uniformly distributed links among nodes for subgraphs with the size $j < N$

- The functional complexity is zero for a fully connected and for a disconnected graph

Functional complexity



LEACH (20 nodes, 4 clusters)



HCC (20 nodes, 4 clusters)

The functional complexity is higher if the green and the blue curves are more far apart

Scalability

- **Scalability** is the capability of the network to adapt to: new nodes joining the network, existing nodes leaving the network, and other nodes migrating from one cluster to another
- We estimate it as the average number of messages sent when a new node joins the network
  - If the average number of messages increases, then the scalability of the network decreases

Energy efficiency

- We calculate the **energy efficiency** of a clustering implementation as the ratio between the average number of intra-cluster connections and the number of links between the base station and each cluster-head in the functional topology
  - If the ratio increases the energy efficiency increases

Complexity, scalability and energy efficiency (LEACH)

- The functional topology has dense local connections (intra-cluster connections), whereas the inter-cluster connections are sparse

- The **dense intra-cluster connections** result in **low scalability** of the algorithm, due to the need of interacting with all nodes in the cluster

#of clusters	3	4	5	6	16	19
C_F	14.35	19.24	22.69	25.55	32.4	31.85
Energy efficiency	1.91	1.08	0.72	0.51	0.07	0.05
Avg. #msg. if node joins	5.33	3.75	2.8	2.16	1	1

Complexity, scalability and energy efficiency (HCC)

- The functional topology has sparse intra and inter cluster connections
- A **sparse connectivity** pattern indicates weak dependencies which result in **high scalability**
  - In order to add a node to the network, the new node needs to establish a connection (send a message) to one of the nodes in the topology and to declare this node as its parent
- For HCC, C_F= 38.31, which is high compared to the functional complexity of the LEACH algorithm, i.e., 19.24 (for the case of 20 nodes divided into 4 clusters)
- For HCC, energy efficiency is 0.61, which is low compared to the LEACH algorithm (which energy efficiency is 1.08) for the same number of nodes which are again divided into four clusters

Summary

- We provided a measurement of the **deviation** of the complex behavior compared to a linear (non-complex) system
- In order to capture this deviation, we analyzed the joint effort of system **sub-parts** which are represented as subgraphs of the functional topology
- **Functional complexity** applied to two clustering algorithms in WSNs **positively correlates with scalability, and negatively with energy efficiency**

Relevant works

1. M. Dzaferagic, N. Kaminski, N. McBride, I. Macaluso, N. Marchetti, “A Functional Complexity Framework for the Analysis of Telecommunication Networks”, *Oxford Academic Journal of Complex Networks*, vol. 6, no. 6, pp. 971-988, Dec 2018

2. K. Pattanayak, A. Chatterjee, M. Dzaferagic, S. Das, N. Marchetti, “A Functional Complexity Framework for Dynamic Resource Allocation in VANETs”, *International Wireless Communications and Mobile Computing Conference (IWCMC)*, Jun 2018

3. M. Dzaferagic, N. Kaminski, I. Macaluso, N. Marchetti, “Relation between Functional Complexity, Scalability and Energy Efficiency in WSNs”, *International Wireless Communications and Mobile Computing Conference (IWCMC)*, Jun 2017

4. I. Macaluso, C. Galiotto, N. Marchetti, L. Doyle, “A Complex Systems Science Perspective on Cognitive Networks”, *Springer Journal of Systems Science and Complexity*, vol. 29, no. 4, pp. 1034–1056, Aug 2016

Ackowledgement

- Dr Irene Macaluso, Cable Television Laboratories Inc. (USA)
- Dr Merim Dzaferagic, Trinity College Dublin