Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Information Theoretical Aspects of Complex Systems

Lecture 2.01

EEU45C09 / EEP55C09
Self Organising Technological Networks

CONNECT
Networks of the Future

Nicola Marchetti
nicola.marchetti@tcd.ie

The Quark and the Jaguar

- “Although [complex systems] differ widely in their physical attributes, they resemble one another in the way they handle information. That common feature is perhaps the best starting point for exploring how they operate.”

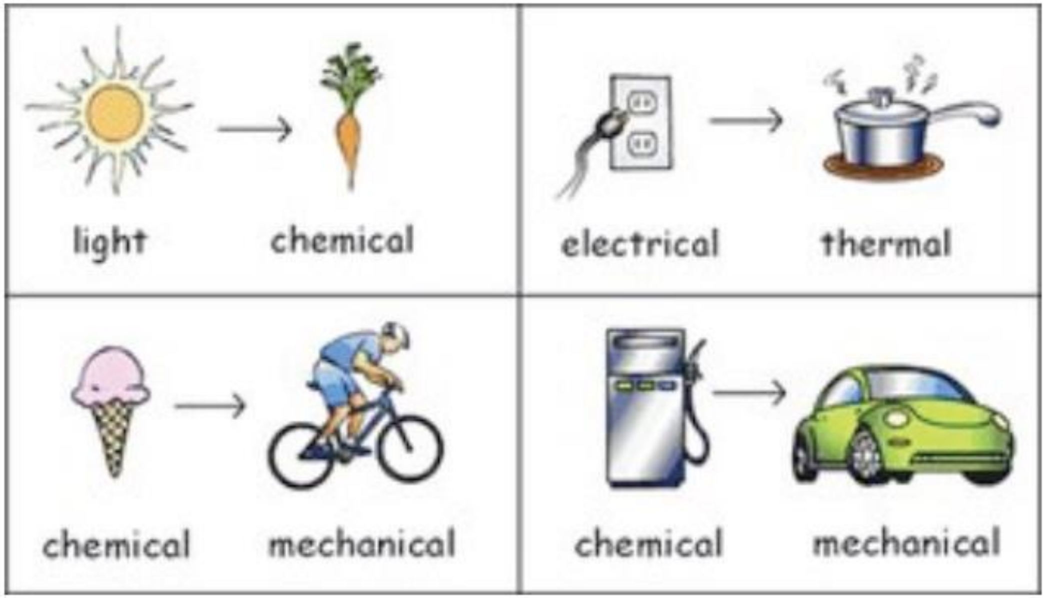
Murray Gell-Mann, “The Quark and the Jaguar”, 1995

The First Law of Thermodynamics

- First law of thermodynamics:
  - *In an isolated system, energy is conserved*

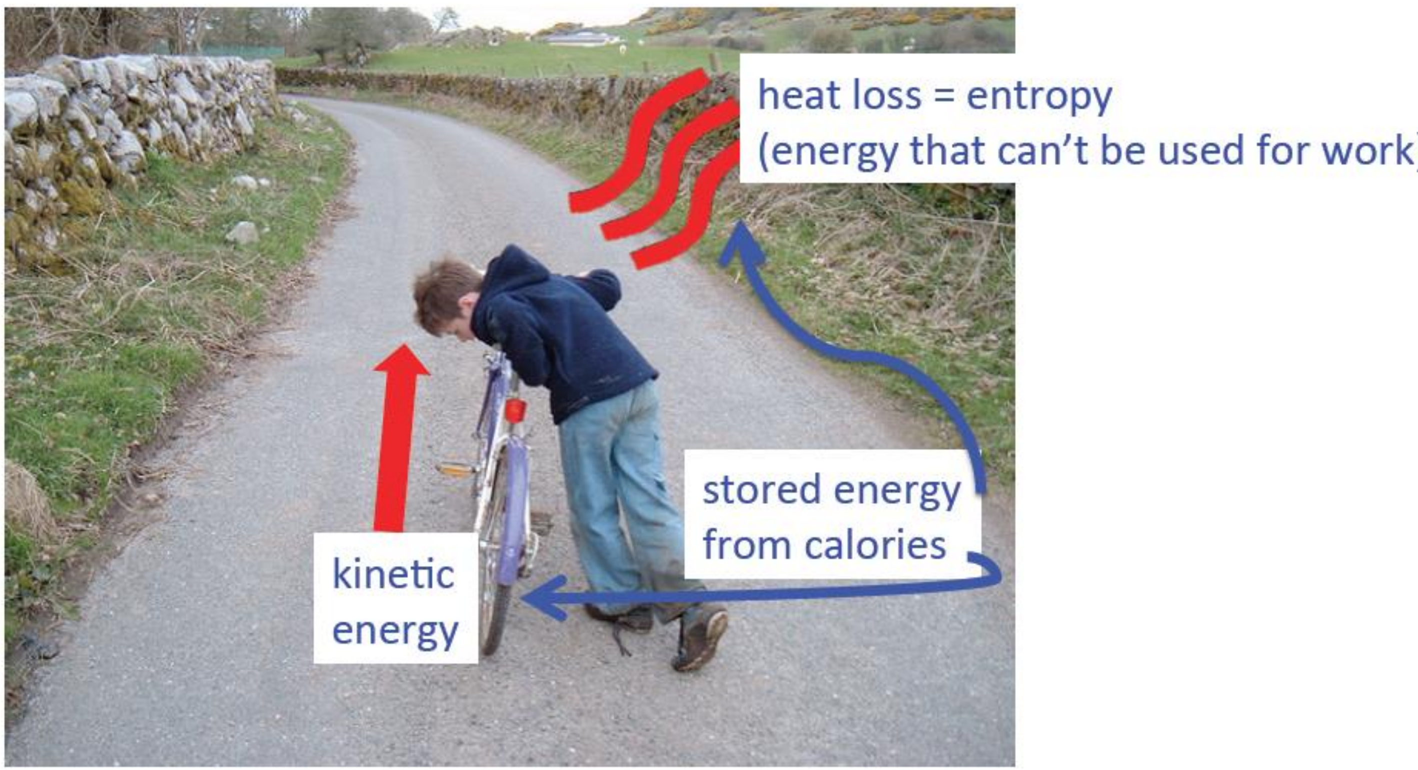
- Energy: A system's potential to do "work"

- Energy can be transformed from one kind into another:



The Second Law of Thermodynamics

- Second law of thermodynamics:
  - *In an isolated system, entropy always increases until it reaches a maximum value*



Third Law of Thermodynamics

- The third law of thermodynamics is sometimes stated as follows, regarding the properties of systems in equilibrium at absolute zero temperature:
  - *The entropy of a perfect crystal at absolute zero is exactly equal to zero*

- A classical formulation by Nernst (actually a consequence of the Third Law) is:
  - It is impossible for any process, no matter how idealized, to reduce the entropy of a system to its absolute-zero value in a finite number of operations

Implications of the Second Law

- Systems are naturally disordered → They cannot become organized without the input of work

- Perpetual motion machines are not possible

- Time has a direction: the direction of increasing entropy

A Bit of Information

- ***Szilard***: A bit of information is the amount of information needed to answer any "yes/no" question

- The field of Computer Science adopted this terminology for computer memory

Leo Szilard, 1898-1964

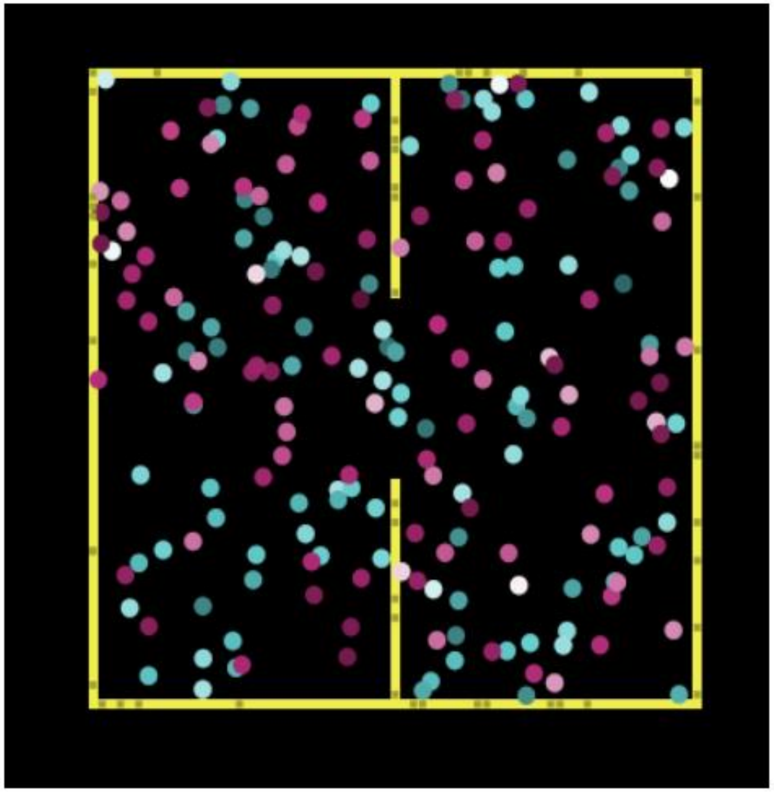
Thermodynamics & Statistical Mechanics

- **Thermodynamics**
  - The study of heat/thermal energy

- **Statistical mechanics**
  - A general mathematical framework that shows how *macroscopic* properties (e.g. heat) arise from statistics of the mechanics of large numbers of *microscopic* components (e.g., atoms or molecules)

Example: Room Full of Air

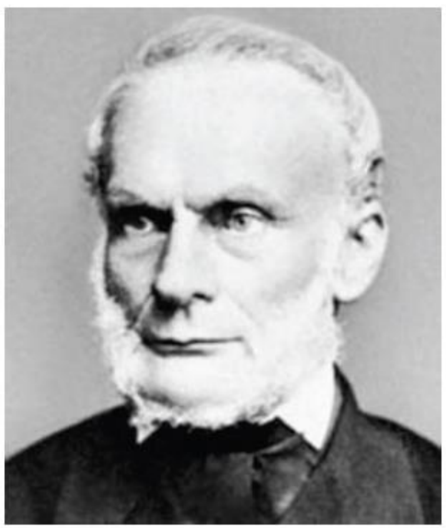
- **Macroscopic property (thermodynamics)**:
  - Temperature, pressure
- **Microscopic property (mechanics)**:
  - Positions and velocities of air molecules
- **Statistical mechanics:**
  - How statistics of positions and velocities of molecules give rise to temperature, pressure, etc.



Thermodynamic entropy measures the amount of heat loss when energy is transformed to work

Heat loss ≈ “disorder”

Theory is specific to heat



Rudolf Clausius, 1822-1888

Statistical mechanics entropy measures the number of possible microstates that lead to a macrostate

Number of microstates ≈ disorder

A more general theory

Ludwig Boltzmann, 1844-1906

A slight sidetrack to learn about microstates and macrostates

Microstates: specific state of the three slot-machine windows

Example microstate: {*cherry*, *lemon*, *apple*}

Note that a microstate here is a triple of fruit values, not a single fruit value. It is a description of the "state" of the slot machine.

Macrostate: Collection (or *set*) of microstates.

Example macrostate: *Win* (collection of microstates that have three of the same fruit showing).

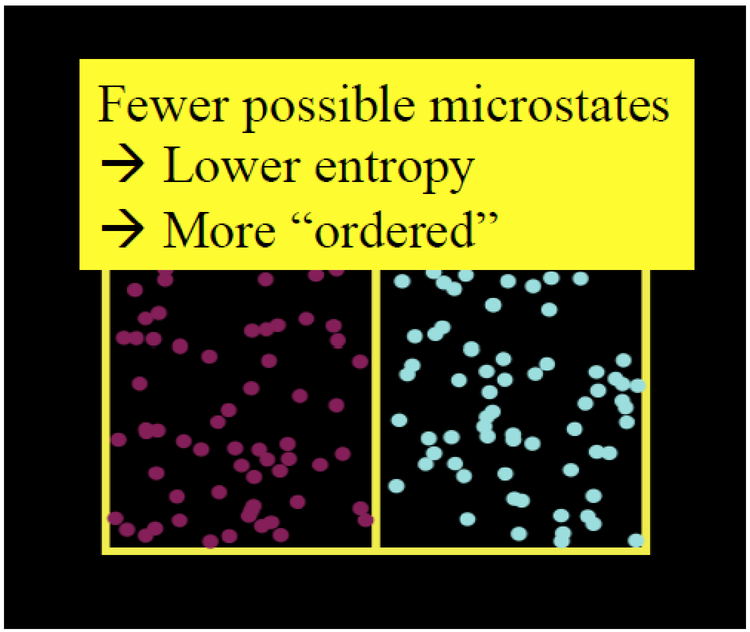
Question 1: How many microstates give rise to the *Win* macrostate?

Question 2: How many microstates give rise to the *Lose* macrostate?

NetLogo Two Gas Model

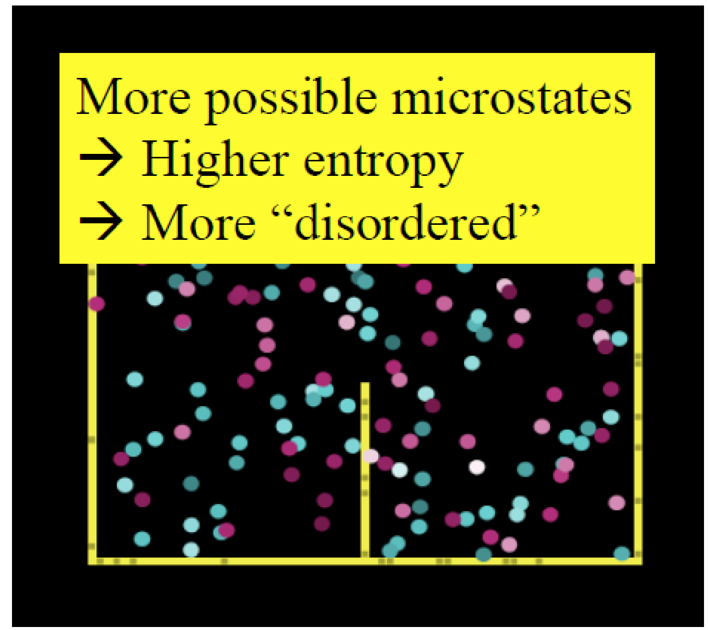
Microstate: Position and velocity of
of every particle

Start



Macrostate: All fast particles are on the right, all slow particles are on the left.

Finish

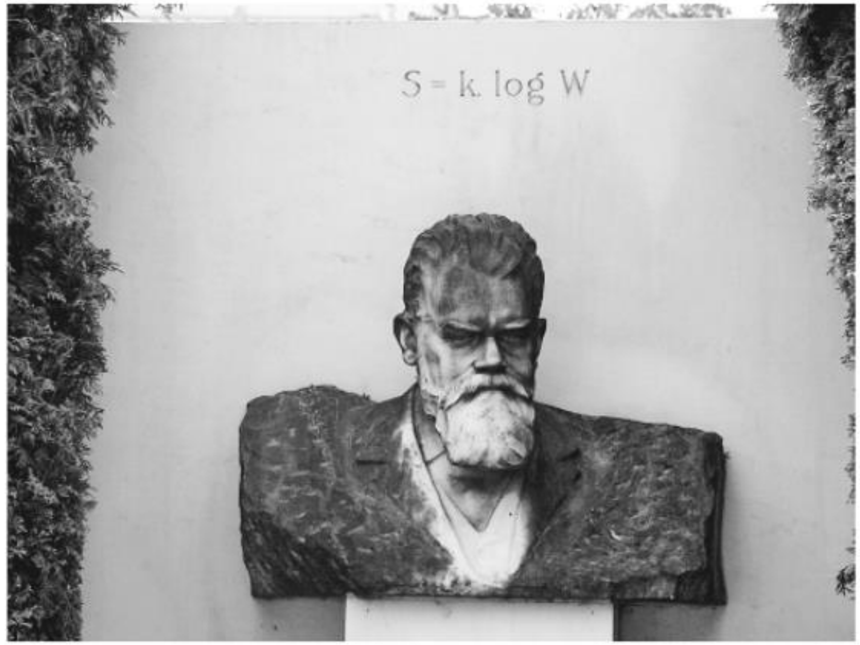


Macrostate: Fast and slow particles are completely mixed.

Second Law of Thermodynamics: In an isolated system, entropy will always increase until it reaches a maximum value.

Second Law of Thermodynamics (Statistical Mechanics Version): In an isolated system, the system will always progress to a macrostate that corresponds to the maximum number of microstates.

Boltzmann Entropy



Boltzmann's tomb, Vienna, Austria

The entropy S of a macrostate is k times the natural logarithm of the number W of microstates corresponding to that macrostate.

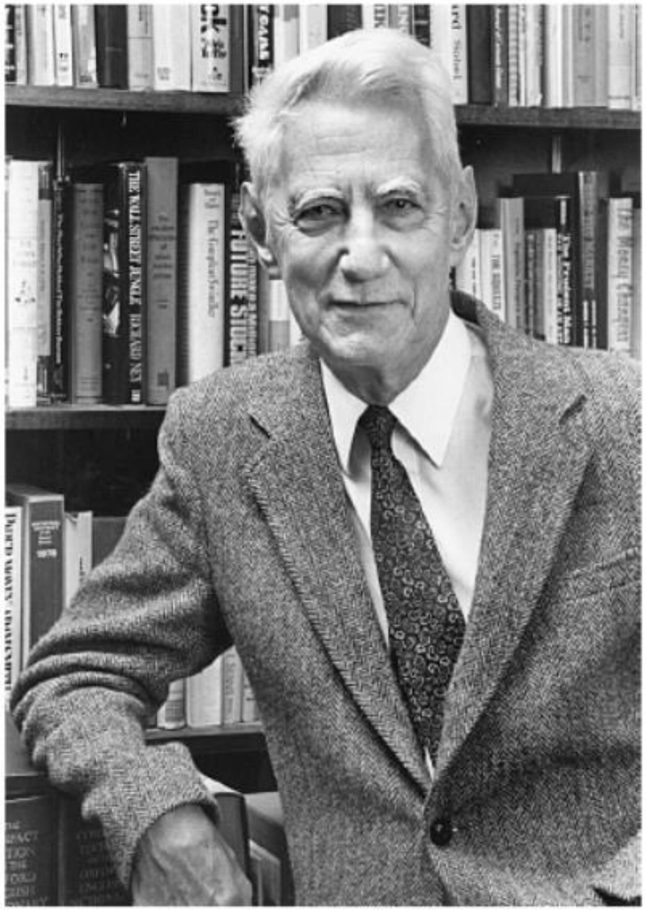
k is called "Boltzmann's constant". This constant and the logarithm are just for putting entropy into a particular units.

General idea: The more microstates that give rise to a macrostate, the more probable that macrostate is. Thus *high entropy = more probable macrostate.*

Second Law of Thermodynamics (Statistical Mechanics Version):

In an isolated system, the system will tend to progress to the **most probable macrostate**.

Shannon Information



Claude Shannon, 1916-2001

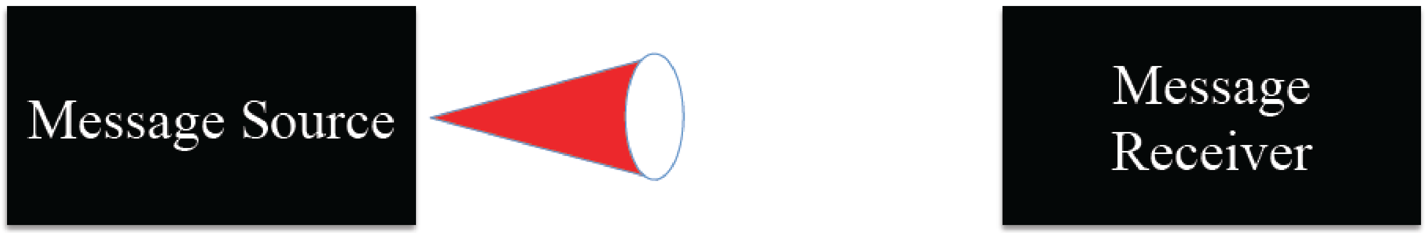
Shannon worked at Bell Labs (part of AT&T)

Major question for telephone communication: How to transmit signals most efficiently and effectively across telephone wires?

Shannon adapted Boltzmann's statistical mechanics ideas to the field of communication.

https://www.imdb.com/title/tt5015534/?ref_=vp_back

Shannon's Formulation of Communication



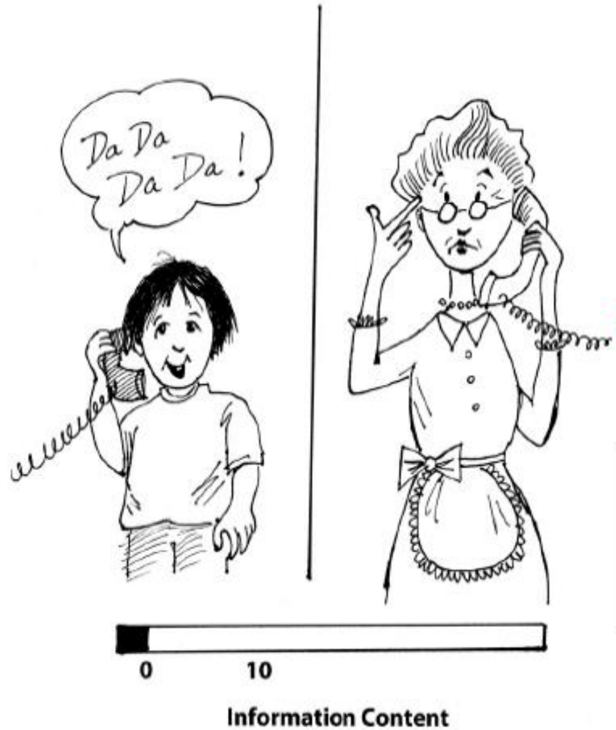
Message (e.g., a word)

Message source : Set of all possible messages this source can send, each with its own probability of being sent next.

Message: E.g., symbol, number, or word

Information content *H* of the message source: A function of the number of possible messages, and their probabilities

Informally: The amount of "surprise" the receiver has upon receipt of each message



No surprise;
no information content

Message source: One-year-old

Messages:
"Da" Probability 1

InformationContent (one-year-old) = 0 bits

More surprise;
more information content

Message source: Three-year-old

Messages: 500 words $(w_1, w_2, \ldots, w_{500})$
Probabilities: $p_1, p_2, \ldots, p_{500}$

InformationContent (three-year-old) > 0 bits

Boltzmann Entropy

Microstate: Detailed configuration of system components (e.g., "apple pear cherry")

Macrostate: Collection of microstates (e.g., "all three the same" or "exactly one apple")

Entropy S : Assumes all microstates are equally probable

$$S(macrostate) = k \log W$$

where W is the number of microstates corresponding to the macrostate.
S is measured in units defined by k (often "Joules per Kelvin")

Shannon Information

Message: E.g., a symbol, number, or word.

Message source: A set of possible messages, with probabilities for sending each possible message

Information content H :
Let M be the number of possible messages. Assume all messages are equally probable.

$$H(message\,source) = \log_2 M$$

H is measured in "bits per message"

General formula for Shannon Information Content

Let M be the number of possible messages, and p_i be the probability of message i. Then

$$H(message\,source) = -\sum_{i=1}^{M} p_i \log_2 p_i$$

In case the M possible messages are equiprobable:

$$H(message\,source) = -\sum_{i=1}^{M} p_i \log_2 p_i$$

$$= -\sum_{i=1}^{M} \frac{1}{M} \log_2 \frac{1}{M}$$

$$= -\log_2 \frac{1}{M}$$

$$= -\log_2 M^{-1}$$

$$= \log_2 M$$

Message source: One-year-old: {“Da”, probability 1}

$$H(\text{one-year-old}) = -\sum_{i=1}^{N} p_i \log_2 p_i = 1 * \log_2 1 = 0 \text{ bits}$$

$M = 1$

$H(\text{one-year-old}) = \log_2 1 = 0$

$M = 3$

$H = \log_2 3 \simeq 1.58$

Message source: Fair coin:
("Heads", probability .5)
("Tails",   probability .5)

$$H(\text{fair coin}) = -\sum_{i=1}^{N} p_i \log_2 p_i$$

$$= -\left[\left(.5 \log_2 .5\right) + \left(.5 \log_2 .5\right)\right]$$

$$= -\left[.5(-1) + .5(-1)\right]$$

$$= 1 \text{ bit} \quad \text{(on average, per message)}$$

Message source: Biased coin:
("Heads", probability .6)
("Tails",   probability .4)

$$H(\text{biased coin}) = -\sum_{i=1}^{N} p_i \log_2 p_i$$

$$= -\left[\left(.6\log_2 .6\right) + \left(.4\log_2 .4\right)\right]$$

$$= .971 \text{ bits} \quad \text{(on average, per message)}$$

Message source: Fair die:

("1", probability 1/6)

("2", probability 1/6)

("3", probability 1/6)

("4", probability 1/6)

("5", probability 1/6)

("6", probability 1/6)

$$H(\text{fair die}) = -\sum_{i=1}^{N} p_i \log_2 p_i$$

$$= -6\left(\frac{1}{6}\log_2 \frac{1}{6}\right)$$

$$\approx 2.58 \text{ bits (per message, on average)}$$

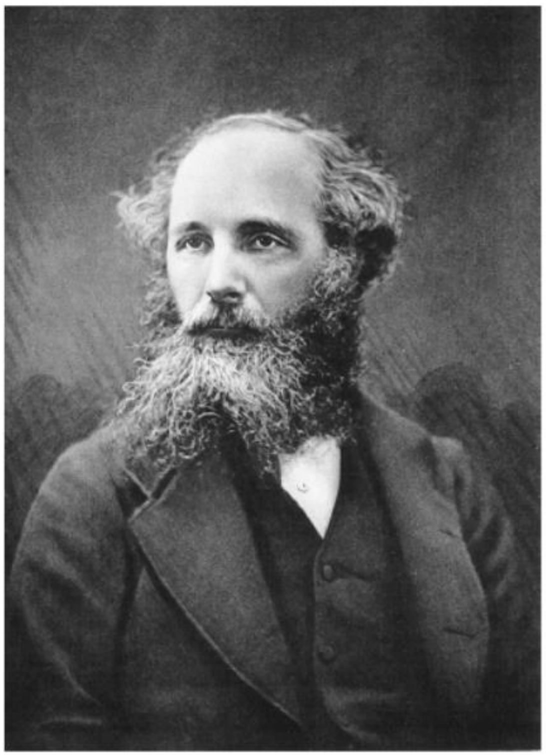
Shannon Information Content versus Meaning

Shannon information content does not capture the notion of the *function* or *meaning* of information.

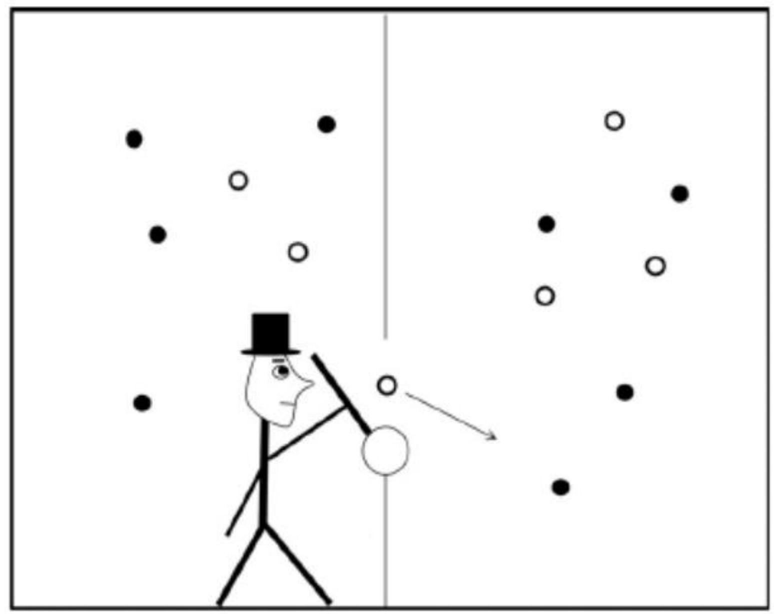
The *meaning* of information comes from **information processing**.

APPENDIX - Maxwell's Demon

https://www.youtube.com/watch?v=11QkX4u6RJg



James Clerk Maxwell, 1831-1879



Maxwell's Demon

APPENDIX - Maxwell's Demon (2)

- In order to contradict the second law of thermodynamics, the demon must open and close the door without doing any work... But how is that possible ?

- The idea is that there is no fundamental physical limit to how little energy is required to open and close the door
  - So you can make it arbitrarily small, and if you make it small enough, the demon still contradicts the Second Law of Thermodynamics

APPENDIX - Maxwell’s Demon (3)

- This definitely isn't apparent at first sight, but this might be one of the first things people checked in attempts to resolve the paradox
  - If it had been this easy to resolve, it wouldn't have stayed open for as long as it did

- The rather non-intuitive resolution of the Maxwell demon paradox is that you need an amount of energy $(\ln 2)kT$ to erase each bit of information from the demon's memory
  - Where k is the Boltzmann constant (approximately 1.38×10^{-23} J/K), T is the temperature of the circuit in kelvins, and $\ln 2$ is the natural logarithm of 2 (approximately 0.69315)
  - See Landauer's Principle

Videos

https://www.youtube.com/watch?v=pHSRHi17RKM

https://www.youtube.com/watch?v=Q8rVJZ-VDKQ

Ackowledgement

- Melanie Mitchell, Santa Fe Institute