1 Patient-Level Time-Expanded Model

We present a patient-level, time-expanded mixed-integer programming model. We track each individual patient across locations over time, explicitly model triage and treatment service times, and use a lexicographic objective: (i) first minimize total service time shortfall, then (ii) minimize a weighted measure of time in system and staff usage.

1.1 Indices and Sets

1. Time periods: $t \in T = \{1, \dots, T_{\max}\}$. In the experiment, we use a 10-minute time interval with $T_{\max} = 44$.
2. Patients: $i \in I = \{1, \dots, I_{\max}\}$.
3. Acuity levels: $l \in L = \{1, 2, 3, 4, 5\}$.
4. Stay-Duration categories: $d \in D = \{1, \dots, 5\}$.
   - $d = 1$: Short stay, $0 \le$ duration < 2 hours
   - $d = 2$: Moderate stay, $2 \le$ duration < 6 hours
   - $d = 3$: Long stay, $6 \le$ duration < 12 hours
   - $d = 4$: Extended stay, $12 \le$ duration < 24 hours
   - $d = 5$: Extremely long stay, duration ≥ 24 hours
5. Nurse 4-hour shift blocks: $u \in U = \{1, \dots, 6\}$.
   - $u = 1$: $07:00 - 11:00$
   - $u = 2$: $11:00 - 15:00$
   - $u = 3$: $15:00 - 19:00$
   - $u = 4$: $19:00 - 23:00$
   - $u = 5$: $23:00 - 03:00$
   - $u = 6$: $03:00 - 07:00$
6. Providers: $j \in J = \{1, \dots, J_{\max}\}$. We assume there are enough providers for all treatment rooms without considering their shifts.
7. Nurse coverage groups (RN assignments): $k \in K$. Each group k consists of a subset of treatment rooms that share an RN; for triage we assign one group per triage room.
8. Locations (states) in the time-expanded network:

$$s \in \mathcal{S} = \mathcal{S}^{\text{tri}} \cup \mathcal{S}^{\text{room}} \cup \mathcal{S}^{\text{queue}} \cup \{\text{Out}\}.$$

   Here:

   - $\mathcal{S}^{\text{tri}}$ = set of triage rooms (e.g., `tri1`, `tri2`, ...).
   - $\mathcal{S}^{\text{room}}$ = set of all treatment rooms and hallways.
   - $\mathcal{S}^{\text{queue}}$ = set of queue states (one per nurse coverage group), for example $q\_\text{triage}, q\_r1, \dots, q\_r20$.

- Out = absorbing state indicating that the patient has completed the visit and has left the CES.

9. For each state $s \in \mathcal{S}$, let $\mathcal{N}(s) \subseteq \mathcal{S}$ denote the set of states that the patient is allowed to occupy in the next time period if it is in state s at the current time. This directed adjacency matches the routing logic encoded in the implementation.

1.2 Parameters

1. $P_{t,l,d}$: number of patient arrivals in period t with acuity l and duration category d (obtained from data). For each patient $i \in I$:
   - $a_i \in T$: arrival period of patient i,
   - $l_i \in L$: acuity level of patient i,
   - $d_i \in D$: stay-duration category of patient i.

   These are constructed so that the arrival counts satisfy

   $$P_{t,l,d} \ = \ \big|\{i \in I : a_i = t,\ l_i = l,\ d_i = d\}\big|.$$

2. $m_{l,d}$: estimated triage service time (in time periods) required for a patient with acuity l and duration d.
3. $M_{l,d}$: estimated treatment service time (in time periods) required for a patient with acuity l and duration d, when both an RN and a provider are available in a treatment room.
4. w_l: priority weight for acuity level l; higher acuity patients have larger w_l:

   $$w_1 = 8, \quad w_2 = 4, \quad w_3 = 2, \quad w_4 = 1, \quad w_5 = 0.5.$$

5. For each time period $t \in T$, let $N_t \subseteq U$ denote the set of nurse 4-hour shifts that are active during period t (this corresponds to the `time_map_nurse` mapping in the implementation).
6. For each nurse group $k \in K$, let $\mathcal{R}_k \subseteq \mathcal{S}^{\text{room}}$ be the subset of treatment rooms (and hallways) that the RN assigned to group k can cover. For triage rooms we define a separate group index for each triage room.
7. For each treatment room $r \in \mathcal{S}^{\text{room}}$ we define its associated nurse group $g(r) \in K$ (the RN whose group covers room r).

1.3 Decision Variables

All decision variables are binary except for the Triage service shortfall and the Treatment service shortfall, which are continuous.

1. Nurse staffing:

   $$y_{u,k} = \begin{cases} 1 & \text{if one RN is scheduled to group } k \text{ in shift } u, \\ 0 & \text{otherwise,} \end{cases} \qquad u \in U,\ k \in K.$$

   (In the code these are the variables `staff[u, r_set]` for both triage rooms and RN groups.)

2. Patient location in the time-expanded network:

$$x_{i,s,t} = \begin{cases} 1 & \text{if patient } i \text{ is in state } s \text{ at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad i \in I,\ s \in \mathcal{S},\ t \in T,\ t \geq a_i.$$

   In particular, $x_{i,\text{Out},t} = 1$ means that patient i has completed the CES visit by time t.

3. Provider assignment to rooms:

$$z_{j,r,t} = \begin{cases} 1 & \text{if provider } j \text{ is assigned to room } r \text{ at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad j \in J,\ r \in \mathcal{S}^{\text{room}},\ t \in T.$$

4. Room serving indicator:

$$h_{r,t} = \begin{cases} 1 & \text{if treatment room } r \text{ is actively serving a patient at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad r \in \mathcal{S}^{\text{room}},\ t \in T.$$

   (This corresponds to `room_service[r_i,t]` in the code.)

5. Triage service indicator:

$$s^T_{i,r,t} = \begin{cases} 1 & \text{if patient } i \text{ is being triaged in triage room } r \text{ at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad i \in I,\ r \in \mathcal{S}^{\text{tri}},\ t \in T.$$

6. Treatment service indicator:

$$s^R_{i,r,t} = \begin{cases} 1 & \text{if patient } i \text{ is being treated in room } r \text{ at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad i \in I,\ r \in \mathcal{S}^{\text{room}},\ t \in T.$$

7. In-system indicator:

$$b_i = \begin{cases} 1 & \text{if patient } i \text{ is still in the system at the horizon } T_{\max}, \\ 0 & \text{if patient } i \text{ has left the CES by } T_{\max}, \end{cases} \qquad i \in I.$$

   (This is `patient_insystem[i]` in the code.)

8. Triage service shortfall:

$$\xi^T_i \geq 0, \quad i \in I,$$

   representing the amount of triage service time (in periods) that patient i does *not* receive relative to the target m_{l_i,d_i}.

9. Treatment service shortfall:

$$\xi^R_i \geq 0, \quad i \in I,$$

   representing the amount of treatment service time (in periods) that patient i does *not* receive relative to the target M_{l_i,d_i}.

1.4 Constraints

Patient State Dynamics

Initial state at arrival. At the arrival time a_i, each patient i either enters the triage queue or occupies one of the triage rooms:

$$x_{i,q_{\text{triage}},a_i} + \sum_{r \in \mathcal{S}^{\text{tri}}} x_{i,r,a_i} = 1, \quad \forall i \in I. \tag{1}$$

Exactly one state per patient per period. For every patient and every time period from arrival until the horizon, the patient occupies exactly one state:

$$\sum_{s \in \mathcal{S}} x_{i,s,t} = 1, \quad \forall i \in I, \ \forall t \in \{a_i, \ldots, T_{\max}\}. \tag{2}$$

Routing along allowed transitions. The directed network of states restricts which states a patient can occupy next. If patient i is in state s at time t, then in the next period $t+1$ the patient must be in one of the successor states $\mathcal{N}(s)$:

$$x_{i,s,t} \ \leq \sum_{s' \in \mathcal{N}(s)} x_{i,s',t+1}, \quad \forall i \in I, \ \forall s \in \mathcal{S}, \ \forall t \in \{a_i, \ldots, T_{\max} - 1\}. \tag{3}$$

Exit or stay in system at the horizon. At the final time $T_{\max}$, each patient either has left the CES (Out) or is counted as still in the system:

$$x_{i,\text{Out},T_{\max}} + b_i = 1, \quad \forall i \in I. \tag{4}$$

Capacity Constraints

Room occupancy. Each physical treatment room (or hallway) can hold at most one patient at any given time:

$$\sum_{i \in I} x_{i,r,t} \ \leq \ 1, \quad \forall r \in \mathcal{S}^{\text{room}}, \ \forall t \in T. \tag{5}$$

Triage nurse capacity. For each triage room $r \in \mathcal{S}^{\text{tri}}$, the number of patients in that room at time t cannot exceed the number of nurses assigned to that triage room at time t:

$$\sum_{i \in I} x_{i,r,t} \ \leq \ \sum_{u \in N_t} y_{u,k(r)}, \quad \forall r \in \mathcal{S}^{\text{tri}}, \ \forall t \in T. \tag{6}$$

Here $k(r)$ denotes the nurse group index associated with triage room r.

Treatment nurse capacity. For each treatment room $r \in \mathcal{S}^{\text{room}}$, the number of patients present at time t cannot exceed the number of nurses assigned to the corresponding room group:

$$\sum_{i \in I} x_{i,r,t} \ \leq \ \sum_{u \in N_t} y_{u,g(r)}, \quad \forall r \in \mathcal{S}^{\text{room}}, \ \forall t \in T. \tag{7}$$

Provider capacity. Each provider can be assigned to at most two treatment rooms in any given time period:

$$\sum_{r \in \mathcal{S}^{\text{room}}} z_{j,r,t} \;\leq\; 2, \quad \forall j \in J,\ \forall t \in T. \tag{8}$$

Room serving indicator. A treatment room can be actively serving a patient only if both a provider and a nurse are available:

$$h_{r,t} \leq \sum_{j \in J} z_{j,r,t}, \qquad \forall r \in \mathcal{S}^{\text{room}},\ \forall t \in T, \tag{9}$$

$$h_{r,t} \leq \sum_{u \in N_t} y_{u,g(r)}, \qquad \forall r \in \mathcal{S}^{\text{room}},\ \forall t \in T. \tag{10}$$

Triage and Treatment Service Time Accounting

Triage service AND logic. The variable $s^T_{i,r,t}$ is 1 if and only if patient i is present in triage room r and there is at least one nurse assigned to that room at time t. This logical AND is enforced by:

$$s^T_{i,r,t} \leq x_{i,r,t}, \qquad \forall i \in I,\ \forall r \in \mathcal{S}^{\text{tri}},\ \forall t \in T, \tag{11}$$

$$s^T_{i,r,t} \leq \sum_{u \in N_t} y_{u,k(r)}, \qquad \forall i \in I,\ \forall r \in \mathcal{S}^{\text{tri}},\ \forall t \in T, \tag{12}$$

$$s^T_{i,r,t} \geq x_{i,r,t} + \sum_{u \in N_t} y_{u,k(r)} - 1, \qquad \forall i \in I,\ \forall r \in \mathcal{S}^{\text{tri}},\ \forall t \in T. \tag{13}$$

Treatment service AND logic. Similarly, $s^R_{i,r,t}$ is 1 if and only if patient i is present in treatment room r and the room is serving at time t:

$$s^R_{i,r,t} \leq x_{i,r,t}, \qquad \forall i \in I,\ \forall r \in \mathcal{S}^{\text{room}},\ \forall t \in T, \tag{14}$$

$$s^R_{i,r,t} \leq h_{r,t}, \qquad \forall i \in I,\ \forall r \in \mathcal{S}^{\text{room}},\ \forall t \in T, \tag{15}$$

$$s^R_{i,r,t} \geq x_{i,r,t} + h_{r,t} - 1, \qquad \forall i \in I,\ \forall r \in \mathcal{S}^{\text{room}},\ \forall t \in T. \tag{16}$$

Triage service time balance. For each patient i, the total number of periods during which the patient is triaged plus the triage shortfall must equal the target triage service time:

$$\sum_{r \in \mathcal{S}^{\text{tri}}} \sum_{t=a_i}^{T_{\max}} s^T_{i,r,t} + \xi^T_i \;=\; m_{l_i,d_i}, \quad \forall i \in I. \tag{17}$$

A patient who receives incomplete triage must remain in the system:

$$\xi^T_i \;\leq\; m_{l_i,d_i}\, b_i, \quad \forall i \in I. \tag{18}$$

Treatment service time balance. Analogously, for treatment:

$$\sum_{r \in \mathcal{S}^{\text{room}}} \sum_{t=a_i}^{T_{\max}} s^R_{i,r,t} + \xi^R_i \;=\; M_{l_i,d_i}, \quad \forall i \in I, \tag{19}$$

with the requirement that incomplete treatment implies the patient remains in the system:

$$\xi^R_i \;\leq\; M_{l_i,d_i}\, b_i, \quad \forall i \in I. \tag{20}$$

Treatment only after full triage. A patient may only occupy a treatment room after completing the required triage time. For any time t and any treatment room $r \in \mathcal{S}^{\text{room}}$,

$$\sum_{r' \in \mathcal{S}^{\text{tri}}} \sum_{\tau=a_i}^{t} s^T_{i,r',\tau} \;\geq\; m_{l_i,d_i}\, x_{i,r,t}, \quad \forall i \in I,\ \forall t \in \{a_i, \ldots, T_{\max}\},\ \forall r \in \mathcal{S}^{\text{room}}. \tag{21}$$

1.5 Objective functions

The model is solved in two stages.

Stage 1: Minimize total service time shortfall

First, we minimize the sum of triage and treatment time shortfalls to make sure patients finish triage and treatment:

$$\min\ Z_1 \;=\; \sum_{i \in I} \left(\xi_i^T + \xi_i^R\right), \tag{22}$$

subject to constraints (1)–(21) and the standard binary/nonnegativity conditions on all variables. Let $Z^\star_{\text{short}}$ denote the optimal value of (22).

Stage 2: Minimize weighted time in system and staff usage

In the second stage we fix the total shortfall at its optimal value from Stage 1 (up to a small tolerance $\varepsilon > 0$):

$$\sum_{i \in I} (\xi_i^T + \xi_i^R) \;\leq\; Z_1^\star + \varepsilon, \tag{23}$$

and minimize a secondary objective that combines patient time in system and staffing costs:

$$\begin{aligned}\min\ Z_2 = &\underbrace{\sum_{i \in I} \sum_{t=a_i}^{T_{\max}} \sum_{s \in \mathcal{S} \setminus \{\text{Out}\}} w_{l_i}\, x_{i,s,t}}_{\text{weighted time in system}} + \underbrace{\sum_{i \in I} b_i - \sum_{i \in I} x_{i,\text{Out},T_{\max}}}_{\text{penalize patients still present at the horizon}} \\ &+ \underbrace{\sum_{i \in I} (\xi_i^T + \xi_i^R)}_{\text{residual shortfall}} + \underbrace{\sum_{u \in U} \sum_{k \in K} y_{u,k}}_{\text{total RN staffing}}\ .\end{aligned} \tag{24}$$

Stage 2 is solved subject to (1)–(21), (23), and integrality constraints.