Problem Set 5

Due Wednesday February 12, 4pm

Data Exercises

(1) The following FRED codes are for monthly U.S. unemployment rates, 1948m1 through current (2025m1), not seasonally adjusted, for individuals in the ages 20+ and the following categories

Men	LNU04000025
Women	LNU04000026
White Men	LNU04000028
White Women	LNU04000029
Black Men	LNU04000031
Black Women	LNU04000032

[The first three characters "LNU" are letters, the remainder are numbers.]

(a) Fit a simple seasonal dummy models for the two series "men" and "women". Plot fitted values for one year. It may be convenient to plot the two on the same graph, or you can plot them separately. These fitted values are the estimated seasonal patterns. Is the seasonality in unemployment rates the same for men and women, or are they different? What is different about the two patterns?

(b) Now fit simple seasonal dummy models for the four series "white men", "white women", "black men", and "black women". Describe the similarities and differences between the estimated seasonal unemployment rate models for these four series.

(2) In the January 2024 Employment Situation, the Bureau of Labor Statistics reported that from December 2023 to January 2024 the seasonally unadjusted number of unemployed **increased** from 5,907,000 to 6,778,000 (about 1 million!) and the unemployment rate increased from 3.5% to 4.1%. Yet over the same period the reported seasonally adjusted number of unemployed **decreased** from 6,268,000 to 6,124,000

(about 100,000) and the seasonally adjusted unemployment rate stayed roughly constant (3.7%). Are these numbers consistent? Was the BLS trying to trick the country about a hidden increase in unemployment? Explain.

Theoretical Questions

(3) Suppose that y_t is a quarterly seasonal time series, with the model

$$y_t = \sum_{i=1}^{4} \gamma_i D_{it} + \varepsilon_t$$

where D_{1t}, D_{2t} are the dummies for the first quarter, second quarter, etc.. The estimates are $\hat{\gamma}_1 = -0.1$, $\hat{\gamma}_2 = 0.5$, $\hat{\gamma}_3 = 0.3$ and $\hat{\gamma}_4 = 0.8$. Construct point forecasts for 2025Q1, Q2, Q3 and Q4.

(4) Explain why the seasonal model

$$S_t = \sum_{i=1}^{4} \gamma_i D_{it}$$

is the same as

$$S_t = \alpha + \sum_{i=1}^{3} \beta_i D_{it}$$

(5) The model is

$$y_t = \beta_0 + \beta_1 Time_t + \sum_{i=1}^{3} \gamma_i D_{it} + \varepsilon_t,$$

where D_{1t}, D_{2t}, and D_{3t} are the dummies for the first, second, and third quarters, respectively.

The estimates are

$$\hat{\beta}_0 = -0.1$$

$$\hat{\beta}_1 = 0.01$$

$$\hat{\gamma}_1 = 0.2$$

$$\hat{\gamma}_2 = -0.7$$

$$\hat{\gamma}_3 = 0.4$$

The time index at the end of sample is $Time_T = T = 200$, and this final observation is the fourth quarter of 2024. Construct point forecasts for 2025Q1, Q2, Q3 and Q4.

(3) Suppose that y_t is a quarterly seasonal time series, with the model

$$y_t = \sum_{i=1}^{4} \gamma_i D_{it} + \varepsilon_t$$

where D_{1t}, D_{2t} are the dummies for the first quarter, second quarter, etc.. The estimates are $\hat{\gamma}_1 = -0.1$, $\hat{\gamma}_2 = 0.5$, $\hat{\gamma}_3 = 0.3$ and $\hat{\gamma}_4 = 0.8$. Construct point forecasts for 2025Q1, Q2, Q3 and Q4.

This is the seosonal regression without intercept, and there are 4 quarters.
For forecasting, we have fitted value $\hat{y} = \sum_{i=1}^{4} \hat{\gamma}_i D_{it}$. Then, we have point forecasts:

$\hat{y}_{2025Q1} = \gamma_1 \cdot D_{1\text{-}2025} + 0 + 0 + 0 = -0.1$ $\qquad$ $D_{it} = 1$ if quarter $= i$.

$\hat{y}_{2025Q2} = 0 + \gamma_2 \cdot D_{2\text{-}2025} + 0 + 0 = 0.5$

$\hat{y}_{2025Q3} = 0 + 0 + \gamma_3 \cdot D_{3\text{-}2025} + 0 = 0.3$

$\hat{y}_{2025Q4} = 0 + 0 + 0 + \gamma_4 \cdot D_{4\text{-}2025} = 0.8$

(4) Explain why the seasonal model

$$S_t = \sum_{i=1}^{4} \gamma_i D_{it}$$

is the same as

$$S_t' = \alpha + \sum_{i=1}^{3} \beta_i D_{it}$$

Intuitively, they're the same because they are both a seosenality model with 4 seasons. In S', α represents the fitted value of the last season (γ_4). $\alpha+\beta_1$, $\alpha+\beta_2$, $\alpha+\beta_3$ for season 1, 2, 3, corresponding to $\gamma_1, \gamma_2, \gamma_3$.

At season 1, $S_t = \gamma_1$ $\quad S_t' = \alpha + \beta_1$ $\qquad$ At season 2, $S_t = \gamma_2$ $\quad S_t' = \alpha + \beta_2$

At season 3, $S_t = \gamma_3$ $\quad S_t' = \alpha + \beta_3$ $\qquad$ At season 4, $S_t = \gamma_4$ $\quad S_t' = \alpha$

Since those value are determined by population. When those 2 model come from the same population, we have corresponding relationships between them.

Alternatively,
Since D_{it} is a seosonal dummy variable, by definition, it follows $\sum_{i=1}^{4} D_{it} = D_{1t} + D_{2t} + D_{3t} + D_{4t} = 1 \quad \forall i = 1,2,3,4$

$$\alpha + \sum_{i=1}^{3} \beta_i D_{it} = \sum_{i=1}^{4} \alpha D_{it} + \left(\sum_{i=1}^{3} \beta_i D_{it}\right) = \sum_{i=1}^{3} (\alpha + \beta_i) \cdot D_{it} + \alpha D_{4t}$$

WLOG, denote $\alpha + \beta_i = \gamma_i$ $(i=1,2,3)$
$\alpha = \gamma_4$
(Since there is no β_4).

$$= \sum_{i=1}^{3} \gamma_i D_{it} + \gamma_4 D_{4t} = \sum_{i=1}^{4} \gamma_i D_{it}.$$

(5) The model is

$$y_t = \beta_0 + \beta_1 Time_t + \sum_{i=1}^{3} \gamma_i D_{it} + \varepsilon_t,$$

where D_{1t}, D_{2t}, and D_{3t} are the dummies for the first, second, and third quarters, respectively.

The estimates are

$$\hat{\beta}_0 = -0.1$$
$$\hat{\beta}_1 = 0.01$$
$$\hat{\gamma}_1 = 0.2$$
$$\hat{\gamma}_2 = -0.7$$
$$\hat{\gamma}_3 = 0.4$$

The time index at the end of sample is $Time_T = T = 200$, and this final observation is the fourth quarter of 2024. Construct point forecasts for 2025Q1, Q2, Q3 and Q4.

For the fitted value, we have $\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 \, Time_t + \sum_{i=1}^{3} \hat{\gamma}_i D_{it} + \varepsilon_t$ 2025 : $Time_{T+1} = T+1 = 201$.

Point forecasts are:

$\hat{Y}_{2025Q4} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 201 + \hat{\gamma}_1 = -0.1 + 0.01 \times 201 + 0.2 = 2.11$

$\hat{Y}_{2025Q4} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 201 + \hat{\gamma}_1 = -0.1 + 0.01 \times 202 - 0.7 = 1.22$

$\hat{Y}_{2025Q4} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 201 + \hat{\gamma}_1 = -0.1 + 0.01 \times 203 + 0.4 = 2.33$

$\hat{Y}_{2025Q4} = \hat{\beta}_0 + \hat{\beta}_1 \cdot 201 + \hat{\gamma}_1 = -0.1 + 0.01 \times 204 = 1.94$