Problem Set 8

Due Wednesday March 19, 4pm

1 Data Exercises

1. The FRED label for the US unemployment rate is `UNRATE`. The newest observation (Feb 2025) is released on March 7. Based on the AR(1) model with an intercept, derive one-, two-, and three-step-ahead point forecasts for the unemployment rate by three different methods we considered: plug-in, iterated, direct. Explain your calculations and discuss the results. (For each method, you should report forecasts for three periods.)

   To do the forecast, firstly, we regress y_t on its lags by using 'fitlm' and 'lagmatrix' to get the model. We call the model with lag 1,2,3 by md1, md2,md3. Then, we do one-, two-, and three-step-ahead point forecasts by 3 different methods. (A one-step-ahead forecast will not involve problems with different methods). For the plug-in method, we use only the $\hat{\alpha}$ and $\hat{\beta}$ we get from md1. Under quadratic loss, We do back substitution so we have:

   $$E[y_{T+2}|\Omega_T] = (1+\beta)\alpha + \beta^2 y_T,$$
   $$E[y_{T+3}|\Omega_T] = (1+\beta+\beta^2)\alpha + \beta^3 y_T.$$

   Since we do not know the exact α and β, we use $\hat{\alpha}$ and $\hat{\beta}$ to replace them. Meanwhile, for the iterated method, we have:

   $$E[y_{T+1}|\Omega_T] = \alpha + \beta y_T$$
   $$E[y_{T+2}|\Omega_T] = \alpha + \beta E[y_{T+1}|\Omega_T] = \alpha + \beta(\alpha + \beta y_T) = (1+\beta)\alpha + \beta^2 y_T$$
   $$E[y_{T+3}|\Omega_T] = \alpha + \beta E[y_{T+2}|\Omega_T] = (1+\beta+\beta^2)\alpha + \beta^3 y_T$$

   Since we do not know the exact α and β, we use $\hat{\alpha}$ and $\hat{\beta}$ to replace them. We find that the plug-in method and iterated method return the same result. Lastly, for the direct method, we use all regressions, following $y_{T+i} = \alpha_i + \beta_i y_T$ for 3 different steps-ahead forecasts.

   Using the plug-in method to forecast, we get 4.1485, 4.1955, and 4.2411 for the one-, two-, three-step-ahead forecast. Using the

iteration method, we get 4.1485, 4.1955, and 4.2411 for the one-, two-, three-step-ahead forecast. Using the direct method, we have 4.1485, 4.1994, and 4.2464 for the one-, two-, three-step-ahead forecast.

2. Take the AR(1) model:

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t, \tag{1}$$

where the errors ε_t are i.i.d. white noise $N(0,1)$.

(a) Set $\alpha = 1$ and $\beta = 0.25$.

i. Calculate the mean $\mu = E(y_t)$. (For this part, you do not need Matlab.)
$\mu = E[y_t] = E[\alpha + \beta y_{t-1} + \epsilon_t] = \alpha + \beta E[y_{t-1}] + E[\epsilon_t] = \alpha + \beta\mu$, **which implies** $\mu = \alpha/(1-\beta) = 1/0.75 = 4/3$

ii. Simulate a series of length $T = 240$. Set the initial value $y_1 = \mu$ to equal the unconditional mean (from part (a)). (This is similar to problem 4 from the last problem set.) Create a time-series plot of your series.
I create a time series plot by creating a series of random epsilon and a for loop. The random seed is 514 and here is the plot:

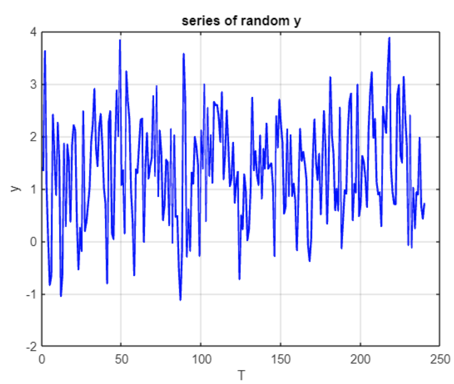

Figure 1:

iii. Estimate an AR(1) model. Are your coefficient estimates close to the true values?
I use fitlm to regress y_t and its lag one. I got 1.0430 for $\hat{\alpha}$ and 0.2322 for $\hat{\beta}$. Those coefficient estimates are close to the true value 1 and 0,25.

(b) Repeat with $\alpha = 10$, $\beta = 0.9$.
Here is the plot of (b). $y_1 = \mu$ **is 100. I got 11.7481 for** $\hat{\alpha}$ **and 0.8827 for** $\hat{\beta}$**. Those coefficient estimates are close to the true value 1 and 0.9.**

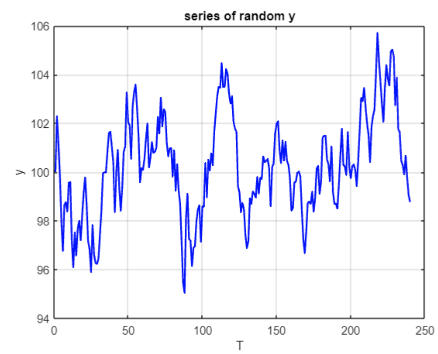

Figure 2:

(c) Repeat with $\alpha = 0$, $\beta = -0.5$.
Here is the plot of (c). $y_1 = \mu$ **is 0. I got 0.0195 for** $\hat{\alpha}$ **and -0.5549 for** $\hat{\beta}$**. Those coefficient estimates are close to the true value 0 and -0.5.**

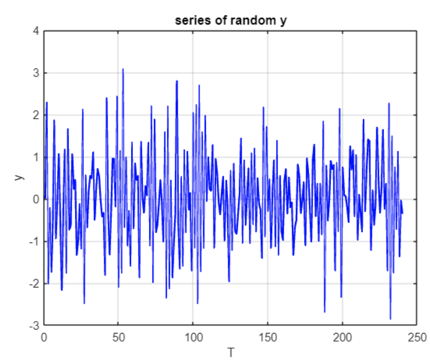

Figure 3: