```
clear all
clc
```

Q1

```
% Load the data
data = readtable('realgdpgrowth.xlsx');

% Extract the 'pdi' series
pdi = data(:,["pdi","date"]);

% Remove missing values if necessary
pdi = rmmissing(pdi);
%discard first 10 observations

% Determine the maximum AR order to consider
maxOrder = 10;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
est=(data.date>=datetime(1950,4,1));
X1=lagmatrix(pdi.pdi,1);
mdl1=fitlm(X1(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,1)=mdl1.ModelCriterion.AIC;

X2=lagmatrix(pdi.pdi,1:2);
mdl2=fitlm(X2(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,2)=mdl2.ModelCriterion.AIC;

X3=lagmatrix(pdi.pdi,1:3);
mdl3=fitlm(X3(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,3)=mdl3.ModelCriterion.AIC;

X4=lagmatrix(pdi.pdi,1:4);
mdl4=fitlm(X4(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,4)=mdl4.ModelCriterion.AIC;

X5=lagmatrix(pdi.pdi,1:5);
mdl5=fitlm(X5(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,5)=mdl5.ModelCriterion.AIC;

X6=lagmatrix(pdi.pdi,1:6);
mdl6=fitlm(X6(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,6)=mdl6.ModelCriterion.AIC;

X7=lagmatrix(pdi.pdi,1:7);
mdl7=fitlm(X7(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,7)=mdl7.ModelCriterion.AIC;

X8=lagmatrix(pdi.pdi,1:8);
mdl8=fitlm(X8(est,:),pdi.pdi(est)); %reg X on Y
```

```
AIC(1,8)=mdl8.ModelCriterion.AIC;

X9=lagmatrix(pdi.pdi,1:9);
mdl9=fitlm(X9(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,9)=mdl9.ModelCriterion.AIC;

X10=lagmatrix(pdi.pdi,1:10);
mdl10=fitlm(X10(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,10)=mdl10.ModelCriterion.AIC;

X11=lagmatrix(pdi.pdi,1:11);
mdl11=fitlm(X11(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,11)=mdl11.ModelCriterion.AIC;

X12=lagmatrix(pdi.pdi,1:12);
mdl12=fitlm(X12(est,:),pdi.pdi(est)); %reg X on Y
AIC(1,12)=mdl12.ModelCriterion.AIC;
disp(AIC);
```

```
   1.0e+03 *

    2.5625    2.5627    2.5647    2.5560    2.5573    2.5564    2.5551    2.5571    2.5591    2.5576    2.5593    2
```

```
[~, bestOrder] = min(AIC);
disp(bestOrder);
```

```
     7
```

Since AR(7) has the least AIC value, we select AR(7) model.

Q2(a)

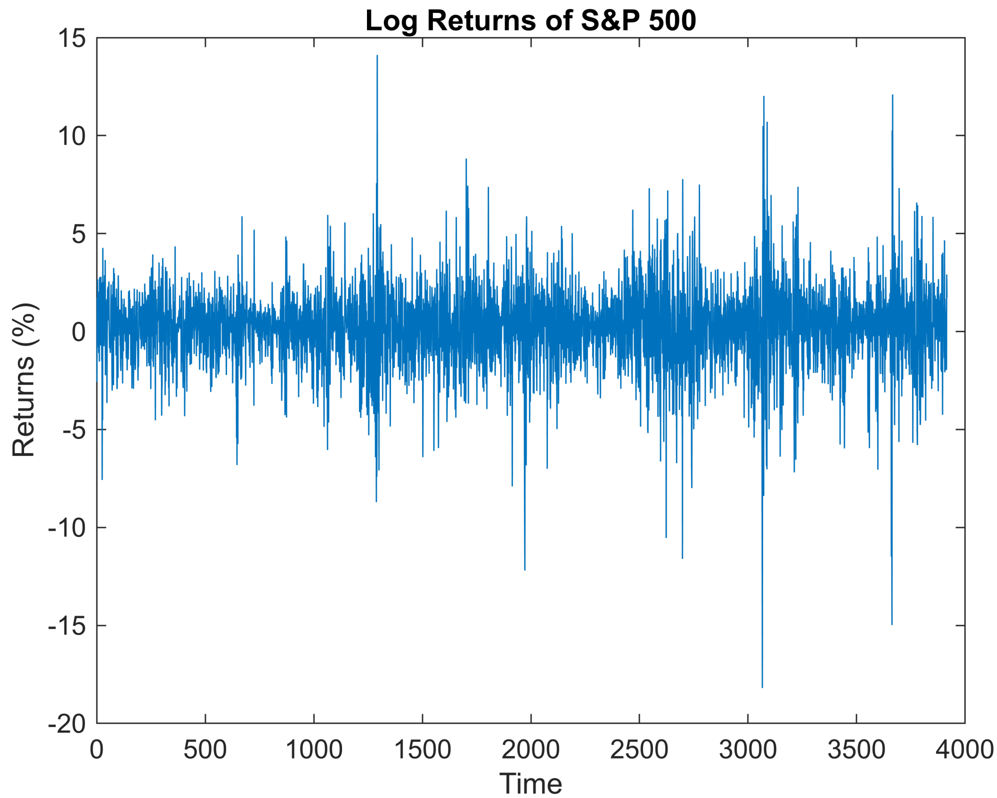
```
% Load Data
data2 = readtable('s&p.csv');
```

```
Warning: Column headers from the file were modified to make them valid MATLAB identifiers before creating
variable names for the table. The original column headers are saved in the VariableDescriptions property.
Set 'VariableNamingRule' to 'preserve' to use the original column headers as table variable names.
```

```
adjClose = data2.adjClose;    % Extract adjusted close prices column

%% Part (a): Transform into returns and plot
% Compute log returns: r_t = 100 * (s_t - s_{t-1}) / s_{t-1}
returns = 100 * diff(adjClose) ./ adjClose(1:end-1);

% Plot Returns
figure;
plot(returns);
title('Log Returns of S&P 500');
xlabel('Time');
ylabel('Returns (%)');
```




```
%% Part (b)-1: Test for Serial Correlation Using Matrix Operations
% Construct the AR(2) Model: r_t = α + β1*r_{t-1} + β2*r_{t-2} + ε_t
OLSmdl= fitlm(lagmatrix(returns,1:2),returns)
```

```
OLSmdl =
Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate        SE        tStat       pValue
                   _________    ________    _______    __________

    (Intercept)      0.17353    0.033859      5.125    3.1195e-07
    x1             -0.040104     0.01598    -2.5096      0.012127
    x2              0.036283     0.01598     2.2705       0.02323


Number of observations: 3914, Error degrees of freedom: 3911
Root Mean Squared Error: 2.1
R-squared: 0.00305,  Adjusted R-Squared: 0.00254
F-statistic vs. constant model: 5.98, p-value = 0.00256
```

```
[EstCov,se,coeff]=hac(OLSmdl,'display','off');
R=[0,1,0;0,0,1];
[h,pValue,stat,cValue]=waldtest(coeff(2:3,1),R,EstCov)
```

```
h = logical
   1
pValue = 0.0190
stat = 7.9214
cValue = 5.9915
```

```
%% Part (b)-2 Compute Heteroscedasticity-Consistent Wald Test
OLSmdl2= fitlm(lagmatrix(returns,1:2),returns) %actually no need for 2 model since coeffients for homo and hetero are same
```

```
OLSmdl2 =
Linear regression model:
    y ~ 1 + x1 + x2

Estimated Coefficients:
                   Estimate       SE        tStat       pValue
                   _________   ________    _______    __________

    (Intercept)      0.17353   0.033859      5.125    3.1195e-07
    x1             -0.040104    0.01598    -2.5096      0.012127
    x2              0.036283    0.01598     2.2705       0.02323


Number of observations: 3914, Error degrees of freedom: 3911
Root Mean Squared Error: 2.1
R-squared: 0.00305,  Adjusted R-Squared: 0.00254
F-statistic vs. constant model: 5.98, p-value = 0.00256
```

```
[EstCov,se,coeff]=hac(OLSmdl2,'display','off',Type="HC");
R=[0,1,0;0,0,1];
[h,pValue,stat,cValue]=waldtest(coeff(2:3,1),R,EstCov)
```

```
h = logical
   0
pValue = 0.0716
stat = 5.2738
cValue = 5.9915
```

Part (c): Compare and Interpret Results

We found that classical Wald test does not reject null hypothesis but the heteroscedasticity-consistent Wald test reject the null hypothesis that 2 coefficient are 0. I think the heteroscedasticity-consistent Wald tests is appropriate since by looking at the residual, we find that they do not follow a homoskedatistic pattern. Commonly, stock market datas are likely to be heteroskedatistic. Heteroskedatistic test is also prefered since they have different result, which means there are some hetero pattern.

Q3(a)

```
data = readtable('realgdpgrowth.xlsx');  % Load the dataset
exports = data.exports;  % Extract the export growth rate column
X=lagmatrix(exports,1:4);
model = fitlm(X, exports);


%%Classical Standard Errors
%classical_SE = model.Coefficients.SE;
%disp(classical_SE);
disp(model.Coefficients);
```

```
                   Estimate       SE        tStat       pValue
                   _________   ________    ________    __________
```

(Intercept)	6.2253	1.1942	5.2131	3.4578e-07
x1	-0.12286	0.057361	-2.1418	0.033012
x2	0.11178	0.056986	1.9616	0.050735
x3	0.0552	0.056672	0.97402	0.33083
x4	-0.051555	0.056496	-0.91253	0.36222

```
%% Compute Robust (Heteroscedasticity-Consistent) Standard Errors
%Robust_Se
[EstCov,se,coeff]=hac(model,'display','off',Type="HC");
disp(se);
```

```
1.4695
0.1004
0.0650
0.0667
0.0524
```

Q3(b)

Yes, two SE are different. And the result follow the general pattern that hetero SE is larger than homo SE.