Problem Set 4

February 2025

1. Extract data on the aggregate level of U.S. quarterly imports, seasonally adjusted, in real 2017 chained dollars, currently available for 1947Q1 through 2024Q4. The FRED label is `IMPGSC1`. Create a time index.

   (a) For the period 1947–2005, plot the level of imports, and its natural log, against time.

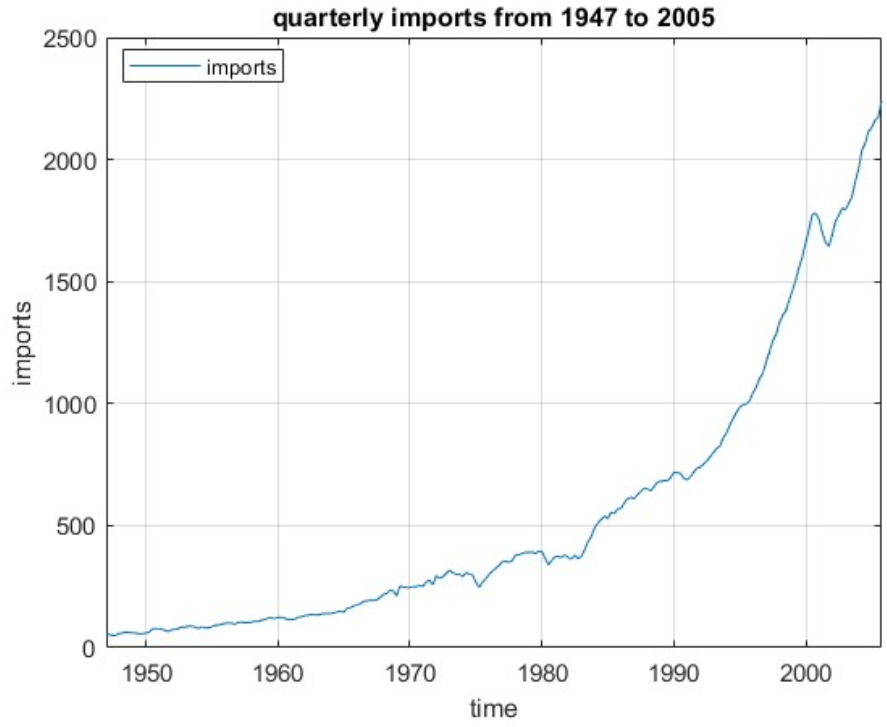

Figure 1: figure for part a

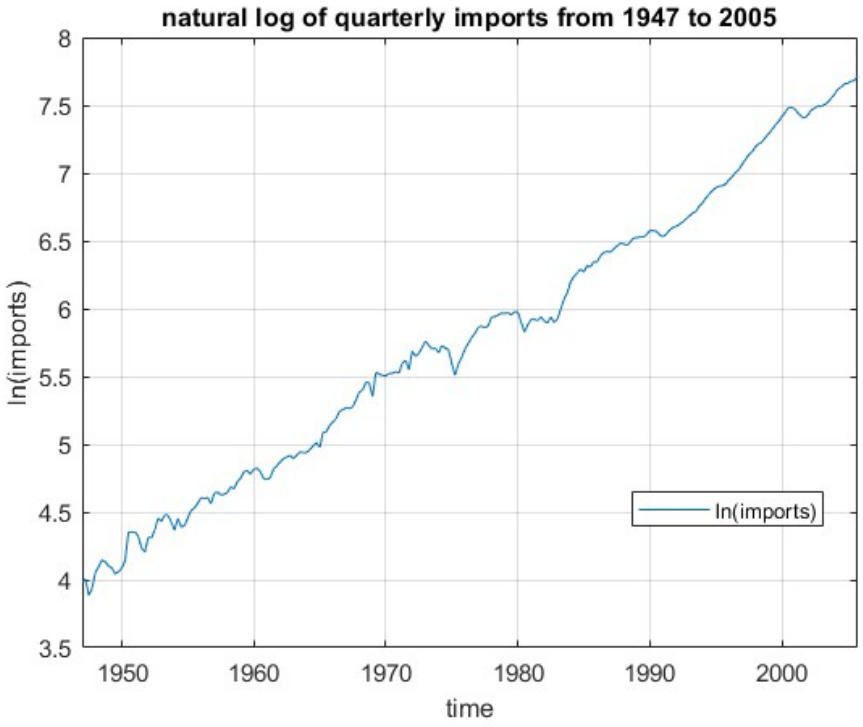

Figure 2: figure for part a

(b) By inspection, determine if the imports series is better represented using a linear or exponential trend.
By inspection, I believe the imports series is better to be represented by using an exponential trend.

(c) Estimate an exponential trend model using the estimation period 1947–2005. Report results.
The $\hat{\beta}_1$ solved in this ln model is the same as the estimator we derived from the direct exponential approach, which is 0.0154. The $\hat{\beta}_0$ in this linear model is actually ln(intercept) in the direct exponential model. Here the intercept is 3.9631 so the intercept in the direct approach is exp($\hat{\beta}_0$)=52.6228

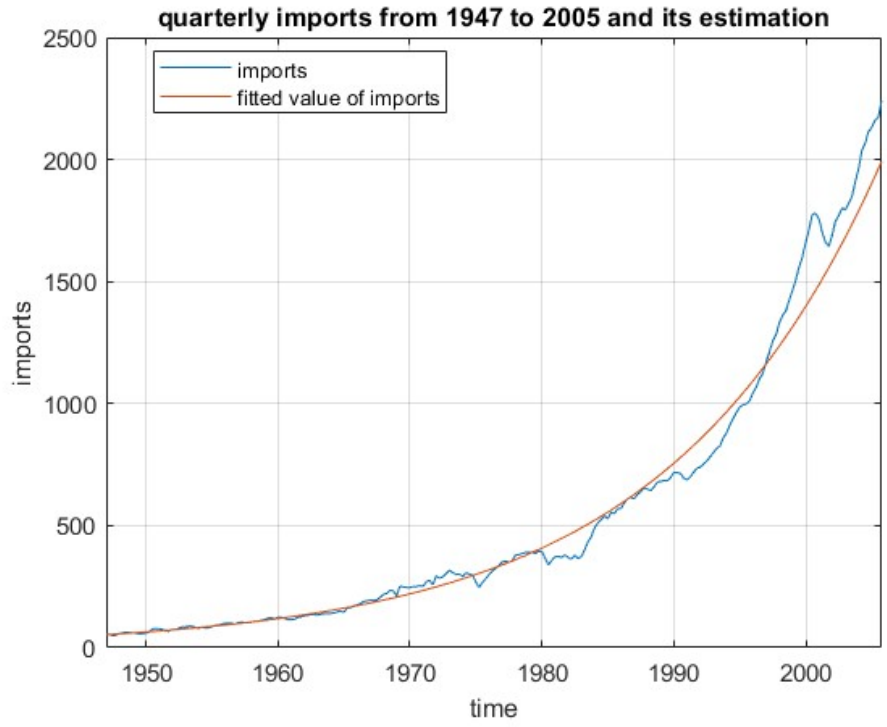

Figure 3: figure for part c

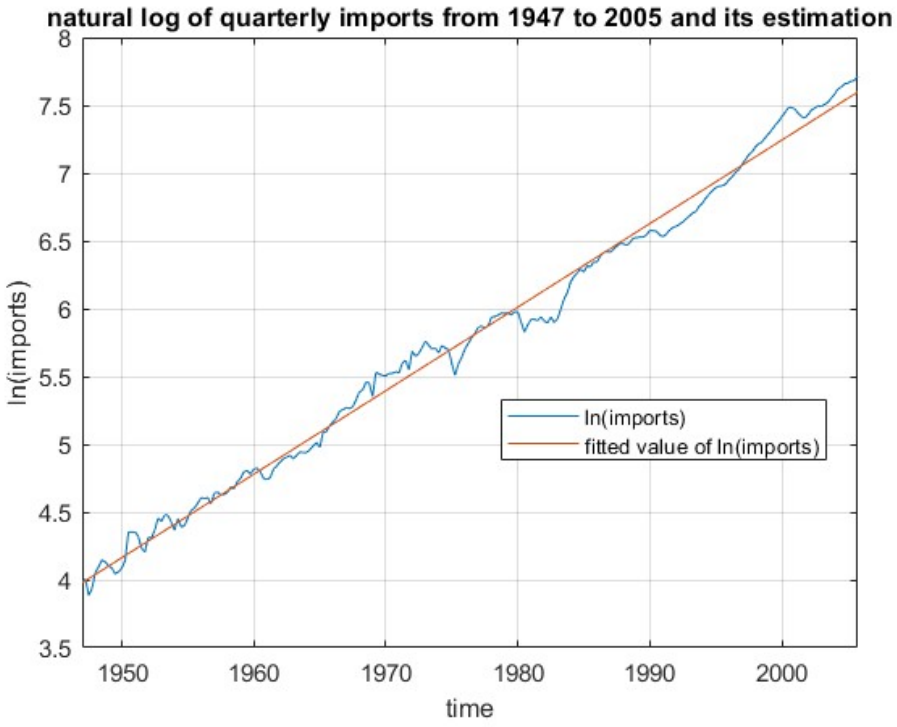

Figure 4: figure for part c

(d) Generate point and 90% interval forecasts for the log-level of imports for 2006Q1–2024Q4. Plot the data (log-levels) up to 2005Q4 and your forecast for 2006Q1–2024Q4. Comment.
Comment: Based on the forecast, the amount of ln(imports), on average, is expected to increase by 0.015399 as time increases by 1. It has a small confidence interval based on the $\hat{\sigma}$ calculated by residuals of forecasting before 2005Q4. Such a small approximation to error may understate the uncertainty in the future.

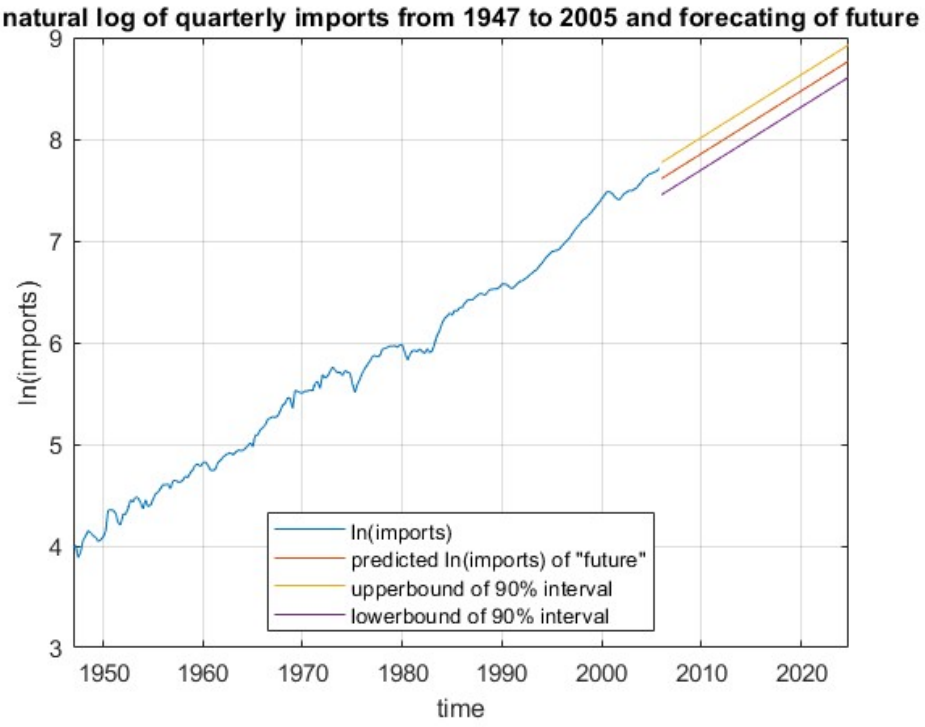

Figure 5: figure for part d

(e) Plot your forecasts against the actual data (log-levels) for 2006Q1–2024Q4. How did the forecast perform?

The interval forecast is good in the short run. In several beginning years from 2006 to 2010, the real ln(imports) mostly stay in the 90% forecasting interval. But in a relatively long run, starting from 2012, the reality starts to deviate more and more away from the forecasting interval.

Similarly, the point forecasting is good in several beginning years. But in the long run, it overestimates the ln(imports)

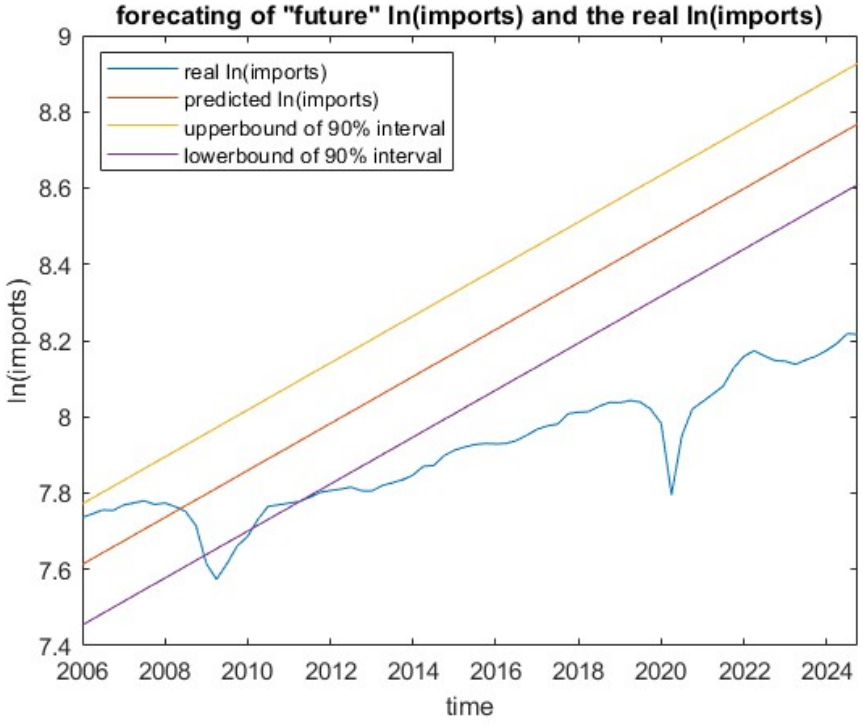

Figure 6: figure for part e

(f) Generate point and 90% interval forecasts for the level of imports for 2006–2024Q4. Plot the data (levels) up to 2005Q4 and your forecast for 2006Q1–2024Q4. Comment.
Comment: As we calculated before, the amount of ln(imports), on average, is expected to increase by 0.015399 as time increases by 1. Thus, in the regular exponential model, the increase rate of imports will increase as time increases. The confidence interval is based on the $\hat{\sigma}$ calculated by residuals of forecasting before 2005Q4. By theorem, to get the forecasting interval in the exponential model, We enlarge the forecasting interval in ln() model by taking an exponential to it. We can see the interval become wider as time increases.

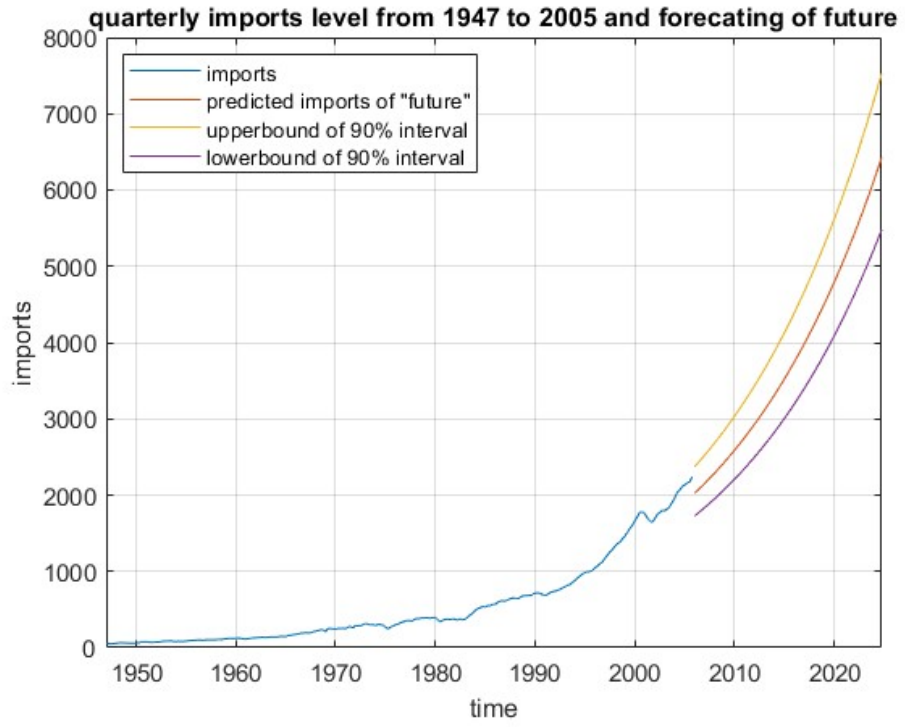

Figure 7: figure for part f

(g) Plot your forecasts against the actual data for 2006Q1–2024Q4. How did the forecast perform?

Similar to part (e), Both the interval forecast and the point forecast are good only in limited years.

In several beginning years from 2006 to 2012, the real ln(imports) mostly stay in the 90% forecasting interval. But in a relatively long run, starting from 2013, the reality starts to deviate more and more away from the forecasting interval.

Similarly, the point forecasting is good in several beginning years. But in the long run, it starts to overestimate the ln(imports).

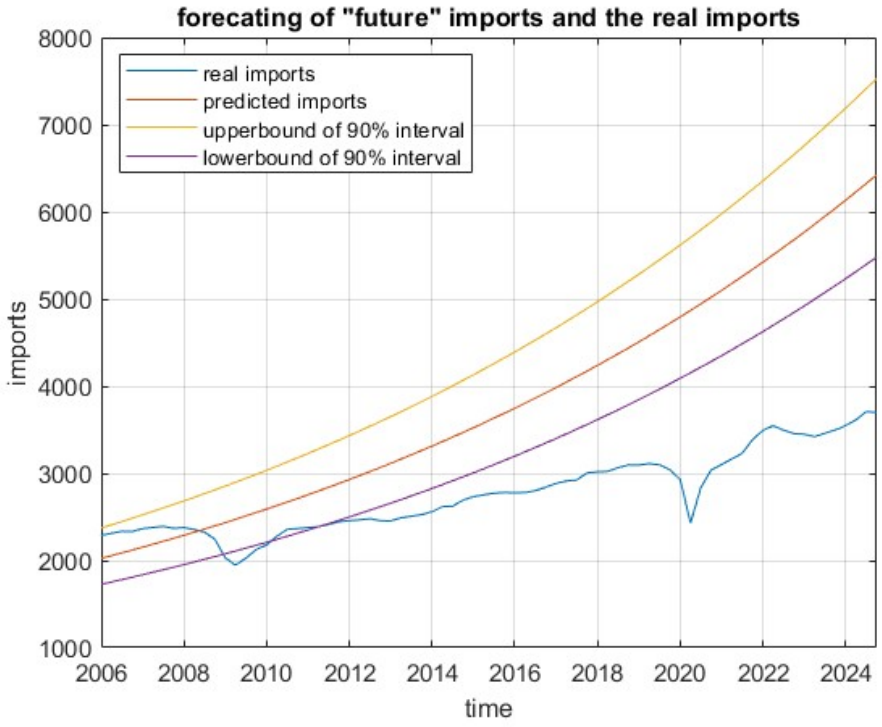

Figure 8: figure for part g

(h) Now re-estimate using the full sample 1947–2024Q4. Report results. Generate point and 90% interval forecasts for the level of imports for the next 16 quarters (4 years).

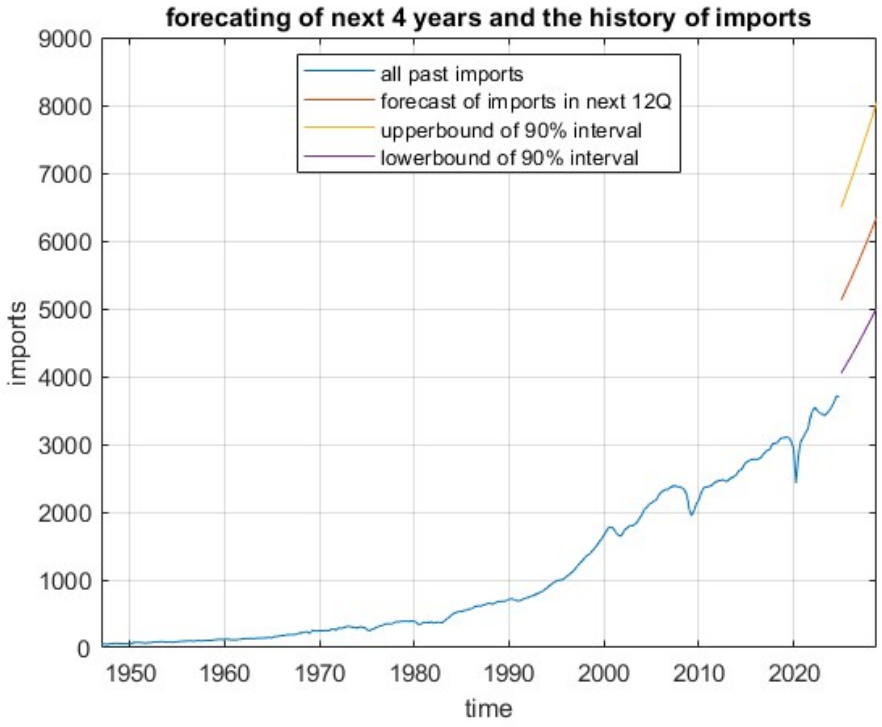

Figure 9: figure for part h

(i) Do the forecasts appear reliable or unreliable?
It is likely to be unreliable. As we have solved, the reality has become deviating from the exponential forecasting starting from 2014. The number of imports in subsequent quarters are likely to be overestimated by point forecasting based on recent years data. And the real imports are likely to be out of the forecasting interval also.