Problem Set 7

Due Wednesday March 5, 4pm

Data Exercises

(1) For this problem you will create simulated data.

Simulate $T = 1000$ observations from each of the four models given below, with ε_t i.i.d. $\mathcal{N}(0,1)$ and $y_0 = 0$. For each series, graph the autocorrelation function, and estimate a MA(2) model. Comment and discuss. For estimation, you can use the commands (where y is the name of the series you created)

Mdl = arima(0,0,2); estimate(Mdl,y)

(a) $y_t = \varepsilon_t$ (white noise)

(b) $y_t = \varepsilon_t + 0.8\varepsilon_{t-1}$

(c) $y_t = \varepsilon_t - 0.6\varepsilon_{t-1} + 0.4\varepsilon_{t-2}$

(d) $y_t = 0.5y_{t-1} + \varepsilon_t$

(2) The series "*pdi_nonresidential*" from the "realgdpgrowth.xlsx" file is real gross private domestic nonresidential investments.

(a) Graph the autocorrelation functions for the series. Is it consistent with MA(q) model? If so, what should be the order q?

(b) Estimate models MA(1)–MA(4). How do your estimates relate to your answer in (a)? Discuss.

Theoretical Questions

(3) Consider the following MA(1) process

$$y_t = 2.3 - 0.95\varepsilon_{t-1} + \varepsilon_t.$$

with $\varepsilon_t \sim$ WN(0,1).

(a) What is the optimal forecast for time periods $T + h$, $h = 1, 2, 3$. Write your answer as a function of $y_1, ..., y_T$ and/or $\varepsilon_1, ..., \varepsilon_T$

(b) Now suppose that $\varepsilon_T = 0.4$ and $\varepsilon_{T-1} = -1.2$. Re-answer part (a)

(4) Consider the following MA(2) process

$$y_t = \varepsilon_t + 4.5\varepsilon_{t-1} + 2\varepsilon_{t-2}.$$

with $\varepsilon_t \sim \text{WN}(0,1)$. Is it invertible?

(5) Suppose that $\{\varepsilon_t\}$ and $\{\eta_t\}$ are both white noise processes. Is $\{\varepsilon_t + \eta_t\}$ a white noise process? Provide a sketch of a proof or a counter-example to support your answer.