Problem Set 9

Due Wednesday March 26, 4pm

Data Exercises

(1) In the file "realgdpgrowth.xlsx" the series *pdi* contains quarterly U.S. seasonally adjusted aggregate investment growth rates, from the BEA. Estimate an AR(4) model for this series. Which lags appear to be most important? Generate a point and 90% interval one-step-ahead forecast for 2024q4.

(2) In the same file the series *exports* is aggregate U.S. exports (growth rates). Estimate an AR(4) model for this series. Which lags appear to be important? Do you notice anything interesting about the coefficients on the relevant lags? Generate a point and 90% interval one-step-ahead forecast for 2024q4.

(3) In the same file the series *pdi_residential* is aggregate residential investment growth rates. Using an AR(4) model, generate point and interval forecasts for 2024q4 through 2025q3. Create a plot of the point and interval forecasts.

(4) The FRED labels for monthly retail sales of sporting goods stores is MRTSSM45111USN. Use it to get the data for 1992m1-2025m1.

 (a) Graph the time series.

 (b) What model should be used for the trend? Seasonal? Cycle?

 (c) Estimate the model for forecast horizons 1 through 12.

 (d) Generate point and 90% interval forecast for each horizon, and plot your forecasts.

Theoretical Questions

(5) Take a stationary AR(p) process

$$y_t = \alpha + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \varepsilon_t$$

Calculate the mean $E(y_t) = \mu$.

(6) Consider AR(3) process

$$y_t = \frac{11}{6}y_{t-1} - y_{t-2} + \frac{1}{6}y_{t-3} + \varepsilon_t.$$

Is y_t stationary? Is $z_t := y_t - y_{t-1}$ stationary?

(7) Consider a components model

$$y_t = T_t + C_t,$$

$$T_t = 1 + 4t,$$

$$C_t = \beta_1 C_{t-1} + \beta_2 C_{t-2} + \varepsilon_t.$$

(a) Rewrite the components model as a regression on the trend and lags of y_t.

(b) Compare trends in your regression when $\beta_1 + \beta_2 < 1$ and when $\beta_1 + \beta_2 = 1$.