# Problem Set 9

Due Wednesday, March 26, 4pm

## Data Exercises

1. In the file `realgdpgrowth.xlsx`, the series `pdi` contains quarterly U.S. seasonally adjusted aggregate investment growth rates from the BEA. Estimate an AR(4) model for this series. Which lags appear to be most important? Generate a point and 90% interval one-step-ahead forecast for 2024Q4.

ARIMA(4,0,0) Model (Gaussian Distribution):

| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| Constant | 6.1137 | 1.278 | 4.7839 | 1.7189e-06 |
| AR{1} | 0.14269 | 0.051047 | 2.7952 | 0.0051869 |
| AR{2} | 0.13073 | 0.046139 | 2.8335 | 0.0046048 |
| AR{3} | -0.05952 | 0.042063 | -1.415 | 0.15706 |
| AR{4} | -0.28709 | 0.041796 | -6.8689 | 6.4705e-12 |
| Variance | 385.2 | 21.032 | 18.315 | 6.2543e-75 |

Figure 1:

**The first, second, and fourth lags are significant. The fourth lag appears to be most important since its coefficient is statistically significant and its magnitude is the biggest one among all coefficients. The point forecast for 2024Q4 is 6.8977, and the 90% forecast interval is [-25.3881, 39.1835].**

2. In the same file, the series `exports` represents aggregate U.S. exports (growth rates). Estimate an AR(4) model for this series. Which lags appear to be important? Do you notice anything interesting about the coefficients on the relevant lags? Generate a point and 90% interval one-step-ahead forecast for 2024Q4.

**The first and second lags are significant. The second lag appears to be most important since its coefficient is statistically significant and its magnitude is the biggest one among all coefficients.**

ARIMA(4,0,0) Model (Gaussian Distribution):

| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| Constant | 5.5794 | 1.2333 | 4.5239 | 6.0725e-06 |
| AR{1} | -0.094329 | 0.046448 | -2.0309 | 0.04227 |
| AR{2} | 0.12891 | 0.052684 | 2.4468 | 0.014413 |
| AR{3} | 0.059564 | 0.054531 | 1.0923 | 0.2747 |
| AR{4} | -0.052503 | 0.061814 | -0.84937 | 0.39567 |
| Variance | 299.59 | 12.974 | 23.093 | 5.4814e-118 |

Figure 2:

**The point forecast for 2024Q4 is 4.5904, and the 90% forecast interval is [-23.8825, 33.0634].**
**It is interesting to see that the coefficient of lags changes from negative, positive, positive, to negative. It is similar to what we think the business cycle is. The cycle should have some negative and positive alternating, but the changing does not follow a clear rule.**

3. In the same file, the series `pdi residential` represents aggregate residential investment growth rates. Using an AR(4) model, generate point and interval forecasts for 2024Q4 through 2025Q3. Create a plot of the point and interval forecasts.

ARIMA(4,0,0) Model (Gaussian Distribution):

| | Value | StandardError | TStatistic | PValue |
|---|---|---|---|---|
| Constant | 2.5922 | 1.1057 | 2.3444 | 0.019058 |
| AR{1} | 0.47255 | 0.034384 | 13.743 | 5.5842e-43 |
| AR{2} | 0.079359 | 0.047017 | 1.6879 | 0.091437 |
| AR{3} | 0.0025396 | 0.051074 | 0.049724 | 0.96034 |
| AR{4} | -0.2311 | 0.05029 | -4.5952 | 4.3229e-06 |
| Variance | 293.43 | 16.162 | 18.156 | 1.1535e-73 |

Figure 3:

**The point forecast and forecast interval are:**

```
Point Forecast: -0.2049
Point Forecast: -1.0189
Point Forecast: 2.7306
Point Forecast: 4.7949

90% Confidence Interval: [-28.3834, -32.1852]
90% Confidence Interval: [-29.5815, -27.9264]
90% Confidence Interval: [27.9736, 30.1473]
90% Confidence Interval: [35.0428, 37.5162]
```

Figure 4:

**Figure 5 is the plot for the forecasts:**

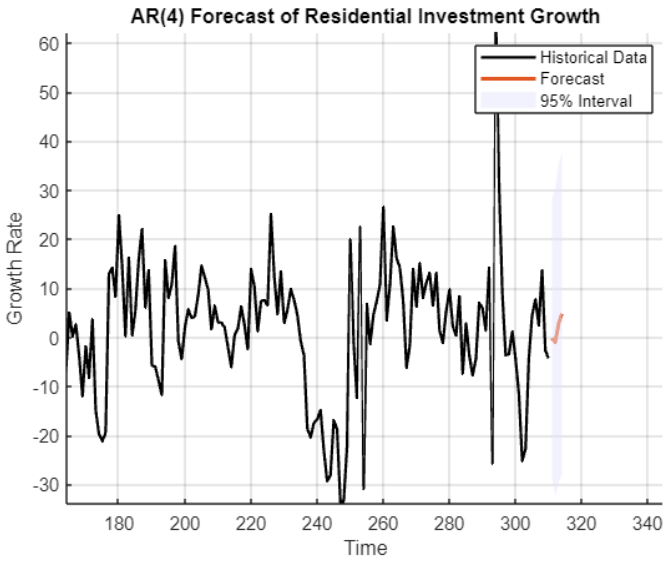


Figure 5:

4. The FRED label for monthly retail sales of sporting goods stores is `MRTSSM45111USN`. Use it to obtain the data for 1992M1-2025M1.

   (a) Graph the time series.
   **Figure 6**

   (b) What model should be used for the trend? Seasonal? Cycle?
   **I would use the model with all trends, seasonal, cycles.**

   (c) Estimate the model for forecast horizons 1 through 12.
   **There will be 12 separate regressions by using the direct method. For horizon h, you will regress $y_t$ on cons, $y_{t-h}...y_{t-h-p+1}$. In AR(12) model, p=12 and h=1:12 for 12 horizons. Please check the file "PS9Q4c" for 12 regression tables.**

   (d) Generate point and 90% interval forecasts for each horizon, and plot your forecasts.
   **Figure 7**

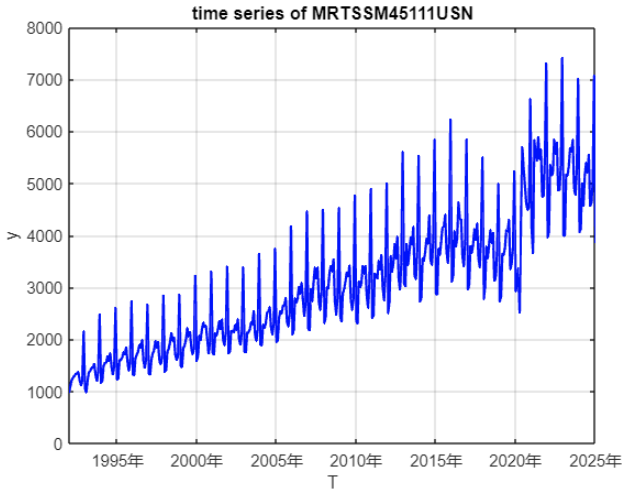


Figure 6:

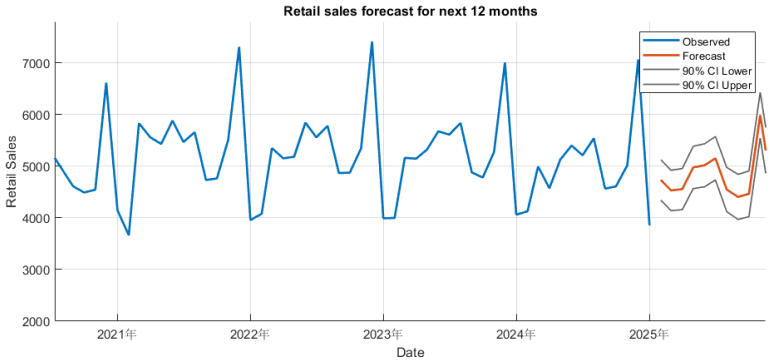


Figure 7: