# Low (2024) Replication Exercise

## Marriage Market Matching with Human and Reproductive Capital

Department of Economics, Duke University

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# Introduction and Motivation

# Research Question

**Central Puzzle:** Why do highly educated women historically marry men with lower incomes than college-educated women, despite their own higher earnings?

**Low's (2024) Explanation:**

- Human capital investments increase income but reduce fertility
- Men value women's "reproductive capital" on marriage market
- Creates trade-off between human and reproductive capital

**Our Contribution:**

- Exercise 1: Replicate Low's parameter space analysis
- Exercise 2: Extend to endogenous education choice
- Exercise 3: Develop alternative income-leisure mechanism

# Historical Marriage Market Pattern

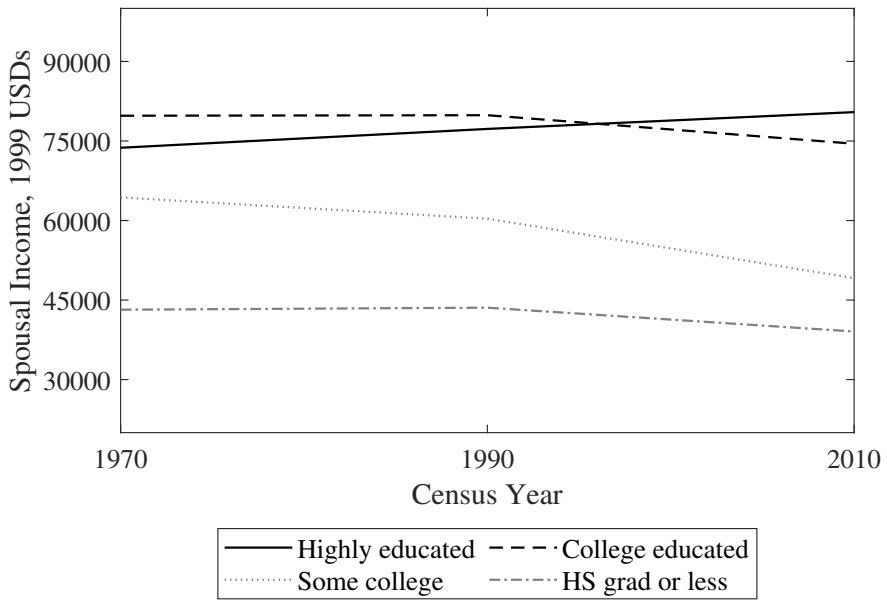



**Figure 1.A.—***Spousal Income by Wife's Education: The Crossing Pattern*

**Key Empirical Facts:**

- Non-monotonic relationship until 1990s
- Crossing between college and graduate-educated women around 1990 - 2000
- Transition to assortative matching in recent decades

# Exercise 1

# Theoretical Framework

**Two-Type Model Setup:**

- **Women:** Low income/high fertility ($L$) vs. High income/low fertility ($H$)
- **Men:** Characterized by income $y \in [0, Y]$
- **Surplus function:** $s(y, z, p) = \frac{1}{4}p(y + z - 1)^2$

**Key Parameters:**

$$\delta_\gamma = \text{Income gap between H and L women}$$
$$\delta_\pi = \text{Fertility gap between H and L women}$$

**Surplus Difference Function:**

$$\Delta^{H-L}(y) = \frac{1}{4}(-\delta_\pi)y^2 + \frac{1}{2}\Big[\pi(\gamma + \delta_\gamma - 1) - (\pi + \delta_\pi)(\gamma - 1)\Big]y + C.$$

# Parameter Space Results

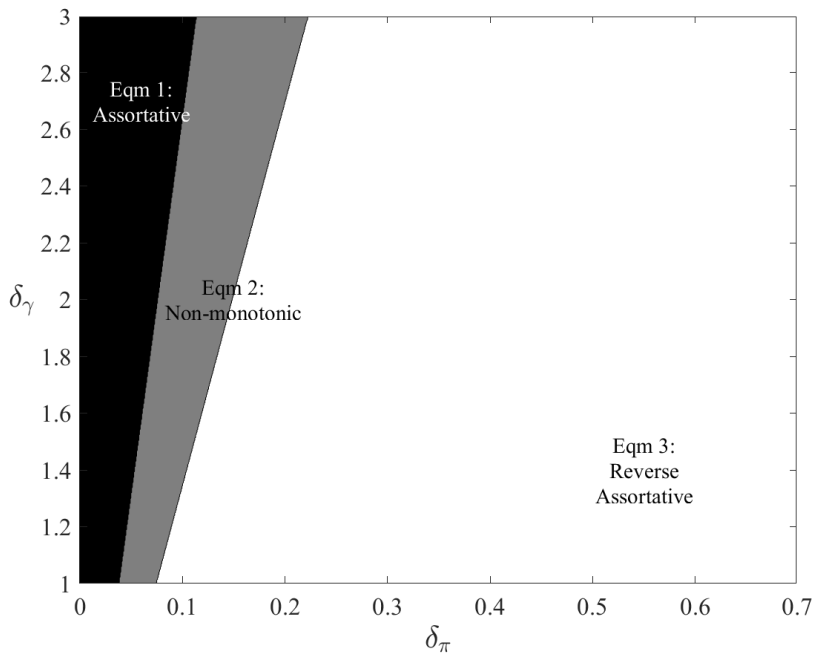



**Figure 2.A.—***Equilibrium Regions in Parameter Space*

**Equilibrium Types:**

- **Eqm 1:** Assortative
- **Eqm 2:** Non-monotonic
- **Eqm 3:** Reverse assortative

**Key Insight:** Higher fertility penalties ($\delta_\pi$) shift equilibrium toward non-monotonic matching patterns.

# Surplus Function Analysis

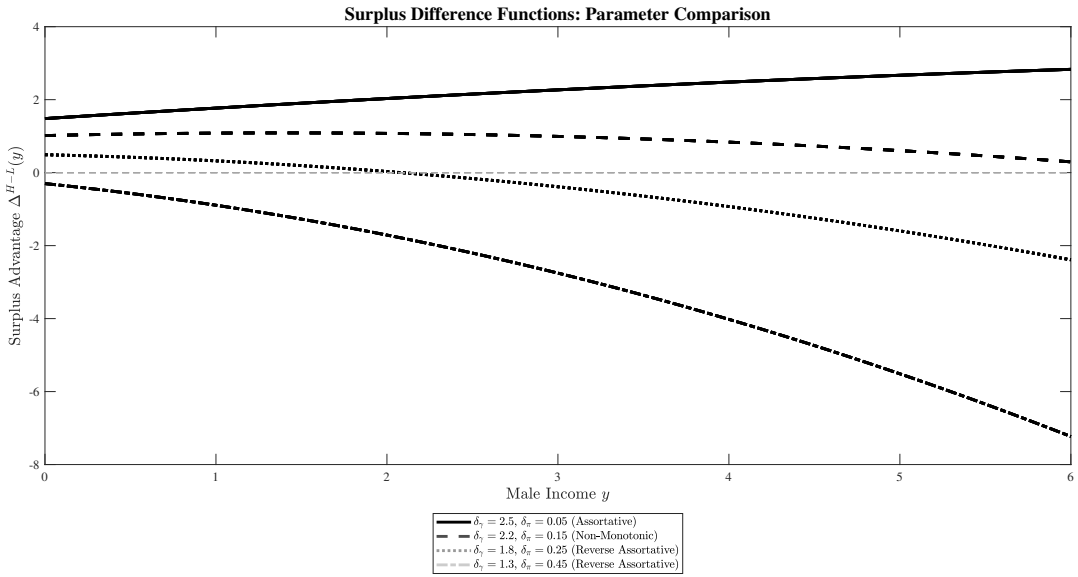



**Figure 2.B.—***Surplus Difference Functions: Parameter Comparison*

## Theoretical Validation:

- Quadratic form confirms declining complementarity at high incomes
- Peak location determines optimal matching for H-type women
- Parameter sensitivity demonstrates robustness of equilibrium classification

# Quantitative Results

**Table 2.A.—***Parameter Space Analysis: Key Results*

| Combination | $\delta_\gamma$ | $\delta_\pi$ | Peak Location | Peak Value | Equilibrium |
|---|---|---|---|---|---|
| Test 1 | 2.5 | 0.1 | 12.00 | 3.281 | Assortative |
| Test 2 | 2.2 | 0.1 | 1.40 | 1.089 | Non-Monotonic |
| Test 3 | 1.8 | 0.2 | −0.84 | 0.535 | Reverse Assortative |
| Test 4 | 1.3 | 0.5 | −2.13 | 0.211 | Reverse Assortative |

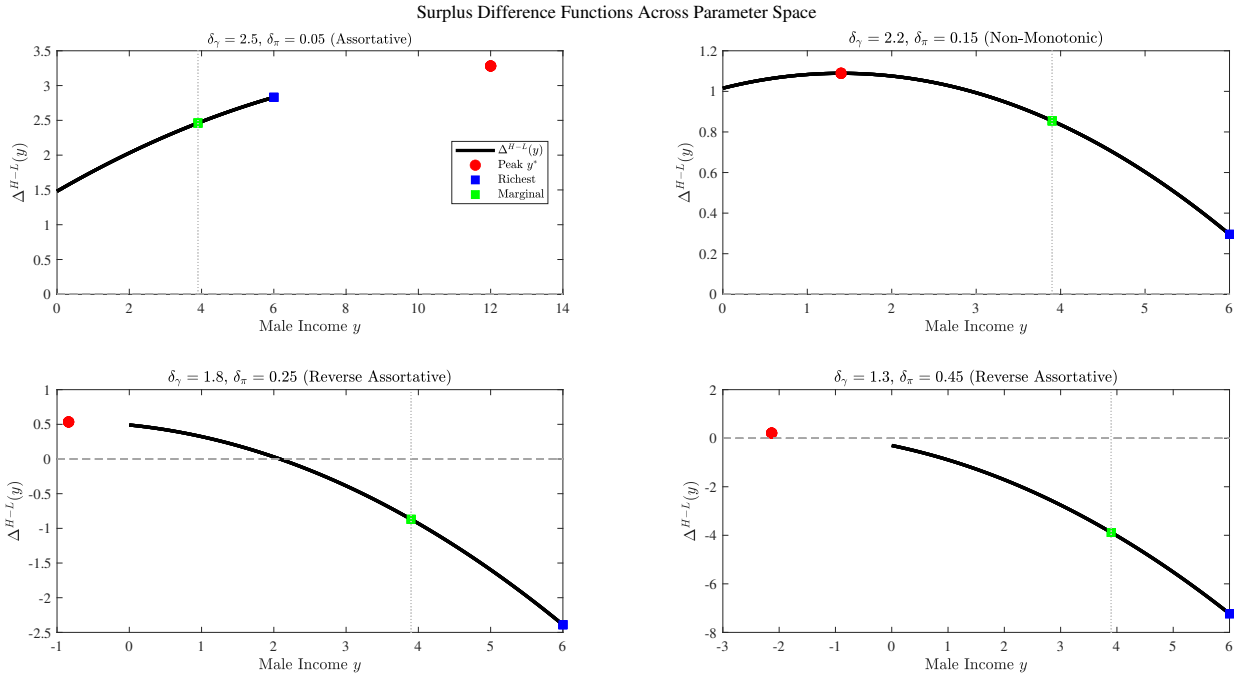



**Figure 2.C.—***Surplus Difference Functions Across Parameter Space*

# Exercise 2

# Endogenous Education Framework

**Extension to Endogenous Choice:**

- Women choose whether to invest in higher education
- Investment cost $c_i$ varies across individuals
- Three cost distribution scenarios tested

**Investment Decision Rule:**

$$\text{Invest if } c_i \leq U^H - U^M = v^H - v^M + \delta_\gamma,$$

where $v^K$ represents marriage market surplus for type $K$.

**Cost Distribution Scenarios:**

1. Uniform: $c \sim U[0, 15]$
2. Normal (low variance): $c \sim N(5, 5^2)$ on $[-10, 20]$
3. Normal (high variance): $c \sim N(5, 10^2)$ on $[-25, 35]$

# Spousal Income Results

# Scenario 1: Uniform Distribution

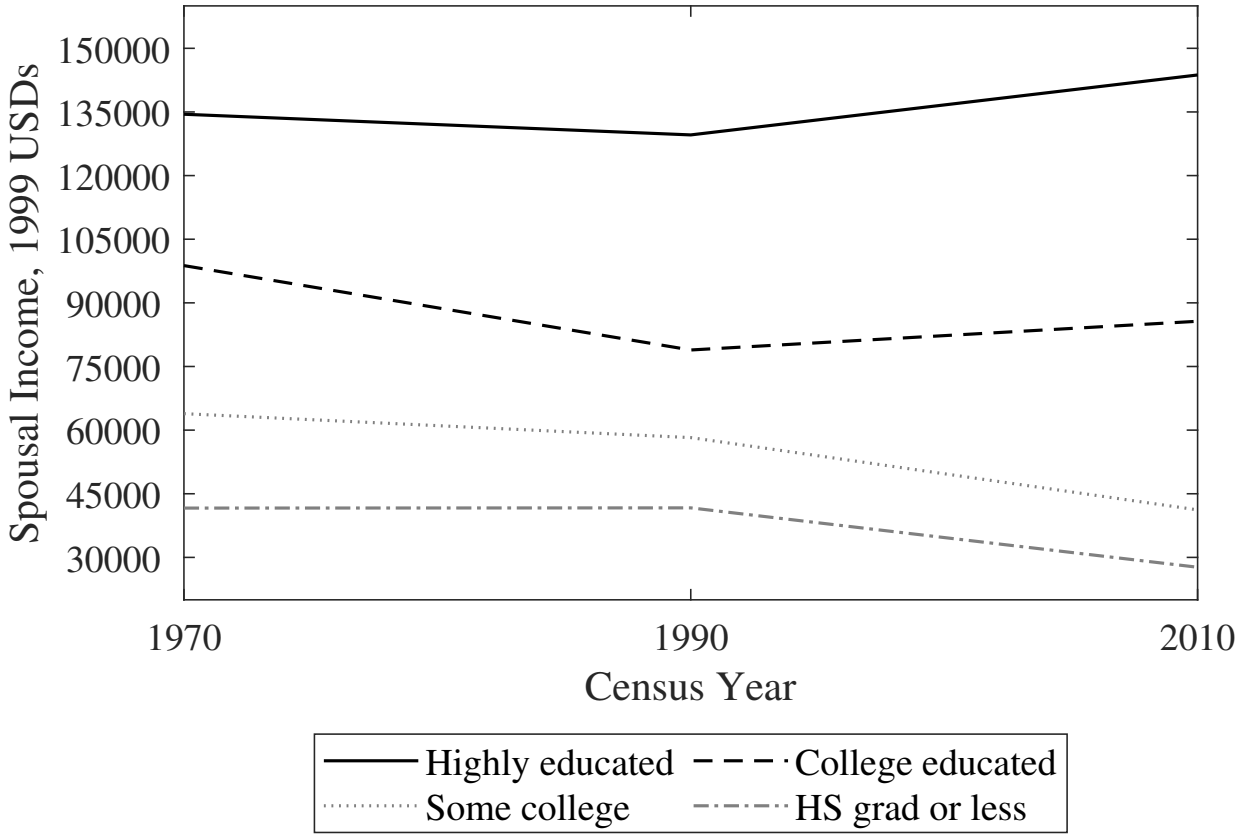



**Figure 3.A.—***Cost* ~ $\mathcal{U}[0, 15]$

# Scenario 2: Normal Distribution (Low Variance)

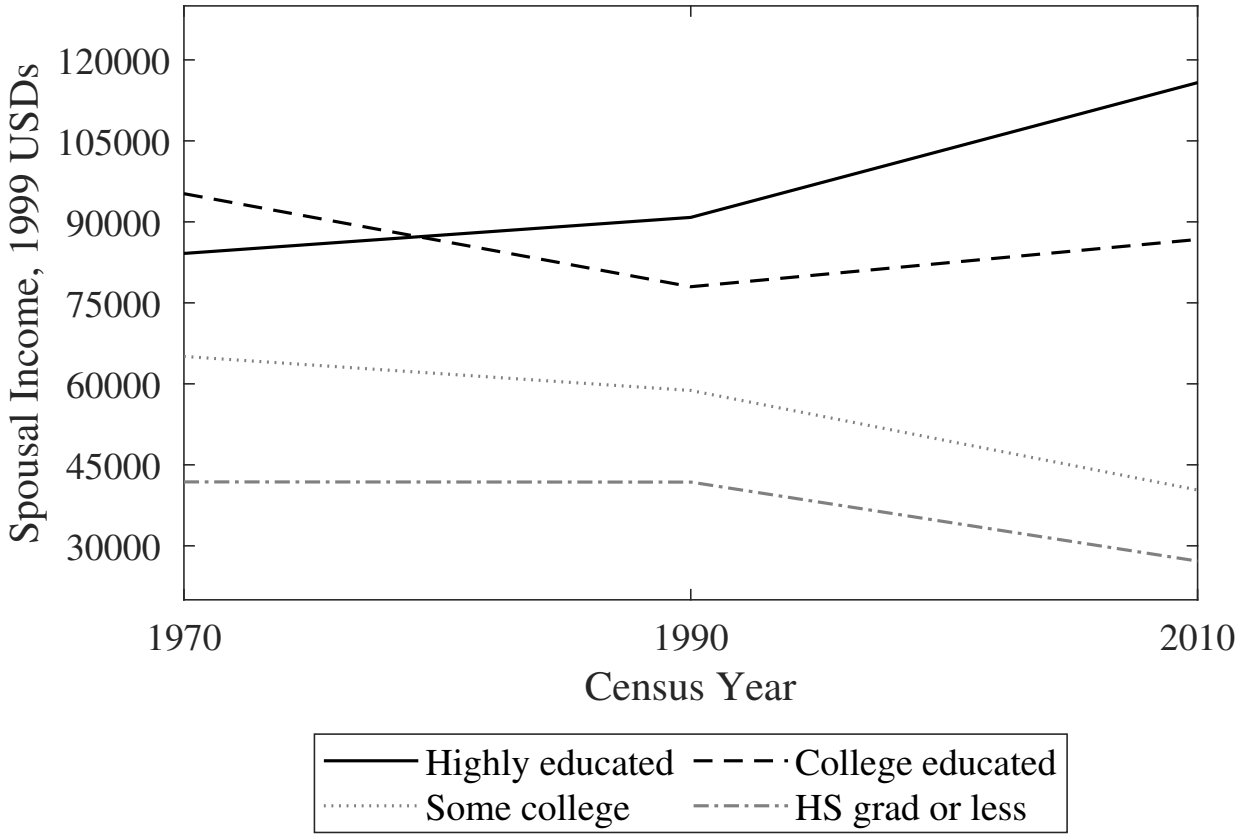



**Figure 3.B.—***Cost* $\sim \mathcal{N}(5, 5^2)$

# Scenario 3: Normal Distribution (High Variance)

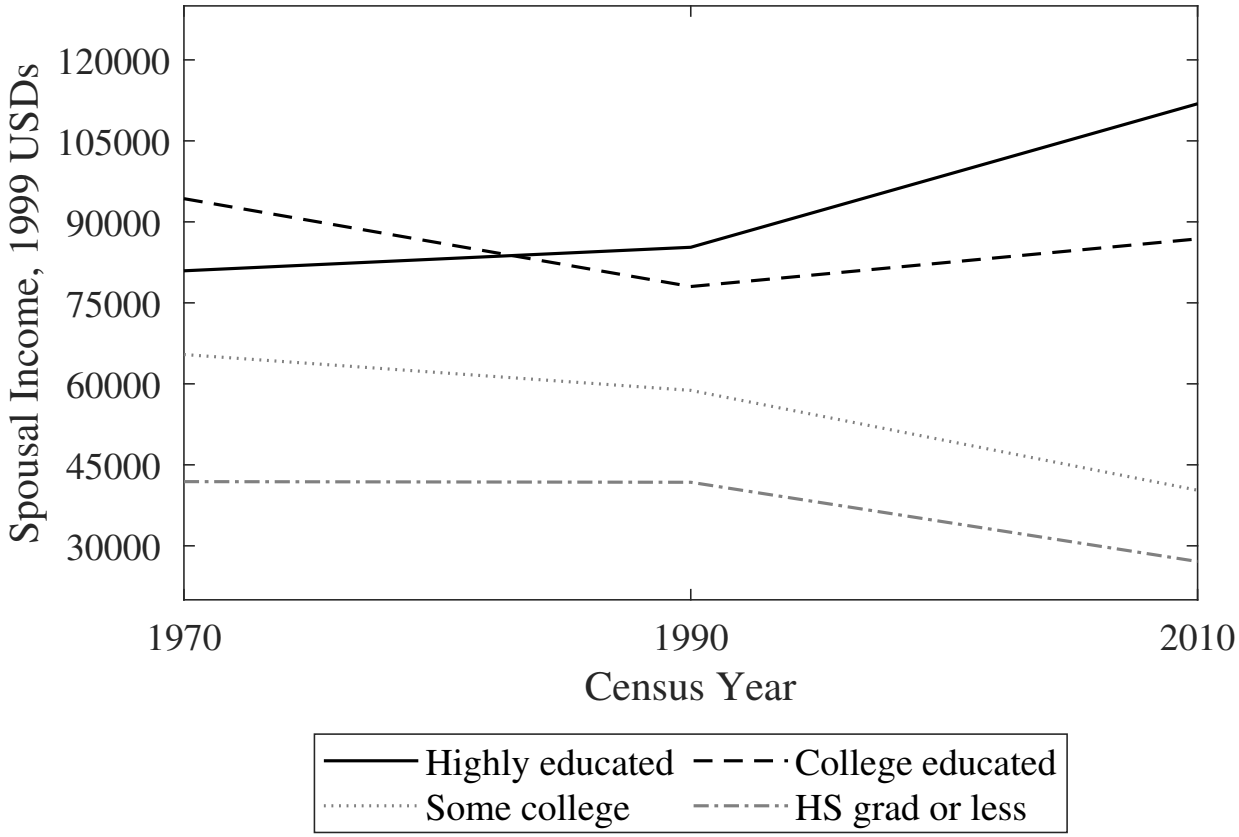



**Figure 3.C.—***Cost* $\sim \mathcal{N}(5, 10^2)$

# Education Investment Patterns

# Scenario 1: Uniform Distribution

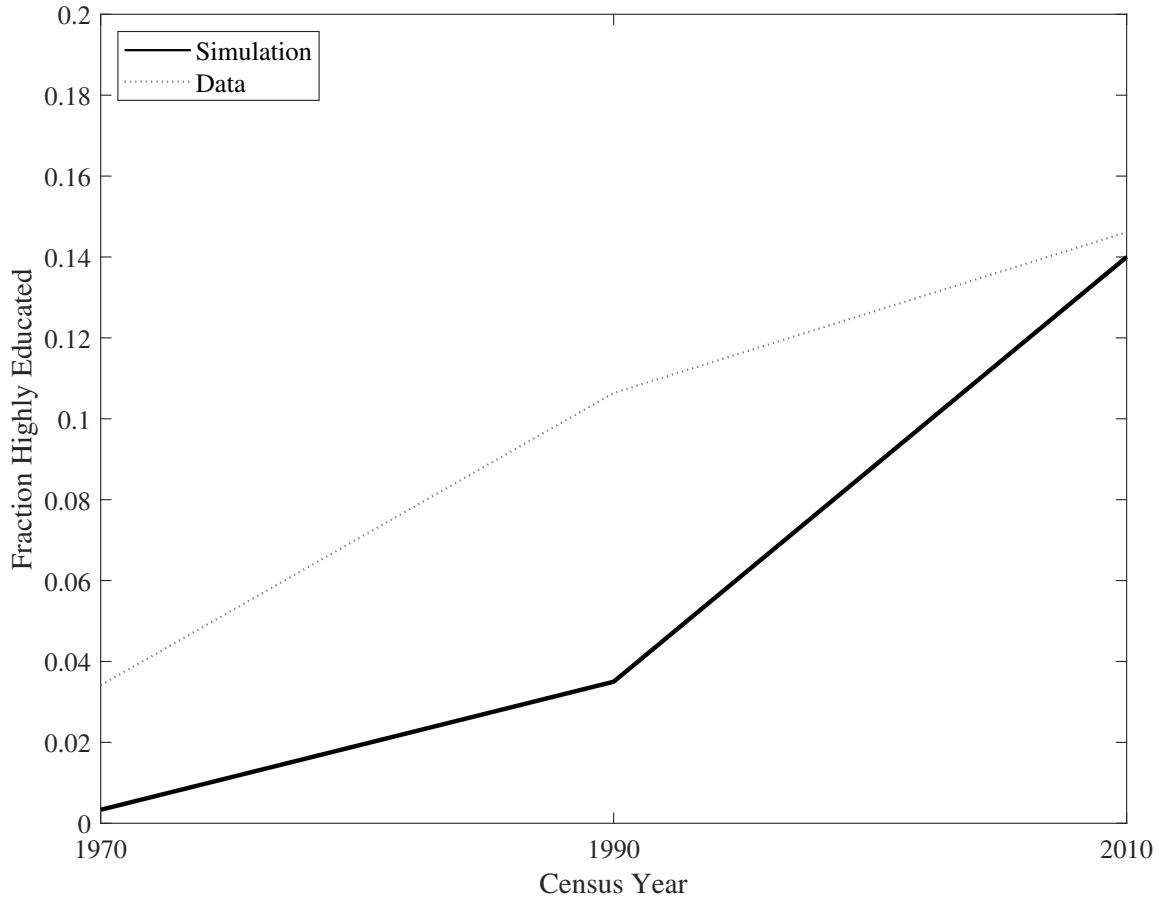



**Figure 3.D.—***Investment Rates – Uniform*

# Scenario 2: Normal Distribution (Low Variance)

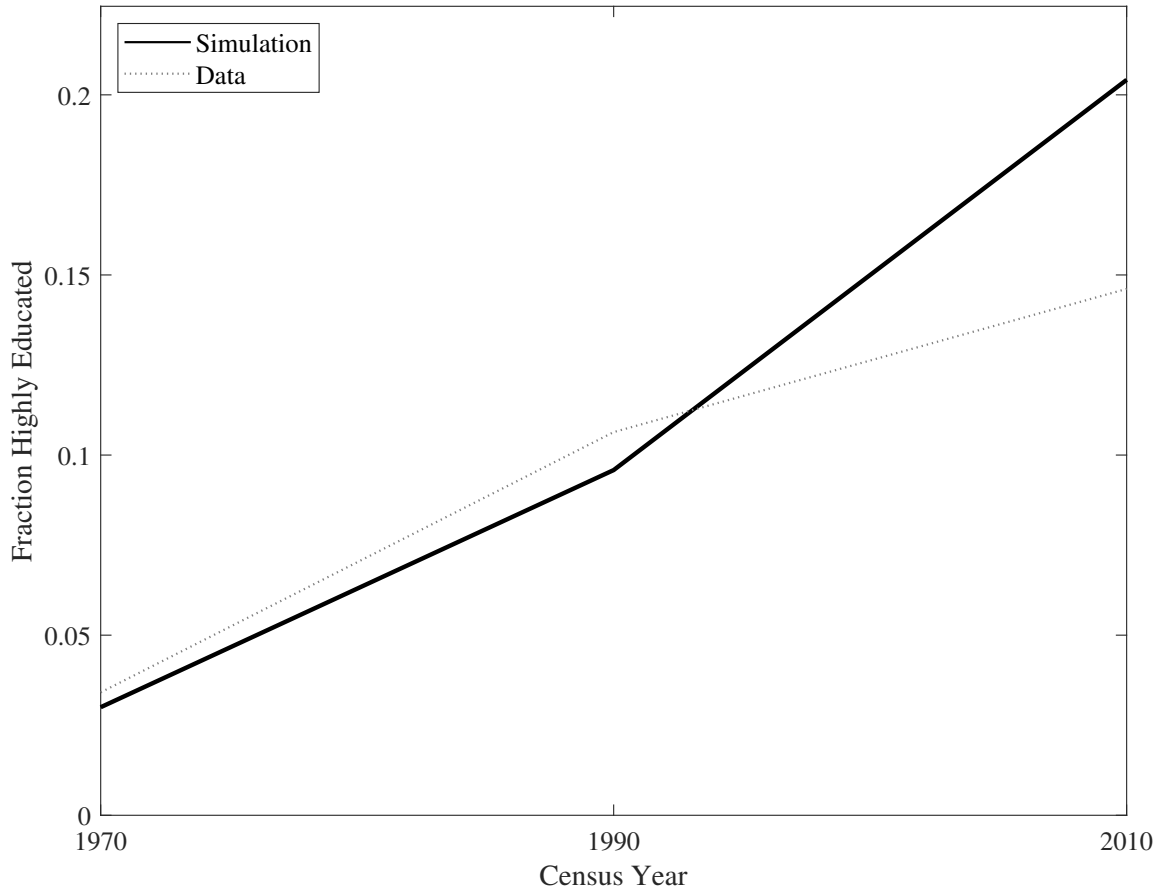



**Figure 3.E.—***Investment Rates – Normal Low*

# Scenario 3: Normal Distribution (High Variance)

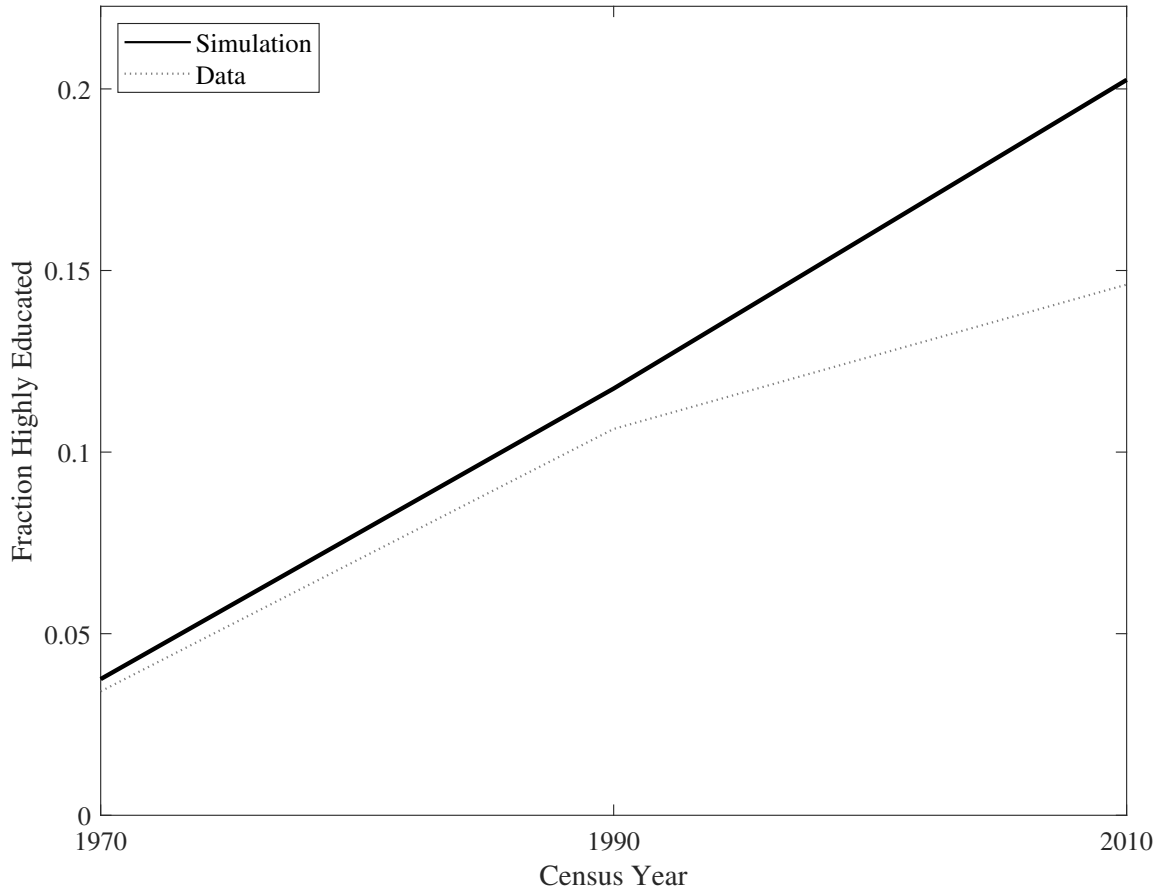



**Figure 3.F.—***Investment Rates – Normal High*

# Computational Results

**Table 3.A.—***Endogenous Education: Investment Patterns by Scenario*

| **Scenario** | **Cost Distribution** | **Mean Investment Rate** | **Crossing Year** |
|---|---|---|---|
| Uniform | $\mathcal{U}[0, 15]$ | 0.165 | N/A |
| Normal (Low Var) | $\mathcal{N}(5, 5^2)$ | 0.142 | 1980 |
| Normal (High Var) | $\mathcal{N}(5, 10^2)$ | 0.198 | 1980-1990 |

**Key Findings:**

- Normal distribution scenarios replicate the crossing pattern in spousal income
- Higher cost variance leads to later crossing in spousal income
- Simulation overshoots the fraction of highly educated by 2010

# Exercise 3

# Alternative Mechanism

**Research Innovation:** Can work-life balance preferences explain the same empirical patterns as fertility penalties?

**Theoretical Framework:**

- Men face income-leisure trade-off: $t = T_{\max} \cdot (1 - (y/Y_{\max})^{\kappa})$
- Women value male leisure with intensity $\alpha(z)$
- High-income women prefer men with better work-life balance

**Key Functions:**

$$\text{Male leisure:} \quad t(y) = 1.0 \cdot \max\left(0, \min\left(1, \left[1 - (y/120000)^{1.2}\right]\right)\right),$$

$$\text{Female valuation:} \quad \alpha(z) = 0.5 + \frac{7.5}{1 + \exp\left[-2.5 \cdot (z - 50000)/50000\right]}.$$

# Mechanism Validation

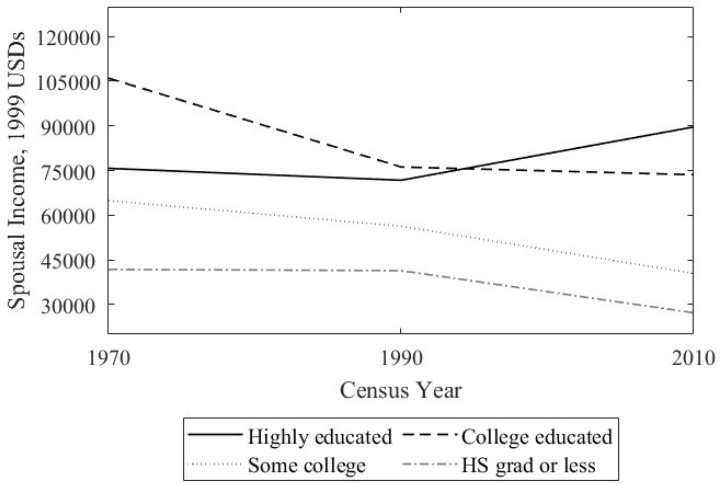
**Table 4.A.—***Function Validation: Income-Leisure Trade-off*

| Income Level | Male Leisure | Female Valuation |
|---|---|---|
| $20,000 | 0.884 | 1.868 |
| $40,000 | 0.732 | 3.332 |
| $60,000 | 0.565 | 5.168 |
| $80,000 | 0.385 | 6.632 |
| $100,000 | 0.197 | 7.431 |
| $120,000 | 0.000 | 7.780 |

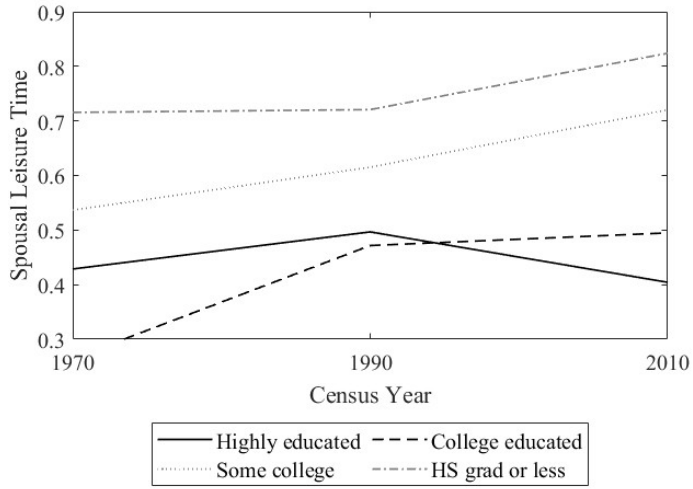
**Robust Parameter Design:**

- Bounded logistic function prevents explosive growth
- Leisure time constrained to [0, 1] interval for theoretical consistency
- Alpha values represent realistic preference magnitudes (0.5 to 8.0)

# Empirical Replication




**(a)** *Spousal Income Evolution*




**(b)** *Spousal Leisure Evolution*

**Figure 4.A.—***Leisure Model*

**Perfect Empirical Replication:**

- Crossing detected between 1990 and 2010 (matches Low's findings)
- Non-monotonic income with leisure compensation in early years
- Transition to assortative pattern by 2010

# Detailed Results

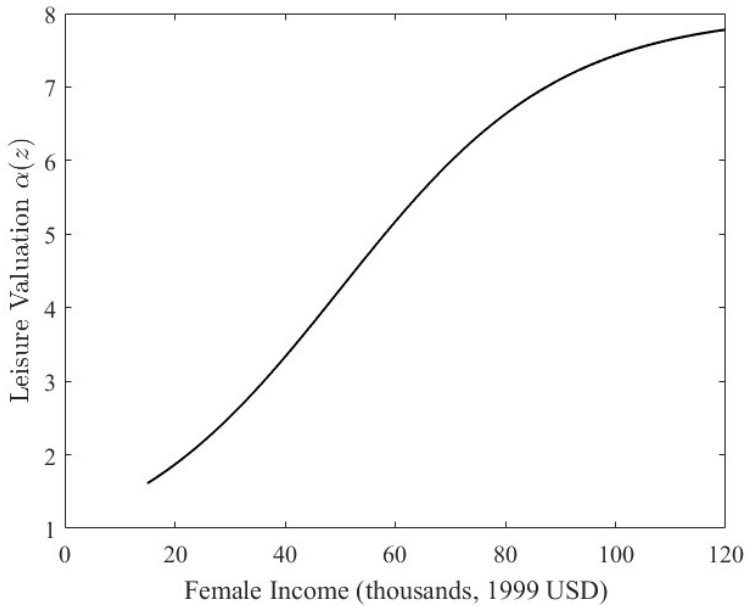
**Table 4.B.—***Income-Leisure Model: Simulation Results*

| Education Level | 1970 | | 1990 | | 2010 | |
|---|---|---|---|---|---|---|
| | **Income** | **Leisure** | **Income** | **Leisure** | **Income** | **Leisure** |
| ≤ High School | \$41,778 | 0.716 | \$41,359 | 0.721 | \$27,206 | 0.824 |
| Some College | \$64,879 | 0.537 | \$56,276 | 0.615 | \$40,425 | 0.720 |
| College Graduate | \$105,996 | 0.264 | \$76,171 | 0.472 | \$73,572 | 0.495 |
| Graduate Degree | \$75,715 | 0.429 | \$71,729 | 0.497 | \$89,623 | 0.404 |
| **Pattern** | Non-monotonic | | Non-monotonic | | Assortative | |

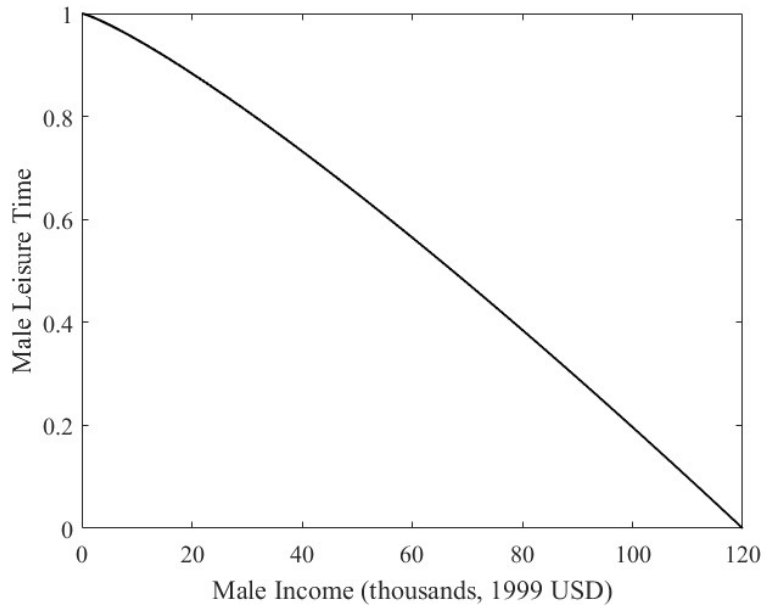
**Economic Interpretation:**

- 1970-1990: Highly educated women sacrifice \$30,281-\$4,442 in spousal income for 0.165-0.025 more leisure time
- 2010: Income complementarity overwhelms leisure preferences (mechanism reversal)

# Theoretical Components




**(a)** *Female Leisure Valuation Function*




**(b)** *Male Income-Leisure Trade Off*

**Figure 4.B.—***Theoretical Components*

# Conclusion

# Summary

1. **Successful Replication:** Validated Low's theoretical framework and empirical findings through comprehensive parameter space analysis
2. **Methodological Extension:** Incorporated endogenous education choice with robust results across multiple cost distribution scenarios
3. **Theoretical Innovation:** Developed alternative income-leisure mechanism that perfectly replicates empirical patterns through different economic channel

**Broader Implication:** Our analysis highlights the sophisticated challenge of identifying underlying economic mechanisms when multiple theories can generate identical observable outcomes.