The Human Capital-Reproductive Capital Trade-Off in Marriage Market Matching

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August 4, 2025

Overview: Research Question

- Does high earnings always yield women wealthier mates?
- Not necessary.
  Human capital investments (education) that yield greater income may also decrease another desirable marriage market trait, the reproductive capital (fertility).
- In this paper, the author wants to link education, income, and fertility together into one model.

Model Setup

- Constructing a bidimensional model where income is negatively correlated with fertility.
- Men characterized by income $(\boldsymbol{y})$
  Women characterized by income $(\boldsymbol{z})$ and fertility $(\boldsymbol{p})$
- The stable match will depend on the trade-off between human capital and reproductive capital.
- The author simulates the model using US Census data on income and fertility.

Section II. Stylized Facts in U.S.

- 1. Older ages for women at first marriage associate with poorer spouses. [1]

  This paper mainly focuses on the second fact.

- 2. Women's human capital (education level) increases earnings but decreases reproductive capital, fertility.

  Therefore, men's valuation of women's fertility can influence both matching patterns and women's incentives to invest in human capital.

[1] Low, Corinne. 2023a. "Pricing the Biological Clock: The Marriage Market Cost of Aging to Women." J. Labor Econ., forthcoming.

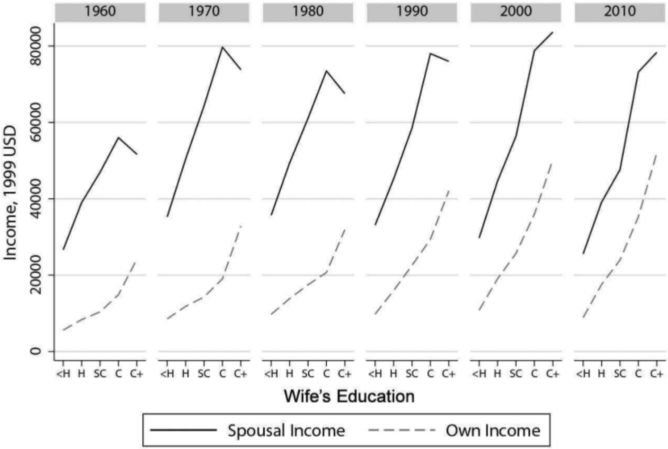


Figure: education v.s income

III. Theoretical Framework

- Assume a fully transferable utility matching model, in which the disaggregate equilibrium will be one and the same as the equilibrium that maximizes total social surplus. (Shapley and Shubik 1971; Becker 1973)
- Men are characterized by income, $\boldsymbol{y}$
- women are characterized by income, $\boldsymbol{z}$, and fertility, $\boldsymbol{p}$, the probability of conceiving.
- Individuals of men and women all value private consumption, $\boldsymbol{q}$, and spending on children, $\boldsymbol{Q}$.

III. Theoretical Framework

- To guarantee the transferable utility function, the utility functions may follow generalized quasilinear (GQL) utility. The author assumes the utility function $q(Q+1)$. Thus, for a family with a man and a woman with fertility p, the optimal solution is

$$T(y,z,p) = p\frac{(y+z+1)^2}{4} + (1-p)(y+z).$$

  T is the joint expected utility from marriage.

- Thus, the joint surplus function, defined by subtracting out the utility from being signal $(-y-z)$, which is:

$$s(y,z,p) = \frac{1}{4}p(y+z-1)^2.$$

  (Allocation inside the family is not considered yet)

III. Theoretical Framework - Supermodular function

The surplus function $s(y, z, p)$ is supermodular in both incomes and fertility.

- Def: For a lattice $(X, \succeq)$, a function $f : X \to \mathbb{R}$ is said to be supermodular if for all $x, x' \in X$,

$$f(x \vee x') + f(x \wedge x') \geq f(x) + f(x')$$

- example:
  u(m, c) is a supermodular utility function where m is the number of computers and c is the number of screen.

$$u(m + \theta, c + \delta) - u(m, c + \delta) \geq u(m + \theta, c) - u(m, c)$$

  Supermodular functions capture complementarity between two factors, preventing either factor from being universally optimal

III. Theoretical Framework - Intuitions from utility function

- When both income and fertility vary, whether the matching is positive or negative assortative on income depends on the distribution of types.
- There is a substitution between husband and wife's income in surplus, but a complementarity between the husband's income and the wife's fertility.
- The marginal benefit of a wife's income relative to fertility decreases as household income rises.

III. Theoretical Framework - Distribution of Types

Women are divided into 3 types:

- **L**: with low income $\gamma - \mu_\gamma$ and high fertility $\pi + \delta_\pi$
- **M**: with medium income γ and high fertility $\pi + \delta_\pi$
- **H**: with high income $\gamma + \delta_\gamma$ and low fertility π

mass of types follows: $g^L + g^M + g^H = 1$
Before Section III.C, we assume the mass of each type is exogenous. In this case, we only consider the matching equilibrium given different value of parameters. (Exercise 1)

Men are continuous and only characterized by income y.

B. Matching Equilibrium

A matching is defined as the probability and utility functions $u(y), v(z, p)$

A matching is stable if two conditions hold for all individuals:

$$\begin{aligned} u(y) + v(z, p) &\geq s(y, z, p), \\ u(y) \geq y, \quad v(z, p) &\geq z. \end{aligned}$$

- The first inequality becomes an equation if the pair is married to each other. **I think** the first equation holds because its negation is not reasonable.
- The second inequality guarantees each individual is better off than being single and consuming their own income.

B. Matching Equilibrium

For men, the surplus benefits from changing types of women as a function of men's income are:

$$\Delta^{\text{M-L}}(y) = s(y, \gamma, \pi + \delta_\pi) - s(y, \gamma - \mu_\gamma, \pi + \delta_\pi)$$

$$= \frac{1}{4}(\pi + \delta_\pi)\mu_\gamma(2y + 2\gamma - \mu_\gamma - 2).$$

$$\Delta^{\text{H-M}}(y) = s(y, \gamma + \delta_\gamma, \pi) - s(y, \gamma, \pi + \delta_\pi)$$

$$= \frac{1}{4}\pi\delta_\gamma(2y + 2\gamma + \delta_\gamma - 2) - \frac{1}{4}\delta_\pi(y + \gamma - 1)^2.$$

$\Delta^{\text{M-L}}(y)$ is monotonic but $\Delta^{\text{H-M}}(y)$ is quadratic. For rich men, there is a tradeoff between women's income and their fertility.

B. Matching Equilibrium

Here, the author did not consider the case that an increase in s from changing types of women results in a decrease in men's allocated utility. **A possible explanation** is in Appendix B.3. The author mentioned that the match results are from a competitive market, so the slope of the husband's utility function is equal to his contribution to surplus, which is monotonic increasing in (y, z, p).

Then, Lemma 1 states that any stable matching will exhibit the following three characteristics.

- (1) All matched men will be higher income than all unmatched men.[2]
- (2) All men matched with M women must be higher income than all men matched with L women.
- (3) The set of men matched with H women must be connected

[2] The author assumes there are more men than women so some of men will be unmatched men.

B. Matching Equilibrium

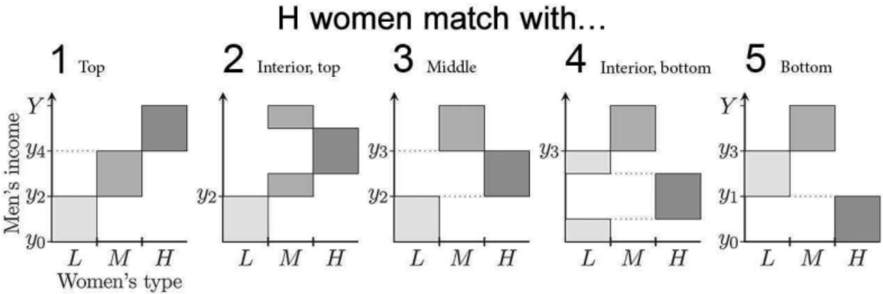


FIG. 3.—Possible matches.

The author assumes that $y_0 = 0$ and $y_1 < y_2 < y_3 < y_4 < Y$.

B. Matching Equilibrium

Follow lemma 1, we have proposition 1: Let Y represent the income of the richest man. For any set of parameters, it is possible to find a Y large enough such that the equilibrium match is nonmonotonic in income. Intuitively, assortive matching requires

$$\Delta^{\text{H-M}}(Y) \geq \Delta^{\text{H-M}}(y_4)$$

Increasing Y sufficiently will cause the condition to be violated, in which case, matching rich men with an H woman cannot be surplus maximizing.

Similarly, by considering the boundary case, one can come up with the threshold value of income premium of H, δ_γ, and fertility loss of H, δ_π.

B. Matching Equilibrium

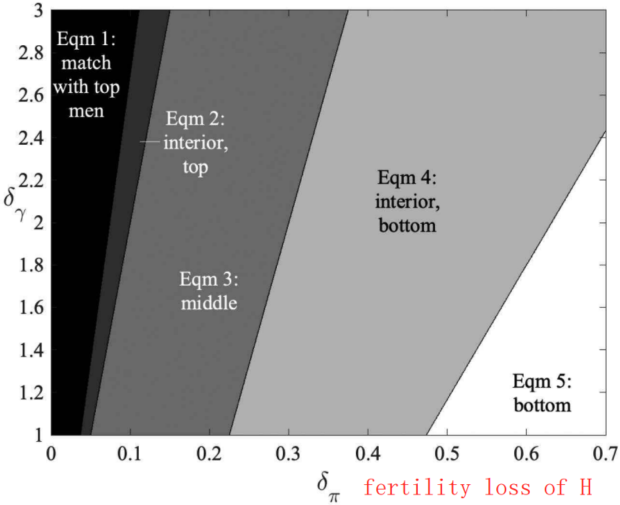


Figure: —Matching equilibrium given different parameters' value

Utility

- Men/women's equilibrium utilities $(u(y),\ v(z,p))$ can be directly solved for:

$$
\begin{aligned}
& v(z,p) = \max_y \{s(y,z,p) - u(y)\} \\
& \text{FOC: } u'(y) = \frac{\partial s(y,z,p)}{\partial y} = \frac{1}{2} p(y + z - 1) \\
& \implies u^K(y) = \frac{1}{4} p^K y \left(y + 2z^K - 2\right) + \mu^K, \quad K \in \{L, M, H\} \\
& \implies v^K = s^K(y) - u^K(y)
\end{aligned}
$$

- Women's equilibrium full utility:

$$
\begin{aligned}
U^{\mathrm{H}} &= \gamma + \delta_\gamma + v^{\mathrm{H}} \\
U^{\mathrm{M}} &= \gamma + v^{\mathrm{M}} \\
U^{\mathrm{L}} &= \gamma - \mu_\gamma + v^{\mathrm{L}}
\end{aligned}
$$

Endogenous Human Capital Investment

- Women start as L or M types
- M types can invest to become H types:
  - Heterogeneous utility cost c_i of investment
  - Invest if
$$c_i \leq U^H - U^M = \delta_\gamma + v^H - v^M$$
- Equilibrium investment cost threshold $\widehat{c}$
  - Fixed point of $c_i = \delta_\gamma + v^H(c_i) - v^M(c_i)$
  - Unique equilibrium where all women with costs $c_i \leq \widehat{c}$ invest

Data Moments

- **Income data**: US Census
  - Men's income: drawn from distribution of married men (ages 41-50) in a given Census year.
  - Women's income: drawn conditional on education for women working full-time.
  - Approximate NPV of lifetime income created
- **Fertility data**: ACS
  - Empirical distribution of number of children at home conditional on education (women ages 38-42).

Simulation

- Surplus:

$$s(y,z,p)=\sum_{c=0}^{4}\left(p_c\left(y+z-\frac{c}{4}\frac{y+z+1}{2}\right)\left(\frac{c}{4}\frac{y+z-1}{2}\right)\right)-y-z$$

  - $c\in\{0,...,4\}$ number of children
  - p_c probability of achieving c children, where $\sum_{c=0}^{4}p_c=1$

Simulation

Procedure:

1. 800 random draws of men and women from empirical income distributions
2. Compute surplus $s(y, z, p)$ for every possible man-woman pair, using empirical fertility distribution and NPV incomes, and create surplus matrix.
3. Use computational matching algorithm (Hungarian method) to identify set of matches that maximizes total marital surplus.
4. For each educational group of women, average the predicted husbands' income
5. Repeat simulation 5 times and average.

Simulation: Result

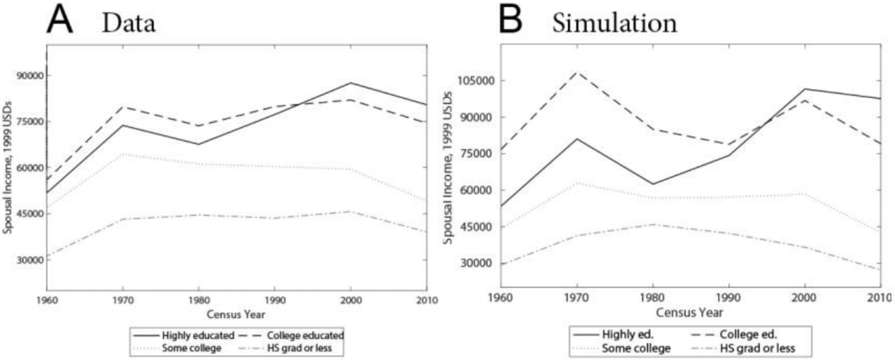


Figure: 7

Simulation: Endogenous Human Capital Investment

Procedure:

- Draw number of college educated women to match the total number of college and highly-educated women.
- Assign heterogeneous utility cost ranging uniformly from -10^{11} to 2×10^{11}.
- Create matrix with investment and no-investment total marital surplus, choose the maximum.
- Continue with previous steps 3-5.

Simulation: Results

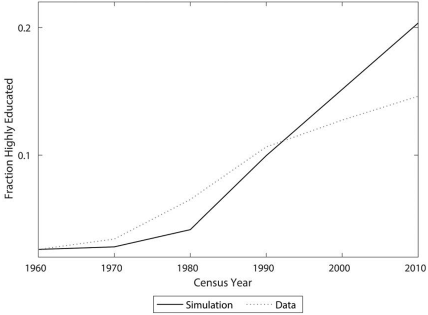
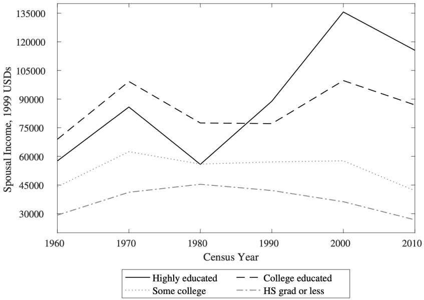


Figure: 8

Figure A7: Predicted Spousal Income with Endogenous Education



Summary

- Bidimensional matching: Women valued for both their income and fertility.
    - Negative relationship between women's income and fertility
    - Graduate education increases income but delays childbearing, reducing fertility.
- Simulation reproduces historical pattern of non-monotonic matching and shift to positive assortative matching as fertility penalty and desired family size changes.