Supplemental Material (not copyedited or formatted) for: Corinne Low. 2024. "The Human Capital-Reproductive Capital Trade-Off in Marriage Market Matching." Journal of Political Economy 132(2). DOI: https://doi.org/10.1086/726238.

Online Appendix

A Appendix: Stylized Facts

Figure A1 shows that the decline in spousal income with age at marriage holds for women currently age 36-45, married up to age 35. Table A1 demonstrates that the difference in spousal income between college and highly educated women is statistically significant. Figure A2 shows that the non-monotonic relationship is also present when restricting to first marriages only, omitting 1990 and 2000 when number of marriages is not available.

Table A1: Spousal income by wife's education level

	Dependent variable: Spousal income, 1999 USD		
	(1)	(2)	(3)
1960 × highly ed.	-3,594**	-3,413**	-3,435**
	(1,209)	(1,209)	(1,209)
1970 × highly ed.	-6,653***	-6,512***	-6,541***
	(1,374)	(1,374)	(1,374)
1980 × highly ed.	-6,463***	-6,473***	-6,468***
	(914)	(914)	(915)
1990 × highly ed.	-2,605*	-2,655*	-2,656*
	(1,036)	(1,037)	(1,037)
2000 × highly ed.	4,225***	4,216***	4,203***
	(1,022)	(1,027)	(1,027)
2010 × highly ed.	6,028***	6,027***	6,018***
	(875)	(875)	(876)
Constant	55,419***	52,176***	49,156***
	(657)	(2,149)	(3,235)
Year FE	Y	Y	Y
YOB FE		Y	Y
Spouse age			Y
Observations	117,013	117,013	117,013

Notes: Regressions of spousal income on wife's education level interacted with year for women with at least a college degree, with "highly educated" constituting all formal education beyond a college degree. No constant or "highly" dummy is included, so coefficients can be interpreted as the additional spousal income for those in the highly educated category in each sample. Source: 1 percent samples of US Census data from 1960, 1970, 1980, 1990 Census, and 2000 and 2010 ACS, married US-born women 41-50 years old, weighted by Census person weights. Robust standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.1

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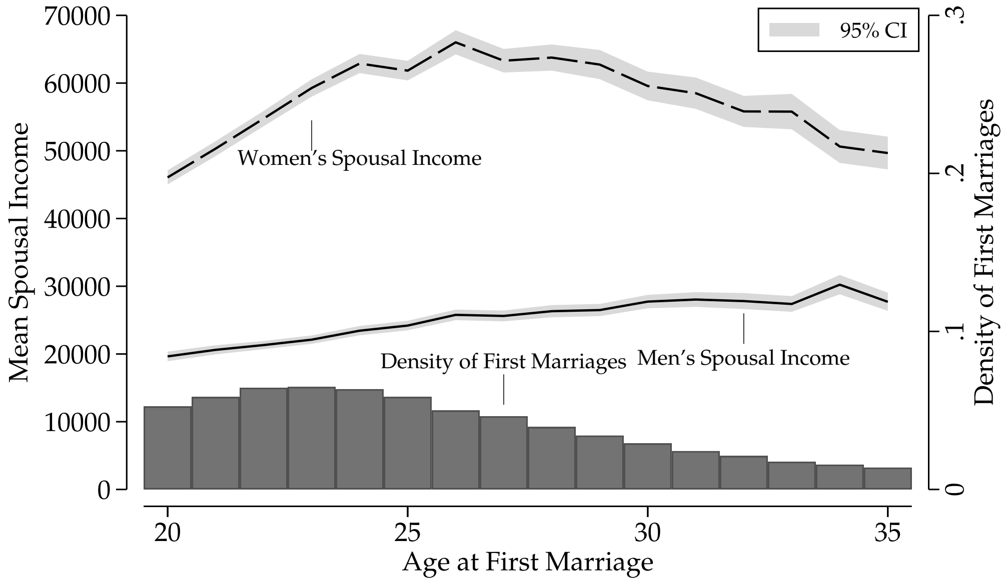
Figure A1: Spousal Income by Age at Marriage, Before Age 35



Notes: Lines represent the average spousal income for first marriages by age at marriage for women versus men. Bars represent the portion of all women's marriages occurring at that age, to check whether selection is driving the effect. Source: 2010 American Community Survey (1 percent sample) marital histories for US-born men and women, 36-45 years old.

Marriage and divorce rates Highly educated women's marriage *rates* have also risen precipitously, shown in Figure A3, another measure of an increasing marriage market premium for highly educated women. Conventional wisdom holds that too-educated, too-high-earning women are punished on the marriage market. However, Figure A3 panel (a) demonstrates that *college* educated women actually always married at rates close to all other educational categories. It is only *highly* educated women who previously had comparatively low rates of marriage, and have now experienced substantial gains. Similarly, as shown in panel (b), highly educated women previously divorced at the highest rates, while college educated women's divorce rates were on par with other educational categories. Since 1990, highly educated women's divorce rates have fallen while college educated divorce rates have leveled off, and all other categories' have risen. These trends are both matched by the model simulation.

B Appendix: Model

B.1 General Model

This section demonstrates that a simple bi-dimensional model where couples care about income and fertility can produce non-monotonic matching in incomes. In particular, matching will always

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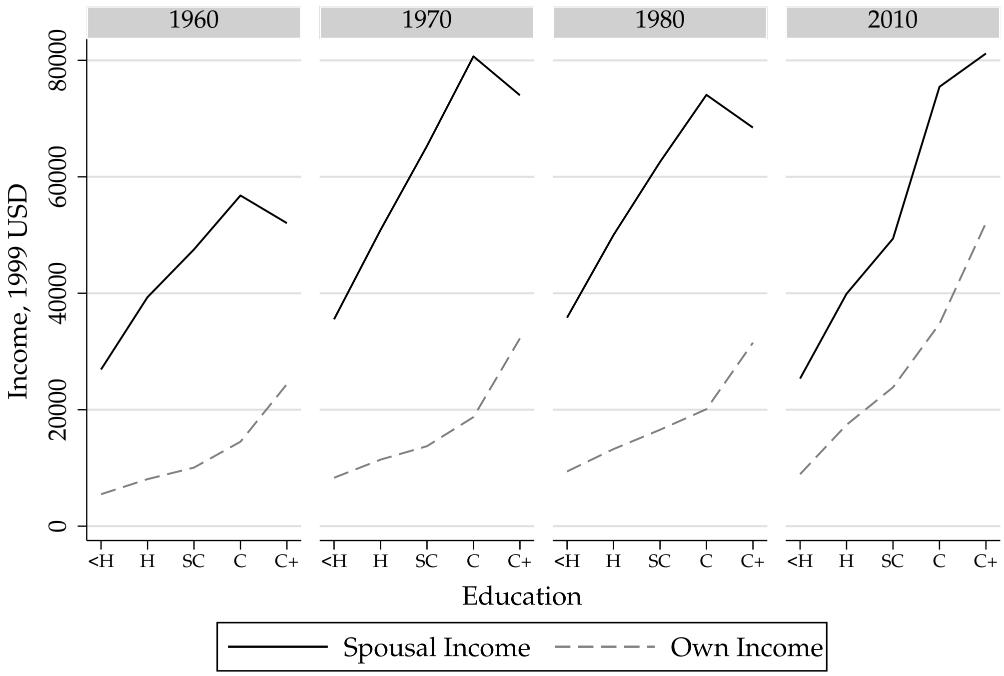
Figure A2: Non-monotonicity in spousal income by wife's education level, First Marriages Only



Notes: Income of spouse based on wife's education level. <H=less than high school, H=high school grad, SC=some college, C=college grad, C+=graduate degree. Source: 1 percent Census data from 1960, 1970, and 1980, and ACS data from 2010. Sample consists of US-born women, ages 41-50 years old, who are on their first marriage.

be assortative when women's income varies, but fertility stays constant, but need not be when income and fertility vary in tandem.

B.1.1 Model Setup

There is a two sided market with a unidimensional side, "men," and a bi-dimensional side, "women." Men are characterized solely by income, $y \in [0,Y]$, and women are characterized by both income and fertility, $(z, p) \in [0,Z] \times [0,1]$.[1]

Individuals care about both children and private consumption, with utility meeting the generalized quasi-linear (GQL) form, which is the necessary and sufficient condition for transferable utility [Bergstrom and Cornes, 1983, Chiappori and Gugl, 2014]. Thus, through maximizing the sum of utility we can identify the household surplus function, $s(y, z, p)$, increasing in all arguments.

[1] Although making men unidimensional is a simplification, it should be noted that other matching models that feature multiple characteristics are actually unidimensional as long as the characteristics can be collapsed to a single index. I focus on income as that is the key factor usually examined in models looking at societal trends in assortative matching. Many of the predictions here would also hold for other factors that could be part of a quality index, such as height or attractiveness.

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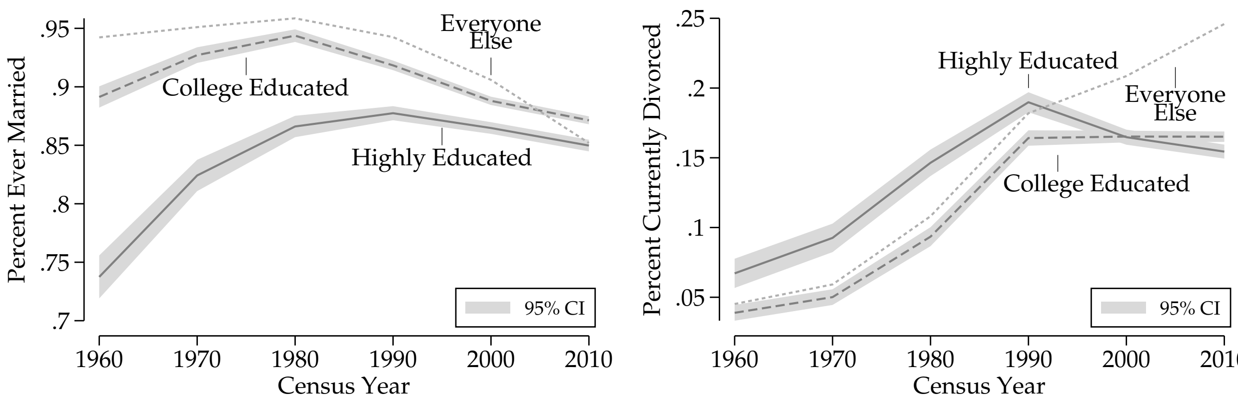
Figure A3: Marriage and divorce rates by education level

(a) Marriage (b) Divorce



Notes: Ever married and currently divorced rates by ages 41-50, for women, based on education level, with "highly educated" constituting all formal education beyond a college degree. Ever divorced rates show a similar pattern, but are not available in all years. Source: 1 percent Census data from 1960, 1970, 1980, and 1990, and American Community Survey from 2000 and 2010. Sample consists of US-born women, ages 41-50 years old, weighted by Census person weights.

Let the surplus exhibit: (1) Supermodularity in incomes: $\frac{\partial s^2}{\partial y \partial z} > 0$, and (2) Supermodularity between men's income and fertility: $\frac{\partial s^2}{\partial y \partial p} > 0$.

B.1.2 Matching Equilibrium

This surplus function will always produce assortative matching in incomes when fertility is equal across women.

Proposition 2. *Let there be two men, y and y', with $y < y'$. For any two women with incomes z and z', $z' > z$, and fertility p, the stable matching matches y' with (z', p) and y with (z, p).*

Proof. With transferable utility, the stable match will maximize total surplus. Thus, supposing by contradiction that y is paired with z' and y' with z, it must be that:

$$s(y, z', p) + s(y', z, p) > s(y', z', p) + s(y, z, p)$$
$$s(y, z', p) - s(y, z, p)) > s(y', z', p) - s(y', z, p)$$

Which would imply that, for a small change in z, $\frac{\partial s(y,\, z,\, p)}{\partial z} > \frac{\partial s(y',\, z,\, p)}{\partial z}$, which would mean s is submodular, contradicting the premise. $\square$

However, when fertility differs, the matching can be positive assortative or negative assortative on incomes, depending on the income-fertility tradeoff for women.

Proposition 3. *Let there be two women* (z, p) *and* (z', p') *with* $z<z'$, $p>p'$. *Let* $\epsilon=z'-z$ *and* $\eta=p-p'$, *both positive. Let* $\lambda=\frac{\epsilon}{\eta}$. *There exists a* λ *such that the stable matching matches* y *with* (z, p) *and* y' *with* (z', p'), *and a smaller* λ *such that the stable matching matches* y *with* (z', p') *and* y' *with* (z, p).

Proof. By total surplus maximization, whenever $s(y, z, p)+s(y', z', p')>s(y, z', p')+s(y', z, p)$, y will be matched with (z, p) and y' with (z', p'), whereas when the opposite holds, y will be matched with (z', p') and y' with (z, p).

Define:

$$\begin{aligned}\bar{\Delta}&=s(y, z, p)+s(y', z', p')-s(y, z', p')-s(y', z, p)\\&=s(y, z, p)-s(y, z', p')-(s(y', z, p)-s(y', z', p'))\end{aligned}$$

For ϵ positive and small enough, the sign is the same as:

$$\begin{aligned}\bar{\Delta}&=-\frac{\partial s(y, z, p)}{\partial z}\epsilon+\frac{\partial s(y, z, p)}{\partial p}\eta-\left(-\frac{\partial s(y', z, p)}{\partial z}\epsilon+\frac{\partial s(y', z, p)}{\partial p}\eta\right)\\&=\left[-\lambda\frac{\partial s(y, z, p)}{\partial z}+\frac{\partial s(y, z, p)}{\partial p}-\left(-\lambda\frac{\partial s(y', z, p)}{\partial z}+\frac{\partial s(y', z, p)}{\partial p}\right)\right]\eta.\end{aligned}$$

Then:

$$\bar{\Delta}=\int_{y}^{y'}\left(\lambda\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial z}-\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial p}\right)\eta\, d\theta$$

Now, define:

$$\begin{aligned}m_z&=\min_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial z},\; M_z=\max_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial z},\\m_p&=\min_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial p},\; M_p=\max_{\theta,z,p\in[0,Y]\times[0,Z]\times[0,1]}\frac{\partial^2 s(\theta, z, p)}{\partial\theta\partial p}.\end{aligned}$$

These are the maximum and minimum complementarities between incomes and income and fertility exhibited over the domain of types. Under the assumptions made, $m_z > 0$ and $m_p > 0$. Then:

- for $\lambda \geq M_p/m_z$, $\bar{\Delta} > 0$, and the first pattern obtains.
- for $\lambda \leq m_p/M_z$, $\bar{\Delta} < 0$, and the second pattern obtains.

□

The intuition for the proof is that the stable match must maximize surplus, and because the surplus is supermodular between both men's income and women's income and fertility, it is always possible for the supermodularity in income-fertility to "outweigh" that in incomes for a sufficiently high quantity of income relative to fertility. In other words, the form of the stable match depends on the distribution of women's traits.

These two propositions together imply non-monotonic matching is possible when some women differ only in income, while others differ in income and fertility.

Lemma 1. *Let z(y) represent the income of the woman matched with man of income y. In a distribution where men vary in income, and women vary in both income and fertility, where fertility is always weakly decreasing in income, it is possible for there to be two men with incomes* y *and* $y' > y$ *such that* $z(y') > z(y)$*, and a third man with income* $y'' > y'$ *where* $z(y'') < z(y')$*.*

Proof. From the proof of Proposition 1, we know that if two women have the same fertility and different income levels, there must be assortative matching. Furthermore we know it is possible to have matching be negative assortative on incomes when women have different fertility levels, for λ low enough. Thus, to have non-monotonic matching simply requires that the the distribution has three women (z, p), (z', p), and (z'', p') where $z'' > z' > z$ and $p' < p$ such that $s(y'', z', p) + s(y', z'', p') > s(y'', z'', p') - s(y', z', p)$, which we know will be true for λ high enough. Note, it is also possible to have assortative matching for the first two women if fertility differs, as long as λ is sufficiently low. □

If the surplus further meets the condition that the relative complementarity of income compared to fertility with men's income goes to zero as men's income increases, then we can guarantee that there will always be a man rich enough such that he matches non-assortatively in the stable match.

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Lemma 2. *For $s(y, z, p)$ such that $\lim_{y\to\infty} \frac{\frac{\partial^2 s(y,z,p)}{\partial y \partial z}}{\frac{\partial^2 s(y,z,p)}{\partial y \partial p}} = 0$, for any λ there exists a Y large enough that the stable match does not match him with the highest-income woman.*

Proof. Assume by contradiction assortative matching everywhere. Then the richest man Y is matched with the richest woman, with income and fertility (z', p'). Define man $\bar{y}$ that is ϵ below Y and matched with woman with income and fertility $(\bar{z}, \bar{p})$, where $z' > \bar{z}$ and $p' < \bar{p}$. Then it must be that $s(Y, z', p') + s(\bar{y}, \bar{z}, \bar{p}) > s(Y, \bar{z}, \bar{p}) + s(\bar{y}, z', p')$. Rearranging, the left-hand side becomes $s(\bar{y}, \bar{z}, \bar{p}) - s(\bar{y}, z', p') - (s(Y, \bar{z}, \bar{p}) - s(Y, z', p'))$, which has the same sign (see proof of Proposition 3) as:

$$\int_{\bar{y}}^{Y} \left(\lambda \frac{\partial^2 s(\theta, z, p)}{\partial \theta \partial z} - \frac{\partial^2 s(\theta, z, p)}{\partial \theta \partial p} \right) \eta \, d\theta.$$

For Y high enough, this will always be negative, since λ is fixed and $\frac{\frac{\partial^2 s(y,z,p)}{\partial y \partial z}}{\frac{\partial^2 s(y,z,p)}{\partial y \partial p}}$ is positive and goes to zero as y increases. Thus the richest man cannot be matched with the richest woman for Y high enough. $\square$

Thus, a matching model that is unidimensional on one side and bi-dimensional on the other can easily match empirical patterns exhibiting non-monotonicty along the "main" trait. In fact, non-monotonicity will be a general feature of the model when the women's traits are negatively correlated and the top of the male income distribution is sufficiently high. Such bi-dimensional matching frameworks may provide a way to reconcile the general tendency toward assortative matching with deviations that appear to suggest some people do not value income. Rather, it is likely that income is negatively correlated with another valuable trait.

B.1.3 Generalizing to Other Settings

While this model is described in terms of income and fertility, its insights can be generalized more broadly. First, in terms of marriage market matching, one can think about insights for any situation in which one side has two salient traits that are negatively correlated and whose contribution to the surplus increases in the other side's quality. This is in contrast to other useful work on bidimensional marriage models, such as Coles and Francesconi [2019], where the value of partner income is separable from own income. The key insight of this work is that even with supermodularity in

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incomes, it is possible to have non-assortative matching depending on the underlying distribution of traits and the change in relative complementarities across the income distribution. This provides predictions in line with the empirical regularity of largely positive assortative matching on income.

The model's insights can also be applicable to settings outside of fertility where one side of the market is bi-dimensional, and the value of both traits is increasing in the other side's characteristics. For example, manufacturers defined by quality and matching with upstream suppliers may care about both quality and speed. If these supplier traits are negatively correlated, non-monotonic matching in qualities is possible depending on the distribution of types. If furthermore the complementarity between qualities relative to the complementarity between quality and speed is decreasing in own quality, the model produces the result that for any distribution of quality and speed among the suppliers, a sufficiently high quality manufacturer will not be matched with the highest quality supplier. This type of model may help explain non-assortative matching in a variety of markets where one expects supermodularity in the most salient trait.

B.2 Parameterized Model Equilibrium

Proposition 4. *The unique stable match is fully characterized by Lemma 1 and the following conditions:*

- *If* $\Delta^{H-L}(y_1) \leq \Delta^{H-L}(y_0)$,

  H women match with poorest men, from y_0 to y_1.

- *If* $\Delta^{H-L}(y_3) < \Delta^{H-L}(y_2)$ *and* $\Delta^{H-L}(y_1) > \Delta^{H-L}(y_0)$,

  H women match with men interior to the set matching with L women, where $\Delta^{H-L}(\underline{y}^*) = \Delta^{H-L}(\underline{y}^* + h)$.

- *If* $\Delta^{H-L}(y_3) \geq \Delta^{H-L}(y_2)$ *and* $\Delta^{H-M}(y_3) \leq \Delta^{H-M}(y_2)$,

  H women match with middle men, from y_2 to y_3.

- *If* $\Delta^{H-M}(Y) < \Delta^{H-M}(y_4)$ *and* $\Delta^{H-M}(y_3) > \Delta^{H-M}(y_2)$,

  H women match with men interior to the set matching with M women, where $\Delta^{H-M}(\underline{y}^*) = \Delta^{H-M}(\underline{y}^* + h)$.

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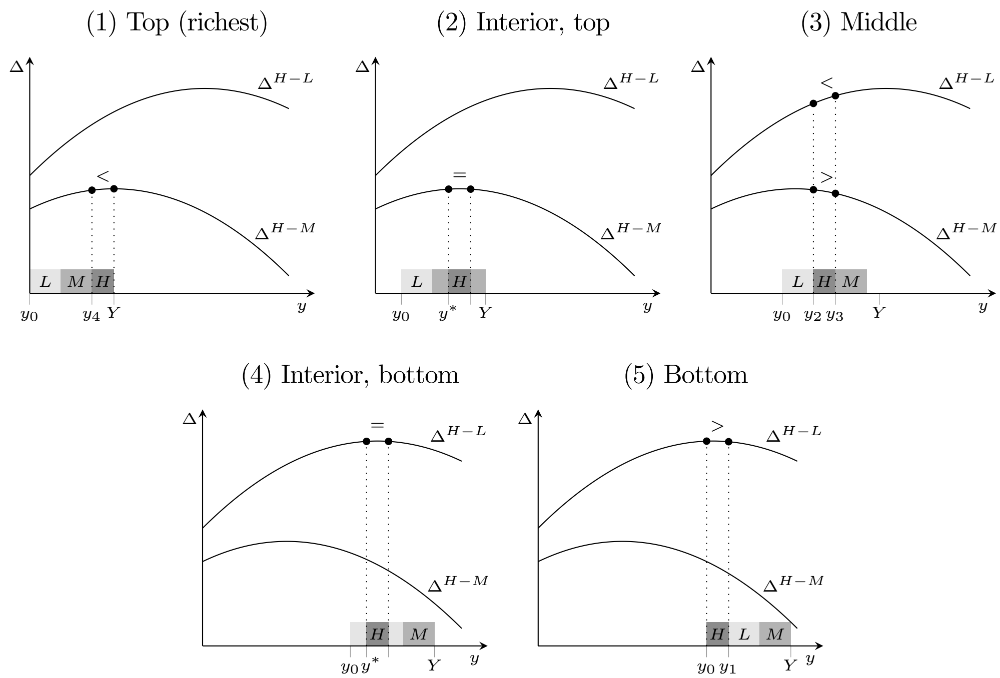
- *If* $\Delta^{H-M}(Y) \geq \Delta^{H-M}(y_4)$,

  H *women match with richest men, from* y_4 *to* Y.

Proof. The conditions in Lemma 1 create a single-variable maximization problem that has a unique solution for any given parameters, as shown in Appendix B.2. The solution is found through the first-order conditions of the problem, and the cutoffs for each equilibrium type is found through the boundaries for corner solutions. □

Figure A4: Illustration of surplus difference conditions for each matching equilibria



These conditions are illustrated in Figure A4. Note that although this figure illustrates the changing equilibria by keeping other parameters fixed and moving the range of y, it is also possible to keep men's distribution fixed and move the distribution of women's types, through changing the human capital return to investment or its fertility penalty, as is likely with technological change.

The matching equilibrium for any set of parameters can be solved for by a single variable optimization problem. Recall the mass of each female type is g^K, where $g^L + g^M + g^H = 1$, and men from y_0 to Y are matched. Define $F(y)$ as a CDF of matched men, where $F(y_0) = 0$ and $F(Y) = 1$.

As labeled in Figure 3, when M-type women match at the top of the distribution, call the lower male income threshold for matching with an M woman y_3. When instead H-type women match at the top, call the male income threshold y_4. When the H-type women match at the bottom of the male income distribution, call the upper male income threshold for matching with an H woman y_1. When instead L-type women match at the bottom, call the male income threshold y_2.

The optimization will be over the bottom man to receive an H-type match: call this $\underline{y}$. Define h as the length of the segment of men who match with H-type women, so that the top man who receives an H-type match will be $\underline{y}+h$.[2] Finally, let $s^K(y)$ represent the surplus obtained from a match with a man of income y and a woman of type $K \in \{L,M,H\}$.

We can now write down the single variable optimization problem to maximize the total surplus by choosing $\underline{y}$:

$$\max_{\underline{y}\in\{y_0,y_4\}} \begin{cases} \max\limits_{\underline{y}\in\{y_0,y_2\}} \int_{y_0}^{\underline{y}} s^L(y)f(y)\mathrm{d}y + \int_{\underline{y}}^{\underline{y}+h} s^H(y)f(y)\mathrm{d}y + \int_{\underline{y}+h}^{y_3} s^L(y)f(y)\mathrm{d}y + \int_{y_3}^{Y} s^M(y)f(y)\mathrm{d}y, \\ \max\limits_{\underline{y}\in\{y_2,y_4\}} \int_{y_0}^{y_2} s^L(y)f(y)\mathrm{d}y + \int_{y_2}^{\underline{y}} s^M(y)f(y)\mathrm{d}y + \int_{\underline{y}}^{\underline{y}+h} s^H(y)f(y)\mathrm{d}y + \int_{\underline{y}+h}^{Y} s^M(y)f(y)\mathrm{d}y. \end{cases}$$

Intuitively, this maximization divides the problem into two cases: one where the segment of men matching with H-type women bisects the segment matching with L-type women (or there is a corner solution), and one where the segment of men matching with H-type women bisects the segment matching with M-type women (or there is a corner solution). Because the surplus gain from switching to an H type is quadratic and concave, we need to find a segment of length h on either side of the maximum benefit from an H match. Thus, the first order conditions for this problem reduce to finding a $\underline{y}$ for which the surplus gain is equal to that of $\underline{y}+h$. When one cannot be found, there is a corner solution, which are the match types 1, 3, and 5. The boundaries for the equilibria are in terms of the surplus gain from switching from either L or M to an H-type woman at the ends of each segment. This provides a full characterization of exactly which form the stable match will take.

The boundaries of each match type in terms of δ_π and δ_γ are shown in Figure 4 of the main text.

B.3 Equilibrium utilities

This section describes the process for calculating the equilibrium utilities, which are needed to back out the payoff to women of investing in human capital. Because the stable match results

[2] $h = F^{-1}(F(\underline{y})+g^H)-\underline{y}$

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from a competitive market, we can recover these utilities as the "prices" associated with each individual. That is, we can calculate the surplus share each individual receives, or the utility over and above their counterfactual single utility.

Because at the stable match the sum of any two individuals' utilities must be greater than or equal to the surplus they could create from marrying one another, we can imagine the matching process as each spouse choosing the partner that maximizes his or her own share of the surplus conditional on keeping his or her spouse happy. That is, for women:

$$v(z, p) = \max_{y}\{s(y, z, p) - u(y)\}.$$

The first order condition of this problem dictates that the slope of the husband's value function must equal the slope of his contribution to the surplus:

$$\begin{aligned} u'(y) &= \frac{\partial s(y, z, p)}{\partial y} \\ &= \frac{1}{2}p(y+z-1). \end{aligned}$$

Because men's partner type does not change locally with their income except at the "boundaries" of a given female type, we can ignore the woman's type and integrate this function to pin the utility down to an additive constant. Then, we know what men's surplus share will be when matched with each of the three types of women:

$$u^K(y) = \frac{1}{4}p^K y(y+2z^K-2) + \mu^K$$ need solve mu by boundary condit

where $K \in L, M, H$, and p^K and z^K refer to the fertility and income of a K type woman.

Women's surplus shares will be a constant for each type, v^K. We can solve for each of the constants and the woman's surplus shares using two sets of restrictions. First, that for each couple the two surplus shares must add up to the surplus produced by the match, and second, that for each male type at a "boundary" between two female types, the utility achieved through each match must be the same. This pins down all values except for the division of surplus between the poorest man and his wife.

Assuming initially that there are more men than women in the market provides this restriction, and allows us to assume the poorest man receives no surplus (since otherwise the unmatched men would compete to take his place), and thus $u(y_0)=0$ (with his total utility simply equaling y_0).

I will now go through an example of this process for equilibrium 3, where high-income women are matched with the middle income men, from y_2 to y_3.

The two "boundary" men, y_2 and y_3, must be indifferent between their possible partners, as otherwise the match will not be stable. Thus we know $u^L(y_2)=u^H(y_2)$ and $u^M(y_3)=u^H(y_3)$. This allows us to pin down the constants μ^M and μ^H relative to μ^L (as a function of y_2 and y_3, but recall these are simple functions of the densities of female types, g^K). To pin down μ^L, we use the assumption that there are more men than women, and thus the lowest-income man earns 0 surplus, and thus $u^L(y_0)=0$.

From here, we can solve for the female surplus shares in each pairing, which will each be a constant simply using the total surplus restriction:

$$v^K = s^K(y) - u^K(y).$$ K=L, M, H

We then have a full characterization of women's and men's surplus shares from marriage, and can further characterize their full utility based on their single utility plus the surplus share, i.e., for men $y+u^K(y)$ and for women z^K+v^K.

Note that a woman's value function responds to fecundity loss through two channels. First, even if the woman's consumption level stayed constant, her utility would be reduced through the lower probability of conceiving, since children directly impact her utility. However, her consumption will also be reduced via the marriage market equilibrium, given that lower fecundity also lowers her husband's utility, and thus he requires a greater share of the available consumption in order to agree to the match.

B.4 Surplus function with multiple children

The following setup expands the model to allow up to four children, with proportionately constrained investments in children for each fertility realization up to four. Let c represent the number of realized children, and investments in children be constrained away from the optimal level

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by the limited number of children to invest in. Here, I make the strong assumption that there is no reallocation of resources to existing children, but weaker assumptions yield very similar results.

The constrained optimal q' and Q' are:

$$Q_c' = \left(\frac{c}{4}\right)\frac{y+z+1}{2}$$
$$q_c' = y+z-\left(\frac{c}{4}\right)\frac{y+z+1}{2}.$$

Let p_c be the probability of achieving each family size c, where $\sum_{c=0}^{4} p_c = 1$. Then the total household surplus will be the weighted average of all possible family size realizations:

$$s(y,\, z,\, p) = \sum_{c=0}^{4}\left[p_c\left(y+z-\frac{c}{4}\frac{y+z+1}{2}\right)\left(\frac{c}{4}\frac{y+z-1}{2}\right)\right]-y-z \tag{2}$$

This surplus has the same properties as the surplus given in equation (1), including the quadratic form of the difference between matching with a high fertility, low income woman and a low fertility, high income woman. If one had data on desired fertility, an even more flexible model could replace four with the specific number of desired children for a given couple.

C Appendix: Simulation

C.1 Simulation Details

Additional details of the simulation is as follows. For men, total income is drawn from the distribution of married men between 41 and 50 in a given Census year, weighted according to person weights. For women, the parameters are the same, but drawn conditional on education, proportional to the weighted education distribution in each year. An approximate NPV of lifetime income is created using a formula of 20 years of discounted income for women and 25 years for men, with a discount rate of 0.08. Given ages are similar, Income is an approximate sufficient statistic for lifetime income, but this scaling is done because the matching algorithm is not scale invariant, so incomes should be the approximate correct order of magnitude. Other methods of calculating the NPV, such as adjusting for total working years from time of marriage, controlling for age, and using different income paths over time yield highly similar results.

Fertility is assigned according to the empirical distribution of the number of children conditional

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on educational category and year, but calculated for 38-42 year old women, given that only number of children at home is available in each year, and this can be distorted for older women whose children may have left home. Results for children ever born, available until 1990, are substantively similar.

To determine the matching, a matrix is created of the surplus for matching each man with each woman. The surplus maximizing match is computed using the Hungarian algorithm. Each simulation uses 800 draws of men and women, and the simulation is repeated 10 times.

For the simulation of endogenous education, the number of college educated women is drawn to match the total number of college and highly educated, and they can choose to become highly educated by paying a heterogenous utility cost ranging uniformly from -10^{11} to 2×10^{11}. A matrix is created with the investment and no investment surplus, the maximum chosen, and then the Hungarian algorithm run to decide on matching, which is equivalent to letting couples match first, then choose the optimal investment decision.

C.2 Fertility

The simulation is performed with the number of children currently in the home, for 38-42 year old women. A different age range is used than the age range for the incomes because of the need to minimize bias from children leaving home. Children born, which would not have this bias, is only available up to 1990, as shown in Figure A5. The numbers are very similar to children at home in terms of the very similar numbers of women who have zero children until the highly educated group. Although there is some increase in the number of lower education groups having more than 4 children, it is unclear if these children survived past early childhood. The results in 1990 show the beginnings of the fertility transition shown in the 2010 data, but that there has not yet been full convergence, as is expected.

The fertility changes shown in Figure 5 are likely partly attributable in falls in desired family size, shown in A6, which could have driven down college educated women's fertility, making it more comparable to highly educated women's.

C.3 Additional Simulations

Endogenous Education A simulation of matching incorporating endogenous education is shown in Figure A7. It still roughly matches the crossing in the data, but the endogenous education

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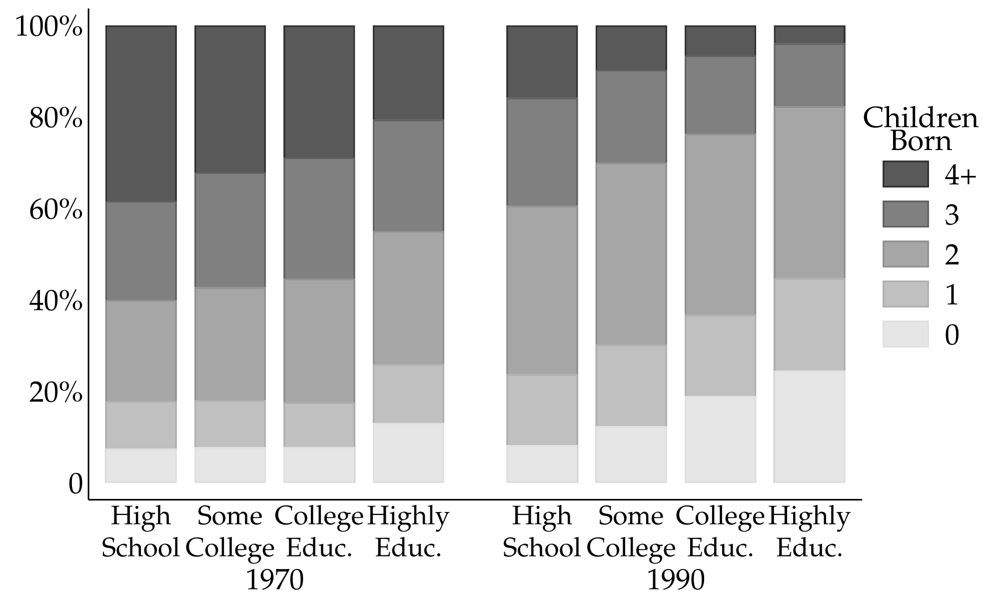
Figure A5: Empirical Distribution of Children Ever Born by Education Level and Year

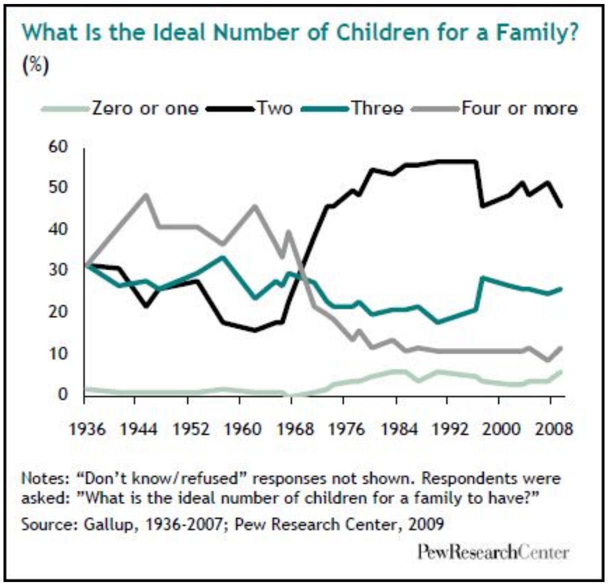


Notes: Children ever born for US-born women 38-42 years old. Source: 1 percent Census data from 1970 and 1990.

exacerbates the differences between groups, since when one group is more preferred to another, it also tends to be the best women who have selected into being in that group. Introducing additional noise into the decision making and matching process would probably improve the fit.

Marriage and Divorce Though the model was not designed to simulate marriage and divorce rates, it can provide a rough approximation of these metrics by adding a shock that causes couples to either not form or to break up once formed. Who breaks up will be highly dependent on the surplus of the unions they are in, since unions with higher surpluses will be more resilient to higher shocks. Figure A8 shows a simulation of marriage (with divorce having a similar prediction, only reversed), based on the surplus of hypothetical unions, shown in panel (a), and then shocks drawn from an extreme value distribution, shown in panel (b). Marriage rates are then simulated in panel (c). As in the data shown in Figure A3, highly educated women's marriage rates start below those of college educated women's, and then converge with those of college educated women over

Figure A6: Desired family size transition



Notes: Figure depicts the rapid transition from four children as the modal desired family size to two children, as evidenced by Gallup polls of men and women. As published in: Pew Center, The New Demography of American Motherhood, August 2010

time.[3] Thus, this simple model also matches the fact that the "reversal of fortune" for educated women on the marriage market was in fact driven by highly educated women.

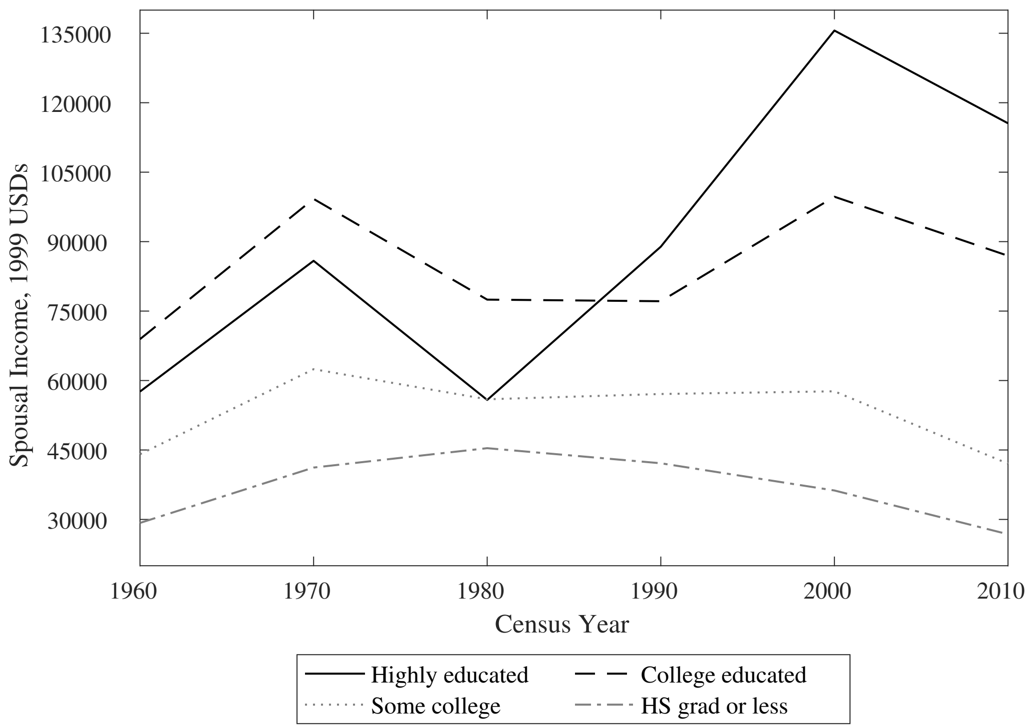
C.4 Alternative Explanations

An alternate possible explanation for the change in spousal income by education group over time is that the selection of women into post-bachelor's education has changed in a way that could align with the observed matching patterns. If women previously pursued graduate education because they had difficulty marrying, rather than because of higher capability, and this force has lessened over time, one would expect graduate educated women to have been historically less positively selected on skill. I directly test for this using the National Longitudinal Surveys in Appendix Table A2, and show that the "aptitude gap" between graduate educated women and college educated women has remained stable over time.

Table A2 examines whether there has been an increasing skill premium among women who attain post-bachelor's education, using data from aptitude scores and educational attainment of three

[3] It fails to match the high-until-recently marriage rates of lower educational groups, demonstrating that perhaps cultural factors are needed to explain these groups' marriage behavior.

Figure A7: Predicted Spousal Income with Endogenous Education



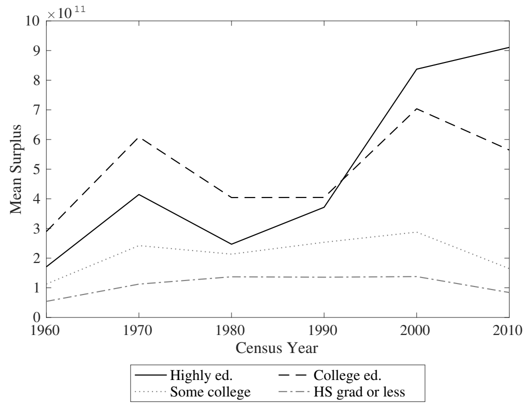
Notes: Model simulation of income matching with endogenous education decision, with uniform education cost, taking NPV of approximate lifetime income conditional on education and fertility as inputs. Matching and education decisions are determined to maximize surplus, as the private education decision will match the efficient equilibrium.

National Longitudinal Surveys NLS cohorts—the 1968 Young Women panel, the 1979 Youth panel, and the 1997 Youth panel. (Unfortunately, the earliest cohort with the necessary test score information only captures the tail end of the non-assortative matching group, but nonetheless, the trend should be informative about whether there are large shifts occurring in underlying selection factors.) If women were previously selecting into post-bachelor's education due to negative selection in other areas, they may be expected to be less positively selected on intelligence and academic potential. The data shows, to the contrary, that highly educated women indeed had substantially higher aptitude scores on average than college educated women even in the earliest cohort, and that the two numbers are not systematically diverging (which would indicate greater skill-driven selection).

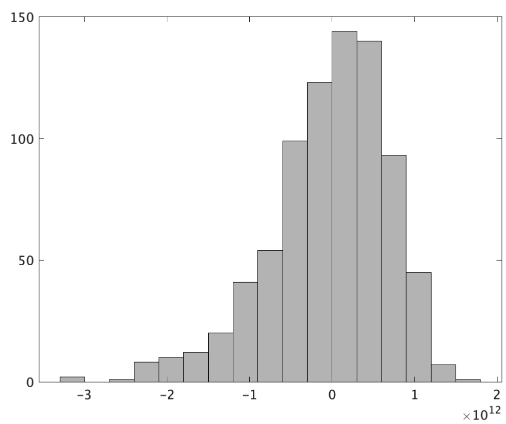
Additionally, as shown in Figure A9, the spousal income gap between college and graduate-educated women does not respond to the percentage of women earning graduate degrees, as would be expected if the gap were primarily caused by negative selection on other traits. Between the

Figure A8: Predicted Marriage Rates with Shocks

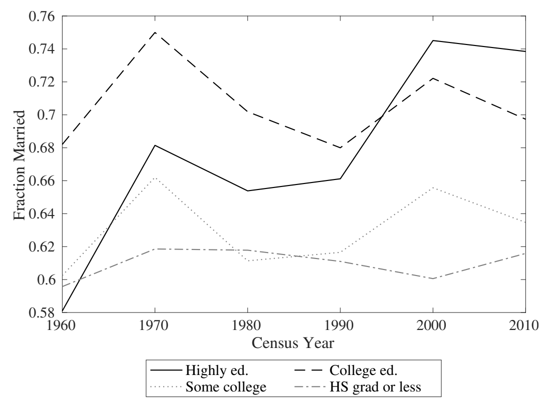
(1) Predicted Surplus



(2) Shocks



(3) Predicted Marriage Rates



Notes: Panel (a) show the total average surplus from the simulation matches shown in Figure 6. Panel (b) shows a distribution of extreme value shocks applied to surpluses to determine whether couples match: couples with negative surplus are assumed to not match. Panel (c) shows the resulting marriage rates by education category.

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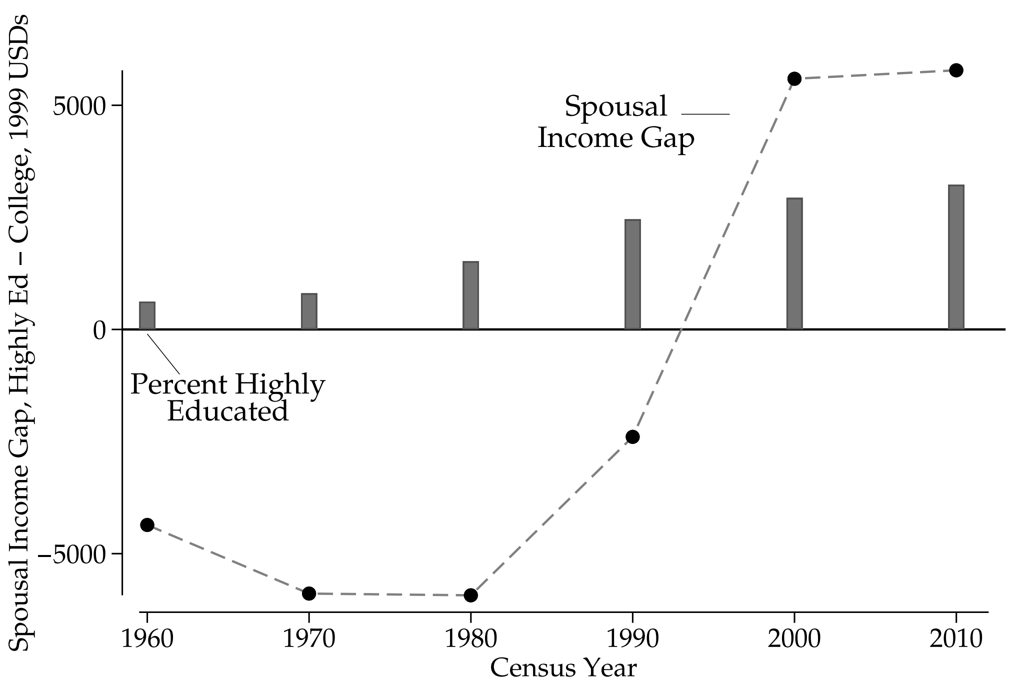
Table A2: Relative college and post-bachelor's average test score percentiles of three NLS cohorts

	NLS Young Women 1944-54 birth cohort	NLS Youth '79 1957-64 birth cohort	NLS Youth '97 1980-84 birth cohort
College graduate	66.5	70.3	63.6
Highly educated	72.0	74.9	69.3

Notes: Numbers represent percentiles for test scores by education group, compared to other women of all education levels with test score information available, in three different National Longitudinal Study cohorts. Young Women test score data is from the SAT converted into an IQ measurement. 1979 and 1997 data is from the Armed Forces Qualification Test. Gap in scores between college and graduate-educated women is large and relatively stable. The difference in percentile at both educational levels between the different years may be attributable to score data being available for a different selection of individuals in different survey rounds (e.g., for the Young Women sample, it was only available for individuals who reached the later years of high school).

1970 and 1980 Census, the number of women who achieved post-bachelor's education approximately doubled, while the "penalty" in spousal income compared to college education remained unchanged. From 1990 to 2000 there is a much more modest change in the "pool" of women with graduate degrees, whereas the spousal income gap showed a rapid reversal. This additionally rules

Figure A9: Rates of women's graduate education versus the spousal income gap



Notes: "Highly educated" constitutes all formal education beyond a college degree. "Spousal income gap" is defined as the average spousal income for highly educated women minus the average spousal income for college educated women. Source: 1 percent Census data from 1970, 1980, and 1990. Sample consists of US-born women, ages 41-50 years old.

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out selection in preferences, such as tastes for children. If graduate degrees became less costly, one would expect women with less extreme tastes to join the pool seeking degrees, which would then moderate the spousal income penalty associated with them. However, the spousal income penalty does not appear to respond to the number of women seeking graduate degrees.

Another possible explanation is that highly educated women prefer lower earning partners. Although non-monotonic matching can reflect women's preference for a higher share of the surplus from a lower quality partner, such patterns would be unlikely to arise solely from high-skill women preferring "low-powered" men. If high-skill women would actually rank lower-earning men above higher-earning men (e.g., due to them being able to spend more time at home), the negative-assortative matching at the top would strengthen, rather than weaken, as female earning power grew. Instead, high-skill women may choose a better relative position with a lower quality partner as a compromise because she cannot command a high "bargaining position" (surplus share) when marrying a high-income man. As the reproductive penalty to career investments dissipate, women can realize more equal partnerships with more assortatively matched mates.[4]

~~D Extension to continuous skill~~

Rather than having three discrete human capital groups, one could imagine women are endowed with continuous skill, and choose whether to invest in increasing their income relative to their skill. This section briefly outlines the adaptations to the model to accommodate this framework, and demonstrates that results are qualitatively similar to the discrete model.

Setup Men and women are each endowed with skill. In the man's case, human capital investment is assumed to be costless, and he thus arrives on the marriage market with a single characteristic, income, y^h, distributed uniformly on $[1,Y]$.[5]

Women, starting with skill s distributed uniformly on $[0,S]$, can choose to improve their level of income, but doing so takes time, and this time is costly in terms of reproductive capital depreciation. As a result, if they choose to make investments, they will have a lower probability

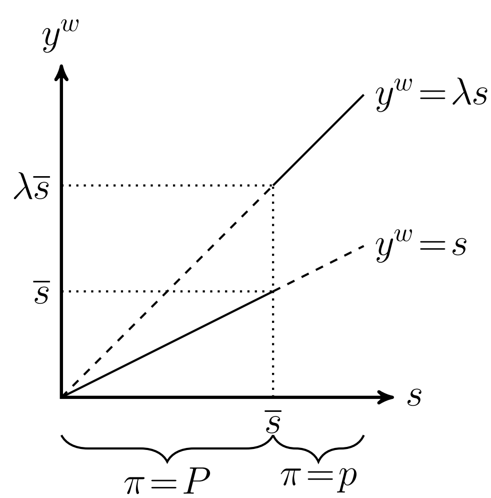
[4] Bertrand et al. [2020] offers gender norms against career women as a possible explanation, suggesting these norms may have dissipated in recent years. My model demonstrates, though, that such a norm shift could at least partly be driven by economic fundamentals.

[5] Starting at 1 simplifies the model by ensuring all individuals want to marry, because marriage is only "profitable" if total income is greater than 1.

of becoming pregnant when they get married. Women are therefore characterized by a pair of characteristics, (y^w,π). This pair is equal to (s,P) if the woman marries without investing and $(\lambda s,p)$ if the woman marries after investing, where $\lambda > 1$ and $P > p$. Note that the "fertility penalty" of investment is the same for all women, whereas the wage difference from investment increases with skill. Thus, higher skilled women may have more to gain from investing.

First, I assume an exogenous skill threshold, $\overline{s}$, above which women invest. After determining the equilibrium in the marriage market conditional on $\overline{s}$, I use this equilibrium to solve backwards for which women would optimally invest in the first stage. Thus, assume women with $s > \overline{s}$ invest, earn income of λs, and have fertility p, whereas women with $s < \overline{s}$ earn income s and have fertility P, as shown in Figure A10.

Figure A10: Women's Income versus Potential Income: Exogenous $\overline{s}$



Notes: Women are endowed with skill, s, shown on the x-axis. Their level of income, y^w, shown on the y axis, is determined by their investment decision. If women invest, they earn income λs, with $\lambda > 1$, but at the cost of reducing their fertility, π from P to $p < P$. In this section, we assume women with $s > \overline{s}$ invest.

After couples match, each has a child with probability π, and allocates their income. This process determines the surplus created by a given marriage, and thus individuals' preferences over different matches. Thus, solving the model requires working backwards from consumption decisions if a child occurs, to the surplus function from marriage, to then determining the optimal match.

As before, married couples can spend income on private consumption given by q^h and q^w and

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a public good, investment in children, denoted by Q:

$$U^h(q^h,Q)=q^h(Q+1)$$
$$U^w(q^w,Q)=q^w(Q+1),$$

with budget constraint $q^h+q^w+Q=y^h+y^w$

Optimal consumption is given by:

$$q^*=\frac{y^h+y^w+1}{2}$$
$$Q^*=\frac{y^h+y^w-1}{2}.$$

(Corner solutions are avoided by restricting $y^h+y^w>1$ based on the distributions of y and s).

The joint expected utility from marriage, T, is a weighted average between the optimal joint utility if a child is born and the fallback position of allocating all income to private consumption:

$$T(y^h,y^w,\pi)=\pi\frac{(y^h+y^w+1)^2}{4}+(1-\pi)(y^h+y^w).$$

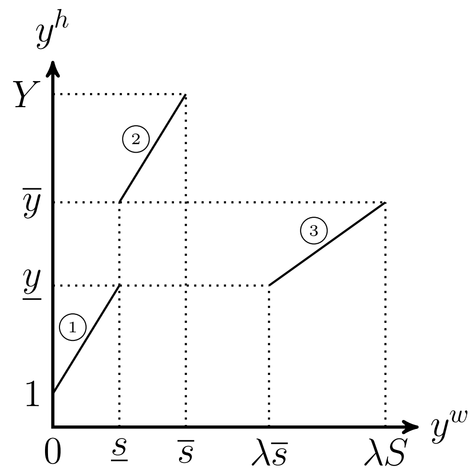
And the surplus function is similarly:

$$s(y^h,y^w,\pi)=\pi\frac{(y^h+y^w-1)^2}{4}.$$

Equilibrium An equilibrium displaying assortative matching for women with the same fertility, but non-assortative matching for women with different fertility levels, is shown in Figure A11. Recall $\overline{s}$ is the skill threshold for women becoming educated. Poor men, from 1 to $\underline{y}$, marry low-skill, fertile women (matching assortatively). On the other side of the threshold, the richest group of women matches assortatively with the middle group of men, from $\underline{y}$ to $\overline{y}$. But the richest men, from $\overline{y}$ to Y, forego matching with the richest women and instead marry the "best of the rest"—the more high-skilled women among those who have not invested and are thus still fertile.[6]

[6] The matching functions in this uniform case are linear—in an arbitrary distribution, their form would be determined by the density of individuals, so that the number of women above any point exactly matches the number of men above that point.

Figure A11: Non-monotonic Equilibrium Match



Notes: Women's income, y^w is on the x-axis, and men's income, y^h on the y-axis. The diagonal lines represent matching between men and women. In this non-monotonic matching equilibrium, women with income between 0 and $\underline{s}$ match with men with income between 1 and $\underline{y}$. Women with incomes between $\underline{s}$ and $\overline{s}$ match with men with incomes between $\overline{y}$ and Y. Women who have invested, and thus have incomes between $\lambda\overline{s}$ and λS, match with men with incomes between $\underline{y}$ and $\overline{y}$.

The equilibrium value functions can be used to show that this is indeed a stable match when λ, the income gain from investing, is high enough to overcome the fertility cost, $\frac{P}{p}-1$, for some men, but not high enough that all men prefer women who have invested. In particular, when $\frac{S-\overline{s}}{S+\overline{s}}(\frac{P}{p}-1)\frac{Y-1}{S}<\lambda<(\frac{P}{p}-1)\frac{Y-1}{S}$, the three-segment match is the unique stable match. For *any* value of S, $\overline{s}$, P, and p, such a λ exists, as $\frac{S-\overline{s}}{S+\overline{s}}<1$. Thus, this model predicts non-monotonic matching.

The matching equilibrium implies that as λ grows relative to $\frac{P}{p}$, the world transitions from one where educated women are penalized for their investment, because the additional income they earn is insufficient to compensate wealthy male partners for their loss in fertility, to one where they are able to compensate, and thus match with, partners similarly high in the income distribution.

The lower bound on women's skill for them to be willing to invest, $\overline{s}$, can be found by using the payoff functions resulting from the matching equilibrium, and finding the point at which the investment payoff dominates the non-investment payoff, with a small fixed cost to investment (note, as here the cost to invest is in this setup a monetary, rather than utility cost, it is also possible that very high-skilled women choose not to invest—i.e., non-monotonicity in investment decisions). To simplify this section, let $Y=2$, $S=1$, and $P=1$.

Using the equilibrium payoff functions, we seek the skill level at which $v_3(\overline{s})=v_2(\overline{s})$, or $\overline{s}^*(\lambda,c,p)$. Although its functional form is complex, $\overline{s}^*$ varies with the parameters in expected ways: it is increasing in c, decreasing in λ, and decreasing in p. In other words, the higher the fixed cost of investment, the higher the skill threshold for pursuing it; the higher the return to investment, the lower the skill threshold; and, the higher the chance of conceiving following investment, the lower the threshold. A higher threshold means fewer women making career investments. A lower threshold means more women making career investments, and this can be spurred by a lower fixed cost of investment, greater returns, or a higher chance of conceiving (e.g., through IVF technology).

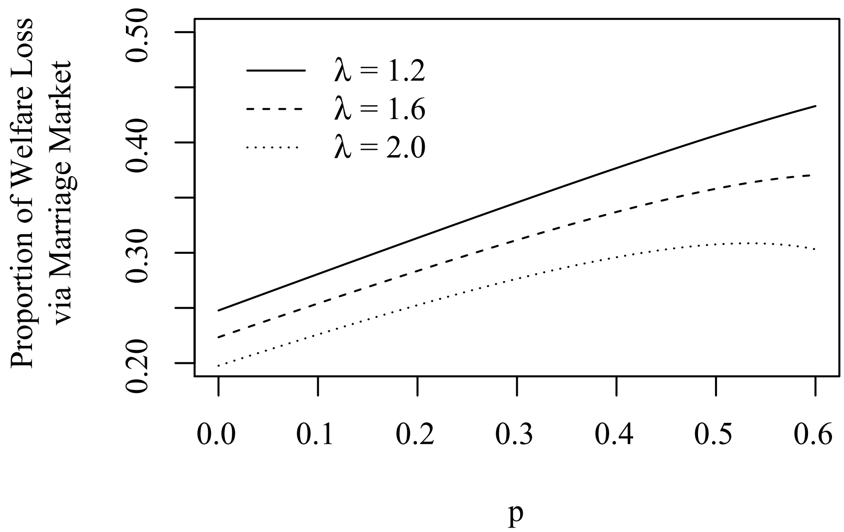
Note that the equilibrium payoff function internalizes not just the individual change in utility from a different fertility level, but also any change in the share of surplus received. This reflects the impact of traits on the overall surplus: someone with traits that yield a large surplus will in exchange receive a favorable match with a high surplus share. Someone with less desirable traits will face a less desirable match and a lower surplus share. Thus, when equalizing the payoff between investing and not investing to find the optimal threshold, both the personal cost of lower fertility and the cost to the marital surplus are considered

Welfare Crucially, the model provides a mechanism through which the biological clock impacts women's welfare through a channel other than her own desire for children. That is, even if a woman did not care at all about having a family, she would still be negatively impacted by her fertility loss through her loss of status on the marriage market.

In fact, a back of the envelope calculation using the model suggests that approximately one-third of the utility cost from the post-investment fertility loss comes through the marriage market, rather than directly through women's utility over children. Figure A12 compares the utility loss of lower fertility from the marriage market alone to the loss including women's personal valuation of fertility. The portion of the welfare loss stemming from being matched with a lower quality spouse and needing to cede more of the marital surplus to that spouse ranges from 20-40% of the total utility cost.[7] This simple calculation highlights that the loss of reproductive capital is an economic loss, just as worker disability is. Because the marriage market creates value for women, the loss of a valuable asset on that market creates real economic impacts.

[7]The calculation is $1 - \frac{v(S|p=P) - v(S|MM)}{v(S|p=P) - v(S)}$ where $v(S|p=P)$ is the woman with skill S's indirect utility if she invests but fertility is unaffected, $v(S)$ is her actual indirect utility, and $v(S|MM)$ is her indirect utility if there is indeed a fertility penalty, but she were to still match with man Y and receive the same *share* of the surplus as if there were no fertility loss.

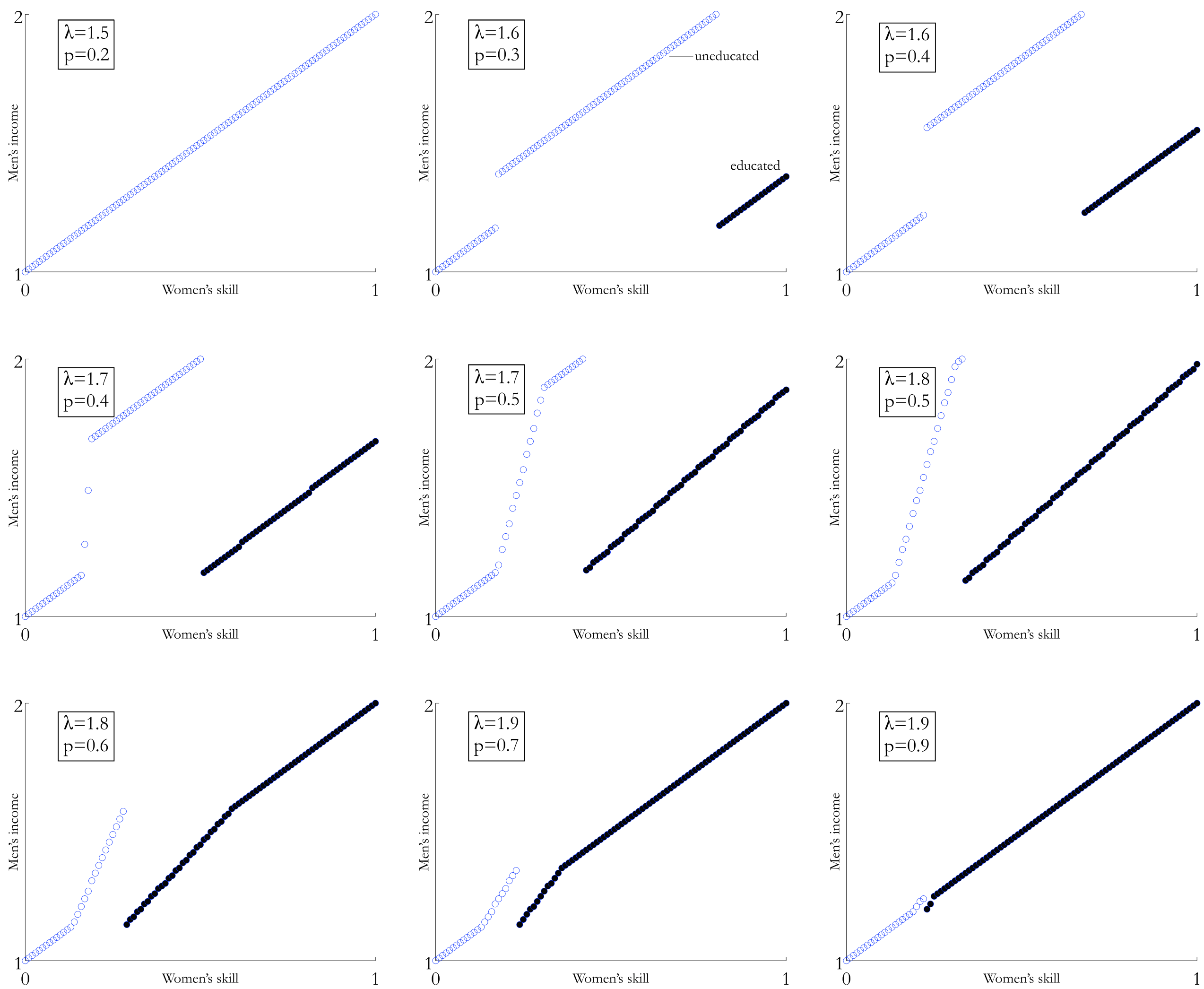
Figure A12: Proportion of Welfare Loss from Time-Limited Fertility due to Marriage Market



Notes: Figure depicts the portion of the welfare loss (y-axis) from lower fertility that comes through the marriage market compared to the total welfare loss, for varying values of λ, across a range of values for p (x-axis). This is shown for the most-skilled woman, with skill-level S, across the range of ps where non-monotonic matching results, for parameter values $Y = 2$, $S = 1$, and $P = 1$, with an exogenous t, investment threshold, of 0.7. The graph is produced by calculating the change in woman's indirect utility between a scenario with zero fertility cost of investment, to one where post-investment fertility equals p, and comparing that to the same effect if her partner and share of the marital surplus were held constant.

Simulation of continuous model Figure A13 simulates the model in the presence of growing returns to women's education and falling fertility costs. The first row of images in Figure A13 show that at first, no woman is willing to risk the marriage market costs of investing, so human capital accumulation by women is limited, and matching is assortative. As λ, the gain from investing, slowly increases while the fertility cost falls (via increasing the success of post-investment conception, p), the education and marriage market transforms. The first women to invest, shown by dark blue dots, are penalized through worse marriage matches, creating the non-monotonic equilibrium exhibited in the early Census data. Over time, as labor market returns to investment rise and the fertility cost falls, the marriage matches of these women gradually improve, as seen in the second row of images. This, in turn, creates a feedback loop, with more women being willing to invest (which also matches the dramatic rise in US women pursuing higher education). Finally, the market becomes essentially assortative, with some "randomization" by the highest earning men: some marry the very richest women, while others still choose the best among the women who have not invested.

Figure A13: Full Two-Stage Optimization Simulation



Notes: Figure depicts the results of a simulation of the investment and matching equilibrium as the value of the return on investment, λ, and post-investment chance of fertility, p, increases. Women's skill is shown on the x-axis and men's income on the y-axis, with dots depicting marriage matches. At first, the returns are low enough—and the potential marriage market cost high enough—that no women invest (and thus matching is assortative). As λ and p rise, some women invest (shown by dark blue dots, but these top-skilled women are penalized on the marriage market, and matching is non-monotonic. As λ and p continue to grow, matching becomes more assortative. Simulation shown for Y=2, S=1, P=1, and c=0.2.

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