CS 241
Data Organization
Binary

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Combinations and Permutations

In English we use the word "combination" loosely, without thinking if the order of things is important. In other words:

- *"My fruit salad is a* **combination** *of apples, grapes and bananas."* In this statement, order does not matter: "bananas, grapes and apples" or "grapes, apples and bananas" make the same salad.
- *"The* **combination** *to the safe is 472."* Here the order is important: "724" would not work, nor would "247".

Combinations and Permutations

In Computer Science and Math, we use more precise language:

- If the order doesn't matter, it is a **Combination**.
- If the order does matter it is a **Permutation**.
    - *Repetition is Allowed*: such as the lock above. It could be "333".
    - *No Repetition*: for example the first three people in a running race. Order does matter, but you can't be first *and* second.

Information in a Binary Signal

1 Bit
2 Permutations

0
1

2 Bits
4 Permutations

0	0
0	1
1	0
1	1

3 Bits
8 Permutations

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

4 Bits
16 Permutations

0000	1000
0001	1001
0010	1010
0011	1011
0100	1100
0101	1101
0110	1110
0111	1111

Numbers in Base Ten and Base Two

Base 10

$$\begin{aligned} 5307 &= 5 \times 10^3 &&+ 3 \times 10^2 &&+ 0 \times 10^1 &&+ 7 \times 10^0 \\ &= 5000 &&+ 300 &&+ 0 &&+ 7 \end{aligned}$$

Base 2

$$\begin{aligned} 1011 &= 1 \times 2^3 &&+ 0 \times 2^2 &&+ 1 \times 2^1 &&+ 1 \times 2^0 \\ &= 8 &&+ 0 &&+ 2 &&+ 1 \end{aligned}$$

Examples of Binary Numbers

0	0	0	0	**1**	0	0	0	**1**	**1**	= 35
512	256	128	64	**32**	16	8	4	**2**	**1**	

0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	= 63
512	256	128	64	**32**	**16**	**8**	**4**	**2**	**1**	

0	0	0	**1**	0	0	0	0	0	0	= 64
512	256	128	**64**	32	16	8	4	2	1	

1	**1**	0	**1**	**1**	0	0	0	**1**	**1**	= 867
512	**256**	128	**64**	**32**	16	8	4	**2**	**1**	

Hexadecimal: Base-16

Hexadecimal (or hex) is a base-16 system that uses sixteen distinct symbols, most often the symbols 09 to represent values zero to nine, and A, B, C, D, E, F to represent values ten to fifteen.

Base 16

$$
\begin{array}{llllll}
0x53AC & = 5\times16^3 & +3\times16^2 & +10\times16^1 & +12\times16^0 \\
 & = 5\times4096 & +3\times256 & +10\times16 & +12\times1 \\
 & = 20{,}480 & +768 & +160 & +12 \\
 & = 21{,}420 & & & \\
\end{array}
$$

Why Hexadecimal?

- Hexadecimal is more compact than base-10
- Hexadecimal is way more compact that base-2
- Since 16 is a power of 2, it is very easy to convert between Binary and Hexadecimal

Base 16							
Four bytes:			0x01239ACF				
01		23		9A		BF	
0000	0001	0010	0011	1001	1010	1011	1111

Hexadecimal Literals

```
#include <stdio.h>
void main(void)
{
   printf("%d\n", 0x1);
   printf("%d\n", 0x2);
   printf("%d\n", 0x3);
   printf("%d\n", 0x8);
   printf("%d\n", 0x9);
   printf("%d\n", 0xA);
   printf("%d\n", 0xB);
   printf("%d\n", 0xC);
   printf("%d\n", 0xD);
   printf("%d\n", 0xE);
   printf("%d\n", 0xF);
   printf("%d\n", 0x10);
   printf("%d\n", 0x11);
   printf("%d\n", 0x12);
}
```

```
1
2
3
8
9
10
11
12
13
14
15
16
17
18
```

Hexadecimal Literals (using %x)

```c
#include <stdio.h>
void main(void)
{
   printf("%x\n", 0x1);
   printf("%x\n", 0x2);
   printf("%x\n", 0x3);
   printf("%x\n", 0x8);
   printf("%x\n", 0x9);
   printf("%x\n", 0xA);
   printf("%x\n", 0xB);
   printf("%x\n", 0xC);
   printf("%x\n", 0xD);
   printf("%x\n", 0xE);
   printf("%x\n", 0xF);
   printf("%x\n", 0x10);
   printf("%x\n", 0x11);
   printf("%x\n", 0x12);
}
```

```
1
2
3
8
9
a
b
c
d
e
f
10
11
12
```

Powers of 2: char, int

```c
#include <stdio.h>
void main(void)
{
   char i=0;
   char a=1;
   unsigned char b=1;
   int c = 1;
   for (i=1; i<22; i++)
   {
      a = a * 2;
      b = b * 2;
      c = c * 2;
      printf("%2d) %4d %3d %7d\n",
                i, a, b, c);
   }
}
```

```
 1)    2   2       2
 2)    4   4       4
 3)    8   8       8
 4)   16  16      16
 5)   32  32      32
 6)   64  64      64
 7) -128 128     128
 8)    0   0     256
 9)    0   0     512
10)    0   0    1024
11)    0   0    2048
12)    0   0    4096
13)    0   0    8192
14)    0   0   16384
15)    0   0   32768
16)    0   0   65536
17)    0   0  131072
18)    0   0  262144
19)    0   0  524288
20)    0   0 1048576
21)    0   0 2097152
```

Powers of 2: int, long

```
#include <stdio.h>
void main(void)
{
  char i=0;
  int c=1;
  long d = 1;
  for (i=1; i<65; i++)
  {
    c = c * 2;
    d = d * 2;
    printf("%2d) %11d %20ld\n",
              i, c, d);
  }
}
```

```
...
29)   536870912            536870912
30)  1073741824           1073741824
31) -2147483648           2147483648
32)           0           4294967296
33)           0           8589934592
...
61)           0  2305843009213693952
62)           0  4611686018427387904
63)           0 -9223372036854775808
64)           0                    0
```

Format code: `ld` for long decimal

Bit Operations

C provides several operators for manipulating the individual bits of a value:

`&`	bitwise AND	`1010 & 0011 = 0010`
`|`	bitwise OR	`1010 | 0011 = 1011`
`^`	bitwise XOR	`1010 ^ 0011 = 1001`
`~`	one's complement	`~1010 = 0101`
`<<`	left-shift	`00000100 << 3 = 00100000`
`>>`	right-shift	`00000100 >> 2 = 00000001`

Shift Operator Example

```c
void main(void)
{
  int i;
  for (i=0; i<8; i++)
  {
    unsigned char n = 1 << i;
    printf("n=%d\n", n);
  }
}
```

Output:

n=1
n=2
n=4
n=8
n=16
n=32
n=64
n=128

Convert 77 to an 8-bit Binary String

$2^7 = 128$ is > 77, put a '0' in the 128s place

0							

$2^6 = 64$ is $<= 77$, put a '1' in the 64s place
and subtract 64: $77 - 64 = 13$

0	1						

$2^5 = 32$ is > 13, put a '0' in the 32s place

0	1	0					

$2^4 = 16$ is > 13, put a '0' in the 16s place

0	1	0	0				

$2^3 = 8$ is $<= 13$, put a '1' in the 8s place
and subtract 8: $13 - 8 = 5$

0	1	0	0	1			

$2^2 = 4$ is $<= 5$, put a '1' in the 4s place
and subtract 4: $5 - 4 = 1$

0	1	0	0	1	1		

$2^1 = 2$ is > 1, put a '0' in the 2s place

0	1	0	0	1	1	0	

$2^0 = 1$ is $<= 1$, put a '1' in the 1s place
and subtract 1: $1 - 1 = 0$

0	1	0	0	1	1	0	1

Convert unsigned char to Binary Array

```c
#include <stdio.h>
void main(void)
{
  char bits[9];
  bits[8] = '\0';
  unsigned char n=83;
  unsigned char powerOf2 = 128;
  int i;
  for (i=0; i<=7; i++)
  { if (n >= powerOf2)
    { bits[i] = '1';
      n = n-powerOf2;
    }
    else bits[i] = '0';
    powerOf2 /= 2;
  }
  printf("%s\n", bits);
}
```

Output:

01010011

The Mask

```c
void main(void)
{
  long mask = 1<<23;
  long x = 25214903917;

  /* Not zero if bit 23 is ON in x. */
  printf("%ld\n", x & mask); /* prints: 8388608 */

  /* Turn ON bit-23. If already ON, x is unchanged. *
  x = x | mask;
  printf("%ld\n", x); /* prints: 25214903917 */

  /* Turn OFF bit 23. If already OFF, x is unchanged.
  x = x & (~mask);
  printf("%ld\n", x); /* prints: 25206515309 */
}
```

Using the Mask: Binary Array

```
#include <stdio.h>
void main(void)
{
  char bits[9];
  bits[8] = '\0';
  unsigned char n=83;
  unsigned char powerOf2 = 128;
  int i;
  for (i=0; i<=7; i++)
  { if(n & powerOf2)
    { bits[i] = '1';
    }
    else bits[i] = '0';
    powerOf2 = powerOf2 >> 1;
  }
  printf("%s\n", bits);
}
```

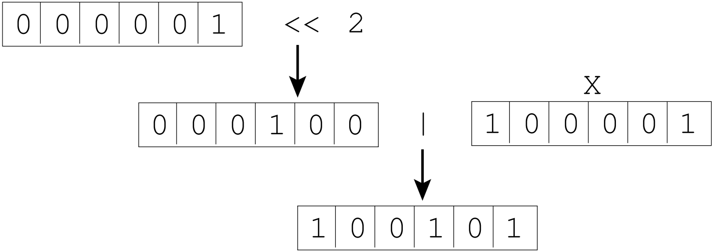
Output:

01010011

In the earlier slide, whenever a power of 2 is found, it is subtracted from n. This method never changes n.

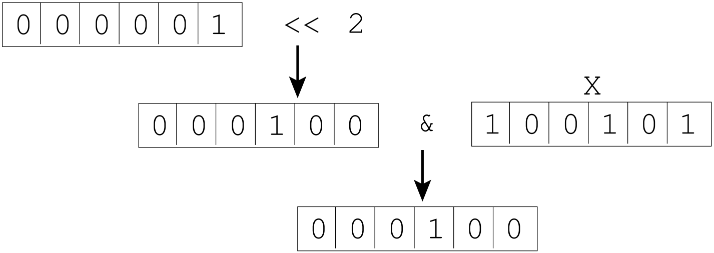
Bit-operations: `x |= (1<<n)`

- Set bit `n` in variable `x`. `(1<<n)` shifts 1 left by `n` bits. The result is OR'ed into `x`.



Bit-operations: `x & (1<<n)`

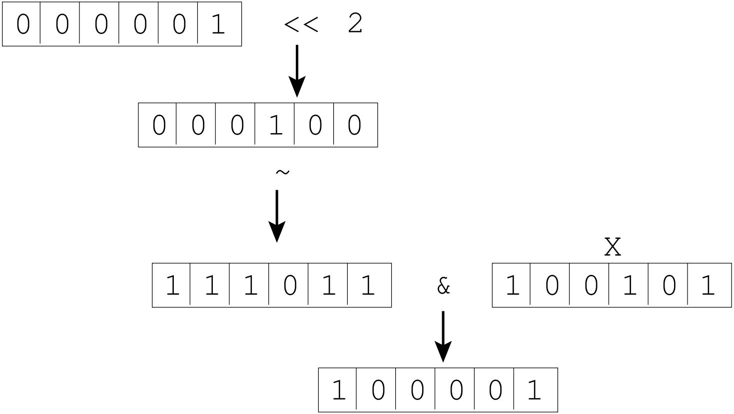
- Test bit `n` in variable `x`. `(1<<n)` creates a value with the appropriate bit set by shifting 1 left by `n` bits. It is AND'ed with `x` to see if that bit is set in `x`.



Bit-operations: `x & ~(1<<n)`

- Clear bit `n` in variable `x`.
- `(1<<n)` creates a value with the appropriate bit set by shifting 1 left by `n` bits.
- The one's complement operation '`~`' flips all the bits in the value, resulting in a value with every bit but the `n`'th set.
- It is AND'ed with `x` to clear the `n`'th bit but leave the rest unchanged.

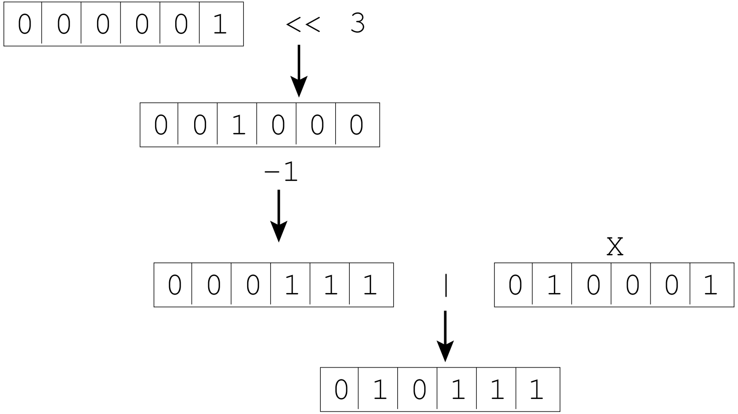
Bit-operations: `x & ~(1<<n)`



Bit-operations: $\boxed{\texttt{x |= (1<<n)-1}}$

- Set n lowest bits in variable `x`.
- We create a mask with all ones in the lower n bits by shifting 1 left by n bits and subtracting 1.
- It is OR'ed with `x` to set the n lowest bits but leave the rest unchanged.

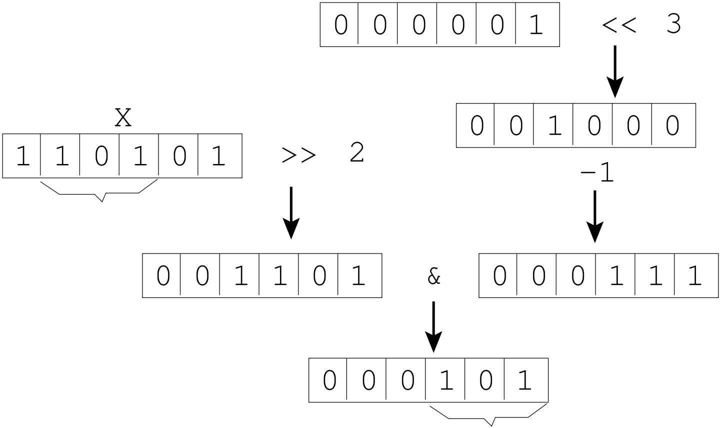
Bit-operations: `x |= (1<<n)-1`



Bit-operations: $\boxed{\texttt{(x>>p)\&((1<<n)-1)}}$

- Suppose we want to extract n bits from `x`, starting at position p.
- We create a mask with all ones in the lower n bits the same way as before: shift 1 left by n bits and subtract 1.
- Next, we shift `x` right by p bits.
- Finally, we AND the mask and `(x>>p)` to strip out any extra high-order bits.

Bit-operations: `(x>>2)&((1<<3)-1)`



Addition: Base 10 and Binary

Base 10

$$
\begin{array}{cccc}
 & & {\scriptstyle 1} & \\
 & & 2 & 9 \\
+ & & 5 & 6 \\
\hline
 & & 8 & 5
\end{array}
$$

Binary

$$
\begin{array}{ccccccccc}
 & & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & & & \\
 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\
+ & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
\hline
 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}
$$

Overflow Addition

```
#include <stdio.h>
void main (void)
{
   char i=0;
   char a = 123, b = 252;
   unsigned char x = 123, y = 252;
   for (i=1; i<=7; i++)
   {
      a++; b++; x++; y++;
      printf("%4d %4d %4d %4d\n", a, x, b, y);
   }
}
```

Output:

124	124	-3	253
125	125	-2	254
126	126	-1	255
127	127	0	0
-128	128	1	1
-127	129	2	2
-126	130	3	3

Two's Complement

From ordinary binary:
Flip the bits and Add 1.

5	0	0	0	0	1	0	1
Flip Bits	1	1	1	1	0	1	0
Add 1	0	0	0	0	0	0	1
-5	1	1	1	1	0	1	1

Ordinary Binary	Decimal
0000 0001	1
0000 0010	2
0000 0011	3
0000 0100	4
0000 0101	5
0000 0010	6
0000 0111	7

Two's Complement	Decimal
1111 1111	-1
1111 1110	-2
1111 1101	-3
1111 1100	-4
1111 1011	-5
1111 1010	-6
1111 1001	-7

Two's Complement Addition

$$
\begin{array}{ccc}
 & & \\
 & 2 & 9 \\
+ & -2 & 9 \\
\hline
 & & 0
\end{array}
\qquad
\begin{array}{cccccccccc}
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \\
 & & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\
+ & & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\
\hline
 & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{cc}
 & \\
 & 7 \\
+ & -4 \\
\hline
 & 3
\end{array}
\qquad
\begin{array}{cccccccccc}
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & & \\
 & & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
+ & & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
\hline
 & & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1
\end{array}
$$