Mathematics Advanced Year 12

Trigonometric Functions Topic Guide

The Mathematics syllabuses are the documents used to inform the scope of content that will be assessed in the HSC examinations.

Topic Guides provide support for the Stage 6 Mathematics courses. They contain information organised under the following headings: Prior learning; Terminology; Use of technology; Background information; General comments; Future study; Considerations and teaching strategies; Suggested applications and exemplar questions.

Topic Guides illustrate ways to explore syllabus-related content and consequently do not define the scope of problems or learning experiences that students may encounter through their study of a topic. The terminology list contains terms that may be used in the teaching and learning of the topic. The list is not exhaustive and is provided simply to aid discussion.

Please provide any feedback to the Mathematics and Numeracy Curriculum Inspector.

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# Topic focus

The topic Trigonometric Functions involves the study of periodic functions in geometric, algebraic, numerical and graphical representations.

A knowledge of trigonometric functions enables the solving of practical problems involving the manipulation of trigonometric expressions to model behaviour of naturally occurring periodic phenomena such as waves and signals and to predict future outcomes.

Study of trigonometric functions is important in developing students’ understanding of periodic functions. Utilising the properties of periodic functions, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals.

# Terminology

|  |  |  |
| --- | --- | --- |
| amplitude  angles of any measure  angular measure  centre of motion  circular measure  constant  complementary angles  composition  degrees  dependent variable derivative  dilation  domain  exact ratio | frequency  graph  horizontal shift  oscillation  trigonometric identity  independent variable  intercept  period  periodic  phase shift  quadrant  radian  range | roots  related angle  sinusoidal functions  sketch  supplementary angles  symmetry properties  transformational shifts  transformations  trigonometric function  unit circle  vertical shift  wavelength  wave form |

# Use of technology

While ‘by-hand’ skills for solving equations and curve sketching are essential for students in this course, graphing technologies are an ideal means of exploring many of the concepts studied in this topic and their use is encouraged in teaching and learning.

In particular, graphing software is useful for investigating the effect of varying the constants and in the graph of where is a trigonometric function.

# Background information

The sine and cosine functions are called sinusoidal functions. They graph wave-forms and are used to describe any physical phenomenon that exhibits a wave-like pattern or periodic behaviour. Examples include the number of daylight hours at a specific location, the oscillation of a pendulum or the amount of energy used to control the temperature in an office.

An early application of sinusoidal functions was to predict the tides, providing important information to those involved in coastal navigation and the fishing industry. The link between the tides and the gravitational pull of the Sun and Moon on the oceans had been known for many centuries. The suggestion that they may also be periodic prompted the use Fourier analysis to build tide-predicting machines.

Sound waves are created through vibrations that consist of wavelength, frequency, velocity and amplitude, and consequently they can also be modelled by sinusoidal functions. Sound waves are characterised as mechanical waves because they are a disturbance that is transported through a medium. They cannot travel through a vacuum.

As the heart beats and blood is pumped through the body, blood flows through the arteries in a pattern similar to a sinusoidal function.

# General comments

The material in this topic builds on the content from the Measurement and Geometry strand of the *K–10 Mathematics* syllabus and related content from the *Mathematics Advanced* syllabus*,* including the Year 11topics of Functions and Trigonometric Functions.

This topic prepares students for many practical applications of trigonometric functions and is essential for many of the more advanced aspects of mathematics.

As with the study of Graphing Techniques in MA-F2, importance must be placed on the order in which the transformations are applied to the original function, and their effect on the position and shape of the graph. This could be investigated dynamically using graphing software.

Both sine and cosine graphs are referred to as sinusoidal graphs, because , and so the graph of is effectively a sine wave with a phase shift of radians.

# Future study

This topic could be taught in conjunction with the Functions topic MA-F2 Graphing Techniques where the transformational shifts of graphs are taught in the context of other functions.

# Subtopics

* MA-T3: Trigonometric Functions and Graphs

## MA-T3: Trigonometric Functions and Graphs

### Subtopic focus

The principal focus of this subtopic is to explore the key features of the graphs of trigonometric functions and to understand and use basic transformations to solve trigonometric equations.

Students develop an understanding of the way that graphs of trigonometric functions change when the functions are altered in a systematic way. This is important in understanding how mathematical models of real-world phenomena can be developed.

### Considerations and teaching strategies

* Review of the following may be needed to meet the needs of students:
  + Angle measures, representations and conversions – This relates to content covered in MA-T1 (T1.2).
  + The graphs of , and – This relates to content covered in MA-T1 (T1.2).
* The original graphs of and are easily developed from the definitions based on the unit circle and could be illustrated using appropriate graphing software applications as a brief review.
* Both radian measure and degrees should be used within this topic.
* The domain and range, period and amplitude of simple trigonometric functions   
  should be noted from graphs plotted using computer software, or ‘by-hand’ methods.
* Initially, examples of sketching related curves of the form should be restricted to changing the value of one variable at a time, for example:

1. Sketch the functions and on the same axes.
2. Sketch the functions and on the same axes.

Once these concepts are understood this can be extended to the consideration of more than one transformation at a time.

* The order in which the transformations are applied should be considered when transformations are combined. This can be effectively investigated using graphing software.
* Equivalent expressions for sinusoidal functions include , where is the period or wavelength, and , where is the frequency.
* Sketches of functions such as and should be drawn by hand, showing the main features.
* Students should be given some practice in using graphs to solve simple equations such as .
* Real-world applications should be discussed, for example data on daily temperatures and other periodic phenomena can be plotted and amplitudes and periods estimated from the graphs.
* It should be noted that real-world tides are a superposition of a number of different sinusoidal functions, with different amplitudes and frequencies, which makes real-world tidal data difficult to model by a single sinusoidal function. However, a useful discussion can be had around this point.   
  More information on tidal constituents can be found on the US National Oceanic and Atmospheric Administration (noaa) website at: [https://tidesandcurrents.noaa.gov/harcon.html?id=9410170](https://www.google.com/url?q=https://tidesandcurrents.noaa.gov/harcon.html?id%3D9410170&sa=D&ust=1503296753343000&usg=AFQjCNGP57nIgXW89w6WizBz0GxlrjN48g).
* Modelling of weather patterns and other geographical data often involves sinusoidal functions. For example, the number of daylight hours on the th day of the year for a particular city can be modelled by a function of the form , where and are constants.
* For sound waves, the loudness of a sound depends on the amplitude of the wave and the pitch of a sound depends on the frequency of the wave.
* Real-world applications of adding sinusoidal graphs together can be found in biorhythms, harmonics, cardiographs, tides, etc.
* Given a graph showing changes in a real-world phenomenon and its equation, students could describe the oscillation in words.

### Suggested applications and exemplar questions

* Which diagram shows the graph of ?

(A)

1

–1

–2

**

3

4

**

3

*x*

*y*

**

3

(B)

1

–1

–4

**

3

2

**

3

–

**

3

*x*

*y*

(C)

1

–5

**

6

7

**

6

*x*

*y*

–1

**

6

(D)

1

–1

–7

**

6

5

**

6

–

**

6

*x*

*y*

* The diagram shows part of the graph of .

2.5

5.5

*y*

*x*

4

**

*O*

**

2

What are the values of and ?



* What is the period of the function ?

A.

B.

C.

D.

* Sketch the curve for .
* (a) Sketch the graph of for .

(b) On the same set of axes, sketch the graph of for .

(c) Find the exact values of the coordinates of the points where the graph of crosses the -axis in the domain .

* Solve for .
* Solve for .
* (a) Draw the graphs of and on the same set of axes for .

(b) Explain why all the solutions of the equation must lie between and .

* (a) Show that is a solution of

(b) On the same set of axes, sketch the graphs of the functions and for .

1. Hence find all solutions of for .
2. Use your graphs to solve for .

* The graph of can be obtained from the graph of by a translation followed by two dilations.

1. Describe each of these three transformations, and give the number of roots of the equation , in the interval , where .
2. Generalise your answer to give the number of roots of the equation   
    in the interval , where , is a positive integer and .
3. How does your answer to part (b) change if is a negative integer?

* A particle moves in a straight line. At time seconds its distance metres from a fixed point in the line is given by .

1. Sketch the graph of as a function of .
2. Find the times when the particle is at rest and the position of the particle at those times.
3. Describe the motion.

* The length of daylight, , is defined as the number of hours from sunrise to sunset, and can be modelled by the equation where is the number of days after 21 December 2015, for .

1. Find the length of daylight on 21 December 2015.
2. What is the shortest length of daylight?
3. What are the two values of for which the length of daylight is 11?

* When humans breathe, they do not inflate their lungs to full capacity. When resting, each inhalation adds approximately 0.5 L of air and this same volume of air is removed upon exhalation. When exhalation is completed, the volume of air that remains in the lungs, called the functional residual capacity, is approximately 2.2 L. On average the time taken to complete an inhale-exhale cycle is approximately 5 seconds.

The volume of air in the lungs can be modelled by the function where is the volume of air in litres and is time in seconds.

1. Use the time for an inhale-exhale cycle to show that the period of this function is .
2. Explain why .
3. Find the value of .
4. Sketch the graph of for using these values of , and .
5. When exercising, the volume of air inhaled and exhaled rises and breathing occurs more rapidly. Explain the effect this would have on the values of , and .
6. Humans have a full lung capacity of approximately 6 L. An athlete who is exercising vigorously inhales approximately 4.6 L of air. Calculate the athlete’s residual lung capacity.