Assignment 2: Evolutionary dynamics on Complex Networks

General remarks:

- Deadline 10-12-2018
- Mail your results to Jelena Grujic <jgrujic@vub.ac.be>. With a subject: "Assignment 2: Evolutionary dynamics on Complex Networks 2018"
- Provide a single (self-contained) with the LastNameFirstName_Affiliation.PDF file, for example: GrujicJelena_VUB.pdf
- Put your name and your affiliation (VUB/ULB) both on the document and in the file name.

Problem description

This assignment combines elements of complex networks and game theory. You will need to generate a Random network and a Scale free networks, analyze their degree distributions and finally simulate agents playing Prisoner's Dilemma on them and study how the network structure changes the cooperation. The report should contain an introduction of the problem, discussion and comments of all the results. You do not need to provide the codes, unless it is specifically asked for it (just for the generation of the networks). You can use any programing language you want.

Part I - Complex Networks

1) Generate Erdos-Renye network (Random networks) [1,2]:

Algorithm :

- Start with N disconnected nodes,
- Connect couples of randomly selected nodes (while checking for multiple connections; two nodes can have maximum one link between them),
- Stop when the total number of edges in the network is equals K.

For a size of the network of N=10000, calculate a K so that each node has in average degree 4.

Do not use existing functions from packages to generate the network !
Generate the network from scratch and present/describe the part of the code you used to generate the network in your document (use pseudo-code notation).

2) Plot the degree distribution of the generated network. Calculate the mean and standard deviation and plot the normal distribution with these same parameters (or in the other word fit the distribution with a Gaussian).

3) Generate a Barabasi-Albert network (Scale Free network) [1,3]:

Algorithm :

- Start with a fully connected network of 4 nodes (N=4),
- At each time step, add one new node with 4 links which you attach to the existing nodes with probability proportional to the number of links they already have.

$$P_i = \frac{k_i}{\sum k_j}$$

  where P_i is probability of attaching the new node to the node existing *i*, k_i is the degree of the node *i*, and j sums over all the nodes in the networks. Check for the multiple links between the same nodes, as for the previous network, each pair of nodes can have maximum one link between them,
- Stop when you have 10000 nodes (N=10000).

Do not use existing functions from packages to generate the network !
Generate the network from scratch and present/describe the code you used to generate the network in your document (use pseudo-code notation).

4) Plot the degree distribution of the generated network using a linear scale on both axes. Plot in the same figure an exponential distribution which looks similar and reports on the parameters of that distribution.

5) Plot the same distribution on log-log scale. Fit the distribution using Least Square fit. You can use existing functions for fitting and plot the fit next to the data. What are the parameters of the fit? How does it fit? Why? Write a paragraph about why we should not use Least Square fit to fit power laws.

6) Plot cumulative distribution and fit it with Least Square Fit, report the obtained parameters and plot of the fitted function.

7) Now fit your distribution using maximum likelihood method described in [4]. You can use any of the packages which has the method developed. I recommend powerlaw library for python https://github.com/jeffalstott/powerlaw

8) Report the parameters of the fit and plot them next to distribution.

9) Compare the power law fit with the exponential fit (using the same package). Report the log likelihood ratio R and the p-value. What do these numbers mean?

10) What is the mathematical formula for scale free distribution you generated? Calculate the mean and the standard deviation of function? What would be the mean and standard deviation if the exponent would be 2.5? Use reference [1] to help you out especially chapter 2.2.2. Scale-free degree distributions. You can calculate the integral using pan and paper and scan it and attach it to the rest of PDF.

Part II – Game Theory on Networks

1) Run a simulation of agents playing Prisoner's Dilemma on the generated networks [5]:

- Each player is playing a Prisoner's Dilemma game with each of their neighbors, playing the same action (Cooperate of Defect) with each one of them. The total payoff is calculated as a sum of the payoffs received from playing all the games.

- In the first round each player plays Cooperate or Defect with 50% probability

- In every other round, the update mechanism is replicator rule: each individual *i* chooses randomly one of their direct neighbors *j* and with a probability P_{ij} they change their action to the action of that neighbor with a probability P_{ij}, where P_{ij} is defined as:

$$P_{ij} = \frac{W_j - W_i}{k_{max} D_{max}}$$

where W_i and W_j are the payoffs of the focal player *i* and its neighbor *j*, k_{max} is the biggest degree between the nodes i and j, and $D_{max} = max(T, R) - min(S, P)$.

- Explain why would we set up the probability P_{ij} like this? Why does it make sense to update your actions like that?

- Use R=1, P=0, S=-0.1 for all simulations and with the following values for T={1.05, 1.1, 1.15, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9}

2) Plot the cooperation level over time for all the values of T and for both networks. Establish after how many rounds the system reaches stationary state (when the level of cooperation does not change too much).

3) Average the stationary cooperation level over 100 simulations (for each T and both networks). Plot the dependence of the stationary cooperation level on the value of T for both networks.

4) Comment the differences of the final cooperation levels.

References

[1] Boccaletti, Stefano, Vito Latora, Yamir Moreno, Martin Chavez, and D-U. Hwang. "Complex networks: Structure and dynamics." *Physics reports* 424, no. 4-5 (2006): 175-308.

[2] Erdos, Paul, and Alfréd Rényi. "On the evolution of random graphs." *Publ. Math. Inst. Hung. Acad. Sci* 5, no. 1 (1960): 17-60.

[3] Albert, Réka, and Albert-László Barabási. "Statistical mechanics of complex networks." *Reviews of modern physics* 74, no. 1 (2002): 47.

[4] Clauset, Aaron, Cosma Rohilla Shalizi, and Mark EJ Newman. "Power-law distributions in empirical data." *SIAM review* 51, no. 4 (2009): 661-703.

[5] Santos, Francisco C., Jorge M. Pacheco, and Tom Lenaerts. "Evolutionary dynamics of social dilemmas in structured heterogeneous populations." *Proceedings of the National Academy of Sciences* 103, no. 9 (2006): 3490-3494.

[6] Roca, Carlos P., José A. Cuesta, and Angel Sánchez. "Evolutionary game theory: Temporal and spatial effects beyond replicator dynamics." *Physics of life reviews* 6.4 (2009): 208-249.