[INFO-F409] Learning Dynamics
First assignment

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1 The Hawk-Dove game

Conventions and notations

First of all, the Hawk-Dove game is modeled by the matrix presented in the Table 1.

	Hawk	Dove
Hawk	$\frac{V-D}{2}$ $\frac{V-D}{2}$	0 V
Dove	V 0	$\frac{V}{2}-T$ $\frac{V}{2}-T$

Table 1 – Payoff matrix of the Hawk-Dove game

The different actions of a player $i \in \{1, 2\}$ are denoted by the set $\mathcal{A} = \{H, D\}$ where H is the hawk action and D the dove action. Moreover, to denote the different actions payoff, we need an utility function of those actions. This utility function is then written :

$$u_i(a_i, a_{-i})$$

such that a_i is an action of player i and a_{-i} is the action of the other player where player i choses a_i. For instance, $u_1(Hawk, Dove) = V$ and $u_2(Hawk, Dove) = 0$).

In the case of mixed strategies, the notion of the **expected value** of a payoff function is used and is written in the general case :

$$U(p_1, ..., p_k) = p_k u(a_k)$$

where p_k is the probability that the other player chose the action a_k. In the case of the Hawk-Dove game, the expected value formula would be written such that $k = 2$ because it exists only 2 actions, i.e. :

$$U(p_1, p_2) = p_1 u(a_1) + p_2 u(a_2)$$

Furthermore, to find Nash equilibria, best responses have to be found. Those one will be highlighted in red for the line player considered as player one and green for the column player considered as player two.

1.1 Question 1 - Nash equilibrium

Statement

Find all the (mixed strategy) Nash equilibria of this game. How do the results change when the order of the parameters V, D and T is changed (V>D, D>T, etc.)?

1.1.1 First case : $V > D$ and $T > \frac{V}{2}$

In the first case, we consider that $V > D$ and $T > \frac{V}{2}$. Therefore, we know that the value of $\frac{V-D}{2}$ will be positive and the value of $\frac{V}{2} - T \leq 0$.

	Hawk	Dove
Hawk	$\frac{V-D}{2}$ $\frac{V-D}{2}$	0 V
Dove	V 0	$\frac{V}{2} - T$ $\frac{V}{2} - T$

Table 2 – Modified payoff matrix of the H-D game for case 1

1.1.2 Second case : $V < D$

1.1.3 Third case :

1.1.4 Fourth case :

2 Which social dilemma ?

3 Games in finite population