Course Project

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Part A:

Pick any two stocks and using 5-years of monthly data estimate the mean and variances of the stocks. Plot the efficient frontier for the pair of stocks and determine the minimum variance portfolio of the two. How does choice of stocks (high correlation, low correlation, negative correlation) change the results? Explore the addition of a risk-free asset. What conclusions can you draw in relation to what we discussed in class?

Summary of the stock:

Two stocks, Apple (AAPL) and IBM, are selected with the time frame from April 1st, 2016 to March 31st, 2021 (in 5 years). The data (closing prices of the stocks) are retrieved in monthly frequency.

Covariance - Variance Matrix

	AAPL	*IBM*
AAPL	**0.006919**	0.001949
IBM	0.001949	**0.004766**

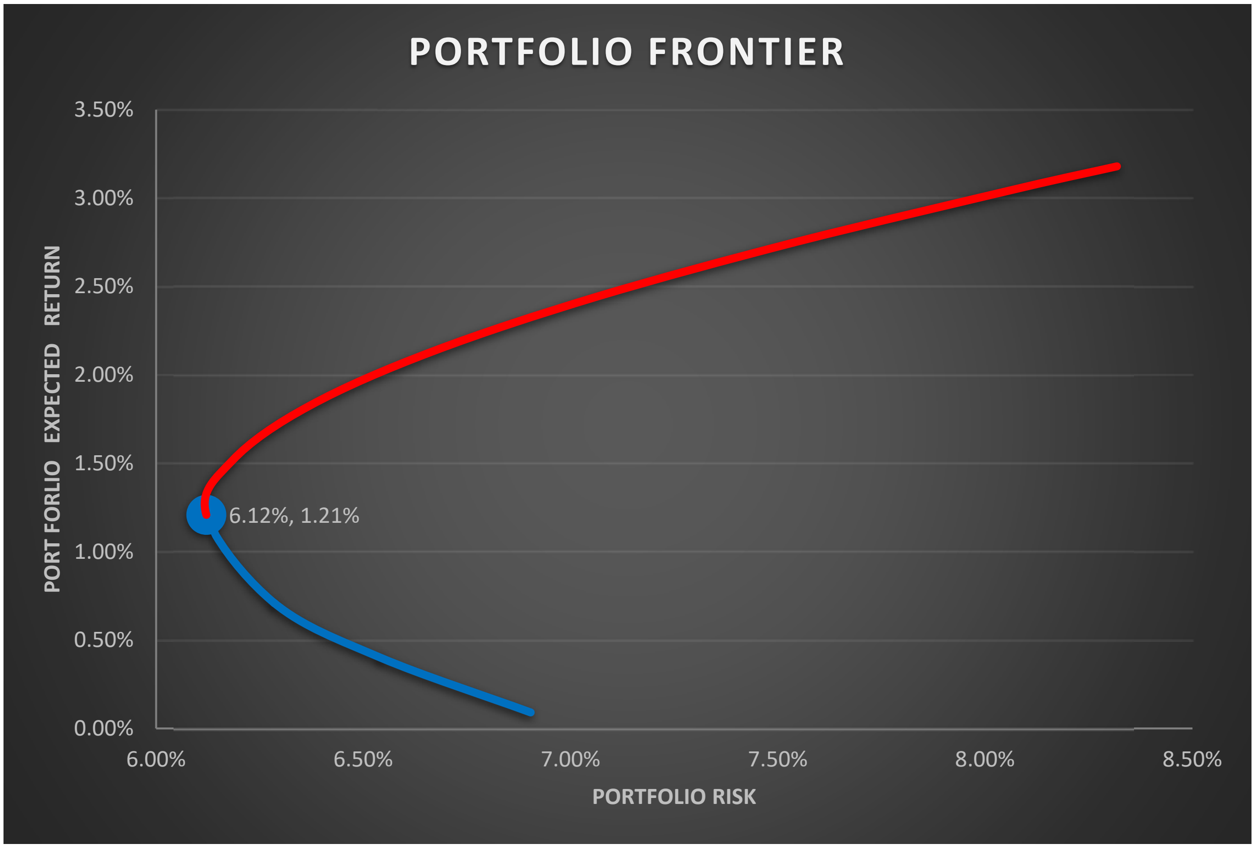
Correlation Matrix

	AAPL	*IBM*
AAPL	**1**	0.339337
IBM	0.339337	**1**

Estimate Expected returns (E[r]), variance (Var) and standard deviation (SD) of each stock:

	E[r]	Var	SD
AAPL	3.18%	0.69%	8.32%
IBM	0.09%	0.48%	6.90%

Efficient frontier:



The point with the data label (r = 1.21%, SD = 6.12%) is the minimum variance portfolio and the red concave line starting from the that point is the efficient frontier**.**

Minimum variance portfolio

	Weights
AAPL	36.18%
IBM	63.82%
Sum	1

Port. Minimum Variance	0.37%
Portfolio Risk - SD	6.12%
Portfolio return - E[r]	1.21%

When the correlation between Apple and IBM is 0.3393, the minimum variance portfolio contains 36.18 % of Apple stock and 63.82% of IBM stock and has the expected return of 1.21% and minimum risk of 6.12%. By diversification, we can achieve a smaller risk than the risk of each individual stock, though the expected return is not as high as the expected return of Apple (3.18%), but higher than the expected return of IBM (0.09%)

Different scenarios of correlation, assuming that short selling is not allowed:
Apple and IBM have a low correlation (**ρ** = 0.339337).

1. <u>If the two stock have positive perfect correlation **ρ = 1:**</u>

	AAPL	IBM
Average Return - r	0.0318	0.0009
Stock Variance - Var	0.006919	0.004766
Stock Standard Deviation - SD	0.08318	0.069036
Correlation ρ	1	

Minimum variance portfolio

	Weights
AAPL	0
IBM	1
Sum	1

Port. Minimum Variance Var	0.004766
Port. Risk - SD	0.069036
Port. Expected Return - E[r]	0.00091

When the correlation between Apple and IBM is 1, the minimum variance portfolio contains no Apple stock and 100% of IBM stock and has the expected return of 0.09% and minimum risk of 6.90%. So, we receive a lower return with a higher risk when choosing high correlated stocks in comparison with two low correlated stocks.

2. <u>If the two stock have no correlation **ρ = 0**</u>

	AAPL	IBM
Average Return - r	0.0318	0.0009
Stock Variance - Var	0.006919	0.004766
Stock Standard Deviation - SD	0.08318	0.069036
Correlation ρ	0	

Minimum variance portfolio

	Weights
AAPL	0.407872
IBM	0.592128
Sum	1

Port. Minimum Variance Var	0.002822
Port. Risk - SD	0.053123
Port. Expected Return - E[r]	0.013509

When the correlation between Apple and IBM is 0, the minimum variance portfolio contains 40.8% Apple stock and 59.2% of IBM stock and has the expected return of 1.35% and minimum risk of 5.31%. As a result, we can receive higher return with lower risk when choosing two independent stocks in comparison with two highly correlated stocks.

3. <u>If the two stock have negative correlation **ρ = -1**</u>

	AAPL	IBM
Average Return - r	0.0318	0.0009
Stock Variance - Var	0.006919	0.004766
Stock Standard Deviation - SD	0.08318	0.069036
Correlation ρ	-1	

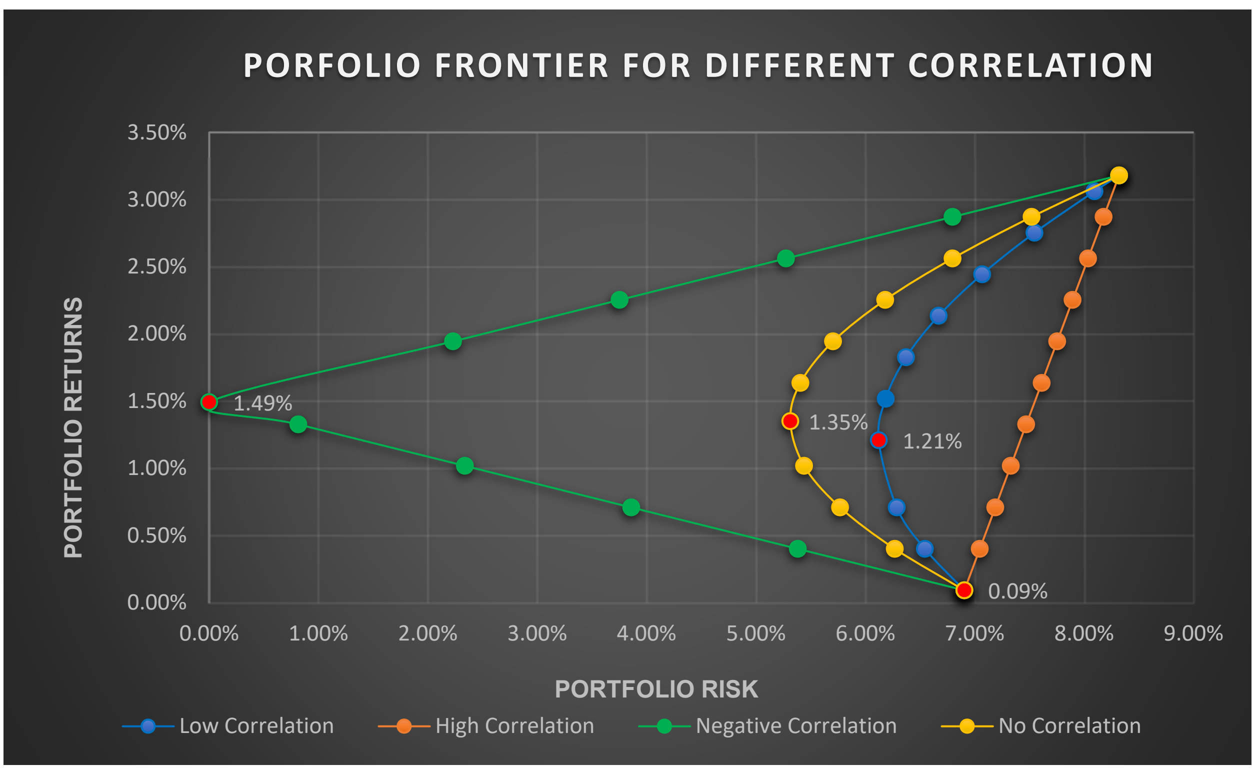
Minimum variance portfolio

	Weights
AAPL	0.453538
IBM	0.546462
Sum	1

Port. Minimum Variance Var	0
Port. Risk - SD	0
Port. Expected Return - E[r]	0.014919

When the correlation between Apple and IBM is -1, the minimum variance portfolio contains 45.4% of Apple stock and 54.6% of IBM stock. It has the expected return of 1.49% and no risk at all. Therefore, we can receive a higher return with no risk when choosing two perfect negative correlated stocks in comparison with two high correlation stocks or two no correlated stocks.

Plot of different correlation scenarios:



The red points with data label are the minimum variance portfolio of each correlation scenario.

<u>Add 1 risk-free asset</u>:

Maximize Sharpe ratio:

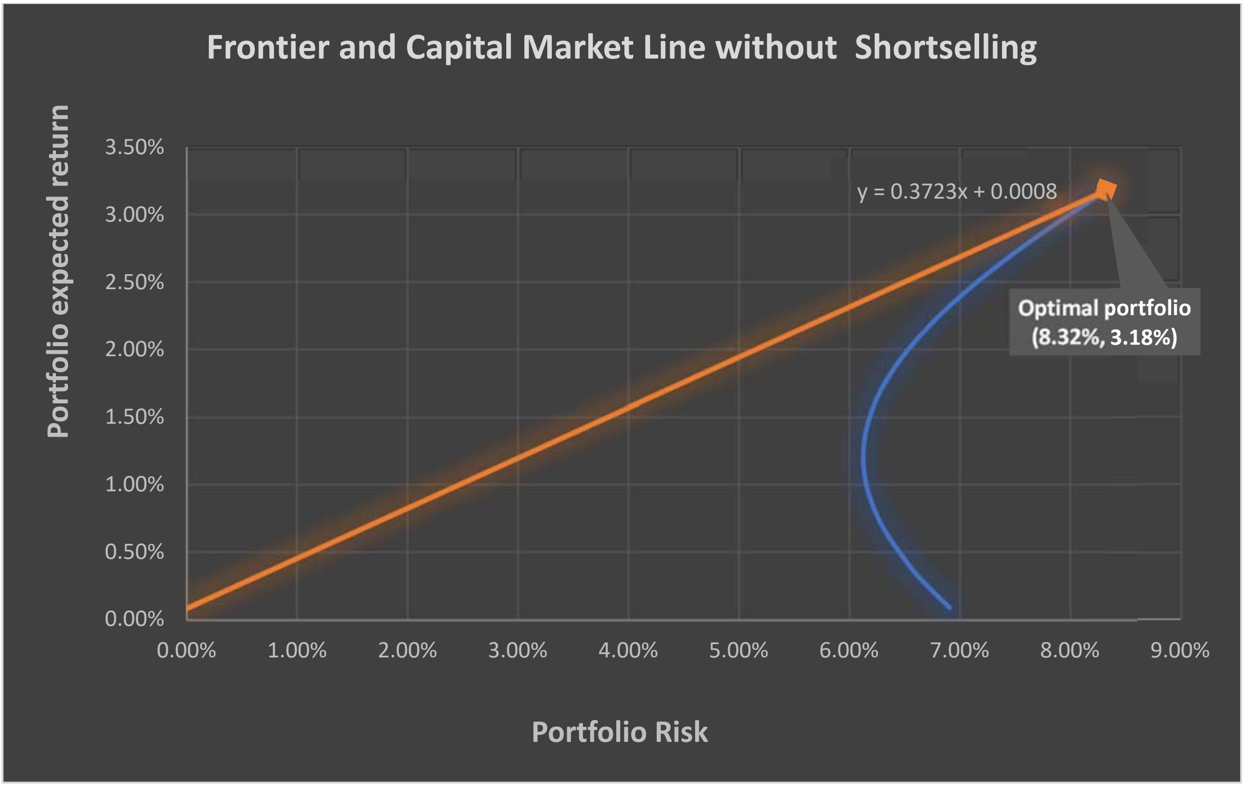
	No short selling	Short selling
	Weights	**Weights**
AAPL	100%	168.3%
IBM	0%	-68.3%
Sum	100%	100%

Min Var of risky asset	0.69%	1.73%
SD of risky asset	8.32%	13.17%
E[r] of risky asset	3.18%	5.29%
Monthly Risk-free rate	0.083%	0.083%
Sharpe Ratio	**0.372263**	**0.395344**
Capital Market Line	**r** = 0.372263***б** + 0.00083	**r** = 0.395344* **б** + 0.00083

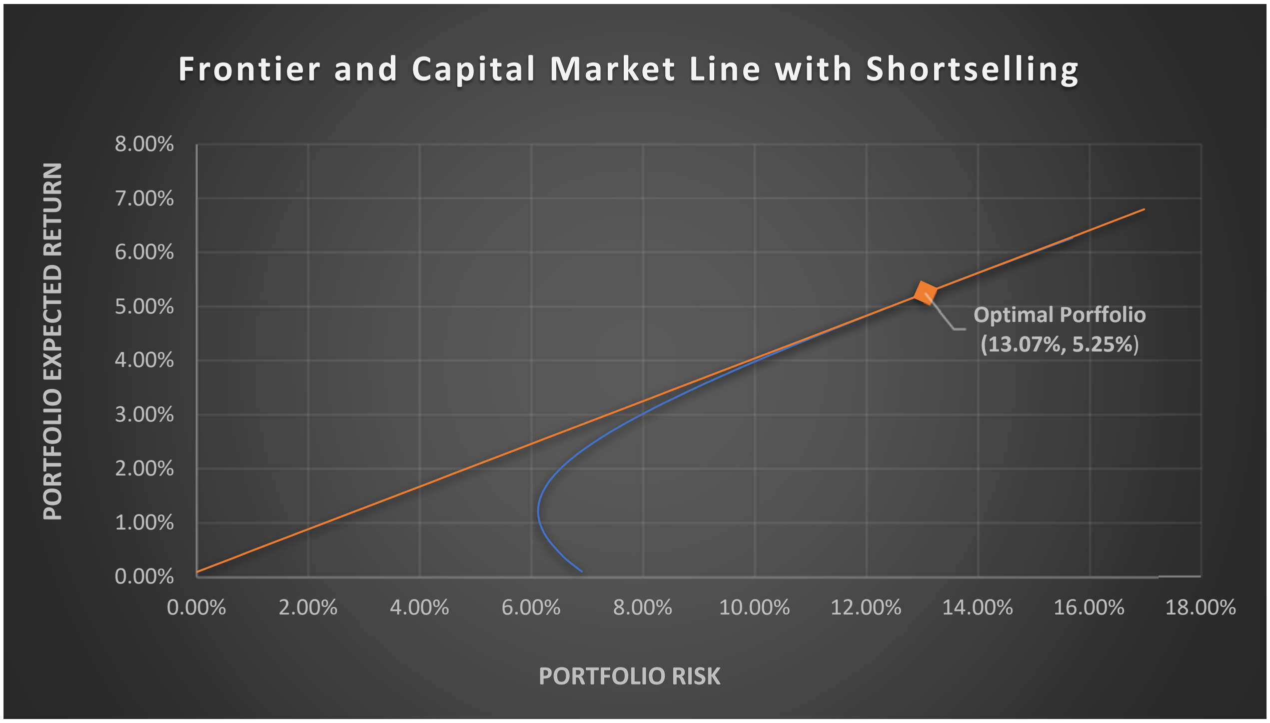
Portfolio of risky assets and a riskless asset:

No Short selling		
	E[r]	**SD**
Risky asset	3.18%	8.32%
Riskless asset	0.083%	0.00%
Correlation	0	

Short selling		
	E[r]	**SD**
Risky asset	5.25%	13.07%
Riskless asset	0.083%	0.00%
Correlation	0	



When short selling is not allowed, the maximum Sharpe ratio is 0.372263, which means that when there is an increase of portfolio risk by 1%, the expected return will proportionally go up by approximately 0.37%. The optimal portfolio is achieved at the tangent points with the expected return of risky asset = 3.18% and risk of risky asset = 8.32%.



When short selling is allowed, we can borrow money at the riskless asset and invest more money in the market portfolio or vice versa. In this case, the Sharpe ratio is 0.395344, which means when there is an increase of portfolio risk by 1%, the expected return will proportionally go up by approximately 0.39%. The optimal portfolio is obtained at the tangent points with the expected return of risky asset = 5.25% and risk of risky asset = 13.07%. Short selling allows us to boost our excess return between the expected return of risky assets and the risk-free rate per unit of risk.

By adding one risk-free asset to the market portfolio, we can identify a combined portfolio between a riskless asset and the optimal portfolio (or tangent portfolio). The combined portfolio can obtain a lower risk portfolio than that of the minimum variance portfolio of 6.12% or a higher expected return than that of the market portfolio of the risky assets.

For example, if we just want the portfolio risk б = 4% and allow short selling, then the expected return of the portfolio is r = 0.395344* 4% + 0.00083 = 1.58%

Let α is the weight of risk - free asset then we have:

$\alpha * 0.083 + (1 - \alpha) * 3.18 = 1.58$, which implies $\alpha = 0.516629$.

The optimal portfolio to achieve the risk of 4% should include 51.7% of a riskless asset and 48.3% of the risky assets. Within 48.3% of the risky assets, we short sell IBM for 68.7 % and invest in AAPL for 168.3%.

***Part B**: Pick any 5 stocks and using 2 years of monthly data estimate the covariances and find sets of portfolio weights that maximize return, minimize risk, and maximize the Sharpe ratio. How does allowing short selling change the results? Write a discussion of the results.*

Summary of the stock:

Five stocks, Walmart (WMT), Netflix (NFLX), Microsoft (MSFT), MC Donald (MCD), and Canadian Tire (CDNAF), are selected with the time frame from Jan 1^{st}, 2019 to Dec 31^{st}, 2020 (in 2 years). The data (closing prices of the stocks) are retrieved in monthly frequency.

Covariance- Variance Matrix

	WMT	*NFLX*	*MSFT*	*MCD*	*CDNAF*
WMT	**0.002046386**	0.000197	0.000762	0.000985	0.001387
NFLX	0.000197397	**0.004726**	0.002233	0.000584	0.001199
MSFT	0.000761535	0.002233	**0.003007**	0.001923	0.002552
MCD	0.000984719	0.000584	0.001923	**0.003615**	0.003893
CDNAF	0.001386863	0.001199	0.002552	0.003893	**0.012518**

Correlation matrix

	WMT	*NFLX*	*MSFT*	*MCD*	*CDNAF*
WMT	1				
NFLX	0.063474601	1			
MSFT	0.307000401	0.592390979	1		
MCD	0.362057156	0.141183146	0.583201736	1	
CDNAF	0.2740086	0.155846639	0.415940767	0.578751	1

Average Expected Return and Variance of each stock:

	E[r]	**Var**	**SD**	**Ratio E[r]/SD**
WMT	1.89%	0.20%	4.52%	0.4181
NFLX	2.28%	0.47%	6.87%	0.3321
MSFT	3.49%	0.30%	5.48%	0.6361
MCD	3.49%	0.36%	6.01%	0.5802
CDNAF	1.42%	1.25%	11.19%	0.1266

Among 5 stocks, MSFT has the highest expected return of 3.49%, while CDNAF has the lowest expected return of 1.42%. Meanwhile, CDNAF has the highest risk of 11.19%, and WMT has the lowest risk of 4.52%.

In general, on considering an investment of an individual stock, based on the normalized expected return of each stock, MCD is the best one since it has the highest return-risk ratio. Meanwhile, CDNAF is the worst stock that should not be invested individually as it has the lowest return-risk ratio.

1. The correlation of stocks is not perfect, so we can enjoy the benefit of diversification by identifying the minimum variance portfolio to minimize the portfolio risk as below:

Minimum variance portfolio

	No short selling	**Short selling**
WMT	55.7%	55.7%
NFLX	23.1%	23.0%
MSFT	3.6%	4.5%
MCD	17.5%	22.1%
CDNAF	0.0%	-5.3%
Sum	100.0%	100.0%

Minimum Variance (Var)	0.14%	0.14%
Risk (SD)	3.72%	3.69%
Port. Expected return (E[r])	2.32%	2.43%

As can be seen from the table above, the minimum variance portfolio includes 55.7% of WMT, 23.1% of NFLX, 17.5% of MCD, 3.6% of NSFT. It excludes CDNAF out of the minimum variance portfolio. This portfolio has the expected return of 2.32% and the minimum risk of 3.72%. This risk is lower than the risk of WMT (which is the stock with the lowest risk of 4.52%).

If short selling is allowed, we can achieve a higher expected return of 2.43% and a lower risk of 2.32% in comparison with the aforementioned portfolio that does not allow short selling. This portfolio contains 55.7% of WMT, 23.0% of NFLX, 22.1% of MCD, 4.5% of NSFT and short sell 5.3% of CDNAF.

2. Let's consider a portfolio that produces maximum return as below:

Maximum Return portfolio

	No short selling weights
WMT	0.0%
NFLX	0.0%
MSFT	52.2%
MCD	47.8%
CDNAF	0.0%
Sum	100.0%

Minimum Variance (Var)	0.26%
Risk (SD)	5.10%
Port. Expected return (E[r])	3.49%

The maximum return portfolio is the combination of 52.2% of MSFT and 47.8% of MCD. This portfolio helps us obtain the highest expected return of 3.49% (the same expected return as that of MSFT and MCD) with a lower risk of 5.1% than risk of the two (5.48% and 6.01% respectively).

When short selling is allowed, Solver cannot find an optimal solution to maximize the expected return of the portfolio.

3. Let's consider a portfolio consists of both a riskless asset with the annual risk-free rate of 1% and risky assets (a portfolio consisting of one or more aforementioned stocks):

Max Sharpe ratio portfolio

	No short selling	**Short selling**
WMT	24.9%	26.5%
NFLX	4.1%	4.8%
MSFT	42.9%	43.7%
MCD	28.2%	45.2%
CDNAF	0.0%	-20.2%
Sum	100.0%	100.0%

Minimum Variance (Var)	0.18%	0.19%
Risk (SD) of risky asset	4.28%	4.40%
E[r] of risky asset	3.04%	3.42%
Monthly Risk-free rate	0.083%	0.083%
Max Sharpe Ratio	0.690726	0.758656
Capital market line	**r** = 0.690726***б** + 0.00083	**r** = 758656***б** + 0.00083

Without short selling

	E[r]	SD
Risky asset	3.04%	4.28%
Riskless asset	0.083%	0%
Correlation	0	

With short selling

	E[r]	SD
Risky asset	3.42%	4.40%
Riskless asset	0.083%	0%
	Correlation	0

When short selling is not allowed, the maximum Sharpe ratio is 0.690726, which means that when there is an increase of portfolio risk by 1%, the expected return will proportionally go up by approximately 0.69%. The optimal portfolio is achieved at the tangent points with the expected return of risky asset = 3.04% and risk of risky asset = 4.28%.

When short selling is allowed, we can borrow money at the riskless asset and invest more money in the market portfolio or vice versa. In this case, the Sharpe ratio is 0.758656, which means when there is an increase of portfolio risk by 1%, the expected return will proportionally go up by approximately 0.75%. The optimal portfolio is gained at the tangent points with the expected return of risky asset = 3.42% and risk of risky asset = 4.40%. Short selling allows us to boost our excess return between the expected return of risky assets and the risk-free rate per unit of risk.

By adding one risk-free asset to the market portfolio, we can identify a combined portfolio between a riskless asset and the optimal portfolio (or tangent portfolio). The combined portfolio can obtain a higher expected return than that of the maximum expected return portfolio (at 3.49%) or a lower risk portfolio than that of the minimum variance portfolio (at 3.69%).

For example, if we want to obtain the portfolio expected return r = 5% and allow short selling,

Let α is the weight of risk - free asset then we have:

$\alpha * 0.083 + (1 - \alpha) * 3.42 = 5$, which implies $\alpha \approx -0.473479$.

To achieve an optimal portfolio with the expected return of 5% , we should short sell 47.3% of a riskless asset and invest 147.3% of the investment fund in the risky assets. Within 147.3% of the risky assets, WMT accounts for 26.5%, NFLX for 4.8%, MSFT for 43.7%, MCD for 45.2%, and CDNAF is short sold at 20.2 %.