SIR equations

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1 Overview of the SIR models used

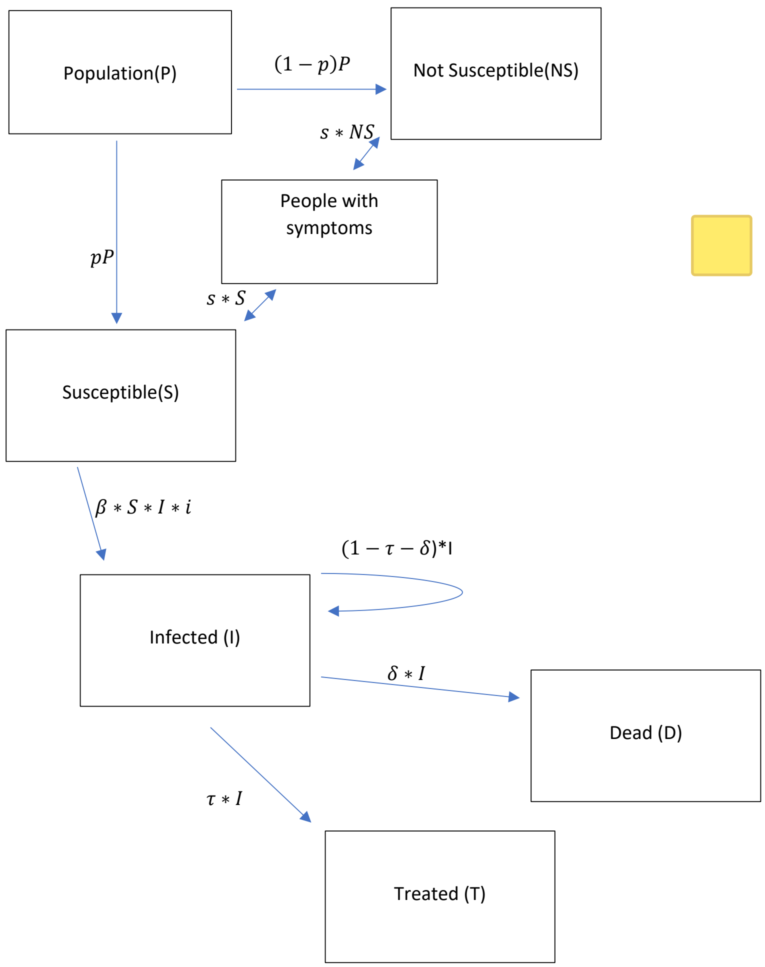
The foundational models for mapping out a disease is the SIR model. The population is divided into three groups (S) Susceptible, (I) Infected, (R) Recovered. There is a Beta term that determines the rate of infection and a gamma term that determines the rate of recovery. So are three equations are as followed:

$$\frac{dS}{dt} = -\beta * I * S$$
$$\frac{dI}{dt} = \beta * I * S - \gamma * I$$
$$\frac{dR}{dt} = \gamma * I$$

To get to an actual equation we have to make some assumptions. One if them is that the whole system is stable with no one leaving the model. This can be represented by the equation: $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$. Now that we have gone over some basics, we will try to make another basic model that we will call the SIRD model. This and other models reach equilibrium state when there are no more people in the infected group.

2 SIRD

This model is different in two ways. For one, it has a component D that will Deaths. Second, we will separate the population into two groups. People who are non-susceptible and people who are. The diagram for this model can be seen below.



Population(P)
$(1-p)P$
Not Susceptible(NS)
$s * NS$
People with symptoms
pP
$s * S$
Susceptible(S)
$\beta * S * I * i$
$(1-\tau-\delta)$*I
Infected (I)
$\delta * I$
Dead (D)
$\tau * I$
Treated (T)

There are going to be a series of differential equations that govern this model. They are

$$\frac{dS}{dt} = -\beta * I * \frac{S}{N}$$
$$\frac{dI}{dt} = \beta * I * \frac{S}{N} - \tau * I - \delta * I$$
$$\frac{dD}{dt} = \delta * I$$
$$\frac{dR}{dt} = \tau * I$$

This model has some initial conditions. The number of people who are non-susceptible will remain constant. Also, another difference is the $\frac{S}{N}$ term. This is the percentage of the population that can still be infected at a given time. Assuming the entire population is susceptible, this proportion will be close to 1 at $t = 0$ (Since you have people initially infected). This model holds the same assumption that $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dD}{dt} + \frac{dR}{dt} = 0$.

The assumptions for this model is as followed:

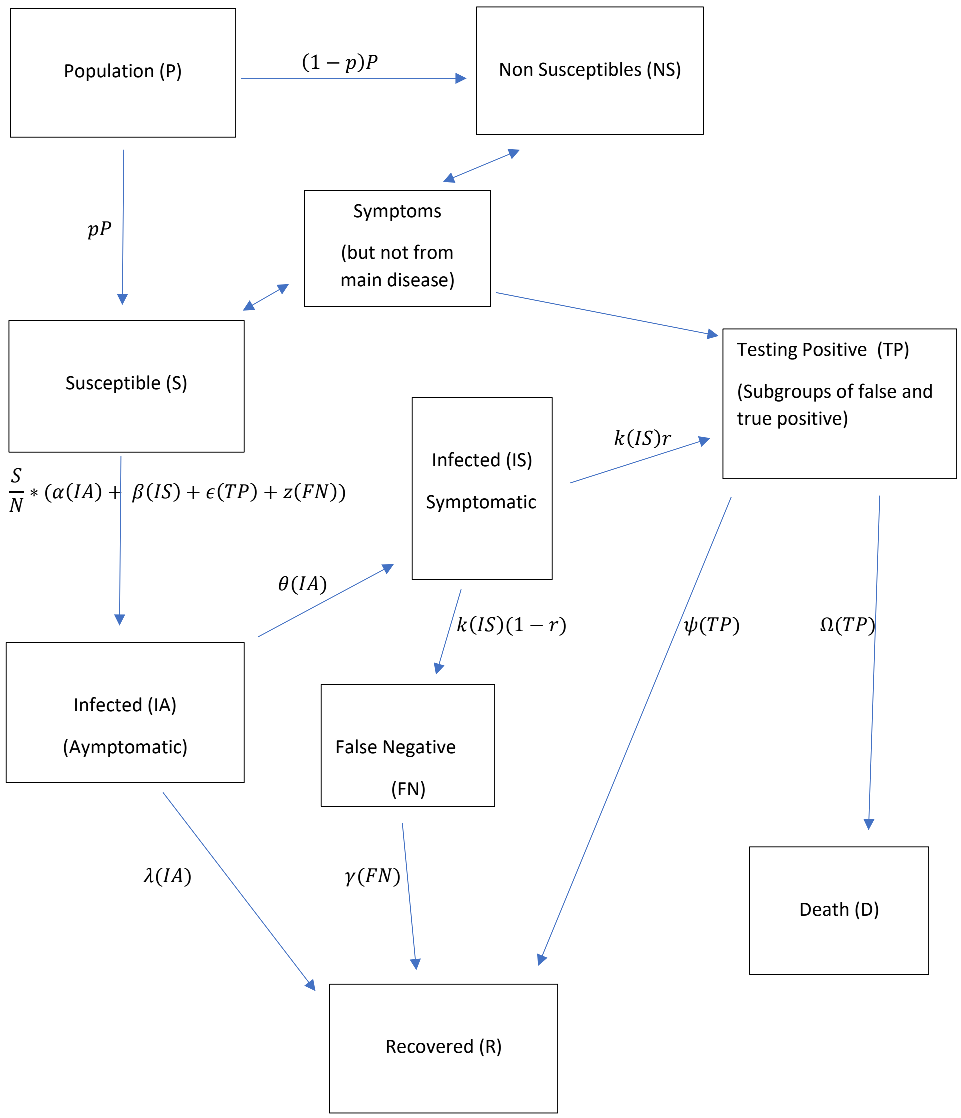
1. Once infected, you are immediately contagious.
2. Once recovered, you are no longer susceptible.

The variable descriptions are listed below:

β: Spread rate of the disease
τ: Treatment rate
δ: death rate
p: Percentage of population that are susceptible
s: Percentage of people who will have symptoms like Covid-19 but not have the disease.

3 Complex SITRD with testing and cost

Finally we have the SITRD model where you have the compartments S (Susceptible), I (Infected), T (Tested), R (Recovered), and D (Death). The group Infected will have two subgroups: Asymptomatic and Symptomatic. The group tested will also have two subgroups: True Positive and False Negative. There will still exist a Population group where only a portion of them are susceptible to the disease. However, some them can still be symptomatic. Below is a diagram that represents how people move between groups.


Population (P)
$(1-p)P$
Non Susceptibles (NS)
pP
Symptoms
(but not from main disease)
Susceptible (S)
Testing Positive (TP)
(Subgroups of false and true positive)
Infected (IS)
Symptomatic
$k(IS)r$
$\frac{S}{N} * (\alpha(IA) + \beta(IS) + \epsilon(TP) + z(FN))$
$\theta(IA)$
$k(IS)(1-r)$
$\psi(TP)$
$\Omega(TP)$
Infected (IA)
(Aymptomatic)
False Negative
(FN)
Death (D)
$\lambda(IA)$
$\gamma(FN)$
Recovered (R)

Now here are the differential equations that govern this model.

$$\frac{ds}{dt} = -\frac{S}{N}(\alpha(IA) + \beta IS + \epsilon(TP) + \zeta(FN)$$
$$\frac{d(IA)}{dt} = \frac{S}{N}(\alpha(IA) + \beta(IS) + \epsilon(TP) + \zeta FN) - \theta(IA) - \lambda(IS)$$
$$\frac{d(IS)}{dt} = \theta(IA) - \kappa(IS)$$
$$\frac{d(FN)}{dt} = \kappa(IS)(1-p) - \gamma(FN)$$
$$\frac{d(TP)}{dt} = \kappa(IA)(p) - \psi(TP) - \Omega(TP)$$
$$\frac{dR}{dt} = \lambda(IA) + \gamma(FN) + \psi(TP)$$
$$\frac{dD}{dt} = \Omega(TP)$$

The assumptions for this model are:
1. Everyone infected at first is asymptomatic.
2. If you test negative but have the disease, there is no chance you will die.
3. You will only die if you test positive and treatment is unsuccessful.
4. Once infected, you are immediately contagious.
5. Only people who experience symptoms will be considered for testing.
6. Everyone who tests positive will receive the treatment.
7. The only change in population can come from people dying.
8. Once recovered, you are no longer susceptible.

Variable descriptions are:

α: Spread rate among Asymptomatic people
β: Spread rate among Symptomatic people
ϵ: Spread rate among people testing positive
ζ: Spread rate among people who test false negative
θ: Transition rate of people who become symptomatic
λ: Recovery rate for asymptomatic people
κ: Percentage of symptomatic people tested each day (Probably 1.00)
r: True positive test rate
γ : Rate at which people who test negative recover
ψ: Success rate of treatment
Ω: Probability of dying among people who test positive.

Moreover, with this model, we will have cost variable that will measure the cost of tests and treatments for a given simulation.