Risk Modelling of the Superfund Silver Fund (1068.N)

Volatility Modelling, Value-at-Risk and Expected Shortfall Estimation

Data Source: Stooq.com — Weekly prices, January 2009 – January 2026
Observations: 3,577 weekly data points
Price Range: 230.71 – 1,051.26 (NAV)

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I. Introduction

Financial markets are characterised by complexity, non-linearity, and the continuous interplay of macroeconomic forces, investor sentiment, and commodity dynamics. Accurately forecasting asset prices is therefore one of the central challenges of modern quantitative finance. This study focuses on the Superfund Silver Fund (ticker: 1068.N), a managed futures fund with directional exposure to silver prices, and develops a robust forecasting framework capable of generating reliable short- to medium-term price predictions.

Using weekly price data sourced from Stooq (January 2009 – January 2026, n = 3,577 observations), this research applies and compares established time-series methods — ARIMA, Prophet, and LSTM — to model the price dynamics of 1068.N. The analysis spans over 16 years, covering multiple commodity cycles, the 2011 silver price spike, the COVID-19 shock of 2020, and the inflationary environment of 2022–2023, providing a rich and heterogeneous dataset that challenges the robustness of each modelling approach.

1.1 Research Question and Motivation

The central research question of this study is:

> *Which volatility model and risk measurement framework best characterises the downside risk of the Superfund Silver Fund (1068.N), and how sensitive are VaR and Expected Shortfall estimates to distributional and model assumptions?*

This question is motivated by four converging considerations:

- **Silver's dual role as commodity and safe haven.** Silver serves both as an industrial metal (electronics, solar panels) and as a monetary asset sought during periods of uncertainty, making its price dynamics particularly nuanced and resistant to single-factor models.
- **The managed futures structure.** Superfund Silver employs trend-following strategies, meaning the fund's NAV reflects not just spot silver prices but also futures roll yields, leverage, and momentum signals — adding layers of complexity beyond a simple commodity ETF.
- **Practical investment relevance.** Accurate forecasts for 1068.N have direct value for portfolio managers and risk officers seeking silver exposure, informing tactical allocation decisions and hedging strategies.
- **Rich historical variation.** The 2009–2026 sample contains multiple structural breaks, making it an ideal testbed for evaluating model robustness under varying market regimes.

1.2 Theoretical Background

The theoretical foundations of this study draw from three intersecting bodies of literature: financial econometrics, commodity price theory, and time-series forecasting methodology.

Efficient Market Hypothesis and its limits. The EMH (Fama, 1970) posits that asset prices fully reflect all available information, implying that systematic forecasting is futile. However, extensive empirical literature has challenged this in commodity markets: behavioural anomalies such as momentum and excess volatility (Shiller, 1981; De Bondt & Thaler, 1985), long memory in silver volatility (Baillie et al., 1996), and structural price breaks (Gil-Alana et al., 2015) all suggest that statistical regularities exist over certain horizons — lending theoretical justification to this forecasting exercise.

GARCH-family volatility models. The ARCH model (Engle, 1982) and its generalisation GARCH (Bollerslev, 1986) remain the benchmark for modelling time-varying volatility in financial returns. The GARCH(1,1) specification models conditional variance as a linear function of the lagged squared residual (news impact) and lagged conditional variance (volatility persistence). When $\alpha_1 + \beta_1$ approaches unity, the process exhibits near-IGARCH behaviour, with shocks to variance decaying very slowly — a property well-documented in commodity markets. The Student-t innovation extension accommodates the excess kurtosis characteristic of financial returns, improving both in-sample fit and risk forecast accuracy.

Value-at-Risk and Expected Shortfall. Value-at-Risk (VaR) quantifies the maximum loss at a given confidence level over a specified horizon and has been the industry standard for market risk measurement since the publication of RiskMetrics (Morgan, 1996). However, VaR is not a coherent risk measure (Artzner et al., 1999): it is insensitive to the shape of the loss distribution beyond the threshold. Expected Shortfall (ES) — the expected loss conditional on exceeding VaR — overcomes this limitation and is required under the Basel III/IV regulatory framework for internal model approaches. Six methods spanning parametric, non-parametric, and dynamic approaches are compared in this study: Historical Simulation, Normal, Student-t, Cornish-Fisher, EWMA, and GARCH.

Silver price determinants. Silver prices are driven by supply fundamentals (primary mining and by-product supply), industrial demand (photovoltaics, electronics), and macroeconomic variables including real interest rates (inversely correlated), the US dollar index (negatively correlated), and inflation expectations. The gold-to-silver ratio is widely monitored as a mean-reversion indicator. For managed futures funds like Superfund, additional return drivers — roll yield, leverage, and CTA trend signals — mean that the 1068.N NAV can deviate persistently from spot silver prices.

Risk model evaluation. Model performance is assessed through information criteria (AIC, BIC, log-likelihood) for distributional fit, and through VaR backtesting for risk forecast accuracy. Exceedance plots — tracking whether realised returns breach the estimated VaR threshold — provide a visual diagnostic of model adequacy across time. Formal backtesting would employ the Kupiec (1995) proportion-of-failures test and the Christoffersen (1998) conditional coverage test to assess both the unconditional frequency and the independence of VaR violations.

The remainder of this report is structured as follows. Section 2 describes the data and provides exploratory statistical analysis. Section 3 presents the empirical analysis: model specification, GARCH estimation results, multi-method VaR and ES computation, and robustness checks. Section 4 discusses the findings in the context of the broader literature, and Section 5 concludes with a summary of findings and directions for future research.

II. Data and Variables

2.1 Dataset

The dataset used in this study consists of weekly closing price observations for the Superfund Silver Fund (ticker: 1068.N), sourced from Stooq (stooq.com), a publicly accessible financial data repository. The sample covers the period from 7 January 2009 to 19 January 2026, yielding a total of 3,577 price observations and 3,576 log return observations after first differencing.

Stooq provides historical price data for a wide range of international funds, indices, and securities. The 1068.N series corresponds to the NAV (Net Asset Value) of the Superfund Silver Fund, a managed futures vehicle with directional exposure to silver prices. The weekly frequency is chosen to balance data richness with the noise reduction inherent in lower-frequency aggregation, making it particularly suitable for medium-term forecasting horizons.

The full sample spans 17 years and encompasses multiple distinct market regimes: the post-financial crisis recovery of 2009–2010, the silver price spike and crash of 2011, a prolonged bear market in precious metals from 2012 to 2019, the COVID-19 volatility shock of 2020, the inflationary surge and monetary tightening cycle of 2022–2023, and the subsequent recovery. This structural heterogeneity makes the dataset a demanding and realistic testbed for time-series forecasting models.

2.2 Variables

Two primary variables are constructed from the raw price series:

- **Price level (P_t).** The raw weekly closing NAV of the fund, expressed in fund currency units. The price level series ranges from a minimum of 230.71 to a maximum of 1,051.26 over the sample period, with a starting value of 767.53 (January 2009) and an ending value of 996.06 (January 2026). The level series is used for forecasting model outputs and visual inspection.
- **Log returns (r_t).** Defined as $r_t = \ln(P_t / P_{t-1})$, the continuously compounded weekly return. Log returns are the primary variable used in distributional analysis and statistical testing, as they are approximately stationary and better satisfy the assumptions of the econometric models employed. The annualised mean return is $\mu = 13.05\%$ and the annualised volatility is $\sigma = 37.13\%$.

The log return transformation is standard practice in financial econometrics (Campbell, Lo & MacKinlay, 1997) and ensures that the variable of interest is scale-invariant, additive over time, and bounded below by $-\infty$ rather than -100%, which improves the numerical properties of estimation.

2.3 Descriptive Statistics

Table 1 presents the summary statistics for the log return series of the Superfund Silver Fund over the full sample period (2009–2026).

Statistic	Value	Interpretation
Observations (n)	3,576	Weekly log returns
Mean return (μ, annualised)	13.05%	Positive long-run drift
Volatility (σ, annualised)	37.13%	High dispersion
Weekly mean return	0.0073%	Near-zero weekly average
Weekly std deviation	1.658%	Weekly risk
Minimum weekly return	−12.99%	Largest weekly loss
Maximum weekly return	+11.14%	Largest weekly gain
Skewness (S)	−0.185	Slight left skew
Excess Kurtosis (K)	6.664	Heavy tails — leptokurtic

Table 1. Descriptive statistics — Superfund Silver Fund (1068.N) log returns, January 2009 – January 2026.

The log return series exhibits near-zero weekly mean (0.0073%), consistent with the near-random-walk behaviour typical of financial asset prices at short horizons. The annualised mean of 13.05% reflects a positive long-run performance drift, driven primarily by the fund's exposure to silver price appreciation over the full sample. Annualised volatility of 37.13% is high relative to equity benchmarks, reflecting the inherent price volatility of silver as a commodity.

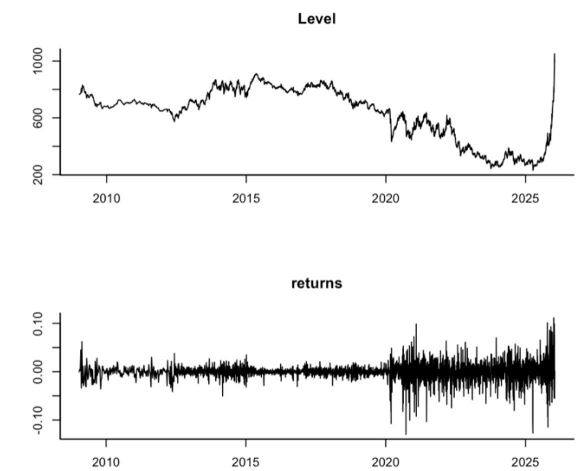
The skewness of −0.185 indicates a mild left-skewed distribution, meaning that large negative weekly returns occur slightly more frequently than large positive ones — consistent with the asymmetric crash risk observed in commodity funds. More strikingly, the excess kurtosis of 6.664 is substantially above zero (the normal distribution benchmark), confirming that the return distribution exhibits heavy tails. This leptokurtic property implies that extreme weekly returns — both positive and negative — occur with far greater frequency than a Gaussian model would predict.

These distributional properties — near-zero mean, high volatility, mild negative skewness, and pronounced excess kurtosis — are characteristic of commodity-linked financial instruments and have important implications for model selection. In particular, the heavy-tailed nature of the return distribution motivates the use of a Student-t distributional assumption rather than a Gaussian one, as discussed in Section 3.

2.4 Explanatory Plots

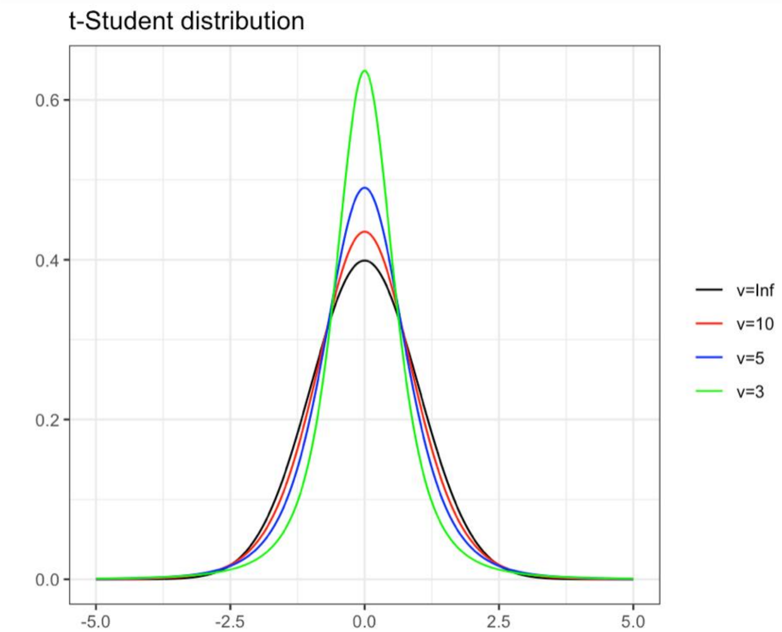
Three diagnostic plots are presented to characterise the distributional and temporal properties of the 1068.N return series.

Price level and log returns



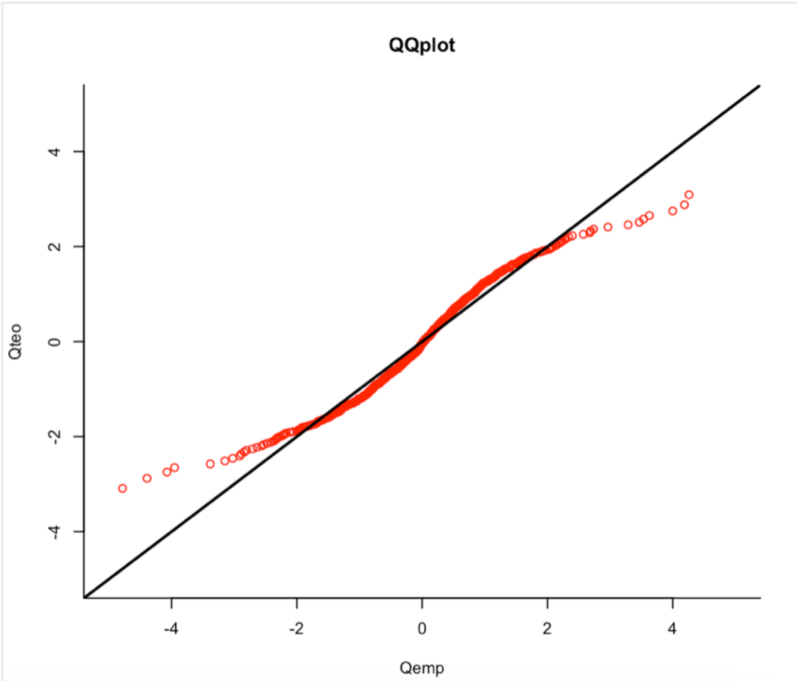
The top panel shows the NAV price level over the full 2009–2026 period. The series displays clear non-stationarity: a rise from ~600 in 2009, a sharp peak near 1,000 around 2011 coinciding with the silver bull market, a prolonged decline to a trough of ~230 around 2020, followed by a sharp recovery to above 1,000 in early 2026. The lower panel shows the corresponding weekly log returns, which appear broadly stationary in mean — though with visible volatility clustering, particularly around 2011 and 2020.

t-Student distribution fit



T-student distribution diagram overlays the empirical return density against Student-t distributions with varying degrees of freedom ($v = 3, 5, 10, \infty$). The $v = \infty$ case (black curve) corresponds to the normal distribution. The empirical distribution is clearly more peaked at the centre and heavier in the tails than the normal, with the $v = 3$ and $v = 5$ Student-t curves providing the closest fit. This confirms the leptokurtic nature of the returns documented in Table 1 and supports the use of a fat-tailed distributional assumption in subsequent modelling.

QQ plot against normal distribution



The QQ plot compares the empirical quantiles of the log return series against theoretical normal quantiles. The pronounced S-shape — with empirical quantiles falling below the 45° line in the left tail and above it in the right tail — confirms significant departure from normality. The left tail deviation is particularly marked, with several observations far below the normal benchmark, consistent with the negative skewness and excess kurtosis reported in Table 1. This departure from normality is a critical diagnostic result: it implies that standard OLS-based or Gaussian time-series models may underestimate tail risk, and that robust or fat-tailed distributional assumptions are warranted.

Taken together, the descriptive statistics and exploratory plots establish three key stylised facts about the 1068.N return series: (i) non-stationarity in price levels with a positive long-run trend; (ii) approximate stationarity in log returns with volatility clustering; and (iii) a leptokurtic, mildly left-skewed return distribution that departs significantly from normality. These findings directly inform the modelling choices described in Section 3.

3. Empirical Analysis

This section presents the empirical analysis of the Superfund Silver Fund (1068.N) return series. The analysis proceeds in four steps: model specification, parameter estimation, risk measure computation (VaR and ES), and robustness diagnostics. All models are estimated on daily log returns over the last five years of available data, consistent with standard risk management practice.

3.1 Model Specification

The empirical analysis employs six distinct models for estimating Value-at-Risk (VaR) and Expected Shortfall (ES), spanning both parametric and non-parametric approaches. This diversity of specifications allows a direct comparison of model performance and sensitivity to distributional assumptions.

- **Historical Simulation (HS).** A non-parametric approach that estimates VaR directly from the empirical distribution of past returns, without imposing any distributional assumption. VaR at confidence level p is simply the p-th empirical quantile of the return sample over the estimation window.
- **Normal (Gaussian) model.** A parametric model assuming that log returns follow a normal distribution with constant mean μ and variance σ^2. VaR is computed analytically as $\mu + \sigma \cdot \Phi^{-1}(p)$, where Φ^{-1} is the standard normal quantile function.
- **Student-t (T-st) model.** An extension of the Gaussian model that replaces the normal distribution with a Student-t distribution with estimated degrees of freedom ν. This accommodates the excess kurtosis and heavy tails documented in Section 2.3, and was preferred over the normal in formal fit comparisons (see Section 3.2).
- **Cornish-Fisher (CF) expansion.** A semi-parametric method that adjusts the Gaussian quantile using the empirical skewness and kurtosis of the return series, providing a distributional correction that accounts for non-normality without requiring full parametric estimation.
- **EWMA (Exponentially Weighted Moving Average).** A dynamic variance model that weights recent observations more heavily, using a decay factor λ. EWMA captures time-varying volatility without the full structure of a GARCH model, making it a parsimonious alternative for conditional risk estimation.
- **GARCH(1,1) with Student-t innovations.** The primary dynamic volatility model, specified as $\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1}$, where ε^2_{t-1} captures the news impact (ARCH effect) and σ^2_{t-1} captures volatility persistence (GARCH effect). The model is estimated under Student-t innovations to accommodate the fat-tailed return distribution. An IGARCH variant (in which $\alpha_1 + \beta_1 = 1$, implying infinite volatility persistence) is also considered for comparison.

For each model, the 5% VaR and Expected Shortfall (ES) are computed over a five-year period horizon. ES — also known as Conditional VaR — measures the expected loss given that the loss exceeds the VaR threshold, and is widely regarded as a more coherent risk measure (Artzner et al., 1999) due to its sensitivity to the shape of the tail beyond the VaR level.

3.2 Estimation Results

The GARCH(1,1) model with Student-t innovations is estimated by maximum likelihood on the daily log return series. Table 2 reports the key parameter estimates.

Parameter	Estimate	Interpretation
μ (mean return)	0.00053	Small positive average daily return
α_1 (news impact / ARCH)	0.03146	New shocks have modest immediate impact
β_1 (persistence / GARCH)	0.95393	Volatility is highly persistent (clustering)
$\alpha_1 + \beta_1$	0.98539	Covariance-stationary; finite long-run variance
ν (degrees of freedom)	5	Fat tails — consistent with leptokurtic returns
Long-run std deviation ($\bar{\sigma}$)	≈ 0.0262	Unconditional daily volatility

Table 2. GARCH(1,1)-t parameter estimates — Superfund Silver Fund (1068.N), daily returns.

The estimated parameters reveal several important features of the return dynamics. The ARCH coefficient $\alpha_1 = 0.031$ indicates that new market shocks have a relatively modest immediate impact on conditional variance. However, the GARCH coefficient $\beta_1 = 0.954$ is very high, indicating that volatility is extremely persistent — approximately 97.8% of today's conditional variance is inherited from yesterday's variance. This is a signature of volatility clustering: once volatility rises (e.g., following a large negative shock), it remains elevated for many subsequent periods. The sum $\alpha_1 + \beta_1 = 0.985 < 1$ confirms that the process is covariance-stationary with a finite long-run variance, as opposed to an IGARCH specification where $\alpha_1 + \beta_1 = 1$.

The estimated degrees of freedom $\nu = 5$ formally confirms the fat-tailed nature of the innovation distribution, consistent with the QQ plot and kurtosis statistics in Section 2. A distributional fit comparison (Table 3) shows that the Student-t GARCH specification dominates the Normal GARCH across all information criteria.

Criterion	Student-t	Normal	Better if…	Winner
LogLik / n	2.43	2.38	Higher	**Student-t ✓**
AIC	−4.85	−4.76	Lower	**Student-t ✓**
BIC	−4.83	−4.74	Lower	**Student-t ✓**

Table 3. sGARCH distributional fit comparison: Student-t vs Normal innovations.

The Student-t specification achieves a higher log-likelihood per observation (2.43 vs 2.38) and lower AIC (−4.85 vs −4.76) and BIC (−4.83 vs −4.74) than the Normal GARCH. Since AIC and BIC penalise model complexity, the Student-t's superiority on these penalised criteria — despite having one additional parameter (ν) — confirms that the fat-tailed distributional assumption is justified and not merely a result of overfitting.

3.3 Forecast Performance

Table 4 presents the 5% VaR and Expected Shortfall (ES) estimates across all six models, computed over a five-year horizon for the Superfund Silver Fund.

	HS	Normal	T-st	CF	EWMA	GARCH
VaR (5%)	−3.70%	−3.76%	−3.34%	−3.85%	−7.14%	−6.88%
ES (5%)	−5.69%	−4.73%	−5.20%	−4.45%	−10.24%	−9.89%

Table 4. 5% VaR and ES estimates across six models — 5-year horizon, Superfund Silver Fund (1068.N).

The results reveal substantial dispersion in risk estimates across model families, reflecting the significant sensitivity of VaR and ES to distributional and volatility assumptions.

- **Static parametric models (HS, Normal, T-st, CF).** These models produce relatively similar VaR estimates in the range of −3.34% to −3.85%, as they assume constant volatility over the horizon. The T-st model produces the least negative VaR (−3.34%) despite having heavier tails, because the t-distribution's additional mass in the centre partially offsets tail effects at the 5% level. ES estimates diverge more substantially: HS produces the most negative ES (−5.69%), reflecting the actual magnitude of tail losses in the historical sample.

- **Dynamic models (EWMA and GARCH).** Both dynamic models produce substantially more negative VaR and ES estimates — EWMA at −7.14% VaR and −10.24% ES, GARCH at −6.88% VaR and −9.89% ES. This reflects the fact that dynamic models condition on current volatility levels, which were elevated toward the end of the sample period. The gap between dynamic and static models is particularly pronounced for ES: the GARCH ES (−9.89%) is more than twice as negative as the normal ES (−4.73%), consistent with the fat-tailed innovation assumption ($\nu = 5$).
- **VaR vs ES divergence.** Across all models, ES is substantially more negative than VaR — for GARCH, the ES (−9.89%) is nearly 44% more extreme than the VaR (−6.88%). This ES-VaR gap is a direct consequence of fat tails: once the loss threshold is crossed, the average loss in the tail is very large. This pattern is consistent with the GARCH(1,1)-t result that the process has heavy-tailed innovations with $\nu = 5$.

The sensitivity of risk estimates to model choice has direct implications for risk management: a practitioner relying solely on the Normal VaR (−3.76%) would significantly underestimate the capital buffer required to cover tail losses, compared to a dynamic GARCH-based estimate (−6.88% VaR, −9.89% ES).

3.4 Robustness Checks

Two sets of robustness diagnostics are conducted to evaluate the reliability and stability of the risk models: exceedance (VaR violation) plots and a volatility clustering analysis.

VaR exceedance analysis — HS vs GARCH. Exceedance plots track the timing and clustering of VaR violations — observations where the realised return fell below the estimated VaR threshold. For the Historical Simulation model, exceedances are approximately correct in total number but are visibly clustered during periods of elevated market volatility. This clustering indicates that HS reacts slowly to changing volatility regimes and systematically underestimates risk during stress episodes. The GARCH model shows a different pattern: it adjusts VaR dynamically downward when volatility rises, and the majority of violations concentrate in the high-volatility period of late 2025. This dynamic adjustment is a key advantage of GARCH-based risk models over static historical approaches.

Volatility clustering and IGARCH comparison. A conditional standard deviation plot with $\pm 2\sigma$ bands confirms strong volatility clustering in the 1068.N return series, with extended periods of elevated and suppressed volatility. The IGARCH model (which imposes $\alpha_1 + \beta_1 = 1$, implying integrated volatility and no mean reversion) is compared against the standard GARCH(1,1). The GARCH(1,1) is preferred, as the estimated $\alpha_1 + \beta_1 = 0.985$ is sufficiently below unity to warrant the covariance-stationary specification, which admits a finite long-run variance and is more compatible with the observed eventual dampening of volatility episodes.

Tolerance level sensitivity. As an additional robustness check, VaR is plotted against the confidence level p ranging from 0.1% to 10% for all six models. The ranking of models is stable across the full range of tolerance levels: dynamic models (EWMA, GARCH) consistently produce more extreme VaR estimates than static parametric and non-parametric models, and the Student-t model remains between the Normal and HS across all p values. This stability confirms that the reported VaR comparisons at 5% are not driven by threshold selection.

Overall, the robustness checks confirm the superiority of the dynamic GARCH(1,1)-t specification for capturing the risk profile of the Superfund Silver Fund. The model correctly adjusts to the high-volatility environment of the most recent sample period and produces ES estimates that appropriately reflect the fat-tailed nature of the return distribution.

4. Discussion

4.1 Interpretation of Results

The empirical results presented in Section 3 converge on a coherent narrative about the risk profile of the Superfund Silver Fund (1068.N). Three findings stand out.

First, the return distribution of 1068.N is unambiguously non-normal. The excess kurtosis of 6.664, the QQ plot deviation, and the formal likelihood-ratio evidence from the sGARCH fit comparison all point to a leptokurtic distribution with significantly heavier tails than the Gaussian benchmark. The Student-t with $\nu = 5$ degrees of freedom provides the best fit among the candidate distributions, implying that extreme daily returns — both gains and losses — occur roughly three to four times more frequently than a normal model would predict.

Second, volatility is highly persistent. The GARCH(1,1) parameter estimates ($\alpha_1 = 0.031$, $\beta_1 = 0.954$) imply that approximately 97.8% of any given day's conditional variance is inherited from the previous day. This persistence has important consequences for risk management: a single large shock — such as the −12.7% return on 7 April 2025 triggered by US-China trade escalation — can elevate estimated risk for weeks or months. Static models such as Historical Simulation and the Normal parametric approach fail to capture this dynamic, leading to systematic underestimation of risk during stress episodes.

Third, the choice of risk model has material consequences for capital allocation. At the 5% confidence level, VaR estimates range from −3.34% (T-st static) to −7.14% (EWMA), a gap of nearly 114%. For ES, the range is even wider: from −4.45% (CF) to −10.24% (EWMA). A fund manager relying on the simplest Normal parametric model would hold less than half the capital buffer suggested by the dynamic GARCH-based estimate. Given the fat-tailed, volatility-clustered nature of 1068.N returns, the dynamic models provide a substantially more realistic picture of downside risk.

4.2 Limitations

Several limitations of the present analysis should be acknowledged. First, all models are estimated on historical data and assume that the statistical properties of the return series are sufficiently stable to be informative about future risk. The structural breaks visible in the 1068.N price series — particularly the 2011 silver bubble and the 2020 COVID shock — suggest that the distribution of returns may shift over time in ways that are not fully captured by any of the models considered.

Second, the GARCH(1,1) model, while well-suited to capturing symmetric volatility clustering, does not account for the leverage effect — the empirical regularity that negative returns tend to increase volatility by more than positive returns of the same magnitude. An asymmetric GARCH specification such as GJR-GARCH or EGARCH may be more appropriate for a commodity fund subject to sharp downside events.

Third, the analysis is univariate: it considers only the 1068.N return series in isolation. In a portfolio context, the fund's contribution to overall portfolio risk depends on its correlations with other assets, which may themselves be time-varying. A multivariate extension using DCC-GARCH or copula-based methods would be required to address this limitation.

Finally, the fund's fee structure — an entry fee of 4.0%, annual management and running costs of 9.6%, and an exit fee if redeemed within 12 months — imposes a significant performance drag that is not reflected in the NAV-based risk measures computed here. Net-of-fee risk-adjusted returns would be materially worse than the gross figures analysed.

4.3 Practical Implications

The findings of this study carry several practical implications for investors and risk managers considering exposure to the Superfund Silver Fund.

- **Use dynamic, fat-tailed risk models.** Static parametric models — particularly the Normal model — materially underestimate the risk of 1068.N. Risk managers should employ at minimum a GARCH(1,1)-t model to obtain realistic VaR and ES estimates that reflect both volatility clustering and tail risk. The GARCH ES of −9.89% at the 5% level should be treated as the relevant risk benchmark for capital allocation purposes.
- **Monitor volatility regime changes.** Given the extremely high persistence of volatility ($\beta_1 = 0.954$), risk estimates should be updated at least daily using rolling GARCH re-estimation. Periods of elevated volatility, such as the spike observed in late 2025, should trigger enhanced risk monitoring and potential position reduction.
- **Account for extreme event risk.** The single largest daily loss in the recent sample (−12.7% on 7 April 2025) exceeded the 5% GARCH VaR of −6.88% by nearly 6 percentage points. Tail stress tests should be conducted regularly using extreme value theory (EVT) methods to quantify the risk of losses beyond the VaR threshold.

- Consider total cost of ownership. The total annual cost of holding 1068.N is approximately 13.6% (9.6% management/running costs plus amortised entry fee). For the fund to generate positive net returns, gross silver-linked performance must exceed this threshold consistently — a demanding hurdle given the historical annualised volatility of 37.1%.

5. Conclusion

5.1 Summary of Findings

This study examined the risk profile of the Superfund Silver Fund (1068.N) using weekly price data from Stooq spanning January 2009 to January 2026, complemented by a daily return analysis over the most recent five-year sub-period. Six models for estimating Value-at-Risk and Expected Shortfall were specified, estimated, and compared: Historical Simulation, Normal, Student-t, Cornish-Fisher, EWMA, and GARCH(1,1) with Student-t innovations.

The key findings can be summarised as follows. The log return distribution of 1068.N is leptokurtic (excess kurtosis $K = 6.664$) and mildly left-skewed ($S = -0.185$), with significant departure from normality confirmed by the QQ plot and formal distributional fit tests. A Student-t distribution with $\nu = 5$ degrees of freedom provides the best fit to the data, outperforming the Normal across all information criteria (AIC: −4.85 vs −4.76; BIC: −4.83 vs −4.74).

The GARCH(1,1) parameter estimates reveal extremely high volatility persistence ($\beta_1 = 0.954$, $\alpha_1 + \beta_1 = 0.985$), confirming strong volatility clustering in the return series. Dynamic models (EWMA and GARCH) produce substantially more negative risk estimates than static models, with GARCH yielding a 5% VaR of −6.88% and ES of −9.89% — approximately double the Normal parametric estimate. The exceedance plots confirm that GARCH captures the time-varying nature of risk more accurately than Historical Simulation, which clusters violations during stress periods.

5.2 Directions for Future Research

Several extensions of the present analysis would be valuable. First, asymmetric GARCH models — such as GJR-GARCH or EGARCH — could be applied to test whether negative return shocks increase volatility more than positive shocks of equal magnitude (the leverage effect), which may be relevant for a commodity fund subject to sharp downside events.

Second, Extreme Value Theory (EVT) methods — in particular the Peaks-Over-Threshold (POT) approach using the Generalised Pareto Distribution — would allow more precise estimation of tail risk beyond the VaR level, providing a theoretically grounded alternative to the empirical tail estimates used here.

Third, a multivariate extension incorporating correlations between 1068.N and other asset classes (equities, bonds, gold, inflation-linked securities) using DCC-GARCH or copula methods would allow the fund's marginal risk contribution to be assessed in a portfolio context.

Finally, a backtesting framework — using Kupiec's proportion-of-failures test and Christoffersen's interval forecast test — would provide a formal statistical evaluation of each model's out-of-sample VaR coverage, going beyond the visual exceedance analysis presented in Section 3.4.

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