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Hedging call options

Meta's stock as the underlying

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1 Introduction

Financial markets usually witness unexpected price movements. Buying and selling financial instruments therefore involves a certain extent of risk and may greatly affect economies. (Strömdahl 2023, 1.) Liu (2024, 1) also emphasises the complexity and uncertainty of financial markets brought about by globalisation and technological advancements. This is when hedging comes into play. Hedging in a financial context is a strategy which is aimed to reduce financial risk by exploiting relationships or correlation between various risky investments and can therefore lead to a better return (Wilmott 2007, 194). Being able to implement hedging is an essential skill for finance professionals, especially those who work closely with risk management or sales of derivatives.

In this assignment, we hedge call options with the help of Delta hedging and Delta-Vega hedging using Meta's stock price as the underlying asset. Hedging using options both offers protection against risk that occurs due to the volatility of a stock and allows flexibility to choose whether the option contract is held (Rachmansyah & Rikumahu 2019, 100). Volatility of a stock is a measure of the uncertainty about the returns provided by the stock and typically ranges between 15% and 60% (Hull 2017, 347).

Both hedging strategies are epitomes of dynamic hedging which entails continuous monitoring and rebalancing by the sale or purchase of the underlying asset (Wilmott 2007, 195). Delta hedging refers to the reduction of directional risk associated with the price fluctuations of an underlying asset, and the goal is to achieve Delta neutral which offsets the risk on the portfolio or option (CFT Education Inc. 2024). Vega hedging, on the other hand, is used to protect traders against volatility moves by adopting a Vega-neutral position that balances out the impact of volatility changes (Chen 2024). Delta-Vega hedging involves minimising the risks associated with changes in both stock price and implied volatility which is volatility implied by Black Scholes model with option prices observed in the market as an input (Hull 2017, 363 & FasterCapital 2024).

2 Single option

2.1 Delta hedging

2.1.1 Pre-hedging: Data and approach

To ensure accurate and reliable hedging, several preprocessing steps and calculations have been implemented in the provided code. These steps are vital for setting up the framework for hedging based on the Greeks of a call option and implied volatility.

The options data is imported from corresponding CSV files which are fetched from Refinitiv workspace. Call options with same underlying (Meta's stock prices), same strike price, same time to maturity, but with 11 distinct expiration dates are chosen to implement the hedging. For single option hedge, the call option with the strike price of 585 (around-the-money calls) is selected, and the time to maturity (45 days) is also filtered to gain meaningful statistical insights into how different hedging strategies and re-hedging frequencies affect the hedge performance. The Time_to_Maturity is calculated based on the number of trading days per year (252 days) as each file includes information on trading days.

It is also worth noting that it is usually not enough in real life to repeat the hedge 10 times with the changing parameter being expiration dates, but in the context of this assignment and because of the time constraints, 10-time hedge is assumed to suffice.

The implied volatility is a crucial input for calculating the option Greeks. It is derived using a built-in R function called EuropeanOptionImpliedVolatility which is imported from the RQuantLib package. The implied volatility is then computed iteratively based on the Black-Scholes model such that the theoretical option price matches the observed price.

An indispensable part of hedging and risk management is to obtain option Greeks. In this case, three option Greeks, including Delta, Vega, and Gamma, are computed. Delta and Vega are in heavy use considering the Delta-hedging and Delta-Vega hedging as key strategies to be employed in this assignment. However, Gamma may be proven useful in some implications of the results as it closely relates to Delta. Delta measures the sensitivity of the option's price to changes in the underlying asset price, Gamma measures the sensitivity of Delta to changes in the underlying asset price, and Vega represents the sensitivity of the option price to changes in implied volatility.

Table 1 below illustrates the output pre-hedging process explained above. The table contains some of the initial datapoints of the maturity date 11 October 2024.

	X <date>	Underlying <dbl>	C585 <dbl>	Strike_585 <dbl>	maturity_date <date>	Time_to_Maturity <dbl>	iv_values_585 <dbl>	Delta <dbl>	Vega <dbl>	Gamma <dbl>
1	2024-08-29	518.220	2.515	585	2024-10-11	0.117808219	0.2670465	0.11323088	34.14556	0.004041561
2	2024-08-30	521.310	2.550	585	2024-10-11	0.115068493	0.2610797	0.11713299	34.77514	0.004259425
3	2024-09-03	511.760	2.110	585	2024-10-11	0.104109589	0.2942543	0.09610124	28.14672	0.003508204
4	2024-09-04	512.740	2.200	585	2024-10-11	0.101369863	0.2982419	0.09934916	28.51360	0.003587412
5	2024-09-05	516.860	2.120	585	2024-10-11	0.098630137	0.2852701	0.10026377	28.54186	0.003797326
6	2024-09-06	500.270	1.150	585	2024-10-11	0.095890411	0.3037215	0.05898149	18.20777	0.002498057
7	2024-09-09	504.790	1.070	585	2024-10-11	0.087671233	0.2985170	0.05792100	17.31870	0.002597018
8	2024-09-10	504.790	0.850	585	2024-10-11	0.084931507	0.2898009	0.04932934	15.00970	0.002393243
9	2024-09-11	511.830	0.990	585	2024-10-11	0.082191781	0.2798990	0.05802450	17.02659	0.002825197
10	2024-09-12	525.600	1.930	585	2024-10-11	0.079452055	0.2784890	0.10169774	26.32128	0.004306188
11	2024-09-13	524.620	1.650	585	2024-10-11	0.076712329	0.2762340	0.09121808	23.83244	0.004086428
12	2024-09-16	533.280	2.740	585	2024-10-11	0.068493151	0.2977363	0.13498908	30.30026	0.005224771
13	2024-09-17	536.315	2.830	585	2024-10-11	0.065753425	0.2933142	0.14185258	30.88361	0.005567291
14	2024-09-18	537.950	3.610	585	2024-10-11	0.063013699	0.3165605	0.16502217	33.52391	0.005807419
15	2024-09-19	559.100	7.800	585	2024-10-11	0.060273973	0.3080150	0.30173388	47.84962	0.008245145
16	2024-09-20	561.350	6.710	585	2024-10-11	0.057534247	0.2779632	0.29439717	46.41350	0.009210307
17	2024-09-23	564.410	6.800	585	2024-10-11	0.049315068	0.2827222	0.30912737	44.16344	0.009943741
18	2024-09-24	563.330	5.700	585	2024-10-11	0.046575342	0.2729538	0.28431603	41.22658	0.010219230
19	2024-09-25	568.310	7.250	585	2024-10-11	0.043835616	0.2820588	0.33655873	43.42682	0.010874969
20	2024-09-26	567.840	6.600	585	2024-10-11	0.041095890	0.2800601	0.32339272	41.34645	0.011141375

Table 1

2.1.2 Hedging: Results

To start with, daily hedging (i.e. hedging frequency = 1) is conducted to obtain an initial picture of the hedging performance before hedging of higher frequencies are implemented.

Figure 1 displays how effective the Delta hedging is when implemented on a daily basis. It can be observed that the change in original portfolio (OP) and replicating portfolio (RE) tend to follow similar trends, indicating that the daily hedging strategy generally works well to replicate changes in the call option portfolio value.

However, when abrupt price movements are present, the Delta hedging becomes imperfect. In other words, the effects of substantial unexpectedness cannot be fully protected against with Delta hedging.

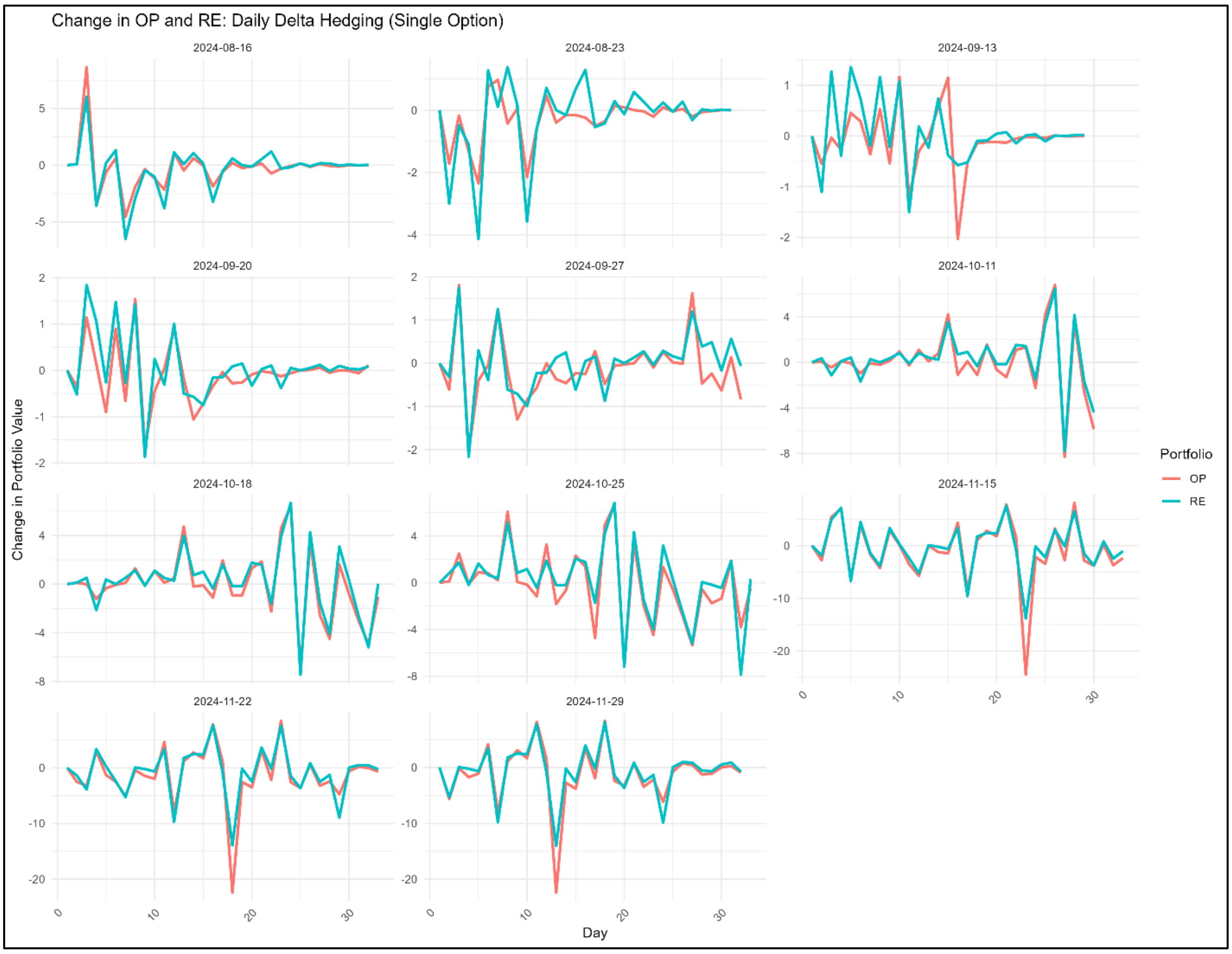

Figure 1

Likewise, figure 2 shows that the Delta hedging implemented every 7 days is relatively effective considering that the change in OP is well replicated by that in RE. However, compared to daily hedging, it seems that there are larger deviations between the change in OP and the change in RE. These deviations also occur more frequently, especially noticeable near both fairly and very sharp peaks and troughs while for daily hedging this challenge remains mostly only when the changes are substantial. This suggests that less frequent hedging leads to increased hedging errors due to the lag in updating the replicating portfolio.

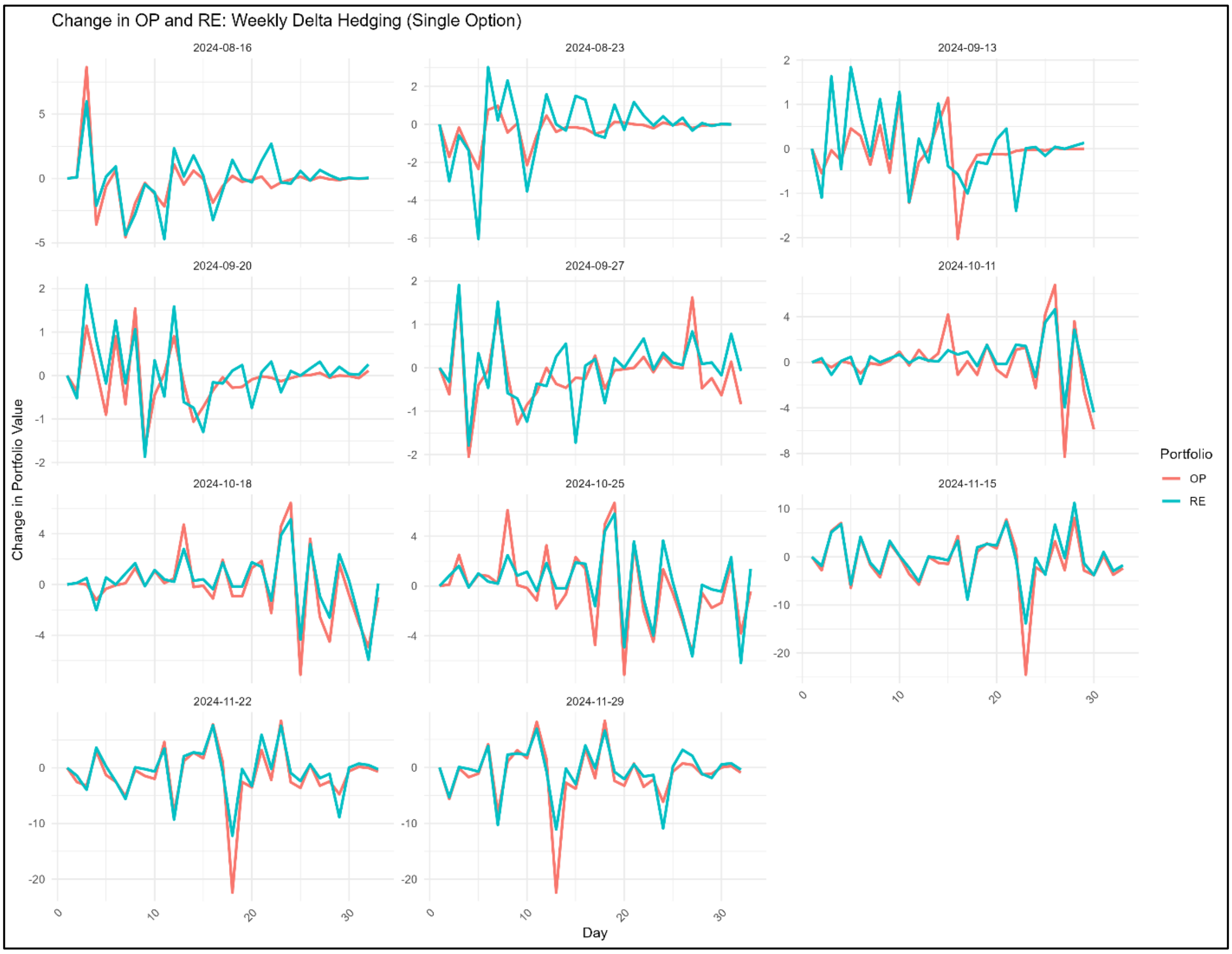

Figure 2

Figure 3 further bolsters the aforementioned statement that more frequent hedging leads to better hedging performance. As can be seen from the figure, the MSE for 7-day hedging (brown line) is consistently higher compared to everyday hedging (orange line) for all the maturity dates. Both strategies show an upward trend as the maturity dates move towards late-October and November.

For options maturing in August, September and early-to-mid October remains relatively low, indicating that both hedging strategies are effective at these times. Daily hedging has a slight edge as the MSE values are closer to zero than those of 7-day hedging are.

A sharp increase in MSE is visible for both strategies from late-October to the end of November. The increase is more pronounced for 7-day hedging where MSE peaks above 6. MSE also grows larger for daily hedging but remains considerably lower than that of 7-day hedging, implying a better performance.

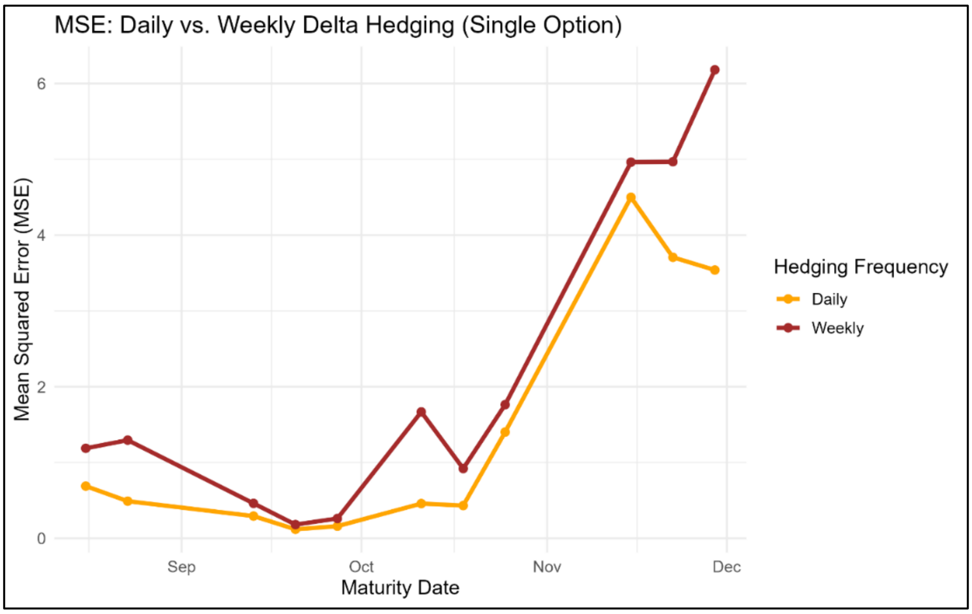

Figure 3

In general, the MSE relative to different maturity dates for different re-hedge frequencies echoes that for daily hedging. The MSE remains low and stable across most frequencies and maturity dates until mid-October. From late-October onwards, the MSE increases significantly across all hedging frequencies, with a clear spike in late-November.

Differences between re-hedge frequencies are minimal in August, September and October, suggesting that hedging accuracy is not much impacted by how frequently the hedge is rebalanced during this period. This implies that more frequent hedging during these periods should be carefully considered and may not be advised because of the transaction costs involved.

In late-October and November the hedging performance tends to become poorer when we hedge less frequently. This means that more frequent hedging can mitigate the errors during this period of elevated market movements or volatility.

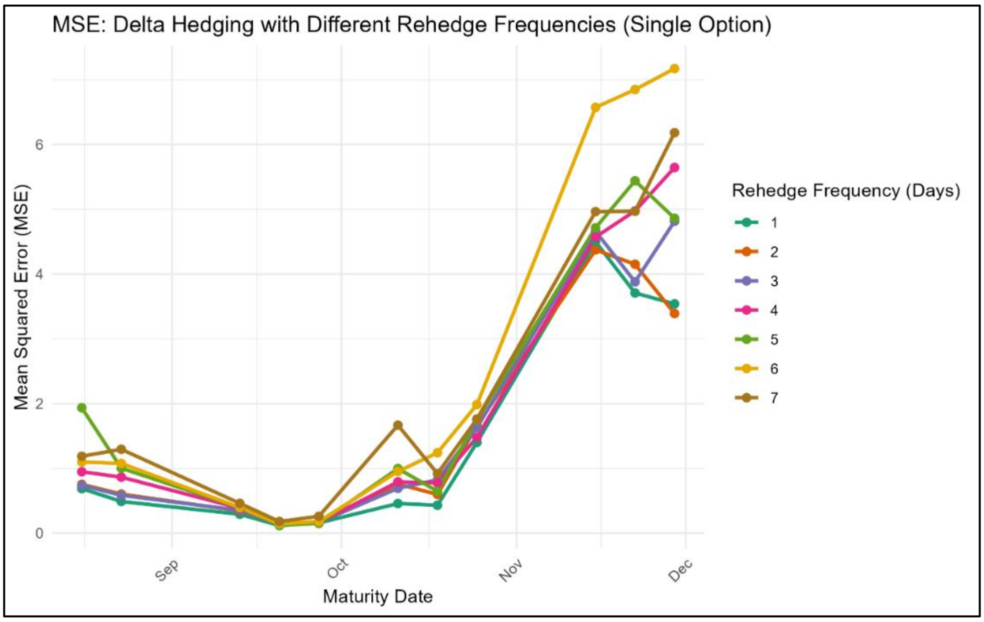

Figure 4

Table 2 illustrates the MSE which is induced by the hedge implemented on the call option mature on the last maturity date – 29 November 2024 with different frequencies. The MSE on this date is also the peak for many re-hedging frequencies. Table 2 is extracted from a larger table as an output on R.

OP_Date	Frequency	MSE
2024-11-29	1	3.5399322
2024-11-29	2	3.3902725
2024-11-29	3	4.8153528
2024-11-29	4	5.6439996
2024-11-29	5	4.8608483
2024-11-29	6	7.1709537
2024-11-29	7	6.1818360

Table 2

The overall result of Delta hedge is summarised in the table 3 below.

Frequency <dbl>	Mean_MSE <dbl>	STD_MSE <dbl>	Min_MSE <dbl>	Max_MSE <dbl>
1	1.435552	1.644970	0.1186510	4.499481
2	1.541398	1.628455	0.1389027	4.372762
3	1.680710	1.837943	0.1375392	4.815353
4	1.886114	2.088686	0.1454155	5.644000
5	2.002763	2.018787	0.1297254	5.439496
6	2.515733	2.843560	0.1451571	7.170954
7	2.168419	2.143733	0.1812099	6.181836

Table 3

2.2 Delta-Vega hedging

2.2.1 Pre-hedging: Data and approach

Our Delta-Vega hedging is based upon the Delta pre-hedging process but with some further data wrangling as follows.

The replicating portfolio is constructed using two components: an α amount of the underlying asset and an η amount of a replicating option. The replicating option we chose is a call option with the same underlying asset and strike price as the original option but with a longer time to maturity. This design leverages the sensitivity of the replicating option's Vega to hedge the original option's exposure to volatility changes.

To obtain the necessary data for both the original option and the replicating option, we merged two option datasets based on trading dates. The option with the longer time to maturity was selected as the replicating option.

A key limitation of the data is the relatively short period of available trading days for Meta's options, typically ranging between 30 to 60 days. As a result, the merged dataset includes only 30 to 42 rows of paired data, corresponding to the number of overlapping trading days. This constraint limits our ability to extend the hedging strategy beyond a maximum of 30-42 days.

Using the merged dataset, we calculated the α and η values for each trading day. These values form the basis for constructing and rebalancing the replicating portfolio.

2.2.2 Hedging: Results

Figure 5 compares the effectiveness of daily Delta-Vega hedging of the option mature on 16 August 2024 with that of weekly Delta-Vega hedging of the same option with the same maturity date. In general, both daily hedging and weekly hedging perform fairly well given the relative alignment between two lines in both graphs. This suggests that Delta-Vega hedge dynamically compensates for price (Delta) and volatility (Vega) changes in the option price.

However, during periods of high volatility, the replicating portfolio responds more effectively to the change in the option price when the hedge is implemented daily than weekly. Delta-Vega hedge is well-calibrated for daily adjustments and can handle sudden market shifts without introducing large mismatches.

Figure 5

Figure 6 indeed confirms that the more frequently the hedge is implemented, the better the hedging performance is. The MSE by daily hedging is consistently lower at all the maturities.

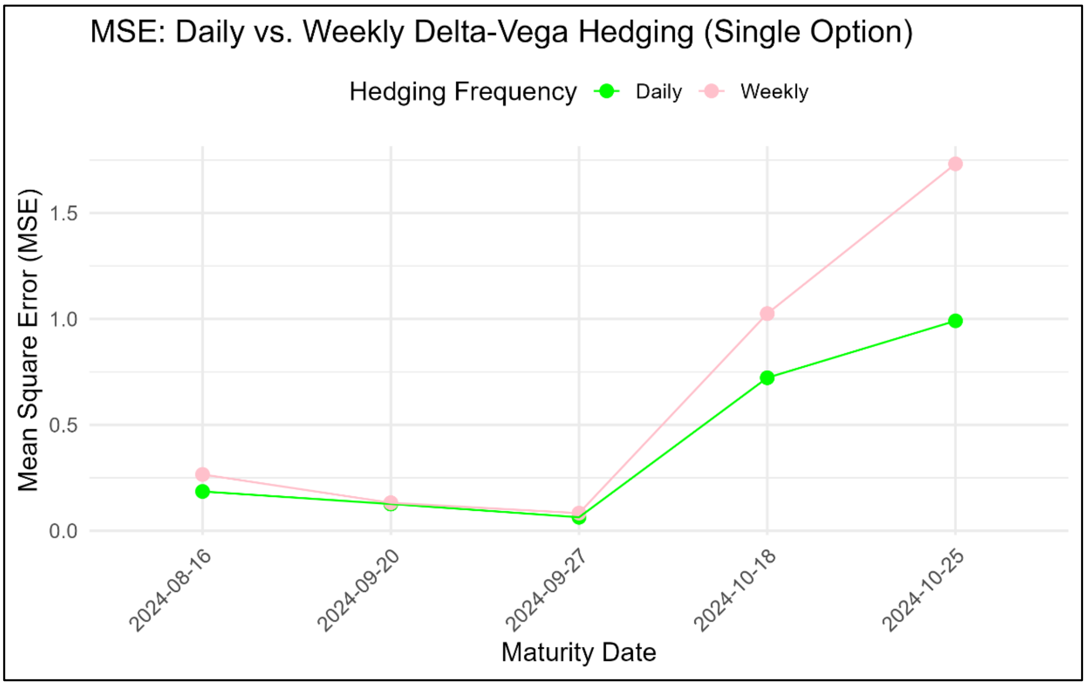

Figure 6

2.3 Discussion

2.3.1 The use of mean square error (MSE)

Mean squared error (MSE) is used as a metric for hedging performances. Mean squared but not mean is used as mean squared reflects better measurement of the accuracy and effectiveness of a (hedging) strategy. When a regular mean is used, positive and negative errors may cancel out each other, potentially leading to a misleading assessment of performance. Squaring the errors ensures that all errors contribute positively to the final measure, providing a full and objective picture of the overall hedging performance. Besides, the goal of hedging is to minimise risk (or reduce the potential for large deviations from the desired outcome), squaring the errors therefore adds extra weight to large discrepancies. This larger penalty ensures that potentially huge risks are paid closer attention to.

As shown in our hedging results, high MSEs in November could be inflated by the large volatility in prices of the Meta's stock price. There may be events expected to occur in November (such as Donald Trump's historic comeback or investor optimism around the TikTok ban), leading to sharp market movements which are hard to predict or hedge against. Gamma risk, which entails the nonlinear change of Delta with the underlying price, may also be a reason for large MSEs in the face of large price swings.

2.3.2 Effects of adding transaction costs

Transaction costs significantly influence the efficiency of hedging strategies by creating a trade-off between the frequency of rebalancing and the cost incurred. In a frictionless market, continuous rebalancing minimizes hedging errors and ensures near-perfect replication of the option's payoff, leading to minimal mean square error (MSE). However, with transaction costs, frequent rebalancing becomes costly, prompting traders to adjust less often to reduce expenses. This reduced frequency increases the tracking error due to mismatches between the hedge and the underlying asset's movements.

The total MSE in hedging comprises two key components: (1) hedging error and (2) transaction costs. As transaction costs rise, the optimal rebalancing strategy minimises total MSE by balancing these competing factors. This nonlinear relationship shows that even small transaction costs can have a substantial impact, particularly in volatile markets or for short-term options requiring frequent rebalancing. The comparison of MSE after Delta hedge is implemented with and without transaction cost is summarised in table 4.

Frequency	Mean_MSE_No_TC	STD_MSE_No_TC	Mean_MSE_TC	STD_MSE_TC
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	1.435552	1.644970	4.627740	3.645867
2	1.541398	1.628455	3.986341	3.723635
3	1.680710	1.837943	3.528844	4.219143
4	1.886114	2.088686	5.079611	5.113791
5	2.002763	2.018787	3.645985	2.998280
6	2.515733	2.843560	4.147892	4.107296
7	2.168419	2.143733	4.166738	3.508478

Table 4

Figure 7 clearly visualises the effects of transaction costs on hedging performance. The MSEs accounting for transaction costs are consistently higher than the MSEs accounting for no transaction costs, reflecting the added cost of frequent rebalancing. When the hedge is implemented more frequently (such as daily or every other day), the impact of transaction costs is more pronounced. As the hedge is implemented less frequently, the gap between Mean_MSE_TC and Mean_MSE_No_TC narrows, but the overall MSE still rises due to increasing hedging errors in both cases. Additionally, the standard deviations of MSE are higher with transaction costs, indicating greater variability and uncertainty in hedging outcomes in this case.

These findings highlight the trade-off between reducing transaction costs and maintaining an accurate hedge, underlining the importance of selecting an optimal rebalancing frequency for cost-effective risk management.

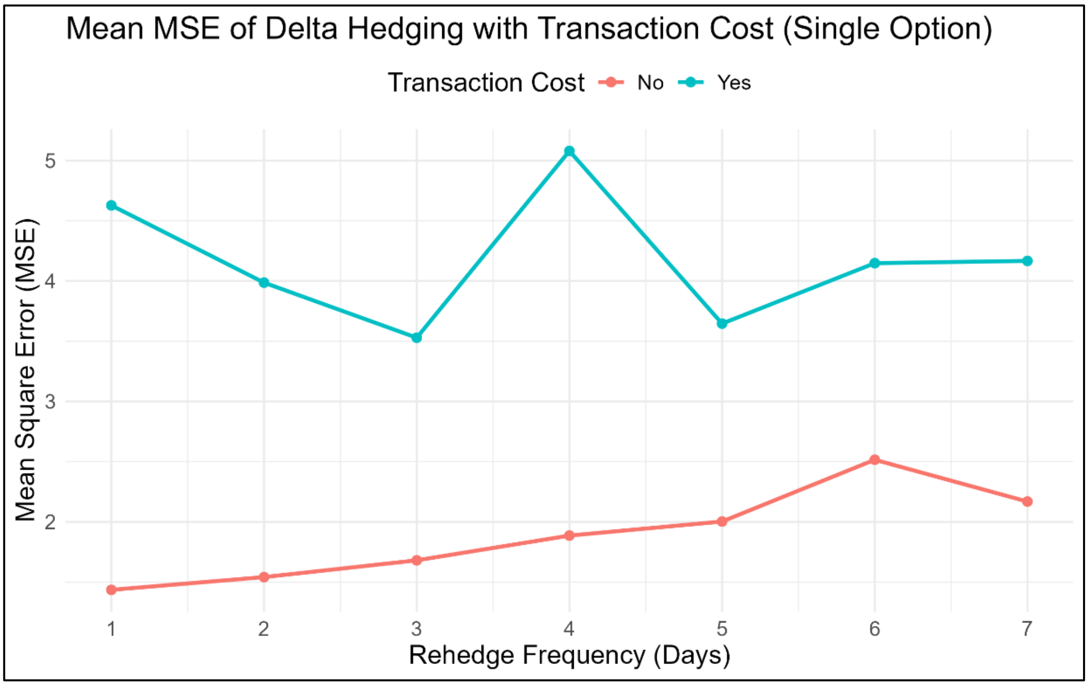

Figure 7

2.3.3 Why do we hold α of the underlying, not Δ in Delta-Vega hedging?

In Delta-Vega hedging, a replicating option is used to address the portfolio's exposure to volatility risk, which cannot be managed solely through the underlying stock. While Delta hedging with the stock protects against price movements, options are also influenced by changes in volatility, known as Vega risk. To hedge both Delta and Vega risks, we incorporate a replicating option with the same underlying asset and strike price but a longer maturity ($T_2 > T$).

Instead of holding Δ of the underlying asset, we hold an amount α because we are implementing a hedge that accounts for both price and volatility risks. The amount α differs from the standard Delta hedge (Δ) due to the additional Delta exposure introduced by the replicating option used for Vega hedging.

Mathematically, this is expressed as: $\alpha(\sigma) = \Delta^{BS}(\sigma) - \frac{\kappa^{BS}(\sigma)}{\kappa^{Rep}(\sigma)}\Delta^{Rep}(\sigma)$

The second term adjusts for the Delta exposure introduced by the replicating option. This adjustment ensures that the portfolio remains hedged against both price changes and volatility fluctuations.

2.3.4 Comparison: Delta hedging vs. Delta-Vega hedging

Figure 8 compares the mean MSE and standard deviation of MSE for Delta hedging and Delta-Vega hedging over a range of re-hedge frequencies from 1 day to 7 days. Delta hedging shows consistently higher mean MSE and standard deviation of MSE than Delta-Vega does. This means that the Delta-Vega hedging outperforms Delta hedging.

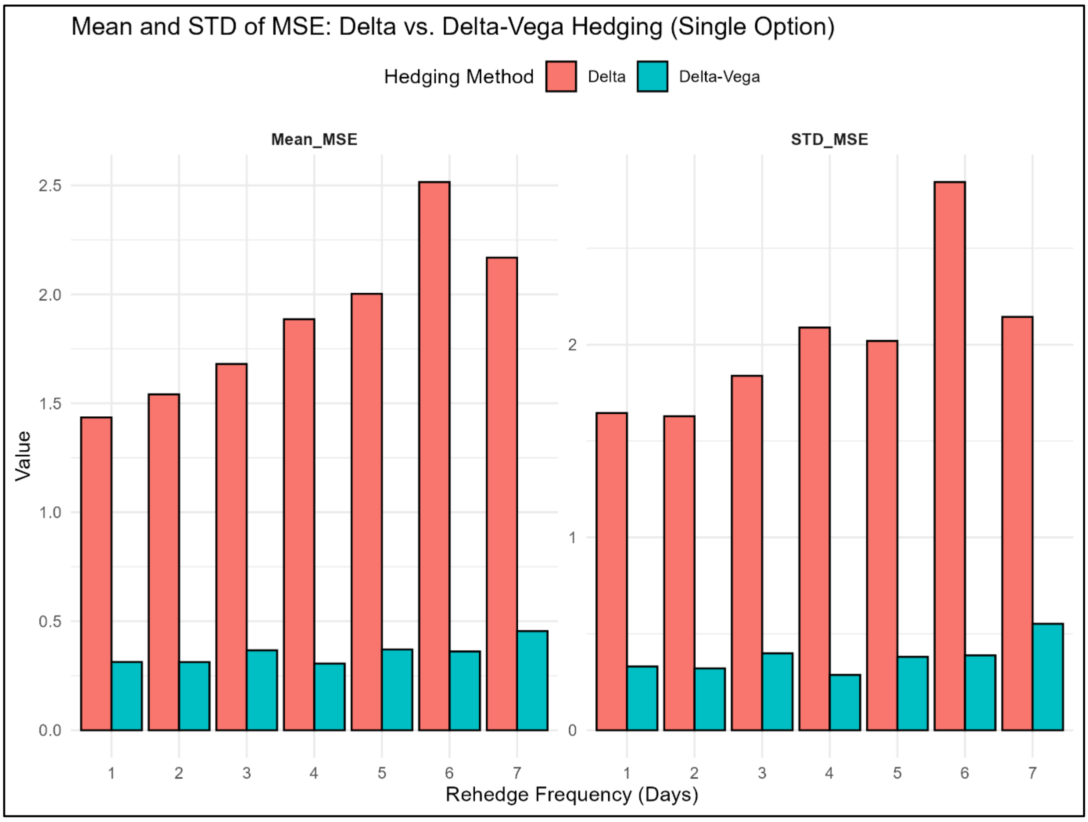

Figure 8

Delta hedging focuses solely on the price changes of the underlying asset, a concept which can lead to significant errors when the market experiences volatility. Delta-Vega hedging, on the other hand, accounts for both price changes and changes in volatility. Options are sensitive to changes in volatility, and ignoring this factor can lead to underperformance. Delta-Vega allows for more dynamic adjustments as market conditions change. By considering both Delta and Vega, traders can better adapt their hedging strategies to reflect current market realities.

3 Portfolio of options

In the portfolio hedging, we choose to hedge with one call option and one put option, both with strike price of 585. Other parameters are kept the same as in the single option hedging described above.

The selection of both a call and a put option at the strike price of 585 (around-the-money) constitutes a straddle strategy, characterised by options with identical strike prices, maturities, and underlying assets. This combination is strategically chosen because ATM options provide maximum Vega sensitivity, making them highly responsive to volatility changes and effective for volatility risk hedging. Additionally, the complementary Delta characteristics of ATM options (approximately +0.5 for calls and -0.5 for puts) create a balanced Delta exposure near the strike price.

Table 5 shows an example of Delta, Vega for the call and put options with strike of 585, maturity date on 29 November 2024 of Meta's stock.

X <date>	Delta_585_call <dbl>	Delta_585_put <dbl>	Vega_585_call <dbl>	Vega_585_put <dbl>
2024-10-15	0.56050945	-0.4397420	97.696388	97.70585
2024-10-16	0.51233268	-0.4847047	96.104913	96.08018
2024-10-17	0.51222760	-0.4851346	95.030439	95.00909
2024-10-18	0.50715311	-0.4884339	93.873137	93.84877
2024-10-21	0.49768092	-0.4959607	90.265520	90.26242
2024-10-22	0.53438696	-0.4640760	89.828370	89.79772
2024-10-23	0.43959629	-0.5765869	85.179223	84.57606
2024-10-24	0.45796345	-0.5353182	85.137482	85.27748
2024-10-25	0.48763438	-0.5111007	85.188151	85.19610
2024-10-28	0.51041652	-0.4888899	82.164245	82.16039

Table 5

3.1 Delta hedging a portfolio of options

Figure 9 illustrates that the portfolio that is effectively Delta-hedged if the hedging is implemented every day. The change in the price of the original portfolio is by and large closely protected by the change in the price of the replicating portfolio. Some challenges persist, however, in late-October and November when swift price movements cannot be fully hedged with this high re-hedge frequency.

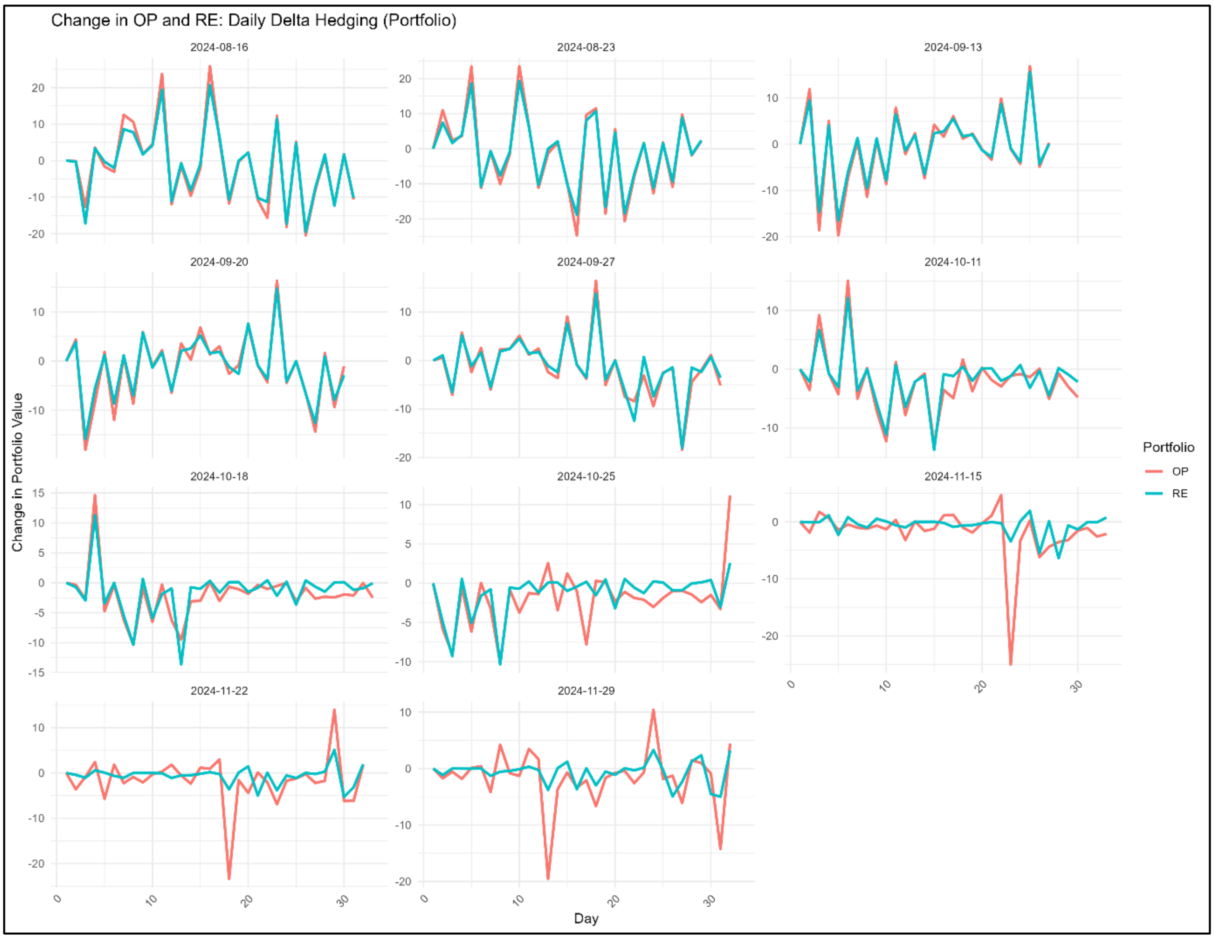

Figure 9

As the re-hedge frequency increases, it becomes more difficult to hedge as shown in figure 10. When this portfolio is Delta-hedged weekly, the challenges of keeping up with the abrupt changes in the price of the original portfolio can be seen.

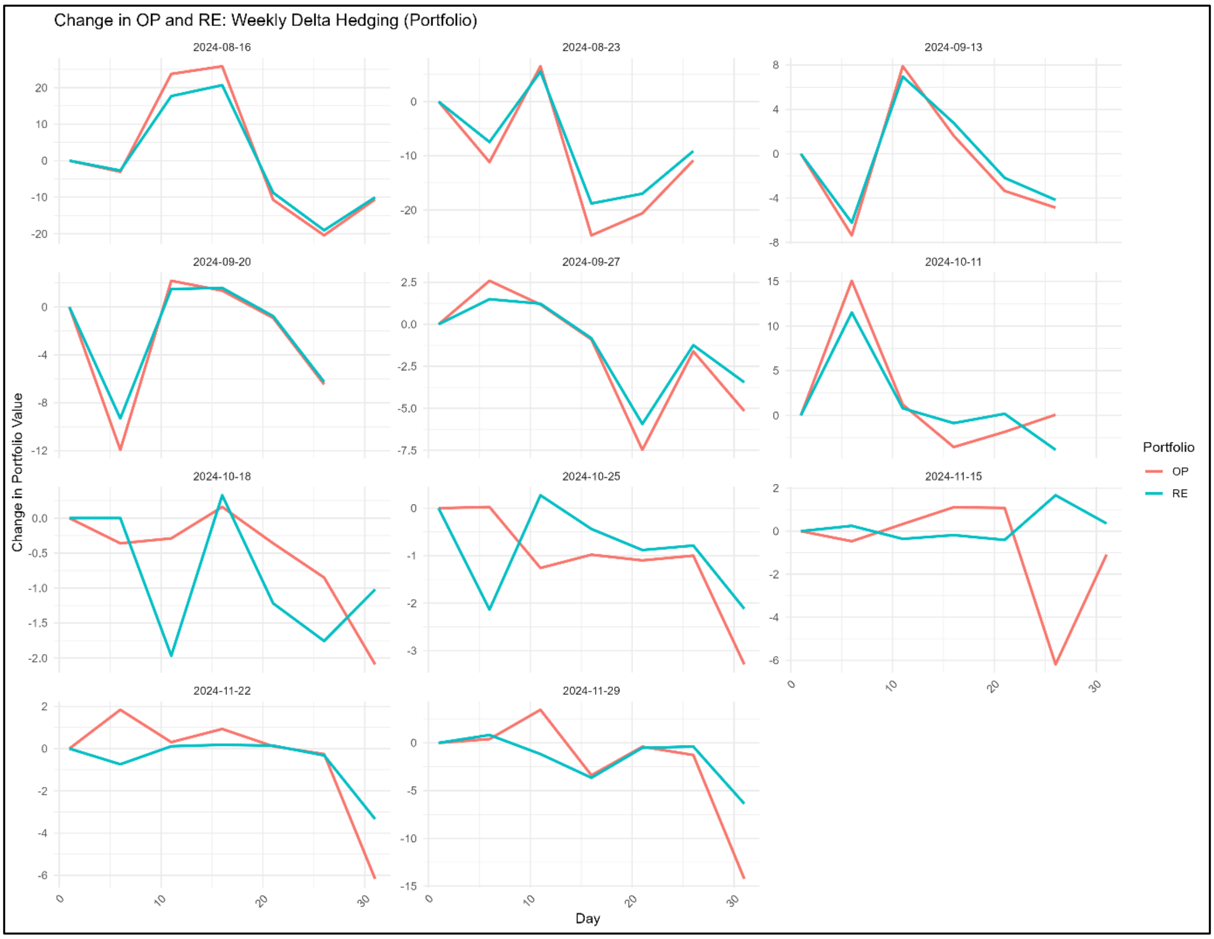

Figure 10

The MSE induced by daily and weekly Delta hedging a portfolio shown in figure 11 is also consistent with that induced by daily and weekly Delta hedging a single option. In other words, the more frequently the hedge is carried out, the better the hedging performance is.

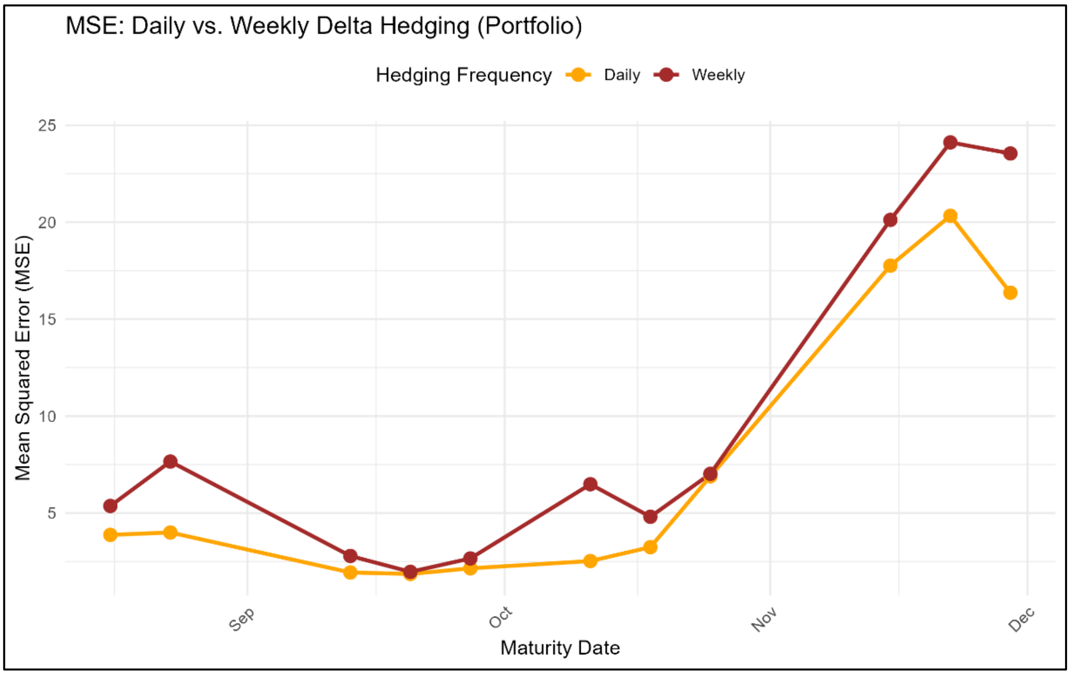

Figure 11

3.2 Delta-Vega hedging a portfolio of options

Challenges remain in our Delta-Vega hedging of this portfolio, both daily and weekly, as can be seen in figure 12. There are visible discrepancies in the change in OP and RE. Large deviations in the price of the portfolio make it hard for the replicating portfolio to catch up with these sharp spikes or drops. Even with such a very high frequency of hedging as daily, the hedging is still underperformed.

Figure 12

Figure 13 also shows the ineffectiveness of Delta-Vega hedging of this portfolio. Daily hedging slightly outperforms weekly hedging, but the values of MSE for both frequencies are remarkably higher than Delta-Vega hedging of a single option.

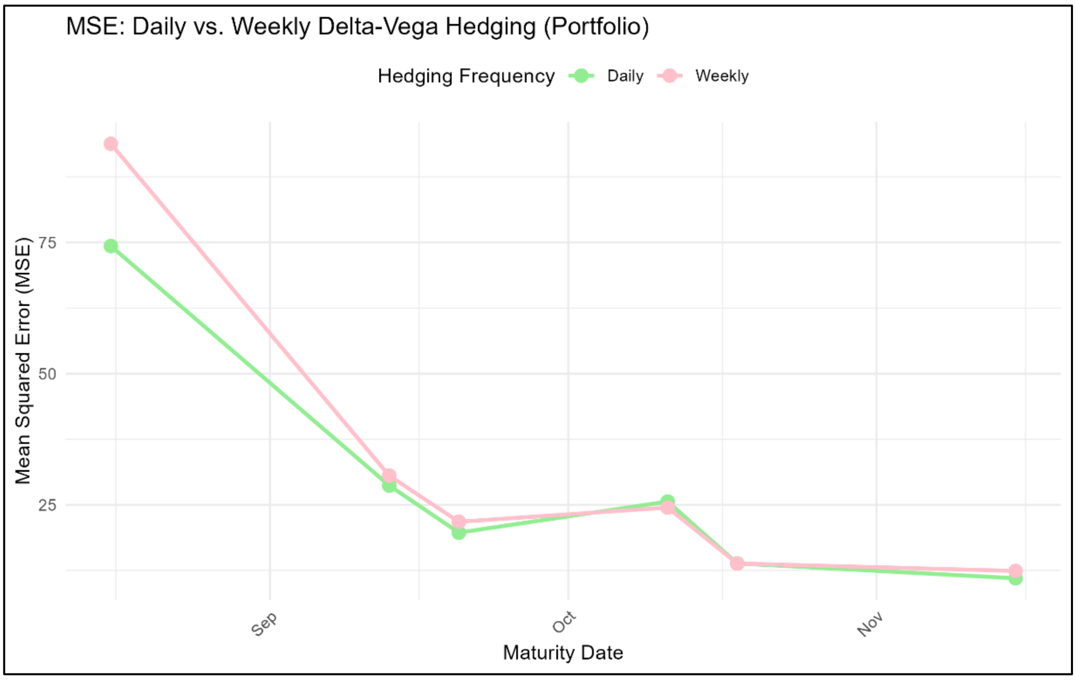

Figure 13

Delta-Vega hedging relies on the implied volatility surface being accurate. If the implied volatility shifts unexpectedly, the hedging model may fail to capture these changes, leading to errors. Portfolio hedging involves multiple other layers of complexities such as nonlinear effects of Gamma and Vanna risks or correlation and diversification effects. Bid-ask spread, liquidity issues, and slippage can also significantly impact hedging accuracy.

4 Conclusion

In this assignment, we have implemented Delta hedging and Delta-Vega hedging for both a single option and a portfolio of options. The results demonstrated that hedging effectiveness depends significantly on the hedging frequency. Higher hedging frequencies generally reduced hedging errors but also increased transaction costs, highlighting the trade-off between precision and cost. Additionally, Delta-Vega hedging has proved more effective in scenarios with significant volatility changes, as it accounts for first-order sensitivity to the underlying's volatility alongside the underlying's price.

However, some challenges persist in terms of Delta-Vega hedging a portfolio. Therefore, it is recommended that some other Greeks (of higher order) be included in the hedging strategy to better capture a high degree of unexpectedness in the financial markets. Besides, we have only investigated the effects of transaction costs on the performance of Delta hedging for a single option. To better capture real-world dynamics, transaction costs should be incorporated into other hedging frameworks.

The analysis underscores the importance of selecting appropriate hedging strategies and parameters tailored to the characteristics of the options and portfolios as well as the underlying market conditions. Although hedging in real life is much more complicated, this hedging assignment has taught us a great deal, namely application of hedging theories learnt in the course Financial Risk Management with Derivatives 1, data analysis and interpretation of hedging results, and coding.

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