CS 577: Introduction to Algorithms — Homework 0

Out: 01/22/18 — **Due: ASAP**

Ground Rules

- The purpose of this homework is to get set up with the course and refresh some of the prerequisite material. **This homework is not to be turned in.**

Ungraded Problems

1. Find a partner for doing homework. (All graded homework problems are to be done in pairs.)
2. Familiarize yourself with the course webpage[1], calendar, and Piazza. Be sure to understand all course policies.
3. Revise the following topics from CS 240 curriculum[2]: induction, solving recurrences, asymptotic notation, and graphs.
4. Prove by induction that the number of leaves in a full binary tree is one more than the number of internal nodes.
5. Prove that every integer (positive, negative, or zero) can be written in the form $\sum_{i \in S} \pm 3^i$ for some finite subset S of nonnegative integers. For example,

$$44 = 3^4 - 3^3 - 3^2 - 3^0$$
$$23 = 3^3 - 3^1 - 3^0$$
$$19 = 3^3 - 3^2 + 3^0$$

6. You are given a $2^n \times 2^n$ chessboard with one square missing (that we will call a hole). Prove by induction on n that regardless of the position of the hole, you can tile the chessboard with L-shaped pieces containing three squares each. That is, you can find an arrangement of the L-shaped tiles such that every square of the chessboard is covered by exactly one tile and the hole is left uncovered.

7. Order the following functions from asymptotically smallest to asymptotically largest.

$$\begin{array}{ccc} \log n & \sqrt{n} & 5^n \\ n^{\log n} & 5^{\sqrt{\log n}} & (\log n)^{\log n} \\ 3^{n+10} & \log(5^n) & \sqrt{5^{\log n}} \end{array}$$

[1] `http://pages.cs.wisc.edu/~shuchi/courses/577-S18/`

[2] You can find lecture notes at `http://pages.cs.wisc.edu/~cs240-1/`