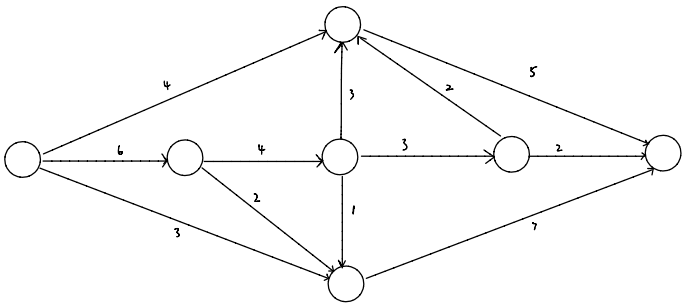
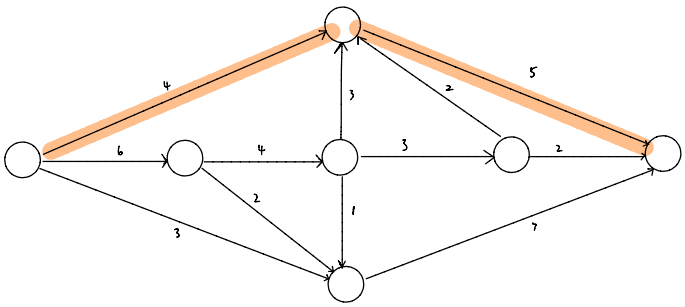
1)

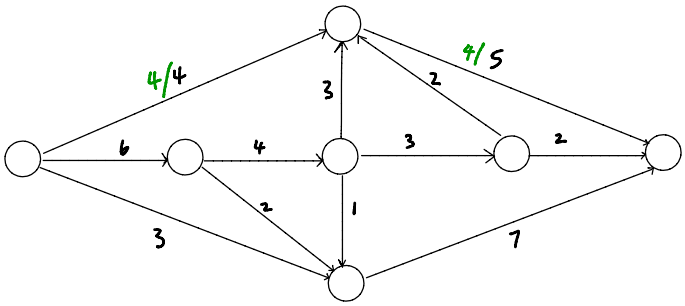
Original

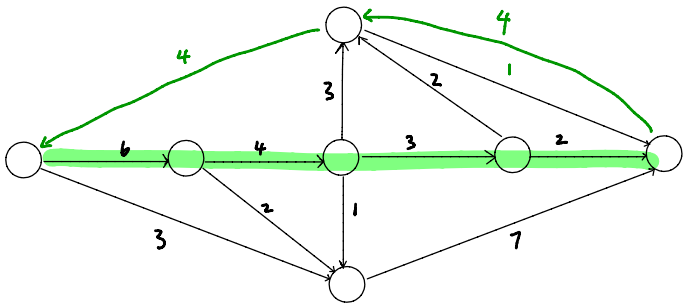
Residual

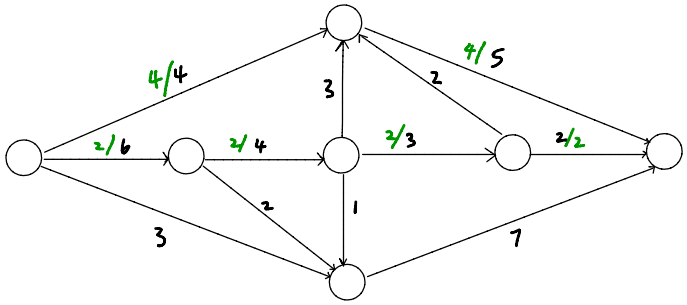
Yunhao Lin

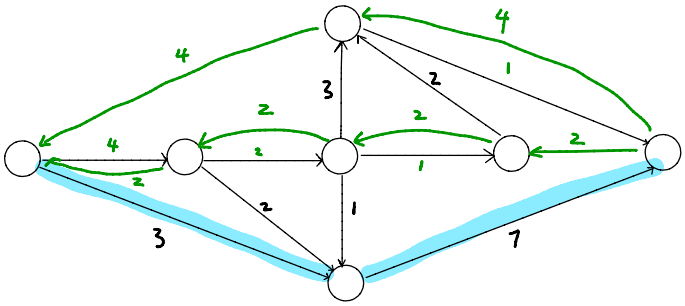


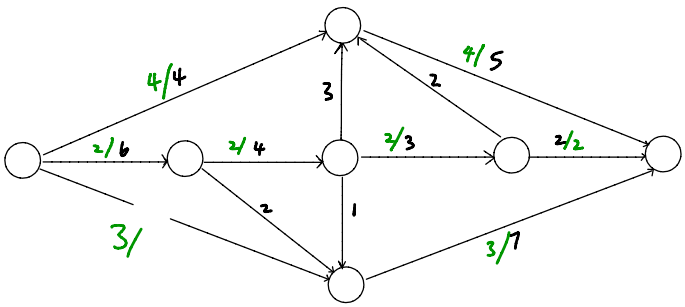


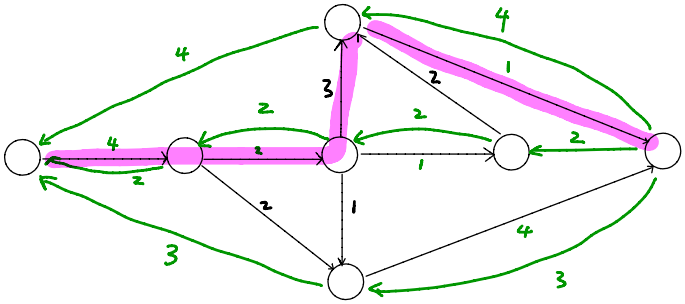


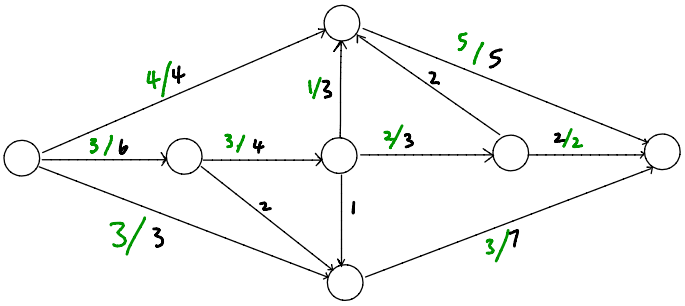


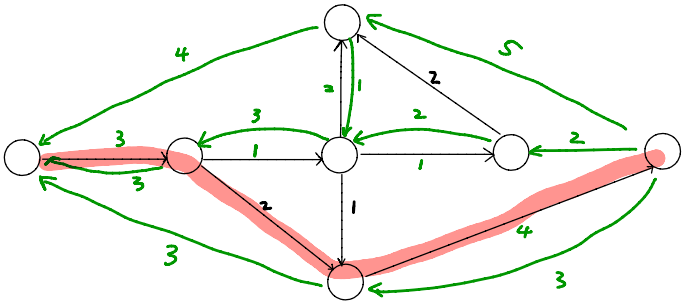


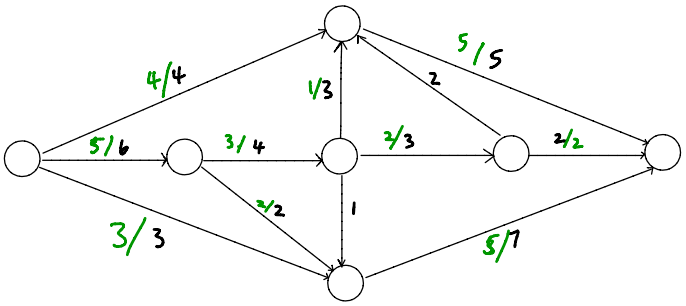


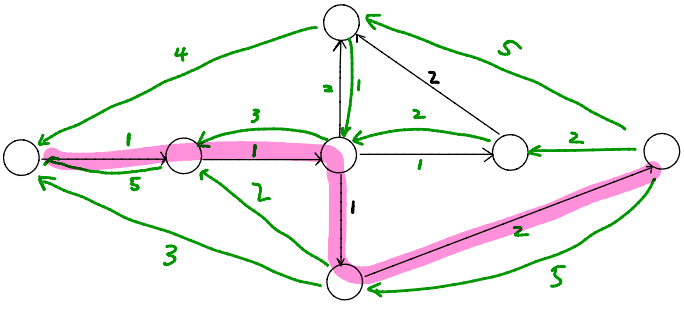


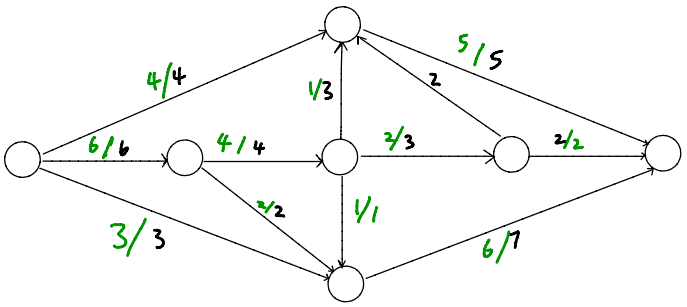


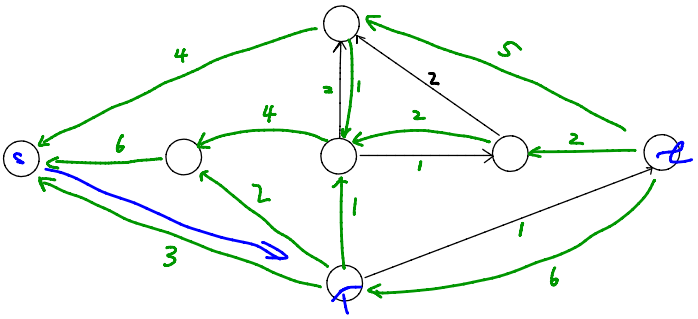






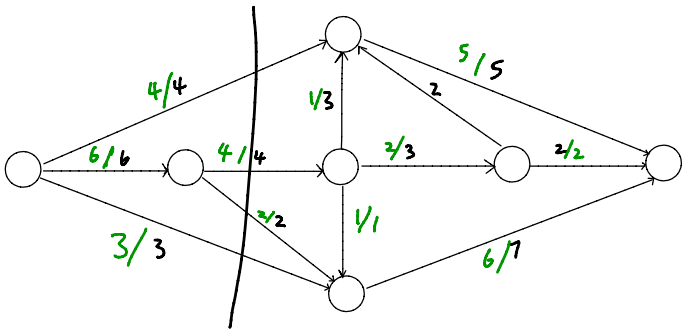






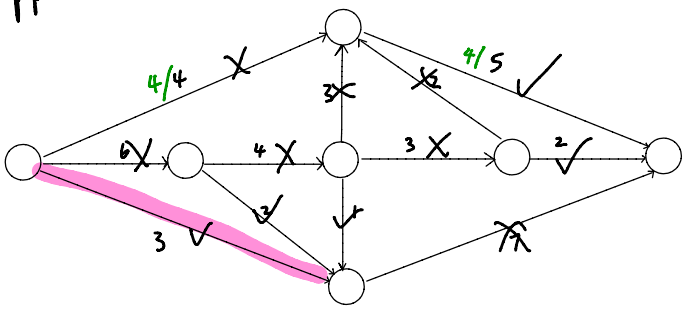
∴ Max s-t flow f^* = flow out of s

= 4 + 6 + 3 = 13

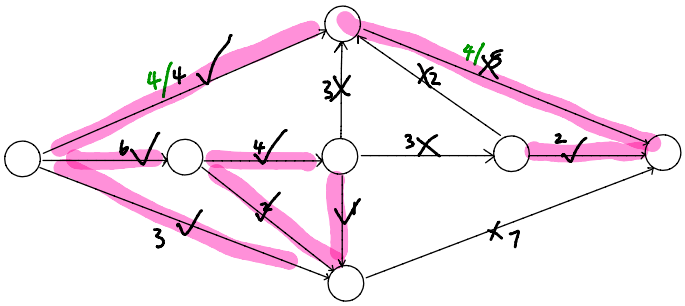


This is a minimum cut

2) upper



low



3) First, we find the Residual Graph G_f

Given G and an s-t flow f^* in G, create G_f:

- The vertex set of G_f is V
- for every $(u,v) \in E$, add to G_f:
  - If $f_{uv} < c_{uv}$, the edge (u,v) with capacity $c_{uv} - f_{uv}$ (forward)
  - if $f_{uv} > 0$, the edge (v,u) with capacity f_{uv}. (reverse)

Then, in the residual graph.

1) BFS to find all the nodes that can reach the s mark each node α.
2) BFS to find all the nodes that is reachable from t, mark each node β.
3) Then do BFS again to find all the nodes that are marked both α and β.

Then check on these nodes. All of these nodes have a edge to s and an edge from s, if one of these edges is not saturated, then this edge is a upper-binding edge.

Code in blue box is provided in class and time complexity in $O(m)$. Then 1, 2, 3 both do BFS and time complexity is $O(m+n)$. Then checking on edges takes at most $O(m+n)$ time, So in general. the total time would be

$$O(m) + 2O(m+n) \approx O(m+n).$$