King Saud University	College of Computer and Information Sciences Department of Computer Science
Data Structures CSC 212	**Final Exam - Spring 2019**
Date: 20/06/2019	Duration: 3 hours

Guidelines: No calculators or any other electronic devices are allowed in this exam.

Student ID:	Name:
Section:	Instructor:

1.1,1.2	1.3	2	3	4	5	6	7	8	Total

Question 1 .... 20 points

(a) (6 points) Choose the most important reason among A to E for each of the following decisions:

A. Guarantees correctness. **B**. Improves worst-case run time. **C**. Improves the best-case run time. **D**. Uses less memory. **E**. Simplifies the code.

1. Add a tail pointer to the class `LinkedQueue`. B
2. Use three values for the status of a cell in linear rehashing (*empty, occupied and deleted*). A
3. In the second phase of heap sort, put the removed element after the end of the heap. D
4. Use a heap instead of a linked priority queue in heap sort. B
5. Use a queue instead of a stack to store the nodes during breadth-first search of a graph. A
6. Use % (modulus) instead of an if-else in the methods `enqueue` and `serve` of `ArrayQuue`. E

(b) (6 points) Choose the run time from A to D for each of the following cases:

A. $O(1)$. **B**. $O(\log n)$. **C**. $O(n)$. **D**. $O(n \log n)$. **E**. $O(n^2)$.

1. The best case of `LinkedList.remove`. A
2. The worst case of `Heap.remove`. B
3. The worst case of `ArrayList.remove`. C
4. The best case of `BST.insert`. A
5. The worst case of `ArrayQueue.enqueue`. A
6. The best case of `AVL.find`. A
7. The worst case of `BST.remove`. C
8. The worst case of heap sort. D
9. The best case of heap sort. D
10. The best case of binary search. A
11. The worst case of `ArrayStack.push`. A
12. The worst case of `ArrayQueue.serve`. A

(c) (8 points) Choose the most appropriate data structure for each of the following tasks.

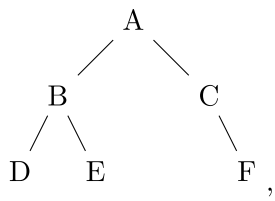
A. LinkedList.	**B**. ArrayList.	**C**. DoubleLinkedList.	**D**. LinkedQueue.
E. LinkedPQueue.	**F**. LinkedStack.	**G**. BT.	**H**. BST.
I. AVL.	**J**. BPlusTree.	**K**. HeapPQueue.	**L**. Graph.

1. An algorithm reads an unknown number of integers from a file, then goes through the numbers and removes any number that is equal to the sum of the one before it and the one after it. **C**
2. An algorithm takes a list of n points $(x_i, y_i), i = 1 \dots n$ and returns the k nearest points to a given point (x_0, y_0). **K**
3. An algorithm takes as input a list of purchases in the form $(UserID, ItemID, Price)$ and computes the total amount spent by every user. **I**
4. An algorithm reads a list of facebook friendship relations in the form $(User_i, User_j)$ and finds out if a post made by a user can reach another user through a sequence of shares (with friends). **L**

Question 2 . . . . 14 points

We can represent the path from the root to any node in a non empty binary tree using a string containing the letters 'L' and 'R'. The letter 'L' indicates going left, whereas 'R' indicates going right.

Example 1. *In this tree, the empty string corresponds to the root 'A', "LL" corresponds to 'D', "LR" corresponds to 'E' and "RR" corresponds to 'F'.*



1. Write the method **private static void** `concat(`**char** `c, List<String> l)`, member of `BT` which concatenates the character `c` at the start of every string in `l`. If the list is empty, nothing happens.

```
1  private static void concat(char c, List<String> l) {
2    if (...) {
3      ...;
4      while (...) {
5        ...;
6        ...;
7      }
8      ...;
9    }
10 }
```

- Line 2:
  - (A) **if** `(!l.first()){`
  - (B) **if** `(l == `**null**`){`
  - (C) **if** `(!l.full()){`
  - (D) **if** `(!l.empty()){`
  - (E) None
- Line 3:
  - (A) `current = head;`
  - (B) `l.current = head;`
  - (C) `l.findNext();`
  - (D) `l.findLast();`
  - (E) **None**
- Line 4:
  - (A) **while** `(!l.first()){`
  - (B) **while** `(l.next != `**null**`){`
  - (C) **while** `(l.last()){`
  - (D) **while** `(!l.last()){`
  - (E) None
- Line 5:
  - (A) `l.update(c);`
  - (B) `l.update(c + l.retrieve());`
  - (C) `l.update(l.retrieve()+ c);`

(D) `l.update(l.retrieve());`

(E) None

- Line 6:

(A) `l.findNext();`

(B) `findNext();`

(C) `current = current.next;`

(D) `l.findFirst();`

(E) None

- Line 8:

(A) `l.update(c + l.retrieve());`

(B) `l.update(l.retrieve()+ c);`

(C) `l.update(c);`

(D) `l.update(l.retrieve());`

(E) None

2. Write the method **public** `T get(String path)`, member of `BT`, which returns the data of the node indicated by `path`. Assume that the tree is not empty and that `path` is valid.

```
1  public T get(String path) {
2    BTNode<T> p = ...;
3    for (...) {
4      if (...)
5        ...;
6      else
7        ...;
8    }
9    return ...;
10 }
```

- Line 2:

(A) `BTNode<T> p = null;`

(B) `BTNode<T> p = root.left;`

(C) `BTNode<T> p = root;`

(D) `BTNode<T> p = root.right;`

(E) None

- Line 3:

(A) `for(int i=0; i<size; i++){`

(B) `for(int i=0; i<path.length(); i--){`

(C) `for(int i=path.length()-1; i>=0; i--){`

(D) `for(int i=0; i<path.length(); i++){`

(E) None

- Line 4:

(A) `if (p.left != null)`

(B) `if (p.data == 'L')`

(C) `if (p.right != null)`

(D) `if (path.charAt(i)= 'L')`

(E) None

- Line 5:

(A) `p = p.left;`

(B) `return p.data;`

(C) `p = root;`

(D) `p = p.parent;`

(E) None

- Line 7:

(A) `p = p.parent;`

(B) `p = p.right;`

(C) `return p.data;`

(D) `p = root;`

(E) None

- Line 9:

(A) `return p.left.data;`

(B) `return p.right.data;`

(C) `return p;`

(D) `return p.data;`

(E) None

3. Write the method **public** `List<String> leafPaths()`, member of `BT`, which returns the paths to all leaf nodes as strings. Use the method **private static void** `concat(char c, List<String> l)` above. If the tree is

empty, empty list is returned. Note that in line 16 below, we are calling the method `concatLists(l1, l2)` which returns the concatenation of the two lists `l1` and `l2`.

```
1  public List<String> leafPaths() {
2    return ...;
3  }
4  private List<String> rf(BTNode<T> t) {
5    List<T> l = new LinkedList<String>();
6    if (...)
7      ...;
8    if (...) {
9      ...;
10     ...;
11   }
12   List<String> l1 = ...;
13   ...;
14   List<String> l2 = ...;
15   ...;
16   return concatLists(l1, l2);
17 }
```

- Line 2:
  - (A) `return rf(t);`
  - (B) `return leafPaths(root);`
  - (C) `return rf(root.left)+rf(root.right);`
  - (D) `return rf(root);`
  - (E) None
- Line 6:
  - (A) `if (t.data == 'R')`
  - (B) `if (t != null)`
  - (C) `if (t.data == 'L')`
  - (D) `if (t == null)`
  - (E) None
- Line 7:
  - (A) `return l.retrieve();`
  - (B) `return 'L';`
  - (C) `return root;`
  - (D) `return l;`
  - (E) None
- Line 8:
  - (A) `if (t.left == null && t.right == null){`
  - (B) `if (t.left != null && t.right != null){`
  - (C) `if (t.left != null || t.right != null){`
  - (D) `if (t.left == null || t.right == null){`
  - (E) None
- Line 9:
  - (A) `l.update("");`
  - (B) `l.findFirst();`
  - (C) `l.insert("L");`
  - (D) `l.insert("");`
  - (E) None
- Line 10:
  - (A) `return l;`
  - (B) `l.insert("R");`
  - (C) `l.findFirst();`
  - (D) `l.findNext();`
  - (E) None
- Line 12:
  - (A) `List<String> l1 = rf(root.left);`
  - (B) `List<String> l1 = rf(t);`
  - (C) `List<String> l1 = rf(t.left);`
  - (D) `List<String> l1 = rf(left);`
  - (E) None
- Line 13:
  - (A) `concat('L', l);`
  - (B) `concat('L', new LinkedList<String>());`
  - (C) `concat('L', l1);`
  - (D) `concat('L', rf(t));`
  - (E) None

- Line 14:
  - (A) `List<String> l2 = rf(right);`
  - (B) `List<String> l2 = rf(t);`
  - (C) `List<String> l2 = rf(t.right);`
  - (D) `List<String> l2 = rf(root.right);`
  - (E) None

- Line 15:
  - (A) `concat('R', rf(t));`
  - (B) `concat('R', new LinkedList<String>());`
  - (C) `concat('R', l2);`
  - (D) `concat('R', l);`
  - (E) None

Question 3 . . . . 12 points

(a) Write the method **private int** `f(BTNode<T> t, T e, int k)`, member of `BT`, which returns the number of nodes in the level `k` of the subtree `t` having data equal to `e`. The root of the subtree (that is `t`) is at level 0.

```
1 private int f(BTNode<T> t, T e, int k) {
2   if(...)
3     return ...;
4   if (...)
5     return ...;
6   return ...;
7 }
```

- Line 2:
  - (A) `if (t.data == e)`
  - (B) `if (t != null)`
  - (C) `if (t == null)`
  - (D) `if (e.equals(t.data))`
  - (E) None

- Line 3:
  - (A) `return 0;`
  - (B) `return k;`
  - (C) `return 1;`
  - (D) `return f(t.left,e,k)&&f(t.right,e,k);`
  - (E) None

- Line 4:
  - (A) `if (!e.equals(t.data))`
  - (B) `if (k == 0 && e.equals(t.data))`
  - (C) `if (e.equals(t.data))`
  - (D) `if (k == 1 && e.equals(t.data))`
  - (E) None

- Line 5:
  - (A) `return f(t.left,e,k)+f(t.right,e,k);`
  - (B) `return 0;`
  - (C) `return 1;`
  - (D) `return 1+f(t.left,e,k)+f(t.right,e,k);`
  - (E) None

- Line 6:
  - (A) `return f(t.left,e,k)+f(t.right,e,k);`
  - (B) `return f(t.left,e,k-1)+f(t.right,e,k-1);`
  - (C) `return f(t,e,k+1)+f(t,e,k+1);`
  - (D) `return f(t.left,e,k+1)+f(t.right,e,k+1);`
  - (E) None

(b) Repeat the same questions as above, but this time as a user.

```
public static <T> int f(BT<T> b, T e, int k) {
  if (...)
    ...;
  ...;
  ...;
```

```
6  }
7  private static <T> int rf(BT<T> b, T e, int k) {
8    if (...)
9      return ...;
10   int n=0;
11   if (...) {
12     ...;
13     ...;
14   }
15   if (...) {
16     ...;
17     ...;
18   }
19 }
```

- Line 2:
  - A `if (b.full())`
  - B `if (b.empty())`
  - C `if (!b.empty())`
  - D `if (e.equals(b.retrieve()))`
  - E None
- Line 3:
  - A `return 1;`
  - B `return rf(b,e,k);`
  - C `return 0;`
  - D `return k;`
  - E None
- Line 4:
  - A `return rf(b,e,k);`
  - B `return e.equals(b.retrieve());`
  - C `b.find(relative.Root);`
  - D `b.find(relative.Parent);`
  - E None
- Line 5:
  - A `return rf(b, e, k - 1);`
  - B `return rf(b,e,k+1);`
  - C `return rf(b,e,k);`
  - D `return rf(b,e,0);`
  - E None
- Line 8:
  - A `if (k==1)`
  - B `if (e.equals(b.retrieve()))`
  - C `if (k==1 && e.equals(b.retrieve()))`
  - D `if (k==0 && e.equals(b.retrieve()))`
  - E None
- Line 9:
  - A `return 0;`
  - B `return 1+rf(b.left,e,k)+rf(b.right,e,k);`
  - C `return 1;`
  - D `return e.equals(b.retrieve());`
  - E None
- Line 11:
  - A `if (b.find(Relative.Parent)){`
  - B `if (b.find(Relative.LeftChild)){`
  - C `if (b.find(Relative.Root)){`
  - D `if (b.left != null){`
  - E None
- Line 12:
  - A `n-=rf(b.left,e,k);`
  - B `n-=rf(b,e,k);`
  - C `n+=rf(b,e,k-1);`
  - D `n+=rf(b,e,k+1);`
  - E None
- Line 13:
  - A `b.find(Relative.RightChild);`
  - B `b.find(Relative.Parent);`
  - C `b.find(Relative.Root);`
  - D `return n+1;`
  - E None
- Line 15:

Ⓐ `if (b.right != null){`

Ⓑ `if (b.find(Relative.RightChild)){`

Ⓒ `if (b.find(Relative.Parent)){`

Ⓓ `if (b.find(Relative.Root)){`

Ⓔ None

- Line 16:

Ⓐ `n+=rf(b,e,k+1);`

Ⓑ `n-=rf(b,e,k);`

Ⓒ `n-=rf(b.right,e,k);`

Ⓓ `n+=rf(b,e,k-1);`

Ⓔ None

- Line 17:

Ⓐ `b.find(Relative.Parent);`

Ⓑ `b.find(Relative.LeftChild);`

Ⓒ `return n+1;`

Ⓓ `b.find(Relative.Root);`

Ⓔ None

Question 4 .... 12 points

(a) (4 points) Choose the most appropriate answer.

1. What is the number of nodes in the last level of a heap containing 1024 nodes?

   Ⓐ **1.** Ⓑ 2. Ⓒ 3. Ⓓ 4. Ⓔ 5.

2. The best case run time for an insert in a heap represented as array is:

   Ⓐ $O(1)$**.** Ⓑ $O(\log n)$. Ⓒ $O(n)$. Ⓓ $O(n \log n)$ Ⓔ $O(n^2)$.

3. The worst case run time for a remove in a heap represented as array is:

   Ⓐ $O(1)$. Ⓑ $O(\log n)$**.** Ⓒ $O(n)$. Ⓓ $O(n \log n)$ Ⓔ $O(n^2)$.

4. Suppose all the keys in a heap have the same value. Then:

   Ⓐ The keys will be served in order of arrival (FIFO). Ⓑ The keys will be served in reverse order of arrival (LIFO). Ⓒ The keys will be served in pre-order. Ⓓ The keys will be served in in-order. Ⓔ **None.**

(b) (8 points) Consider the following heap represented as an array: 3, 5, 6, 9, 7, 8, 10, 15, 11, 12. Choose the correct answer for every operation (all operations are done on the above heap).

1. Heap after inserting 4:

   Ⓐ 3, 5, 6, 9, 7, 8, 10, 15, 11, 12, 4 Ⓑ **3, 4, 6, 9, 5, 8, 10, 15, 11, 12, 7** Ⓒ 3, 6, 4, 9, 5, 8, 10, 15, 11, 12, 7 Ⓓ 3, 4, 5, 6, 9 7, 8, 10, 15, 11, 12 Ⓔ None

2. Heap after inserting 2 then 13: Ⓐ 2, 3, 6, 9, 5, 8, 10, 11, 12, 7, 13, 15 Ⓑ 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15 Ⓒ 2, 3, 6, 9, 5, 8, 10, 11, 12, 7, 15, 13 Ⓓ **2, 3, 6, 9, 5, 8, 10, 15, 11, 12, 7, 13** Ⓔ None

3. Heap after deleting one key: Ⓐ 5, 6, 9, 7, 8, 10, 15, 11, 12 Ⓑ 5, 7, 6, 9, 8, 10, 15, 11, 12 Ⓒ **5, 7, 6, 9, 12, 8, 10, 15, 11** Ⓓ 5, 6, 7, 8, 9, 10, 11, 12, 15 Ⓔ None

4. Heap after deleting two keys: Ⓐ **6, 7, 8, 9, 12, 11, 10, 15** Ⓑ 6, 7, 8, 9, 10, 11, 12, 15 Ⓒ 6, 7, 9, 12, 8, 10, 15, 11 Ⓓ 6, 7, 9, 12, 8, 10, 11, 15 Ⓔ None

Question 5 . . . . 12 points

(a) (4 points)

Remark 1. *In what follows the height of tree is the number of levels in the tree. Hence, an empty tree has height 0, whereas a tree with 1 node has height 1.*

Choose the most appropriate answer:

1. The maximum height of an AVL tree with 4 nodes is:

   (A) 1. (B) 2. **(C) 3.** (D) 4. (E) 5.

2. The cost of one rotation in an AVL tree is:

   (A) $O(1)$. (B) $O(\log n)$. (C) $O(n)$. (D) $O(n \log n)$ (E) $O(n^2)$.

3. The maximum number of rotations caused by an insert in an AVL tree with n nodes and height h is (a single rotation is counted 1; a double rotation is counted 2):

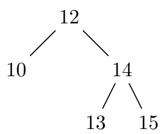
   (A) 1. **(B) 2.** (C) h. (D) $2h$ (E) n.

4. The worst case height of an AVL tree with n nodes is:

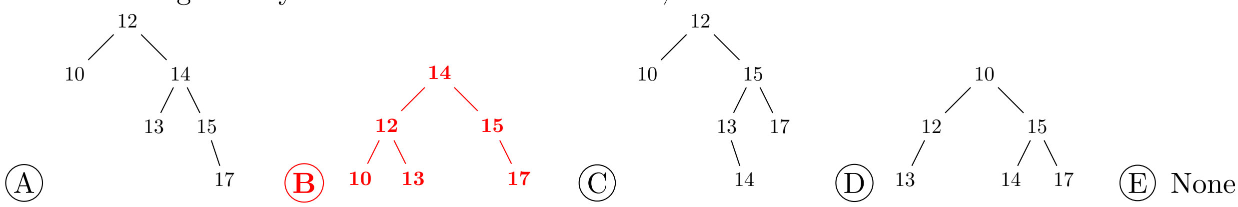
   (A) $O(1)$. **(B)** $O(\log n)$. (C) $O(n)$. (D) $O(n \log n)$ (E) $O(n^2)$.

(b) (8 points) Choose the correct result in each of the following cases (follow the the convention of replacing with the smallest key in the right sub-tree when necessary):

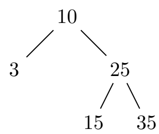
1. After inserting the key 17 in the AVL



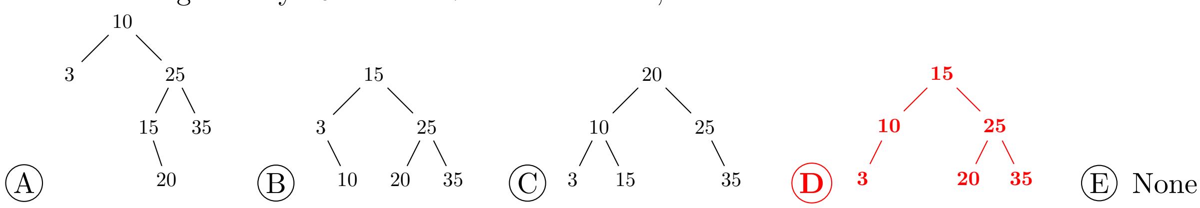
, the tree becomes:



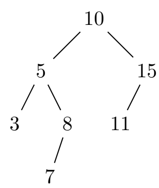
2. After inserting the key 20 in the AVL



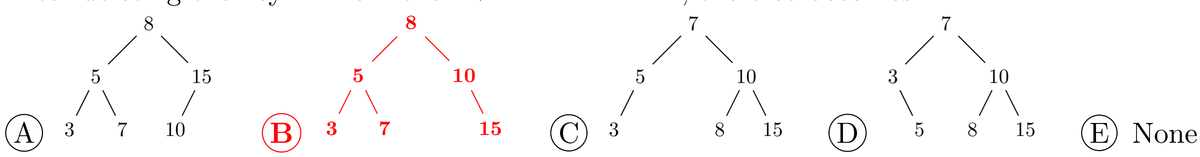
, the tree becomes:

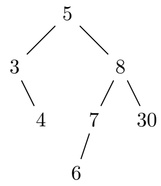


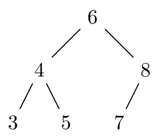
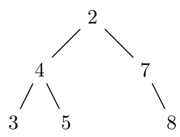
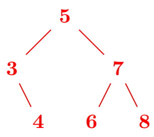
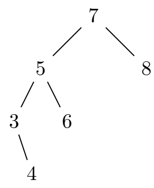
3. After deleting the key 11 from the AVL



, the tree becomes:



4. After deleting the key 30 from the AVL  , the tree becomes:

Ⓐ  Ⓑ  **Ⓒ**  Ⓓ  Ⓔ None

Question 6 .... 12 points

(a) (4 points) Choose the most appropriate answer:

1. The insert operation in a hash table using linear rehashing has a worst case run time:

   Ⓐ $O(1)$ Ⓑ $O(\log n)$ **Ⓒ $O(n)$** Ⓓ $O(n \log n)$ Ⓔ None

2. You want to store at most 16 keys in hash table using the % hash function. Choose the most appropriate table size:

   Ⓐ 14 Ⓑ 15 Ⓒ 16 **Ⓓ 17** Ⓔ 18

3. How many keys can a hash table that uses folding on a single digit store if the key contains 4 digits?

   Ⓐ 4 Ⓑ 36 **Ⓒ 37** Ⓓ 39 Ⓔ 40

4. Consider the following hash function: select the two rightmost digits then apply % 11 on the corresponding number. Which of the following couples of keys cause a collision?

   Ⓐ 3848 and 4756 Ⓑ 3973 and 1258 Ⓒ 162 and 35476 Ⓓ All of the above. **Ⓔ None of the above.**

(b) (8 points) Use the hash function $H(key) = key\%5$ to store the sequence of keys $22, 15, 12, 27, 18$ in a hash table of size 5. Use the following collision resolution strategies:

1. Linear rehashing (c=1). Fill in the following table:

Key	22	15	12	27	18
Position	2	0	3	4	1
Number of probes	1	1	2	3	4

2. External chaining. Fill in the following table:

Key	22	15	12	27	18
Index of the list	2	0	2	2	3

3. Coalesced chaining with cellar size 2 and address region size 5. Fill in the following table (put -1 if there is no next element):

Key	22	15	12	27	18
Position	2	0	6	5	3
Index of next element	6	-1	5	-1	-1

Question 7 . . . . 12 points

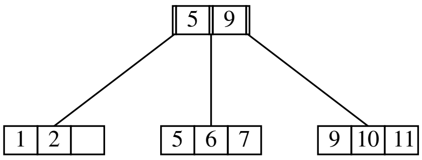
(a) (4 points)

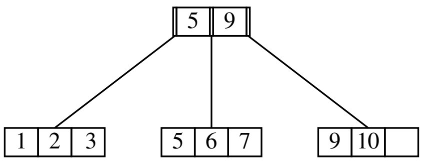
Remark 2. *In what follows the height of tree is the number of levels in the tree. Hence, an empty tree has height 0, whereas a tree with 1 node has height 1. Recall also that a B+ tree has two parameters, m: the maximum number of children and l: the maximum number of elements in a leaf node.*

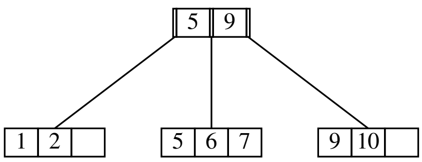
Choose the most appropriate answer:

1. The leaves of a B+ tree with $l = 7$ can contain the following number of data elements:
   (A) 3 to 7 elements (B) **4 to 7 elements** (C) 5 to 7 elements (D) 0 to 7 elements (E) None
2. In a B+ tree with $m = 4$, $l = 3$, and height h, the worst number of comparisons required to find a key is:
   (A) h (B) $4h$ (C) $\log h$ (D) $3h$ (E) None
3. The maximum number of data elements in a B+ tree with $m = l = 3$ and height 2 is:
   (A) 3 (B) 6 (C) 9 (D) 12 (E) None
4. The height of a B+ tree containing n different keys is:
   (A) $O(\log n)$ (B) $O(n)$ (C) $O(n \log n)$ (D) $O(n^2)$ (E) None

(b) (8 points) Choose the correct result in each of the following cases (when possible, always borrow and transfer to the left):

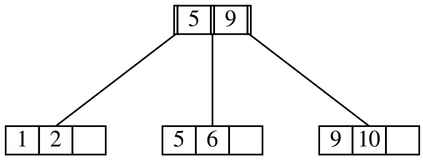
1. After inserting the key 12 to the B+ tree 

, the **root** of the tree becomes:
   (A) 5 | 9 (B) 5 | 9 | 11 (C) 9 | (D) 6 | 10 (E) None
2. After inserting the key 4 to the B+ tree 

, the **root** of the tree becomes:
   (A) 4 | (B) 5 | 9 (C) 4 | 7 (D) 7 | 9 (E) **None**
3. After deleting the key 1 from the B+ tree 

, the **root** of the tree becomes:

(A) [6 | 9] (B) [7 |] (C) [6 |] (D) [5 | 7] (E) None

4. After deleting the key 1 from the B+ tree  , the **root** of the tree becomes:

(A) [2 | 9] (B) [5 | 9] (C) [6 |] **(D)** [9 |] (E) None

Question 8 . . . . 6 points

(a) (2 points) Choose the most appropriate answer:

1. If you apply DFS on a binary tree, the nodes will be visited in the same order as:

   (A) Preorder. (B) Inorder. (C) Postorder. (D) BFS. (E) None.

2. You apply BFS from a given node and you find out that **not** all nodes were visited. This means:

   (A) Your BFS implementation has a bug. (B) Some nodes have no edges. (C) The graph contains cycles. **(D) The graph is disconnected.** (E) None.

(b) (4 points) Given the following graph adjacency list, answer the questions below.

a	$\to b \to c$
b	$\to a \to c \to e \to d$
c	$\to a \to b \to d$
d	$\to b \to c \to e \to f$
e	$\to b \to d \to f$
f	$\to e \to d$

1. Which of the following sequences are paths in this graph? Answer by T (true) or F (false).

   (a) (a, c, e, f) **F**

   (b) (a, c, b, d, f, e) **T**

   (c) (a, b, e, c) **F**

   (d) (f, e, b, d) **T**

2. Answer by T (true) or F (false).

   (a) The graph is connected. **T**

   (b) The number of edges in the graph is 8. **F**

   (c) (b, e, f, b) i a cycle. **F**

   (d) The number of 1s in the adjacency matrix of this graph is 18. **T**

3. The BFS traversal of this graph starting from a is (insert neighbors in the data structure in increasing alphabetic order):

   (A) a, b, d, c, e, f. **(B) a, b, c, d, e, f.** (C) a, c, d, f, e, b. (D) a, b, f, c, d, e. (E) a, b, c, f, e, d.

4. The DFS traversal of this graph starting from a is (insert neighbors in the data structure in increasing alphabetic order):

   (A) a, b, d, c, f, e. (B) a, b, d, c, e, f. (C) a, b, c, d, e, f. **(D) a, c, d, f, e, b.** (E) a, b, f, c, d, e.