



Question 1 ..... 12 points

Choose the most appropriate data structure for each of the following tasks.

- |               |              |                     |                |                |           |
|---------------|--------------|---------------------|----------------|----------------|-----------|
| A. LinkedList | B. ArrayList | C. DoubleLinkedList | D. LinkedQueue | E. AVL         | F. Stack  |
| G. BT.        | H. BST.      | I. LinkedPQueue.    | J. BPlusTree.  | K. HeapPQueue. | L. Graph. |

- ✓ 1. An application that analyzes the communication patterns on an online social network. Graph
- ✓ 2. An application in a restaurant that keeps track of all tables assigned to every waiter. Linked Queue
- ✓ 3. An algorithm that decides on the order of surgical procedures at a hospital. Linked Priority Queue
- ✓ 4. An application that keeps track of people that have been vaccinated and those who have not. B Plus Tree
- ✓ 5. A video play list that allows forward and backward navigation between the videos. DoubleLinkedList
- ✓ 6. An algorithm that receives a character and returns its ASCII code. AVL

Question 2 ..... 12 points

Write the method `public static int ls(List<Boolean> l)` which takes as input a non-empty list `l` of Booleans and returns the length of the longest contiguous sequence of `true` in `l`.

Example 1. If  $l = \{0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1\}$ , `ls(l)` returns 4.

```

1 public static int ls(List<Boolean> l) {
2     int maxL = ...;
3     ...
4     while (true) {
5         ...
6         while (true) {
7             if (...)
8                 ...
9             else
10                ...
11            if (...)
12                break;
13            else
14                ...
15        }
16        if (...)
17            ...
18        if (...)
19            break;
20        else
21            ...
22    }
23    return maxL;
24 }

```

1. Line 2:

- ☐ (A) `int maxL = -1;`  
☐ (B) `int maxL = 1;`  
☐ (C) `int maxL = l.retrieve();`  
☒ (D) `int maxL = 0;`  
☐ (E) None

2. Line 3:

- ☐ (A) `l.insert(true);`  
☐ (B) `l.update(false);`  
☐ (C) `l.findNext();`  
☐ (D) `maxL = l.length();`  
☒ (E) None `l.findFirst`

3. Line 5:

- ☐ (A) int cpt = 1;
- ☐ (B) int cpt = -1;
- ☒ (C) int cpt = 0;
- ☐ (D) int cpt = l.retrieve();
- ☐ (E) None

4. Line 7:

- ☒ (A) if (l.retrieve() > cpt)
- ☐ (B) if (l.retrieve() > maxL)
- ☒ (C) if (l.retrieve())
- ☐ (D) if (l.last())
- ☒ (E) None *MAXIMUM = 1*

5. Line 8:

- ☒ (A) maxL++;
- ☐ (B) cpt--;
- ☒ (C) cpt++;
- ☐ (D) l.findFirst();
- ☐ (E) None

6. Line 10:

- ☐ (A) continue;
- ☒ (B) break;
- ☐ (C) l.findFirst();
- ☒ (D) l.findNext();
- ☐ (E) None

7. Line 11:

- ☐ (A) if (l.retrieve())
- ☒ (B) if (l.last())
- ☐ (C) if (maxL == 0)
- ☐ (D) if (cpt == 0)
- ☐ (E) None

8. Line 14:

- ☐ (A) maxL = 0;
- ☐ (B) maxL = cpt;
- ☐ (C) l.findFirst();
- ☒ (D) l.findNext();
- ☐ (E) None

9. Line 16:

- ☐ (A) if (cpt == maxL)
- ☐ (B) if (cpt < maxL)
- ☒ (C) if (cpt > maxL)
- ☐ (D) if (cpt == 0)
- ☐ (E) None

10. Line 17:

- ☐ (A) cpt = maxL;
- ☐ (B) maxL++;
- ☒ (C) maxL = cpt;
- ☐ (D) cpt++;
- ☐ (E) None

11. Line 18:

- ☐ (A) if (maxL < 0)
- ☒ (B) if (l.last())
- ☐ (C) if (maxL > 0)
- ☐ (D) if (l.retrieve())
- ☐ (E) None

12. Line 21:

- ☐ (A) l.findFirst();
- ☒ (B) l.findNext();
- ☐ (C) cpt++;
- ☐ (D) maxL++;
- ☐ (E) None

Question 3 ..... 14 points

- (a) Write a constructor for the class `BT` which builds the tree from its array representation `a`. The root element is at position 1, and the left and right children of a node at position `i` are located at positions `2*i` and `2*i+1` respectively. If a position contains `null`, then the corresponding node does not exist.



```

1 public BT(T[] a) {
2     current = root = ...;
3 }
4 private BTNode<T> rfa(T[] a, int i) {
5     if (...)
6         return ...;
7     BTNode<T> nnode = ...;
8     ...
9     ...
10    ...
11 }

```

1. Line 2:

- ☐ (A) rfa(a, 0)  
☐ (B) null  
☐ (C) rfa;  
☒ (D) rfa(a, 1)  
☐ (E) None

2. Line 5:

- ☐ (A) a[i] < 0  
☒ (B) i >= a.length || a[i] == null  
☐ (C) i < a.length && a[i] < 0  
☐ (D) a[i] != null && i > a.length  
☐ (E) None

3. Line 6:

- ☐ (A) current;  
☒ (B) null  
☐ (C) i  
☐ (D) a[i/2]  
☐ (E) None

4. Line 7:

- ☐ (A) root;  
☐ (B) a[i];  
☐ (C) new BTNode<T>(i)  
☒ (D) new BTNode<T>( a[i] )  
☐ (E) None

5. Line 8:

- ☐ (A) current = current->left;  
☐ (B) current.left = rfa(a, i);  
☒ (C) nnode.left = rfa(a, i+2);  
☐ (D) current.right = rfa(a, i+2);  
☐ (E) None

6. Line 9:

- ☐ (A) current = current->right;  
☒ (B) nnode.right = rfa(a, i+2+1);  
☐ (C) current.right = rfa(a, i+2+1);  
☐ (D) current.left = rfa(a, i+2+1);  
☐ (E) None

7. Line 10:

- ☐ (A) return current;  
☐ (B) return current.rfa();  
☐ (C) return null;  
☒ (D) return nnode;  
☐ (E) None

(b) Repeat the same question as user of BT.

```

1 public static <T> BT<T> fa(T[] a) {
2     BT<T> bt = new BT<T>();
3     if (...) {
4         ...
5         ...
6     }
7     return bt;
8 }
9 private static <T> void rfa(BT<T> bt, T[] a,
10    int i) {
11     if (...) {
12         ...
13         ...
14     }

```

```

15     if (...) {
16         ...
17         ...
18         ...
19     }
20 }

```

1. Line 3:

- ☒ (A) if (a.length > 1 && a[1] != null){  
☐ (B) if (a.length > 0 || a[0] != null){  
☐ (C) if (a.length > 0 && a[0] != null){  
☐ (D) if (a.length > 1){

- (E) None
2. Line 4:
- (A) `bt.insert(a[0], relative.Root);`
  - (B) `rfa(bt, a, 1);`
  - (C) `bt.insert(a[ar.length/2], relative.Root);`
  - (D) `bt.insert(a[1], relative.Root);`
  - (E) None
3. Line 5:
- (A) `rfa(bt, a, ar.length/2);`
  - (B) `rfa(bt, a, 1);`
  - (C) `rfa(bt, a, 0);`
  - (D) `bt.insert(a[1], relative.Parent);`
  - (E) None
4. Line 10:
- (A) `if (a[2*i] != null){`
  - (B) `if (2*i < a.length && a[2*i] != null){`
  - (C) `if (2*i < a.length){`
  - (D) `if (2*i < a.length || a[2*i] != null){`
  - (E) None
5. Line 11:
- (A) `bt.insert(a[2*i], Relative.LeftChild);`
  - (B) `bt.insert(a[i/2], Relative.LeftChild);`
  - (C) `rfa(bt, a, 2*i);`
  - (D) `bt.find(Relative.Parent);`
  - (E) None
6. Line 12:
- (A) `bt.insert(a[i+1], Relative.LeftChild);`
  - (B) `bt.insert(a[2*i], Relative.LeftChild);`
  - ⚡ (C) `bt.find(Relative.Parent);`
  - (D) `rfa(bt, a, 2*i);`
  - (E) None

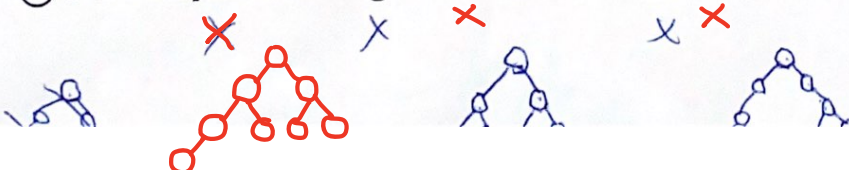
7. Line 13:
- ⚡ (A) `rfa(bt, a, 2*i);`
  - (B) `bt.find(Relative.Parent);`
  - (C) `bt.find(Relative.Root);`
  - (D) `rfa(bt, a, i/2);`
  - (E) None
8. Line 15:
- (A) `if (2*i+1 < a.length){`
  - (B) `if (a[2*i+1] != null){`
  - (C) `if (2*i+1 < a.length || a[2*i+1] != null){`
  - (D) `if (2*i+1 < a.length && a[2*i+1] != null){`
  - (E) None
9. Line 16:
- (A) `bt.find(Relative.Parent);`
  - (B) `bt.insert(a[2*i+1], Relative.RightChild);`
  - (C) `bt.insert(a[(i+1)/2], Relative.RightChild);`
  - ;
  - (D) `rfa(bt, a, 2*i+1);`
  - (E) None
10. Line 17:
- ⚡ (A) `bt.find(Relative.Parent);`
  - (B) `bt.insert(a[i+2], Relative.RightChild);`
  - (C) `bt.insert(a[2*i+1], Relative.RightChild);`
  - (D) `rfa(bt, a, 2*i+1);`
  - (E) None
11. Line 18:
- ⚡ (A) `rfa(bt, a, 2*i+1);`
  - (B) `bt.find(Relative.Root);`
  - (C) `bt.find(Relative.Parent);`
  - (D) `rfa(bt, a, (i+1)/2);`
  - (E) None

Question 4 ..... 14 points

Choose the most appropriate answer.

1. Suppose  $T$  is a binary tree with 8 nodes and height 4. Which of the following is surely true?

- (A)  $T$  is a complete tree. (B)  $T$  is a min-heap. (C)  $T$  is a max-heap. (D)  $T$  is not a heap. ?





$$2^6 - 1 = 63 = \text{complete!}$$

(E) None.

2<sup>something</sup> - 1 = complete tree

$$64 - 1 = 63$$

2. Suppose  $T$  is a binary tree with 63 nodes and height 6. Which of the following is surely true?

(A)  $T$  is a complete tree.

(B)  $T$  is a min-heap.

(C)  $T$  is a max-heap.

(D)  $T$  is not a heap.

(E) None.

{3, 10, 8, 12, 11, 20, 14, 5, 6}

3. Given a complete binary tree  $T$  and its array representation  $A$ , which traversal algorithm visits the nodes of  $T$  in the same order as they have in the array  $A$ ?

(A) Pre-order

(B) In-order

(C) Post-order

(D) BFS

(E) DFS

(F) None

4. Given the heap  $H = [3, 10, 8, 12, 11, 20, 14]$ , what is  $H$  after inserting 5 and then 6?

(A) 3, 4, 5, 8, 6, 12, 10, 11, 14, 20

(B) 3, 5, 8, 6, 11, 20, 14, 10, 12

(C) 3, 5, 6, 8, 10, 11, 12, 14, 20

(D) 3, 5, 6, 10, 11, 8, 14, 20, 12

(E) None

5. Given the heap  $H = [3, 10, 8, 12, 11, 20, 14]$ , what is  $H$  after deleting one key?

(A) 8, 14, 12, 10, 11, 20

(B) 8, 10, 11, 12, 14, 20

(C) 8, 10, 12, 11, 20, 14

(D) 8, 10, 14, 12, 11, 20

(E) None

6. Given the heap  $H = [3, 6, 8, 9, 23, 10, 13, 11]$ , what is  $H$  after deleting two keys?

(A) 3, 6, 8, 9, 23, 10.

(B) 8, 9, 23, 10, 13, 11.

(C) 8, 9, 10, 11, 13, 23.

(D) 8, 9, 10, 11, 23, 13.

(E) None

7. Given two heaps,  $H_1 = [6, 10, 18, 20, 22, 24, 30]$  and  $H_2 = [8, 12, 14, 19, 23]$ , what is the result of merging them along with the key 13?

(A) 6, 10, 8, 13, 18, 12, 14, 20, 22, 24, 30, 19, 23

(B) 6, 8, 12, 14, 13, 18, 19, 23, 20, 22, 24, 30

(C) 6, 8, 12, 13, 14, 18, 19, 20, 22, 23, 24, 30

(D) 6, 8, 14, 10, 23, 12, 20, 22, 24, 30, 18, 19

(E) None

Question 5 ..... 14 points

(a) (4 points)

**Remark 1.** In what follows the height of tree is the number of levels in the tree. Hence, an empty tree has height 0, whereas a tree with 1 node has height 1.

Choose the most appropriate answer:

1. The maximum depth of an AVL tree with 5 nodes is:

(A) 1.

(B) 2.

(C) 3.

(D) 4.

(E) 5.

2. The worst case run time for insert in AVL tree is:

(A)  $O(1)$ .

(B)  $O(n)$ .

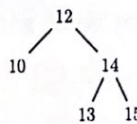
(C)  $O(\log n)$ .

(D)  $O(n \log n)$ .

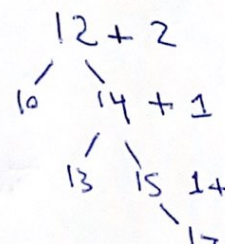
(E)  $O(n^2)$ .

(b) (10 points) Choose the correct result in each of the following cases (follow the the convention of replacing with the smallest key in the right sub-tree when necessary):

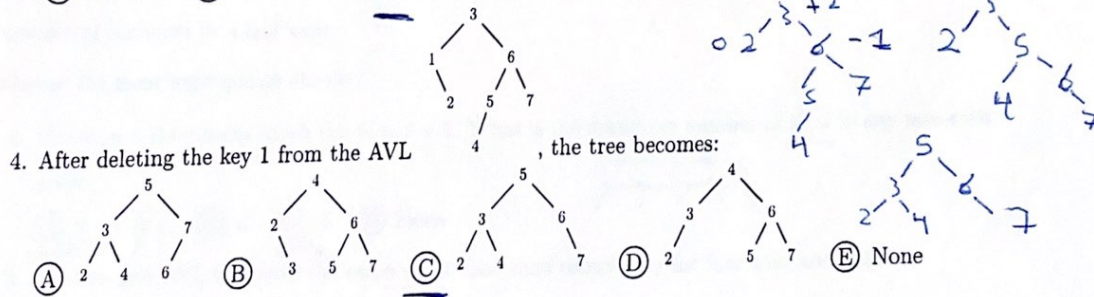
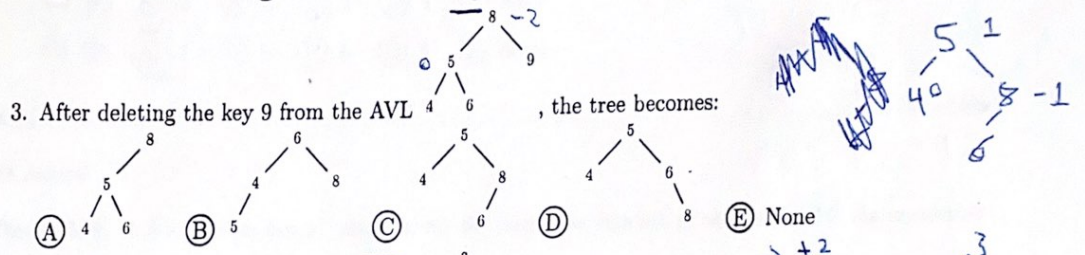
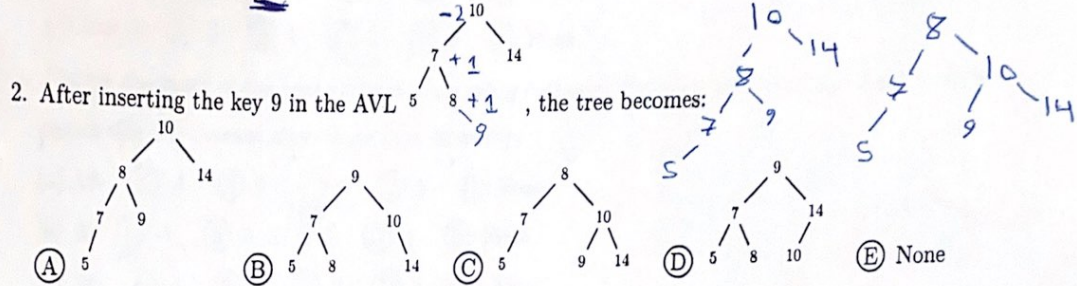
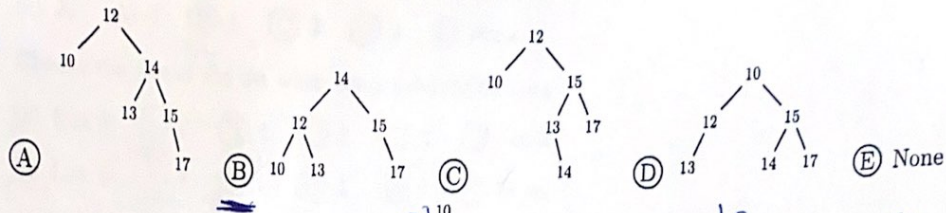
1. After inserting the key 17 in the AVL



14







Question 6 ..... 14 points

(a) (4 points) Choose the most appropriate answer:

1. The insert operation in a hash table using external chaining has a worst case run time:

(A)  $O(1)$  (B)  $O(\log n)$  (C)  $O(n)$  (D)  $O(n \log n)$  (E) None

2. Consider the following hash function: The product of the rightmost digit and the leftmost digit

:  $17 \times 11$  then apply  $\% 11$  to the result. Which of the following couples of keys cause a collision?

(A) 9 and 101 (B) 162 and 370 (C) 523 and 259 (D) All of the above. (E) None of the above.

(b) (10 points) Use the hash function  $H(key) = key \% 6$  to store the sequence of keys 16, 5, 22, 18, 31 in a hash table of size 6. Use the following collision resolution strategies:

1. Choose the number of probes when using linear rehashing with  $c=1$ :

(a) 5: (A) 0 (B) 1 (C) 2 (D) 3 (E) None

(b) 22: (A) 0 (B) 1 (C) 2 (D) 3 (E) None

16 5 22 18 31

4	5	6	0	1					
1	1	3	1	1					



- (c) 31: (A) 0 (B) 1 (C) 2 (D) 3 (E) None
2. Choose the size of the list when using external chaining.
- (a) List 0: (A) 0 (B) 1 (C) 2 (D) 3 (E) None
- (b) List 2: (A) 0 (B) 1 (C) 2 (D) 3 (E) None
- (c) List 4: (A) 0 (B) 1 (C) 2 (D) 3 (E) None

16	5	22	18	31
4	5	4	0	1

3. Choose the index of the next element when using coalesced chaining with cell size 2 and address region size 4 (-1 means there is no next element):

- (a) 16: (A) -1 (B) 0 (C) 3 (D) 5 (E) None
- (b) 5: (A) -1 (B) 0 (C) 3 (D) 4 (E) None
- (c) 22: (A) -1 (B) 1 (C) 2 (D) 4 (E) None
- (d) 18: (A) -1 (B) 1 (C) 5 (D) 6 (E) None

16	5	22	18	31
4	5	<del>4</del>	0	1
<del>3</del>	-1	-1	-1	-1

clap = ~~3~~ 3, 2

Question 7 ..... 14 points

(a) (6 points)

**Remark 2.** A B+ tree has two parameters,  $m$ : the maximum number of children and  $l$ : the maximum number of elements in a leaf node.

Choose the most appropriate answer:

1. Consider a B+-tree in which the  $m = l = 5$ . What is the minimum number of keys in any non-root node :

- (A) 2 (B) 1 (C) 4 (D) 3 (E) None



2. B+-tree and AVL tree have the same worst case time complexity for insertion and deletion:

- (A) True (B) False

3. The best case running time for checking if a key exists in a B+-tree is:

- (A)  $O(1)$  (B)  $O(\log n)$  (C)  $O(n)$  (D)  $O(n \log n)$  (E) None

- (b) (8 points) Choose the correct result in each of the following cases (when possible, always borrow and transfer to the left). The order of the tree is  $m = 3$ :

1. After inserting the key 7 in the B+ tree 

3	5	9
---	---	---

, the root of tree becomes:

- (A) 

5	7
---	---

 (B) 

7	
---	--

 (C) 

5	
---	--

 (D) 

9	
---	--

 (E) None



2. After inserting the key 10 in the B+ tree 

3	4	5
---	---	---

7	8	9
---	---	---

12	13	
----	----	--

, the root of the tree becomes:

- (A) 

7	10
---	----

 (B) 

7	9
---	---

 (C) 

8	12
---	----

 (D) 

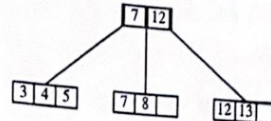
7	12
---	----

 (E) None





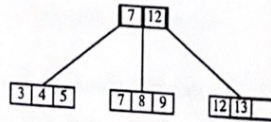
3. After deleting the key 12 from the B+ tree becomes:



, the root of the tree

- (A) [7 | 13] (B) [7] (C) [13] (D) [7 | 8] (E) None

4. After deleting the key 12 from the B+ tree becomes:



, the root of the tree

- (A) [7] (B) [5] (C) [7 | 9] (D) [7 | 13] (E) None

Question 8

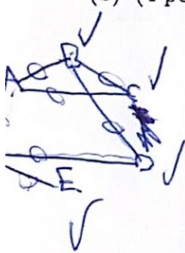
(a) (2 points) Choose the most appropriate answer:

1. What is the maximum length of any simple path in a graph containing 100 nodes?  
(A) 10. (B) 100. (C) 99. (D) 101. (E) None.

2. Which of the following is true for DFS and BFS graph traversal?

- (A) Both DFS and BFS use Stack. (B) DFS uses Queue and BFS uses Stack. (C) BFS uses Queue. (D) DFS uses Stack. (E) C and D.

(b) (4 points) Given the following graph adjacency list, answer the questions below.



A	→ B → C → F
B	→ A → C → D
C	→ A → B
D	→ B → F
E	→ F
F	→ A → D → E

(b) The number of edges in the graph is 7.

I

(c) In the graph, (A, B, D, F, A) is a cycle.

I

(d) The neighbors of node C are A, B, and D.

F

3. Which of the following is true for this graph?

- (A) The graph is a tree. (B) The graph is acyclic. (C) The graph is disconnected. (D) The graph is connected. (E) None.

4. The DFS traversal of this graph starting from A is (insert neighbors in the data structure in increasing alphabetic order):

- (A) A, C, B, E, D, F. (B) A, F, D, E, C, B. (C) A, F, B, C, E, D. (D) A, F, E, D, C, B. (E) None.

1. Which of the following sequences are paths in this graph? Answer by T (true) or F (false).

- (a) (F, B, D, A) F  
(b) (A, B, D, F, A) T  
(c) (A, B, C, F, D) F  
(d) (F, A, B, C) T

2. Answer by T (true) or F (false).

- (a) The graph is a tree. F



A F E D C B



A F E D C B