Useful math facts

Big-Oh notation

$O(g(n))$ is the set of all functions with a smaller or same order of growth as $g(n)$.
$f(n) \in O(g(n)) \Leftrightarrow \exists c > 0, n_0 \geq 0 | f(n) \leq cg(n)$ for all $n \geq n_0$.

- $\sum_{i=0}^{m} a_i n^i \in O(n^m)$.
- $f(n) \in O(g_1(n))$ and $h(n) \in O(g_2(n)) \Rightarrow f(n) + h(n) \in O(MAX(g_1(n), g_2(n)))$.
- $f(n) \in O(g_1(n))$ and $h(n) \in O(g_2(n)) \Rightarrow f(n) \cdot h(n) \in O(g_1(n) \cdot g_2(n))$.
- $f(n) = \sum_{i=0}^{k} a_i n^i, h(n) = \sum_{j=0}^{l} b_j n^j, a_k > 0, b_l > 0 \Rightarrow \frac{f(n)}{h(n)} \in O\left(\frac{n^k}{n^l}\right)$.

Logarithms and exponents

- $a^{\log_a b} = b$.
- $\log_a b = \frac{\log_x b}{\log_x a}$.
- $\log(ab) = \log a + \log b$.
- $\log(\frac{a}{b}) = \log a - \log b$.
- $\log(x^a) = a \log x$.
- $x^a \cdot x^b = x^{a+b}$.
- $\frac{x^a}{x^b} = x^{a-b}$.
- $(x^a)^b = (x^b)^a = x^{ab}$.

Summations

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Basic asymptotic classes:

1 (constant).

$\log n$ (logarithmic).

n (linear).

$n \log n$ (n-log-n).

n^2 (quadratic).

n^3 (cubic).

2^n (exponential).

$n!$ (factorial).