Answer the following questions:

1. Show that the following conditional statements **(¬p ∧ (p ∨ q)) → q** is a tautology by using:

    a) Truth tables.
    b) Applying a chain of logical equivalences.

2. Determine the truth value of the following propositions for the given universe of discourse.

	Universe of discourse	Truth value
$\exists x \ (x+1>2x)$	$\mathbb{Z}$	
$\forall x \ (x+1>2x)$	$\mathbb{Z}$	
$\exists x \ (x^2 = 2)$	$\mathbb{R}$	
$\exists x \ \exists y \ (x + y = x - y)$	$\mathbb{Z}$	
$\forall x \ \exists y \ (x + y = x - y)$	$\mathbb{Z}$	
$\forall y \ \exists x \ (x + y = x - y)$	$\mathbb{Z}$	
$\forall x \ \exists y \ (x - 2y = 0)$	$\mathbb{R}$	
$\forall x \ (x<10) \rightarrow \forall y \ (y<x) \rightarrow y<9$	$\mathbb{Z}$	
$\forall x \ (x<10) \rightarrow \forall y \ (y<x) \rightarrow y<9$	$\mathbb{R}$	

3. For the following statements, write "True" or "False":
    a. $\forall x \ (P(x) \land Q(x)) \equiv \forall x \ P(x) \land \forall x \ Q(x)$
    b. $\forall x \ (P(x) \lor Q(x)) \equiv \forall x \ P(x) \lor \forall x \ Q(x)$
    c. $\exists x \ (P(x) \land Q(x)) \equiv \exists x \ P(x) \land \exists x \ Q(x)$
    d. $\exists x \ (P(x) \lor Q(x)) \equiv \exists x \ P(x) \lor \exists x \ Q(x)$

4. Let L(x,y)="x likes y". Express the following statements using predicates and quantifiers:
    a. Everyone likes Khaled.
    b. There is someone who Fahad doesn't like.
    c. Everyone likes someone.
    d. There is someone whom everyone likes.
    e. There is someone whom no one likes.