April-2019

KING SAUD UNIVERSITY

COLLEGE OF COMPUTER & INFORMATION SCIENCES

DEPT OF COMPUTER SCIENCE

$\boxed{26 \times 26 \times 26 \times 9999}$

CSC281 Discrete Mathematics for CS Students

Practice Questions (Final)

1. The car plate board in Saudi Arabia has the form: A #, where # is any number 1 to 9999, and A is a three letter in the range A-Z. How many car plate boards can you have?

2. Find the coefficient of the smallest and largest power of x in the expansion of $(3x + 2x^2 + 4/x)^{100}$.

3. How many base 3 strings of length 10 that starts with 000, or 222?

4. How many positive integers between 25 and 134 that are divisible by 4 and by 6 at the same time?

5. English alphabet has 21 consonants and 5 vowels. How many strings of length 6 of lowercase letters that has no vowels? Exactly 2 vowels? At least 2 vowels?

6. Solve the recurrence relation: $a_n = 3a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 0$, $a_1 = 5$.

7. Convert the number 123 (in base 6) to a number in base 9.

8. Write the sequence generated by the GF $\dfrac{1+x+x^2}{(1-x)^2}$.

9. Derive the recurrence relation to count the number of bit strings of length n that has the pattern 101.

10. Prove using induction that for all $n \geq 1$,

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}.$$

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5 * no vowels $= 21^6$

* exactly two vowels $= \binom{6}{2} \times 5^2 \times 21^4$

$\boxed{\text{6 اماكن}}$

الامكانات في هذه الخانة

جميع الترتيبات الممكنة لباقي الحروف

at least 2 vowels $= \sum_{k=2}^{6} \binom{6}{k} * 5^k * 21^{6-k}$

final practice

A	#
$26 \times 26 \times 26$	

①

1 digit or
2 digits or
3 digits or
4 digits

$26 \times 26 \times 26 \times 9$ +

$26 \times 26 \times 26 \times 9 \times 10$ → $10 \to 99$

$+\ 26 \times 26 \times 26 \times 9 \times 10 \times 10$ → $100 \to 999$

$+\ 26 \times 26 \times 26 \times 9 \times 10 \times 10 \times 10$ → $1000 \to 9999$

$= 26^3 \times (9 + 90 + 900 + 9000)$

$= 26^3 \times 9999$

final practice 2

$$\left(3x+2x^2+\frac{4}{x}\right)^{100}$$

$$=\sum_{n_1+n_2+n_3=100}\binom{100}{n_1,n_2,n_3}(3x)^{n_1}\cdot(2x^2)^{n_2}\cdot\left(\frac{4}{x}\right)^{n_3}$$

n_1	n_2	n_3
0	0	100
⋮	⋮	⋮
0	100	0

$\rightarrow \frac{(100)!}{0!\,0!\,(100)!}\cdot(3x)^0\cdot(2x^2)^0\left(\frac{4}{x}\right)^{100}$

$= 4^{100}\cdot x^{-100}$

~~smallest~~ coefficient of ~~4^{100}~~ smallest power $= 4^{100}$

$\frac{(100)!}{(0!)(100!)(0!)}(3x)^0\cdot(2x^2)^{100}\cdot\left(\frac{4}{x}\right)^0$

$= 2^{100}\ x^{200}$

$\therefore$ coefficient of largest power $=$ ~~2^{100}~~ 2^{100}

Practice ③

base 3 = { 0, 1, 2 }

all strings starts with 000 = | 0 0 0 | anything | = 3^7

all strings starts with 222 = | 2 2 2 | any thing | = 3^7

∴ all strings = $3^7 + 3^7 = 2 \cdot (3^7)$

$\boxed{\text{practice 4}}$

$$\left\lfloor \frac{134}{Lcm(4,6)} \right\rfloor - \left\lfloor \frac{25}{Lcm(4,6)} \right\rfloor$$

$$= \left\lfloor \frac{134}{12} \right\rfloor - \left\lfloor \frac{25}{12} \right\rfloor$$

$$= 11 - 2 = 9$$

check

هذا غير مطلوب في الحل

all divisible on 4 = {28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, 104, 108, 112, 116, 120, 124, 128, 132}

divisible on 6 = {30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132}

وهكذا مباشرة نوجد جميع الأعداد بين 25/134 التي تقبل القسمة على 12

= {36, 48, 60, 72, 84, 96, 108, 120, 132

practice 6

$a_n = 3a_{n-1} + 2a_{n-2}$, $a_0 = 0, a_1 = 5$

char. eq.

$1r^2 - 3r - 2 = 0$

$a = 1, \quad b = -3, \quad c = -2$

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r_{1,2} = \frac{3 \pm \sqrt{9 - 4(1)(-2)}}{2}$$

$$= \frac{3 \pm \sqrt{17}}{2}$$

$\therefore r_1 = \frac{3 - \sqrt{17}}{2}$, $r_2 = \frac{3 + \sqrt{17}}{2}$

$r_1 \neq r_2$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$\because a_0 = 0 \Rightarrow \alpha_1 + \alpha_2 = 0 \Rightarrow \boxed{\alpha_1 = -\alpha_2}$

$\because a_1 = 5$

$\therefore \alpha_1 \left(\frac{3 - \sqrt{17}}{2}\right) + \alpha_2 \cdot \left(\frac{3 + \sqrt{17}}{2}\right) = 5$

$\because \alpha_1 = -\alpha_2$

$\therefore -\alpha_2 \left(\frac{3 - \sqrt{17}}{2}\right) + \alpha_2 \cdot \left(\frac{3 + \sqrt{17}}{2}\right) = 5$

practice 6 ع٢

$$\alpha_2\left(\frac{\cancel{-3}+\sqrt{17}}{2}+\frac{3+\sqrt{17}}{2}\right)=5$$

$$\frac{\cancel{2}\sqrt{17}}{\cancel{2}}\alpha_2=5$$

$$\alpha_2=\frac{5}{\sqrt{17}}\ ,\ \alpha_1=\frac{-5}{\sqrt{17}}$$

$$\therefore\ a_n=\frac{-5}{\sqrt{17}}\cdot\left(\frac{3-\sqrt{17}}{2}\right)^n+\frac{5}{\sqrt{17}}\left(\frac{3+\sqrt{17}}{2}\right)^n$$

practice 7

$$\overset{6^2\,6^1\,6^0}{(123)_6} = 1\times 6^2 + 2\times 6^1 + 3\times 6^0$$

$$= 36 + 12 + 3$$

$$= (51)_{10}$$

$$\begin{array}{r|l l}
9 & 51 & 6 \\
9 & 5 & 5 \\
 & 0 &
\end{array}$$

$$(123)_6 = (51)_{10} = (56)_9$$

practice 8

$<1, 1, 1, \ldots> \rightarrow \frac{1}{1-x}$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left(1 + x + x^2 + x^3 + \ldots\right)$$

$$= 0 + 1 + 2x + 3x^2 + \ldots$$

$A(x)$

$$= \left(\frac{1}{(1-x)^2}\right) = <1, 2, 3, 4, \ldots>$$

$B(x) = 1 + x + x^2 = <1, 1, 1, 0, 0, 0, \ldots>$

$C(x) = A(x) \cdot B(x)$

$= (1 + \cancel{1x} + x^2) * (1 + 2x + 3x^2 + 4x^3, 5x^4 \ldots>$

$= 1, 2 + 1, 3 + 2 + 1, 4 + 3 + 2, 5 + 4 + 3, \ldots$

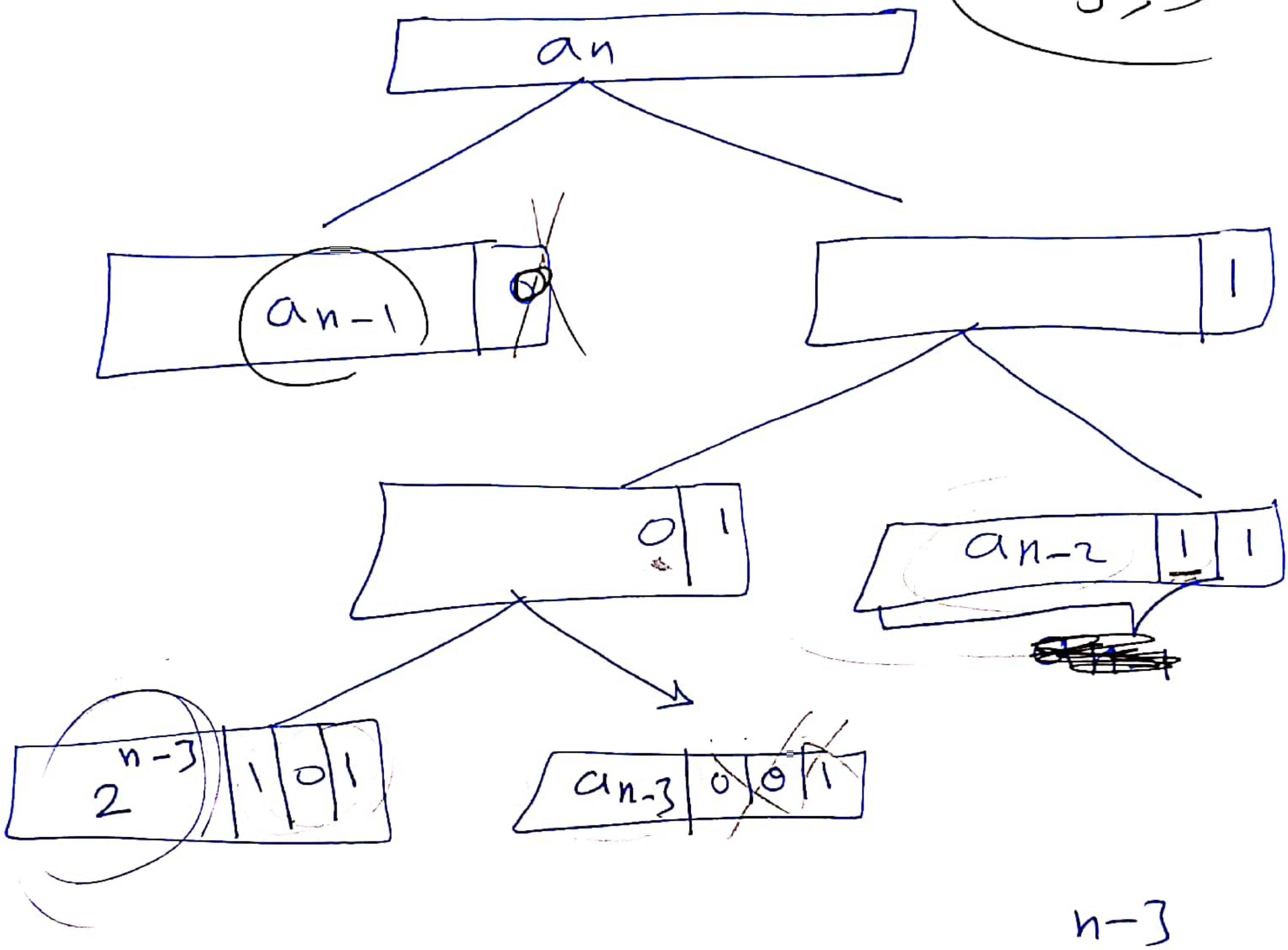
$= <1, 3, 6, 9, 12, 6+5+4 \ldots>$

$+2 \quad +3 \quad +3 \quad +3 \quad +3$

$= 1 + 3x + 6x^2 + 9x^4 + 12x^5 + 15x^6 \ldots$

Practice 9

حل ا لدكتور عقيل



$$a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}, \quad n \geq 4$$

$a_1 = 0$

$a_2 = 0$

$a_3 = 1$

$\{0, 1\}$

$\{00, 01, 10, 11\}$

$a_4 = 1 + 0 + 0 + 2^1 = 3$

$a_5 = a_4 + a_3 + a_2 + 2^2 = 8$

$ 3 \quad\; 1 \quad\; 0$

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$a_3 = 1$

(10) $n \geq 1$

$$P(n): \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

بـ1

Base: $P(1)$

$$L.H.S = \sum_{k=1}^{1} \frac{1}{k(k+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$$

$$R.H.S = \frac{1}{1+1} = \frac{1}{2}$$

$\therefore L.H.S = R.H.S \Rightarrow P(1)$ True

~~Ind~~ Inductive step let $P(k)$ True

$$P(r): \sum_{k=1}^{r} \frac{1}{k(k+1)} = \frac{r}{r+1}$$

proof: we want to prove: $P(r+1)$ True

Required المطلوب

$$\sum_{k=1}^{r+1} \frac{1}{k(k+1)} = \frac{r+1}{r+2} \text{ ??}$$

practice 10 Q4

$$L.H.S = \sum_{k=1}^{r+1} \frac{1}{k(k+1)}$$

$$= \sum_{k=1}^{r} \frac{1}{k(k+1)} + \sum_{k=r+1}^{r+1} \frac{1}{k(k+1)}$$

$$\frac{r}{r+1} + \frac{1}{(r+1)(r+2)}$$

$$\frac{r(r+2)}{(r+1)(r+2)} + \frac{1}{(r+1)(r+2)}$$

$$= \frac{r^2+2r+1}{(r+1)(r+2)} = \frac{\cancel{(r+1)}(r+1)}{\cancel{(r+1)}(r+2)}$$

$$= \frac{r+1}{r+2}$$

$\therefore$ by Math Ind. $P(r+1)$ True

$\therefore$ $P(n)$ True

Solving RR

Linear
Homogenous
Degree = K
Constant Coefficient

Linear → $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

$c_i \in R$, $c_k \neq 0$

$b_n = 3b_{n-1}^2$ (not linear)
$c_n = c_{n-1} + 5$ (not Homo.)
$d_n = d_{n-1} + d_{n-3}$ (degree = 3)
$e_n = n e_{n-1}$ (not const. coeff.)

???? (degree = 2)

$a_n = c_1 a_{n-1} + c_2 a_{n-2}$; $c_1, c_2 \in R$
$c_2 \neq 0$

Characteristic Eq.
$r^2 - c_1 r - c_2 = 0$

Two different roots $r_1 \neq r_2$
$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

one root $r_1 = r_2 = r$
$a_n = (\alpha_1 + \alpha_2 n) r^n$

$(\alpha_1 + \alpha_2 n) r^n$

Calculate from I.C

Ex. Fibonacci Seq.

$f_n = f_{n-1} + f_{n-2}$ RR

I.C. $f_0 = 0$, $f_1 = 1$

~~f_n~~

Char. eq. $r^2 - r - 1 = 0$

root $= \dfrac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \dfrac{1 \pm \sqrt{5}}{2}$

$f_n = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$

$f_0 = \alpha_1 + \alpha_2 = 0$ $\Rightarrow \alpha_2 = -\alpha_1$

$f_1 = \alpha_1 \left(\frac{1+\sqrt{5}}{2}\right) - \alpha_1 \left(\frac{1-\sqrt{5}}{2}\right) = 1$

$\alpha_1 \left(\frac{1+\sqrt{5} - 1 + \sqrt{5}}{2}\right) = 1$

$\alpha_1 = 1/\sqrt{5}$

$f_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2}\right)^n$

Ex(1) $a_n = 3a_{n-2}$, $a_0 = a_1 = 1$

char. eq.

$r^2 - \cancel{0r} - 3 = 0$

$r^2 - 3 = 0$

$r^2 = 3 \Rightarrow r = \pm\sqrt{3}$

$\therefore r_1 = -\sqrt{3}$, $r_2 = \sqrt{3}$

$r_1 \neq r_2$

$\therefore a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$a_n = \alpha_1 (-\sqrt{3})^n + \alpha_2 (\sqrt{3})^n$,

$\because a_0 = 1 \Rightarrow n = 0$, $a_n = 1$

$1 = \alpha_1 \cdot 1 + \alpha_2 \cdot 1$

$\boxed{\alpha_1 + \alpha_2 = 1} \Rightarrow \boxed{\alpha_2 = 1 - \alpha_1} \rightarrow (1)$

$\because a_1 = 1 \Rightarrow n = 1$, $a_n = 1$

$\boxed{1 = \alpha_1 \cdot (-\sqrt{3}) + \alpha_2 \cdot \sqrt{3}} \rightarrow (2)$

من (1) بالتعويض في (2)

$1 = -\sqrt{3}\,\alpha_1 + \sqrt{3}\,(1 - \alpha_1)$

الشكل العام

$a_n = c_1 a_{n-1} + c_2 a_{n-2}$

char. eq.

$r^2 - c_1 r^1 - c_2 = 0$

CamScanner الممسوحة ضوئيا بـ

Ex1 Q5

$$1 = -\sqrt{3}\,\alpha_1 - \sqrt{3}\,\alpha_1 + \sqrt{3}$$

$$1-\sqrt{3} = -2\sqrt{3}\,\alpha_1$$

$$\alpha_1 = \frac{1-\sqrt{3}}{-2\sqrt{3}} = \frac{\sqrt{3}-1}{2\sqrt{3}}$$

$$\alpha_2 = 1 - \frac{\sqrt{3}-1}{2\sqrt{3}}$$

مطلوب

$$a_n = \frac{\sqrt{3}-1}{2\sqrt{3}} \cdot (-\sqrt{3})^n + \left(1-\frac{\sqrt{3}-1}{2\sqrt{3}}\right)(\sqrt{3})^n$$

غير مطلوب

check: $a_0 = \cancel{\frac{\sqrt{3}-1}{2\sqrt{3}} + \left(1-\frac{\sqrt{3}-1}{2\sqrt{3}}\right)}$

$$a_1 = \cancel{\frac{\sqrt{3}-1}{2\sqrt{3}}(-\sqrt{3}) + \left(1-\frac{\sqrt{3}-1}{2\sqrt{3}}\right)(\sqrt{3})^n}$$

Ex: $a_n = 3a_{n-2}$ $\qquad a_0 = a_1 = 1$

char. $r^2 - 3 = 0$ $\longleftarrow a_n = 0a_{n-1} + 3a_{n-2}$

$r^2 - 0r - 3 = 0$

roots $= \pm\sqrt{3}$

$a_n = \alpha_1 (\sqrt{3})^n + \alpha_2 (-\sqrt{3})^n$

~~$a_0 = \alpha_1(\sqrt{3})^0 + \alpha_2(-\sqrt{3})^0$~~

$a_0 = \alpha_1 + \alpha_2 = 1 \Rightarrow \alpha_2 = 1 - \alpha_1$

~~$a_1 = \alpha_1 + \alpha_2$~~

$a_1 = \alpha_1(\sqrt{3}) + (1 - \alpha_1)(-\sqrt{3}) = 1$

$\alpha_1 - (1 - \alpha_1) = \frac{1}{\sqrt{3}}$

$2\alpha_1 = \frac{1}{\sqrt{3}} + 1$

$\alpha_1 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{3}}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{3}}$

$a_n = \frac{\sqrt{3}+1}{2\sqrt{3}} (\sqrt{3})^n + \left(1 - \frac{\sqrt{3}+1}{2\sqrt{3}}\right)(-\sqrt{3})^n$

Ex: $a_n = 6a_{n-1} - 9a_{n-2}$

I.C $a_0 = 1,\ a_1 = 1$

Char eq.

$r^2 - 6r + 9 = 0$

roots $= \frac{6 \pm \sqrt{36 - 4 \times 1 \times 9}}{2}$

$r_1 = r_2 = 3$

$a_n = (\alpha_1 + \alpha_2 n) \cdot 3^n$

I.C $\quad \therefore a_0 = 1$

$\alpha_1 \cdot 3^0 = 1 \Rightarrow \boxed{\alpha_1 = 1}$

$a_1 = 1$

$(\alpha_1 + \alpha_2 * 1) \times 3^1 = 1 \Rightarrow \alpha_1 + \alpha_2 = \frac{1}{3}$

$\alpha_2 = \frac{1}{3} - 1 = -\frac{2}{3}$

$\boxed{a_n = (1 - \frac{2}{3}n) \cdot 3^n}$ هذا المطلوب

Genarating Function (GF)

Sequence $\xrightarrow{as}$ Function

$a_0, a_1, a_2, a_3, \dots$

$GF(x) = a_0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots$

$= \sum_{k=0}^{\infty} a_k x^k$ (open form)

Sequence

$\langle 0, 0, 0, \dots \rangle \longrightarrow 0 + 0x + 0x^2 + 0x^3 + \dots = 0$

$\langle 1, 1, 1, \dots \rangle \xrightarrow{\text{place holder}} \underbrace{1 + x + x^2 + x^3 + \dots}_{\text{open form}} = \frac{1}{1-x}$ ✓ closed form

$\sum_{k=0}^{\infty} x^k = \frac{x^{n+1}-1}{x-1}$ $\boxed{|x|<1}$

$= \frac{1}{1-x}$

$|x|<1$

$\langle 1, -1, 1, -1, \dots \rangle \longrightarrow 1 - x + x^2 - x^3 + \dots = \frac{1}{1+x}$ closed form

$\langle 1, a, a^2, a^3, \dots \rangle \longrightarrow 1 + ax + a^2x^2 + a^3x^3 + \dots = \frac{1}{1-ax}$

Operation on GF.

Add, Multiply, Scale, Differentiate, shift

Scaling

$\langle 1, 1, 1, \dots \rangle \longmapsto \frac{1}{1-x}$ $*3$ $3 + 3x + 3x^2 + \dots$

$3(1 + x + x^2 \dots)$

$\langle 3, 3, 3, 3, \dots \rangle \longmapsto \frac{3}{1-x}$

$*2$ وتبديل x ب $-x$

$\langle 2, -2, 2, -2, \dots \rangle \longrightarrow \frac{2}{1+x}$

Add

$\langle 1,1,1,1,\dots\rangle \longrightarrow \frac{1}{1-x}$

$\langle 1,-1,1,-1,\dots\rangle \longrightarrow \frac{1}{1+x}$

$\boxed{\begin{aligned} &2+\cancel{0x}+2x^2+\cancel{0x^3}+2x^4+\cdots \\ &2(1+x^2+x^4+\cdots) \\ &\frac{2}{1-x^2} \end{aligned}}$

+ المطلوب

scaling $\langle 2,0,2,0,\dots\rangle \longrightarrow \frac{1}{1-x}+\frac{1}{1+x}=\frac{2}{1-x^2}$

$\div 2$

$\langle 1,0,1,0,\dots\rangle \longrightarrow \frac{1}{1-x^2}$

shift

$\langle 1,1,1,1,\dots\rangle \longrightarrow \frac{1}{1-x}$

$\langle 0,1,1,1,\dots\rangle \longrightarrow \frac{x}{1-x}$

$0+x+x^2+x^3\dots$
$x(1+x+x^2+\dots)$

$\langle 0,0,1,1,1,1,\dots\rangle \longrightarrow \frac{x^2}{1-x}$

$\langle 0,0,0,1,1,1,\dots\rangle \longrightarrow \frac{x^3}{1-x}$

what is GF for seq. $\langle 1,1,1,0,0,0,0,\dots\rangle$

$\langle 1,1,1,1,\dots\rangle \longrightarrow \frac{1}{1-x}$

$-\langle 0,0,0,1,1,\dots\rangle \longrightarrow \frac{x^3}{1-x}$

المطلوب $\langle 1,1,1,0,0,0,\dots\rangle \longrightarrow \frac{1-x^3}{1-x}$

$\frac{1}{1-x}-\frac{x^3}{1-x}$

وعلى طول مباشرة $1+x+x^2$

Differentiable

$\frac{1}{1-x} \rightarrow \langle 1,1,1,\dots\rangle$

$\frac{d}{dx}\left(\frac{1}{1-x}\right)=\frac{1}{(1-x)^2}$

$\frac{d}{dx}(1+x+x^2+x^3+\dots)$
$=0+1+2x+3x^2+\dots$

$\rightarrow \langle 1,2,3,4,\dots\rangle$

Scanned with CamScanner

Multiply

$A(x) = a_0 + a_1 x + a_2 x^2 + \dots$

$B(x) = B_0 + b_0 x + b_1 x^2 + \dots$

$C(x) = A(x) \cdot B(x)$

$c_0 = a_0 \cdot b_0 \rightarrow$ X~ يعني

$c_1 = a_0 b_1 + a_1 b_0$

$c_2 = a_0 b_2 + a_1 b_1 + a_2 b_0$

$c_k = a_0 b_k + a_1 b_{k-1} + \dots + a_k b_0$

$a_1 b_{k-1} \rightarrow$ مجموع K

Using GF in counting

$a + b + c = 17$

Count # solutions if $2 \le a \le 5$

$4 \le b \le 10$

$5 \le c \le 7$

Coefficient of x^{17} in the expansion of

$(x^2 + x^3 + x^4 + x^5) \cdot (x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}) \cdot (x^5 + x^6 + x^7)$

$\boxed{?}\ x^{17}$

How many ways to give 8 cookies for 3 kids such that each kid get at least 2 and no more than 4

$(x^2 + x^3 + x^4) \cdot (x^2 + x^3 + x^4) \cdot (x^2 + x^3 + x^4)$

$(x^2 + x^3 + x^4)^3$

Coeff. of x^8 =