CSC281: Discrete Math for Computer Science

Computer Science Department — First Semester 1442

King Saud University — Tutorial 4: Set Theory and Functions

Question 1. Use set builder notation to give a description of each of these sets:

a) $\{0, 1, 4, 9, 16, 25\}$ $\{x^2 \mid x \in N \wedge x \leq 5\}$

b) $\{0, 00, 10, 000, 010, 100, 110 \dots\}$ $\{x \mid x$ is a bit string with 0 as the rightmost bit$\}$

Question 2. What's the cardinality of each of these sets where a and b are distinct elements?

a) $\mathcal{P}(\{a, b, \{a, b\}\})$ let $A = \{a, b, \{a, b\}\}$ $|A| = 3$ $|P(A)| = 2^3 = 8$

b) $\mathcal{P}(\{\phi, a, \{a\}, \{\{a\}\}\})$ let $A = \{\phi, a, \{a\}, \{\{a\}\}\}$, $A = 4$ therefore $|P(A)| = 2^4 = 16$

c) $\mathcal{P}(\mathcal{P}(\phi))$ let $A = P(\phi) = \{\phi\}$, $|A| = 1$, $|P(A)| = 2^1 = 2$

Question 3. What's the truth set for the predicate F(x): x can fly, where the domain is the set of mammals. Is $\exists x F(x)$ true? Is $\forall x F(x)$? The truth set of F is $\{$bats$\}$. Since it is not empty, $\exists x F(x)$ is T. $\forall x F(x)$ is F since the truth set is not equivalent to the domain

Question 4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find the following:

a) $A \cup B$. All elements.

b) $A \cap B$. $\{A, b, c, d, e\}$

c) $A - B$. $\{A, b, c, d, e\}$

d) $B - A$. $\{f, g, h\}$

Question 5. Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$:

a) by showing each side is a subset of the other side.

b) using a membership table.

Question 6. Determine whether f is a function from the set of all bit strings to the set of integers if:

0 Possibly S: 1, 2, 5, -1

be for 2 or

a) f(S) is the position of a 0 bit in S. Not. each preimage can possibly be assigned to more than one image.

b) f(S) is the number of 1 bits in S. Function.

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c) f(S) is the smallest integer i such that the ith bit of S is 1 and f(S) $= 0$ when S is the empty string, the string with no bits. Not. Some elements in the domain are undefined.

Question 7. Let $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from **R** to **R**. Find the following:

$f+g = x^2+x+3$

$fg = x^3+2x^2+x+2$

a) $f \circ g$ $f(g) = f(x+2) = (x+2)^2 + 1 = x^2 + 4x + 4 + 1 = x^2 + 4x + 5$

b) $g \circ f$ $g(f)$ $g(x^2+1) = (x^2+1) + 2 = x^2 + 3$

Question 8. How many bytes are required to encode n bits of data where n equals

Byte → bits

a) 4? $\lceil 4/8 \rceil = 1$

b) 10? $\lceil 10/8 \rceil = 2$

c) 500? $\lceil 500/8 \rceil = 63$

d) 3000? $\lceil 3000/8 \rceil = 375$

Question 9. Show that if a set S has cardinality m, where m is a positive integer, then there is a one-to-one correspondence (*bijection*) between S and the set $\{1, 2, \dots, m\}$. Next, call that function f, will f be invertible?

Question 10. For each of the following functions, specify whether they are one-to-one (*injections*), onto (*surjections*) and/or one-to-one correspondence (*bijections*):

a) Let A be the students in discrete mathematics class, and Y is the possible grades $\{A, B, C, D, F\}$. $f : A \to Y$ such that $f(a) = y$ means student a got grade y. Note, every grade was taken by at least one student.

b) Let $f : Z \to Z$ where $f(x) = 3x$. — one-one only

but from $\mathbb{R} \to \mathbb{R}$ bi (1-1 correspondence)

c) Let $f : Z^+ \to Z$ where $f(x) = x^2$.

1-1
not onto
(5 don't have preimage)

$f(x) = 3x$

$\mathbb{Z}$	$\mathbb{Z}$
3	3
2	2
1	1
0	0
-1	-1
-2	-2

a-)

more than one student can get the same grade, hence, it is not 1-1. Since every grade is taken by at least one student, then it is onto. (The codomain is the same as the range)

5)

SECOND PART Let $x \in \overline{A} \cup \overline{B} \cup \overline{C}$

Using the definition of the union, x is the union of the sets if x is in one of the sets (or both).

$$x \in \overline{A} \vee x \in \overline{B} \vee x \in \overline{C}$$

Using the definition of the complement, x is in the complement of the set when x is not in the set:

$$\neg(x \in A) \vee \neg(x \in B) \vee \neg(x \in C)$$

Use De Morgan's law for propositions:

$$\neg(x \in A \wedge x \in B \wedge x \in C)$$

Using the definition of the intersection, x is in the intersection of two sets when it is both sets.

$$\neg(x \in A \cap B \cap C)$$

Using the definition of the complement, x is in the complement of the set when x is not in the set:

$$x \in \overline{A \cap B \cap C}$$

We have then shown $\overline{A} \cup \overline{B} \cup \overline{C} \subset \overline{A \cap B \cap C}$

CONCLUSION We obtained $\overline{A \cap B \cap C} \subseteq \overline{A} \cup \overline{B} \cup \overline{C}$ and $\overline{A} \cup \overline{B} \cup \overline{C} \subseteq \overline{A \cap B \cap C}$, thus the two sets have to be equal.

$$\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$$

(b) If x is an element, then 1 represents that the element is in the set and 0 represents that the element is not in the set.

A	B	C	$A \cap B \cap C$	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}}$	$\overline{\mathbf{A}} \cup \overline{\mathbf{B}} \cup \overline{\mathbf{C}}$
0	0	0	0	1	1	1	**1**	**1**
0	0	1	0	1	1	0	**1**	**1**
0	1	0	0	1	0	1	**1**	**1**
0	1	1	0	1	0	0	**1**	**1**
1	0	0	0	0	1	1	**1**	**1**
1	0	1	0	0	1	0	**1**	**1**
1	1	0	0	0	0	1	**1**	**1**
1	1	1	1	0	0	0	**0**	**0**

Since the last two columns have the same values, the two expressions are equal.

$$\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$$