1. [Marks 14]

Circle the correct option.

a. For an inverse of a function to exist, the function must be:

1. One-to-One
2. Onto
3. One-to-One correspendence
4. Neither

b. If function $f : \mathbb{R} \to \mathbb{R}$, where $f(x) = 3x - 5$, then $f^{-1}(x)$ is given by,

1. 1/(3x − 5)
2. (x + 5)/3
3. Does not exist since f is not one-to-one
4. None of the above

c. A function $f : X \to Y$ is one-to-one if

1. $f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in X$. $\forall x_1 >$
2. $f(x_1) = f(x_2)$ then $x_1 = x_2$ for all $x_1, x_2 \in X$.
3. $f(x_1) = f(x_2)$ for all $x_1, x_2 \in X$.
4. None of the above

ok

d. Suppose we have 100 sets: $A_1, A_2, \ldots, A_{100}$. If $A_1 \subset A_2, A_2 \subset A_3, \cdots, A_{99} \subset A_{100}$. Suppose the cardinality of set $|A_k| = k + 1$. Then $\left|\bigcup_{k=1}^{100} A_k\right|$ is:

1. 99. so $A_{100} = 100 + 1 = 101$
2. 100.
3. 101.
4. 102.

$A \subset B \rightarrow$ A has less elements than B and those element are in B as well

e. Let the set be A= {a, b, c, {a,b}} then which of the following is *false*?

1. {a, b} ∊ A.
2. a ∊ A.
3. {a} ∊ A. {a} is not an element of A ; (element ∊ set) it's a set
4. b, c ∊ A.

f. If A is {{Ø}, {Ø, {Ø}}}, then the power set of A has how many elements?

1. 2.
2. 4. 2^2 2^2
3. 6.
4. 8.

g. Let statement S(x,y) = "x is a student in y university". Then, the statemen "Every university in the world has students" is expressed as:

1. $\exists x \exists y S(x,y)$.
2. $\forall x \exists y S(x,y)$. $\forall y \exists x$ $\forall y \exists x\ S(x,y)$
3. $\exists y \forall x S(x,y)$.
4. $\forall y \exists x S(x,y)$.

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2. **[Marks 10]**
Determine if $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

$$(p \wedge q) \rightarrow (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$$
$$\equiv (\neg p \vee \neg q) \vee (p \vee q)$$
$$\equiv (\neg p \vee p) \vee (\neg q \vee q)$$
$$\equiv T \vee T$$
$$\equiv T$$

3. **[Marks 11]**
Use Induction to show that for every positive integer n,
$$\sum_{i=1}^{n} i \cdot (i+1) = \frac{n(n+1)(n+2)}{3}.$$ Show all the details.

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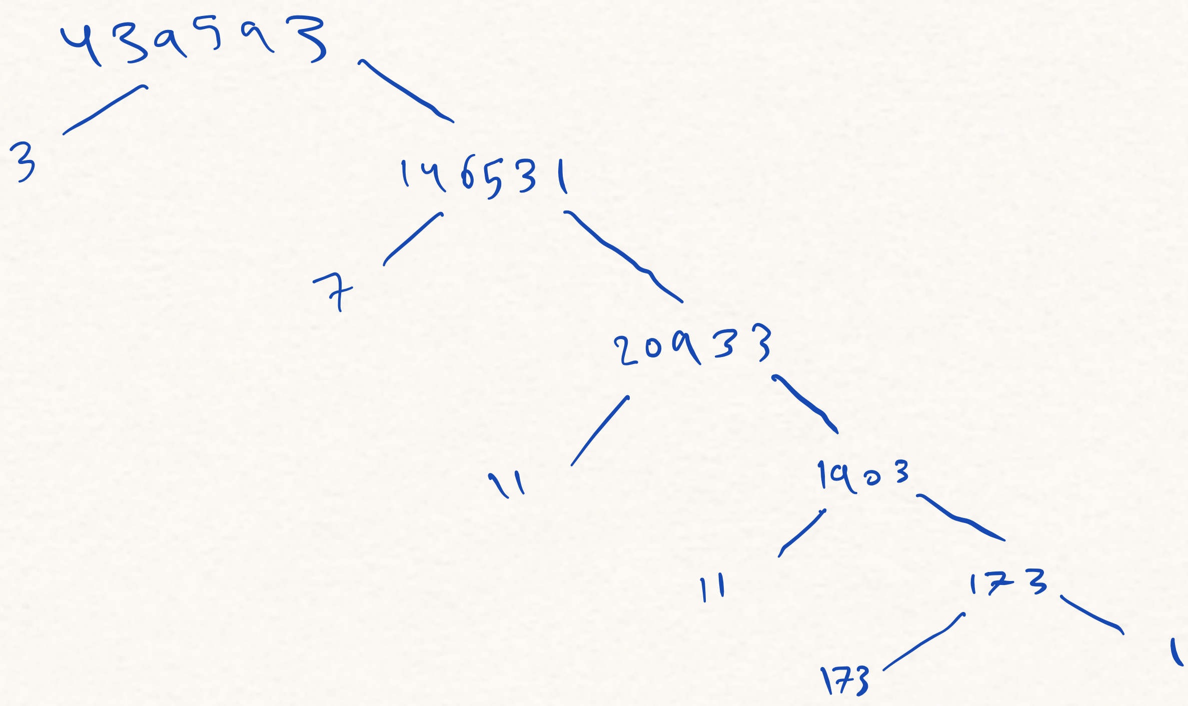
4. [Marks 10]
Find the prime factorization of the number 439593.



$\sqrt{173} = 13$

so if 13 or less

does not devide 173

$\Rightarrow$ 173 is prime

$439597 = 3 \cdot 7 \cdot 11^2 \cdot 173$

5. [Marks 10]
Consider the following sequence: $a_1 = 6, a_2 = 3, a_3 = \frac{3}{2}, a_4 = \frac{3}{4}, \ldots$ Derive the formula to determine a_k (k-th term) of the sequence. Show all details.

$a_1 = 6 \,, r = \frac{1}{2} \Rightarrow a_k = 6\left(\frac{1}{2}\right)^{n-1}$

6. [Marks 15]

Calculate the summation of $\sum_{i=1}^{10}\sum_{j=1}^{20}(2i-j)$. Show details.

$$\ast \quad \sum_{j=1}^{20}(2i-j) = 2\sum_{j=1}^{20} i - \sum_{j=1}^{20} j$$

$$= 2(20i) - \frac{20(21)}{2}$$

$$= 40i - 210$$

$$\sum_{i=1}^{10} 40i - 210 = 40\sum_{i=1}^{10} i - \sum_{i=1}^{10} 210$$

$$= 40\,\frac{10(11)}{2} - (10)(210)$$

$$= 2200 - 2100 = \boxed{100}$$

8. [Marks 10]
Determine if the numbers: 22, 35, and 63 are pairwise relatively prime. ×

Since $\gcd(35,63) = 7 \neq 1$

then 22, 35 and 63 are not pairwise relatively prime

9. [Marks 10]
Prove that for any integer n, the integer $n(n+1)$ is even.

$2k+1$

Case 1 : n is even

let $n = 2k \quad ; k \in \mathbb{Z}$

$$n(n+1) = 2k(2k+1)$$
$$= 4k^2 + 2k$$
$$= 2(\underbrace{2k^2+k}_{m \in \mathbb{Z}})$$
$$= 2m \text{ (even)}$$

Case 2 : n is odd

let $n = 2k+1 \quad ; k \in \mathbb{Z}$

$$n(n+1) = (2k+1)(2k+2)$$
$$= 4k^2 + 4k + 2k + 2$$
$$= 4k^2 + 6k + 2$$
$$= 2(\underbrace{2k^2+3k+1}_{m \in \mathbb{Z}}) = 2m \text{ (even)}$$

7. **[Marks 10]**
In a school, one bell rings every 30 minutes, and the other bell rings every 35 minutes. If both bells ring together at 8:15 am. At what time will they ring together again? Argue your point.

30 min - bill

8:15 → 8:45 → 9:15 → 9:45 → 10:15 → 10:45 ...

so whenever the 35min-clock reaches x:15 or x:45
the will meet

35 min - bill

8:15 → 8:50 → 9:25 → 10:00 → 10:35 → 11:10 → $\boxed{11:45}$

So the will meet on 11:45 am