

1. [Marks 20]

Consider the propositions,

$p$ : You drive over 120 km per hour.

q: You get caught by Saher camera.

Express the following propositions using  $p$  and  $q$  and logical connectives, i.e.  $\wedge$  (and),  $\vee$  (or), and  $\neg$  (not).

A	You drive over 120 km per hour, <u>but</u> you do not get caught by Saher camera.	$P \wedge \neg Q$
B	You will get caught by Saher camera if you drive over 120 km per hour.	$P \rightarrow Q$
C	If you do not drive over 120 km per hour, then you will not get caught by Saher camera.	$\neg P \rightarrow \neg Q$
D	Driving over 120 km per hour is sufficient for getting caught by Saher camera.	$P \rightarrow Q$
E	You did get caught by Saher camera, but you did not drive over 120 km per hour.	$Q \wedge \neg P$

**2. [Marks 10]**

Show that  $(p \wedge q) \rightarrow (p \vee q)$  is a tautology.

**3. [Marks 10]**

Consider the set  $S = \{\emptyset, a, 2, (b, 3)\}$ . Write the subset of the power set of  $S$  where each element has a cardinality of 3.

**4. [Marks 10]**

Let  $f(x) = 1/x^2$  and  $g(x) = x/\sqrt{x+1}$ . Compute:

a.  $(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{\sqrt{x+1}}\right) = \frac{\frac{x}{\sqrt{x+1}}}{\left(\frac{x}{\sqrt{x+1}}\right)^2} = \frac{x+1}{x^2}$

b.  $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x^2}\right) = \frac{1}{\left(\frac{1}{x^2}\right)^2} = x^4$

**5. [Marks 10]**

Find the prime factorization of the number 197351.

$$= 7 \cdot 11^2 \cdot 233$$

**6. [Marks 10]**

You are given the sequence  $a_{35} = 45, a_{36} = 53, a_{37} = 61$  and  $a_{38} = 69$ . Find the

sum  $\sum_{k=10}^{20} a_k$ . Show all the details.

7. [Marks 10]

Determine if the numbers: 22, 35, and 63 are pairwise relatively prime.

if gcd for all pairs is  $= 1$  then they are

Pairwise relatively prime

**8. [Marks 10]**

Calculate the summation,  $\sum_{i=1}^n \prod_{j=1}^i c$ . Show all the details.

$$\uparrow \quad \underbrace{c \cdot c \cdot c \dots}_i$$

$$\rightarrow \sum_{i=1}^n C^i = \frac{C^{n+1} - C}{C - 1}$$

Q2:  $(p \wedge q) \longrightarrow (p \vee q)$

tenth table

Q3:  $S = \{\emptyset, a, 2, (b, 3)\}$

Subset of  $P(S) \Rightarrow \text{Cardinality} = 3$

$$\{ \{ \emptyset, a, 2 \}, \{ \emptyset, a, (b, 3) \}, \{ a, 2, (b, 3) \}, \{ \emptyset, 2, (b, 3) \} \}$$

Q6:  $a_{35} = 45$ ,  $a_{36} = 53$ ,  $a_{37} = 61$ ,  $a_{38} = 69$

$$\sum_{k=10}^{20} a_k = a_{10} + a_{11} + a_{12} + \dots + a_{20}$$

$$a_k = a_1 + 8(k-1)$$

$K = \$5$

$$45 = a_1 + 8(34)$$

$$a_1 = 45 - 8(34)$$

$$a_1 = -227$$

$$* a_k = -227 + 8(k-1)$$

$$\begin{aligned} \therefore \sum_{k=10}^{20} -227 + 8(k-1) &= \sum_{k=10}^{20} -227 + 8 \sum_{k=10}^{20} (k-1) \\ &\downarrow \\ &= -227 \cdot 11 + 8 \sum_{k=10}^{20} k - \sum_{k=10}^{20} 8 \\ 20-10+1 &\quad \quad \quad \uparrow \\ &= 8 \left[ \sum_{k=10}^{20} k - \sum_{k=1}^{10} k \right] - 8 \cdot (20-10+1) \\ &= 8 \left[ \frac{1}{2} \cdot 20 \cdot 21 - \frac{1}{2} \cdot 9 \cdot 10 \right] - 8 \cdot 11 \\ &= -2497 + 1232 = -1265 \end{aligned}$$