

# CSC281: Discrete Math for Computer Science

Computer Science Department  
King Saud University

First Semester 1442

## Tutorial 6: Integer Representations and Algorithms

**Question 1.** Convert each of the following integers from a binary expansion to octal and hexadecimal expansions.

a)  $(11111)_2$

Octal:  $\begin{array}{ccc} 11 & 111 & \\ 21 & 421 & \\ (37)_8 & & \end{array}$  hexadecimal:  $\begin{array}{ccc} 1 & 1111 & \\ 1 & 3721 & \\ (1F)_{16} & & \end{array}$

b)  $(100000001)_2$

Octal:  $\begin{array}{ccc} 1 & 000 & 000 & 001 \\ & (1001)_8 & & \end{array}$  hexa:  $\begin{array}{ccc} 10 & 0000 & 0001 \\ & (201)_{16} & & \end{array}$

**Question 2.** Find the sum and the product of each of these pairs of numbers. Express your answers as a binary expansion.

a)  $(1000111)_2$  ,  $(1110111)_2$

Sum:  $\begin{array}{r} 1000111 \\ + 1110111 \\ \hline 10111110 \end{array}$  Product:  $\begin{array}{r} 1000111 \\ \times 1110111 \\ \hline 1000111 \\ 0000111 \\ 0000111 \\ 0000111 \\ 1000111 \\ \hline 1000111111 \end{array}$

b)  $(11101111)_2$  ,  $(10111101)_2$

**Question 3.** Use Algorithm 5 to find  $7644 \bmod 645$ .

Handwritten calculation for  $7644 \bmod 645$  using Algorithm 5, showing the binary expansion of 7644 and the subtraction of multiples of 645.

**Question 2.** Find the sum and the product of each of these pairs of numbers. Express your answers as a binary expansion.  
a)  $(1000111)_2$  ,  $(1110111)_2$   
b)  $(11101111)_2$  ,  $(10111101)_2$   
Solution: a)  $1000111 + 1110111 = 10111110$   
b)  $11101111 + 10111101 = 101011100$

The last question solution is correct as the expansion should be done to the power not to the mod

So,  $644 = (10\ 1000\ 0100)_2$

and the full trace is here:

When  $a_i = 1$  then  $x$  is first multiplied by the power and reduced modulo 645.  
Then on each iteration the power is multiplied by itself and reduced modulo 645.

i	$a_i$	x	power
0	0	1	$(7*7) \bmod 645 = 49$
1	0	1	$(49*49) \bmod 645 = 466$
2	1	$(1*466) \bmod 645 = 466$	$466^2 \bmod 645 = 436$
3	0	466	$436^2 \bmod 645 = 466$
4	0	466	$466^2 \bmod 645 = 436$
5	0	466	466
6	0	466	436
7	1	$(466*436) \bmod 645 = 1$	466
8	0	1	436
9	1	$436 \bmod 645$	466

The final value of  $x$  is 436

b)  $\begin{array}{r} 11101111 \\ + 10111101 \\ \hline 110101100 \end{array}$

Handwritten calculation for the sum and product of  $(11101111)_2$  and  $(10111101)_2$ , showing the binary expansion of the sum and the product.