Permutations and combinations:

1. How many bit strings of length 12 contain
   a) exactly three 1s?   C(12,3)
   b) at most three 1s?   C(12,3)+ C(12,2)+ C(12,1)+ C(12,0)
   c) at least three 1s?   C(12,3)+ C(12,4)+ C(12,5)+…+ C(12,12)
   d) an equal number of 0s and 1s?   C(12,6)

2. How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other? P(10,10) x P(11,6) = 10! x 11! / 5!

3. How many ways are there for four men and five women to stand in a line so that
   a) all men stand together?   4! 6!
   b) all women stand together?   5! 5!
   c) all men stand together and all women stand together? 2(4! 5!)

4. Seven women and nine men are on the faculty in the mathematics department at a school.
   a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee? C(16,5)-C(9,5)
   b) How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee? C(16,5)-C(9,5)-C(7,5)

5. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women? C(10,3) x C(15,3)

6. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men? C(15,6) + C(15,5)xC(10,1) + C(15,4)xC(10,2)

7. How many ways are there to assign three jobs to five employees if each employee can be given more than one job? 5^3

8. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose:

    a) a dozen croissants?   C(6+12-1,12)

    b) two dozen croissants with at least two of each kind? C(6+12-1,12)

    c) two dozen croissants with no more than two broccoli croissants?
    C(6+24-1,24) – C(6+21-1,21)

9. How many solutions are there to the equation: $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$, where x_i is a nonnegative integer such that:

    a) $x_i > 1$ for i = 1, 2, 3, 4, 5, 6?   C(6+17-1,17)

    b) $x_1 \le 5$?   C(6+29-1,29) – C(6+23-1,23)

10. How many different strings can be made from the letters in MISSISSIPPI, using all the letters? 11!/(4!4!2!)

11. How many different strings can be made from the letters in AARDVARK, using all the letters, if all three As must be consecutive? 6!/2!

The Binomial Theorem:

1. How many terms are there in the expansion of $(x + y)^{100}$ after like terms are collected? 101 terms

2. What is the coefficient of x^9 in $(2 - x)^{19}$? $C(19,9)$ x 2^{10} x -1^9

3. What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x - 3y)^{200}$? $C(200,99)$ x 2^{101} x -3^{99}

4. Use the binomial theorem to find the coefficient of x^6y^9 in the expansion of $(5x^2 + 2y^3)^6$? j= 9/3. → $C(6,3)$ x 5^3 x 2^3

5. Use the binomial theorem to find the coefficient of x^2y^{15} in the expansion of $(5x^2 + 2y^3)^6$? j= 15/3. → $C(6,5)$ x 5 x 2^5

6. What is the coefficient of x^{12} in $(3x - 5/x^2)^{30}$
$C(30,j) \cdot (3x)^{30-j} \cdot (-5x^{-2})^j$
$C(30,j) \cdot 3^{30-j} \cdot -5^j \cdot (x)^{30-3j}$
So, 12=30-3j, and j=6
Solution: C(30,6) x 3^{24} x -5^6