CSC281: Discrete Math for Computer Science

Computer Science Department   First Semester 1442

King Saud University   Tutorial 5: Sequences + Divisibility and Modular Arithmetic

Question 1.   Show that the sequence a_n is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if:

a)$a_n = -n + 2$

b)$a_n = 5(-1)^n - n + 2$

Question 2.   Find the solution to each of these recurrent relations and initial conditions. Use iterative aproach.

a)$a_n = 3a_{n-1}$, $a_0 = 2$

b)$a_n = a_{n-1} + 2$, $a_0 = 3$

Question 3.   Suppose that the number of bacteria in a colony triples every hour.

a) Set up a recurrence relation for the number of bacteria after n hours have elapsed.

b) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

Question 4.   Let $\{a_n\}$ be an arithmetic sequence:

a) What's the value of a_{30} if the value of the initial term is -23 and the common difference is 7?

b) Express the sequence as a recurrence relation.

c) Express a summation for this sequence, and find a closed form formula for it.

Question 5.   Show that if $a|b$ and $b|a$,where a and b are integers, then $a = b$ or $a = -b$.

Question 6.   Show that if a, b, and c are integers, where $a \neq 0$ and $c \neq 0$,such that $ac|bc$,then $a|b$.

Question 7.   Show that if $n|m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.

Question 8.   Find counterexamples to each of these statements about congruences.

a)If $ac \equiv bc \pmod{m}$,where a,b,c,and m are integers with $m \geq 2$, then $a \equiv b \pmod{m}$.

b)If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, where a, b, c, d, and m are integers with c and d positive and $m \geq 2$,then $ac \equiv bd \pmod{m}$.