

Exercise 1

Question: Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

- We solved this before. We found: $\sum_{n=6}^8 36^n - 26^n$
- $P_6 = \text{\#passwords with 1 digit} + \text{\#passwords with 2 digits} + \dots + \text{\#passwords with 6 digits}$
- $P_6 = \binom{6}{1}10 \cdot 26^5 + \binom{6}{2}10^2 \cdot 26^4 + \binom{6}{3}10^3 \cdot 26^3 + \dots + \binom{6}{6}10^6$
- $P_6 = \sum_{k=1}^6 \binom{6}{k}10^k \cdot 26^{6-k}$
- $P_6 + P_7 + P_8 = \sum_{n=6}^8 \sum_{k=1}^n \binom{n}{k}10^k \cdot 26^{n-k}$

Exercise 2

Question: There are five people.

1. How many ways to photograph them in groups of three?
 - Solve using permutations: $P(5,3) = 5 \cdot 4 \cdot 3 = 60$
 - Solve using combinations: $C(5,3) \cdot 3! = 5 \cdot 4 \cdot 3 = 60$
2. Suppose out of these 5 people, there is one person must be in each picture?
 - Solve using permutations: $P(4,2) \cdot 3 = 4 \cdot 3 \cdot 3 = 36$
 - Solve using combinations: $C(4,2) \cdot 3! = 4 \cdot 3 \cdot 3 = 36$