Answer the following questions:

1. Show that the following conditional statements **(¬p ∧ (p ∨ q)) → q** is a tautology by using:

a) Truth tables.

b) Applying a chain of logical equivalences.

2. Determine the truth value of the following propositions for the given universe of discourse.

	Universe of discourse	Truth value
$\exists x\ (x+1>2x)$	$\mathbb{Z}$	T
$\forall x\ (x+1>2x)$	$\mathbb{Z}$	F
$\exists x\ (x^2 = 2)$	$\mathbb{R}$	T
$\exists x\ \exists y\ (x + y = x - y)$	$\mathbb{Z}$	T
$\forall x\ \exists y\ (x + y = x - y)$	$\mathbb{Z}$	T
$\forall y\ \exists x\ (x + y = x - y)$	$\mathbb{Z}$	F
$\forall x\ \exists y\ (x - 2y = 0)$	$\mathbb{R}$	T
$\forall x\ (x<10) \rightarrow \forall y\ (y<x) \rightarrow y<9$	$\mathbb{Z}$	T
$\forall x\ (x<10) \rightarrow \forall y\ (y<x) \rightarrow y<9$	$\mathbb{R}$	F

3. For the following statements, write "True" or "False":

a. $\forall x\ (P(x) \wedge Q(x)) \equiv \forall x\ P(x) \wedge \forall x\ Q(x)$ T

b. $\forall x\ (P(x) \vee Q(x)) \equiv \forall x\ P(x) \vee \forall x\ Q(x)$ F

c. $\exists x\ (P(x) \wedge Q(x)) \equiv \exists x\ P(x) \wedge \exists x\ Q(x)$ F

d. $\exists x\ (P(x) \vee Q(x)) \equiv \exists x\ P(x) \vee \exists x\ Q(x)$ T

4. Let L(x,y)="x likes y". Express the following statements using predicates and quantifiers:

a. Everyone likes Khaled.

b. There is someone who Fahad doesn't like.

c. Everyone likes someone.

d. There is someone whom everyone likes.

e. There is someone whom no one likes.