**CSC281: Discrete Math for Computer Science**

| | |
|---|---|
| Computer Science Department | First Semester 1442 |
| King Saud University | Tutorial 1: Propositional and Predicate Logic |

---

**Question 1.** Find a proposition for the given truth table:

| p | q | $(p \land q)$ | $\neg p \land q$ | $(\neg q \to p)$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | F |
| F | T | F | T | T |
| F | F | F | F | F |

**Question 2.**

1. Write a proposition with three variables, which is true iff only one variable is true, and false otherwise. $(p \land \neg q \land \neg r) \lor (\neg p \land q \land \neg r) \lor (\neg p \land \neg q \land r)$

   for ex 1) $(p=T, q=F, r=F)$ it should be true or 2) $(p=F, q=T, r=F)$ then it should be true or 3) $(p=T, q=T, r=F)$ then it should be false

2. Write a proposition with three variables, which is never true (unsatisfiable).
   $(p \land \neg p) \lor (q \land \neg q) \lor (r \land \neg r)$

**Question 3.** Determine whether $p \to (q \to r)$ and $(p \to q) \to r$ are equivalent.

**Question 4.** Determine whether the following propositions are tautologies, if yes, proof them using propositional equivalences:

1. $(q \land (p \to \neg q)) \to \neg p$
2. $((p \to q) \land \neg p) \to \neg q$ No. let p: false, q true. (Put a counter example)

**Question 5.** Write a proposition equivalent to $p \lor \neg q$ that uses only $p, q, \neg$, and the connective $\land$.

$\neg(\neg p \land q)$

**Question 6.** Let the variable x represent students and y represent courses, with: F(x): x is a freshman, A(x): x is a part-time student, and T(x,y): x is taking y. Translate these statements into English:

1. $\exists x(A(x) \land \neg F(x))$ Some part time students are not a freshmen
2. $\forall x\ \exists y\ T(x, y)$ Every student is taking some course
3. $\exists x\ \forall y\ T(x, y)$ There is a student who takes all courses

$\mathbb{Z}^+$ $\geq 0$

**Question 7.** Let $p(m, n)$ mean $m \leq n$, where n and m are non-negative integers. Determine whether the following statements are true:

$m \leq n$ $m, n \geq 0$

1. $\forall n\ p(0, n)$ ✓
2. $\exists n\ \forall m\ p(m, n)$ $m \leq n$ ✗ → [illegible]
3. $\forall m\ \exists n\ p(m, n)$ $m \leq n$ ✓

**Question 8.** Let the variable x denote people, and S(x): x is smart, T(x): x is tall, and W(x): x is worried. Write these statements using the given predicates and any needed quantifiers:

1. Some people are not worried.
   $\exists x\ \neg W(x)$

Non negative integer is the set of all integers without the negative integers. It includes 0 (zero) and other positive integers like +1, +2, +3... All the integers in this set are greater than or equal to 0.

Positive integers is the set of all integers greater than 0 (zero) and includes +1, +2, +3... All the integers in this set are strictly greater than 0.

$\forall x\,(T(x) \to S(x))$

2.All tall people are smart.

3.No smart people are worried. $\forall x\,(S(x) \to \neg W(x))$

4.Some people are tall and smart, but they are worried. $\exists x\,(T(x) \land S(x) \land W(x))$

5.If a person is smart, then that person is not worried.

$\forall x\,(S(x) \to \neg W(x))$

In Rules we use $\forall$, even if it didn't say exceplity.

$\forall$ usualy with $\to$

$\exists$ Usually with $\land$

Q3

$$p \to (q \to r) \overset{?}{\equiv} (p \to q) \to r$$

| $p$ | $q$ | $r$ | $p \to q$ | $(p \to q) \to r$ | $q \to r$ | $p \to (q \to r)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | T | F | T | F | F | F |
| T | F | T | F | T | T | T |
| T | F | F | F | T | T | T |
| F | T | T | T | T | T | T |
| F | T | F | T | $\boxed{F}$ | F | $\boxed{T}$ |
| F | F | T | T | T | T | T |
| F | F | F | T | $\boxed{F}$ | T | T |

They are not equivlant.

Q4:-

$$(q \land (p \to \neg q)) \to \neg p \overset{?}{\equiv} T$$

$$(q \land (p \to \neg q)) \to \neg p$$
$$\equiv (q \land (\neg p \lor \neg q)) \to \neg p$$
$$\equiv ((q \land \neg p) \lor (q \land \neg q)) \to \neg p$$
$$\equiv ((q \land \neg p) \lor F) \to \neg p$$
$$\equiv (q \land \neg p) \to \neg p$$
$$\equiv \neg (q \land \neg p) \lor \neg p$$
$$\equiv (\neg q \lor p) \lor \neg p$$
$$\equiv \neg q \lor (p \lor \neg p)$$
$$\equiv \neg q \lor T$$
$$\equiv T$$