CSC281: Discrete Math for Computer Science

Computer Science Department — First Semester 1442

King Saud University — Tutorial 4: Set Theory and Functions

Question 1. Use set builder notation to give a description of each of these sets:

a) $\{0, 1, 4, 9, 16, 25\}$

b) $\{0, 00, 10, 000, 010, 100, 110 \dots\}$

Question 2. What's the cardinality of each of these sets where a and b are distinct elements?

a) $\mathcal{P}(\{a, b, \{a, b\}\})$

b) $\mathcal{P}(\{\phi, a, \{a\}, \{\{a\}\}\})$

c) $\mathcal{P}(\mathcal{P}(\phi))$

Question 3. What's the truth set for the predicate F(x): x can fly, where the domain is the set of mammals. Is $\exists x F(x)$ true? Is $\forall x F(x)$?

Question 4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find the following:

a) $A \cup B$.

b) $A \cap B$.

c) $A - B$.

d) $B - A$.

Question 5. Show that if A, B, and C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$:

a) by showing each side is a subset of the other side.

b) using a membership table.

Question 6. Determine whether f is a function from the set of all bit strings to the set of integers if:

a) f(S) is the position of a 0 bit in S.

b) f(S) is the number of 1 bits in S.

c) f(S) is the smallest integer i such that the ith bit of S is 1 and f(S) $= 0$ when S is the empty string, the string with no bits.

Question 7. Let $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $\mathbf{R}$ to $\mathbf{R}$. Find the following:

a) $f \circ g$

b) $g \circ f$

Question 8. How many bytes are required to encode n bits of data where n equals

a) 4?

b) 10?

c) 500?

d) 3000?

Question 9. Show that if a set S has cardinality m, where m is a positive integer, then there is a one-to-one correspondence (*bijection*) between S and the set $\{1, 2, \dots, m\}$. Next, call that function f, will f be invertible?

Question 10. For each of the following functions, specify whether they are one-to-one (*injections*), onto (*surjections*) and/or one-to-one correspondence (*bijections*):

a) Let A be the students in discrete mathematics class, and Y is the possible grades $\{A, B, C, D, F\}$. $f : A \to Y$ such that $f(a) = y$ means student a got grade y. Note, every grade was taken by at least one student.

b) Let $f : Z \to Z$ where $f(x) = 3x$.

c) Let $f : Z^+ \to Z$ where $f(x) = x^2$.