The Foundations: Logic and Proofs

Chapter 1

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Propositional Logic

Section 1.1

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Section Summary

Propositions

Connectives

- Negation
- AND
- OR
- Exclusive OR
- Implication; contrapositive, inverse, and converse
- Biconditional

Truth Tables

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Propositions

A ***proposition*** is a declarative sentence that is either true or false.

- Examples:
  a) Riyadh is the capital city of KSA. Proposition
  b) Jeddah is the capital city of KSA. Proposition
  c) Sit down! Not proposition
  d) What time is it? Not proposition
  e) $1 + 0 = 1$ Proposition
  f) $0 + 0 = 2$ Proposition
  g) $x + 1 = 2$ Not proposition
  h) $x + y = z$ Not proposition

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Propositional Logic

Constructing Propositions

- Propositional Variables: *p*, *q*, *r*, *s*, …
- Compound Propositions: constructed from logical connectives and other propositions
  - Negation $\neg$
  - Conjunction (AND) $\wedge$
  - Disjunction (OR) $\vee$
  - Exclusive OR $\oplus$
  - Implication $\rightarrow$
  - Biconditional $\leftrightarrow$

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Negation

The *negation* of a proposition p is denoted by $\neg p$ and has this truth table:

p	$\neg p$
T	F
F	T

Example:

- if p denotes "The weather is hot."
- then $\neg p$ denotes "It is not the case that the weather is hot."
  - or more simply "The weather is not hot."

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Conjunction (AND)

The *conjunction* of propositions p and q is denoted by $p \land q$ and has this truth table:

p	q	$P \land q$
T	T	T
T	F	F
F	T	F
F	F	F

Example:

- If p denotes "I am at home." and q denotes "It is raining."
- then $p \land q$ denotes "I am at home <u>and</u> it is raining."

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Disjunction (OR)

The *disjunction* of propositions p and q is denoted by $p \vee q$ and has this truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example:

- If p denotes "I am at home." and q denotes "It is raining."
- then $p \vee q$ denotes "I am at home <u>or</u> it is raining."

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The Exclusive Or

In English "or" has two distinct meanings.

- "Inclusive Or" - In the sentence "Students who have taken CSC113 or Math151 may take this class," we assume that students need to have taken one of the prerequisites, but may have taken both. This is the meaning of disjunction. For $p \vee q$ to be true, either one or both of p and q must be true.
- "Exclusive Or" - When reading the sentence "Soup or salad comes with this entrée," we do not expect to be able to get both soup and salad. This is the meaning of Exclusive Or (Xor). In $p \oplus q$, one of p and q must be true, but not both. The truth table for $\oplus$ is:

p	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

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Implication

If p and q are propositions, then $p \to q$ is a *conditional statement* or *implication* which is read as "if p, then q" and has this truth table:

p	q	$P \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: If p denotes "I am at home." and q denotes "It is raining." then $p \to q$ denotes "If I am at home then it is raining."

In $p \to q$, p is the *hypothesis* and q is the *conclusion*.

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Understanding Implication

One way to view the logical conditional is to think of an obligation or contract.

- "If you get 100% on the final, then you will get A+."

If the student gets 100% and does not get A+, then the student can say that the professor has broken the pledge.

This corresponds to the case where p is true and q is false.

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Different Ways of Expressing $p \rightarrow q$

- **if** p, **then** q
- **if** p, q
- p **implies** q
- p **only if** q
- q **if** p
- q **when** p
- q **unless** $\neg p$

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Converse, Contrapositive, and Inverse

From $p \rightarrow q$ we can form new conditional statements .

- The **converse** is $q \rightarrow p$
- The **contrapositive** is $\neg q \rightarrow \neg p$
- The **inverse** *is* $\neg p \rightarrow \neg q$

Example: Find the converse, inverse, and contrapositive of "If $\underbrace{\text{it snows}}_{p}$, $\underbrace{\text{the traffic moves slowly}}_{q}$."

Solution:

converse: if $\underbrace{\text{the traffic moves slowly}}_{q}$, then $\underbrace{\text{it snows}}_{p}$

contrapositive: if $\underbrace{\text{the traffic doesn't move slowly}}_{\neg q}$, then $\underbrace{\text{it doesn't snow}}_{\neg p}$.

inverse: If $\underbrace{\text{it doesn't snow}}_{\neg p}$, $\underbrace{\text{the traffic moves quickly}}_{\neg q}$.

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Biconditional

If p and q are propositions, then we can form the *biconditional* proposition $p \leftrightarrow q$, read as "p if and only if q."
$p \leftrightarrow q$ has this truth table:

p	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

If p denotes "I am at home." and q denotes "It is raining." then $p \leftrightarrow q$ denotes "I am at home <u>if and only if</u> it is raining."

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Truth Tables of Compound Propositions

Construct the truth table for $(p \vee \neg q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

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Truth Tables of Compound Propositions

Construct the truth table for $(p \vee q) \rightarrow \neg r$

p	q	r	$\neg r$	$p \vee q$	$(p \vee q) \rightarrow \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

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