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KING SAUD UNIVERSITY

COLLEGE OF COMPUTER & INFORMATION SCIENCES

DEPT OF COMPUTER SCIENCE

CSC281 Discrete Mathematics for CS Students
Second Semester 1439/1440 AH (Spring 2019)
First midterm Examination: Sun 24.02.2019 C.E. (Time: 5:30-7:00 pm)
Instructor: Dr. Aqil Azmi

[redacted]

Name: [redacted] **S/N:** [redacted]

1. **[Marks 12]**
Determine the Truth of the following propositions for the given universe of discourse. NOTE: $\mathbb{N}$ (set of non-negative integers); $\mathbb{Z}$ (set of all integers); and $\mathbb{R}$ (set of real numbers).

1,2,3,4,5   -2,-1,0,1,2

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{R}$
$\forall x \exists y\ 2x - y = 0$	true	true	true
$\forall x \exists y\ x - 2y = 0$	false	true ✗	true
$\forall x\ (x < 10) \rightarrow (\forall y\ y < x \rightarrow y < 9)$	false ✗	true	true ✗
$\forall x \exists y\ [(y > x) \wedge \exists z\ (y + z = 100)]$	false	false ✗	false ✗

أي رقم اضربه 2 وضعه على الأقل رقم
اضربه منه مكون الثاني .

$x=1$ $y=\frac{1}{2}$

-5

2. **[Marks 8]**
Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

$$
\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) \\
&\equiv \neg p \vee \neg q \vee p \vee q \\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) \\
&\equiv T \vee T \\
&\equiv T
\end{aligned}
$$

✓

$A = \{1, 2\}$
$B = \{2, 3, 7\}$

3. **[Marks 10]**
Let $A \times B = \{(1,2),(1,3),(1,7),(2,2),(2,3),(2,7)\}$. Write the following sets:
(a) $A \cup B = \{1, 2, 3, 7\}$
(b) $A \cap B = \{2\}$
(c) $A - B = \{1\}$

4. **[Marks 10]**
Find the prime factorization of the number 196885.

ANSWER	$196885 = 5 \times 13^2 \times 233$

$5 \times 13^2 \times 233$

$\boxed{\sqrt{196885} = 443}$

$2 \nmid 196885$
$3 \nmid 196885$
$5 \mid 196885 \rightarrow \frac{196885}{5} = 39377$
$5 \nmid 39377$
$7 \nmid 39377$
$11 \nmid 39377$
$13 \mid 39377 \rightarrow \frac{39377}{13} = 3029$
$13 \mid 3029 \rightarrow \frac{3029}{13} = 233$

5. **[Marks 5]**
Determine if the numbers: 22, 35, and 63 are pairwise relatively prime. (gcd of them = 1)

$\boxed{gcd(63, 35)}$
$63 = \boxed{1} * 35 + \boxed{28}$
$35 = \boxed{1} * 28 + \boxed{7} \leftarrow gcd(63, 35) = 7$
$28 = \boxed{4} * 7 + \boxed{0}$

∴ so they are not pairwise relativey prime cause $gcd(63, 35) = 7 \neq 1$

$\boxed{gcd(63, 22)}$
$63 = \boxed{2} * 22 + \boxed{19}$
$22 = \boxed{1} * 19 + \boxed{3}$
$19 = \boxed{6} * 3 + \boxed{1} \leftarrow gcd(63, 22) = 1$

$gcd(35, 22)$
$35 = \boxed{1} * 22 + \boxed{13}$
$22 = \boxed{1} * 13 + \boxed{9}$
$13 = \boxed{1} * 9 + \boxed{4}$
$9 = \boxed{2} * 4 + \boxed{1} \leftarrow gcd(35, 22) = 1$

★

6. [Marks 15=5+10]

Solve the following summation. NOTE: $\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6$.

a. Calculate the sum: $\sum_{k=1}^{3}\left(\sum_{m=k}^{3} mk\right)$

$$\sum_{K=1}^{3}\left(\sum_{m=1}^{3} mK - \sum_{m=1}^{-K} mK\right)$$

$$\sum_{K=1}^{3}\left(K\sum_{m=1}^{3} m - K\sum_{m=1}^{-K} m\right)$$

$$\sum_{K=1}^{3}\left(K * \frac{3(4)}{2} - K * \frac{-K(-K+1)}{2}\right)$$

$$\sum_{K=1}^{3}\left(6K + \frac{K^2(-K+1)}{2}\right)$$

$$\sum_{K=1}^{3}\left(6K + \frac{(-K^3)+K^2}{2}\right)$$

$$2\sum_{K=1}^{3} -K^3 + K^2 + 3K$$

$$2\left(\sum_{K=1}^{3} -K^3 + \sum_{K=1}^{3} K^2 + 3\sum_{K=1}^{3} K\right) = 2\left(\left(\frac{3(4)}{2}\right)^2 + \frac{3(4)(7)}{2} + \frac{3(3)(4)}{2}\right)$$

$$= 43432$$

-3

b. Find the general formula for the summation: $\sum_{k=1}^{n}\left(\sum_{m=k}^{n} k\right)$.

ANSWER	Formula = $\cancel{\frac{1}{2}\left(nK(K+1) + K(K+1)(2K+1)\right)}$

$$= \frac{1}{2}\left(n^2(n+1) + n(n+1)(2n+1)\right)$$

$$= \frac{1}{2}\left(3n^3 + 4n^2 + n\right)$$

$$\sum_{K=1}^{n}\left(\sum_{m=1}^{n} K - \sum_{m=1}^{-K} K\right)$$

$$\sum_{K=1}^{n}\left(nK + K^2\right)$$

$$n\sum_{K=1}^{n} K + \sum_{K=1}^{n} K^2$$

$$\cancel{n * \frac{K(K+1)}{2} + \frac{K(K+1)(2K+1)}{2}}$$

$$\cancel{\frac{1}{2}\left(nK(K+1) + K(K+1)(2K+1)\right)}$$

$$n * \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{2}$$

Cancel

$$\frac{1}{2}\left[n^2(n+1) + n(n+1)(2n+1)\right]$$

$$\frac{1}{2}\left(n^3+n^2 + (n^2+n)(2n+1)\right)$$

$$\frac{1}{2}\left(n^3+n^2+2n^3+2n^2+n^2+n\right)$$

$$\frac{1}{2}\left(3n^3+4n^2+n\right)$$

-4

7. [Marks 10]

Given a sequence. If $\sum_{i=1}^{5} a_i = 7$, and $\sum_{i=6}^{12} a_i = 25$. Calculate $\sum_{i=1}^{12}(1-a_i)$.

ANSWER	Sum = -20

$$\sum_{i=1}^{12}(1-a_i) = \sum_{i=1}^{12} 1 - \sum_{i=1}^{12} a_i$$

$$= \sum_{i=1}^{12} 1 - \left(\sum_{1}^{5} a_i + \sum_{6}^{12} a_i\right)$$

$$= 12 - (7+25)$$

$$= -20$$

✓

8. [Marks 10]

Express the gcd(65, 1326) using the linear combination of its arguments.

ANSWER	gcd(65, 1326) = 13

gcd(65, 1326)

$1326 = \boxed{20} * 65 + \boxed{26}$

$65 = \boxed{2} * 26 + \boxed{13}$ ← gcd

$26 = \boxed{2} * 13 + \boxed{0}$

✓ -5

9. [Marks 20]
Solve using the Chinese Remainder Theorem: $x \equiv 2 \bmod 6$ and $x \equiv 4 \bmod 13$.

ANSWER	$x =$ 56

①

$X \equiv 2 \bmod 6$

$X \equiv 4 \bmod 13$

Pairwise relativly Prime

$gcd(13, 6)$

$13 = \boxed{?} * 6 + \boxed{?}$ gcd

②

$m = m_1 * m_2$

$m = 6 * 13$

$m = 78$

③ $M_1 = \frac{m}{m_1} = \frac{78}{6} = 13$

$M_2 = \frac{m}{m_2} = \frac{78}{13} = 6$

④

$M_1 y_1 \equiv 1 \bmod m_1$

$13 y_1 \equiv 1 \bmod 6$

$1 y_1 \equiv 1 \bmod 6$

$y_1 = 1$

$M_2 y_2 \equiv 1 \bmod m_2$

$6 y_2 \equiv 1 \bmod 13$

$y_2 = 11$

$0 < X < 78$

$X = (a_1 M_1 y_1 + a_2 M_2 y_2) \bmod m$

$X = [(2 \times 13 \times 1) + (4 \times 6 \times 11)] \bmod 78$

$X = [26 + 204] \bmod 78$

$X = 290 \bmod 78$

$X = 56 \bmod 78$

in genera solution

$X = 56 + 78K \; ; K \in Z$