1. **[Marks 10]**

Compute $2^{2000} \bmod 991$. Show all the steps.

2. **[Marks 15]**

Suppose we have 20 white socks, 20 black socks, 20 brown and 20 red socks. We have three boys and they all want to wear the same color socks (e.g. all three want brown socks). Assume the room is dark and the socks are all mixed up. (a) How many socks do they need to pick up so to guarantee that all 3 boys were the same colored sock. (b) How many socks do they need to pick up so to guarantee that all 3 boys wear blacks socks.

3. **[Marks 10]**

Prove that if n is an integer and $n^2 + 5$ is odd, then n is even using:

   a. An indirect proof.
   b. A proof by contradiction.

4. **[Marks 10]**

Prove that if n is not divisible by 5, then n^2 leaves a remainder of 1 or 4 when divided by 5.

5. **[Marks 10]**

Use induction to show that $2^n < n!$ for all $n \geq 4$.

6. **[Marks 10]**

Give a recursive definition of the sequence $a_n, n = 1, 2, 3, \ldots$ if

   a. $a_n = 3n^2$.
   b. $a_n = 1/n$.

7. **[Marks 5]**

Consider the statement, “for every prime number p, the number $p+2$ is also a prime”. Give a counter example.

8. **[Marks 20]**

How many positive integers less than 500 that are:

   a. Are divisible by 3.
   b. Are divisible by 3 but not by 4.
   c. Are divisible by either 3 or 4.
   d. Are divisible by 3 but not by 9.