CSC281: Discrete Math for Computer Science

Computer Science Department — First Semester 1442

King Saud University — Tutorial 3: Valid Arguments and Proofs

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Question 1. What rule of inference is used in each of these arguments?

a) Kangaroos live inAustralia and are marsupials. Therefore, kangaroos are marsupials.

b) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.

Question 2. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a) If I eat ice cream, then I will be sick. I take medicine if I am sick. I did not take medicine.

b) Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean.

Question 3. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x(\neg P(x) \wedge Q(x) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is true.

Question 4. For each of these arguments determine whether the argument is valid or invalid and explain why.

a) Everyone enrolled in the university has lived in a Riyadh. Mai has never lived in a Riyadh. Therefore, Mai is not enrolled in the university.

b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

c) Lina likes all action movies. Lina likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.

d) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is a positive real number, then a is a positive real number.

e) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$, then $a \neq 0$.

Question 5. What is wrong with this argument? Let H(x) be "x is happy." Given the premise $\exists x H(x)$, we conclude that H(Lola). Therefore, Lola is happy.

Question 6. Use a direct proof to show that the sum of two odd integers is even.

Question 7. Let $x \in Z$. If $x^2 - 6x + 5$ is even, then x is odd. Show that using proof by contrapositive.

Question 8. Suppose $a \in Z$. If a^2 is even, then a is even. Show that using proof by contradiction.

Question 9. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

a) a proof by contraposition.

b) a proof by contradiction.