show that these conditional statements are a Tautology using truth table

① $(p \land q) \to p$
② $p \to (p \lor q)$
③ $\neg p \to (p \to q)$
④ $(p \land q) \to (p \to q)$
⑤ $\neg(p \to q) \to p$
⑥ $\neg(p \to q) \to \neg q$

①

p	q	$p \land q$	$(p \land q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

②

p	q	$p \lor q$	$p \to (p \lor q)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

③

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

④

p	q	$p \land q$	$p \to q$	$(p \land q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

⑤

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

6

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

عال 281

التاريخ: / / هـ Using Law الموضوع:

① $(p \wedge q) \rightarrow p$

$\Rightarrow \neg(p \wedge q) \vee p$

$\Rightarrow (\neg p \vee \neg q) \vee p$

$\Rightarrow (\neg p \vee p) \vee (\neg q \vee p)$

$\Rightarrow T \vee (\neg q \vee p)$

$\Rightarrow T$

② $p \rightarrow (p \vee q)$

$\Rightarrow \neg p \vee (p \vee q)$

$\Rightarrow (\neg p \vee p) \vee (\neg p \vee q)$

$\Rightarrow T \vee (\neg p \vee q)$

$\Rightarrow T$

③ $\neg p \rightarrow (p \rightarrow q)$

$\Rightarrow \neg(\neg p) \vee (p \rightarrow q)$

$\Rightarrow \neg(\neg p) \vee (\neg p \vee q)$

$\Rightarrow p \vee (\neg p \vee q)$

$\Rightarrow (p \vee \neg p) \vee (p \vee q)$

$\Rightarrow T \vee (p \vee q)$

$\Rightarrow T$

④ $(p \wedge q) \rightarrow (p \rightarrow q)$

$\Rightarrow \neg(p \wedge q) \vee (p \rightarrow q)$

$\Rightarrow (\neg p \vee \neg q) \vee (\neg p \vee q)$

$\Rightarrow (\neg q \vee \neg p) \vee (\neg q \vee q)$

$\Rightarrow (\neg q \vee \neg p) \vee T$

$\Rightarrow T$

⑤ $\neg(p \rightarrow q) \rightarrow p$

$\Rightarrow \neg(\neg p \vee q) \rightarrow p$

$(p \wedge \neg q) \rightarrow p$

$\Rightarrow \neg(p \wedge \neg q) \vee p$

$(\neg p \vee q) \vee p \Rightarrow (\neg p \vee q) \vee (\neg p \vee p)$

$\Rightarrow T$

⑥ $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg(\neg p \vee q) \rightarrow \neg q$

$(p \wedge \neg q) \rightarrow \neg q$

$\neg(p \wedge \neg q) \vee \neg q$

$(\neg p \vee q) \vee \neg q$

$(\neg p \vee q) \vee (q \vee \neg q)$

$(\neg p \vee q) \vee T$

$\Rightarrow T$

التاريخ : / / هـ

(3) show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent

let $p(x)$ be "x spends more than 5 hours every weekday in class", where the domain of x is all students, Express in english

ⓐ $\exists x\, p(x)$, ⓑ $\forall x\, p(x)$, ⓒ $\exists x \neg p(x)$, ⓓ $\forall x \neg p(x)$

ⓐ there is Exist a studen who spends more than 5 hours every weekday in class.

ⓑ every students . .

ⓒ there a student that doesn't spends

ⓓ every student doesn't spends ---

---

let $p(x)$ be "x speaks Russian", $Q(x)$ be "x knows C++"
express, using quantifaction and logical Quantifirs

ⓐ there is a student who speak Russian and knows

$\exists x\, [p(x) \wedge Q(x)]$

ⓑ "   "   "   but doesn't know C++

$\exists x\, [P(x) \wedge \neg Q(x)]$

ⓒ Every student either speaks Russian or knows C++

$\forall x\, [P(x) \vee Q(x)]$

ⓓ No "   " knows "   and "

$\forall x\, [\neg P(x) \wedge \neg Q(x)]$

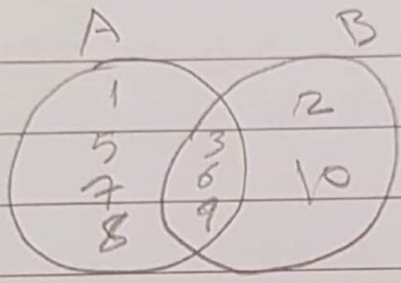
let $P(x,y)$ be "student x has taken class y"
Domain of x is all student in the class
and domain of y consists of all CS courses.
Express in quantification and logical connectives
ⓐ Exist a student x who has taken course y

ⓑ Every student x has taken at least one course y.

a) $\exists x\, \exists y\, P(x,y)$

b) $\forall x\, \exists y\, P(x,y)$

Find A and B if $A - B = \{1, 5, 7, 8\}$
$B - A = \{2, 10\}$
$A \cap B = \{3, 6, 9\}$



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let $A = \{a, b, c\}$
$B = \{x, y\}$
$C = \{1, 0\}$
Find $A \times B \times C$

$A \times B \times C = \{ax1, ax0, ay1, ay0, bx1, bx0, by1, by0, cx1, cx0, cy1, cy0\}$

$\frac{210}{5}$

Use chinese reminder theorem to find all sol. of the sys. of cong.

$x \equiv 1 \mod 2$

$x \equiv 2 \mod 3$

$x \equiv 3 \mod 5$

$x \equiv 4 \mod 11$

$m = 2 \times 3 \times 5 \times 11$

$= 330$

$\frac{330}{2}$ $M_1 = 165$

$\frac{330}{3}$ $M_2 = 110$

$\frac{330}{5}$ $M_3 = 66$

$\frac{330}{11}$ $M_4 = 30$

$M_1 = 165 \mod 2 \Rightarrow 165 \cdot 1 \equiv 1 \mod 2$

$y \equiv 1 \mod 2$ $\boxed{y_1 = 1}$

$M_2 = 110 \mod 3 \Rightarrow 110 \cdot 2 \equiv 1 \mod 3$

$\boxed{y_2 = 2}$

$M_3 = 66 \mod 5 \Rightarrow 66 \cdot 3 \equiv 1 \mod 5$

$\boxed{y_3 = 1}$

$M_4 = 30 \mod 11 \Rightarrow 30 \cdot 4 \equiv 1 \mod 11$

$8y \equiv 1 \mod 5$ $\boxed{y_4 = 7}$

التاريخ: / / هـ

الموضوع:

if the product of 2 integers is $2^7 \cdot 3^8 \cdot 5^2 \cdot 7^{11}$, and their gcd is $2^3 \cdot 3^4 \cdot 5$ what is their lcm?

$2^3 \cdot 3^4 \cdot 5 \cdot 7^{11}$

show that if $a|b$ and $b|a$ where a and b are integers, then $a = b$ or $a = -b$.

$a = bk \quad : k \in \mathbb{Z}$

$b = ac \quad : c \in \mathbb{Z}$

$\cancel{ab} = \cancel{ab}ck$

$1 = ck$

$c = \pm 1$ or $k = \pm 1$

$a = b(-1) \Rightarrow a = -b$

or

$a = b(1) \Rightarrow a = b$

find gcd (3000, 197) using Eculid's Algorith

3000, 197 = 1

$(3000, 197) = \boxed{15} \times 197 + \boxed{45}$

$(197, 45) = 4 \times 45 + 17$

$(45, 17) = 2 \times 17 + 11$

$(17, 11) = 1 \times 11 + 6$

$(11, 6) = 1 \times 6 + 5$

$(6, 5) = 1 \times 5 + \boxed{1} \longrightarrow (5, 1) = 5 \times 1 + 0$

(ii) gcd (270, 192) = 6

$(270, 192) = 1 \times 192 + 78$

$(192, 78) = 2 \times 78 + 36$

$(78, 36) = 2 \times 36 + \boxed{6}$

$(36, 6) = 6 \times 36 + 0$

التاريخ: / / الموضوع:

Determine these function from Z to Z are one to one

① $f(n) = n^2+1$

② $f(n) = n^3$

③ $f(n) = \lceil \frac{n}{2} \rceil$

① $n = f(n)^2+1$

$f(n)^2 = n-1$

$f(n) = \pm\sqrt{n-1}$

not 1-1

② $n = f^3(n)$

$f(n) = \sqrt[3]{n}$

1-1

③ not 1-1

cause ① ②

$\frac{1}{2}$, $\frac{2}{2} = 1$

② $a_n = a_1 + (n-1)d$

$a_n = 15 + (n-1)(-7)$

$a_n = 22 - 7n$

① → 3.2^{n-1}

k	1	2	3	4
3^k	3	6	12	24

For each list on Z provide a formula rule

① 3, 6, 12, 24, ...

② 15, 8, 1, -6, -13, ...

find all formula $\mathbb{Z} < 30$ that are relatively prime to 30,

$1, 7, 11, 13, 17, \ldots, 29$

Sum of Geometric Progression

(a) $\sum_{j=0}^{8} 3 \cdot 2^j = 1533$

formula

$\frac{r^{n+1} - 1}{r - 1}$

(b) $\sum_{j=1}^{8} 2^j = 510$

find the Prime factorization of 899

$899 = 31 \times 29$

$\sqrt{899} =$

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$f \circ g$ $\quad g \circ f$

let $f(x) = ax + b$, $g(x) = cx + d$ $\left\{ \begin{array}{l} a, b, c, d \\ \text{are constant} \end{array} \right.$

What are the necessary and sufficient condition for $f \circ g = g \circ f$

$f(g(x)) = a(cx + d) + b \quad = acx + ad + b$

$g(f(x)) = c(ax + b) + d \quad = cax + cb + d$

$ad + b = bc + d$

Compute (i) $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j) = 78$

(ii) $\sum_{i=0}^{2} \sum_{j=0}^{3} i \cdot j \quad - 18$