Answer the following questions from the book (section 6.1):

3. A multiple-choice test contains 10 questions. There are four possible answers for each question.

a) How many ways can a student answer the questions on the test if the student answers every question? 4^{10}

b) How many ways can a student answer the questions on the test if the student can leave answers blank? 5^{10}

7. How many different three-letter initials can people have? 26^3

8. How many different three-letter initials with none of the letters repeated can people have? 26x25x24

9. How many different three-letter initials are there that begin with an A? 26^2

16. How many strings are there of four lowercase letters that have the letter x in them? $26^4 - 25^4$ or (25^3X4 + 25^2X6 + 25X4 + 1)

25. How many PINs of three decimal digits that

a) do not contain the same digit three times? 10^3-10=990

b) begin with an odd digit? 5x10x10=500

c) have exactly two digits that are 4s? 3x9=27

44. In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if:

a) the bride must be in the picture?
6x(1x9x8x7x6x5)=P(9,5)x6

b) both the bride and groom must be in the picture?
6x5x(1x1x8x7x6x5)=P(8,4)x6x5

c) exactly the bride and not the groom in the picture? Part(a) – Part(b) <u>or</u> P(8,5)x6

d) exactly one of the bride and the groom is in the picture? 2 x Part(c)

67. How many ways are there to arrange the letters a, b, c, and d such that a is not followed immediately by b? 4! – 3! = 18

The Pigeonhole Principle:

1. There are six professors teaching the introductory discrete mathematics class at a university. The same final exam is given by all six professors. If the lowest possible score on the final is 0 and the highest possible score is 100, how many students must there be to guarantee that there are two students with the same professor who earned the same final examination score? 101 x 6 = 606 +1 = 607 students

2. Show that among any group of five integers, there are two with the same remainder when divided by 4. There are only n possible remainders [0,1,..,n-1] when an integer is divided by n. By the pigeonhole principle, if we have n+1 remainders, then at least two must be the same.
In general, among any group of d+1 integers there are two integers with exactly the same remainder when they are divided by d.

3. How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee that at least one pair of these numbers add up to 16? 5

4. Assuming that no one has more than 1,000,000 hairs on their head and that the population of Riyadh was 7.67 million in 2018, show there had to be at least eight people in Riyadh in 2018 with the same number of hairs on their heads. $\left\lceil \frac{7.67M}{1M} \right\rceil = 8$

Question 1: Suppose we have the following letters: [**A,B,C,D,E**]. Answer the following:

1. How many words of length 5, if duplicate letters are allowed? 5^5
2. How many words of length 4, if duplicate letters are allowed? 5^4
3. How many words of length 5, if duplicate letters are not allowed? 5!=P(5,5)
4. How many words of length 4, if duplicate letters are not allowed? 5x4x3x2=P(5,4)
5. How many words of length 3, if duplicate letters are not allowed? 5x4x3=P(5,3)
6. How many words of length 5, if duplicate letters are not allowed and A,B together? 4! x 2! → 2! For (AB and BA)
7. How many words of length 5, if duplicate letters are not allowed and A,B are not together? 5! – (4! x 2!)
8. How many words of length 5, if duplicate letters are not allowed and A,B,C together? 3! X 3! → Second 3! For (ABC, ACB, …, CBA)

Question 2: How many positive integers between 1 and 100 that are:

1. divisible by 6? 6k <= 100 → $\left\lfloor \frac{100}{6} \right\rfloor$ = 16.
2. divisible by 12 **and** 18? $\left\lfloor \frac{100}{lcm(12,18)} \right\rfloor$ = 2. Check 36,72
3. divisible by 12 **or** 18? $\left\lfloor \frac{100}{12} \right\rfloor + \left\lfloor \frac{100}{18} \right\rfloor - \left\lfloor \frac{100}{lcm(12,18)} \right\rfloor$ = 11
4. divisible by 12 **or** 18, and for positive integers between 150 and 500? Count numbers between 1-500 (minus) numbers between 1-149

$$\left\lfloor \frac{500}{12} \right\rfloor + \left\lfloor \frac{500}{18} \right\rfloor - \left\lfloor \frac{500}{lcm(12,18)} \right\rfloor - (\left\lfloor \frac{149}{12} \right\rfloor + \left\lfloor \frac{149}{18} \right\rfloor - \left\lfloor \frac{149}{lcm(12,18)} \right\rfloor)$$