King Saud University

College of Computer & Information Sciences

Dept of Computer Science

$\frac{98}{100}$

CSC281 Discrete Mathematics for CS Students
First Semester 1440/1441 AH (Fall 2019)
First midterm Examination: Sun 20.10.2019 C.E. (Time: 6:00-7:30 pm)
Instructor: Prof. Aqil Azmi

Name: [redacted] **ID:** [redacted]

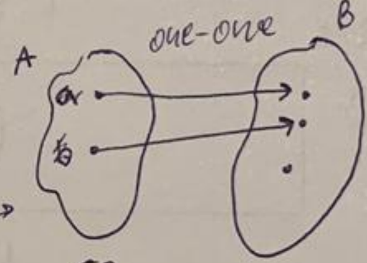
1. **[Marks 10]**
Determine the Truth of the following propositions for the given universe of discourse. NOTE: $\mathbb{N}$ (set of non-negative integers); $\mathbb{Z}$ (set of all integers); and $\mathbb{R}$ (set of real numbers).

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{R}$
$\forall x\ (x^2 \geq x)$	T	~~F~~ T	$(\frac{1}{2})$ F
$\forall x \exists y\ (x+y=10)$	(10) F	T	T
$\exists x \forall y\ (x+y=10)$	F	F	F ✓

2. **[Marks 10]**
Sets A, B. $f : A \rightarrow B$. Write the *mathematical* definition of one-one function.

$\forall a,b \in A$ $f(a) = f(b) \Longleftrightarrow a = b$ ✓

-2



→ كل نقطة من B يدخل عليها سهم ، واحد على الأكثر، لا بد يطلع عليها شيء .

3. **[Marks 10]**
Determine if this statement $(p \rightarrow q) \vee (q \rightarrow r)$ is a tautology? YES / NO.

$(p \rightarrow q) \vee (q \rightarrow r)$

$(\neg p \vee q) \vee (\neg q \vee r)$

$(q \vee \neg q) \vee (\neg p \vee r)$

$T \vee (\neg p \vee r) \equiv \underline{\underline{T}}$ ✓

4. **[Marks 10]**
Let the set $A = \{\varnothing, \{a\}, (1,2)\}$. Write $P(A)$, the powerset of A. $2^{|A|} = 2^{|3|} = 8$

ANSWER	$P(A) = \{\phi, \{\phi\}, \{\{a\}\}, \{(1,2)\}, \{\{a\}, (1,2)\}, \{\phi, \{a\}\}, \{\phi, (1,2)\}, \{\phi, \{a\}, (1,2)\}\}$

5. **[Marks 15=5+10]**
Calculate the following summation.

a. Calculate the sum: $\sum_{\substack{p \\ \text{prime}<30}} p$

ANSWER	Sum of primes < 30 = $2+3+5+7+11+13+17+19+23+29 = 129$

b. Find general formula for the summation: $\sum_{k=1}^{n}\left(\sum_{m=0}^{k} r^m\right)$. Show all details.

$r=2$

$1+2+4+8$

$k=8$
$n=4$
$r=2$

ANSWER	Formula = $\frac{1}{r-1}\left(\frac{r^{n+2}-r^2}{r-1} - n\right)$

56

$$\sum_{k=1}^{n}\sum_{m=0}^{k} r^m \rightarrow \sum_{k=1}^{n}\left(\frac{r^{k+1}-1}{r-1}\right)$$

$$\frac{1}{r-1}\left(\sum_{k=1}^{n} r^{k+1} - 1\right) \rightarrow \frac{1}{r-1}\left(\sum_{k=1}^{n} r^k \cdot r - \sum_{k=1}^{n} 1\right)$$

$$\frac{1}{r-1}\left(r\sum_{k=1}^{n} r^k - \sum_{k=1}^{n} 1\right) \rightarrow \frac{1}{r-1}\left(r\left(\frac{r^{n+1}-r}{r-1}\right) - n\right)$$

$$\frac{1}{r-1}\left(\frac{r^{n+2}-r^2}{r-1} - n\right)$$

35

23415

7350

910

6. **[Marks 15]**
Express the gcd(10!, 57085) as a linear combination of its arguments.

ANSWER	gcd(10!, 57085) = -373 * 10! + 23711 * 57085

$10! = 3628800$

~~3596355~~

$\gcd(10!, 57085)$

$10! = \boxed{63} \times 57085 + \boxed{32445}$

$57085 = \boxed{1} \times 32445 + \boxed{24640}$

$32445 = \boxed{1} \times 24640 + \boxed{7805}$

$24640 = \boxed{3} \times 7805 + \boxed{1225}$

$7805 = \boxed{6} \times 1225 + \boxed{455}$

$1225 = \boxed{2} \times 455 + \boxed{315}$

$455 = \boxed{1} \times 315 + \boxed{140}$

$315 = \boxed{2} \times 140 + \boxed{35}$

$\cancel{140 = \boxed{4} \times 35 + \boxed{0}}$

$35 = 1\cdot 315 - 2\cdot 140$
$= 1\cdot 315 - 2(1\cdot 455 - 1\cdot 315)$
$= -2\cdot 455 + 3\cdot 315$
$= -2\cdot 455 + 3(1\cdot 1225 - 2\cdot 455)$
$= 3\cdot 1225 - 8\cdot 455$
$= 3\cdot 1225 - 8(1\cdot 7805 - 6\cdot 1225)$
$= -8\cdot 7805 + 51\cdot 1225$
$= -8\cdot 7805 + 51(1\cdot 24640 - 3\cdot 7805)$
$= 51\cdot 24640 - 161\cdot 7805$
$= 51\cdot 24640 - 161(1\cdot 32445 - 1\cdot 24640)$
$= -161\cdot 32445 + 212\cdot 24640$
$= -161\cdot 32445 + 212\cdot(1\cdot 57085 - 1\cdot 32445)$
$= 212\cdot 57085 - 373\cdot 32445$
$= 212\cdot 57085 - 373(1\cdot 10! - 63\cdot 57085)$
$= -373\cdot 10! + 23711\cdot 57085$

7. **[Marks 10]**
Solve the congruent equation: $8x + 2 \equiv 5 \bmod 15$. Show all the steps.

ANSWER	$x = 6$

$8x \equiv 3 \bmod 15$

$x \equiv 6 \bmod 15$

unique solution

$x = 6 + 15k \ : \ k \in \mathbb{Z}$ (General Solution)

relatively prime

$\gcd(15, 8)$

$15 = \boxed{1} \times 8 + \boxed{7}$

$8 = \boxed{1} \times 7 + 1$

$1 = 1\cdot 8 - 1\cdot 7$
$= 1\cdot 8 - 1(1\cdot 15 - 1\cdot 8)$
$= -1\cdot 15 + 2\cdot 8$

8. **[Marks 20]**
Solve using the Chinese Remainder Theorem: $x \equiv 2 \bmod 5$, $x \equiv 3 \bmod 9$, and $x \equiv 4 \bmod 13$. Show all the calculations.

35/40

ANSWER	$x = 147$

$x \equiv 2 \bmod 5$
$x \equiv 3 \bmod 9$
$x \equiv 4 \bmod 13$

5, 9 & 13 are pairwise relatively prime

$m = m_1 \times m_2 \times m_3 = 5 \times 9 \times 13 = 585$

$M_1 = m/m_1 = \frac{585}{5} = 117$
$M_2 = m/m_2 = \frac{585}{9} = 65$
$M_3 = m/m_3 = \frac{585}{13} = 45$

$x \equiv (2 \times 117 y_1 + 3 \times 65 y_2 + 4 \times 45 y_3) \bmod 585$

$117 y_1 \equiv 1 \bmod 5$ $\quad$ $65 y_2 \equiv 1 \bmod 9$ $\quad$ $45 y_3 \equiv 1 \bmod 13$

gcd(117, 5)
$117 = \boxed{23} \times 5 + \boxed{2}$
$5 = \boxed{2} \times 2 + \boxed{1}$
$1 = 1 \cdot 5 - 2 \cdot 2$
$= 1 \cdot 5 - 2(1 \cdot 117 - 23 \cdot 5)$
$= -2 \cdot 117 + 47 \cdot 5$
$y_1 = 3$

gcd(65, 9)
$65 = \boxed{7} \times 9 + \boxed{2}$
$9 = \boxed{4} \times 2 + \boxed{1}$
$1 = 1 \cdot 9 - 4 \cdot 2$
$= 1 \cdot 9 - 4(1 \cdot 65 - 7 \cdot 9)$
$= -4 \cdot 65 + 29 \cdot 9$
$y_2 = 5$

gcd(45, 13)
$45 = \boxed{3} \times 13 + \boxed{6}$
$13 = \boxed{2} \times 6 + \boxed{1}$
$1 = 1 \cdot 13 - 2 \cdot 6$
$= 1 \cdot 13 - 2(1 \cdot 45 - 3 \cdot 13)$
$= -2 \cdot 45 + 7 \cdot 13$
$y_3 = 11$

$x \equiv (2 \times 117 \times 3 + 3 \times 65 \times 5 + 4 \times 45 \times 11) \bmod 585$
$\equiv 3657 \bmod 585$
$\equiv 147 \bmod 585$
unique solution

$x = 147 + 585k \quad k \in \mathbb{Z}$ (General Solution)