Answer the following questions:

1. Show that the following conditional statements **(¬p ∧ (p ∨ q)) → q** is a tautology by using:

a) Truth tables.

b) Applying a chain of logical equivalences.

2. Determine the truth value of the following propositions for the given universe of discourse.

	Universe of discourse	**Truth value**
$\exists x\ (x+1>2x)$	$\mathbb{Z}$	
$\forall x\ (x+1>2x)$	$\mathbb{Z}$	
$\exists x\ (x^2 = 2)$	$\mathbb{R}$	
$\forall x\ \exists y\ (x - 2y = 0)$	$\mathbb{R}$	
$\forall x\ (x<10) \rightarrow \forall y\ (y<x) \rightarrow y<9$	$\mathbb{Z}$	
$\forall x\ (x<10) \rightarrow \forall y\ (y<x) \rightarrow y<9$	$\mathbb{R}$	

3. Let L(x,y)="x likes y". Express the following statements using predicates and quantifiers:

a. Everyone likes Khaled.
b. There is someone who Fahad doesn't like.
c. Everyone likes someone.
d. There is someone whom everyone likes.
e. There is someone whom no one likes.

4. Prove using direct proof that "the sum of two rational numbers is rational".

5. Prove using indirect proof that "if n is an integer and n^3+5 is odd, then n is even".

6. prove that "every odd integer is the difference of two squares".

7. Prove or disprove that "if a and b are rational numbers, then a^b is also rational".