1. [Marks 10]

Write the inverse y of all elements x in modulo 10. That is $x * y \equiv 1 \bmod 10$.

x	1	2	3	4	5	6	7	8	9
y	1	—	7	—	—	—	3	—	9

$\gcd(2,10) \neq 1$ $\gcd(4,10) \neq 1$
$\gcd(5,10) \neq 1$
$\gcd(6,10) \neq 1$
$\gcd(8,10) \neq 1$

2. [Marks 10]

Solve the equation $x^2 \equiv 3 \bmod 11$. Find all the solutions.

ANSWER

$x = \pm 5$	$x = \pm 6$

x	x^2	$x^2 \bmod 11$	
1	1	1	
2	4	4	
3	9	9	
4	16	5	
5	25	3	→ Solution $x = 5, x = 6$
6	36	3	→
7	49	5	
8	64	9	
9	81	4	
10	100	1	

3. [Marks 20]
Use the Chinese Remainder Theorem to solve the following equations:

$x \equiv 2 \bmod 5$

$x \equiv 3 \bmod 8$

$x \equiv 5 \bmod 9.$

ANSWER

$x = 347$	General solution $x =$ $347 + 360n$

5, 8, 9 are relatively prime.

$m = 5 \times 8 \times 9 = 360$, $M_1 = \frac{360}{5} = 72$, $M_2 = \frac{360}{8} = 45$

$M_3 = \frac{360}{9} = 40$

$M_1 y_1 \equiv 1 \bmod 5 \Rightarrow 72 y_1 \equiv 1 \bmod 5 \quad y_1 = 3$

$M_2 y_2 \equiv 1 \bmod 8 \Rightarrow 45 y_2 \equiv 1 \bmod 8 \quad y_2 = 5$

$M_3 y_3 \equiv 1 \bmod 9 \Rightarrow 40 y_3 \equiv 1 \bmod 9 \quad y = 7$

$X = (a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3) \bmod m$

$= (2 \times 72 \times 3 + 3 \times 45 \times 5 + 5 \times 40 \times 7) \bmod 360$

$= 2507 \bmod 360 = 347$

$X = 347$

$X = 347 + n(360)$

4. [Marks 15]

Express the gcd(128, 81) as a linear combination of 128 and 81.

ANSWER $\boxed{\gcd(128,81) = (-31 \times 128) + (49 \times 81)}$

$128 = 1 \times 81 + 47 \rightarrow 47 = 128 - 1 \times 81$

$81 = 1 \times 47 + 34 \rightarrow 34 = 81 - 1 \times 47$

$47 = 1 \times 34 + 13 \rightarrow 13 = 47 - 1 \times 34$

$34 = 2 \times 13 + 8 \rightarrow 8 = 34 - 2 \times 13$

$13 = 1 \times 8 + 5 \rightarrow 5 = 13 - 1 \times 8$

$8 = 1 \times 5 + 3 \rightarrow 3 = 8 - 1 \times 5$

$5 = 1 \times 3 + 2 \rightarrow 2 = 5 - 1 \times 3$

$3 = 1 \times 2 + 1 \rightarrow 1 = 3 - 1 \times 2$

$2 = 2 \times 1 + 0$

$1 = 3 - 1 \times 2 \Rightarrow = 3 - 1 \times (5 - 1 \times 3) \Rightarrow = -1 \times 5 + 2 \times 3$

$= -1 \times 5 + 2 \times (8 - 1 \times 5) = 2 \times 8 - 3 \times 5 = 2 \times 8 - 3(13 - 1 \times 8)$

$= -3 \times 13 + 5 \times 8 = -3 \times 13 + 5 \times (34 - 2 \times 13) = 5 \times 34 - 13 \times 13$

$= 5 \times 34 - 13 \times (47 - 1 \times 34) = -13 \times 47 + 18 \times 34 = -13 \times 47 + 18 \times (81 - 1 \times 47)$

$= 18 \times 81 - 31 \times 47 = 18 \times 81 - 31 \times (128 - 1 \times 81)$

$= (-31 \times 128) + (49 \times 81)$