Solution. a) Let p be the proposition Kangaroos live in Australia. and q be the proposition Kangaroos are marsupials.

$p \wedge q$

q Simplification

b) Let p be the proposition Hotter than 100 degrees today. and q be the proposition Pollution is dangerous.

$\neg p$

$p \vee q$

q Disjunctive syllogism

c) Let p be the proposition Linda is an excellent swimmer. and q be the proposition Can work as a lifeguard.

p

$p \to q$

q Modus ponens

d) Let p be the proposition Steve will work at a computer company this summer. and q be the proposition Steve will be a beach bum.

p

$p \vee q$ Addition

Solution. a) Predicates: $E(x)$: I eat ice cream on day x $S(x)$: I am sick on day x $M(x)$: I take medicine on day x

Premises: $E(x) \to S(x)$

$S(x) \to M(x)$

$\neg M(x)$

Conclusion: $\neg E(x)$

$S(x) \to M(x)$ and $\neg M(x) \therefore \neg S(x)$ [modus tollens] (not sick)

$E(x) \to S(x)$ and $\neg S(x) \therefore \neg E(x)$ [modus tollens] (did not eat ice cream on the day before)

b) Predicates : $J(x) =$ x lives in New Jersey $O(x) =$ x lives within 50 miles of the ocean $S(x)$ = x has seen the ocean

Premise:

$\forall x(J(x) \rightarrow O(x))$

$\exists x(J(x) \wedge \neg S(x))$

Conclude:

$\exists x(O(x) \wedge \neg S(x))$

Step:

1 $\exists x(J(x) \wedge \neg S(x))$ Premise 2

2 $J(y) \wedge \neg S(y)$ Existential Instantiation on (1) (y is an element of the domain)

3 $J(y)$ Simplification on (2)

4 $\forall x(J(x) \rightarrow O(x))$ Premise 1

5 $J(y) \rightarrow O(y)$ Universal Instantiation on (4)

6 $O(y)$ Modus Ponens on (3) and (5)

7 $\neg S(y)$ Simplification on (2)

8 $O(y) \wedge \neg S(y)$ Conjunction on (6) and (7)

$\exists x(O(x) \wedge \neg S(x))$ Existential generalizing on (8)

Question 3. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x(\neg P(x) \wedge Q(x) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is true.

Solution. 1. $\forall x(P(x) \vee Q(x))$ Premise
2. $P(c) \vee Q(c)$ Universal instantiation using (1)
3. $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ Premise
4. $(\neg P(c) \wedge Q(c)) \rightarrow R(c)$ Universal instantiation using (3)
5. $\neg(\neg P(c) \wedge Q(c)) \vee R(c)$ Implication using (4)
6. $P(c) \vee \neg Q(c) \vee R(c)$ De Morgans law using (5)
7. $P(c) \vee R(c)$ Resolution using (2) and (6)
8. $\neg R(c) \rightarrow P(c)$ Implication using (7)
9. $\forall x(\neg R(x) \rightarrow P(x))$ Universal generalization using (8)

Solution. $\boxed{\text{a)}\textbf{(Answer: Valid)}}$

Let $p(x)$ be the proposition x is enrolled in the university. And $q(x)$ be the proposition x has lived in a dormitory.

Step Reason

(1)$\forall x(p(x) \to q(x))$ Premise

(2)$p(Mai) \to q(Mai)$ Universal instantiation

(3)$\neg q(Mai)$ Premise.

(4)$\neg p(Mai)$ MT, (2), (3).

Namely, Mai is not enrolled in the university. The argument is valid.

b)**Answer:** $\boxed{\textbf{Invalid}}$

Let $P(x)$ be x is a convertible car. And $Q(x)$ be x is fun to drive.

The premises are: $\forall x(P(x) \to Q(x))$, and $\neg P(Isaacscar)$. From these premises, we cannot conclude $\neg Q(Isaacscar)$. The argument is invalid.

c) Lina likes all action movies. Lina likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.

d) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is a positive real number, then a is a positive real number.

e) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$, then $a \neq 0$.

c)**Answer: Invalid**

False, Quincy might like other kinds movies and all action movies.

d)**Answer: Invalid**

Take $a = -1$ for a counterexample.

e)**Answer: Valid**

This argument is valid. It is an application of universal instantiation.

Question 5. What is wrong with this argument? Let H(x) be "x is happy." Gi
premise $\exists x H(x)$, we conclude that H(Lola). Therefore, Lola is happy.

*Solution.*We are not sure that lola belong to the domain $\exists x\ H(x)$
then $H(Lola)$ mistake: we selected arbitrary $x(Lola)$

Question 6. Use a direct proof to show that the sum of two odd integers is even.

*Solution.*Let a and b be odd integers. By definition of odd we have that $a = 2n + 1$ and $b = 2m + 1$.

Consider the sum $a + b = (2n + 1) + (2m + 1) = 2n + 2m + 2 = 2k$, where $k = n + m + 1$ is an integer. Therefore by definition of even we have shown that a + b is even and my hypothesis is true.

Question 7. Let $x \in Z$. If $x^2 - 6x + 5$ is even, then x is odd. Show that using proof by contrapositive.

*Solution.*Proof. **(Contrapositive)** Suppose x is not odd. Thus x is even, so $x = 2a$ for some integer a.
So $x^2 - 6x + 5 = (2a)^2 - 6(2a) + 5 = 4a^2 - 12a + 5 = 4a^2 - 12a + 4 + 1 = 2(2a^2 - 6a + 2) + 1$. Therefore $x^2 - 6x + 5 = 2b + 1$, where b is the integer $2a^2 - 6a + 2$. Consequently $x^2 - 6x + 5$ is odd. Therefore $x^2 - 6x + 5$ is not even. ∎

Question 8. Suppose $a \in Z$. If a^2 is even, then a is even. Show that using proof by contradiction.

*Solution.*Proof. For the sake of contradiction suppose a^2 is even and a is not even. Then a^2 is even, and a is odd.
Since a is odd, there is an integer c for which $a = 2c + 1$. Then $a2 = (2c + 1)^2 = 4c^2 + 4c + 1$ $=2(2c^2 + 2c) + 1$,so a^2 is odd. Thus a^2 is even and a^2 is not even, a contradiction. (And since we have arrived at a contradiction, our original supposition that a^2 is even and a is odd could not be true.) ∎