Answer the following questions:

1. Determine the truth of the following statements:

a. $\emptyset \in \{\emptyset\}$
b. $\emptyset \in \{\emptyset, \{\emptyset\}\}$
c. $\{\emptyset\} \in \{\emptyset\}$
d. $\{\emptyset\} \in \{\{\emptyset\}\}$
e. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
f. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
g. $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$
h. $|\{1,2,1\}|=3$
i. $|A \times B| = |B \times A|$
j. $A \times \emptyset = A$
k. If $A \cup C = B \cup C$, then $A = B$
l. If $A \cap C = B \cap C$, then $A = B$

2. Prove that the following sets are equal to each other. You are free to use set-builder notation, or membership table.

a. $A \cup \emptyset = A$

b. $\overline{(A - B) \cup B} = \bar{A} \cap \bar{B}$

3. Determine if the following functions [one-to-one, onto] from $\mathbb{Z} \to \mathbb{Z}$.

a. f(n) = 0
b. f(n) = 2n+1
c. f(n) = 2n+1 [$\mathbb{R} \to \mathbb{R}$]
d. f(n) = |n-2|

4. Consider the following functions defined on real numbers $\boldsymbol{x} \in \mathbb{R}$. Let $\boldsymbol{f(x) = \lfloor x \rfloor}$, and $\boldsymbol{g(x) = -x}$. Define a function $\boldsymbol{h(x) = (g \circ f \circ g)(x)}$.

a. Write the function $\boldsymbol{h(x)}$.
b. What is the value of $\boldsymbol{h(-1.4)}$?

5. Compute the sum of the first 120 numbers in the list: {3, 7, 11, 15, 19, 23, …}
Show all the details.

6. Evaluate the following summation:

$$\sum_{k=1}^{n}\sum_{j=1}^{m}\left(\frac{k}{nm}\right)$$

7. Evaluate the following summation:

$$\sum_{i=1}^{n}\sum_{j=i}^{m} i$$