

**Answer the following questions:**

**1.** Show that the following conditional statements  $(\neg p \wedge (p \vee q)) \rightarrow q$  is a tautology by using:

- a) Truth tables.
- b) Applying a chain of logical equivalences.

**2.** Determine the truth value of the following propositions for the given universe of discourse.

	Universe of discourse	Truth value
$\exists x (x+1 > 2x)$	$\mathbb{Z}$	
$\forall x (x+1 > 2x)$	$\mathbb{Z}$	
$\exists x (x^2 = 2)$	$\mathbb{R}$	
$\forall x \exists y (x - 2y = 0)$	$\mathbb{R}$	
$\forall x (x < 10) \rightarrow \forall y (y < x) \rightarrow y < 9$	$\mathbb{Z}$	
$\forall x (x < 10) \rightarrow \forall y (y < x) \rightarrow y < 9$	$\mathbb{R}$	

**3.** Let  $L(x,y)$  = "x likes y". Express the following statements using predicates and quantifiers:

- a. Everyone likes Khaled.
- b. There is someone who Fahad doesn't like.
- c. Everyone likes someone.
- d. There is someone whom everyone likes.
- e. There is someone whom no one likes.

**4.** Prove using direct proof that "the sum of two rational numbers is rational".

**5.** Prove using indirect proof that "if  $n$  is an integer and  $n^3+5$  is odd, then  $n$  is even".

**6.** prove that "every odd integer is the difference of two squares".

**7.** Prove or disprove that "if  $a$  and  $b$  are rational numbers, then  $a^b$  is also rational".