## CSC281: Discrete Math for Computer Science

Computer Science Department

Second Semester 1441/1442

King Saud University

Tutorial 11: The Basics of Counting+ The Pigeonhole Principle

Question 1. There are 18 mathematics majors and 325 computer science majors at a college.

- a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
- b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

Question 2. How many strings of five ASCII characters contain the character @ (at sign) at least once? [Note: There are 128 different ASCII characters.]

Question 3. How many positive integers less than 1000\*

- a) are divisible by 7?
- b) are divisible by 7 but not by 11?
- c) are divisible by both 7 and 11?
- d) are divisible by either 7 or 11?
- e) are divisible by exactly one of 7 and 11?
- f) are divisible by neither 7 nor 11?
- g) have distinct digits?
- h) have distinct digits and are even?

(a) We need to use the **product rule**, because the first event is picking a mathematics major and the second event is picking a computer science major

 $18 \cdot 325 = 5850$ 

(b) We need to use the **sum rule**, because the event is picking a mathematics major OR picking a computer science major (while nobody falls in both categories, thus non-overlapping)

18 + 325 = 343



Strings of length 5 with at least one @ Strings of length 5 with at least one @ are strings of length 5 that are not strings of length 5 without an @  $128^5 - 127^5 = 34,359,738,368 - 33,038,369,407 = 1,321,368,961$ 

Question 4. How many license plates can be made using either three digits followed by three uppercase English letters or three Arabic letters followed by three digits?\*

Question 5. How many strings of eight English letters are there

- a) that contain no vowels, if letters can be repeated?
- b) that contain no vowels, if letters cannot be repeated?
- c) that start with a vowel, if letters can be repeated?
- d) that start with a vowel, if letters cannot be repeated?
- e) that contain at least one vowel, if letters can be repeated?
- f) that contain exactly one vowel, if letters can be repeated?
- g) that start with X and contain at least one vowel, if letters can be repeated?
- h) that start and end with X and contain at least one vowel, if letters can be repeated?

**Question 6.** Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.\*

**Question 7.** A room contains 10 men and 10 women. A manger selects members of team at random without looking at them.

a) How many member must the manger select to be sure of having at least three members of the same gender?



There are 26 possible letters and 10 possible digits

First letter= 26 ways Second letter= 26 ways Third letter= 26 ways First digit= 10 ways Second digit= 10 ways

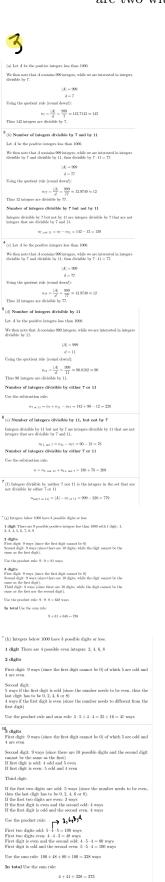
Third digit= 10 ways Order = 2 ways (because letters are before the digits or the digits are before the letters)

Using the product rule

 $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 2 = 26^3 \cdot 10^3 \cdot 2 = 35,152,000$ 

b) How many member must the manger select to be sure of having at least three women in the team?\*

**Question 8.** Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.





There are 30 students in the class. Assume that each student has at least a first name and a last name. Each last name has to begin with a letter of the alphabet. objects = first letter of each last name = 30 = N holes = letters of the alphabet = 26 = k

| 30 | 26 | = 2
| So by the Pigeonhole Principle, there are at least two students who have a last name that begin with the same letter.

| Question 7. A room contains 10 men and 10 women. A manger selects members of team at random without looking at them.
| a) How many member must the manger select to be sure of having at least three members of the same gender?
| b) How many member must the manger select to be sure of having at least three women in the team?\*

| Solution. | There are 20 persons, 10 men and 10 women. | must be equal to three, and the least positive integer to satisfy the equation is 5.
| b) The first 10 choices many be all men, so the manager needs to choose at least 13 member to be sure that at least 3 of them are women.

Question 8. Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

Solution.

There are 4 possible values of remainder: 0, 1, 2, 3

And we have 5 integers. Therefore, by pigeonhole principle, at least 2 of them must give the same reminder when divided by 4.