CSC281: Discrete Math for Computer Science

Computer Science Department — First Semester 1443

King Saud University — Tutorial 12: Counting (6.3, 6.4, 6.5 and 8.5)

Question 1. How many bit strings of length 12 contain

a) exactly three 1s?

b) at most three 1s?

c) at least three 1s?

d) an equal number of 0s and 1s?

Question 2. Thirteen people on a softball team show up for a game.

a) How many ways are there to choose 10 players to take the field?

b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?

c) Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

Question 3. What is the coefficient of x^7 in $(1+x)^{11}$?

Question 4. Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$

a) using a combinatorial argument.

b) by algebraic manipulation.

Question 5. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29,$$

where $x_i, i = 1, 2, 3, 4, 5, 6$, is a nonnegative integer such that

a) $x_i > 1$ for $i = 1, 2, 3, 4, 5, 6$?

b) $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4, x_5 \geq 5$ and $x_6 \geq 6$?

c) $x_1 \leq 5$?

d) $x_1 < 8$ and $x_2 > 8$?

Question 6. How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

Question 7. A survey of households in the United States reveals that 96% have at least one television set, 98% have telephone service, and 95% have telephone service and at least one television set. What percentage of households in the United States have neither telephone service nor a television set?

Question 8. How many students are enrolled in a course either in calculus, discrete mathematics, data structures, or programming languages at a school if there are 507, 292, 312, and 344 students in these courses, respectively; 14 in both calculus and data structures; 213 in both calculus and programming languages; 211 in both discrete mathematics and data structures; 43 in both discrete mathematics and programming languages; and no student may take calculus and discrete mathematics, or data structures and programming languages, concurrently?