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KING SAUD UNIVERSITY

COLLEGE OF COMPUTER & INFORMATION SCIENCES

DEPT OF COMPUTER SCIENCE

CSC281 Discrete Mathematics for CS Students
First Semester 1440/1441 AH (Fall 2019)
Second midterm Examination: Sun 24.11.2019 C.E. (Time: 6-7:30pm)
Instructor: Prof. Aqil Azmi [redacted]

Name: [redacted] **ID:** [redacted]

1. **[Marks 10]**
   Prove using contradiction the following statement. If $a, b \in \mathbb{Z}$, then $a^2 - 4b \neq 2$.

~~Assume $a,b \notin \mathbb{Z}$~~ Assume $a^2 - 4b = 2$

$a^2 = 2 + 4b \Rightarrow \cancel{a = \sqrt{2+4b}}$

$a^2 = 2(1+2b)$ $\therefore a^2$ is even.

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$\leadsto$ a is even!

$\cancel{4b = a^2 - 2 \Rightarrow b = \frac{a^2-2}{4} = \frac{(2k)^2-2}{4} = \frac{4k^2-2}{4} = \frac{2(2k^2-1)}{4} = \frac{2k^2-1}{2}}$

$\cancel{a^2 - 4\left(\frac{2k^2-1}{2}\right) = 2}$

$\cancel{a^2 - 2(2k^2-1) = 2 \Rightarrow a^2 - 4k^2 + 2 = 2}$

$\cancel{a^2 = 4k^2}$

?

~~So it contradicts that~~ $b \in \mathbb{Z}$

$4b = a^2 - 2 \Rightarrow b = \frac{a^2-2}{4} = \frac{2k-2}{4} = \frac{2(k-1)}{4} = \frac{k-1}{2}$

$a^2 - 4\left(\frac{k-1}{2}\right) = 2 \Rightarrow a^2 - 2k + 2 = 2 \Rightarrow a^2 = 2k$

Let $a = 2k$
$(2k)^2 = 2(1+2b)$
$4k^2 = 2(1+2b)$
$2k^2 = 2b+1$
$\therefore$ contradiction, since even number is not equal to odd number.

$\cancel{a^2 = 2(2+k-1)}$
$\cancel{= 4+2k-2}$
$\cancel{= 2+2k}$
$\cancel{= 2(1+k)}$

$a^2 \to$

$4 \mid 12$
$x = 2$
$y = 3$
$z = 4$
$4 \nmid 3$
$4 \mid 4$

2. **[Marks 10]**
   Disprove the following theorem using counter example, "For all integers $x, y, z \in \mathbb{Z}$ and $x \neq 0$, if $x^2 \mid yz$, then $x \mid y$ and $x \mid z$."

ANSWER	$x =$ 2 $y =$ 3 $z =$ 4

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$\left\lceil \frac{N}{4} \right\rceil = 2$

$\left\lceil \frac{N}{4} \right\rceil = 3$

3. **[Marks 15]**

A box contains 10 red balls, 10 green balls, 10 yellow balls, and 10 white balls. A blind folded boy picks 5 balls. Mark the following statements True/False. Mark true only if it is fully guaranteed and give reason,

		True/False	Reason
a	One of the balls is red?	~~T~~ F	~~not always will be one of the balls red.~~
b	At least 2 balls of same color?	T	$\left\lceil \frac{N}{4} \right\rceil = 2$, and 5 is true for that.
c	At least 3 balls of same color?	F	$\left\lceil \frac{N}{4} \right\rceil = 3$, therefor we have to pick 7 balls to get atleast 3 ball of same color.

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4. **[Marks 15]**

Compute 6^{199} mod 79. Show all the details.

ANSWER	47 mod 79

$6^{78} \equiv 1 \bmod 79$

$6^{199} \bmod 79$

$(\overbrace{6^{78}}^{=1} \times 6^{121}) \bmod 79$

$(6^{64} \times 6^{32} \times 6^{16} \times 6^{8} \times 6) \bmod 79$

$(4 \times 2 \times 9 \times 76 \times 6) \bmod 79$

$\underline{\underline{47 \bmod 79}}$

$6 \equiv 6 \bmod 79$

$6^2 \equiv 36 \bmod 79$

$6^4 \equiv 32 \bmod 79$

$6^8 \equiv 76 \bmod 79$

$6^{16} \equiv 9 \bmod 79$

$6^{32} \equiv 2 \bmod 79$

$6^{64} \equiv 4 \bmod 79$

$6^{128} \equiv 16 \bmod 79$

5. **[Marks 15]**
How many solutions does the following equation have: $w + x + y + z = 15$, where all the variables w, x, y, z are integers ≥ 1. Show all your calculations.

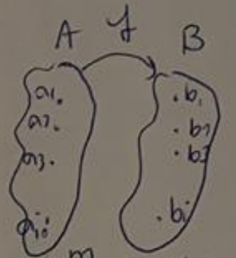
ANSWER	Number of solutions are: 364

let us say $(w'+1) + (x'+1) + (y'+1) + (z'+1) = 15$

$w' + x' + y' + z' = 11$

$$\binom{4+11-1}{11} = \binom{14}{11} = 364$$

6. **[Marks 20]**
Given two sets A, B. Let the function $f : A \to B$. If $|A| = 10$, and $|B| = 15$. Count the following (always show your arguments):



a. The number of functions f?

ANSWER	$m^n = 15^{10}$

a_i ~~has m choices~~

$= m \times m \times m \cdots$

$= m^n$

b. If f is a One-One function. How many functions do we have?

$\forall a, b \in A \ a = b \Leftrightarrow f(a) = f(b)$

ANSWER	~~15~~ ~~10!~~ 15! $15!/5!$

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at bad ~~situation~~ condition each element of A will go to element from B, ~~then it will~~

7. **[Marks 15]**

What is the *coefficient* of x^{23} in the expansion of $(2x^2 + 3x^3)^{10}$. Show all your calculations.

ANSWER	Coefficient of x^{23} = 414720

$$(2x^2+3x^3)^{10} = \sum_{k=0}^{10} \binom{10}{k} \cdot \underbrace{(2x^2)^{10-k} \cdot (3x^3)^k}_{}$$

$$(2^{10-k}) \cdot (x^{20-2k}) \cdot (3^k) \cdot (x^{3k})$$

$$2^{10-k} \cdot 3^k \cdot x^{\cancel{20}k+20}$$

$$k+20 = 23$$
$$k = 3$$

$$\text{coeff. of } x^{23} = \binom{10}{3} \cdot 2^7 \cdot 3^3 = 414720$$