Dec - 2020

King Saud University

Practice problems for final

الاجابة في الصفحات التالية

00966508793242

00201157933898

1. What is the co-efficient of $x^3y^4z^3$ when expanding $(x + 2y + 3z)^{10}$.

2. Calculate the value of $\sum_{k=0}^{n}\prod_{i=0}^{k} 3$.

3. Prove using Induction that for all positive integer n, then
$$\sum_{k=0}^{n} k\cdot(k!) = (n+1)! - 1.$$

4. Use Induction to show that, $n! > 2^n$ for all $n \geq 4$.

5. Solve the recurrence relation $a_n = a_{n-1} - 3a_{n-2}$ with initial conditions $a_0 = 1, a_1 = 6$.

6. How many passwords of length 7 can you make using following symbols: a-z, A-Z, @, and 0-9. Each password must have at least one capital letter, and at least one digit.

7. Suppose we have three sets: X, Y, and Z of sizes n, m, ℓ respectively. Let set $W = X \times Y$ (cross-product of two sets), and let $E = P(W)$, that is the power set of W. Count the number of functions $f : Z \mapsto E$.

8. Solve using the Chinese remained theorem the system of equations,
$x \equiv 2 \bmod 9$
$x \equiv 3 \bmod 50$
$x \equiv 6 \bmod 49$

9. How many different words can you make by re-arranging the letters of the name, *MOHAMMAD*. What if we insist that the first letter must be "M", how many different words can you make by re-arranging the other letters.

10. How many ways can you distribute 6 identical toys to 5 children if each child must get at least one toy. What if the toys are different?

11. Express the gcd of the numbers 245 and 363 as a linear combination of both numbers.

12. Calculate $\binom{-1/3}{5}$.

$$= \frac{-\frac{1}{3}\left(-\frac{1}{3}-1\right)\left(-\frac{1}{3}-2\right)\left(-\frac{1}{3}-3\right)\left(-\frac{1}{3}-4\right)}{5!}$$

$$= \frac{-\frac{1}{3} \times -\frac{4}{3} \times -\frac{7}{3} \times -\frac{10}{3} \times \frac{-13}{3}}{120}$$

$$= -0.1248285$$

Dec 2020

حل لرقم 6

6

$a \to z \quad : 26$

$A \to Z \quad : 26$

$@ \quad \to 1$

$0 - 9 \to 10$

المجموع $= 63$

let x = passwords have at least 1 capital

y = 〃 〃 〃 〃 1 digit

Required المطلوب $= x \cap y$ = all passwords $- \overline{x \cap y}$

all passwords $= 63^7$

$\overline{x \cap y} = \bar{x} \cup \bar{y}$

$\bar{x}$ = pass have no capital $= (26+10+1)^7 = 37^7$

$\bar{y}$ = pass have no digits $= (26+26+1)^7 = 53^7$

$\therefore |\overline{x \cap y}| = |\bar{x} \cup \bar{y}| = |\bar{x}| + |\bar{y}| - |\bar{x} \cap \bar{y}|$

$= 37^7 + 53^7 - 27^7$ ليس عنده حروف كابتال ولا ارقام

$\therefore$ pass that have at least 1 capital and at least 1 digit

$= 63^7 - (37^7 + 53^7 - 27^7)$

$= 2.6798 \times 10^{12}$

Dec-2019 = practice 9 = practice 7 Dec 2020

X, Y, Z : sets

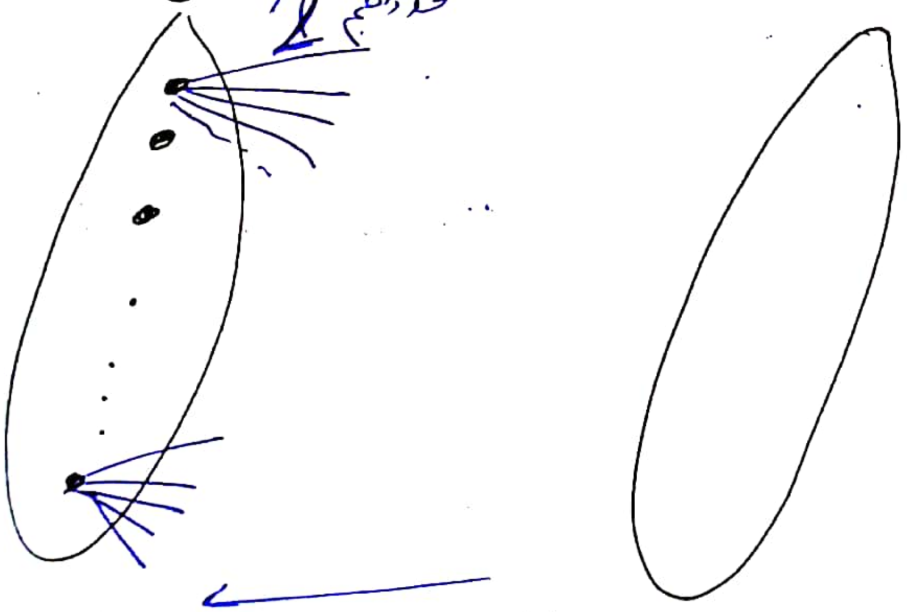
$|X| = n \qquad |Y| = m \qquad |Z| = \ell$

$$W = X \times Y \Rightarrow |W| = |X| \cdot |Y| = nm$$

$$E = p(W) \Rightarrow |E| = 2^{|W|}$$

$$E = p(X \times Y) = 2^{nm}$$

$Z \qquad 2^{nm}$ الاسهم



$\ell = Z$ عدد $=$ الاسهم

$$\underbrace{2^{nm} * 2^{nm} * \cdots * 2^{nm}}_{\ell}$$

$$\therefore \left(2^{nm}\right)^{\ell} = 2^{nm\ell}$$

practice 9 Dec 2020

all words = $\dfrac{8!}{3! \times 1! \times 1! \times 2! \times 1!}$

if we insist the the Letter must be M

∴ all words

$$= \frac{7!}{1! \times 1! \times 2! \times 2! \times 1!}$$

practice 10

children x_1, x_2, x_3, x_4, x_5

$$x_1 + x_2 + x_3 + x_4 + x_5 = 6$$

عدد الالعاب

we will find ceffecient of x^6 in the following expansion

$$\underbrace{(x^1 + x^2 + x^3 + x^4 + x^5 + x^6)}_{1^{s}\text{ child}} \cdot \underbrace{(x^1 + x^2 + x^3 + \cdots + x^6)}_{2^{nd}\text{ child}}$$

$+ \cdots \cdots$ 5th child

expansion in $(x^1 + x^2 + x^3 + x^4 + x^5 + x^6)^5$

$$= x^1 \cdot x^1 \cdot x^1 \cdot x^1 \cdot x^1 \cdot x^2 +$$
$$x^1 \cdot x^1 \cdot x^1 \cdot x^1 \cdot x^2 \cdot x^1 +$$
$$x^1 \cdot x^1 \cdot x^1 \cdot x^2 \cdot x^1 \cdot x^1 +$$
$$x^1 \cdot x^1 \cdot x^2 \cdot x^1 \cdot x^1 \cdot x^1 +$$
$$x^1 \cdot x^2 \cdot x^1 \cdot x^1 \cdot x^1 \cdot x^1 +$$
$$x^2 \cdot x^1 \cdot x^1 \cdot x^1 \cdot x^1 \cdot x^1$$

$$= 6x^6$$

$\therefore$ 6 ways

practicell

$$363 = 1(245) + 118$$
$$245 = 2(118) + 9$$
$$118 = 13(9) + 1 \rightarrow gcd$$
$$9 = 9(1) + 0$$

$$1 = 1\cdot(118) - 13(9)$$
$$= 1\cdot(118) - 13(1\cdot 245 - 2\cdot(118))$$
$$= 1\cdot(118) - 13\cdot(245) + 26(118)$$
$$1 = 27(118) - 13(245)$$
$$= 27(363 - 245) - 13(245)$$
$$= 27\cdot(363) - 27(245) - 13(245)$$
$$1 = 27(363) - 40(245)$$

#

CamScanner الممسوحة ضوئيا بـ