

CSC281: Discrete Math for Computer Science

Computer Science Department
King Saud University

First Semester 1442
Tutorial 9: Cryptography + Mathematical Induction

Question 1. Let $P(n)$ be the statement that $1^3 + 2^3 + \dots + n^3 = (\frac{n(n+1)}{2})^2$ for the positive integer n .

- a) What is the statement $P(1)$?
- b) Show that $P(1)$ is true, completing the basis step of the proof.
- c) What is the inductive hypothesis?
- d) What do you need to prove in the inductive step?
- e) Complete the inductive step, identifying where you use the inductive hypothesis.
- f) Explain why these steps show that this formula is true whenever n is a positive integer.

Question 2. Prove that $3 + 3 \times 5 + 3 \times 5^2 + \dots + 3 \times 5^n = 3 \times \frac{5^{n+1} - 1}{4}$ whenever n is a nonnegative integer.

Question 3. Prove that $2^n > n^2$ if n is an integer greater than 4.

Question 4.

- a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.
- b) Prove your answer to (a) using the principle of mathematical induction. Be sure to state explicitly your inductive hypothesis in the inductive step.
- c) Prove your answer to (a) using strong induction. How does the inductive hypothesis in this proof differ from that in the inductive hypothesis for a proof using mathematical induction?