KING SAUD UNIVERSITY

COLLEGE OF COMPUTER & INFORMATION SCIENCES

DEPT OF COMPUTER SCIENCE

CSC281 Discrete Mathematics for CS Students

Practice Questions (Final)

1. The car plate board in Saudi Arabia has the form: A #, where # is any number 1 to 9999, and A is a three letter in the range A-Z. How many car plate boards can you have?

$26^3 \times 9999$

2. Find the coefficient of the smallest and largest power of x in the expansion of $(3x + 2x^2 + 4/x)^{100}$.

smallest power of x is $\frac{1}{x^{100}}$ coeff = 4^{100}

largest power of x when expanding above is x^{200} and coeff = 2^{100}

3. How many base 3 strings of length 10 that starts with 000, or 222?

2×3^7

4. How many positive integers between 25 and 134 that are divisible by 4 and by 6 at the same time?

positive integers divisible by 4 and 6 at same time $= \lfloor \frac{134}{12} \rfloor - \lfloor \frac{25}{12} \rfloor = 9$

where $12 = \text{lcm}(4,6)$

5. English alphabet has 21 consonants and 5 vowels. How many strings of length 6 of lowercase letters that has no vowels? Exactly 2 vowels? At least 2 vowels?

string with no vowels $= 21^6$

exactly two vowels $= \binom{6}{2} \times 5^2 \times 21^4$

at least 2 vowels $= \sum_{k=2}^{6} \binom{6}{k} \cdot 5^k \cdot 21^{6-k}$

6. Solve the recurrence relation: $a_n = 3a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 0$, $a_1 = 5$.

Char eq. $r^2 - 3r - 2 = 0$

roots $= \frac{3 \pm \sqrt{17}}{2}$ $\Rightarrow a_n = \alpha_1 \left(\frac{3+\sqrt{17}}{2}\right)^n + \alpha_2 \left(\frac{3-\sqrt{17}}{2}\right)^n$

Solve for initial condition. we get

$$a_n = \frac{5}{\sqrt{17}} \left(\frac{3+\sqrt{17}}{2}\right)^n - \frac{5}{\sqrt{17}} \left(\frac{3-\sqrt{17}}{2}\right)^n$$

7. Convert the number 123 (in base 6) to a number in base 9.

$123_6 = 3 + 2 \times 6 + 1 \times 6^2 = 51$ (in base 10)

$51_{10} \Rightarrow 56$ in base 9

so $123_6 = 56_9$

8. Write the sequence generated by the GF $\frac{1+x+x^2}{(1-x)^2}$.

Sequence generated $= 1, 3, 6, 9, 12, \ldots$

9. Derive the recurrence relation to count the number of bit strings of length n that has the pattern 101.

Let $a_n =$ # bit string of length n with 101

$a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}$ for $n \geq 4$

$a_1 = 0$

$a_2 = 0$

$a_3 = 1$

10. Prove using induction that for all $n \geq 1$,

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}.$$

Base case $n = 1$

$$\left.\begin{aligned} LHS &= \frac{1}{1 \times (1+1)} = \frac{1}{2} \\ RHS &= \frac{1}{2} \end{aligned}\right\} \text{ equal.}$$

Inductive case

Assume it is true for some n, then for $n+1$

$$LHS = \sum_{K=1}^{n+1} \frac{1}{K(K+1)}$$

$$= \underbrace{\sum_{K=1}^{n} \frac{1}{K(K+1)}}_{=\frac{n}{n+1} \text{ by inductive hypothesis}} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{1}{n+1}\left[n + \frac{1}{n+2}\right]$$

$$= \frac{1}{n+1} \times \frac{n^2+2n+1}{n+2} \qquad = (n+1)^2$$

$$= \frac{n+1}{n+2}$$

$= RHS$