1. **[Marks 10]**
Fill-in the *missing* numbers in the following row in Pascal's triangle,

1 7 21 35 35 21 7 1

✓

2. ~~**[Marks 10]**~~
~~How many distinct words can you make by re-arranging the letters in the word "ABBAS".~~

~~*ANSWER*~~

cancel

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3. **[Marks 15]**
Show that if any 14 integers are selected from the set $A = \{1, 2, 3, ..., 25\}$, there are at least two integers whose sum is 26. **HINT:** Use pigeonhole principle. Explain.

1 2 3 4 5 ~~1 2 3 4 5 6 7 8 9~~

10 9 8 7 6 1 2 3 4 5 6 7 8 9 10 11

25 24 23 22 21 20 19 18

11 12 13 14 15

20 19 18 17 16

25 24 23 22 21

2	3	4	5	6	7	8	9	10	11	12
24	21	22	19	20	17	18	15	16	13	14
6	24	26	24	26	24	26	26	26	24 ?	26

-7

4. [Marks 15]

Compute $25^{666} \bmod 61$. Show all the steps.

ANSWER

$25^{60} = 1 \bmod 61 \Rightarrow (25^{60})^{10} \equiv 25^{600} \equiv 1 \bmod 61$

$25^{666} \equiv \underbrace{25^{600} \bmod 61}_{1} \cdot 25^{66} \bmod 61$ ✓

$25 \equiv 25 \bmod 61 \equiv 25$

$25^2 \equiv 625 \bmod 61 \equiv 15$

$25^4 \equiv 225 \bmod 61 \equiv 42$

$25^8 \equiv 1764 \bmod 61 \equiv 56$

$25^{16} \equiv 56^2 \bmod 61 \equiv 25$

$25^{32} \equiv 25^2 \bmod 61 \equiv 15$

$25^{64} \equiv 15^2 \bmod 61 \equiv 42$

$25^{66} \equiv (42 \times 15) \bmod 61$

$\equiv 20$ ✓

5. [Marks 15]

Use induction to show that $2^n < n!$ for all $n \geq 4$.

Baus caus$(n=4)$ let $P(n)$

RHS $2^4 = 16$; LHS $4! = 24$

$16 < 24$ The baus caus is true

inducteve caus Asume baus caus is truy for some n we show than for next N

let $P(n+1)$

RHS$= 2^{n+1} \Rightarrow 2^n \cdot 2$; LHS $(n+1)!$

frome Baus caus $2^n < n!$ and becous that

$2^n \cdot 2 < (n+1)!$ × (-6)

indecteve is true and that what we n

5. [Marks 10]

If p, q are two different primes, show that if $a \equiv b \bmod p$ and $a \equiv b \bmod q$, then $a \equiv b \bmod pq$.

$a \equiv b \bmod p \Rightarrow p \mid a-b \Rightarrow a-b = px$

$a \equiv b \bmod q \Rightarrow q \mid a-b \Rightarrow a-b = qy$

So we have $px = qy$ for some ints x, y

Now

$p \mid qy$

but $p \nmid q$

$\Rightarrow p \mid y$

$q \mid px$

but $q \nmid p$

$\Rightarrow q \mid x$

let $p \mid y \Rightarrow y = pz$ for some int z.

So $a-b = qy = pqz \Rightarrow pq \mid a-b$

or $a \equiv b \bmod pq$

6. [Marks 10]

Suppose x is rational, and y is irrational. Use proof by contradiction to show that $x + y$ is irrational. ↓ let $x = a/b$ a, b integers

Assume $x+y$ is rational.

So, let $x+y = c/d \ni c, d \in \mathbb{Z}$

$$\text{then } y = \frac{c}{d} - x$$

$$= \frac{c}{d} - \frac{a}{b} = \frac{bc-ad}{bd} \begin{array}{l} \leftarrow \text{integer} \\ \leftarrow \text{integer} \end{array}$$

that is y is rational, a contradiction.

6. [Marks 15]

Find the coefficient of x^{10} in the expansion of $(2+3x^2)^{15}$.

ANSWER

$(2+3x^2)^{15}$ $\sum_{k=0}^{15}\binom{15}{k} a^{k} \cdot b^{n-k}$

$2^{k} \cdot (3x^2)^{n-k} \Rightarrow 3^{n-k} \cdot (x^2)^{n-k} \to x^{2n-2k}$

x^{30-2k} $30-2k=$ [illegible] $.10 \Rightarrow k=10$

~~K = 5 = -30 2K = 12.5~~
~~-2~~

~~We can't becaus is have boint~~ [illegible]

$x^{10}=\binom{15}{10}2^{10}\cdot 3^{5}$ ✓

7. **[Marks 20=6+7+7]**
How many bit strings of length 8 contains (no need to calculate):

a. Exactly 3 zeros.
ANSWER

$$P(n,r) \Rightarrow \frac{n!}{r!(n-r)!} =$$

$$= \frac{8!}{3!(8-3)!} = 56$$

✓

b. Exactly 3 zeros where one of the zeros must be at the rightmost bit.
ANSWER

$$P(n,r) \qquad \frac{8!}{(8-3)!} = 336$$

$$P(8,3)$$

✗

-7

c. Odd number of zeros.
ANSWER

$$k=1 \qquad \frac{8!}{1!(8-1)!} + \frac{8!}{3!(8-3)!} + \frac{8!}{5!(8-5)!} +$$

✓ ✓ ✓ ✓

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