

1. Consider the statement, if $\gcd(a,b) = 1$ then either a or b must be prime. If this is a True statement then prove it, and if it is False give a counter example.
2. What are the possible remainders of $[n^3 \bmod 9]$ for all positive values of n. In other words, what are the possible values of x in: $n^3 \equiv x \pmod{9}$.
3. Solve: $4x^2 \equiv 3 \pmod{9}$.
4. Find the value of $(2021^{2021} \bmod 21)$. Show all the steps.
5. Use the Chinese Remainder Theorem to solve the system of congruences:
$$\begin{aligned}x &\equiv 2 \pmod{9} \\x &\equiv 5 \pmod{26} \\x &\equiv 3 \pmod{55} \\x &\equiv 6 \pmod{49}\end{aligned}$$