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Computer Science Department

CSC 281 Final – 1st Semester 2016/2017

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Question 1 (10)

For each of the following statements, determine whether it is true or false. Write “true” or “false”. Do not write “T”, “t”, ”F” or “f”.

	STATEMENT	True/False
(a)	$\neg(A \wedge B)$ and $(\neg A \wedge \neg B)$ are logically equivalent	False
(b)	$\neg\forall x\exists y\ (F(x,y) \rightarrow P(x))$ is logically equivalent to $\exists x\forall y\ \neg F(x,y) \vee P(x)$	False
(c)	The statement $(t \wedge q \wedge \neg q \wedge r) \rightarrow (\neg p \vee q \vee t \vee r)$ is a tautology	True
(d)	Given two sets A={1,2,3,4} and B={3}, B-A={1,2,4}	True
(e)	$\sum_{k=0}^{3}(3x2^{k})$ is equal to 45	True
(f)	If f *is* the function from $\{a,b,c,d\}$ to {1,2,3} defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$ then f is a bijection	False
(g)	If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $2a+d \equiv 2b+c \pmod{m}$ where m is a positive number	True
(h)	$2n^2+3n+1$ is $O(n^4)$	True
(i)	$3^{443} \bmod 11 = 5$	True
(j)	If $a=b-3m$, then $a \equiv b \pmod{m}$, where m is a positive integer	True

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Question 2 (10)

A) Solve the congruence 9x ≡ 15 (mod 23) Hint use the inverse of 9 modulo 23. (5)

Let's first use Euclid's algorithm to find gcd(23, 9).

$$\begin{aligned} 23 &= 9(2) + 5 \\ 9 &= 5(1) + 4 \\ 5 &= 4(1) + \boxed{1} \longleftarrow \gcd(23, 9) \\ 4 &= 1(4) + 0 \end{aligned}$$

Using back substitution we obtain

$$\begin{aligned} 1 &= 5 - \mathbf{4} \\ &= 5 - \boxed{(9-5)} = \mathbf{5}(2) - 9 \\ &= \boxed{(23 - 9(2))}(2) - 9 = 23(2) - 9(5) \end{aligned}$$

The last equation implies that

$$9(-5) = 1 + 23(-2) \longrightarrow 9(-5) \equiv 1 \pmod{23}.$$

We conclude that $x_0 = -5$ is an inverse of 9 modulo 23.

Multiplying equation (3) by 15 we obtain

$$9(-75) \equiv 15 \pmod{23}.$$

Then, for any integer k, $\boxed{x = -75 + 23k}$ are solutions of the linear congruence (2). The unique solution in $0 \leq x < 23$ is

$$x = -75 \bmod 23 = 17.$$

B) Prove that for every positive integer n,

1 * 2 + 2 * 3 + 3 * 4 + · · ·+n(n + 1) = n(n + 1)(n + 2) / 3. (5)

Basis step: Let n = 1. Then

Left-hand side = 2
Right-hand side = 1(2)(3)/3 = 2

Inductive hypothesis: Assume that for some positive integer k

1 * 2 + 2 * 3 +…+ k (k+1) = k (k+1) (k+2) / 3

Inductive step:

1 * 2 + 2 * 3+…+k (k+1) + (k+1) (k+2) = k (k+1) (k+2) / 3 + (k+1) (k+2)

= k (k+1) (k+2) / 3 + 3(k+1) (k+2) / 3

by taking (k+1)(k+2) as a common factor, then the above is equal to

(k+1) (k+2) (k+3) / 3 = right-hand side done

Question 3 (10)

A) Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers:
$1 = 2^0$, $2 = 2^1$, $3 = (2^0 + 2^1)$, $4 = 2^2$ and so on.

[Hint: For the inductive step, separately consider the case where k + 1 is even and where it is odd. When it is even, note that (k + 1)/2 is an integer.] (5)

We will prove this statement using strong induction on n. Let n be a positive integer. We will prove that n can be written as the sum of distinct powers of 2. Let P(n) be the statement that we can write n as the sum of distinct powers of 2.

Base step: P(1) is true because $1 = 2^0$.

Inductive step: Assume for $1 \le j \le k$ for some integer k that P(k) is true. That is, assume we can write all integers from 1 to k as the sum of distinct powers of 2. We have two cases to consider.

Case 1: k is even. Then, k + 1 is odd. Now, by the inductive hypothesis, we have that k has an expansion

$$k = 2^{m1} + 2^{m2} + \ldots + 2^{mn}$$

where $m_1 > m_2 > \ldots > m_n$. (Note: we need to write it in general terms because we don't know which powers of 2 are needed for the integer. There is no reason that k would have to have 2 or 4 or any other particular power of 2 in it's expansion.). Notice that since k is even, there is no 2^0 in the expansion; so, from the expansion for k we add 2^0 to yield

$$k+1 = 2^{m1} + 2^{m2} + \ldots + 2^{mn} + 2^0$$

to give the expansion for k+1.

Case 2: k is odd. Then k + 1 is even. Since k + 1 is even, then $(k+1)/2 \in Z$ and therefore by the inductive hypothesis, has an expansion consisting of the sum of distinct powers of 2. Let the expansion be

$$(k +1)/2 = 2^{m1} + 2^{m2} + \ldots + 2^{mn}$$

Then, we can multiply both sides by 2, yielding

$$2\,(k+1)/2 = 2(2^{m1} + 2^{m2} + \dots + 2^{mn})$$ $$k+1 = 2^{m1+1} + 2^{m2+1} + \dots + 2^{mn+1}$$ which completes the inductive step. So, by strong induction, we have shown that if P(j) is true for $1 \leq j \leq k$, then P(k + 1) is true as well.	

B) How many strings of eight uppercase English letters are there? (5)

A	**That contain no vowels, if letters can be repeated?(1)**
	21^8
B	**That contain no vowels, if letters cannot be repeated? (1)**
	21*20*19*18*17*16*15*14
C	**That start with a vowel, if letters can be repeated? (1)**
	$5*26^7$
D	**That contain at least one vowel, if letters can be repeated? (1)**
	$26^8 - 21^8$
E	**That contain exactly one vowel, if letters can be repeated? (1)**
	$8 * 5 * 21^7$

(Note: A, E, I, O and U are vowels)

Question 4 (10)

C) One hundred tickets, numbered 1, 2, 3. . . 100 are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). How many ways are there to award the prizes if (10)

A	**There are no restrictions?**
	100 * 99 * 98 * 97
B	**The person holding ticket 47 wins the grand prize?**
	99 * 98 * 97
C	**The person holding ticket 47 wins one of the prizes?**
	99 * 98 * 97 * 4
D	**The person holding ticket 47 does not win a prize?**
	99 * 98 * 97 * 96
E	**The people holding tickets 19 and 47 both win prizes?**
	98 * 97 * C(4,2) * 2
F	**The people holding tickets 19, 47, and 73 all win prizes?**
	C(4,3) * 97 * 3!
G	**The people holding tickets 19, 47, 73, and 97 all win prizes?**
	4 * 3 * 2 * 1
H	**None of the people holding tickets 19, 47, 73, and 97 wins a prize?**
	96 * 95 * 94 * 93
I	**The grand prize winner is a person holding ticket 19, 47, 73, or 97?**
	4 * 99 * 98 * 97
J	**The people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?**
	96 * 95 * C(4,2) * 2