STACKS

CS212: Data Structure

Stacks

- A stack is a container of objects that are inserted and removed according to the **last-in-first-out (LIFO)** principle.

- Objects can be inserted at any time, but only the last (the most-recently inserted) object can be removed.
- Inserting an item is known as “Pushing” onto the stack. “Popping” off the stack is synonymous with removing an item
- Used in Operating system to implement method calls, and in evaluating Expressions.

ADT Stack: Specification

Elements: The elements are of a generic type <Type>. (In a linked implementation an element is placed in a node)

Structure: the elements are linearly arranged, and ordered according to the **order of arrival**, most recently arrived element is called top.

Domain: the number of elements in the stack is bounded therefore the domain is finite. Type of elements: Stack

ADT Stack: Specification

Operations:

All operations operate on a stack S.

1. **Method** push (Type e)
   requires: Stack S is not full.
   input: Type e.
   results: Element e is added to the stack as its most recently added elements.
   output: none.
2. **Method** pop (Type e)
   requires: Stack S is not empty.
   input: none
   results: the most recently arrived element in S is removed and its value assigned to e.
   output: Type e.
3. **Method** empty (boolean flag)
   input: none
   results: If Stack S is empty then flag is true, otherwise false.
   output: flag.

ADT Stack: Specification

Operations:

4. **Method** Full (boolean flag).

   requires:

   input: none

   results: If S is full then Full is true, otherwise Full is false.

   output: flag.

```
public static void print(Stack<T> st){
    Stack<T> ts = new LinkedStack<>();
    while(!st.empty()){
        T e = st.pop();
        ts.push(e);
    }
    while(!ts.empty()){
        st.push(ts.pop());
    }
}
```

```
public static <T> void reverse(Stack<T> st){
    StackTsts = new LinkedStack<>();
    while(!st.empty())
        ts.push(st.pop());
    st = ts;
}
```

→ st will be empty

→ WRONG

→ changing the local variable != changing the actual variable

```
queue<T> temp = new LinkedStack<>();
while(!st.empty()){
    T e = st.pop();
    temp.enqueue(e);
}
while(temp.size != 0){
    T e = temp.dequeue();
    st.push(e);
}
```

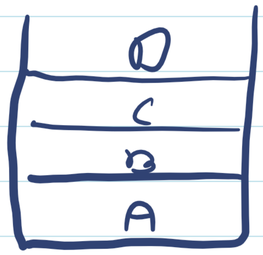
↓

QUEUE

```
public static <T> void reverse(Stack<T> st){
    if(st.empty())
        return;
```

```
public static void print(Stack<T> st) {
    if (st.empty())
        return;
    T e = st.pop();
    s.o.p(e);
    print(ST);
    st.push(e);
}
```

recursx



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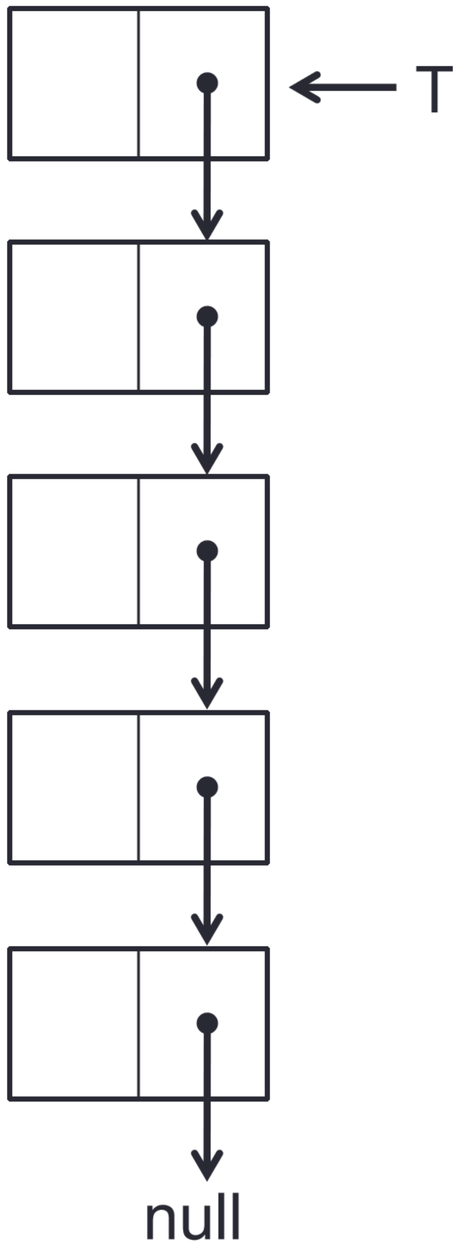
Stack Interface

```
public interface Stack<T>{
public T pop( );
public void push(T e);
public boolean empty( );
public boolean full( );
}
```

ADT Stack (Linked-List)



ADT Stack (Linked-List): Element

```
public class Node<T> {
  public T data;
  public Node<T> next;

  public Node () {
        data = null;
        next = null;
  }

  public Node (T val) {
        data = val;
        next = null;
  }

  // Setters/Getters?
}
```

ADT Stack (Linked-List): Implementation

```
public class LinkedStack<T> implements Stack<T> {
 private Node<T> top;

 /* Creates a new instance of LinkStack */
 public LinkedStack() {
      top = null;
 }
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ADT Stack (Linked-List): Implementation

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 public LinkedStack() {
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 }
```

null ⟵ T

ADT Stack (Linked-List): Implementation

```
public boolean empty(){
    return top == null;
}

public boolean full(){
    return false;
}
```

ADT Stack (Linked-List): Implementation

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public boolean empty(){
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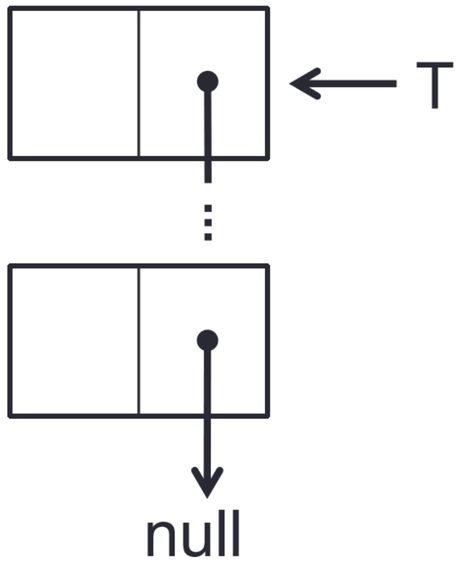
```

false




```
public boolean full(){
    return false;
}
```

true

ADT Stack (Linked-List): Implementation

```
public void push(T e){ O(1)
    Node<T> tmp = new Node<T>(e);
    tmp.next = top;
    top = tmp;
}
```

ADT Stack (Linked-List): Implementation

```
public void push(T e){
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null ⟵ T

Example #1

ADT Stack (Linked-List): Implementation

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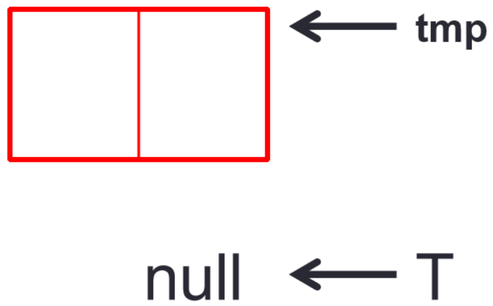

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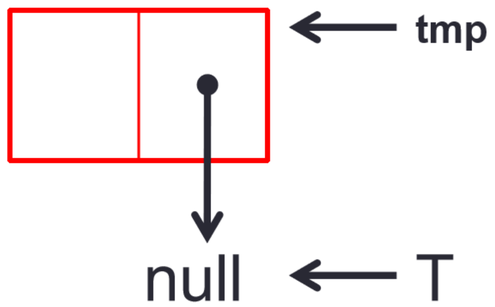

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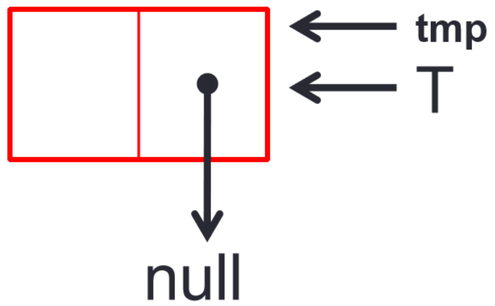

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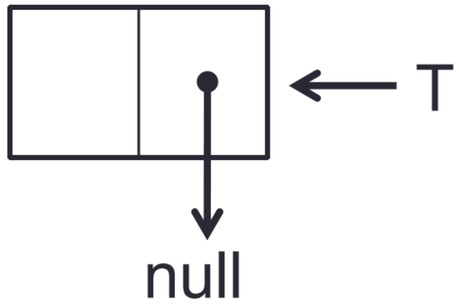

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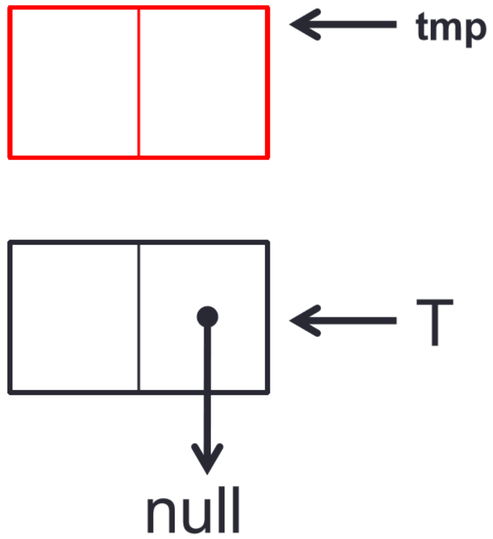

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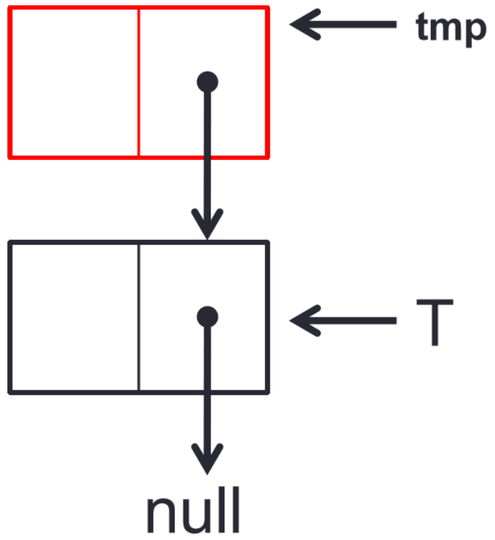

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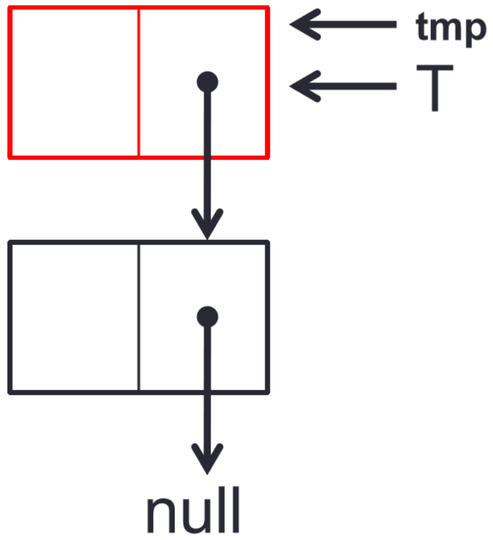

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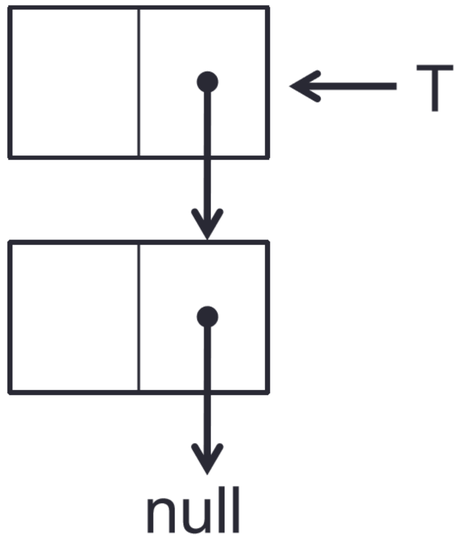

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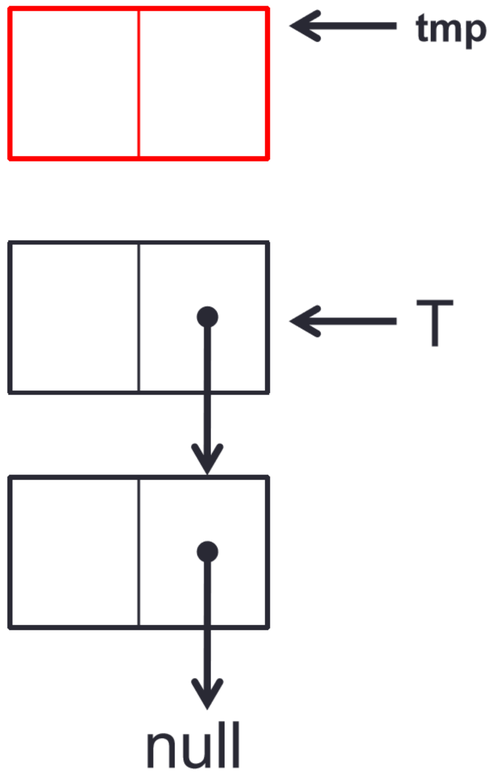

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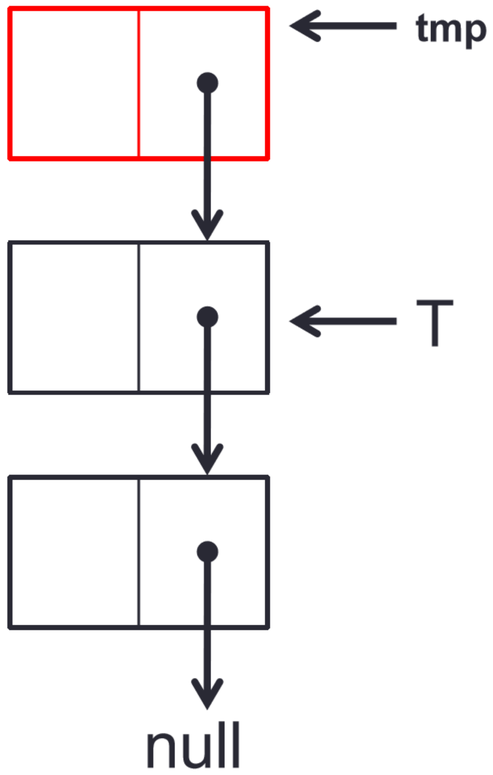

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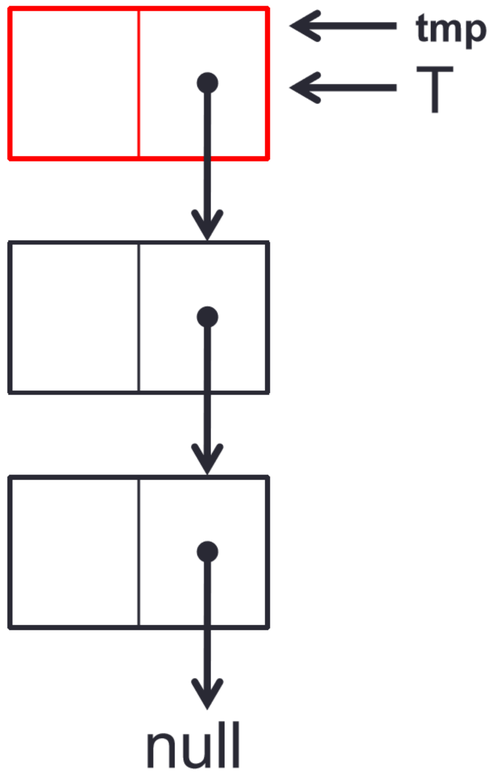

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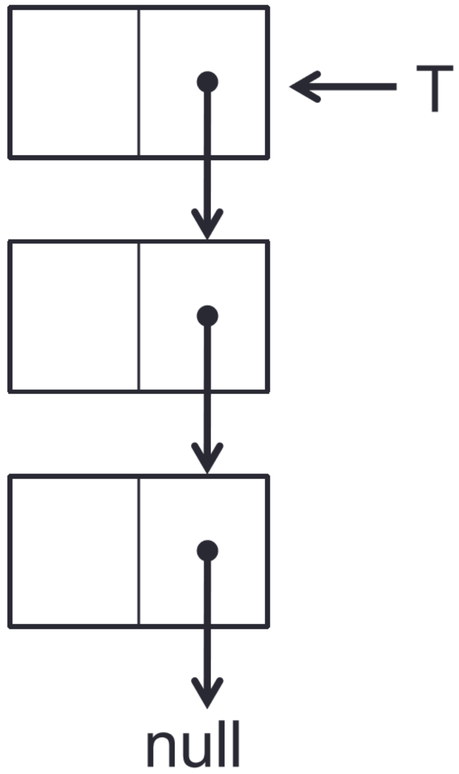

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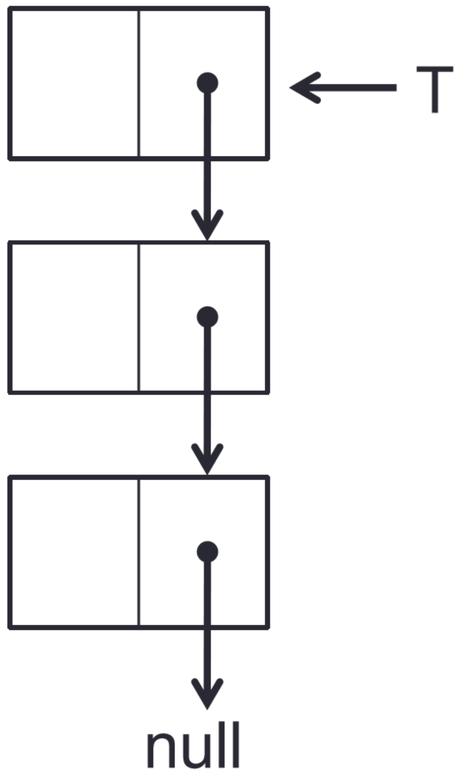

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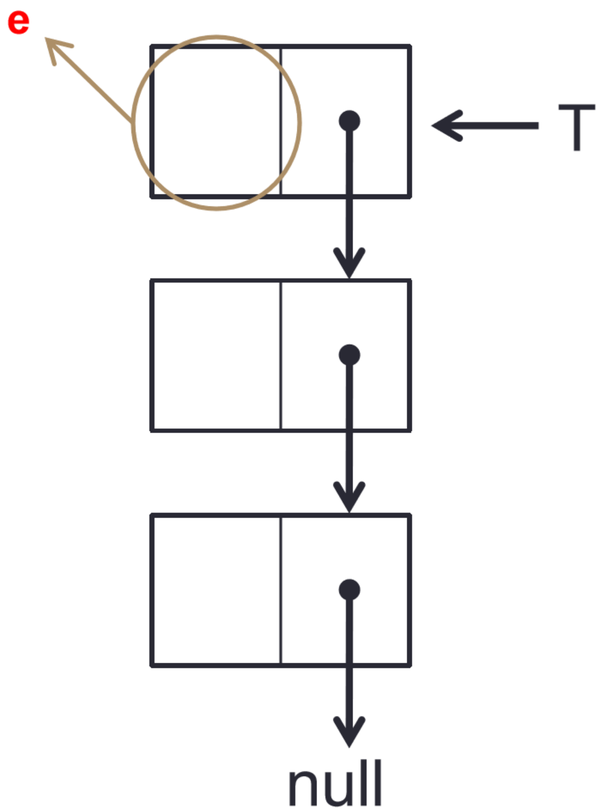

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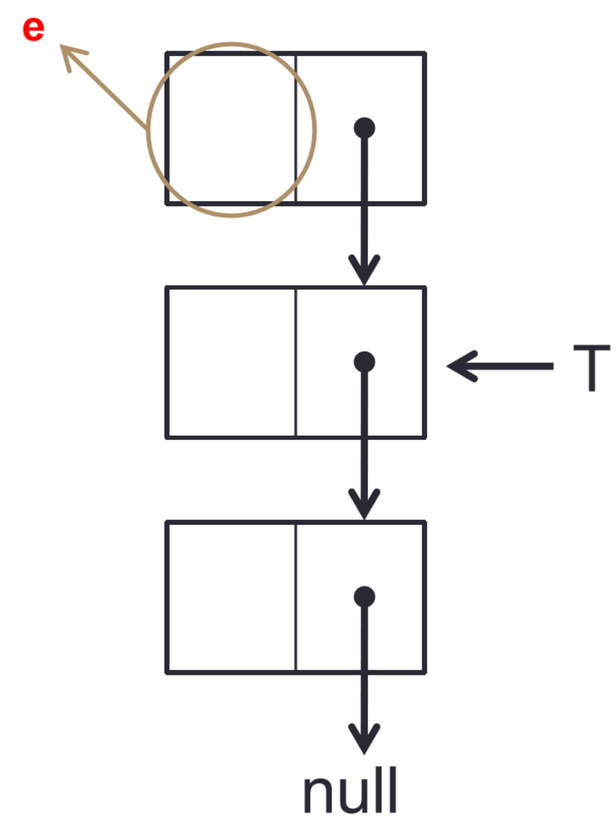

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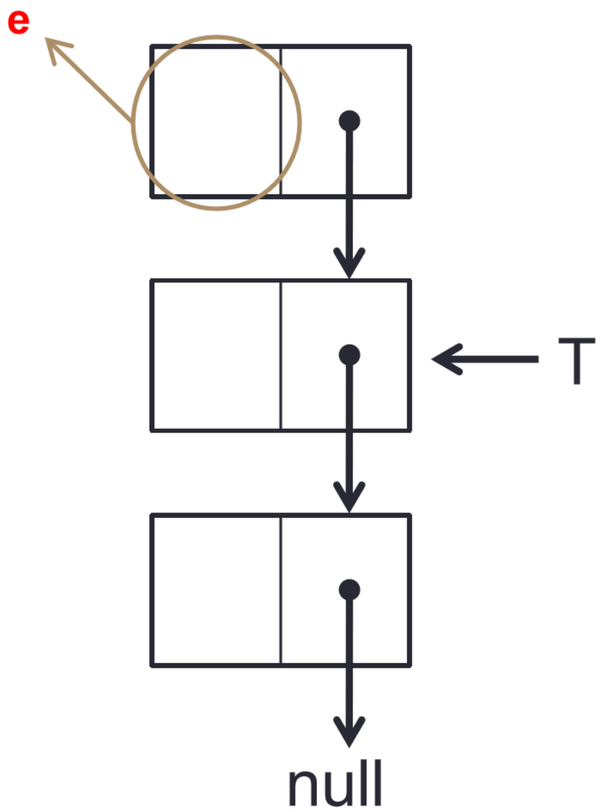

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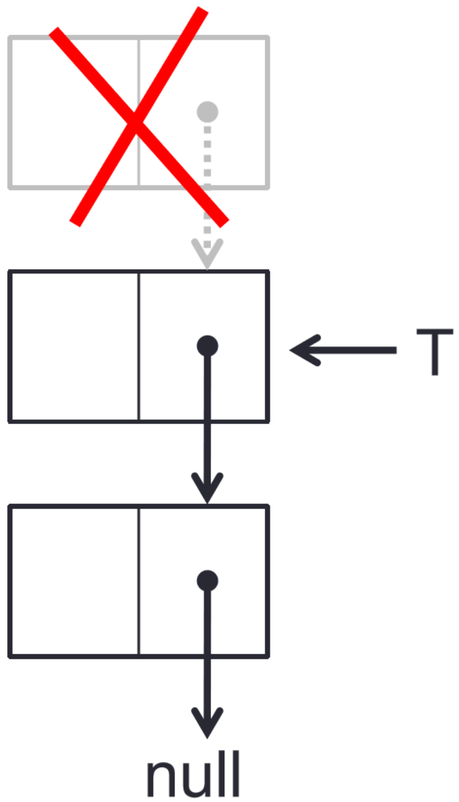

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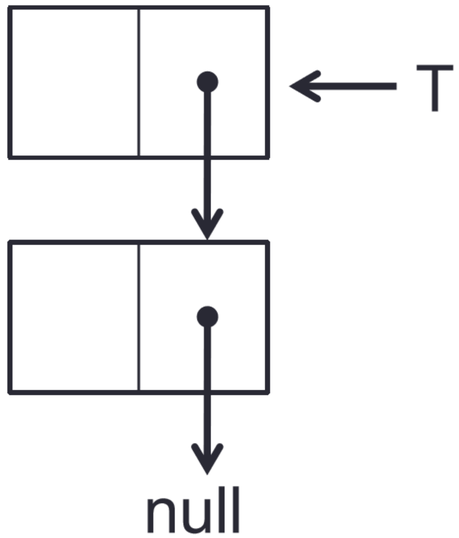

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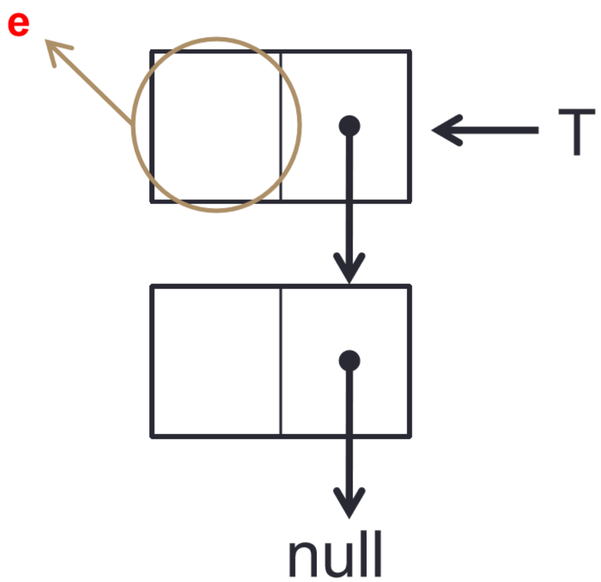

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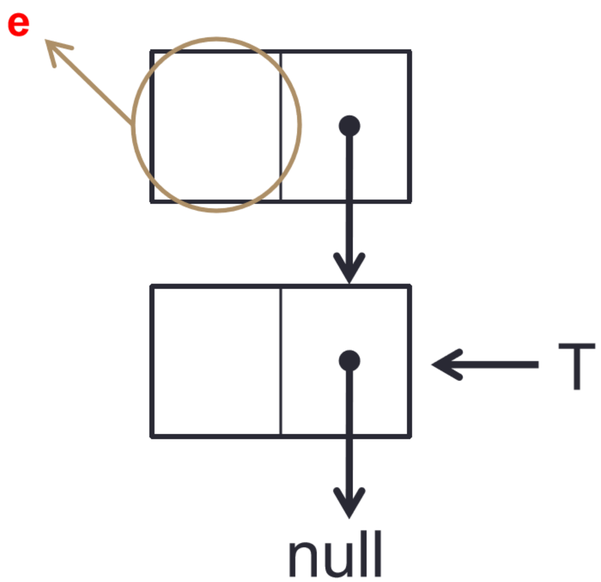

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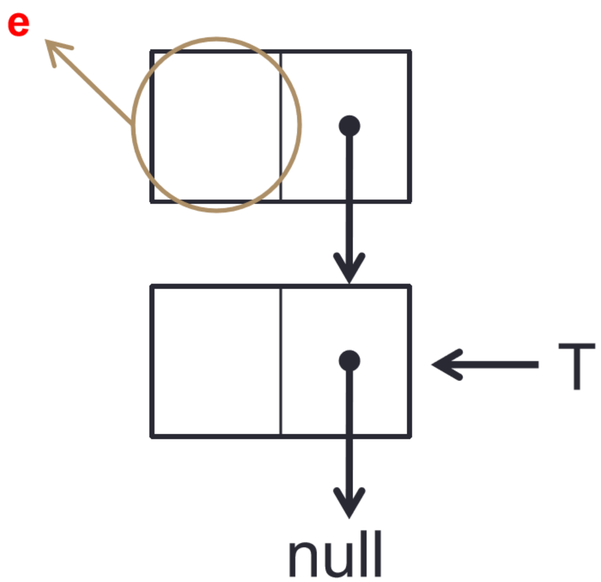

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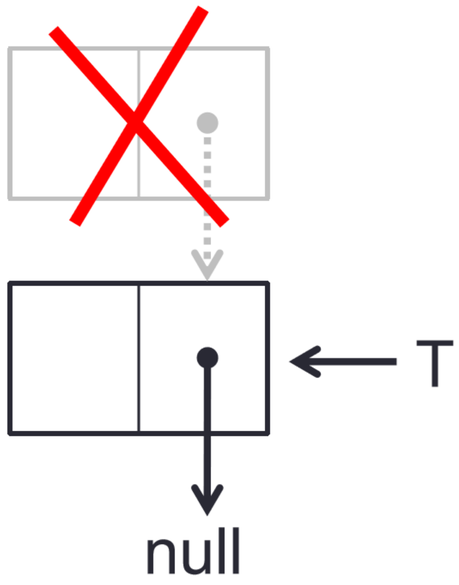

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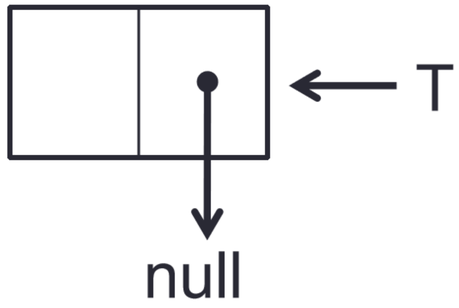

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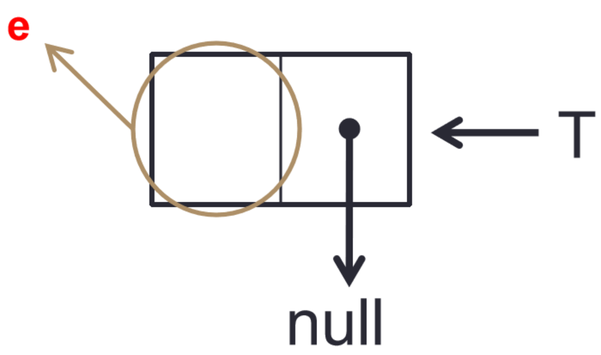

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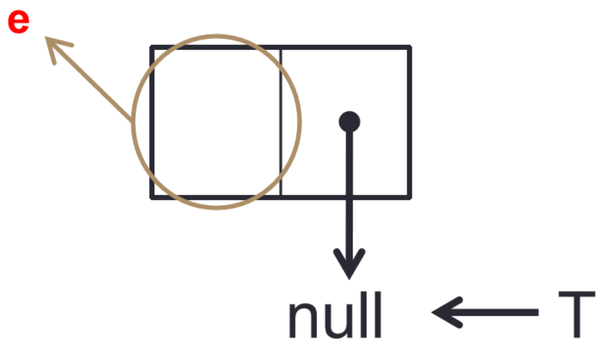

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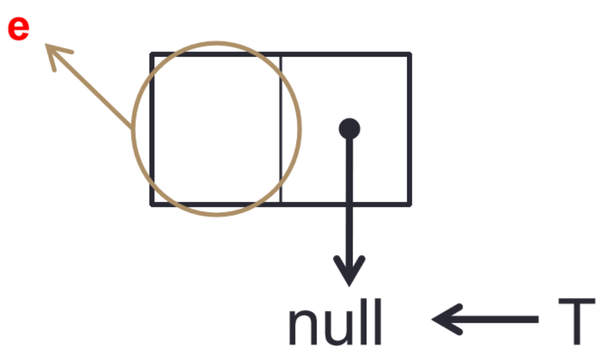

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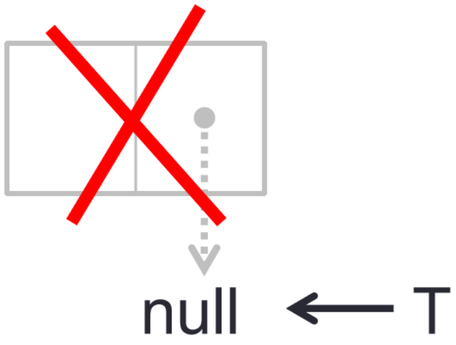

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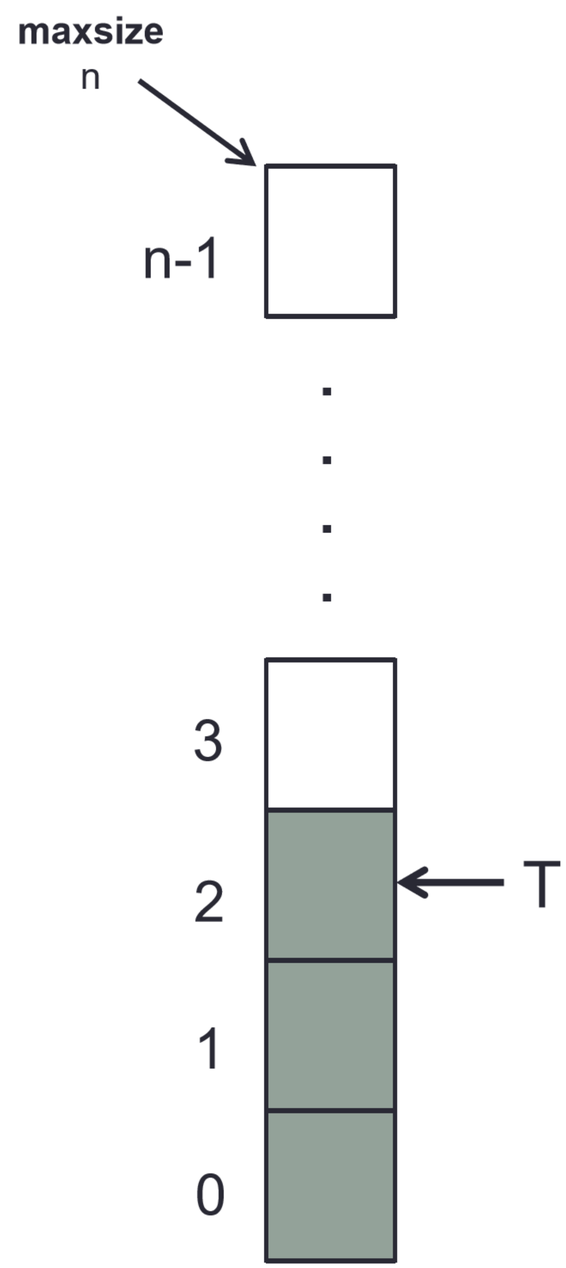
```

null ⟵ T

Example #3

ADT Stack (Array)



ADT Stack (Array): Representation

```
public class ArrayStack<T> implements Stack<L>  {
  private int maxsize;
  private int top;
  private T[] nodes;

  /** Creates a new instance of ArrayStack */
  public ArrayStack(int n) {
        maxsize = n;
        top = -1;
        nodes = (T[]) new Object[n];
  }
```

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```
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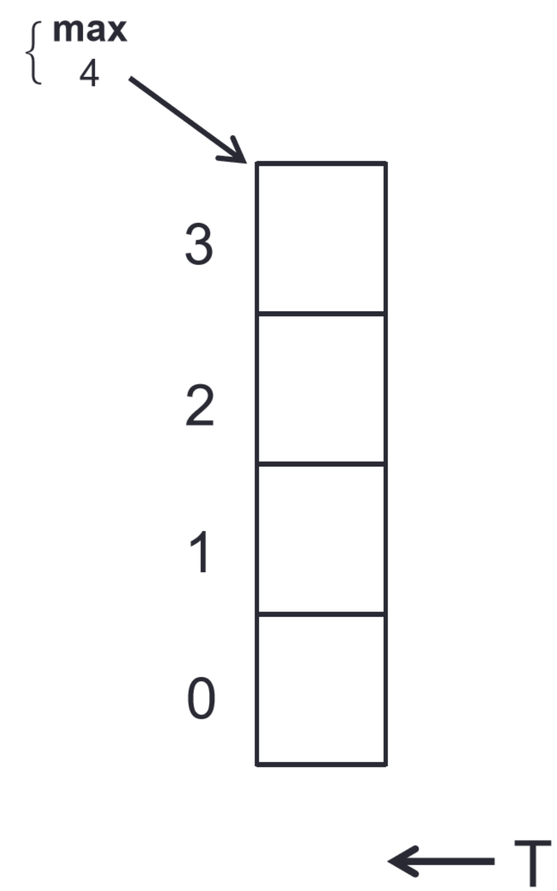


ADT Stack (Array): Implementation

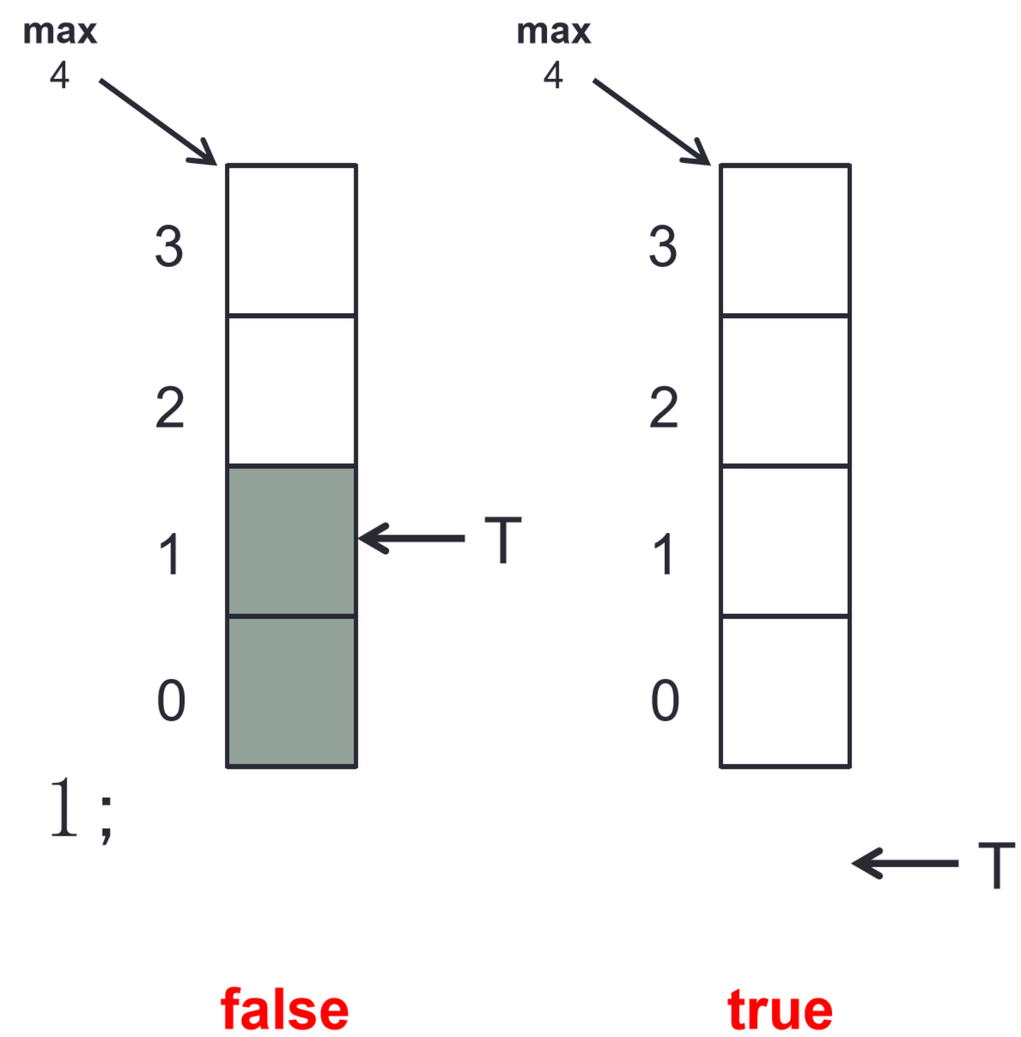
```
public boolean empty(){
    return top == -1;
}
```

top == 0 ?

```
public boolean full(){
    return top == maxsize -1;
}
```

ADT Stack (Array): Implementation

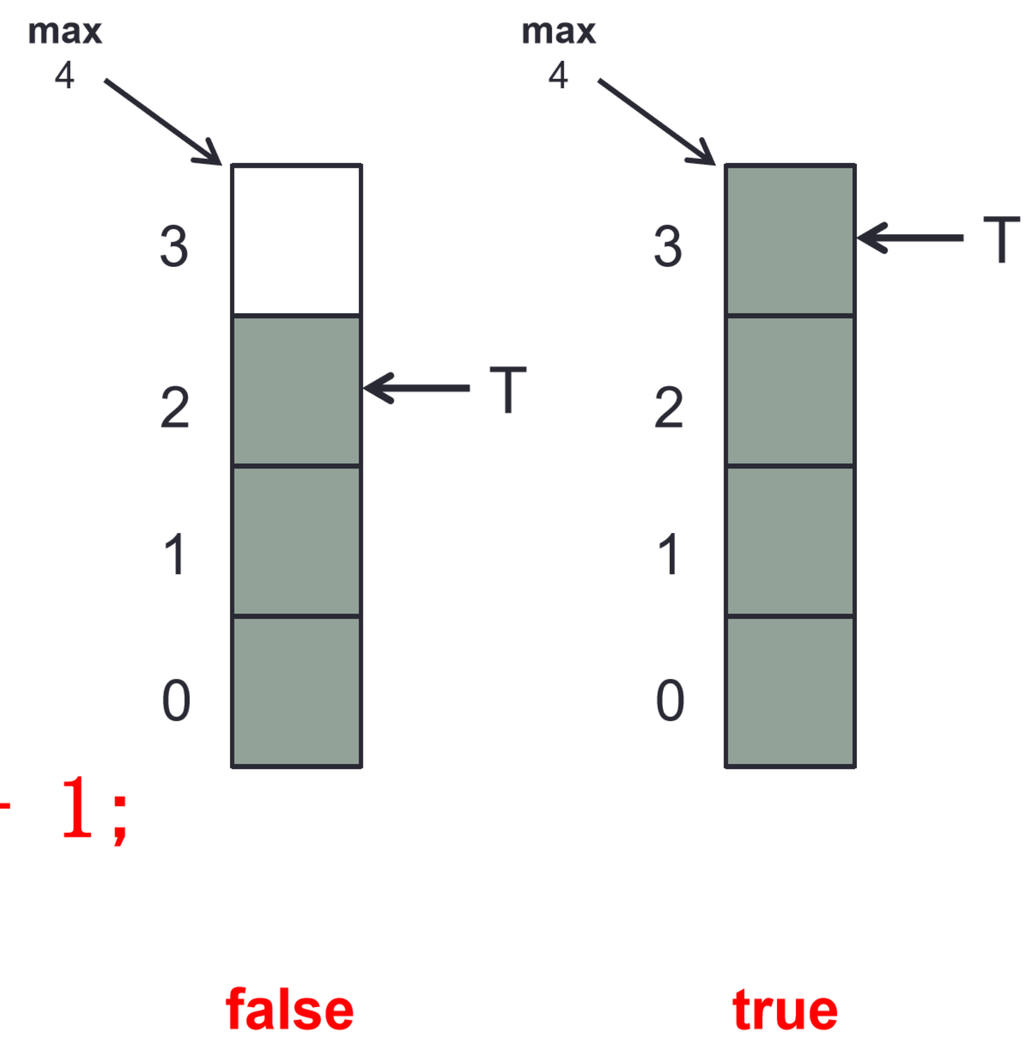
```
public boolean empty(){
    return top == -1;
}

public boolean full(){
    return top == maxsize - 1;
}
```



ADT Stack (Array): Implementation

```
public boolean empty() {
    return top == -1;
}
```

```
public boolean full() {
    return top == maxsize - 1;
}
```



ADT Stack (Array): Implementation

```
public void push(T e){
    nodes[++top] = e;
}

 public T pop(){
    return nodes[top--];
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}
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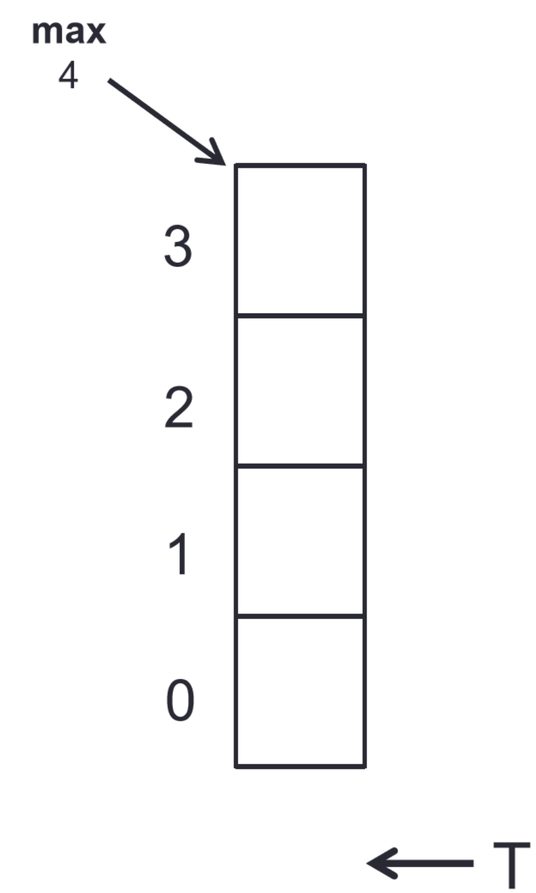

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```
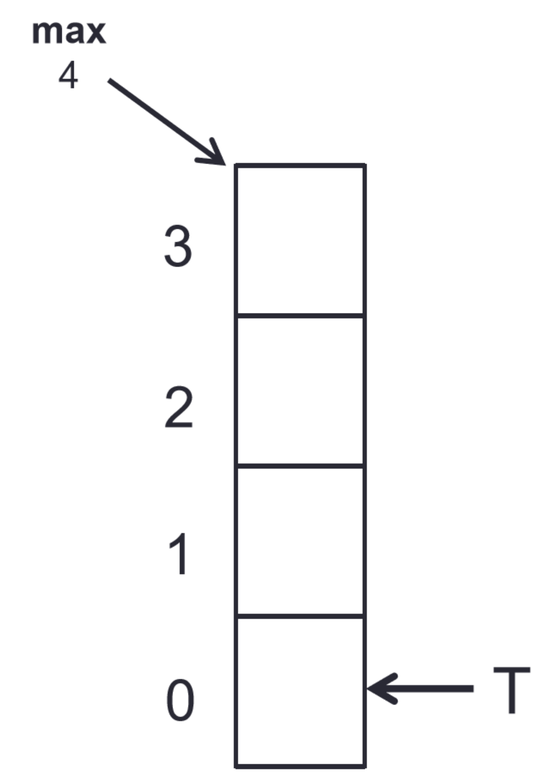

S1 ⟵ ++top
nodes[S1] = e



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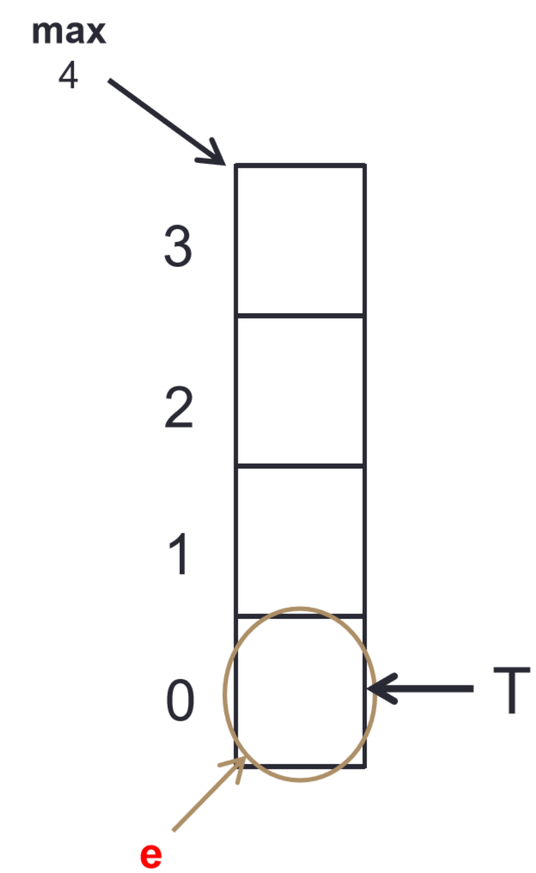

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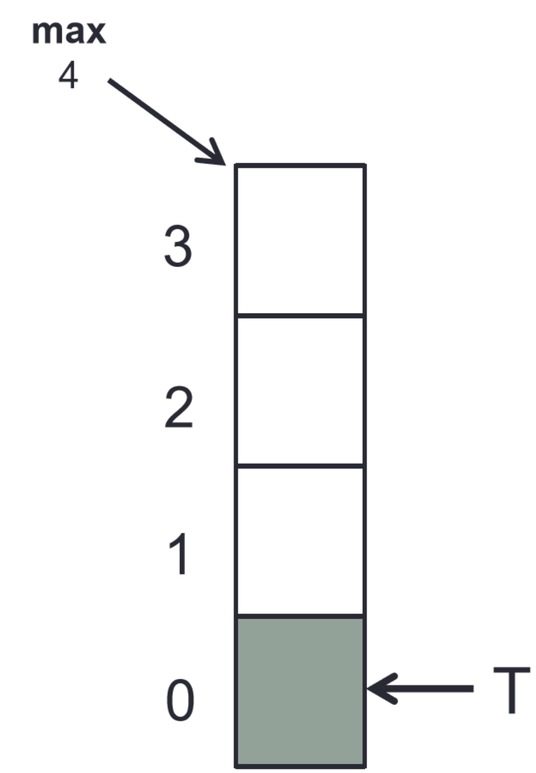

Example #2

ADT Stack (Array): Implementation

```
public void push(T e){
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```

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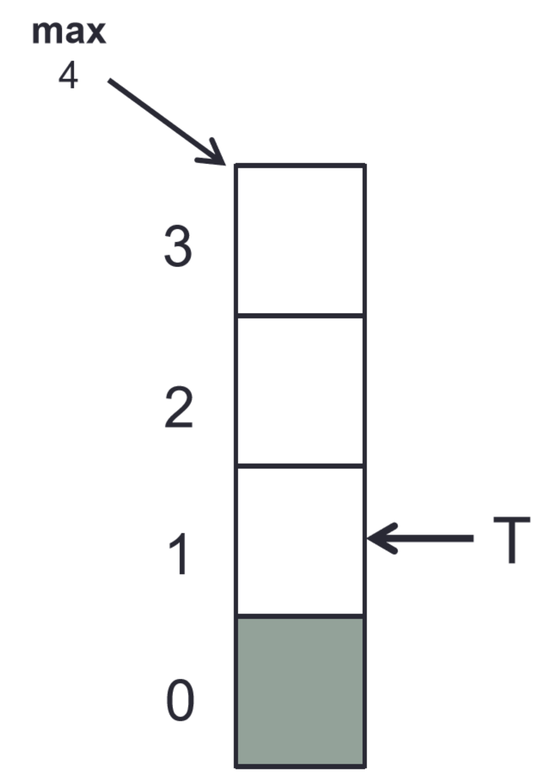

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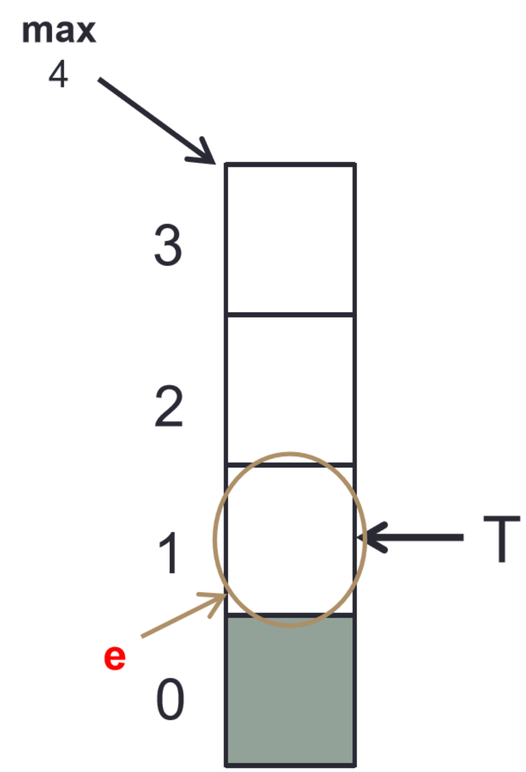

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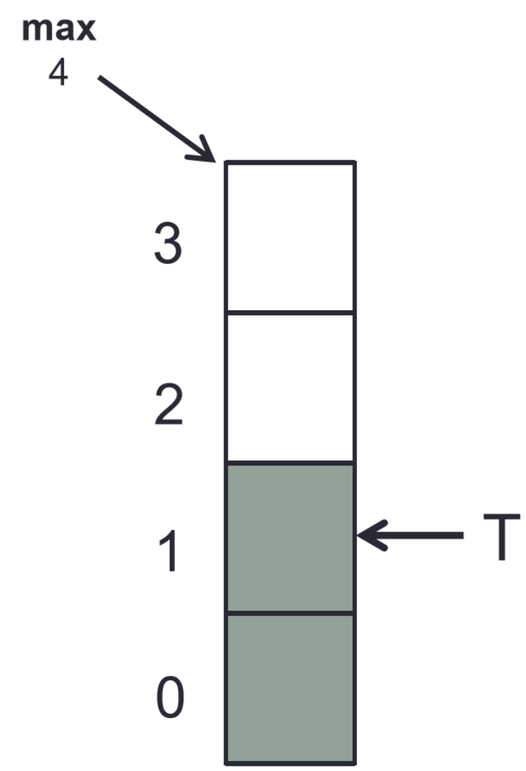


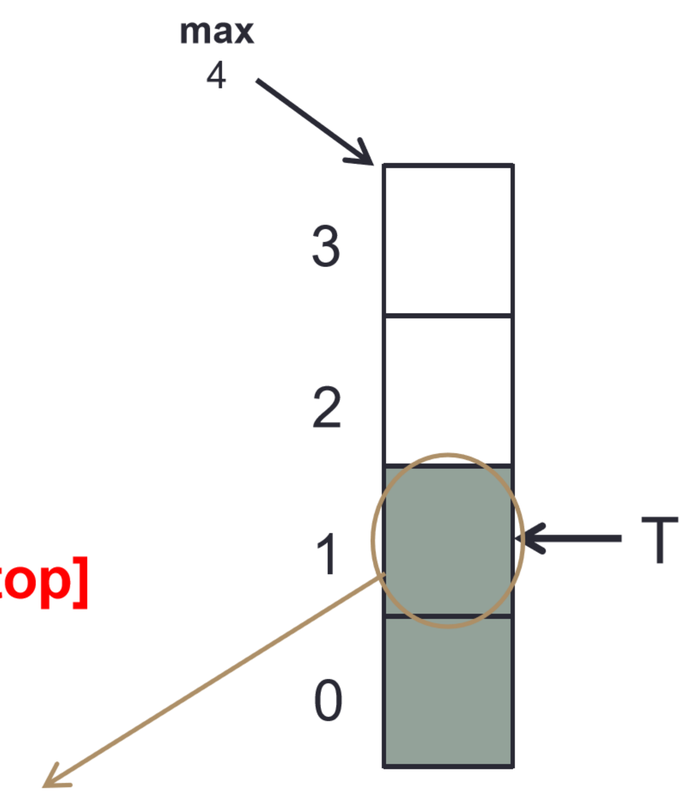
Example #3

ADT Stack (Array): Implementation

```
public void push(T e){
        nodes[++top] = e;
}

  public T pop(){
        return nodes[top--];
  }
}
```

S1 ⟵ nodes[top]
top--
return S1



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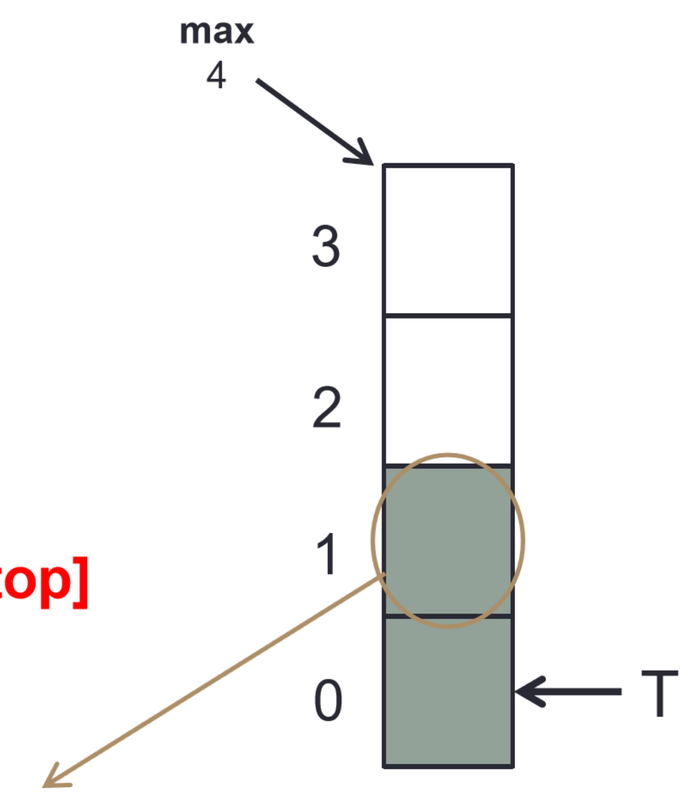

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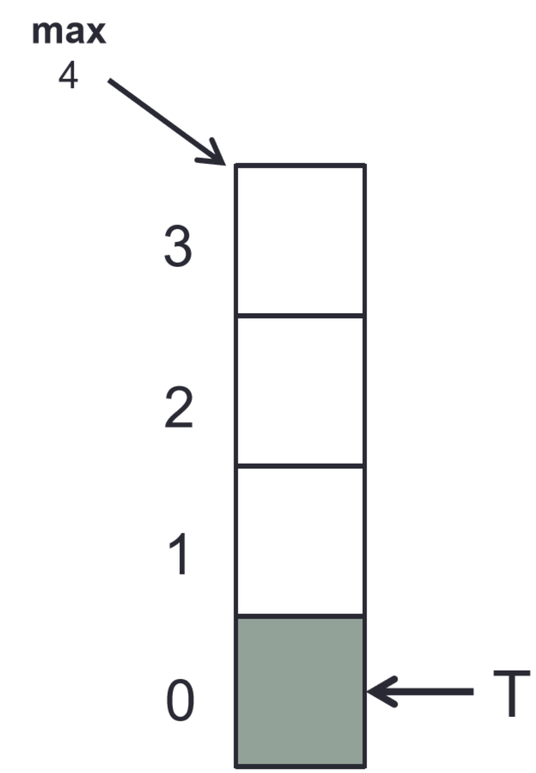

Example #4

ADT Stack (Array): Implementation

```
public void push(T e){
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}

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  }
}
```

S1 ⟵ nodes[top]
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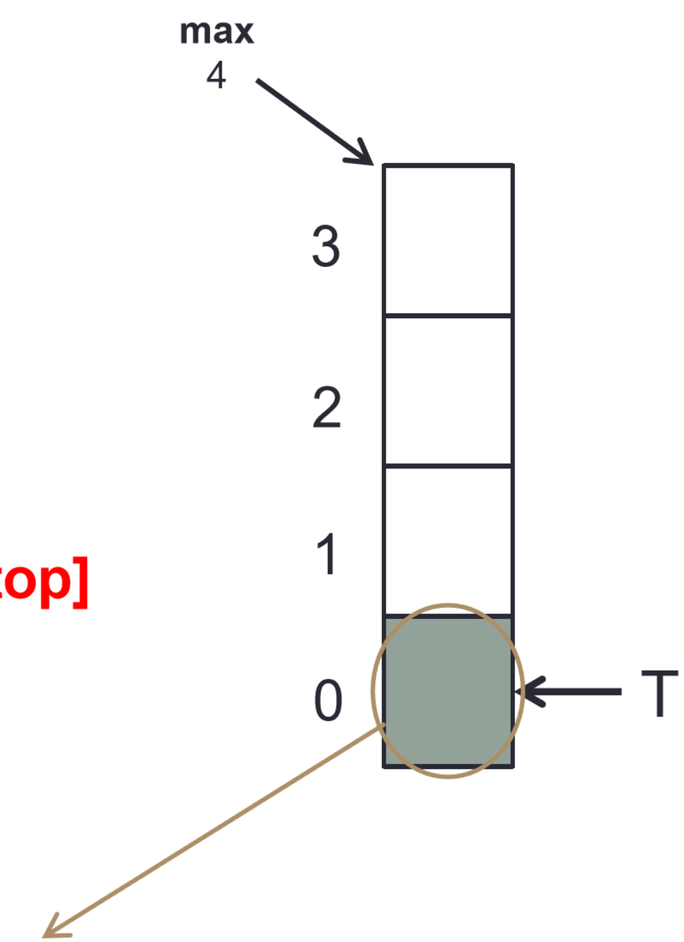

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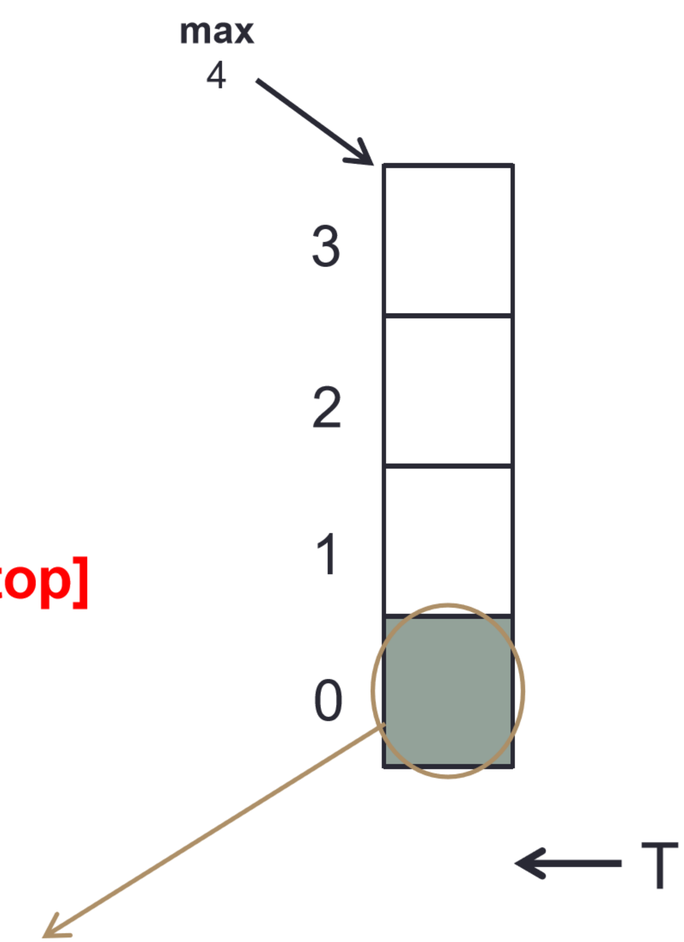
return S1



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  }
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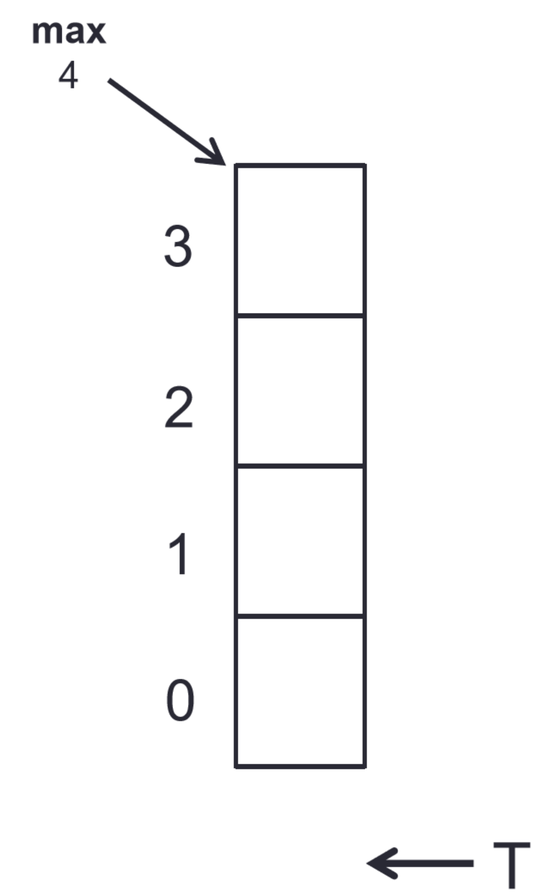


Example #4

Applications of Stacks

- Some applications of stacks are:
  - Balancing symbols.
  - Computing or evaluating postfix expressions.
  - Converting expressions from infix to postfix.

1. Balancing Symbols → not closing parenthesis

- Expressions: mathematical (a + ((b-c)*d)) or programs have **delimiters**.

```
begin
        S1
        S2
        begin
                S3
                begin
                ....
                end
        end
end
```

```
{
        S1
        {
                S2
                S3
        }
        S4
}
```

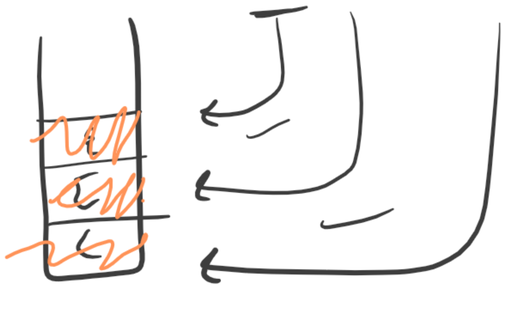
1. Balancing Symbols

- Delimiters must be balanced.
- One of the common use of the stacks is to parse certain kinds of expressions or string text.
- Write a program that verifies the delimiters in a line of text or expression typed by the user.
  - a*(b+c) //This expression is right
  - b/[a*(b+c)] //This expression is right
  - {a*(b+c]} //This expression is wrong

1. Balancing Symbols

(a+[b/c+{a*y}]+d)

if the stack is empty



- Read characters from the start of the expression to the end.

  ([{

  - If the token is a starting delimiter, then push on to the stack.

)] }

  - If the token is a closing delimiter, then pop from the stack.
    - If symbol from this pop operation matches the closing delimiter, then we carry on.
    - If not, or the stack was empty, then we have unbalanced symbols (report an error).
- If stack is empty at the end of expression, we have balanced symbols.
- If not (stack is not empty), then we have unbalanced symbols (report an error).

1. Balancing Symbols

- **Input** : expression
- **Output**: True if and only if delimiters are balanced
- Let S be empty Stack
- Let n be number of characters
- for i=0→ n-1
  - If expression[i] is a **Opening delimiter**, then
    - S.push(expression[i]).
  - else If expression[i] is a **closing delimiter,** then
    - **If** the S is empty
      - return false unbalanced symbols
    - symbol=S.pop().
    - If symbol does not matches the closing delimiter
      - return false unbalanced symbols.
- If S is empty
  - return true balanced symbols.
- else
  - return false unbalanced symbols

```
boolean isEmpty(Token[] tmp, int n) {
stack<Token> st = new LinkedStack(<>);
for(int i=0; i<n; i++){
if(tmp[i].equals("<"||"["||"("))
    st.push(tmp[i]);
if(tmp[i].equals(">"||"]"||")")){
    if(st.empty())
        return false;
Token e=st.pop();
if(!e.equals(tmp[i]))
    return false
}
}
return st.empty();
```

2. Postfix Expressions

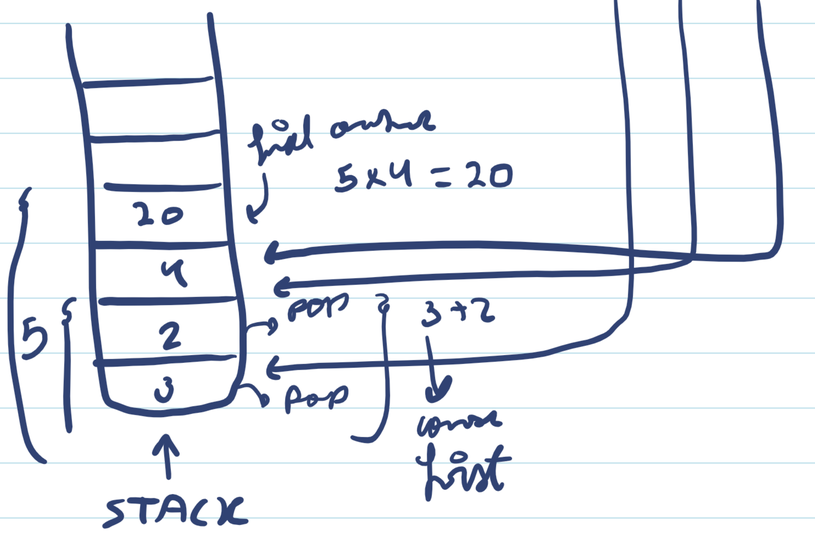
- Evaluating Postfix Expressions:
  - Infix expression: 4.99*1.06+5.99+6.99*1.06 → infix "normal"
    normal way
  - Value 18.69 correct← parenthesis used.
  - Value 19.37 incorrect ← no parenthesis used.
  - In postfix form, above expression becomes:

    4.99 1.06 * 5.99 + 6.99 1.06*+ → operands then expression

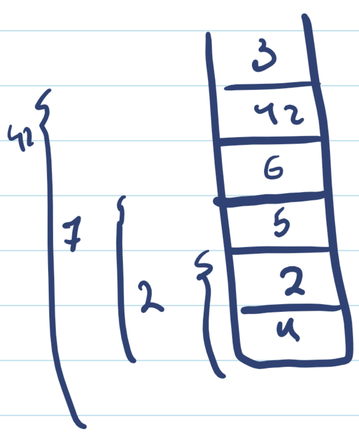
→Advantage: no brackets are needed and a stack can be used to compute the expression.

Post → after the numbers

Postfix: $(3+2)\times 4 \rightarrow 3\sim 2+4\times$



$4\sim 2-5+6\times 3-$



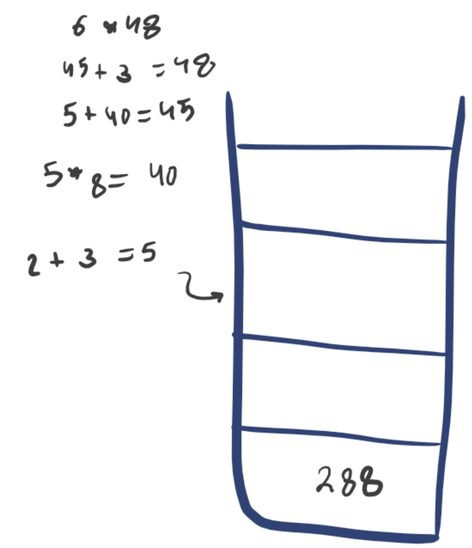
$42 - 3 = 39$

7×6

$2+5=7$

$4-2=2$

2. Postfix Expressions



- Example:
  - infix: 6*(5+((2+3)*8)+3)
  - postfix: 6 5 2 3 + 8 * + 3 + *.
- Algorithm to compute postfix expression:
  - Read the postfix expression left to right.
  - When a number is read push it on the stack.
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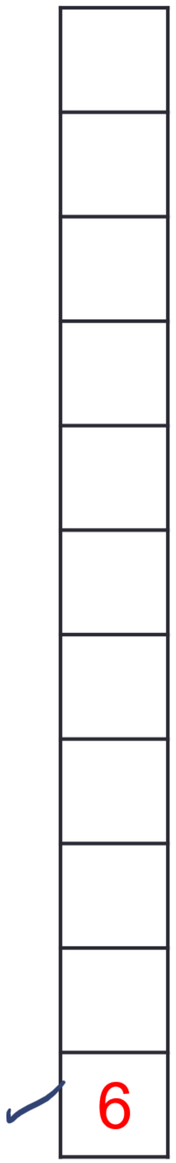

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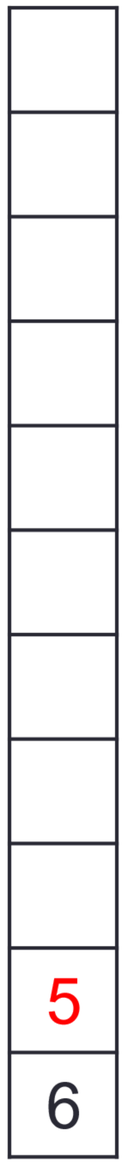

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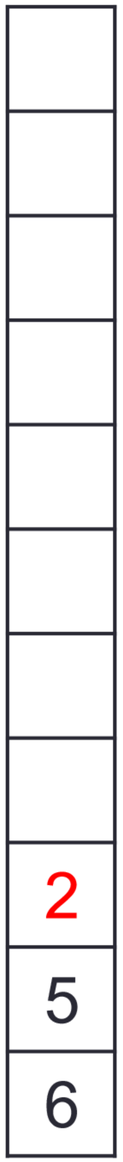

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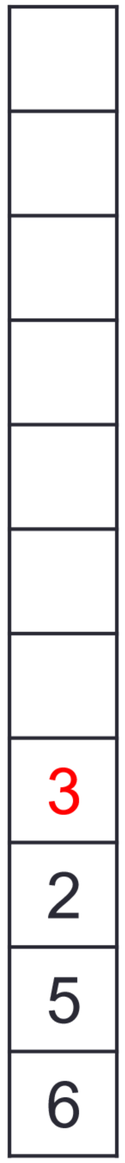

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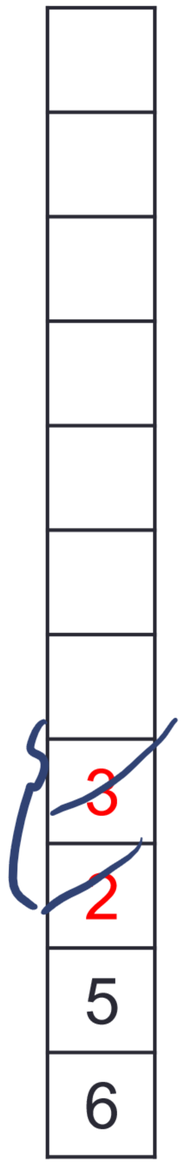

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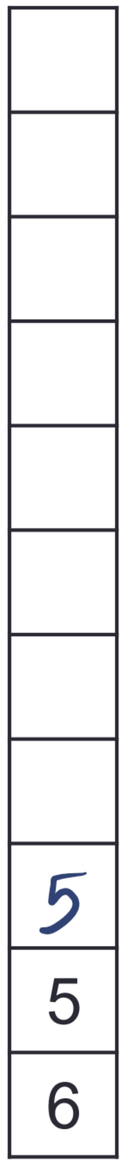
2 + 3 = 5



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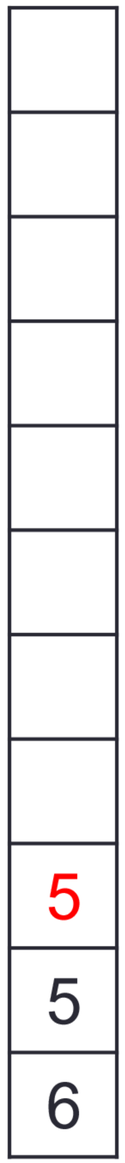

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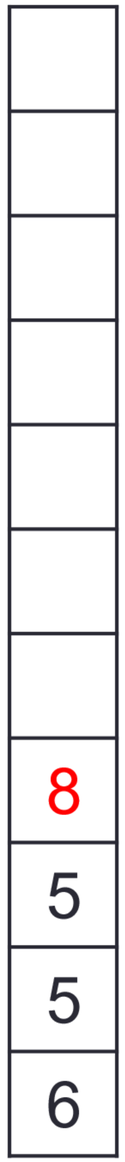

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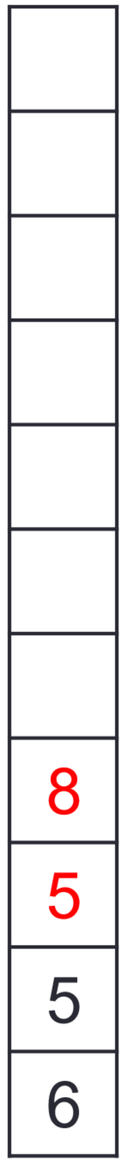

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5 * 8 = 40

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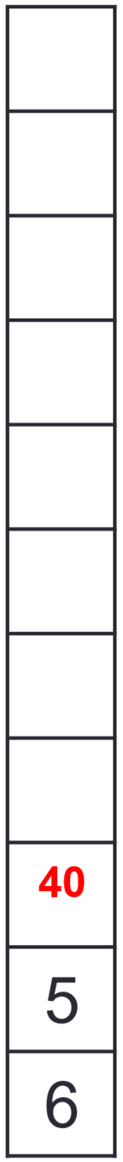
5 * 8 = 40



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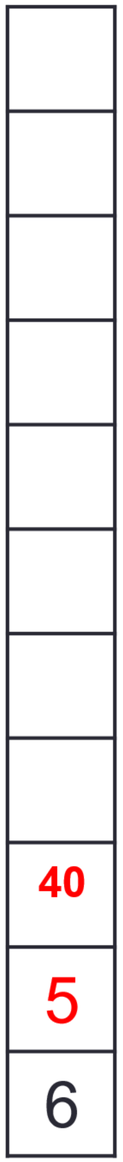

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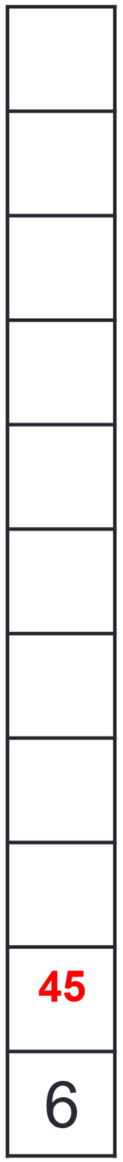

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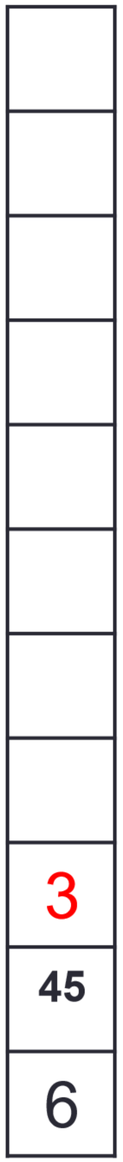

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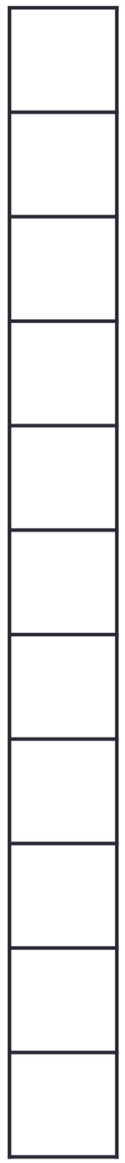

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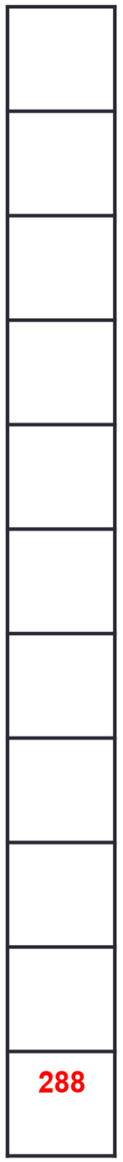

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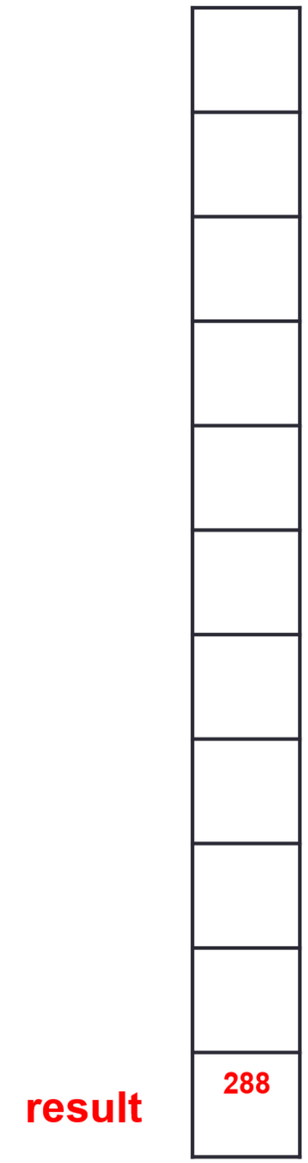
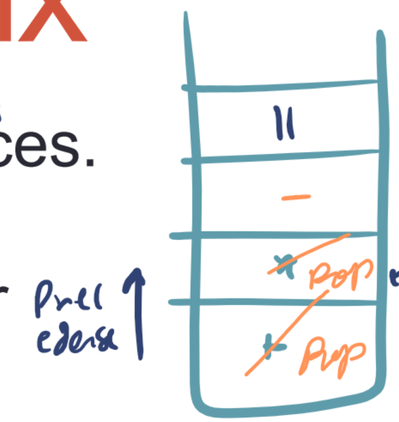


2+4 * 3 - 11

243 * + 11 -

3. Converting from infix to postfix



Assume the infix expression is a string of tokens delimited by spaces. The operator tokens are *, /, +, and -, along with the left and right parentheses, (with). The operand tokens are the single-character identifiers A, B, C, and so on.

The following steps will produce a string of tokens in postfix order.

1. Create an **empty stack** called opstack for keeping operators. Create an **empty list** for output.
   to hold the operations
2. Scan the token list from left to right.
   - If the token is an operand, **append** it to the end of the **output list**.
   - If the token is a left parenthesis, **push** it on the **opstack**.
   - If the token is a right parenthesis, **pop** the **opstack** until the corresponding left parenthesis is removed. Append each operator to the end of the output list.
   - If the token is an operator, *, /, +, or -, push it on the opstack. However, first remove any operators already on the opstack that have higher or equal precedence and **append** them to the **output list**.
3. When the input expression has been completely processed, check the opstack. Any operators still on the stack can be removed and appended to the end of the output list.

2*(3+4)+2

234+*2+

STACK OPERATIONS

CS212: Data Structure

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine → stack!
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

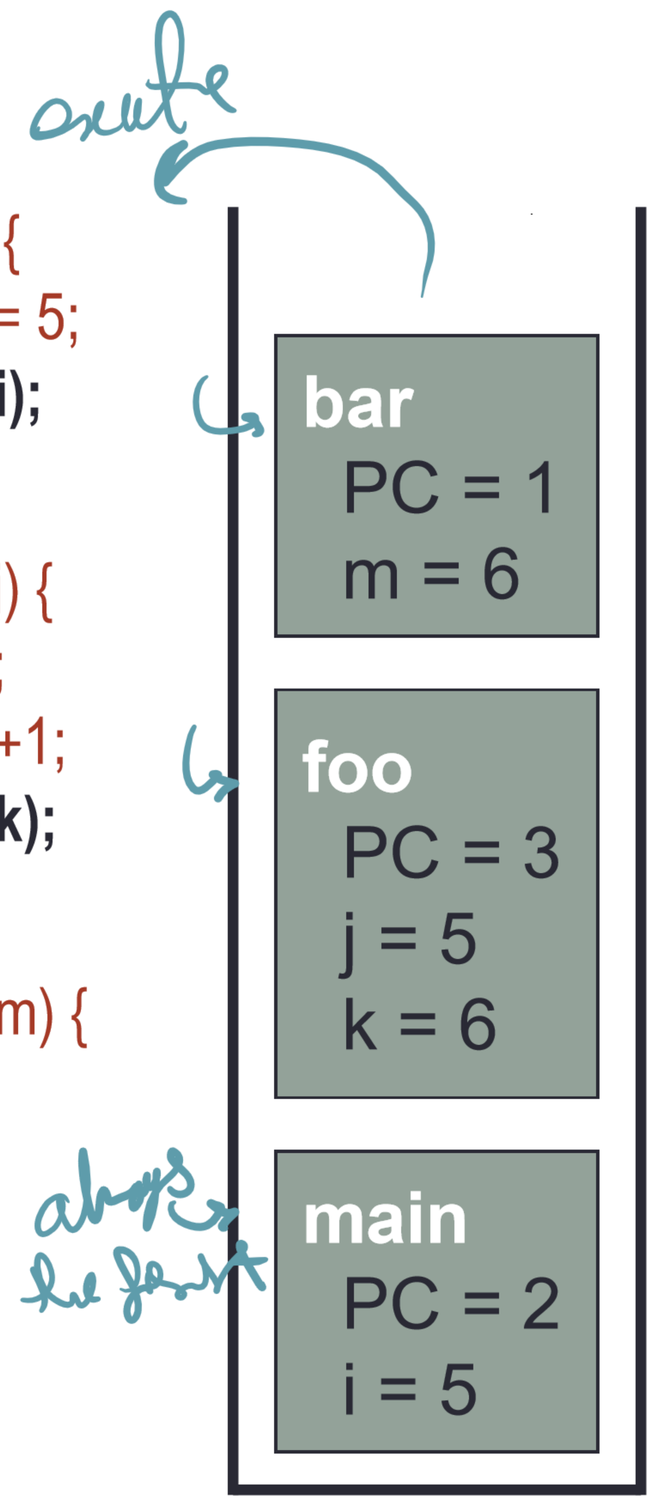
Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed

line we stopped at

- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for recursion

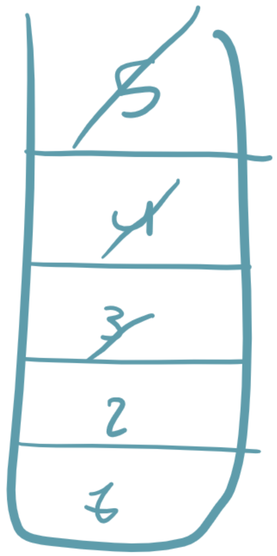
```
main() {
    int i = 5;
    foo(i);
}

foo(int j) {
    int k;
    k = j+1;
    bar(k);
}

bar(int m) {
    ...
}
```



Reverse a List using Stack

```
public class Tester {

  // ··· other methods here

  public void intReverse(List<Integer> l) {
      Stack<Integer> s = new Stack<Integer>();

      l.findFirst();
      while(!l.empty()) {
          s.push(l.retrieve());
          l.remove();
      }

      while(!s.empty())
          l.insert(s.pop());
  }
}
```

1 → 2 → 3 → 4 → 5



5 → 4 → 3 → 2 → 1

Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “]”
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )))){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

```
Algorithm ParenMatch(X, n):
  Input: An array X of n tokens, each of which is either a grouping symbol,
  a variable, an arithmetic operator, or a number
  Output: true if and only if all the grouping symbols in X match
  Let S be an empty stack
  for i=0 to n-1 do
        if X[i] is an opening grouping symbol then
              S.push(X[i])
        else if X[i] is a closing grouping symbol then
              if S.isEmpty() then
                    return false {nothing to match with}
              if S.pop() does not match the type of X[i] then
                    return false {wrong type}
  if S.isEmpty() then
        return true {every symbol matched}
  else
        return false {some symbols were never matched}
```

HTML Tag Matching

For fully-correct HTML, each <name> should pair with a matching </name>

```
<body>
  <center>
        <h1> The Little Boat </h1>
  </center>
  <p> The storm tossed the little
  boat like a cheap sneaker in an
  old washing machine. The three
  drunken fishermen were used to
  such treatment, of course, but
  not the tree salesman, who even as
  a stowaway now felt that he
  had overpaid for the voyage.</p>
  <ol>
        <li> Will the salesman die? </li>
        <li> What color is the boat? </li>
        <li> And what about Naomi? </li>
  </ol>
</body>
```

The Little Boat

The storm tossed the little boat like a cheap sneaker in an old washing machine. The three drunken fishermen were used to such treatment, of course, but not the tree salesman, who even as a stowaway now felt that he had overpaid for the voyage.

1. Will the salesman die?
2. What color is the boat?
3. And what about Naomi?

Evaluating Arithmetic Expressions

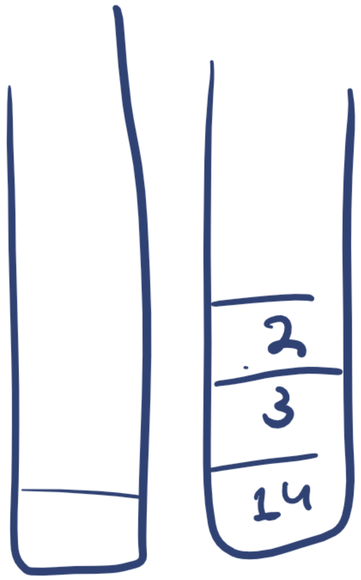
14 – 3 * 2 + 7 = (14 – (3 * 2)) + 7

14 3 2

3 * 2

14 – 6 = 8

8 + 7 = 15



Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: (x – y) + z rather than x – (y + z)

14 3 2 * - 7 +

Idea: push each operator on the stack, but first pop and perform higher and *equal* precedence operations.

Algorithm for Evaluating Expressions

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

when you reach it you have to do all operations

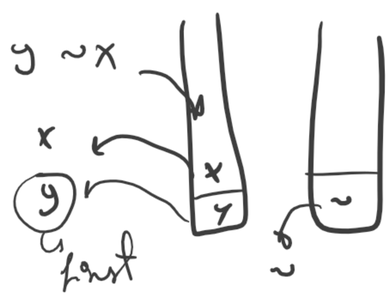
Algorithm doOp()

x ← valStk.pop();

y ← valStk.pop();

op ← opStk.pop();

valStk.push(y **op** x)



Algorithm repeatOps(refOp):

while (valStk.size() > 1 ∧ → and

not empty

prec(refOp) ≤ prec(opStk.top())

doOp() → of the next precedence ≤ the current

→ do the operation

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

Output: the value of the expression

while there's another token z → number → op.

if isNumber(z) **then**

valStk.push(z)

else

repeatOps(z);

opStk.push(z)

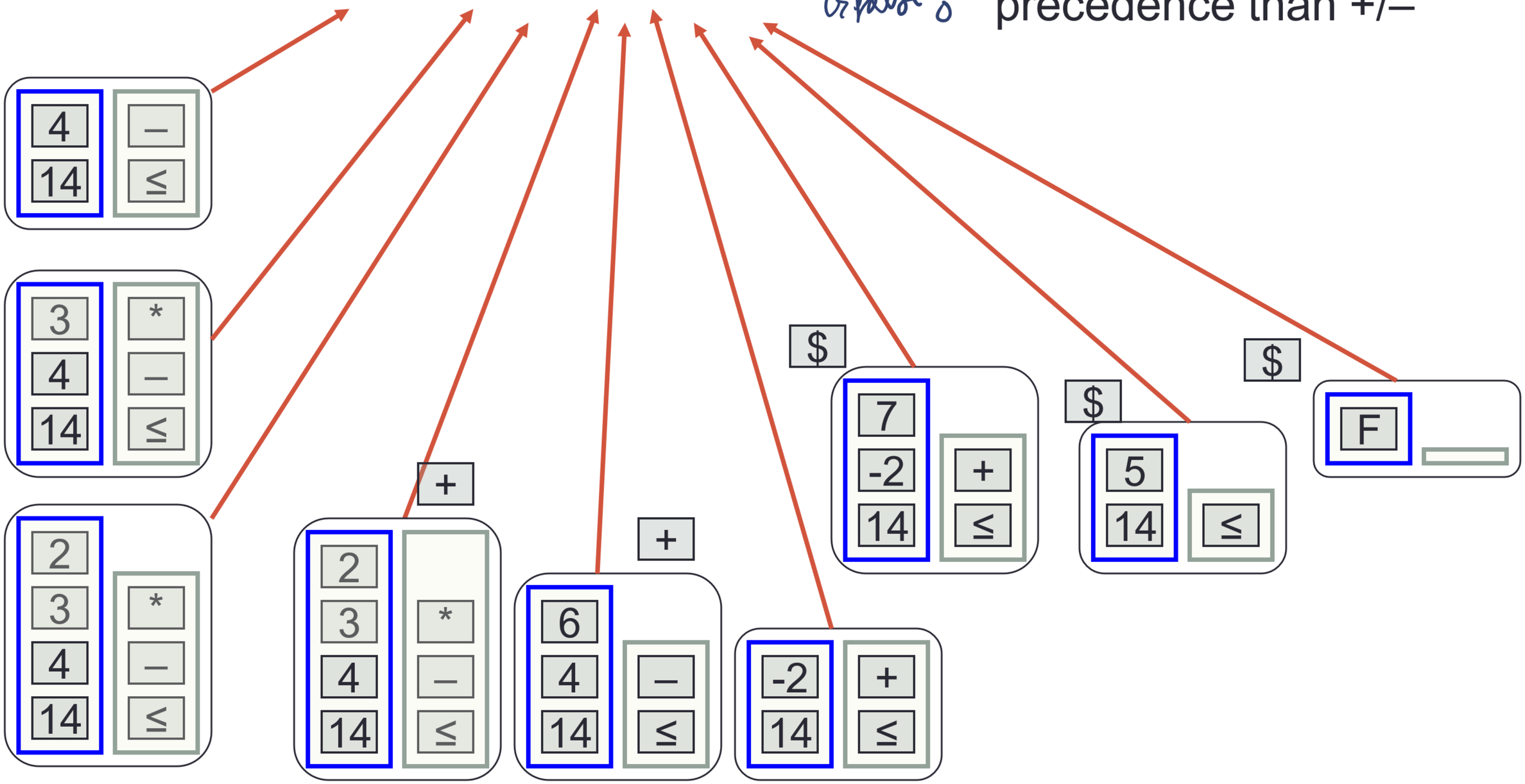
repeatOps($); → to complete all the other operation

return valStk.top()

returns the value of the top elt! w/o changing the stack

Algorithm on an Example Expression

14 ≤ 4 – 3 * 2 + 7

3*2=6
4 - 6 = -2
-2+7 = 5
14 ≤ 5
false!

Operator ≤ has lower precedence than +/–