
HOMework 1

Problem1:

1-

function	Big oh	order
$4n \log n + 2n$	$O(n \log n)$	6
2^{10}	$O(1)$	1
$2^{\log n} = n^{\log 2} = n$	$O(n)$	2
$3n + 100 \log n$	$O(n)$	3
$4n$	$O(n)$	4
2^n	$O(2^n)$	9
$n^2 + 10n$	$O(n^2)$	7
n^3	$O(n^3)$	8
$n \log n$	$O(n \log n)$	5

2-

$$\log n^{2n} + n^2 \leq cn^2 \quad \forall n \geq n_0$$

$$\log n^{2n} \leq c^2 - n^2$$

$$\log n^{2n} \leq (c - 1)n^2$$

$$\frac{\log n^{2n}}{(c-1)} \leq n^2$$

which is true for $c = 2$ when $n_0 = 4$

3-

$$\sum_{i=1}^5 i^3 \leq c(1)$$

$$(n(n+1)/2)^2 = 225 \leq c$$

which is true for $c = 225$ when $n_0 = 1$

4-

$$\sum_{i=1}^n [\log i] = \log(1) + \log(2) + \log(3) + \dots + \log(n-1) + \log(n)$$

From Logarithm rules: $\log(ac) = \log(a) + \log(c)$

$$\sum_{i=1}^n [\log i] = \log(1 * 2 * 3 * \dots * (n-1) * n) = \log(n!) \leq c(n \log n)$$

Which is true for $c=1$ when $n_0=1$

5-

$$10n + 5\log n \leq cn$$

$$5\log n \leq (c - 10)n$$

$$\frac{5\log n}{c-10} \leq n$$

Which is true for $c=15$ when $n_0 = 1$

Another solution:

$$10n + 5\log n \leq (10 + 5)n$$

$$10n + 5\log n \leq 15n$$

For $c=15$ when $n_0 = 1$

Problem2:

1-

Since the algorithm chooses $\log n$ elements, and each element needs $O(n)$ time calculation, so the worst case is: **$O(n \log n)$** , where **$\log n$** represent the number of elements, and **n** represent the time calculation for each element.

2-

The worst case when all array elements are even, which is **$O(n^2)$** , where **n** represent the number of even elements, and the **other n** represent the time calculation for each even number.

3-

Since algorithm D calls algorithm E for each element in the array, the time computation will be the summation of $\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$ which gave us the worst case with **$O(n^2)$** .

Problem3:

	statements	S/E	freq	total
1	public void func1 (int n) {	0	0	0
2	for (int i = 0; i < n * log(n); i++) {	1	$n\log n + 1$	$n\log n + 1$
3	System . out . println (i);	1	$n\log n$	$n\log n$
4	for (int j = 2; j < n; j++) {	1	$n\log n(n-2+1)$	$n^2\log n - n\log n$
5	System .out . println (j);	1	$n\log n(n-2)$	$n^2\log n - 2n\log n$
6	}	0	0	0
7	System .out . println (" Goodbye !");	0	0	0
8	}	1	1	1
9		0	0	0
	Total operation		$2n^2\log n - n\log n + 2$	
	Big oh		$O(n^2\log n)$	

	statements	S/E	freq	total
1	public void func2 (int n) {	0	0	0
2	for (int i = 0; i < n * n; i++) {	1	$n^2 - 0 + 1$	$n^2 + 1$
3	System . out . println (i);	1	n^2	n^2
4	for (int j = 2 * n; j > n; j --) {	1	$n^2(2n - n + 1)$	$2n^3 - n^3 + n^2 = n^3 + n^2$
5	System .out . println (j);	1	$n^2(2n - n)$	$2n^3 - n^3 = n^3$
6	}	0	0	0
7	System . out . println (" Goodbye !");	0	0	0
8	}	1	1	1
9		0	0	0
	Total operation		$2n^3 + 3n^2 + 2$	
	Big oh		$O(n^3)$	

	statements	S/E	freq	total
1	public void func3 (int n) {	0	0	0
2	for (int i = n; i > 0; i --) {	1	$n - 0 + 1$	$n + 1$
3	System . out . println (i);	1	n	n
4	for (int j = 0; j < i; j++) {	1	$\frac{n(n+1)}{2} + n$	$\frac{n^2 + n}{2} + n$
5	System .out . println (j);	1	$\frac{n(n+1)}{2}$	$\frac{n^2 + n}{2}$
6	}	0	0	0
7	System .out . println (" Goodbye !");	0	0	0
8	}	1	1	1
9		0	0	0
	Total operation		$n^2 + 4n + 2$	
	Big oh		$O(n^2)$	

	statements	S/E	freq	total
1	void func4 (int n) {	0	0	0
2	int m = 1;	1	1	1
3	while (m <= n) {	1	$n-1+1+1$	$n+1$
4	system . out . println (m);	1	n	n
5	i = n;	1	n	n
6	while (i > 0) {	0	$n(\log n+2)$	$n\log n+2n$
7	system .out . println (i);	0	$n(\log n+1)$	$n\log n+n$
8	i = i / 2;	1	$n(\log n+1)$	$n\log n+n$
9	}	0	0	0
10	m++;		n	n
11	}		0	0
12	}		0	0
	Total operation		$3n\log n+8n+2$	
	Big oh		$O(n\log n)$	

Problem3:

1-

```

public class Test {
    public static void main (String [] args){

        long selection,bubble,quick, time1,time2;

        for (int i=10000; i<= 50000; i=i+10000){

            double[] selectionSort= new double[i];
            double[] bubbleSort = new double [i];
            double[] quickSort = new double [i];
            selection=0;
            bubble=0;
            quick=0;

            for(int j=0; j<i; j++){

                selectionSort[j]=bubbleSort[j]=quickSort[j]=Math.random();;
            }

            for (int x=0 ; x<100 ; x++){
                time1=System.nanoTime();
                Sort.selectionSort(selectionSort,i);
                time2= System.nanoTime();
                selection = selection+(time2-time1);

                time1=System.nanoTime();
                Sort.bubbleSort(bubbleSort,i);
                time2= System.nanoTime();
                bubble = bubble+(time2-time1);

                time1=System.nanoTime();
                Sort.quickSort(quickSort,i);
                time2= System.nanoTime();
                quick = quick +(time2-time1);
            }
        }
    }
}

```

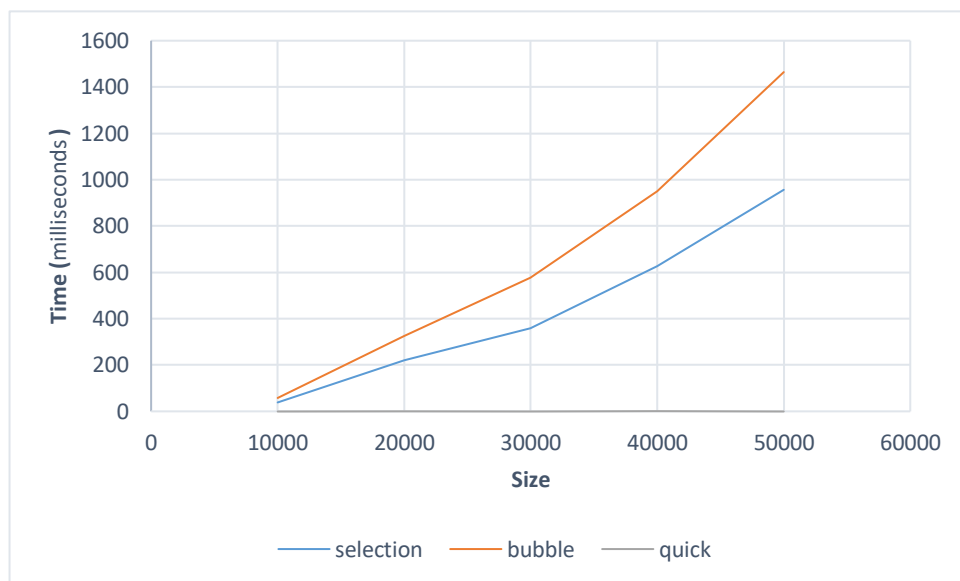
```

        System.out.println("Average of selection with size:" + i+ " is: " +
((double)selection/100)/1000000);
        System.out.println("Average of bubbleSort with size:" + i+ " is: " +
((double)bubble/100)/1000000);
        System.out.println("Average of quick with size:" + i+ " is: " +
((double)quick/100)/1000000);
    }
}
}

```

2-

Size\sort type	Selection sort	Bubble sort	Quick sort
10000	38.5	57.79	0.11
20000	220.46	324.78	0.09
30000	359.51	575.7	0.12
40000	625.48	951.5	0.9
50000	956.38	1464.28	0.13



3-

Quicksort is the fastest since it takes the shortest time to sort the array.

4-

Selection sort is faster since it takes shorter time to sort the array with different sizes, bubble sort has larger growth rate than the selection sort because bubble sort take longer time to sort the array, even they have the same time complexity with big oh (n^2) .