RECURSION

CSC212: Data Structures

Recursion

- **Sometimes, certain statements in an algorithm are repeated on different sizes of an input instance.**
- **Repetition can be achieved in two different ways.**
  - **Iteration: uses for and while loops**
  - **Recursion: function calls itself**

Example -1:

- Factorial Function
- Factorial function of any integer n is defined as

$$n! = \begin{cases} 1 \text{ if} & n = 0 \\ n.(n-1).(n-2) \sim\sim\sim\sim\sim\sim 3.2.1 \text{ if } n \geq 1 \end{cases}$$

- 4! =4*3*2*1
- 4! =4 * 3!

Recursion

Example -1:

−Factorial function of any integer n is defined as

$$n! = \begin{cases} 1 \text{ if } n = 0 & \leftarrow \text{ Base Case} \\ n(n-1)! \text{ if } n \geq 1 & \leftarrow \text{ Recusion Case} \end{cases}$$

– This is recursive definition. It consists of two parts:

i. **Base case**

ii. **Recursive case**

Important:

Every recursion must have at least one base case, at which the recursion does not recur

Recursion

Example -1 (Continue) :

- It can be written as:

$$\mathrm{fact}(n) = \begin{cases} 1 \text{ if } n = 0 & \leftarrow \text{ Base Case} \\ n\mathrm{fact}(n-1)! \text{ if } n \geq 1 & \leftarrow \text{ Recusion Case} \end{cases}$$

where $\mathrm{fact}(n)$ is the function that calculates $n!$.

Recursion

Example -1 (Continue) :

– Implementation

Recursive:

```
public static int recursiveFact(int n)
{
    if(n==0)return 1;
    else
       return n*recursiveFact(n-1);
}
```

Iterative:

```
public static int iterativeFact(int n)
{
    int fact = 1;
    for(i = 1; i <= n; i++)
       fact=fact*I;
    return fact;
}
```

Recursive Trace

- A graphical representation of recursive calls.
- It is used to analyze the algorithm.

Example -2:

  - Recursive trace for recursiveFact(4)

```
public static int recursiveFact(int n)
{
        if(n==0)
          return 1;
        else
          return n*recursiveFact(n-1);
}
```

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- Recursive trace for recursiveFact(4)

recursiveFact(4)

4*?=?

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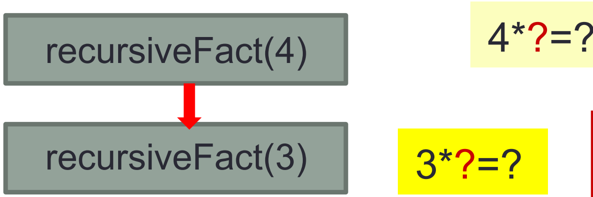

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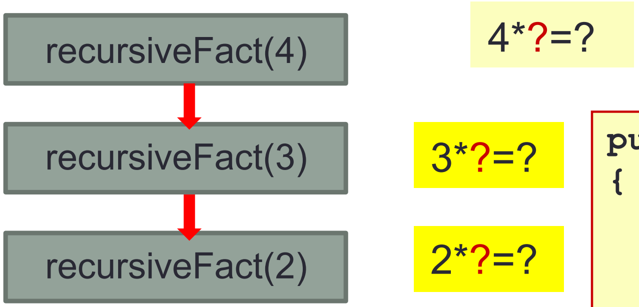

recursiveFact(4)

4*?=?

recursiveFact(3)

3*?=?

recursiveFact(2)

2*?=?

```
public static int recursiveFact(int n)
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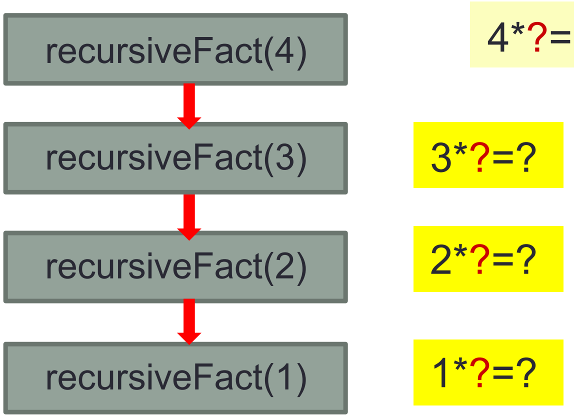

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4*?=?

recursiveFact(3)

3*?=?

recursiveFact(2)

2*?=?

recursiveFact(1)

1*?=?

```
public static int recursiveFact(int n)
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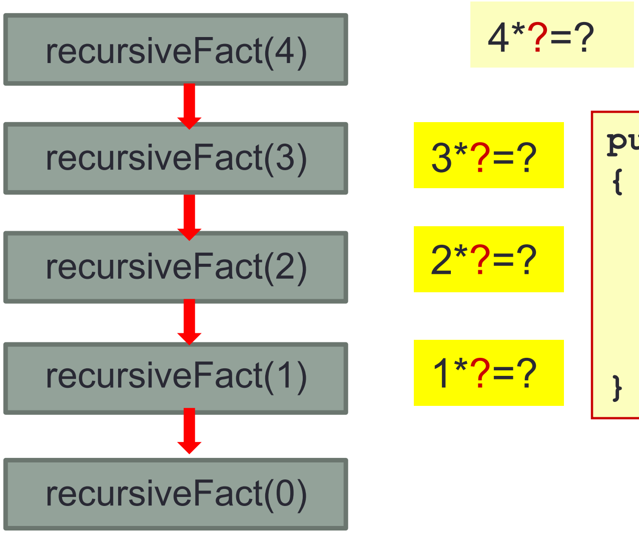

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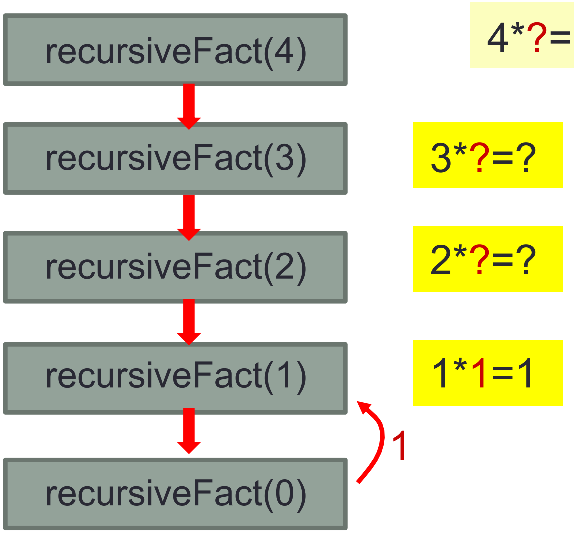

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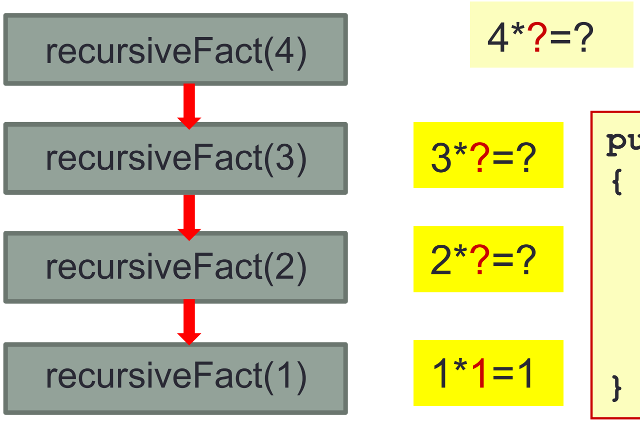

recursiveFact(4)

4*?=?

recursiveFact(3)

3*?=?

recursiveFact(2)

2*?=?

recursiveFact(1)

1*1=1

```
public static int recursiveFact(int n)
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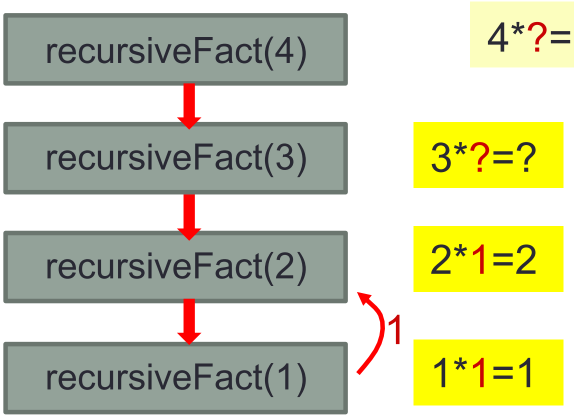

recursiveFact(4)

4*?=?

recursiveFact(3)

3*?=?

recursiveFact(2)

2*1=2

recursiveFact(1)

1*1=1

```
public static int recursiveFact(int n)
{
         if(n==0)
           return 1;
         else
           return n*recursiveFact(n-1);
}
```

Recursive Trace

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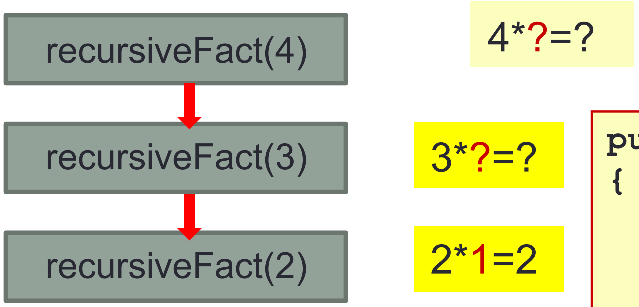
Example -2:

– Recursive trace for recursiveFact(4)



```
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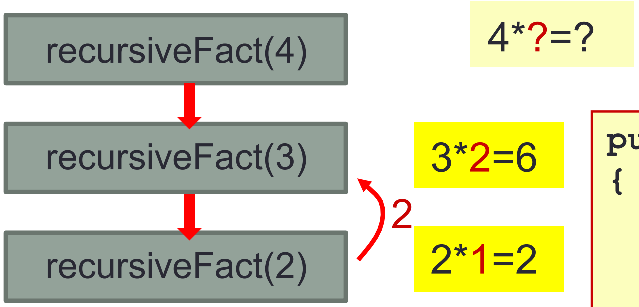

```
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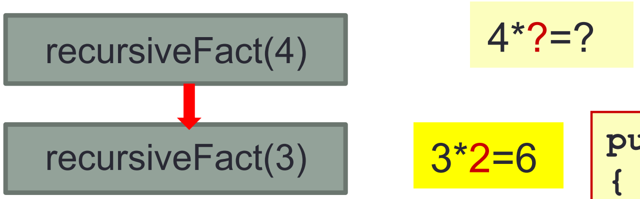

recursiveFact(4)

4*?=?

recursiveFact(3)

3*2=6

```
public static int recursiveFact(int n)
{
        if(n==0)
          return 1;
        else
          return n*recursiveFact(n-1);
}
```

Recursive Trace

- A graphical representation of recursive calls.
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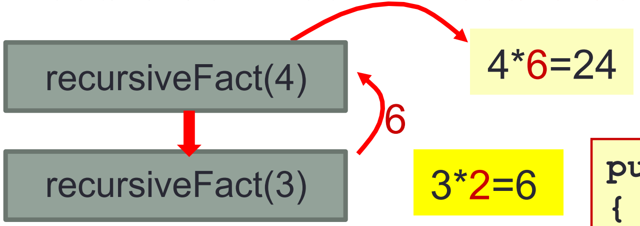

recursiveFact(4)

4*6=24

6

recursiveFact(3)

3*2=6

```
public static int recursiveFact(int n)
{
          if(n==0)
            return 1;
          else
            return n*recursiveFact(n-1);
}
```

Recursive Trace

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Example -2:

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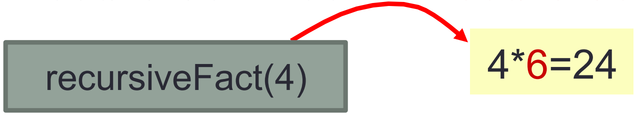

recursiveFact(4)

4*6=24

```
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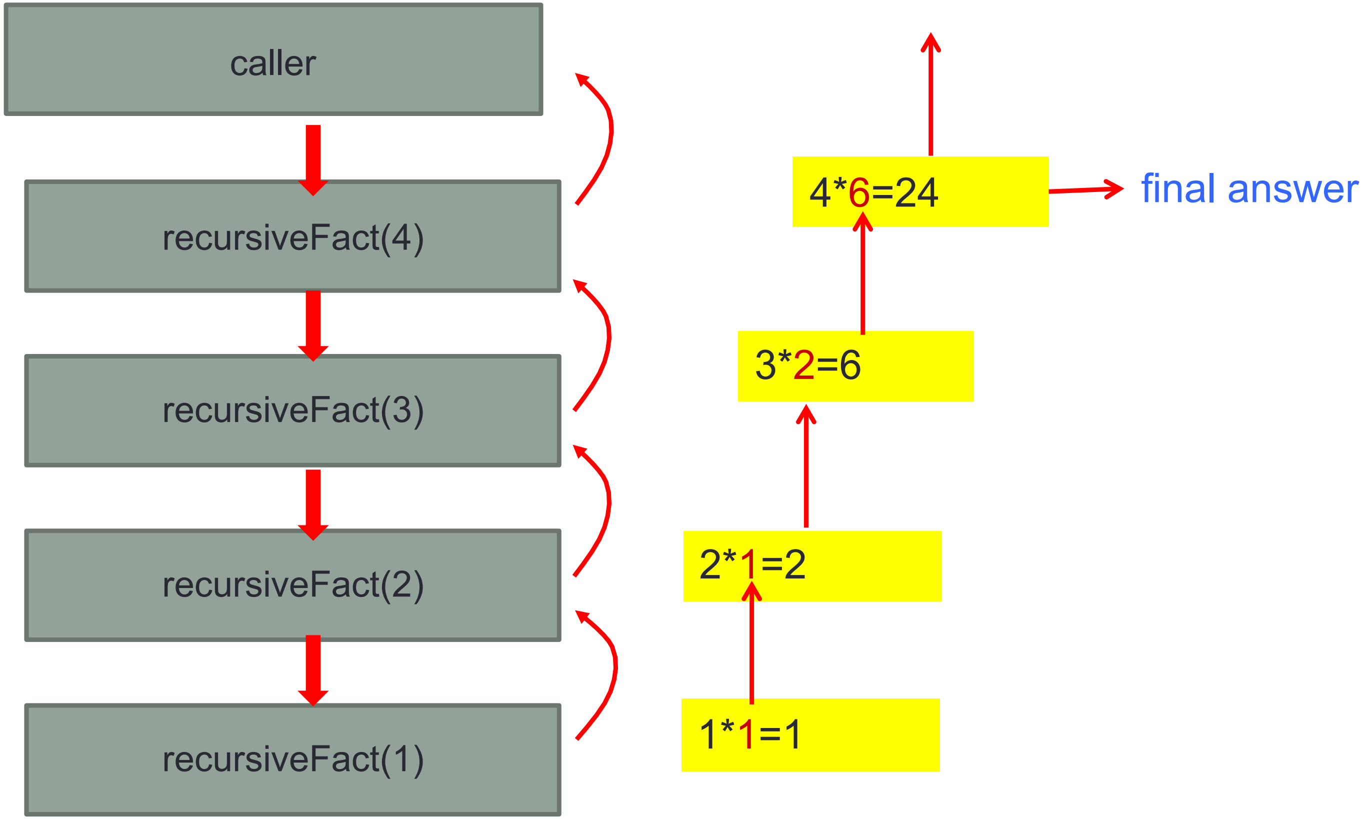
}
```

Recursive Trace



caller

recursiveFact(4)

recursiveFact(3)

recursiveFact(2)

recursiveFact(1)

1*1=1

2*1=2

3*2=6

4*6=24

final answer

Recursive Exercise

Calculate x^n using both iteration and recursion. (Assume x > 0 and n >= 0)

```
Public int Power(x,n){
        result =0;
    if (n==0)
        return result=1;
    else
 for( int i=1, i≤n, i++){
        result =* x ; }
    return result ; }
```

```
Public int re_Power(x,n){
        if (n==0)
           return 1;
  else
    return x * re-Power(x,n-1);
```

Main Types of Recursion

- Linear Recursion
- Binary Recursion

Linear Recursion

In this case a recursive method makes at most one recursive call each time it is invoked.

Example – 3:

- **Problem:** Given an array A of n integers, find the sum of first n integers.
- **Observation:** Sum can be defined recursively as follows:

$$\mathrm{Sum}(n) = \begin{cases} \mathrm{A}[0] \text{ if } n = 1 & \leftarrow \text{ Base Case} \\ \mathrm{Sum}(n-1) + \mathrm{A}[n-1] \text{ if } n > 1 & \leftarrow \text{ Recusive Case} \end{cases}$$

```
Public int Sum (A, n) {
    if (n == 1)
        return A[0];
    else
    return A[n-1] + Sum( A, n-1) }
```

Linear Recursion

Example – 3 (Continued):

- Algorithm

Sum(A, n)

Input: An integer array A and an integer *n* ≥1, such that A has at least *n* elements

Output: The sum of the first *n* integers in A.

Processing:

```
if n = 1;
    return A[0];                      → base case
else
    return Sum(A, n-1) + A[n-1];      → recursive case.
```

Note:

- Base case should be defined so that every possible chain of recursive calls eventually reach a base case.
- Algorithm must start by testing a set of base cases.
- After testing for base cases perform a single recursive call.

Linear Recursion

Example – 3 (Continued):

- Recursive trace for sum(A ,n), where A ={4,3,6,2,5}, n=5

	A
0	4
1	3
2	6
3	2
4	5

Sum(A, n)

Input: An integer array A and an integer *n* ≥1, such that A has at least *n* elements

Output: The sum of the first *n* integers in A.

Processing:

```
    if n = 1;
        return A[0];                → base case
    else
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```

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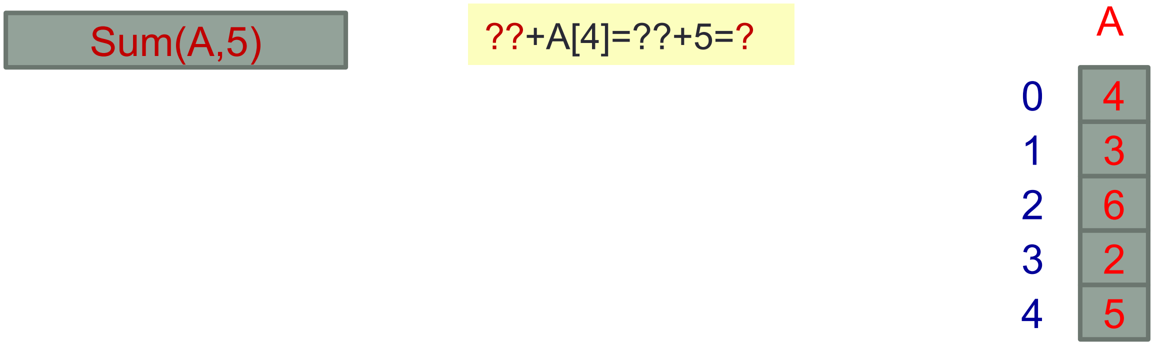

Sum(A,5)

??+A[4]=??+5=?

A

	A
0	4
1	3
2	6
3	2
4	5

Sum(A, n)

Input: An integer array A and an integer $n \geq 1$, such that A has at least n elements

Output: The sum of the first n integers in A.

Processing:

```
    if n = 1;
        return A[0];            → base case
    else
    return Sum(A, n-1) + A[n-1];          → recursive case.
```

Linear Recursion

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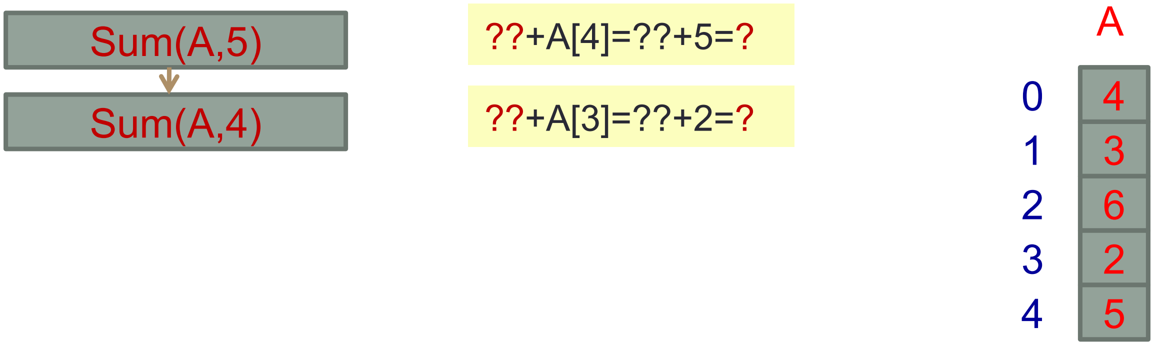

Sum(A,5)

Sum(A,4)

??+A[4]=??+5=?

??+A[3]=??+2=?

A

0	4
1	3
2	6
3	2
4	5

```
Sum(A, n)
    Input: An integer array A and an integer $n \geq 1$, such that A has at least $n$ elements
    Output: The sum of the first $n$ integers in A.
  Processing:
        if n = 1;
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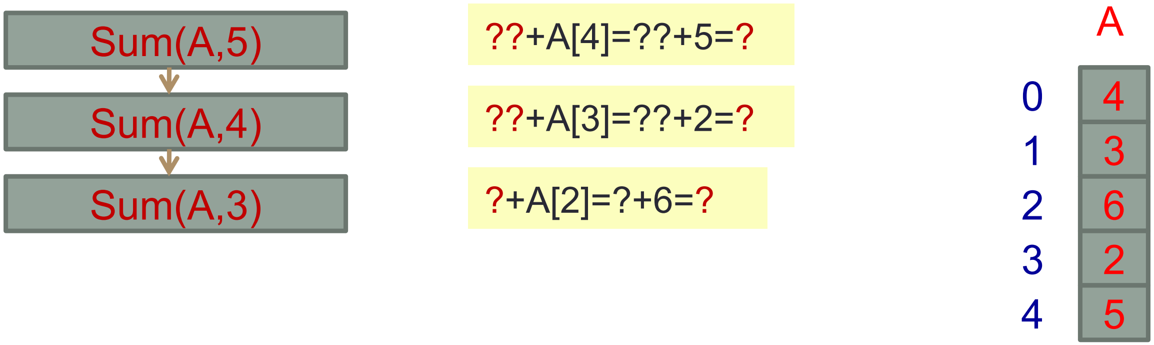

Sum(A,5)

Sum(A,4)

Sum(A,3)

??+A[4]=??+5=?

??+A[3]=??+2=?

?+A[2]=?+6=?

A

	A
0	4
1	3
2	6
3	2
4	5

Sum(A, n)

Input: An integer array A and an integer *n* ≥1, such that A has at least *n* elements

Output: The sum of the first *n* integers in A.

Processing:

```
if n = 1;
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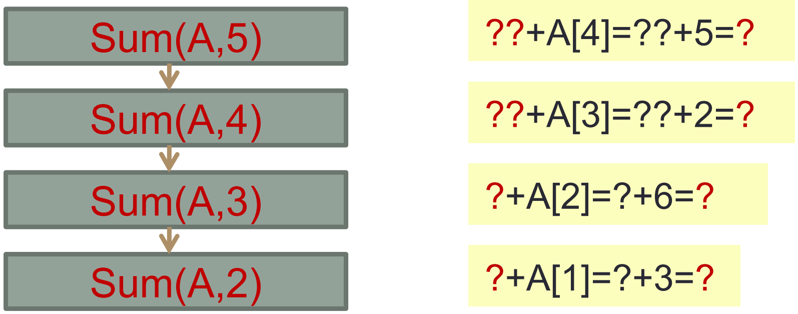

Sum(A,5)

Sum(A,4)

Sum(A,3)

Sum(A,2)

??+A[4]=??+5=?

??+A[3]=??+2=?

?+A[2]=?+6=?

?+A[1]=?+3=?

	A
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1	3
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Sum(A, n)
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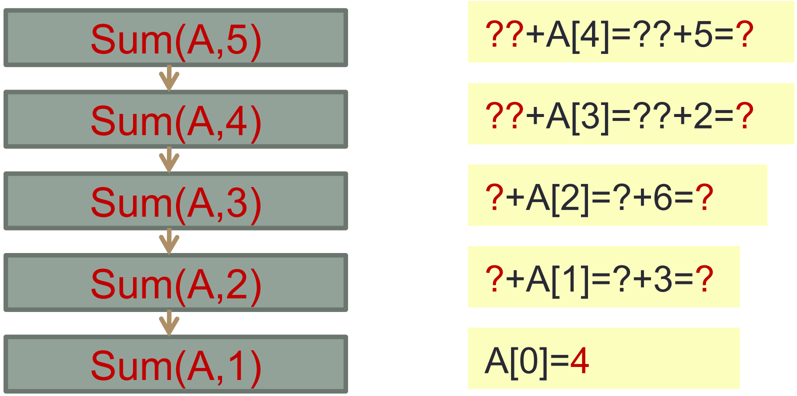

Sum(A,5)

Sum(A,4)

Sum(A,3)

Sum(A,2)

Sum(A,1)

??+A[4]=??+5=?

??+A[3]=??+2=?

?+A[2]=?+6=?

?+A[1]=?+3=?

A[0]=4

	A
0	4
1	3
2	6
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Sum(A, n)

Input: An integer array A and an integer *n* ≥1, such that A has at least *n* elements

Output: The sum of the first *n* integers in A.

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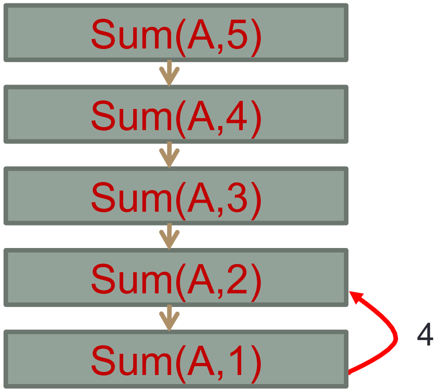

Sum(A,5)

Sum(A,4)

Sum(A,3)

Sum(A,2)

Sum(A,1)

4

??+A[4]=??+5=?

??+A[3]=??+2=?

?+A[2]=?+6=?

4+A[1]=4+3=7

A[0]=4

	A
0	4
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2	6
3	2
4	5

Sum(A, n)

Input: An integer array A and an integer *n* ≥1, such that A has at least *n* elements

Output: The sum of the first *n* integers in A.

Processing:

```
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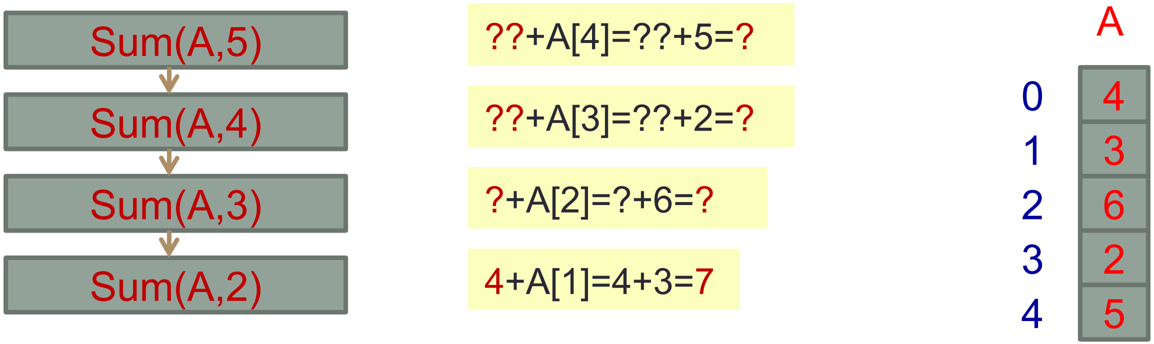

Sum(A,5)

Sum(A,4)

Sum(A,3)

Sum(A,2)

??+A[4]=??+5=?

??+A[3]=??+2=?

?+A[2]=?+6=?

4+A[1]=4+3=7

A

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Sum(A, n)

Input: An integer array A and an integer *n* ≥1, such that A has at least *n* elements

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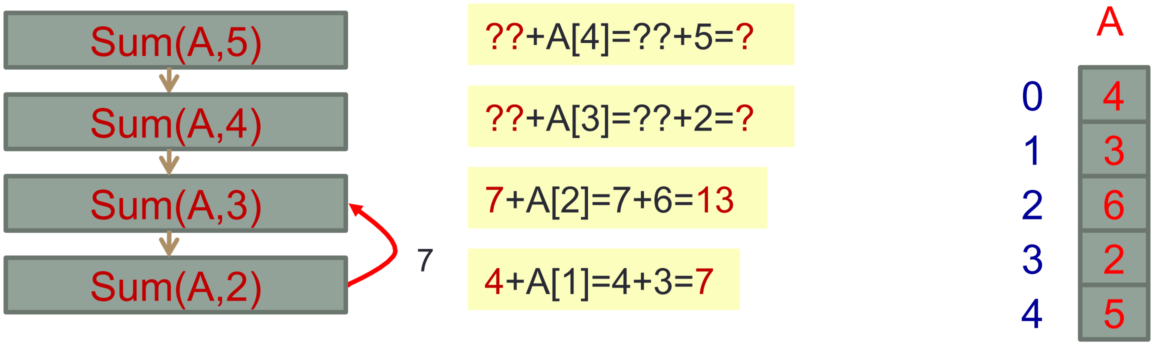

Sum(A,5)

Sum(A,4)

Sum(A,3)

Sum(A,2)

7

??+A[4]=??+5=?

??+A[3]=??+2=?

7+A[2]=7+6=13

4+A[1]=4+3=7

A

0 4

1 3

2 6

3 2

4 5

Sum(A, n)

Input: An integer array A and an integer $n \geq 1$, such that A has at least n elements

Output: The sum of the first n integers in A.

Processing:

```
if n = 1;
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Linear Recursion

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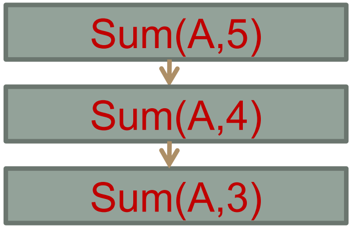

Sum(A,5)

Sum(A,4)

Sum(A,3)

??+A[4]=??+5=?

??+A[3]=??+2=?

7+A[2]=7+6=13

	A
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Sum(A, n)

```
    Input: An integer array A and an integer n ≥1, such that A has at          least n elements
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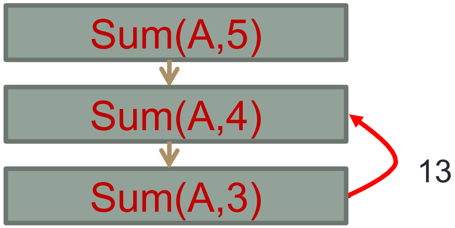

??+A[4]=??+5=?

13+A[3]=13+2=15

7+A[2]=7+6=13

	A
0	4
1	3
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Sum(A, n)

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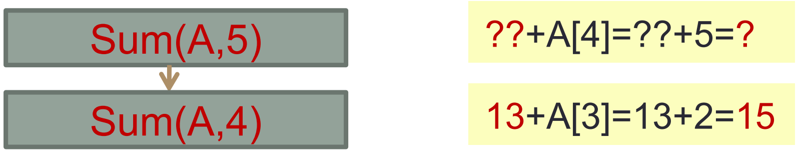
Example – 3 (Continued):

- Recursive trace for sum(A ,n), where A ={4,3,6,2,5}, n=5



??+A[4]=??+5=?

13+A[3]=13+2=15

A

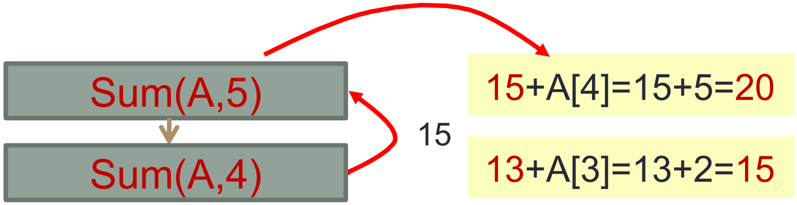
	A
0	4
1	3
2	6
3	2
4	5

```
Sum(A, n)
    Input: An integer array A and an integer n ≥1, such that A has at    least n elements
    Output: The sum of the first n integers in A.
  Processing:
        if n = 1;
            return A[0];                → base case
        else
        return Sum(A, n-1) + A[n-1];            → recursive case.
```

Linear Recursion

Example – 3 (Continued):

- Recursive trace for sum(A ,n), where A ={4,3,6,2,5}, n=5



A

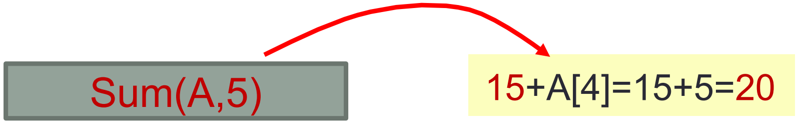
0	4
1	3
2	6
3	2
4	5

```
Sum(A, n)
    Input: An integer array A and an integer n ≥1, such that A has at        least n elements
    Output: The sum of the first n integers in A.
  Processing:
        if n = 1;
            return A[0];            → base case
        else
        return Sum(A, n-1) + A[n-1];          → recursive case.
```

Linear Recursion

Example – 3 (Continued):

– Recursive trace for sum(A ,n), where A ={4,3,6,2,5}, n=5



Sum(A,5)

15+A[4]=15+5=20

	A
0	4
1	3
2	6
3	2
4	5

Sum(A, n)

Input: An integer array A and an integer *n* ≥1, such that A has at least *n* elements

Output: The sum of the first *n* integers in A.

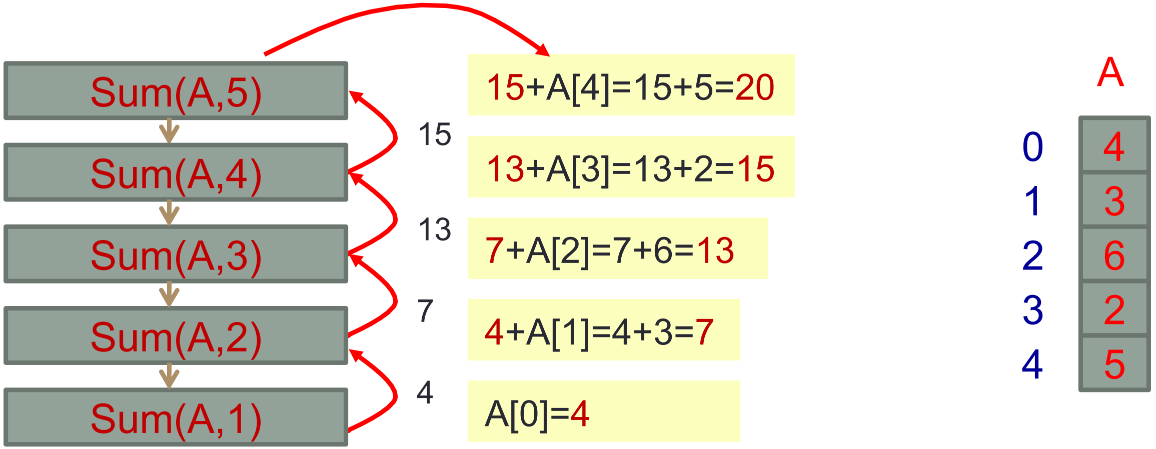
Processing:

```
if n = 1;
    return A[0];              → base case
else
return Sum(A, n-1) + A[n-1];          → recursive case.
```

Linear Recursion

Example – 3 (Continued):

- Recursive trace for sum(A ,n), where A ={4,3,6,2,5}, n=5



Note:

- For an array of size *n*, Sum(A, *n*) makes n calls.
- Each spends a constant amount of time.
- So time complexity is O(*n*).

Binary Recursion

In this case, a recursive algorithm makes two recursive calls.

Example – 4

- **Problem:** Find the sum of n elements of an integer array A.
- **Algorithm:**
  - Recursively find the sum of elements in the first half of A.
  - Recursively find the sum of elements in the second half of A.
  - Add these two values

	A
0	6
1	5
2	3
3	2
4	8

BinarySum(A ,i ,n)

Input: An integer array A and an integer $n \geq 1$, such that A has at least n elements

Output: The sum of the first n integers in A.

Processing:

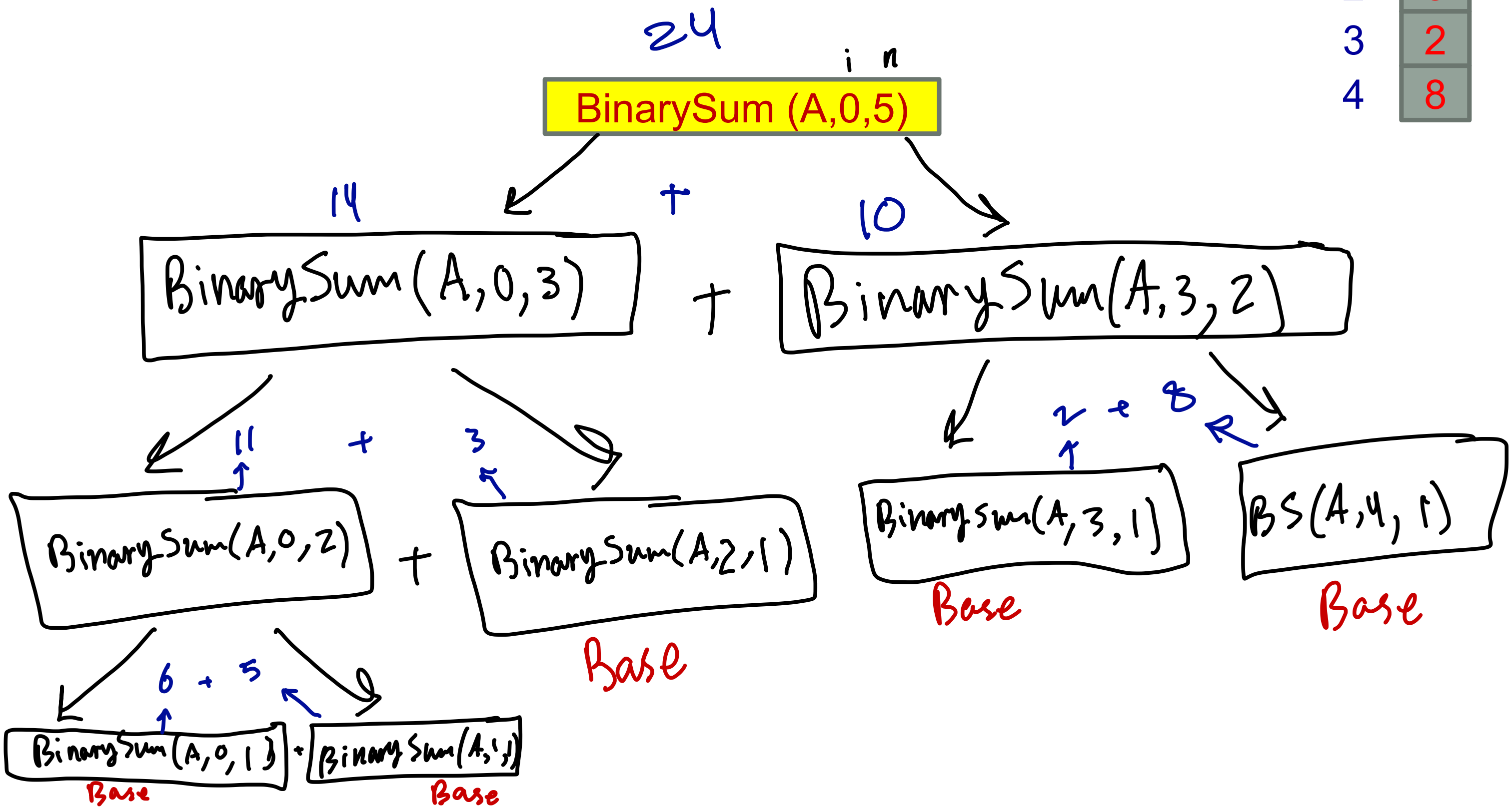
```
if n =1
    return A[i];
Else
                                   Ceiling                                flooring
return BinarySum(A ,i,⌈n/2⌉)+BinarySum(A ,i+⌈n/2⌉,⌊n/2⌋);
```

Binary Recursion

Example – 4 (Continued):

- Recursive trace

	A
0	6
1	5
2	3
3	2
4	8



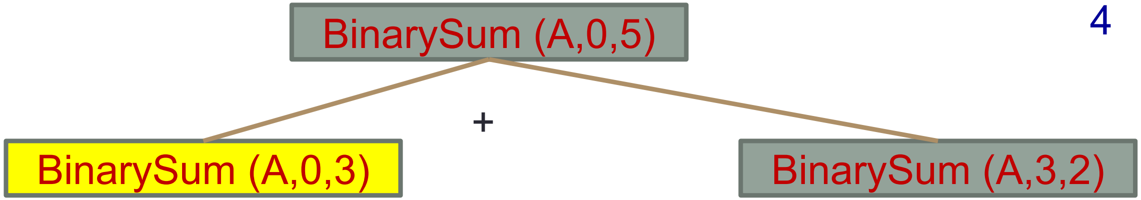
BinarySum (A,0,5)

Binary Recursion

Example – 4 (Continued):

- Recursive trace

	A
0	6
1	5
2	3
3	2
4	8



BinarySum (A,0,5)

+

BinarySum (A,0,3)

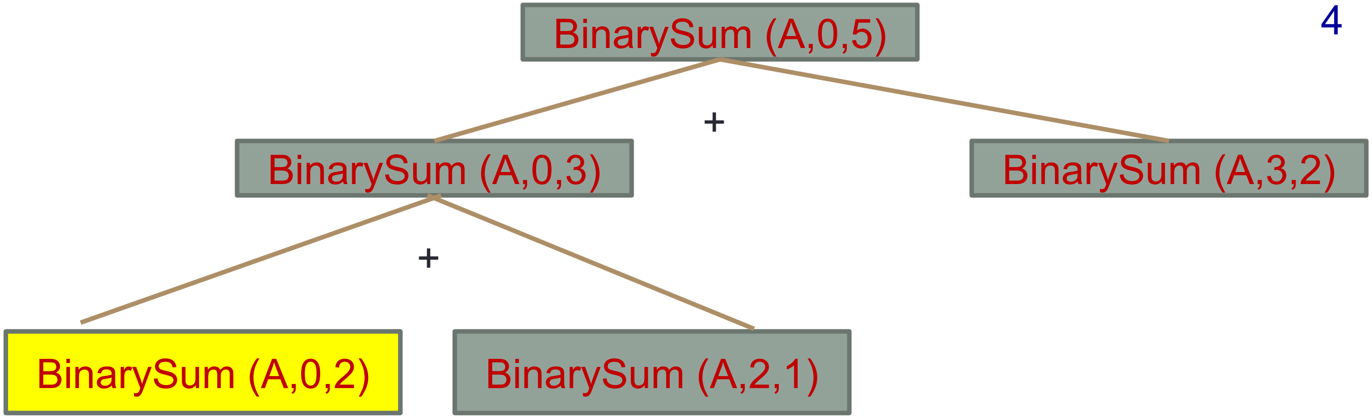
BinarySum (A,3,2)

Binary Recursion

Example – 4 (Continued):

- Recursive trace

	A
0	6
1	5
2	3
3	2
4	8

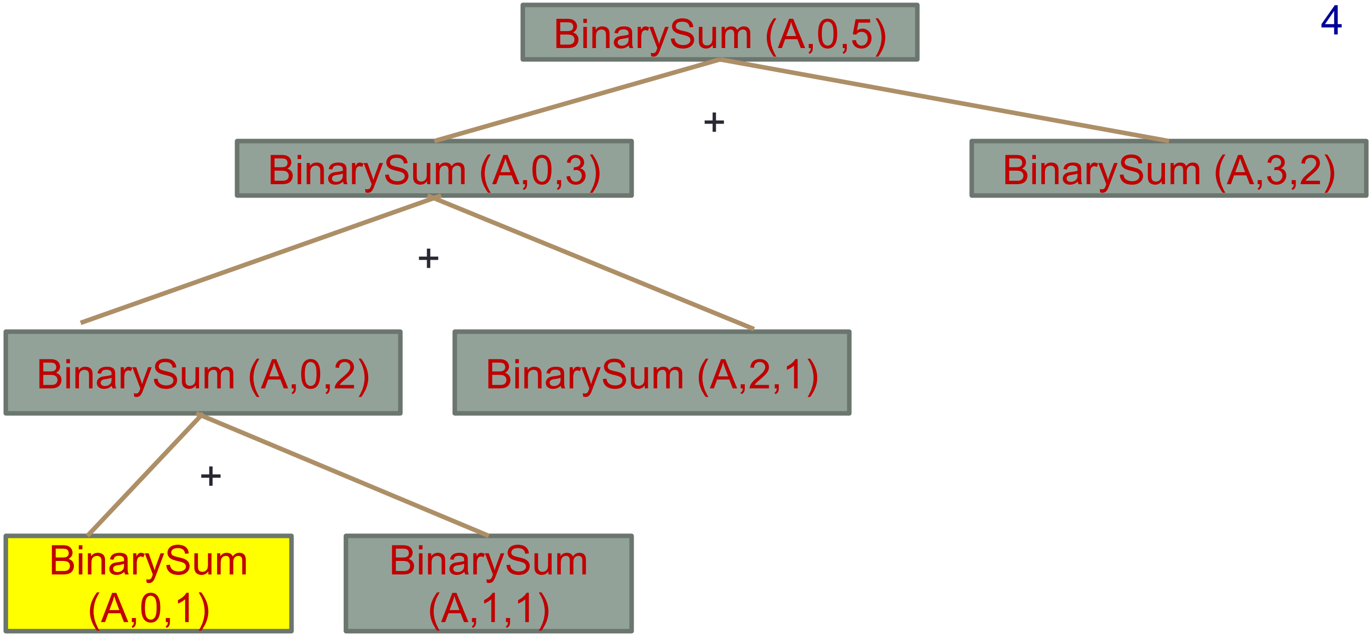


Binary Recursion

Example – 4 (Continued):

- Recursive trace

	A
0	6
1	5
2	3
3	2
4	8



BinarySum (A,0,5)

+

BinarySum (A,0,3)

BinarySum (A,3,2)

+

BinarySum (A,0,2)

BinarySum (A,2,1)

+

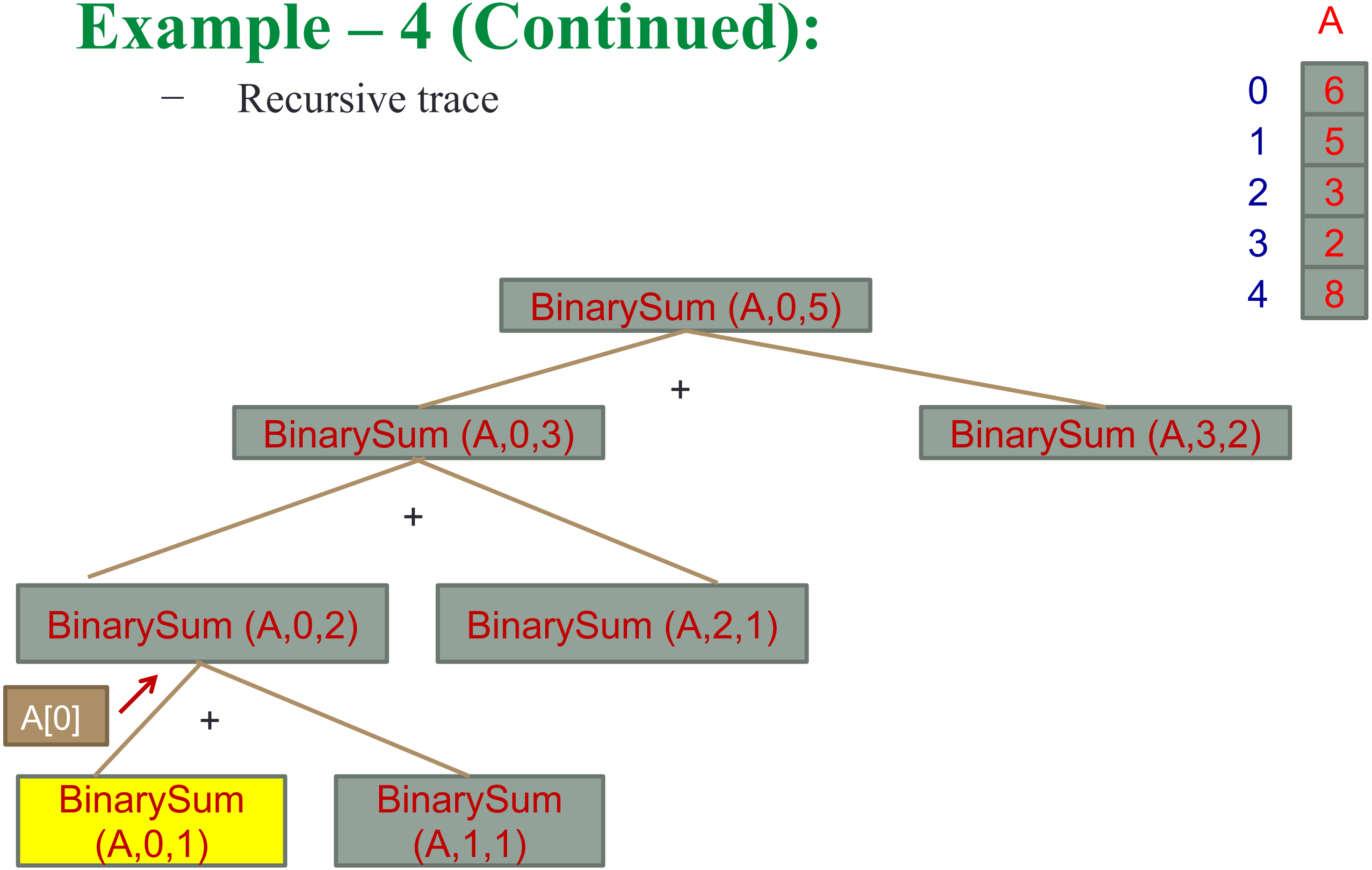
BinarySum (A,0,1)

BinarySum (A,1,1)

Binary Recursion

Example – 4 (Continued):

- Recursive trace

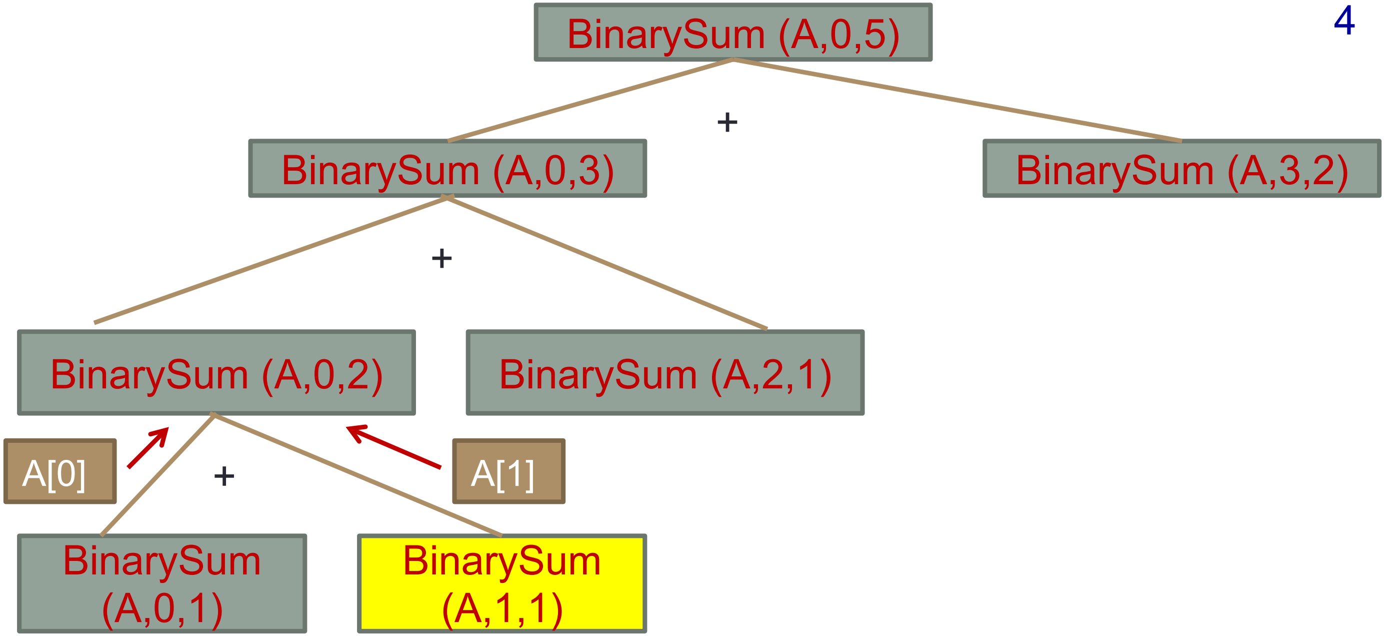


Binary Recursion

Example – 4 (Continued):

- Recursive trace

	A
0	6
1	5
2	3
3	2
4	8



BinarySum (A,0,5)

+

BinarySum (A,0,3)

BinarySum (A,3,2)

+

BinarySum (A,0,2)

BinarySum (A,2,1)

A[0]

+

A[1]

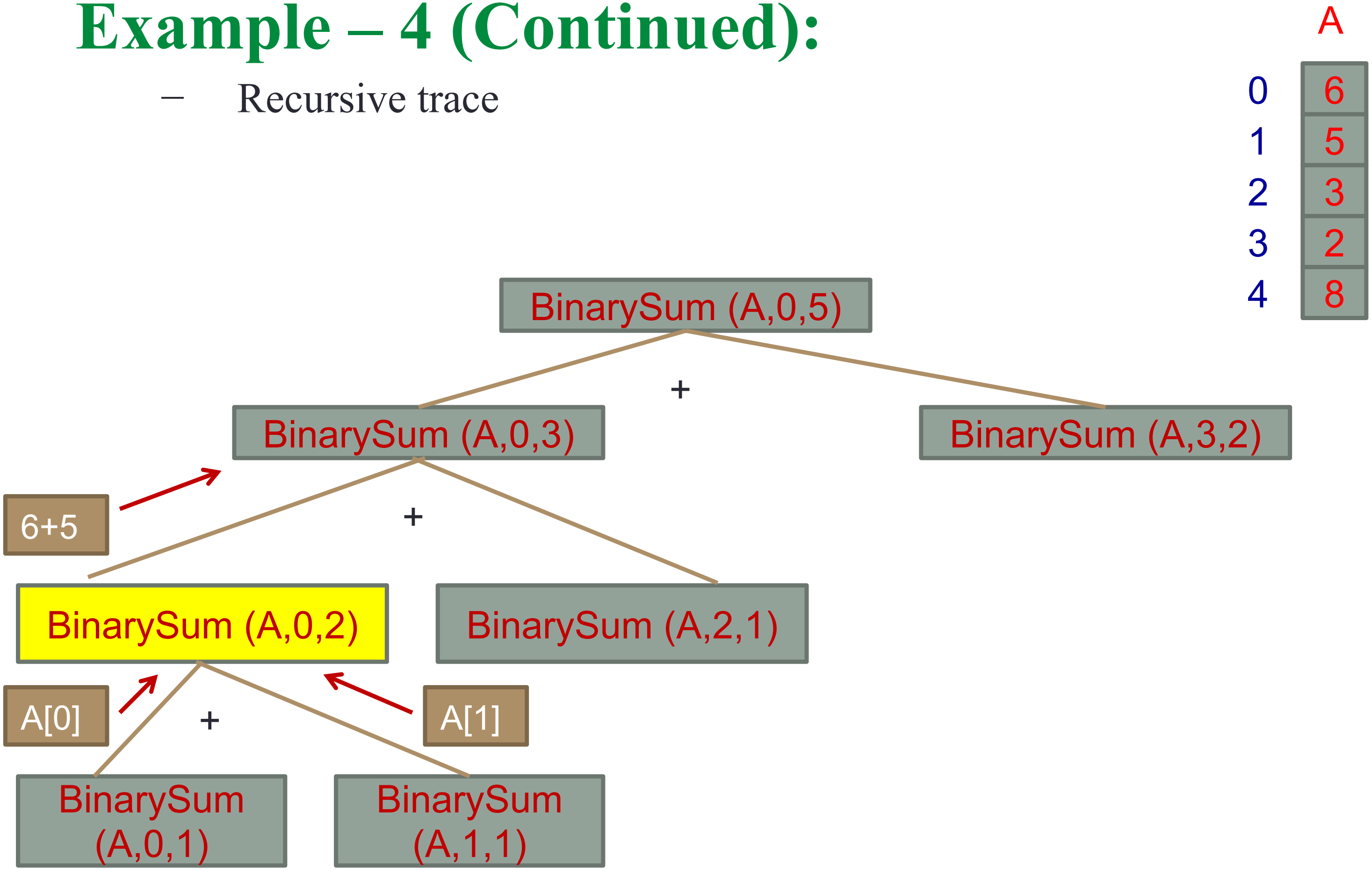
BinarySum (A,0,1)

BinarySum (A,1,1)

Binary Recursion

Example – 4 (Continued):

- Recursive trace

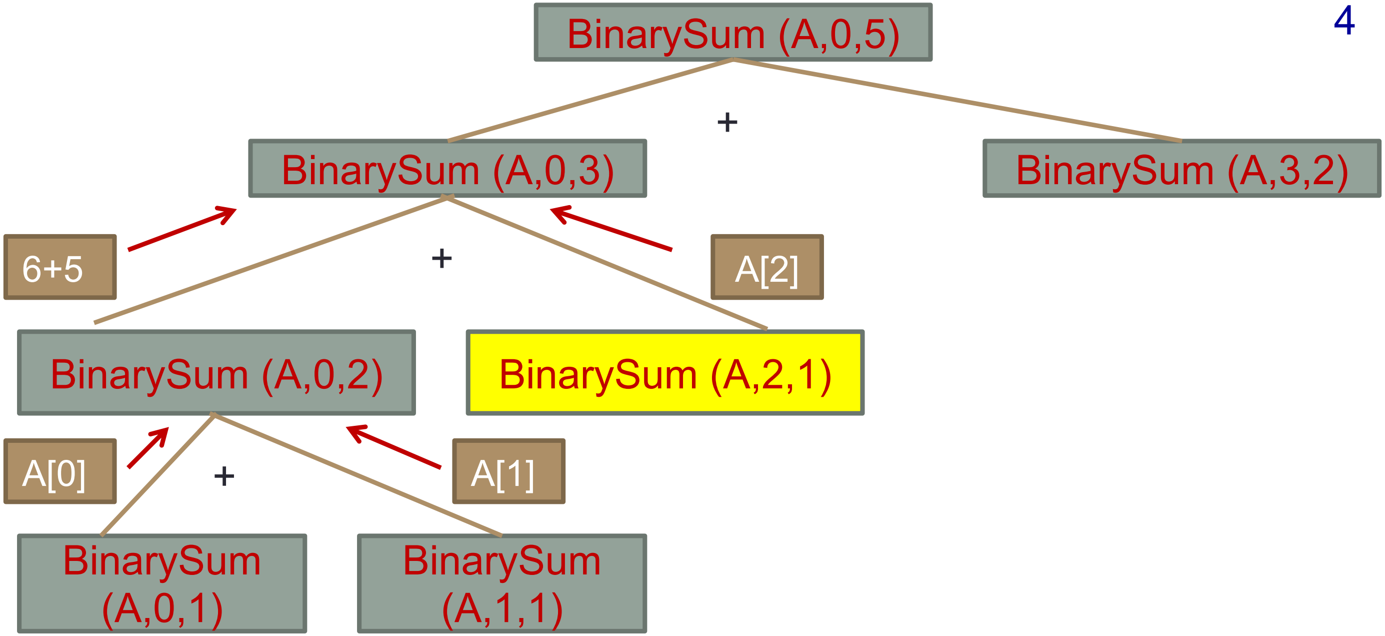


Binary Recursion

Example – 4 (Continued):

- Recursive trace

	A
0	6
1	5
2	3
3	2
4	8



BinarySum (A,0,5)

+

BinarySum (A,0,3)

BinarySum (A,3,2)

6+5

+

A[2]

BinarySum (A,0,2)

BinarySum (A,2,1)

A[0]

+

A[1]

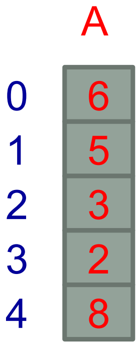
BinarySum (A,0,1)

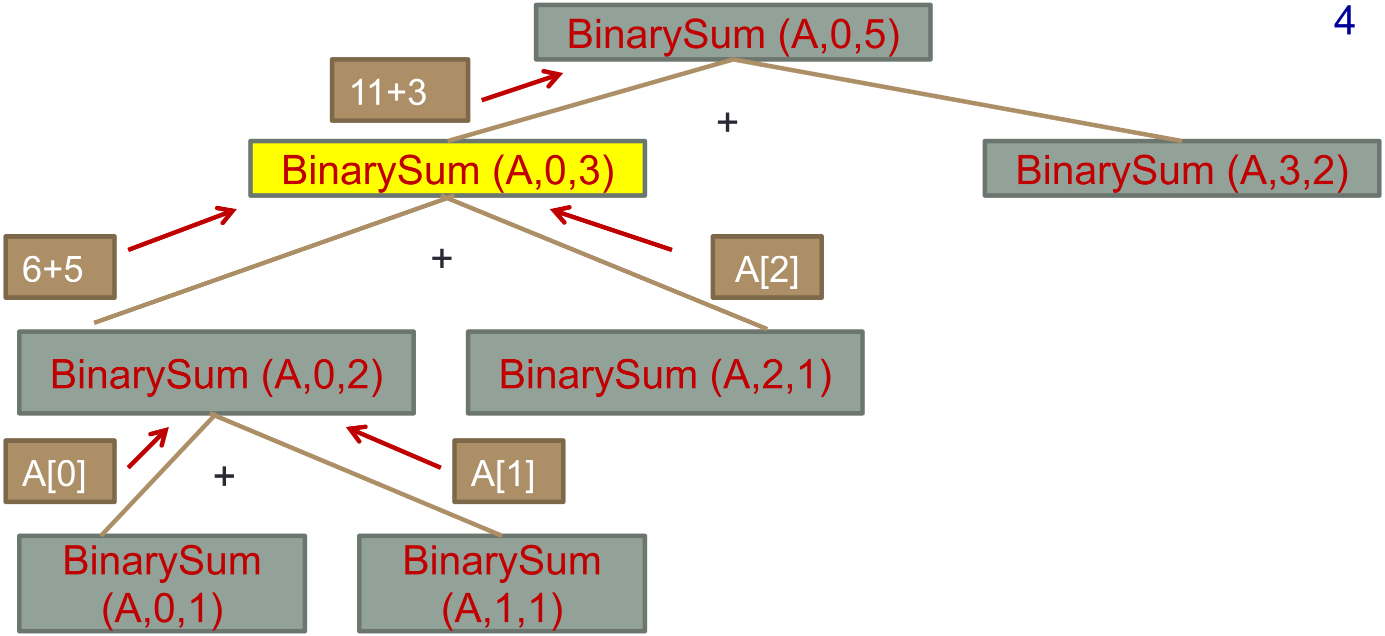
BinarySum (A,1,1)

Binary Recursion

Example – 4 (Continued):

- Recursive trace



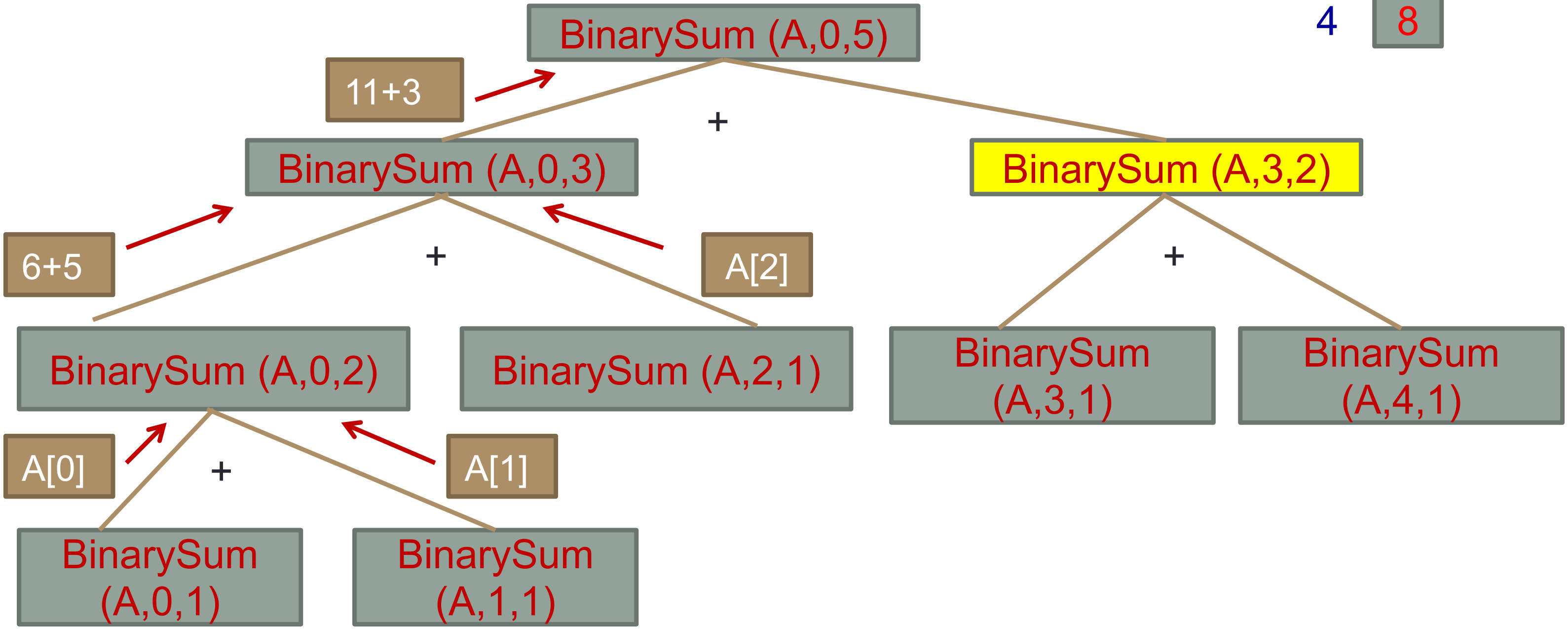


Binary Recursion

Example – 4 (Continued):

- Recursive trace

	A
0	6
1	5
2	3
3	2
4	8



BinarySum (A,0,5)

11+3

+

BinarySum (A,0,3)

BinarySum (A,3,2)

6+5

+

A[2]

+

BinarySum (A,0,2)

BinarySum (A,2,1)

BinarySum (A,3,1)

BinarySum (A,4,1)

A[0]

+

A[1]

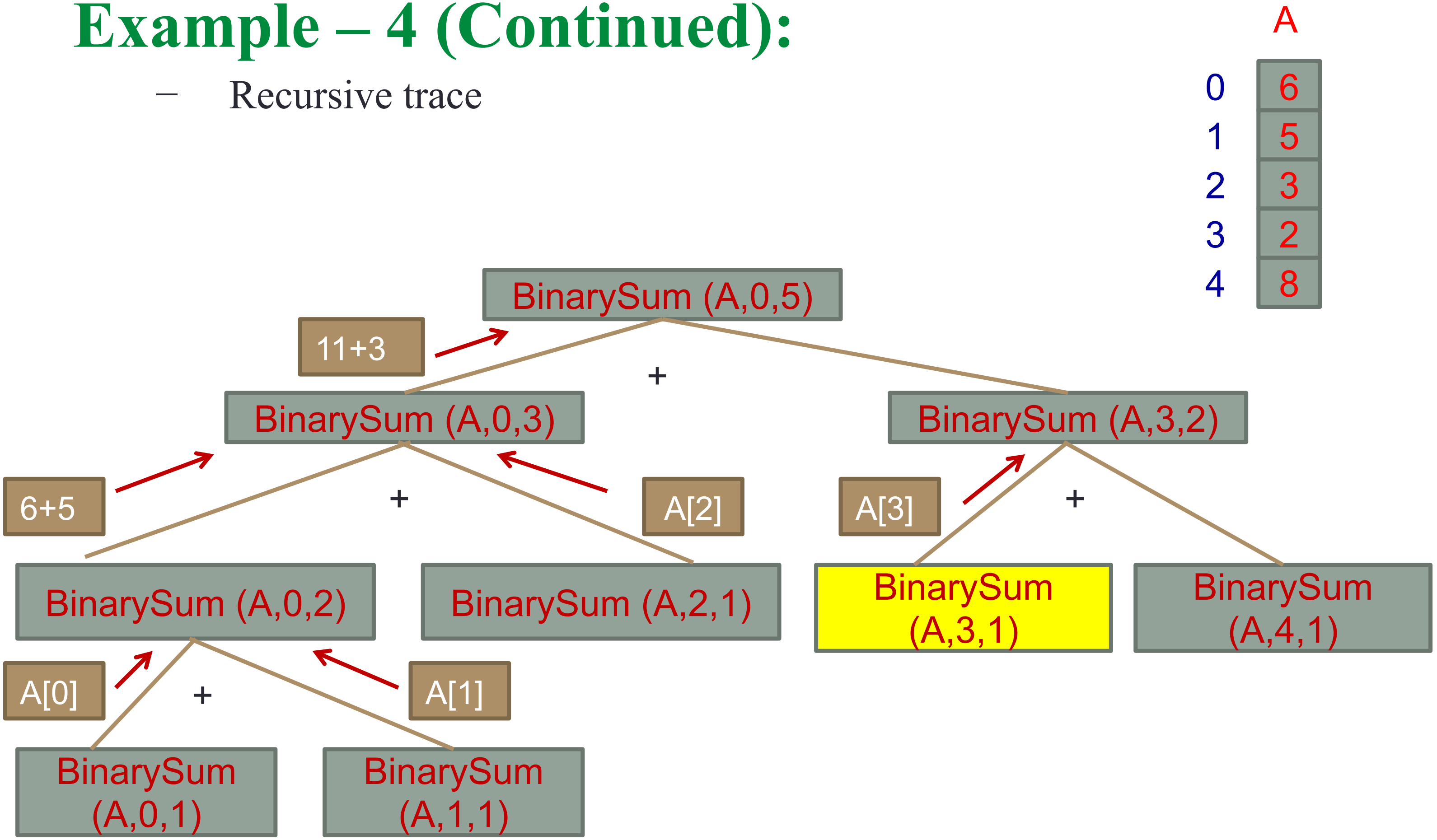
BinarySum (A,0,1)

BinarySum (A,1,1)

Binary Recursion

Example – 4 (Continued):

- Recursive trace



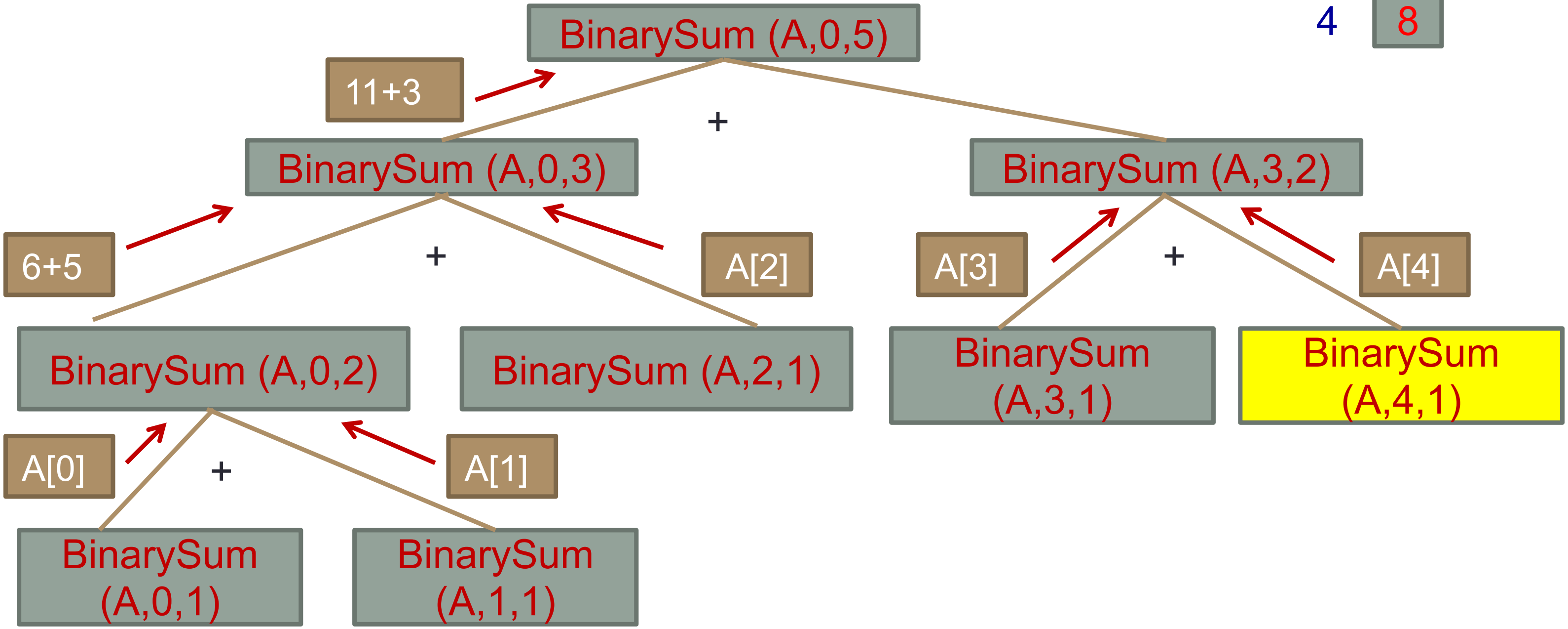
	A
0	6
1	5
2	3
3	2
4	8

Binary Recursion

Example – 4 (Continued):

- Recursive trace

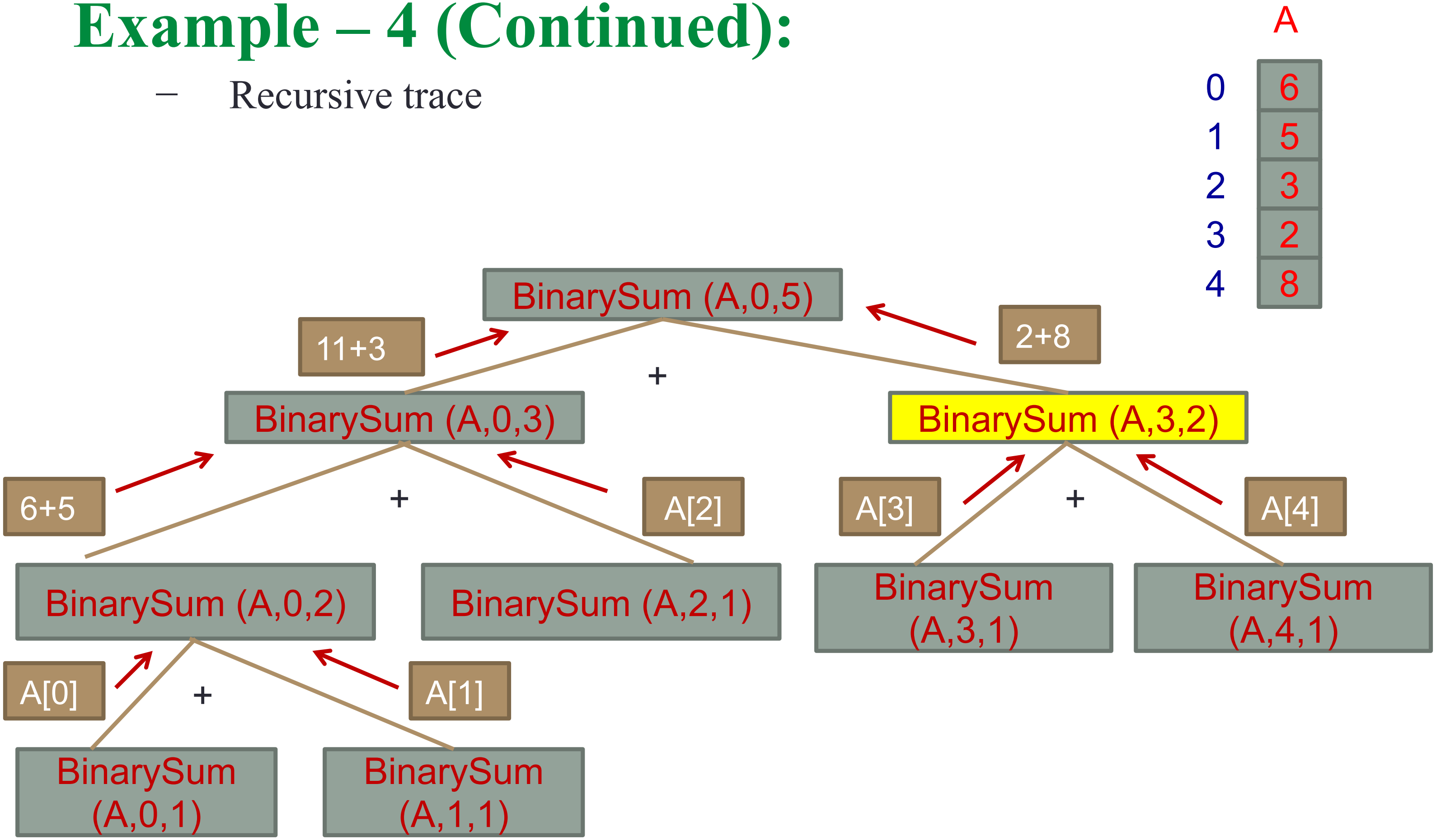
	A
0	6
1	5
2	3
3	2
4	8



Binary Recursion

Example – 4 (Continued):

- Recursive trace

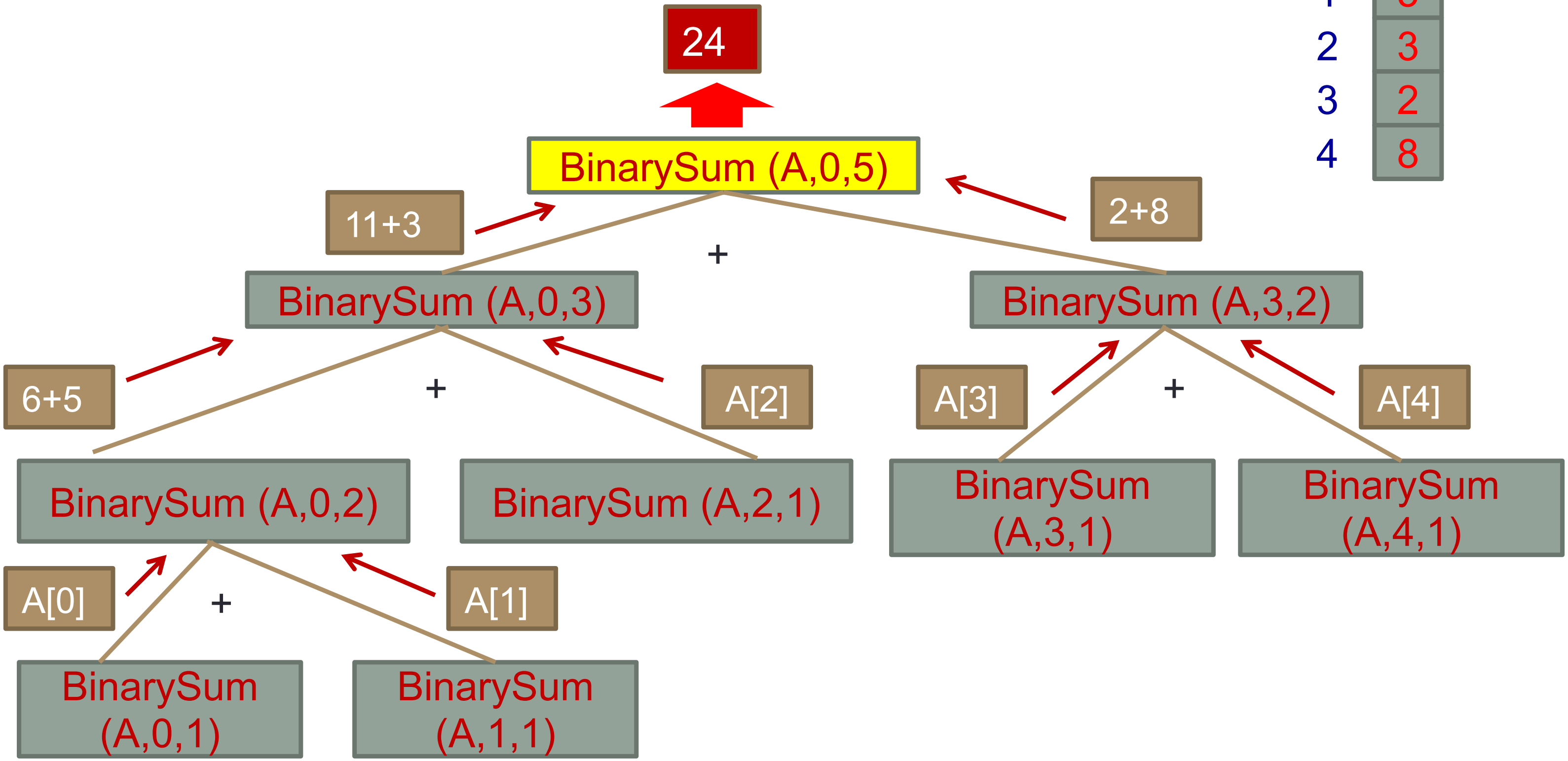


Binary Recursion

Example – 4 (Continued):

- Recursive trace

A	
0	6
1	5
2	3
3	2
4	8



24

BinarySum (A,0,5)

11+3

2+8

+

BinarySum (A,0,3)

BinarySum (A,3,2)

6+5

+

A[2]

A[3]

+

A[4]

BinarySum (A,0,2)

BinarySum (A,2,1)

BinarySum (A,3,1)

BinarySum (A,4,1)

A[0]

+

A[1]

BinarySum (A,0,1)

BinarySum (A,1,1)

Binary Recursion

Example – 5

- **The Fibonacci Number**
  1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
- Each number after the second number is the sum of the two preceding numbers.
- These numbers can naturally be defined recursively :

$$\mathrm{F}(n)=\begin{cases}1 \text{ if } n=0 & \leftarrow \text{ Base Case-1}\\ 1 \text{ if } n=1 & \leftarrow \text{ Base Case-2}\\ \mathrm{F}(n-1)+\mathrm{F}(n-2) \text{ if } n>1 & \leftarrow \text{ Recursive Case}\end{cases}$$

Binary Recursion

Example – 5 (Continued)

- Recursive Implementation of Fibonacci Function

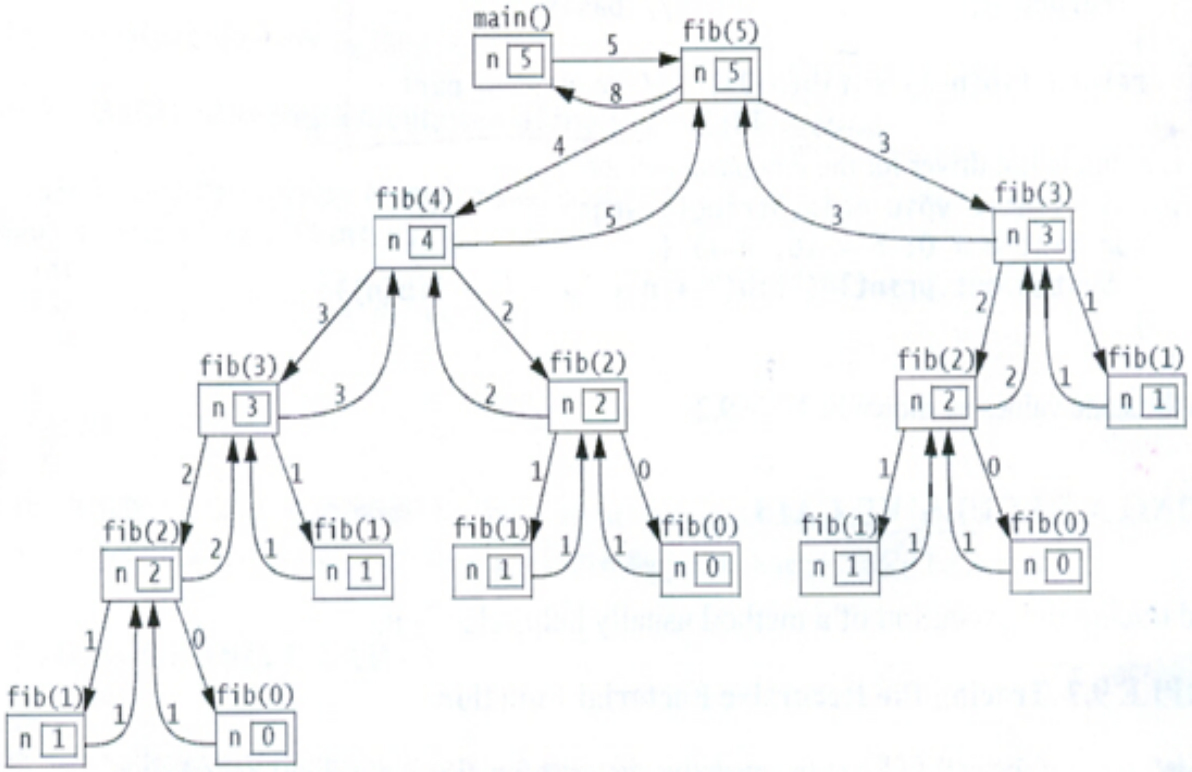
```
public static int fib(int n)
{
        if (n < 2)
                return 1;                               // base cases
        else
                return  fib(n-1)+fib(n-2);   // recursive part

}
```

Binary Recursion

Example – 5 (Continued)

- Recursive Trace of Fibonacci Function: fib(5)



Linear Recursion

Example – 6: Binary Search

- **Problem**: Given $S=\{s_0, s_1, \ldots , s_{n-1}\}$ is a sorted sequence of *n* integers, and an integer x. Search whether x is in S.
- **Binary Search Algorithm:**
  - ○ If the sequence is empty, return -1. Base case
  - ○ Let s_i be the middle element of the sequence.
    - ▪ If $s_i = x$, return its index i. Base case
    - ▪ If $s_i < x$, apply the algorithm on the subsequence that lies above s_i. recursive case
    - ▪ Otherwise, apply the algorithm on the subsequence of S that lies below s_i. recursive case

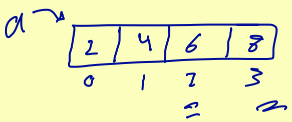
```
BinarySearch (A, i, x) {
  if (A[i] == x)
    return i;
 else if (A[i] > x)
    return BinarySearch (A, i+1, x);
 else
  return BinarySearch (A, i-1, x); }
```

Linear Recursion

Example – 6 (continued): Binary Search

- Implementation:

```
                                                starting Point
public static int search(int a[], int lo, int hi, int x)
                                                 end Point          x = 6
{
    if (lo > hi)  return -1; // Basis
    else{                    // Recursive part
        int i = (lo+hi)/2;   → الإلمنت الي بالنص
        if(a[i] == x) return i;
        else if(a[i] < x)
                return search (a, i+1, hi, x)
        else x الإلمنت الي بالنص أكبر من
                return search (a, lo,i-1, x);
    }
}
```



Linear Recursion

Example – 6 (continued): Binary Search

a	2	5	7	11	14	20

- Implementation:

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
     else{                       // Recursive part
           int i = (lo+hi)/2;
           if(a[i] == x) return i;
           else if(a[i] < x)
                      return search (a, i+1, hi, x)
           else
                      return search (a, lo,i-1, x);
     }
}
```

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

i=2

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
     else{                        // Recursive part
           int i = (lo+hi)/2;
           if(a[i] == x) return i;
           else if(a[i] < x)
                       return search (a, i+1, hi, x)
           else
                       return search (a, lo,i-1, x);
     }
}
```

search(a, 0, 5, 14)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20
					i=4	

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
     else{                          // Recursive part
           int i = (lo+hi)/2;
           if(a[i] == x) return i;
           else if(a[i] < x)
                      return search (a, i+1, hi, x)
           else
                      return search (a, lo,i-1, x);
     }
}
```

search(a, 0, 5, 14)

search(a, 3, 5, 14)

Linear Recursion

Example – 6 (continued): Binary Search

- Implementation:

a	2	5	7	11	14	20

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
     else{                          // Recursive part
           int i = (lo+hi)/2;
           if(a[i] == x) return i;
           else if(a[i] < x)
                      return search (a, i+1, hi, x)
           else
                      return search (a, lo,i-1, x);
     }
}
```

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

i=2

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                       // Recursive part
         int i = (lo+hi)/2;
         if(a[i] == x) return i;
         else if(a[i] < x)
                   return search (a, i+1, hi, x)
         else
                   return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 2)

Linear Recursion

Example – 6 (continued): Binary Search

a	2	5	7	11	14	20
	i=0					

– Implementation:

```
public static int search(int a[], int lo, int hi, int x)
{
         if (lo > hi)  return -1; // Basis
        else{                        // Recursive part
                int i = (lo+hi)/2;
                if(a[i] == x) return i;
                else if(a[i] < x)
                                return search (a, i+1, hi, x)
                else
                                return search (a, lo,i-1, x);
        }
}
```

search(a, 0, 5, 2)

search(a, 0, 1, 2)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

i=2

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                     // Recursive part
         int i = (lo+hi)/2;
         if(a[i] == x) return i;
         else if(a[i] < x)
                   return search (a, i+1, hi, x)
         else
                   return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 5)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20
	i=0					

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                          // Recursive part
         int i = (lo+hi)/2;
         if(a[i] == x) return i;
         else if(a[i] < x)
                   return search (a, i+1, hi, x)
         else
                   return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 5)

search(a, 0, 1, 5)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20
		i=0				

```
public static int search(int a[], int lo, int hi, int x)
{
       if (lo > hi)  return -1; // Basis
      else{                              // Recursive part
             int i = (lo+hi)/2;
             if(a[i] == x) return i;
             else if(a[i] < x)
                           return search (a, i+1, hi, x)
             else
                           return search (a, lo,i-1, x);
      }
}
```

search(a, 0, 5, 5)

search(a, 0, 1, 5)

search(a, 1, 1, 5)

Linear Recursion

Example – 6 (continued): Binary Search

a	2	5	7	11	14	20

– Implementation:

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
     else{                           // Recursive part
           int i = (lo+hi)/2;
           if(a[i] == x) return i;
           else if(a[i] < x)
                      return search (a, i+1, hi, x)
           else
                      return search (a, lo,i-1, x);
     }
}
```

Linear Recursion

Example – 6 (continued): Binary Search

- Implementation:

a	2	5	7	11	14	20

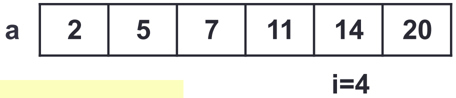
i=2

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                     // Recursive part
        int i = (lo+hi)/2;
        if(a[i] == x) return i;
        else if(a[i] < x)
                return search (a, i+1, hi, x)
        else
                return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 21)

Linear Recursion

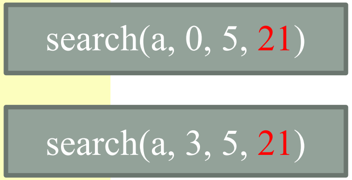
Example – 6 (continued): Binary Search



a	2	5	7	11	14	20
					i=4	

– Implementation:

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                      // Recursive part
         int i = (lo+hi)/2;
         if(a[i] == x) return i;
         else if(a[i] < x)
                   return search (a, i+1, hi, x)
         else
                   return search (a, lo,i-1, x);
    }
}
```



search(a, 0, 5, 21)

search(a, 3, 5, 21)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

i=5

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                         // Recursive part
          int i = (lo+hi/2);
          if(a[i] == x) return i;
          else if(a[i] < x)
                    return search (a, i+1, hi, x)
          else
                    return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 21)

search(a, 3, 5, 21)

search(a, 5, 5, 21)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                                  // Recursive part
        int i = (lo+hi)/2;
        if(a[i] == x) return i;
        else if(a[i] < x)
                return search (a, i+1, hi, x)
        else
                return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 21)

search(a, 3, 5, 21)

search(a, 5, 5, 21)

search(a, 6, 5, 21)

Linear Recursion

Example – 6 (continued): Binary Search

- Implementation:

a	2	5	7	11	14	20

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
     else{                      // Recursive part
           int i = (lo+hi)/2;
           if(a[i] == x) return i;
           else if(a[i] < x)
                      return search (a, i+1, hi, x)
           else
                      return search (a, lo,i-1, x);
     }
}
```

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

i=2

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                     // Recursive part
         int i = (lo+hi)/2;
         if(a[i] == x) return i;
         else if(a[i] < x)
                   return search (a, i+1, hi, x)
         else
                   return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 12)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

i=4

```
public static int search(int a[], int lo, int hi, int x)
{
     if (lo > hi)  return -1; // Basis
    else{                     // Recursive part
         int i = (lo+hi)/2;
         if(a[i] == x) return i;
         else if(a[i] < x)
                   return search (a, i+1, hi, x)
         else
                   return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 12)

search(a, 3, 5, 12)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20
				i=3		

```
public static int search(int a[], int lo, int hi, int x)
{
    if (lo > hi)  return -1; // Basis
    else{                    // Recursive part
        int i = (lo+hi)/2;
        if(a[i] == x) return i;
        else if(a[i] < x)
                return search (a, i+1, hi, x)
        else
                return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 12)

search(a, 3, 5, 12)

search(a, 3, 3, 12)

Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
    else{                               // Recursive part
          int i = (lo+hi)/2;
          if(a[i] == x) return i;
          else if(a[i] < x)
                    return search (a, i+1, hi, x)
          else
                    return search (a, lo,i-1, x);
    }
}
```

search(a, 0, 5, 12)

search(a, 3, 5, 12)

search(a, 3, 3, 12)

search(a, 4, 3, 12)

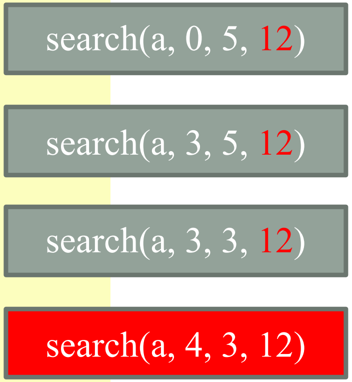
Linear Recursion

Example – 6 (continued): Binary Search

– Implementation:

a	2	5	7	11	14	20

```
public static int search(int a[], int lo, int hi, int x)
{
      if (lo > hi)  return -1; // Basis
      else{                            // Recursive part
            int i = (lo+hi)/2;
            if(a[i] == x) return i;
            else if(a[i] < x)
                        return search (a, i+1, hi, x)
            else
                        return search (a, lo,i-1, x);
      }
}
```



search(a, 0, 5, 12)

search(a, 3, 5, 12)

search(a, 3, 3, 12)

search(a, 4, 3, 12)