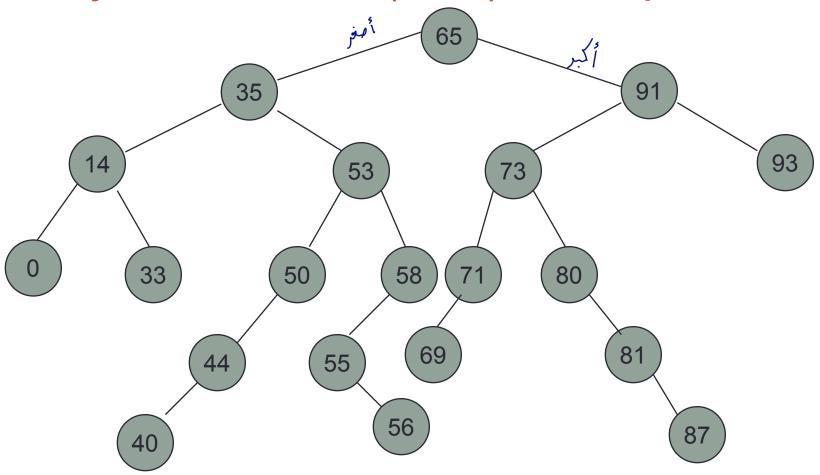
BINARY SEARCH TREES (BSTS)

CSC212: Data Structures

Binary Search Trees (BSTs)

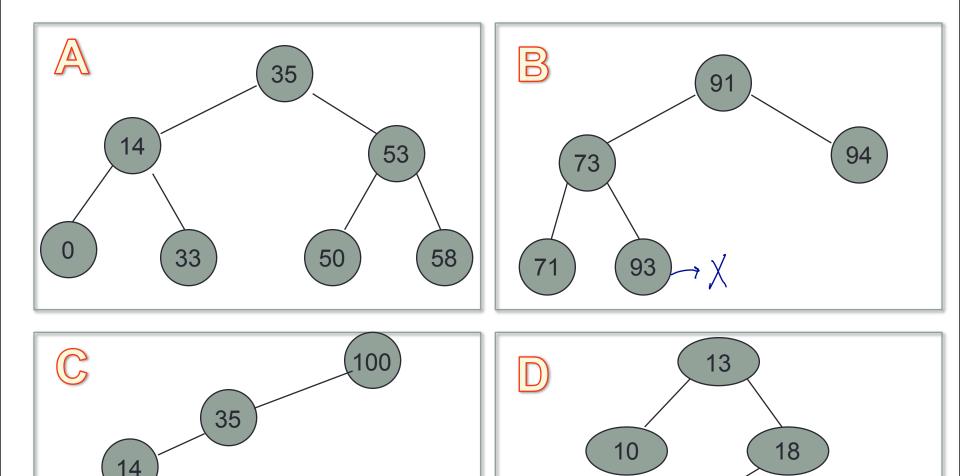
- A Binary Search Tree (BST) is a binary tree such that for each node, say N, the following statements are true:
 - If L is any node in the left subtree of N, then L is <u>less</u> than N.
 - 2. If R is any node in the right subtree of N, then R is <u>greater</u> than N.

Binary Search Tree (BST): Example



Binary Search Trees (BSTs)

- Consider the search operation FindKey: find an element of a particular key value in a binary tree.
 - In binary tree this operation is O(n).
 - In a binary tree of 10^6 nodes $\rightarrow 10^6$ steps required.
 - In a Binary Search Tree (BST) this operation can be performed very efficiently: O(log₂n).
 - A binary search tree of 10^6 nodes $\rightarrow \log_2(10^6) \cong 20$ steps only are required.
 - In average case



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Which of the above Trees is not BST?

Elements: the elements are nodes (BSTNode), each node contains the following data type: Type,Key and has LeftChild and RightChild references. .

Structure: hierarchical structure; each node can have two children: left or right child; there is a root node and a current node. If N is any node in the tree, nodes in the left subtree < N and nodes in the right subtree > N.

Domain: the number of nodes in a BST is bounded; type/class name is BST

Operations:

Method FindKey (int tkey, boolean found).

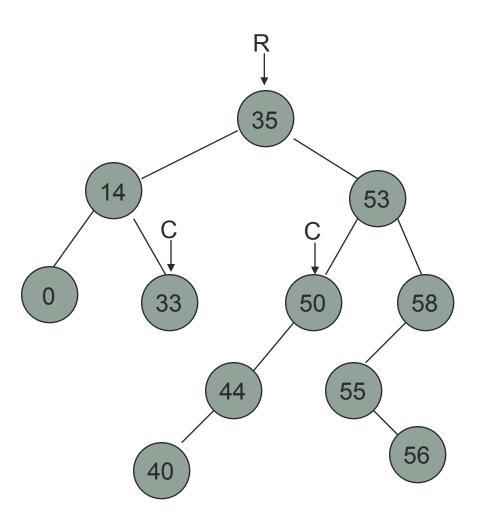
requires: none.

input: tkey.

results: If bst contains a node whose key value is tkey, then that node is made the current node and found is set to true; otherwise found is set to false and either the tree is empty or the current node is the node to which the node with key = tkey would be attached as a child if it were added to the BST.

output: found.

Find



find (50) find (30)

2. Method Insert (int k, Type e, boolean inserted)

requires: Full (bst) is false. input: key, e.

results: if bst does not contain k then inserted is set to true and node with k and e is inserted and made the current element; otherwise inserted is set to false and current value does not change. output: inserted.

Method Remove_Key (int tkey, boolean removed) input: tkey

results: Node with key value tkey is removed from the bst and removed set to true. If BST is not empty then root is made the current. **output**: removed

Method Update(int key, Type e, boolean updated)

requires: Empty(bst) is false. input: key, e. results: current node's element is replaced with e. Output: updated.

These operations have the same specification as ADT Binary Tree.

- **5. Method** Traverse (Order ord)
- **6. Method** DeleteSub ()
- 7. **Method** Retrieve (Type e)
- 8. Method Empty (boolean empty).
- 9. Method Full (boolean full)

ADT Binary Search Tree: Element

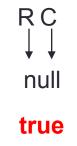
```
public class BSTNode <T> {
 public int key;
 public T data;
 public BSTNode<T> left, right;
 /** Creates a new instance of BSTNode */
 public BSTNode(int k, T val) {
        key = k;
        data = val;
        left = right = null;
 public BSTNode(int k, T val, BSTNode<T> 1, BSTNode<T> r) {
        kev = k;
        data = val;
        left = 1:
        right = r;
```

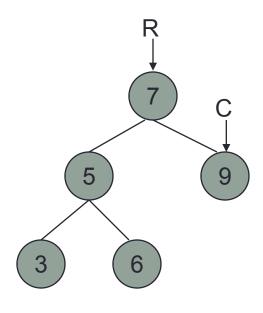
```
public class BST <T> {
 BSTNode T> root, current;
 /** Creates a new instance of BST */
 public BST() {
        root = current = null;
 public boolean empty() {
        return root == null;
 public boolean full() {
        return false;
 public T retrieve () {
        return current. data;
```

```
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 BSTNode T> root, current;
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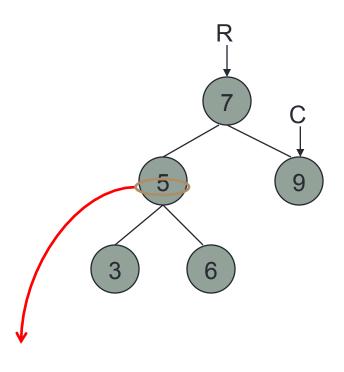
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false

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BST: Searching

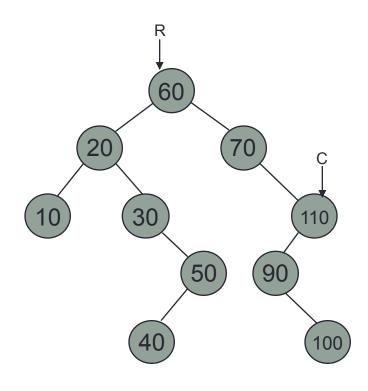
 The search operation in a binary search tree can be carried out as:

```
While (the target element is not found <u>and</u> there is more tree to search) <u>do</u>

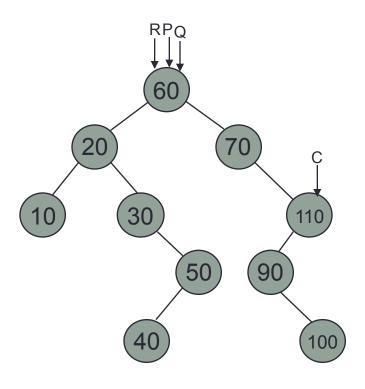
<u>if</u> the target element is "less than" the current element <u>then</u> search the left subtree <u>else</u> search the right subtree.
```

```
public boolean findkey(int tkey) {
        BSTNode<T> p = root, q = root;
        if(empty())
                  return false;
        while(p != null) {
                  q = p;
                  if(p. key == tkey)  {
                            current = p;
                            return true;
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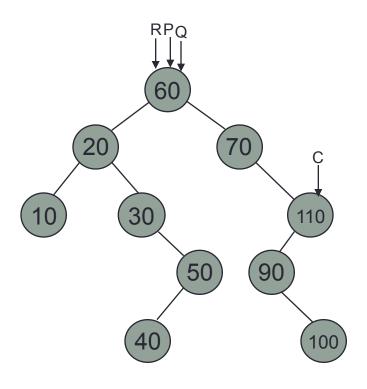
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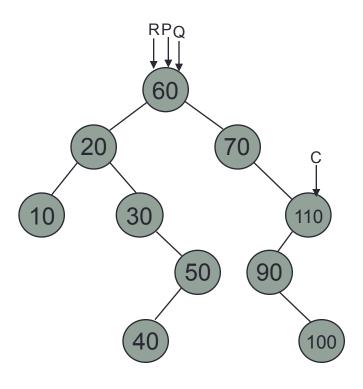
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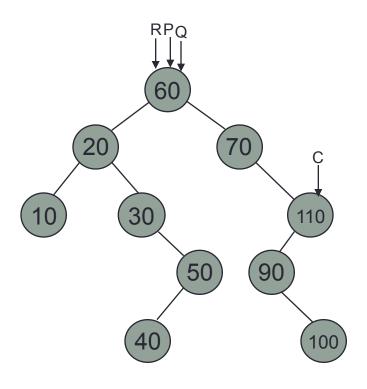
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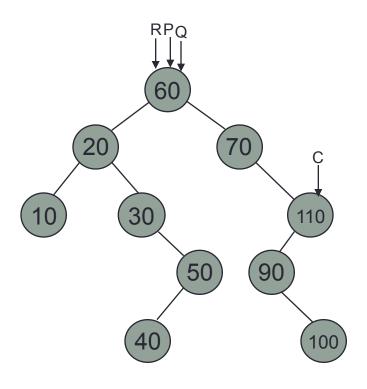
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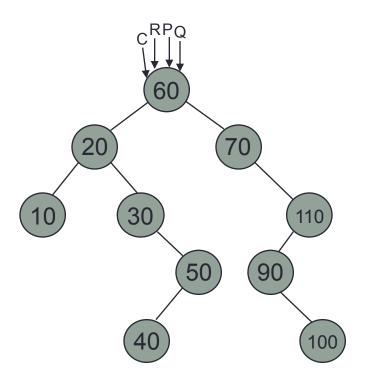
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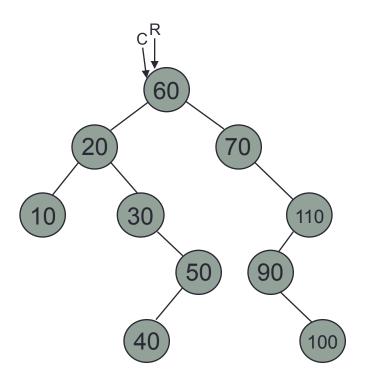
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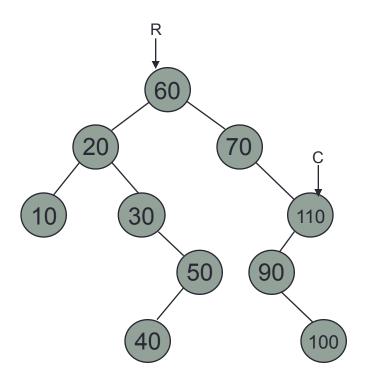
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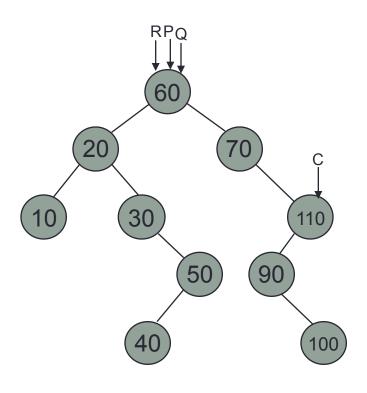
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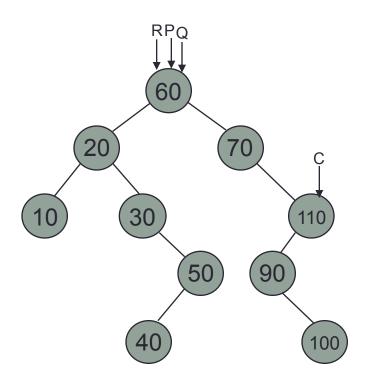
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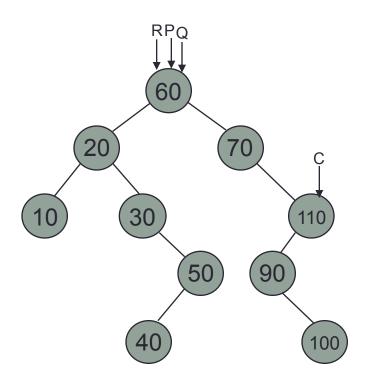
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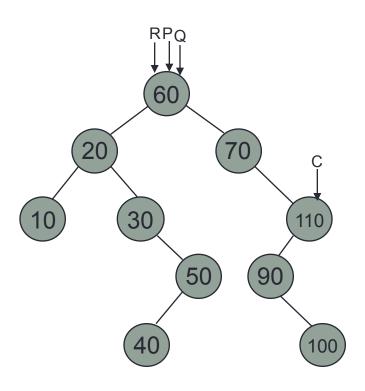
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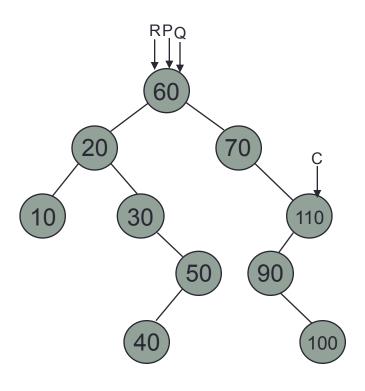
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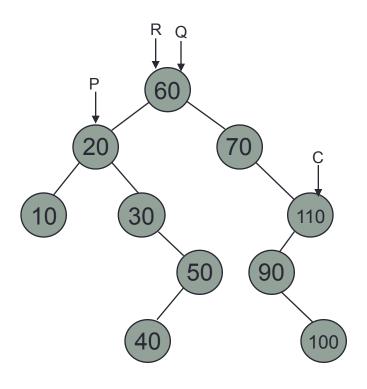
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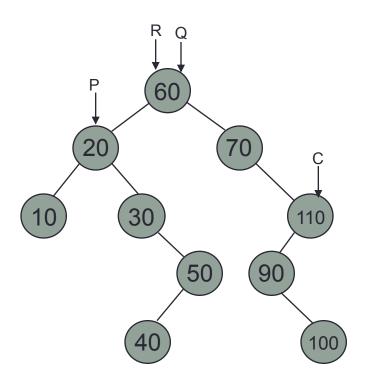
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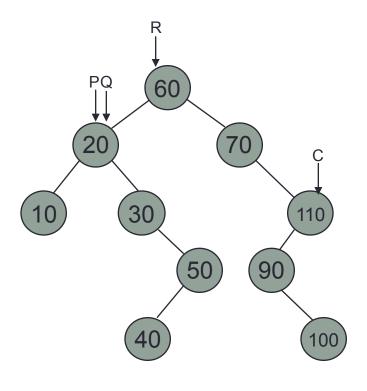
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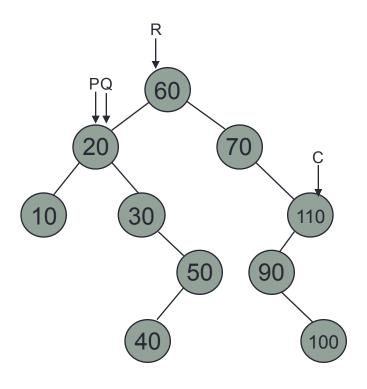
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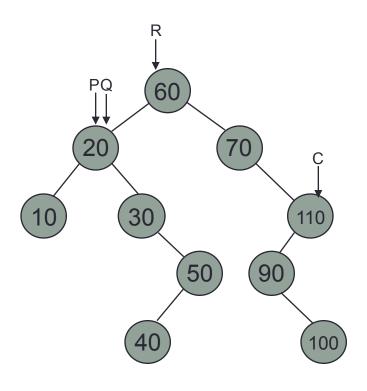
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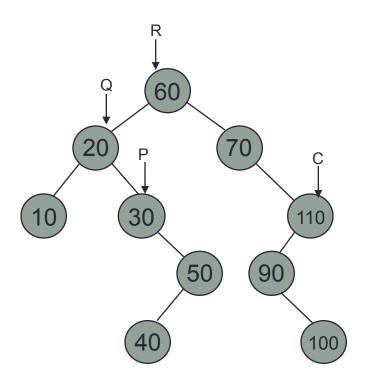
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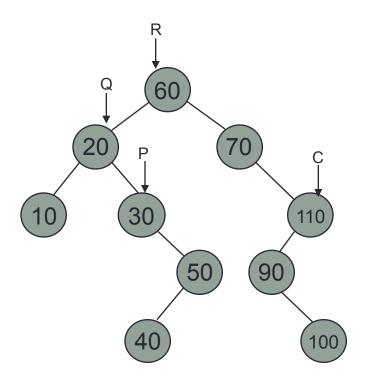
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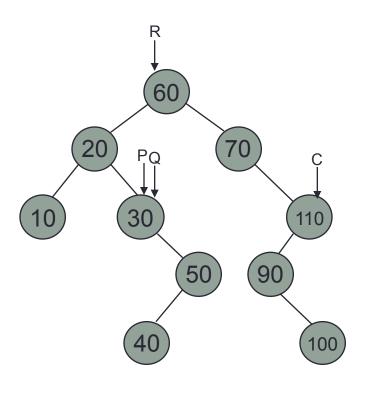
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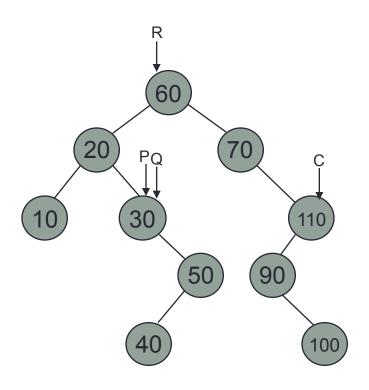
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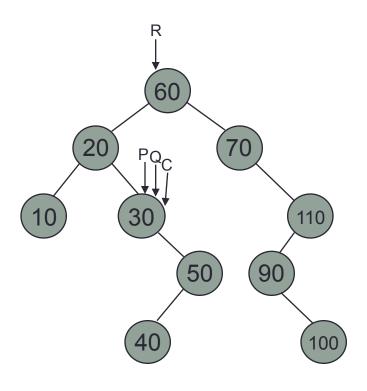
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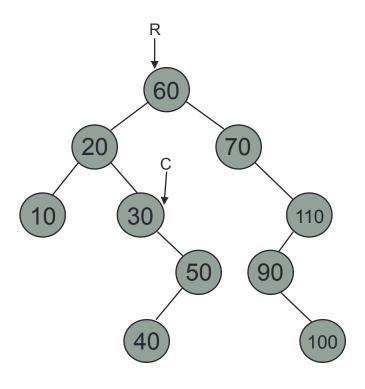
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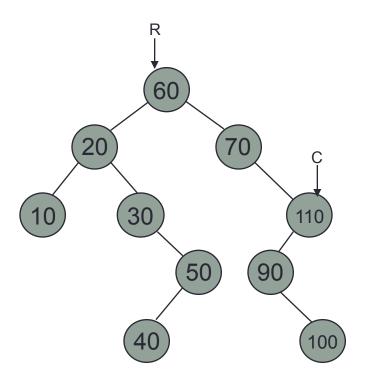
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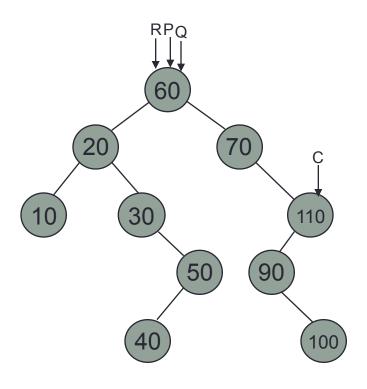
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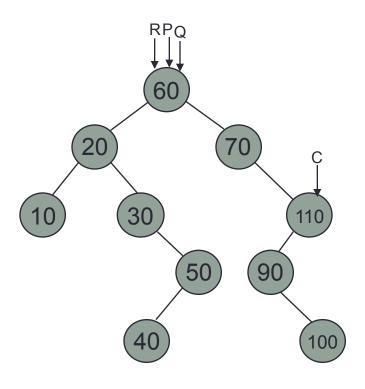
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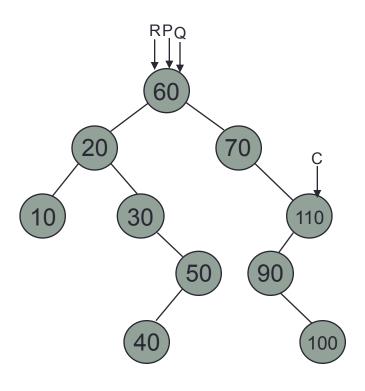
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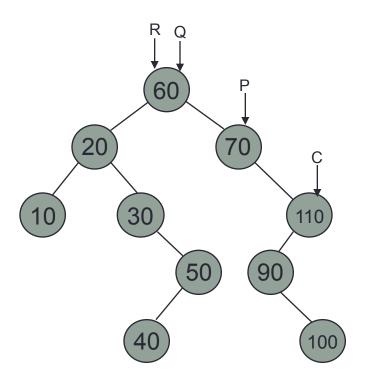
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                  else
                            p = p. right;
        current = q;
        return false:
```



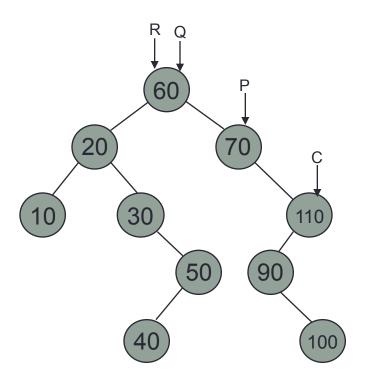
```
public boolean findkey(int tkey) {
        BSTNode<T> p = root, q = root;
        if(empty())
                  return false;
        while(p != null) {
                  q = p;
                  if(p. key == tkey) {
                            current = p;
                            return true;
                  else if(tkey < p. key)
                            p = p. left;
                  else
                            p = p. right;
        current = q;
        return false:
```



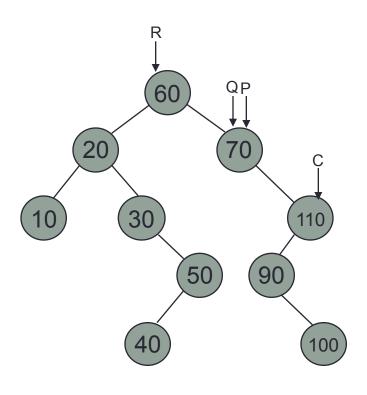
```
public boolean findkey(int tkey) {
        BSTNode<T> p = root, q = root;
        if(empty())
                  return false;
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                  q = p;
                  if(p. key == tkey)  {
                            current = p;
                            return true;
                  else if(tkey < p. key)
                            p = p. left;
                  else
                            p = p.right;
        current = q;
        return false:
```



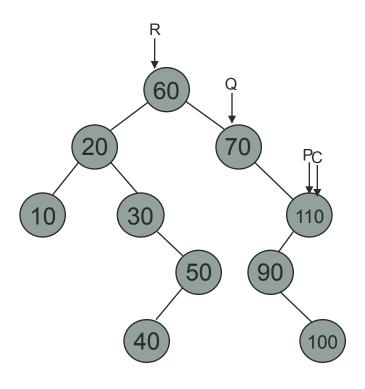
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
                   return false;
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                   q = p;
                   if(p. key == tkey)  {
                              current = p;
                              return true;
                   else if(tkey < p. key)
                              p = p. left;
                   else
                              p = p. right;
        current = q;
        return false:
```



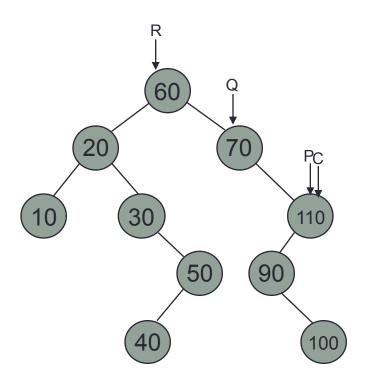
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
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                   q = p;
                   if(p. key == tkey) {
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                              return true;
                   else if(tkey < p. key)
                              p = p. left;
                   else
                              p = p. right;
        current = q;
        return false:
```



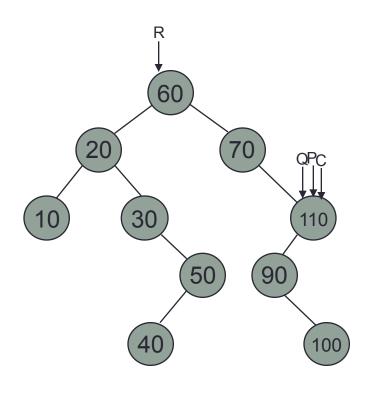
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        BSTNode\langle T \rangle p = root, q = root;
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                   else
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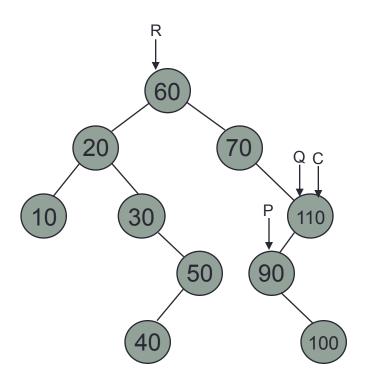
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        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
                   return false;
        while(p != null) {
                   q = p;
                   if(p. key == tkey)  {
                              current = p;
                              return true;
                   else if(tkey < p. key)
                              p = p. left;
                   else
                              p = p. right;
        current = q;
        return false:
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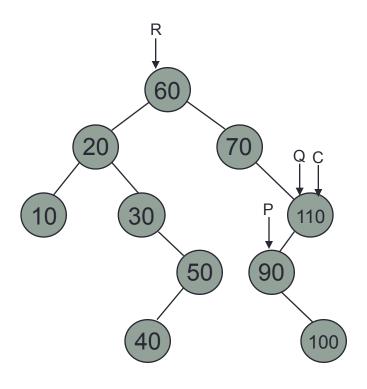
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
                   return false;
        while(p != null) {
                   q = p;
                   if(p. key == tkey) {
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                              return true;
                   else if(tkey < p. key)
                              p = p. left;
                   else
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        current = q;
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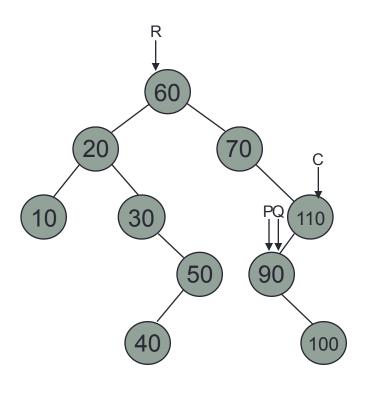
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public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
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                   q = p;
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                              p = p. left;
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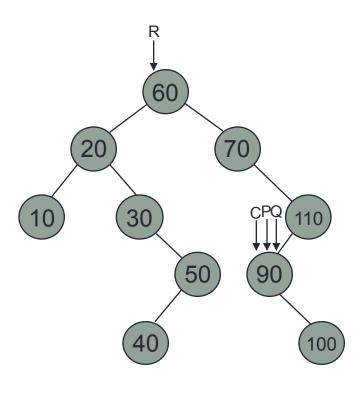
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public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
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                   q = p;
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                   else
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        current = q;
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```



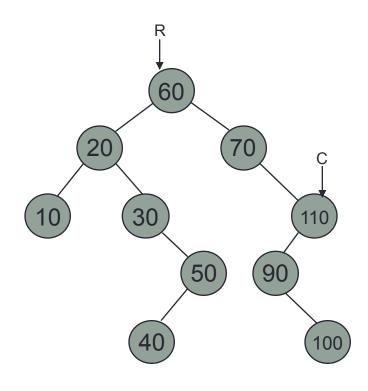
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public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
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                   q = p;
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        current = q;
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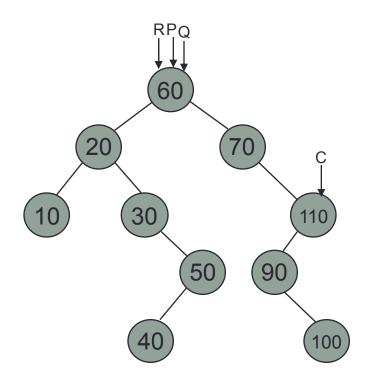
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
                   return false;
        while(p != null) {
                   q = p;
                   if(p. key = tkey) {
                              current = p;
                              return true;
                   else if(tkey < p. key)
                              p = p. left;
                   else
                              p = p. right;
        current = q;
        return false:
```



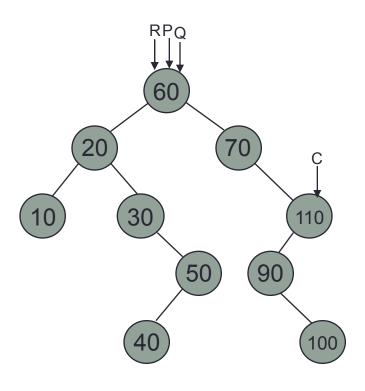
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
                   return false;
        while(p != null) {
                   q = p;
                   if(p. key == tkey)  {
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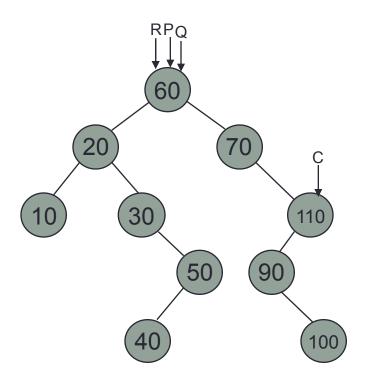
```
public boolean findkey(int tkey) {
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        if(empty())
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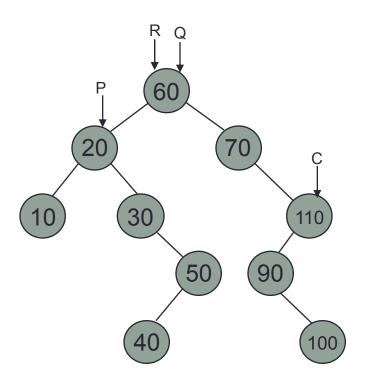
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        BSTNode<T> p = root, q = root;
        if(empty())
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                  else
                            p = p. right;
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```



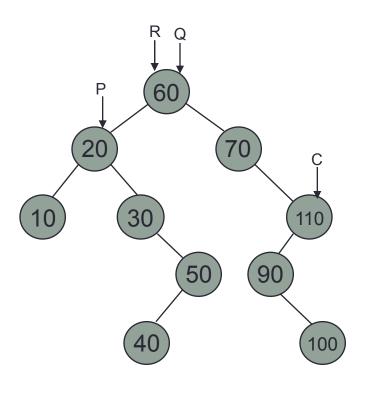
```
public boolean findkey(int tkey) {
        BSTNode<T> p = root, q = root;
        if(empty())
                  return false;
        while(p != null) {
                  q = p;
                  if(p. key == tkey) {
                            current = p;
                            return true;
                  else if(tkey < p. key)
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                  else
                            p = p. right;
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        return false:
```



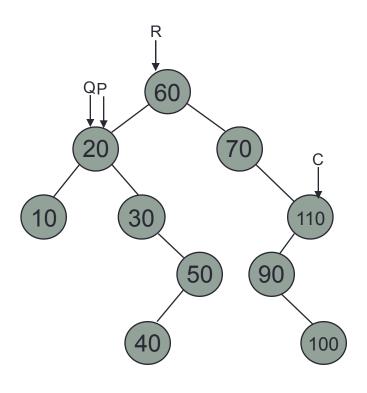
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
        if(empty())
                   return false;
        while(p != null) {
                   q = p;
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                              current = p;
                              return true;
                   else if(tkey < p. key)
                              p = p. left;
                   else
                              p = p. right;
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```



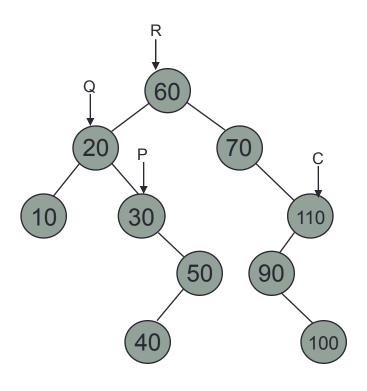
```
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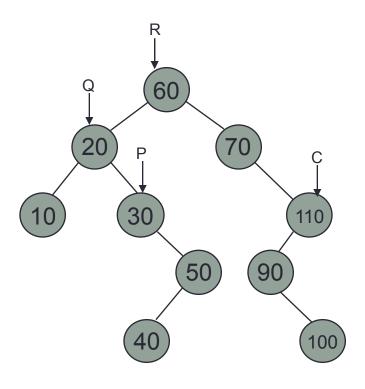
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public boolean findkey(int tkey) {
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                   else if(tkey < p. key)
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                   else
                              p = p. right;
        current = q;
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```



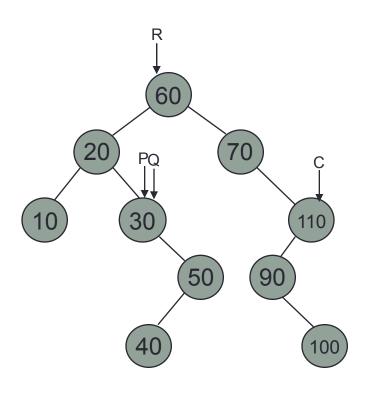
```
public boolean findkey(int tkey) {
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        if(empty())
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                  else
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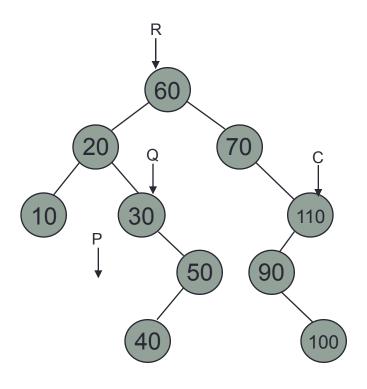
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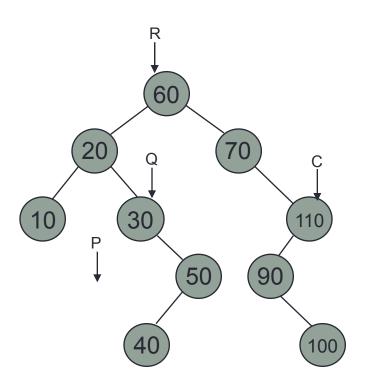
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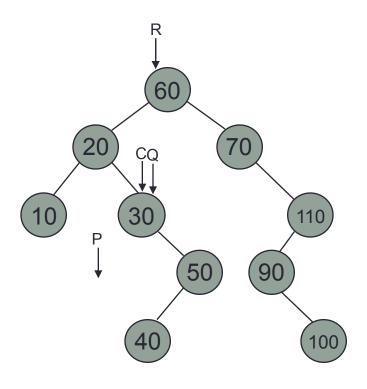
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
         if(empty())
                   return false;
        while(p != null) {
                   q = p;
                    if(p. key == tkey)  {
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                    else if(tkey < p. key)</pre>
                              p = p. left;
                    else
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        current = q;
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```



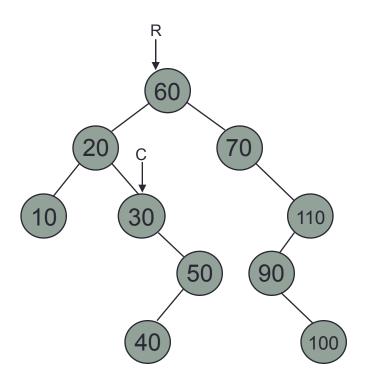
```
public boolean findkey(int tkey) {
        BSTNode\langle T \rangle p = root, q = root;
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                              p = p. left;
                   else
                              p = p. right;
        current = q;
        return false:
```



```
public boolean insert(int k, T val) {
       BSTNode <T> p, q = current;
        if(findkey(k)) {
                  current = q: // findkey() modified current
                 return false; // key already in the BST
        p = new BSTNode < T > (k, val);
        if (empty()) {
                 root = current = p;
                 return true;
        else {
                 // current is pointing to parent of the new key
                  if (k < current. key)
                           current. left = p;
                  else
                            current.right = p;
                  current = p;
                 return true:
```

```
Example #1
public boolean insert(int k, T val) {
                                                                  k = 60
       BSTNode <T> p, q = current;
        if(findkey(k)) {
                  current = q; // findkey() modified current
                 return false: // key already in the BST
                                                                   null
        p = new BSTNode < T > (k, val);
        if (empty()) {
                 root = current = p;
                 return true;
        else {
                 // current is pointing to parent of the new key
                  if (k < current. key)
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                            current.right = p;
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Example #1

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                  else
                           current.right = p;
                  current = p;
                 return true:
```

```
Example #2
public boolean insert(int k, T val) {
                                                                  k = 20
       BSTNode <T> p, q = current;
        if(findkey(k)) {
                 current = q: // findkey() modified current
                 return false; // key already in the BST
        p = new BSTNode < T > (k, val);
        if (empty()) {
                 root = current = p;
                 return true;
        else {
                 // current is pointing to parent of the new key
                  if (k < current. key)
                           current. left = p;
                  else
                           current.right = p;
                  current = p;
                 return true:
```

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       BSTNode<T> p, q = current;
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                                                                  c Ro
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                 return true;
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                  else
                            current.right = p;
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```

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public boolean insert(int k, T val) {
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        BSTNode <T> p, q = current;
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                                                                    C<sub>RQ</sub>
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public boolean insert(int k, T val) {
                                                                    k = 20
        BSTNode <T> p, q = current;
        if(findkey(k)) {
                                                                    C<sub>RQ</sub>
                  current = q; // findkey() modified current
                  return false; // key already in the BST
                                                                     60
        p = new BSTNode < T > (k, val);
        if (empty()) {
                  root = current = p;
                  return true;
        else {
                  // current is pointing to parent of the new key
                  if (k < current. key)
                            current. left = p;
                  else
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Example #2
public boolean insert(int k, T val) {
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       BSTNode <T> p, q = current;
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```
Example #3
public boolean insert(int k, T val) {
                                                                  k = 70
       BSTNode <T> p, q = current;
        if(findkey(k)) {
                 current = q: // findkey() modified current
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        p = new BSTNode < T > (k, val);
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Example #4

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public boolean insert(int k, T val) {
                                                                   k = 30
       BSTNode <T> p, q = current;
        if(findkey(k)) {
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```

```
Example #4
public boolean insert(int k, T val) {
                                                                  k = 25
       BSTNode <T> p, q = current;
        if(findkey(k)) {
                 current = q: // findkey() modified current
                 return false: // key already in the BST
                                                                   60
        p = new BSTNode < T > (k, val);
        if (empty()) {
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public boolean insert(int k, T val) {
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        p = new BSTNode \langle T \rangle (k, val);
        if (empty()) {
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        else {
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```
Example #5
public boolean insert(int k, T val) {
                                                                  k = 90
       BSTNode <T> p, q = current;
        if(findkey(k)) {
                 current = q: // findkey() modified current
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                                                                   60
        p = new BSTNode < T > (k, val);
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public boolean insert(int k, T val) {
                                                                  k = 10
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        if(findkey(k)) {
                  current = q: // findkey() modified current
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                                                                   60
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public boolean insert(int k, T val) {
                                                                  k = 30
       BSTNode <T> p, q = current;
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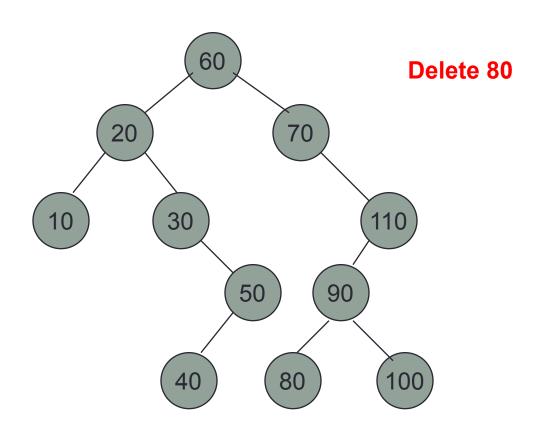
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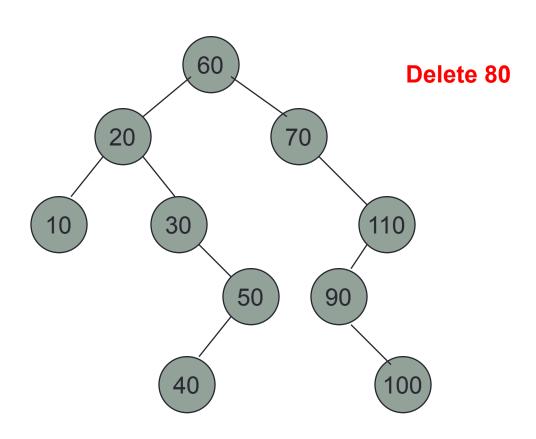
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```

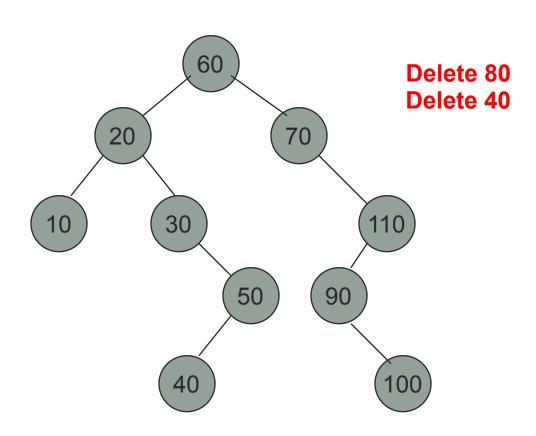
BST Node Deletion

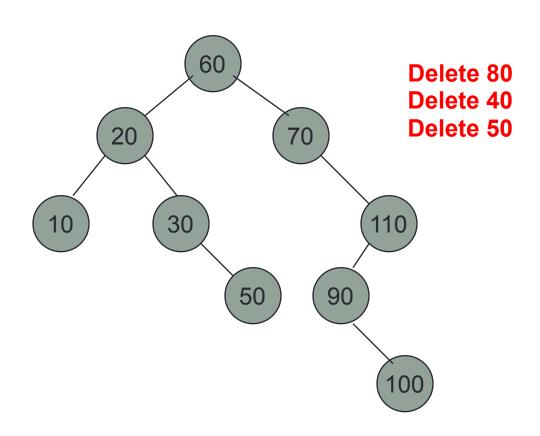
- There are three cases:
 - Case 1: Node to be deleted has no children.
 - Case 2: Node to be deleted has one child.
 - Case 3: Node to be deleted has two children.
- In all these case it is always a leaf node that gets deleted.

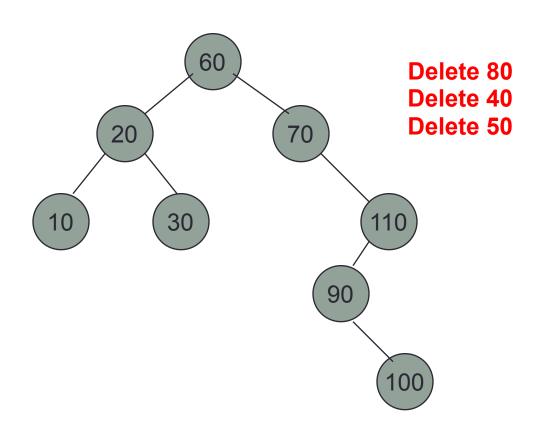
- Node to be deleted has no children.
- Simplest case. Unlink the node from its parent.
- The parent will be linked with null in the place of the deleted node.



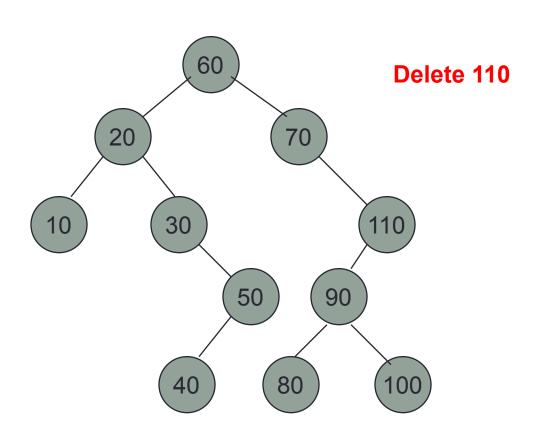


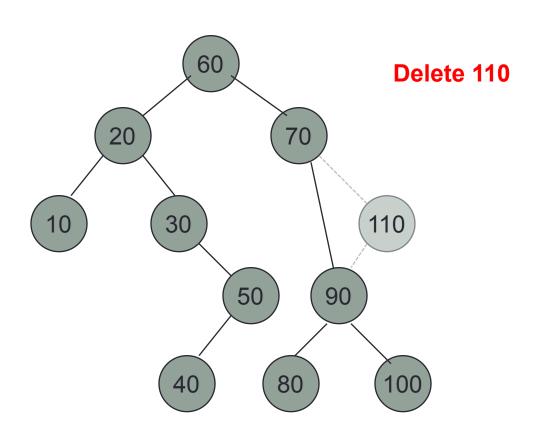


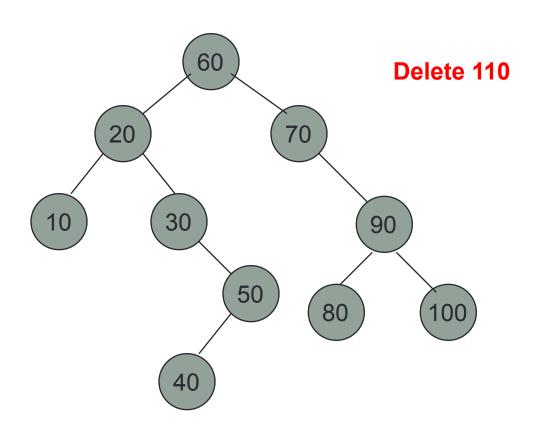


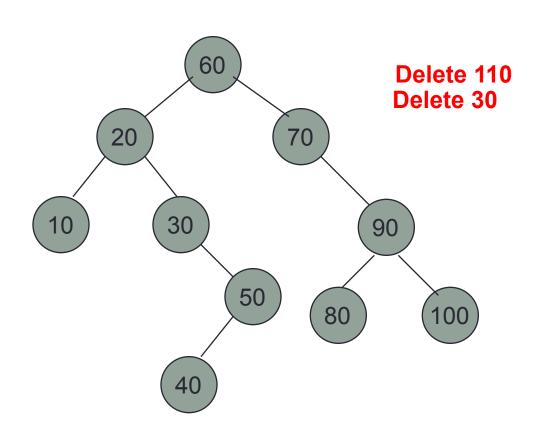


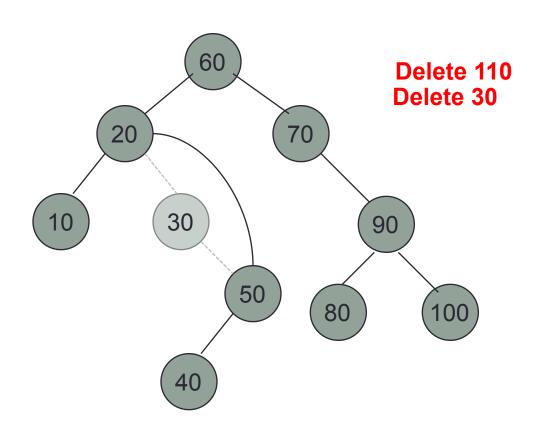
- Node to be deleted has one child.
- Remove the node, and place its child (along with its subtree) in its place.
- The parent will be linked with the child of the deleted node.

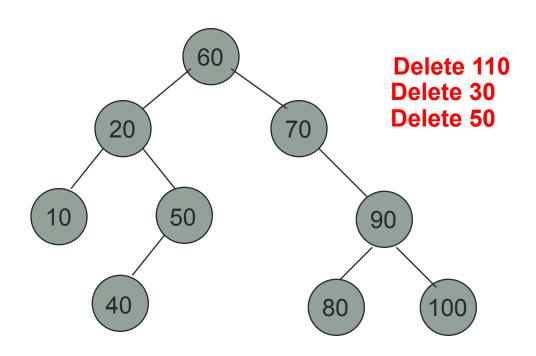


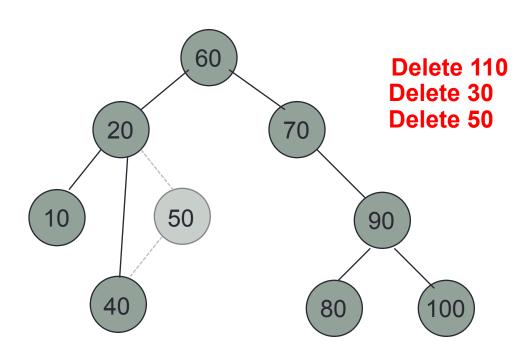


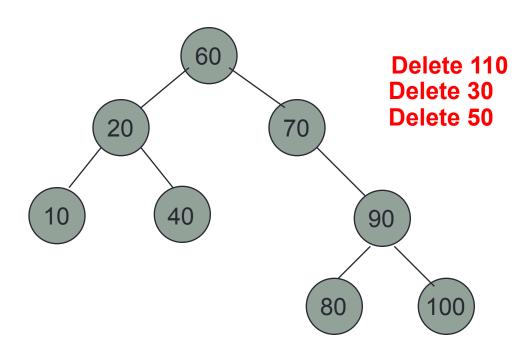




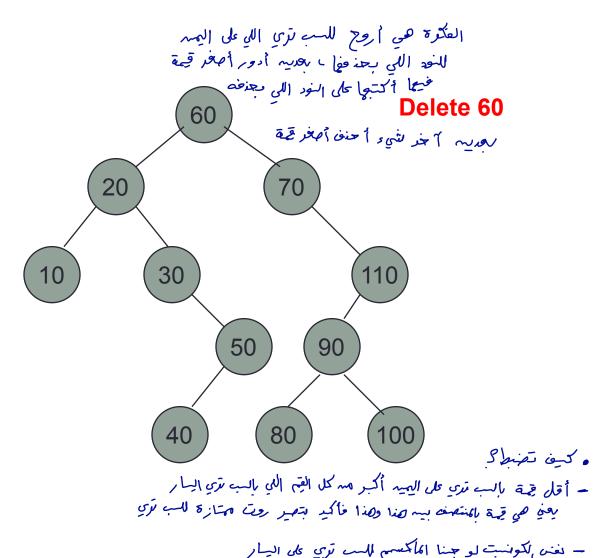


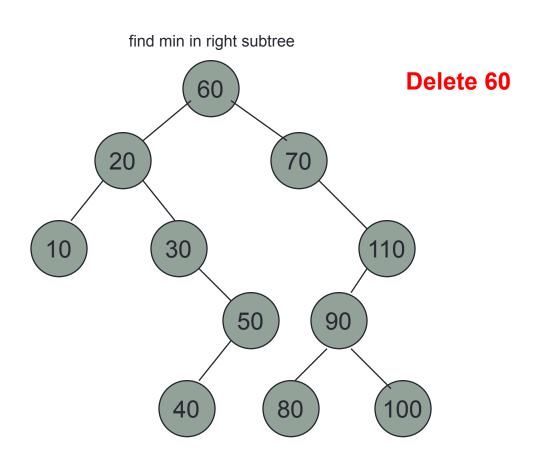


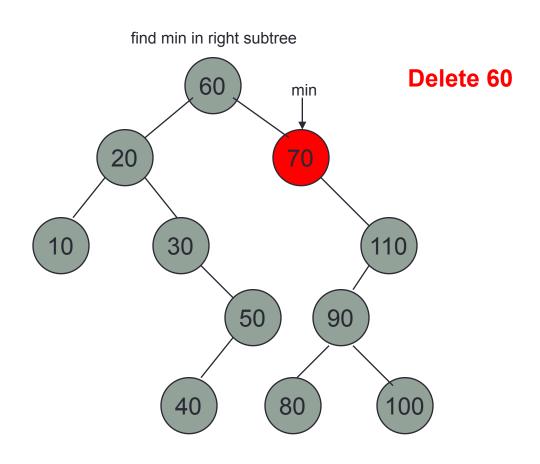


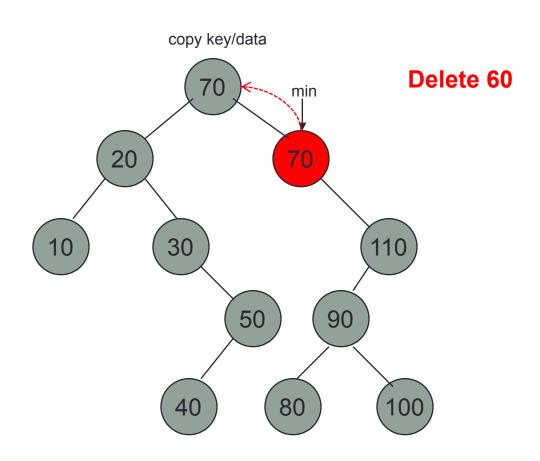


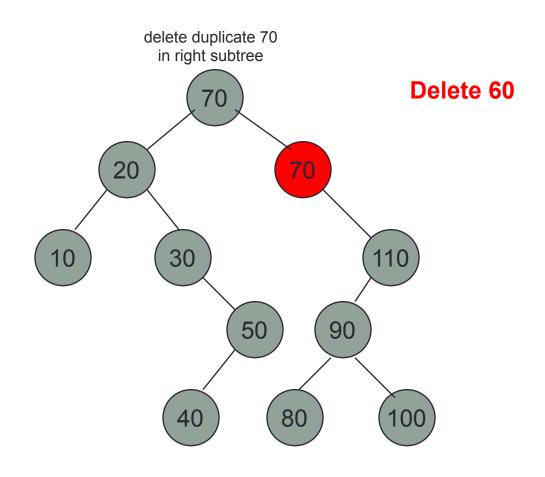
- Node to be deleted has two children.
- Complex case:
 - Find the node with the minimum key in the right subtree (left-most node in the right subtree).
 - Copy its key/data over the node to be deleted.
 - Delete the duplicate node (using either Case 1 or 2)
- The node will be overwritten by the minimum node in the right subtree. Then that duplicate node will be deleted.

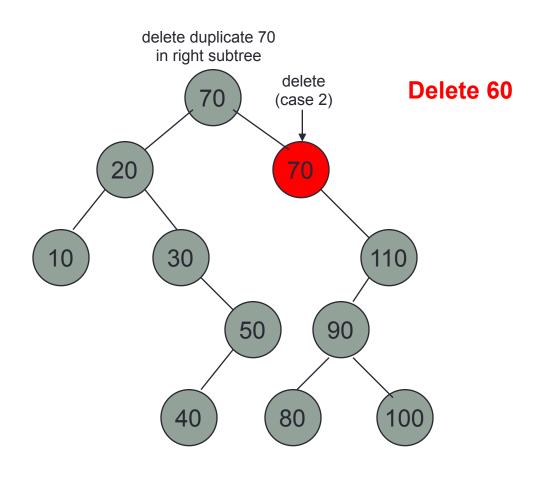


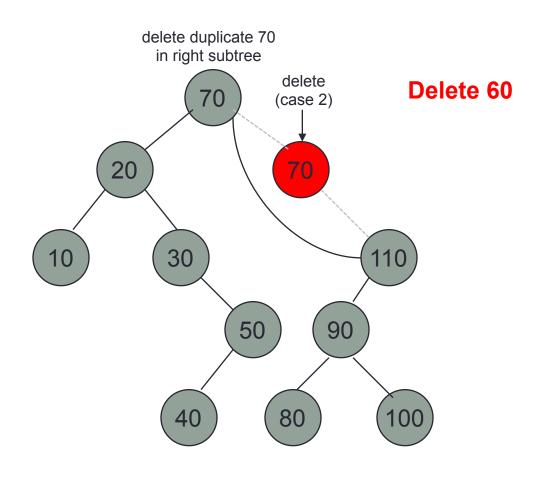


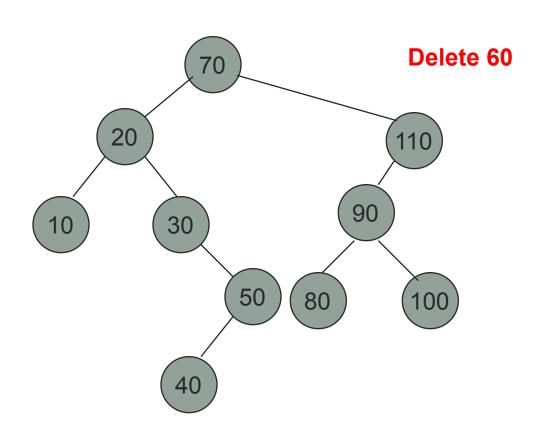


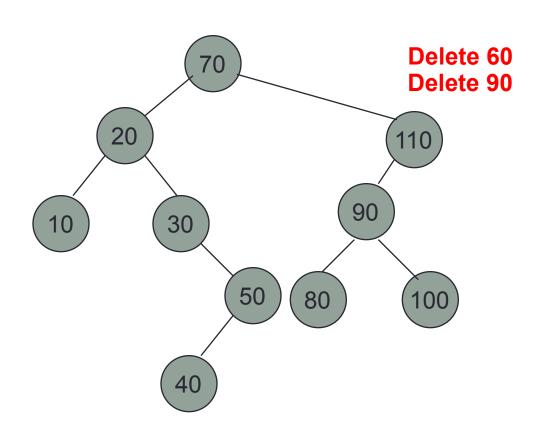


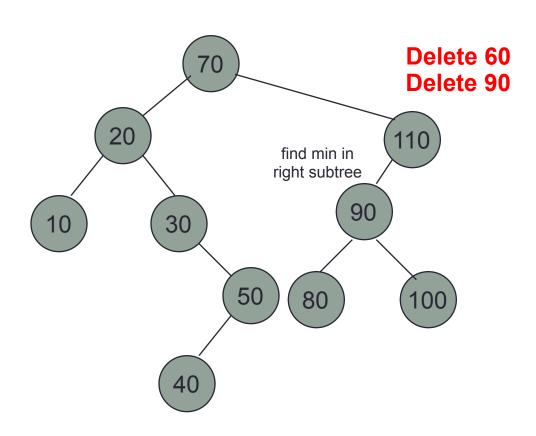


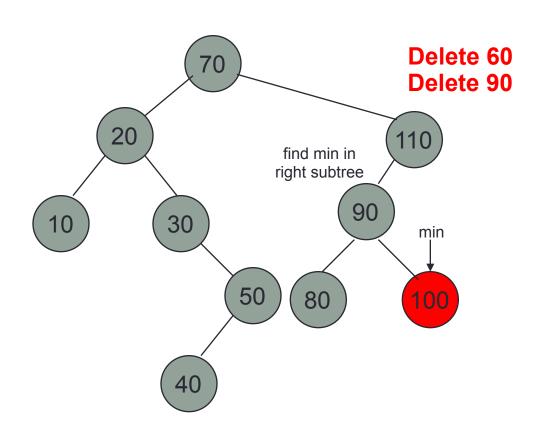


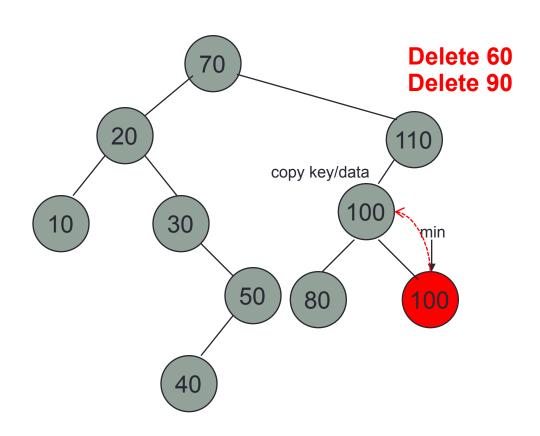


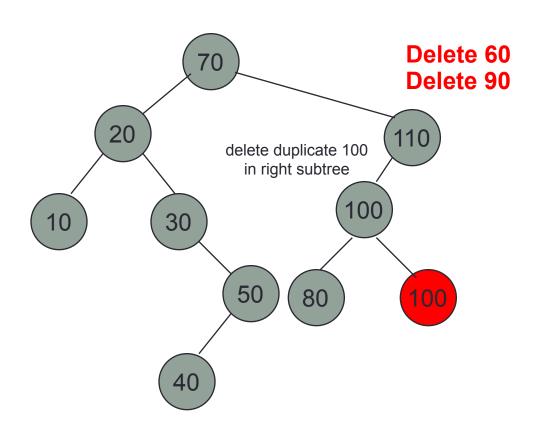


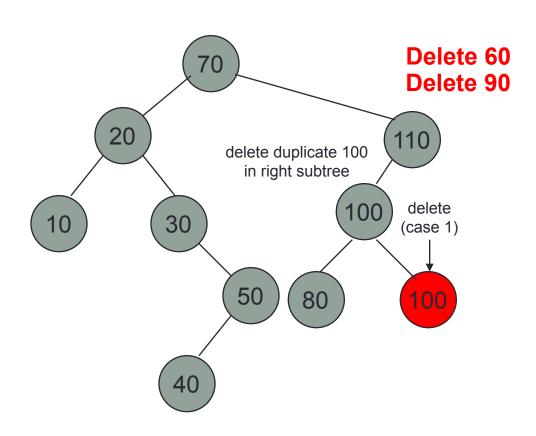


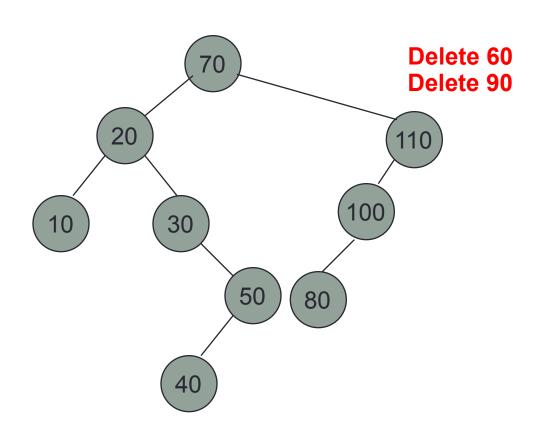


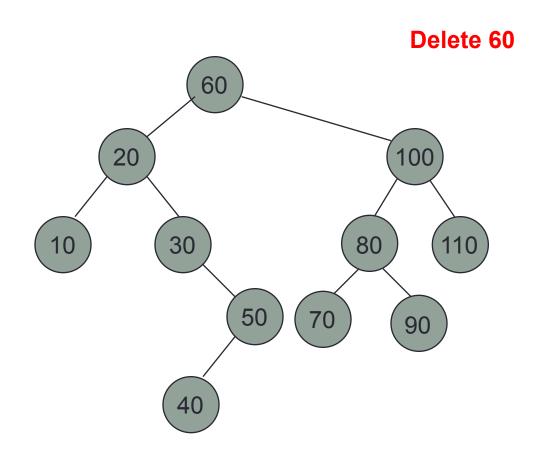


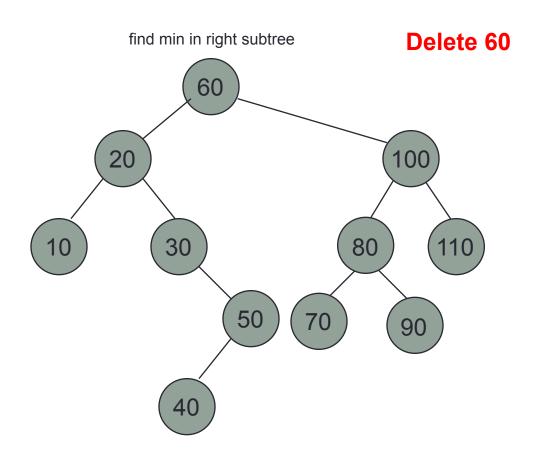


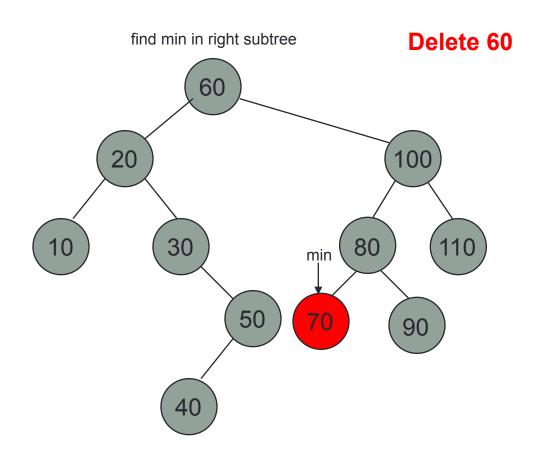


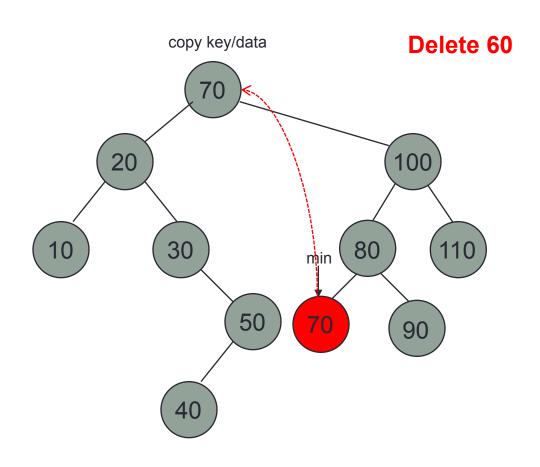


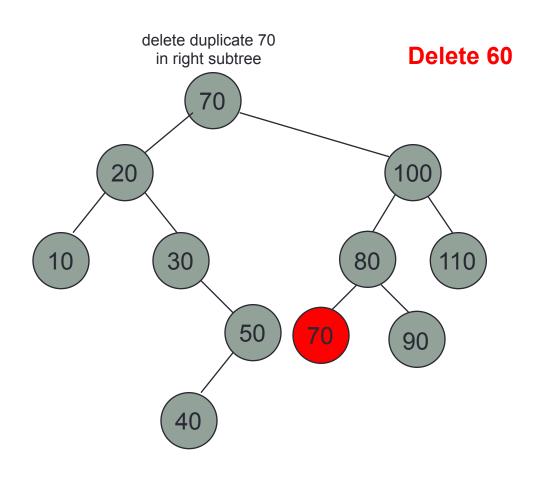




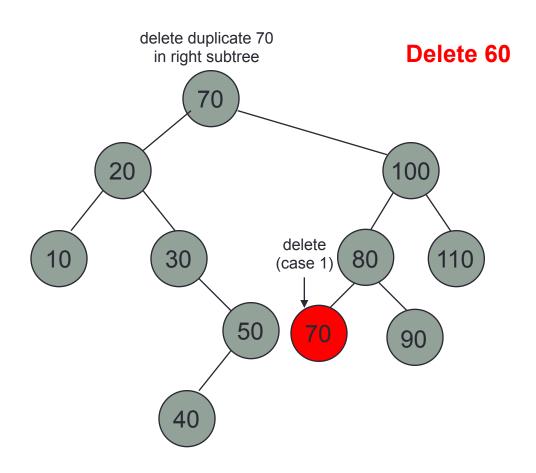




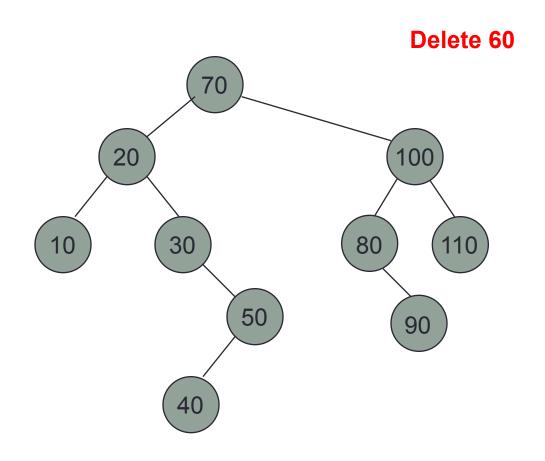




BST Deletion: Case 3



BST Deletion: Case 3



```
public boolean remove_key (int tkey) {
    Boolean removed = new Boolean(false);
    BSTNode<T> p;
    p = remove_aux(tkey, root, removed);
    current = root = p;
    return removed;
}
```

```
public boolean remove_key (int tkey) {
BooleanWrapper removed = new BooleanWrapper(false);
BSTNode<T> p;
p = remove_aux(tkey, root, removed);
current = root = p;
return removed.get();
}
```

Traverse the tree to find the key and handle remove cases (all 3 cases). If found, it will *remove* and set removed to (true). Otherwise, *removed* will not change (false). The method will return the modified tree.

```
private BSTNode<T> remove aux(int key, BSTNode<T> p, BooleanWrapper
flag) {
      BSTNode<T> q, child = null;
      if(p = null)
               return null;
      if(key < p. key)
               p. left = remove_aux(key, p. left, flag); //go left
      else if (key > p. key)
               p. right = remove aux(key, p. right, flag); //go right
      else {
               flag. set ( true);
               if (p. left != null && p. right != null) { //two children
                       q = find_min(p.right);
                       p. key = q. key;
                       p. data = q. data;
                       p. right = remove aux(q. key, p. right, flag);
```

```
else {
               if (p. right == null) //one child
                       child = p. left;
               else if (p. left == null) //one child
                       child = p. right;
               return child;
return p;
```

```
private BSTNode<T> find_min(BSTNode<T> p) {
    if(p == null)
        return null;

    while(p.left != null) {
        p = p.left;
    }

    return p;
}
```

Find left-most node (minimum key node) in any tree p

```
public boolean update(int key, T data) {
    remove_key(current.key);
    return insert(key, data);
}
```

To update the current key/value:

- 1). Remove the current node.
- 2). Insert a new node with the new key/data.

 Note: The new node will be set the current after insert.

```
//Method removeKey: iterative
public boolean removeKey(int k) {
      // Search for k
      int k1 = k;
      BSTNode < T > p = root;
      BSTNode<T> q = null; // Parent of p
      while (p != null) {
         if (k1 < p.key) {
            q = p;
            p = p.left;
         } else if (k1 > p.key) {
            q = p;
            p = p.right;
```

```
else { // Found the key
            // Check the three cases
            if ((p.left != null) && (p.right != null)) {
               // Case 3: two children
               // Search for the min in the right subtree
               BSTNode<T> min = p.right;
               q = p;
               while (min.left != null) {
                  q = min;
                  min = min.left;
               p.key = min.key;
               p.data = min.data;
               k1 = min.key;
               p = min;
               // Now fall back to either case 1 or 2
```

```
// The subtree rooted at p will change here
      if (p.left != null) { // One child
         p = p.left;
      } else { // One or no children
         p = p.right;
      if (q == null) { // No parent for p, root must change
         root = p;
      } else {
         if (k1 < q.key) {
            q.left = p;
         } else {
            q.right = p;
      current = root;
      return true;
return false; // Not found
```

Ex

Insert the following keys into an empty BST and show the BST after each insertion: 10, 12, 8, 17, 11, 14, 9, 4, 3,20, 5.

```
BST<String> bst;
                                            Print the BST
 bst = new BST<String>();
                                             nodes using
bst.insert(10, "L");
                                          in_order travers
bst.insert(12, "N");
bst.insert(8, "E");
bst.insert(17, ":");
bst.insert(11, "E");
                                                 find(9)
bst.insert(14, "T");
                                                 find(6)
bst.insert(9, "L");
bst.insert(4, "X");
bst.insert(3, "E");
                                         Try to delete node
bst.insert(20, ")");
bst.insert(5, "C");
                                             with key 10
```