QUEUE

CSC212:Data Structure

Queue

- Queue: First In First Out (FIFO).
 - Used in operating systems, simulations etc.
- Priority Queues: Highest priority item is served first.
 - Used in operating systems, printer servers etc.

ADT Queue: Specification

Elements: The elements are of generic type <Type> (The elements are placed in nodes for linked list implementations).

Structure: the elements are linearly arranged, and ordered according to the order of arrival. Most recently arrived element is called the <u>back or tail</u>, and least recently arrived element is called the <u>front or head</u>.

<u>Domain:</u> the number of elements in the queue is bounded therefore the domain is finite. Type of elements: Queue

ADT Queue: Specification

Operations:

1. **Method** Enqueue (Type e)

requires: Queue Q is not full. input: Type e.

results: Element e is added to the queue at its tail. output: none.

2. **Method** Serve (Type e)

requires: Queue Q is not empty. input: none

results: the element at the head of Q is removed and its value

assigned to e. output: Type e.

3. **Method** Length (int length)

requires: none. input: none

results: The number of element in the Queue Q is returned.

output: length.

ADT Queue: Specification

Operations:

4. **Method** Full (boolean flag).

requires: none. input: none

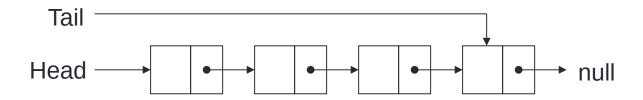
results: If Q is full then flag is set to true, otherwise flag is set to

false. output: flag.

Queue Interface

```
public interface Queue<T>{
public T serve();
public void enqueue(T e);
public int length();
public boolean full();
}
```

ADT Queue (Linked-List)



ADT Queue (Linked-List): Element

```
public class Node<T> {
 public T data;
 public Node<T> next;
 public Node() {
 data = null;
 next = null;
 public Node(T val) {
 data = val;
 next = null;
// Setters/Getters?
```

ADT Queue (Linked-List): Representation

```
public class LinkedQueue<T> implements Queue<L>{
   private Node<T> head, tail;
   private int size;

/** Creates a new instance of LinkedQueue */
   public LinkedQueue() {
     head = tail = null;
     size = 0;
}
```

Size = 0

ADT Queue (Linked-List): Representation

```
public class LinkedQueue<T> {
   private Node<T> head, tail;
   private int size;

/** Creates a new instance of LinkedQueue */
   public LinkedQueue() {
      head = tail = null;
      size = 0;
   }
```

```
public boolean full() {
    return false;
}

public int length () {
    return size;
}
```

```
public void enqueue(T e) {
   if(tail == null) {
      head = tail = new Node<T>(e);
   }
   else {
      tail.next = new Node<T>(e);
      tail = tail.next;
   }
   size++;
}
```

```
public void enqueue(T e) {
   if(tail == null){
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #1
                 Size = 0
   size++;
                   HT
                    null
```

```
public void enqueue(T e) {
   if(tail == null){}
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #1
                 Size = 0
   size++;
                   HT
                          null
```

```
public void enqueue(T e) {
   if(tail == null){
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #1
                 Size = 1
   size++;
                   HT
                           null
```

```
public void enqueue(T e) {
   if(tail == null){
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #2
                 Size = 1
   size++;
                   HT
                           null
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public void enqueue(T e) {
   if(tail == null){
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   else {
       tail.next = new Node<T>(e);
       tail = tail.next;
                              Example #2
                 Size = 1
   size++;
                   HT
                                  null
```

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public void enqueue(T e) {
   if(tail == null){
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #2
                 Size = 1
   size++;
                   Н
                                    null
```

```
public void enqueue(T e) {
   if(tail == null){
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #2
                 Size = 2
   size++;
                   Н
                                    null
```

```
public void enqueue(T e) {
   if(tail == null){
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #3
                 Size = 2
   size++;
                   Н
                                    null
```

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public void enqueue(T e) {
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                              Example #3
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public void enqueue(T e) {
   if(tail == null){
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       tail = tail.next;
                               Example #3
                 Size = 2
   size++;
                   Н
                                            null
```

```
public void enqueue(T e) {
   if(tail == null){
       head = tail = new Node < T > (e);
   else {
       tail.next = new Node < T > (e);
       tail = tail.next;
                               Example #3
                 Size = 3
   size++;
                   Н
                                             null
```

```
public T serve() {
    T x = head.data;
    head = head.next;
    size--;
    if(size == 0)
        tail = null;
    return x;
}
```

```
public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
                                Example #1
   return x;
                   Size = 3
                     Н
                                              null
```

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public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
   return x;
                                Example #1
                   Size = 3
}
                    Н
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public T serve() {
   T x = head.data;
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   if(size == 0)
       tail = null;
   return x;
                                Example #1
                  Size = 3
}
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   T x = head.data;
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   return x;
                                Example #1
                   Size = 2
}
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                                              null
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   T x = head.data;
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                                Example #1
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}
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   T x = head.data;
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   size--;
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                                Example #1
                   Size = 2
}
                               Н
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public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
   return x;
                                Example #2
                   Size = 2
                               Н
                                              null
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public T serve() {
   T x = head.data;
   head = head.next;
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   if(size == 0)
       tail = null;
   return x;
                                Example #2
                   Size = 2
}
                               Н
                                              null
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```
public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
                               Example #2
   return x;
                  Size = 2
}
                                       HT
                                              null
```

```
public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
   return x;
                                Example #2
                  Size = 1
}
                                       HT
                                              null
```

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public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
   return x;
                                Example #2
                  Size = 1
}
                                       HT
                                              null
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public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
   return x;
                                Example #3
                   Size = 1
}
                                       HT
                                              null
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public T serve() {
   T x = head.data;
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   size--;
   if(size == 0)
       tail = null;
   return x;
                               Example #3
                  Size = 1
}
                                       HT
                                              null
```

```
public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
   return x;
                               Example #3
                  Size = 1
```

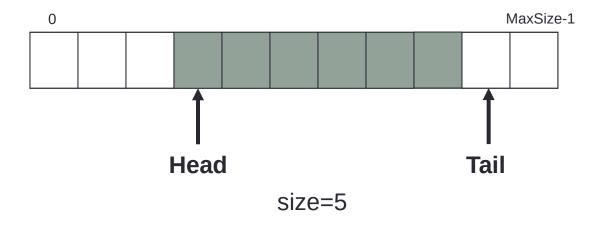
```
public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
   return x;
                               Example #3
                  Size = 0
```

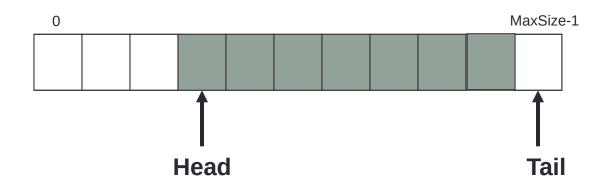
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   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
       tail = null;
                               Example #3
   return x;
                  Size = 0
}
                                             HT
```

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public T serve() {
   T x = head.data;
   head = head.next;
   size--;
   if(size == 0)
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   return x;
                               Example #3
                  Size = 0
}
                                             HT
```

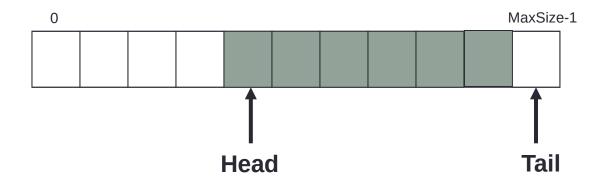
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   head = head.next;
   size--;
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       tail = null;
   return x;
                               Example #3
                  Size = 0
                                              HT
                                              null
```

- A fixed size array is used to store the data elements.
- As data elements are enqueued & served the queue crawls through the array from low to high index values.
- As the queue crawls forward, it also expands and contracts.

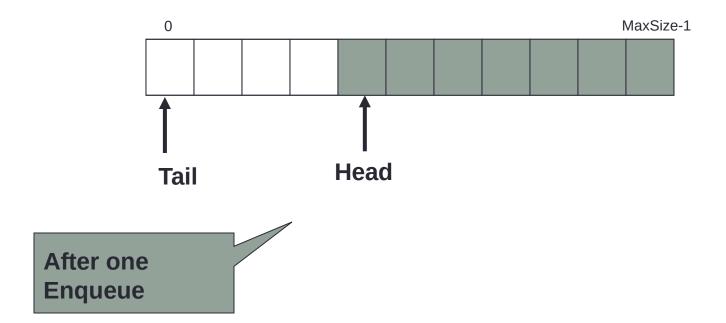


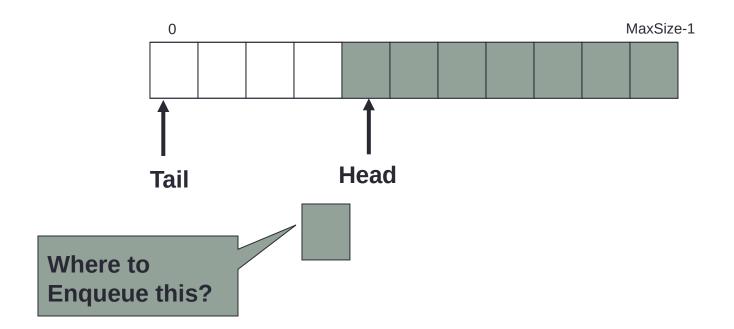


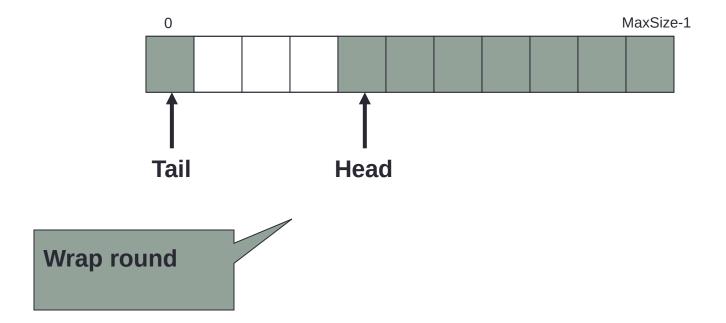
After one Enqueue

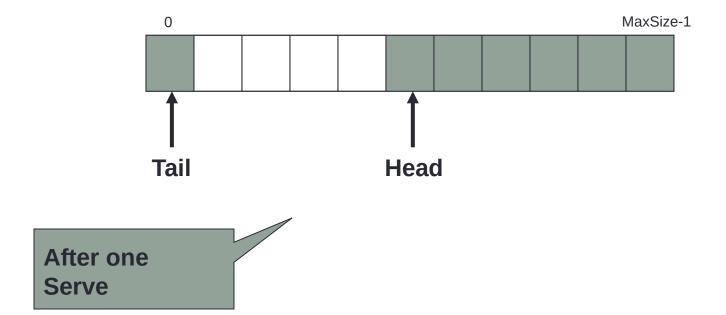


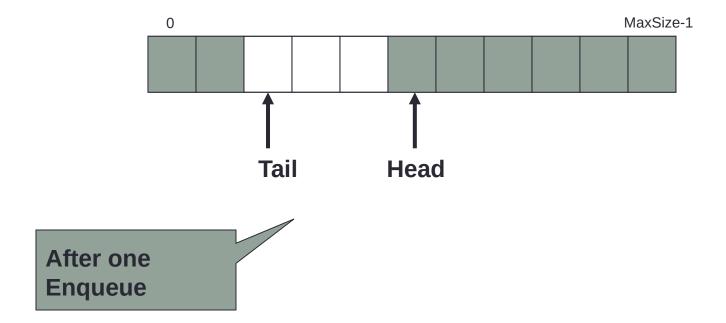
After one Serve

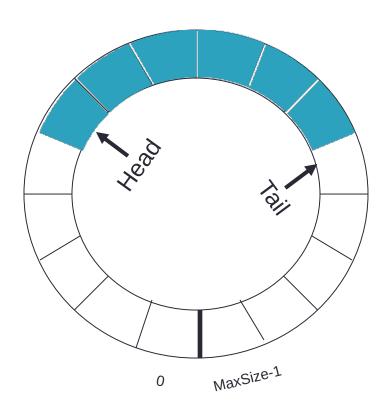












ADT Queue (Array): Representation

```
public class ArrayQueue<T> implements Queue<T>{
 private int maxsize;
 private int size;
 private int head, tail;
 private T[] nodes;
 /** Creates a new instance of ArrayQueue */
 public ArrayQueue(int n) {
    maxsize = n;
    size = 0;
    head = tail = 0;
    nodes = (T[])new Object[n];
```

ADT Queue (Array): Representation

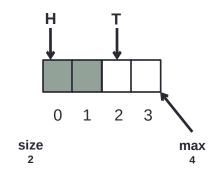
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 private int maxsize;
 private int size;
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 /** Creates a new instance of ArrayQueue */
 public ArrayQueue(int n) {
                                          HT
    maxsize = n;
    size = 0;
    head = tail = 0;
                                           0 1 2
    nodes = (T[])new Object[n];
                                        size
                                                    max
```

```
public boolean full () {
    return size == maxsize;
}

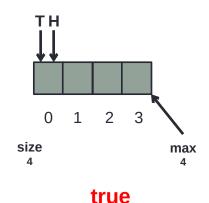
public int length () {
    return size;
}
```

```
public boolean full () {
    return size == maxsize;
}

public int length () {
    return size;
}
```

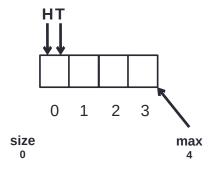


false

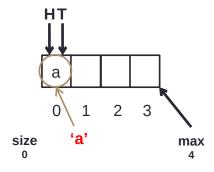


```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

```
public void enqueue(T e) {
    nodes[tail] = e;
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}
```



```
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}
```



```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

Example #1 (0 + 1) % 4 = 1 % 4 = 1 HT a 0 1 2 3

max

size

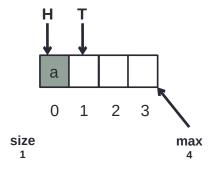
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Example #1 (0 + 1) % 4 = 1 % 4 = 1 H T a 0 1 2 3

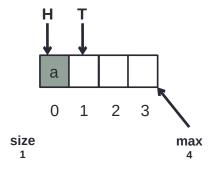
max

size

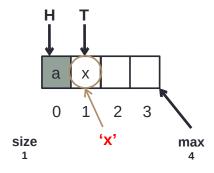
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    nodes[tail] = e;
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}
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    nodes[tail] = e;
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```



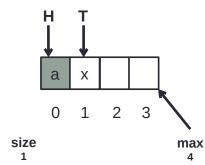
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    nodes[tail] = e;
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    size++;
}
```



```
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    nodes[tail] = e;
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}
```

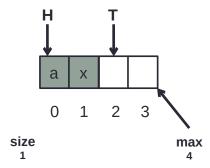
Example #2

(1 + 1) % 4 = 2 % 4 = 2

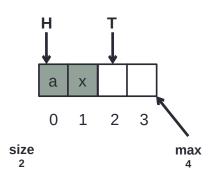


```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

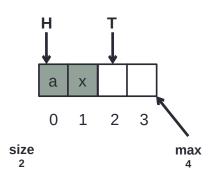
$$(1 + 1) \% 4 = 2 \% 4 = 2$$



```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

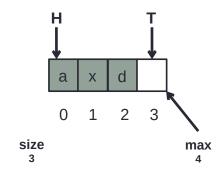


```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```



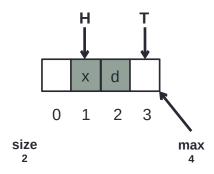
```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

After one Enqueue



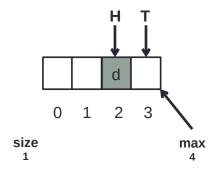
```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

After one Serve

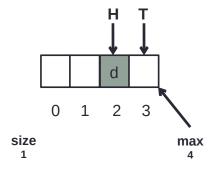


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public void enqueue(T e) {
    nodes[tail] = e;
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```

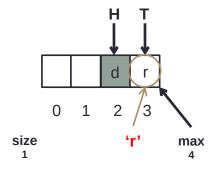
After another Serve



```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

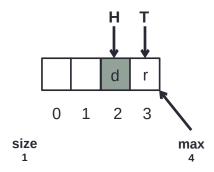


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public void enqueue(T e) {
    nodes[tail] = e;
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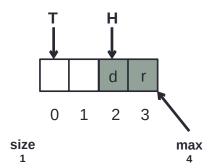
```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

$$(3 + 1) \% 4 = 4 \% 4 = 0$$

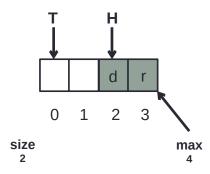


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public void enqueue(T e) {
    nodes[tail] = e;
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    size++;
}
```

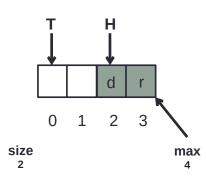
$$(3 + 1) \% 4 = 4 \% 4 = 0$$



```
public void enqueue(T e) {
    nodes[tail] = e;
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}
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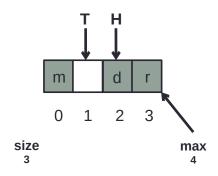


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    nodes[tail] = e;
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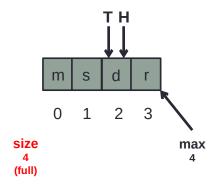
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```

After one Enqueue



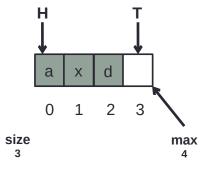
```
public void enqueue(T e) {
    nodes[tail] = e;
    tail = (tail + 1) % maxsize;
    size++;
}
```

After another Enqueue



```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

```
public T serve () {
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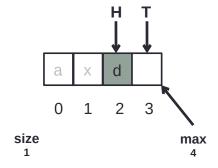
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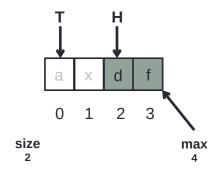
```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

After another Serve



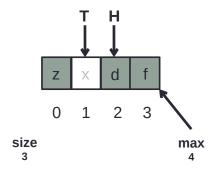
```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

After one Enqueue



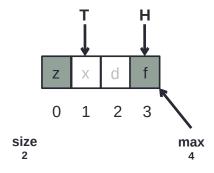
```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

After another Enqueue

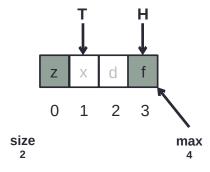


```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

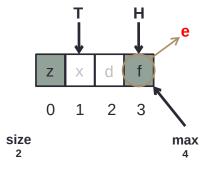
After one Serve



```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

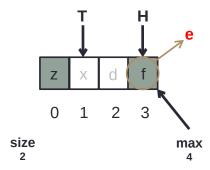


```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

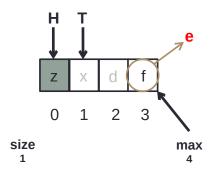


```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

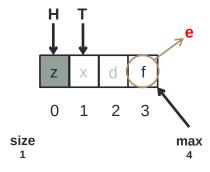
$$(3 + 1) \% 4 = 4 \% 4 = 0$$



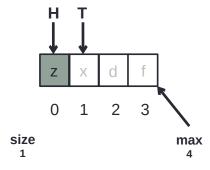
```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```



```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```



```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```



```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}

Example #3

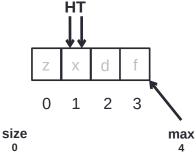
(0+1) % 4 = 1 % 4 = 1

    vertical formula in the size of the si
```

```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```

```
public T serve () {
    T e = nodes[head];
    head = (head + 1) % maxsize;
    size--;
    return e;
}
```



Exercice

- Write the method enquiry that returns the data at the head of the queue without changing it.
 - As user of the ADT Queue<T>
 - As a member of the class LinkedQueue<T>
 - As a member of the class ArrayQueue<T>

Static Method Enquiry (LinkedQueue/ArrayQueue)

```
public static<T> T enquiry(Queue<T> q) {
    T data = q.serve();
    q.enqueue(data);
    for(int i = 0; i < q.length() - 1; i++)
        q.enqueue(q.serve());
    return data;
}</pre>
```

Member Method Enquiry (LinkedQueue)

```
public T enquiry() {
    return head.data;
}
```

Member Method Enquiry (ArrayQueue)

```
public T enquiry() {
    return nodes[head];
}
```

Priority Queue

- Each data element has a priority associated with it. Highest priority item is served first.
- Real World Priority Queues: hospital emergency rooms (most sick patients treated first), events in a computer system, etc.
- Priority Queue can be viewed as:
 - View 1: Priority queue as an ordered list.
 - View 2: Priority queue as a set.

Elements: The elements are of type PQNode. Each node has in it a data element of generic type <Type> and a priority of type Priority (which could be int type).

Structure: the elements are linearly arranged, and may be ordered according to a priority value, highest priority element is called the <u>front</u> or <u>head</u>.

<u>Domain:</u> the number of nodes in the queue is bounded therefore the domain is finite. Type of elements: PriorityQueue

Operations:

Method Enqueue (Type e, Priority p)

requires: PQ is not full. input: e, p.

results: Element e is added to the queue according to its priority.

output: none.

Method Serve (PQElement<Type> pqe)

requires: PQ is not empty. input: None

results: the element and the priority at the head of PQ is

removed and returned. output: pqe.

3. **Method** Length (int length)

input: **results**: The number of element in the PQ is returned.

output: length.

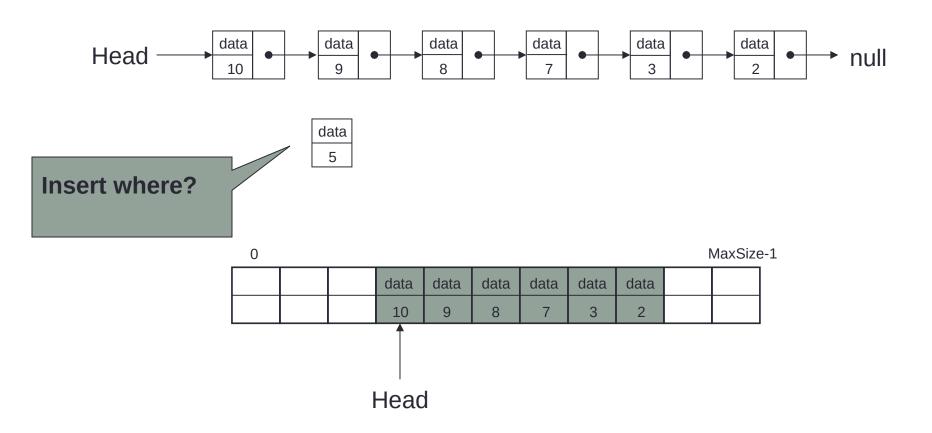
Operations:

4. **Method** Full (boolean flag).

requires: input:

results: If PQ is full then flag is set to true, otherwise

flag is set to false. output: flag.



ADT Priority Queue (Linked-List): Element

```
public class PQNode<T> {
 public T data;
 public Priority priority;
 public PQNode<T> next;
 public PQNode() {
     next = null;
 }
 public PQNode(T e, Priority p) {
     data = e;
     priority = p;
 }
 // Setters/Getters?
```

ADT Priority Queue (Linked-List): Element (int Priority)

```
public class PQNode<T> {
 public T data;
 public int priority;
 public PQNode<T> next;
 public PQNode() {
     next = null;
 }
 public PQNode(T e, int p) {
     data = e;
     priority = p;
 }
 // Setters/Getters?
```

```
public class LinkedPQ<T> {
    private int size;
    private PQNode<T> head;

/* tail is of no use here. */
    public LinkedPQ() {
        head = null;
        size = 0;
    }
```

```
public class LinkedPQ<T> {
   private int size;
   private PQNode<T> head;

/* tail is of no use here. */
   public LinkedPQ() {
      head = null;
      size = 0;
   }
```

```
Size = 0
H
↓
null
```

```
public int length (){
    return size;
}

public boolean full () {
    return false;
}
```

```
public void enqueue(T e, int pty) {
    PQNode < T > tmp = new PQNode < T > (e, pty);
    if((size == 0) || (pty > head.priority)) {
        tmp.next = head;
        head = tmp;
    else {
        PQNode < T > p = head;
        PQNode < T > q = null;
        while((p != null) && (pty \leq p.priority)) {
             q = p;
             p = p.next;
        tmp.next = p;
        q.next = tmp;
    size++;
}
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                 p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
H
↓
null
```

Example #1

```
Н
                                             tmp
public void enqueue(T e, int pty) {
    null
    if((size == 0) || (pty > head.priority)) {
         tmp.next = head;
         head = tmp;
    else {
         PQNode < T > p = head;
         PQNode < T > q = null;
                                                         Example #1
         while((p != null) && (pty <= p.priority)) {
              q = p;
              p = p.next;
         tmp.next = p;
         q.next = tmp;
    size++;
```

```
Н
                                                    tmp
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                                  null
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #1
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
                                                    tmp
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                                  null
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                 Example #1
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
tmp H
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                                  null
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #1
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
tmp H
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                                  null
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #1
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
5
                                                                   null
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #2
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty); data
                                                                  null
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
                                                   tmp
           PQNode < T > q = null;
                                                                  Example #2
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
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public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
5
                                                                  null
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           tmp.next = head;
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     else {
           PQNode < T > p = head;
                                                   tmp
           PQNode < T > q = null;
                                                                  Example #2
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                                  null
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
                                                 10
     else {
           PQNode < T > p = head;
                                                   tmp
           PQNode < T > q = null;
                                                                  Example #2
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                                  null
                                                      5
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
                                                   tmp
           PQNode < T > q = null;
                                                                  Example #2
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                                  null
                                                      5
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
                                                   tmp
           PQNode < T > q = null;
                                                                  Example #2
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #2
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty); data
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                 Example #3
           while((p != null) && (pty <= p.priority)) {
                q = p;
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           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
                                                     10
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                                                                 Example #3
           while((p != null) && (pty <= p.priority)) {
                q = p;
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           tmp.next = p;
           q.next = tmp;
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```

```
public void enqueue(T e, int pty) {
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                                                     data
     if((size == 0) || (pty > head.priority)) {
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           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                 Example #3
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty \leq p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
qHp
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                 Example #3
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
qH
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
qH
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty \leq p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
     if((size == 0) || (pty > head.priority)) {
                                                      10
           tmp.next = head;
           head = tmp;
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           PQNode < T > p = head;
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                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
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                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
     if((size == 0) || (pty > head.priority)) {
                                                      10
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
     if((size == 0) || (pty > head.priority)) {
                                                      10
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p!= null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
                                                                             null
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
                                                                             null
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #3
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #4
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty); data
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                 Example #4
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
                                                     10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                 Example #4
           while((p != null) && (pty <= p.priority)) {
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                                                                  Example #4
           while((p != null) && (pty \leq p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
qHp
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                 Example #4
           while((p != null) && (pty <= p.priority)) {
                q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
qH
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
                                                                 data
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #4
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

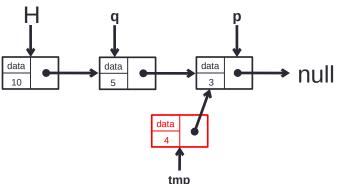
```
qH
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
                                                                 data
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
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           q.next = tmp;
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```
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                                                                  Example #4
           while((p != null) && (pty <= p.priority)) {
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           tmp.next = p;
           q.next = tmp;
     size++;
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```
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public void enqueue(T e, int pty) {
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                                                      data
                                                                                        null
                                                      10
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                                                                  Example #4
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                p = p.next;
           tmp.next = p;
           q.next = tmp;
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```
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           while((p != null) && (pty \leq p.priority)) {
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           tmp.next = p;
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```
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           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                 p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```



Example #4

null

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #4
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

null

```
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                     data
                                                      10
     if((size == 0) || (pty > head.priority)) {
           tmp.next = head;
           head = tmp;
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           PQNode < T > p = head;
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                                                                  Example #4
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

```
Н
public void enqueue(T e, int pty) {
     PQNode<T> tmp = new PQNode<T>(e, pty);
                                                      data
     if((size == 0) || (pty > head.priority)) {
                                                      10
           tmp.next = head;
           head = tmp;
     else {
           PQNode < T > p = head;
           PQNode < T > q = null;
                                                                  Example #4
           while((p != null) && (pty <= p.priority)) {
                 q = p;
                p = p.next;
           tmp.next = p;
           q.next = tmp;
     size++;
```

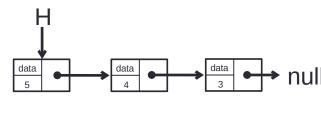
```
public PQElement<T> serve(){
PQNode < T > node = head;
PQElement<T> pqe=new PQElement<T>(node.data,node.p);
head = head.next;
size--;
return pge;
public class PQElement<T>
public T data;
public Priority p;
public PQElement(T e, Priority pr){
data=e;
p=pr;
```

```
public PQElement<T> serve(){
    PQNode<T> node = head;
    PQElement<T> pqe=new PQElement<T>(node.data,node.p);
    head = head.next;
    size--;
    return pqe;
}
```

```
public PQElement<T> serve(){
    PQNode<T> node = head;
    PQElement<T> pqe=new PQElement<T>(node.data,node.p);
    head = head.next;
    size--;
    return pqe;
}
```

```
public PQElement<T> serve(){
    PQNode<T> node = head;
    PQElement<T> pqe=new PQElement<T>(node.data,node.p);
    head = head.next;
    size--;
    return pqe;
}
```

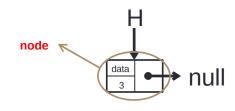
```
public PQElement<T> serve(){
    PQNode<T> node = head;
    PQElement<T> pqe=new PQElement<T>(node.data,node.p);
    head = head.next;
    size--;
    return pqe;
}
```



```
public PQElement<T> serve(){
   PQNode<T> node = head;
   PQElement<T> pqe=new PQElement<T>(node.data,node.p);
   head = head.next;
   size--;
   return pqe;
}
```

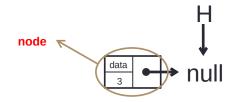
```
data data 3 • null
```

```
public PQElement<T> serve(){
    PQNode<T> node = head;
    PQElement<T> pqe=new PQElement<T>(node.data,node.p);
    head = head.next;
    size--;
    return pqe;
}
Yet another serve
```



```
public PQElement<T> serve(){
    PQNode<T> node = head;
    PQElement<T> pqe=new
    PQElement<T> (node.data,node.p);
    head = head.next;
    size--;
    return pqe;
}
```

Example #3



```
public PQElement<T> serve(){
        PQNode<T> node = head;
        PQElement<T> pqe=new PQElement<T>(node.data,node.p);
        head = head.next;
        size--;
    return pqe;
}
```

```
node data nul
```

```
node data null
```



```
public PQElement<T> serve(){
        PQNode<T> node = head;
        PQElement<T> pqe=new PQElement<T>(node.data,node.p);
        head = head.next;
        size--;
    return pqe;
}
```

ADT Priority Queue

- Implementations
 - Linked List: Enqueue is O(n), Serve is O(1).
 - Array Implementation: Enqueue is O(1), Serve is O(1).
 - Heap: Enqueue is O(log n), Serve is O(log n) ☐ Heaps to be discussed later.

- Double ended queue (or a deque) supports insertion and deletion at both the front and the tail of the queue.
- The first element is called head and the last element is called tail.
- Supports operations: addFirst(), addLast(), removeFirst() and removeLast().
- Can be used in place of a queue or a stack.

Operations: (Assume all operations are performed on deque DQ)

Method addFirst (Type e)

requires: DQ is not full. input: e.

results: Element e is added to DQ as first element. output: none.

2. **Method** addLast (Type e)

requires: DQ is not full. input: e

results: Element e is added to DQ as last element. output: none.

3. **Method** removeFirst (Type e)

requires: DQ is not empty. **input**: none **results**: Removes and returns the first element of DQ. **output**: e.

- 4. **Method** removeLast (Type e)
 - requires: DQ is not empty. input: none.
 - results: Removes and returns the last element of DQ. output: e.
- Method getFirst (Type e)
 - requires: DQ is not empty. input: none
 - results: Returns the first element of DQ. output: e.
- **Method** getLast (Type e)
 - requires: DQ is not empty. input: none
 - results: Returns the last element of DQ. output: e
- 7. **Method** size (int x)
 - **input**: none **results**: Returns the number of elements in DQ.
 - output: X

8. **Method** empty (boolean x)

input: none results: if DQ is empty returns x as true otherwise

false. **output**: x

Complexity so far?

Operation	Queue (LL)	Queue (CA)	Priority Queue (LL)	Priority Queue (CA)
Full	O(1)	O(1)	O(1)	O(1)
Length	O(1)	O(1)	O(1)	O(1)
Enqueue	O(1)	O(1)	O(n)	O(n)
Serve	O(1)	O(1)	O(1)	O(1)

Complexity so far?

Operation	Double-Ended Queue (LL)	Double-Ended Queue (CA)	Double-Ended Queue (DLL)
AddFirst			
AddLast			
RemoveFirst			
RemoveLast	O(n)		O(1)
GetFirst			
GetLast			
Size			
Empty			

ToDo

- Read 5.2, 5.3 of the Textbook.
- Add "int length()" method in the LinkedQueue class with O(n) complexity.
- Add "int length(ArrayQueue<T> q)" in the Test class of ArrayQueue. The Queue must remain unchanged after the operation.
- Add "T enquiry(ArrayQueue<T> q)" in the Test class of ArrayQueue. It should return the data of the head without changing the queue at the end of the call.
- Implement DQueue (Double-ended queue) using a Java class using Linked-List.
- Test this DQueue using a test Class.