5 (a)

geometric series

$$a + ar^1 + ar^2 + \dots + ar^n$$

$$= \sum_{i=0}^{n} ar^i = \frac{a(r^{n+1} - 1)}{r - 1}$$

Arithmetic sequence Example

$$(a, \quad a+1d, \quad a+2d, \quad a+3d, \quad \dots)$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$a_1 \quad a_2 \quad a_3 \quad a_4$$

$$\vdots$$

$$\boxed{a_n = a + (n-1)d}$$ closed formula

recurrence formula

$$a_n = a_{n-1} + d$$

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6

(3, 5, 7, 9, 11, 13, - - -)

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6$

+2 +2 +2

closed formula | recurrance formula

$a_n = a_{n-1} + 2$

$a_n = 3 + (n-1) \cdot 2$

$= 3 + 2n - 2$

$\boxed{a_n = 2n + 1}$

$a_6 = 2(6) + 1 = 13$

$a_6 = ?$

$a_1 = \_$

$a_2 = \_$

$a_3 = \_$

$a_4 = \_$

$a_5 = \_$

$a_6 = \_$

$a_{100} = 2(100) + 1$

$= 201$

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بالتعويض في الخطوة k

$$T(n) = T(1) + c \cdot \log_2 n$$

$$\boxed{T(n) = a + c \log_2 n}$$

$\therefore T(n)$ is $\Theta(\log_2 n)$

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solving recurrence relation

المقصود به انه يعطى لنا recurrence relation

ونوجد ال closed formula المقابلة لها

أو شيء قريب منها في درجتها

لأن هدفنا الأساسي حساب ال complexity

Big-O   Big-Ω   Big-Θ

Example

using substitution method

solve $T(n) = T(n/2) + c$

$$\boxed{T(1) = a}$$

sol

$T(n) = T\left(\frac{n}{2}\right) + c$

$T\left(\frac{n}{2}\right) = T\left(\frac{n}{2^2}\right) + c$

$T\left(\frac{n}{2^2}\right) = T\left(\frac{n}{2^3}\right) + c$

$$T(n) = \boxed{T\left(\frac{n}{2}\right)} + c \rightarrow (1)$$

$$T(n) = \boxed{T\left(\frac{n}{2^2}\right) + c} + c \rightarrow (2)$$

$$T(n) = T\left(\frac{n}{2^3}\right) + c + c + c \rightarrow (3)$$

$\vdots$

$$T(n) = T\left(\frac{n}{2^k}\right) + \underbrace{c + c + \cdots + c}_{k \text{ عدد}}$$

$$\boxed{T(n) = T\left(\frac{n}{2^k}\right) + ck} \rightarrow k \rightarrow (k) \text{ خطوة}$$

$\because T(1) = a$

put $\frac{n}{2^k} = \frac{1}{1} \Rightarrow n = 2^k$

$$\log_2 n = \log_2 2^k = k$$

$$\boxed{k = \log_2 n}$$

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$$B = 2^{k-1}c + 2^{k-2}c + \cdots + 2^{1}c + 2^{0}c$$

$$= c\left(2^{0} + 2^{1} + \cdots + 2^{k-1}\right)$$

$$= c\sum_{i=0}^{k-1} 2^{i}$$

$$= c\,\frac{(2^{k} - 1)}{(2 - 1)}$$

$$= c\,(2^{k} - 1)$$

$\therefore T(n) = A + B =$

الخطوة

$$T(n) = 2^{k}\,T\left(\frac{n}{2^{k}}\right) + c\,(2^{k} - 1) \rightarrow *$$

$\because T(1) = c$

put $T\left(\frac{n}{2^{k}}\right) = T(1) \Rightarrow \frac{n}{2^{k}} = 1$

$\therefore 2^{k} = n \Rightarrow k = \log n$

$$\therefore T(n) = nT(1) + c(n-1)$$

$$= cn + cn - c$$

$$T(n) = 2cn - c \qquad \Theta(n)$$

general form:

$$\underline{T(n)} = a\,\underline{T(n/b)} + \underline{f(n)}, \quad \boxed{T(1) = c}$$

Time to solve problem of size n

subproblem

time to solve subproblems

overhead

a, b integer

$b \geq 2$

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هو #steps لأي دالة تعمل
بطريقة recursion

Examples

$n \geq 0$

① $T(n) = \begin{cases} 0 & \\ T(n-1) + c & n > 0 \end{cases}$

$n = 0$

② $T(n) = \begin{cases} 0 & \\ T(n-1) + n & n > 0 \end{cases}$

③ $T(n) = \begin{cases} 2T(\frac{n}{2}) + c & n > 1 \\ c & n = 1 \end{cases}$

④ $T(n) = \begin{cases} aT(\frac{n}{b}) + cn & \text{if } n > 1 \\ c & n = 1 \end{cases}$

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Mergesort Analysis

$$T(n) = 2T\left(\frac{n}{2}\right) + \underline{n}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n \longrightarrow 1$$

$$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$\boxed{T(n) = 2^2 T\left(\frac{n}{2^2}\right) + n + n \longrightarrow 2}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n}{2}$$

$$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$$

$$T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right) + n + n$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + n + n + n \longrightarrow 3$$

$$\vdots$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn \longrightarrow k$$

$$\because T(1) = 1 \Rightarrow \frac{n}{2^k} = 1 \Rightarrow n = 2^k$$

$$\Rightarrow k = \log n$$

$$T(n) = n \cdot 1 + n\log n$$

$$\Rightarrow \underline{\underline{\Theta(n\log n)}}$$

Example

solve $T(n) = 2T\left(\frac{n}{2}\right) + c$

$T(1) = c$

sol

$$T(n) = 2\boxed{T\left(\frac{n}{2}\right)} + c \longrightarrow (1)$$

$$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + c\right) + c$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2^1 c + c \longrightarrow (2)$$

$$= 2^2\left(2T\left(\frac{n}{2^3}\right) + c\right) + 2c + c$$

$$T(n) = \underbrace{\boxed{2^3 T\left(\frac{n}{2^3}\right)}}_{A} + \underbrace{\boxed{2^2 c + 2^1 c + c}}_{B} \longrightarrow (3)$$

.

.

.

$$T(n) = \underbrace{\boxed{2^k T\left(\frac{n}{2^k}\right)}}_{A} + \underbrace{\boxed{2^{k-1} c + 2^{k-2} c + \cdots\cdots + 2^0 c}}_{B} \quad (k)$$

التعويض

$T(n) = 2T\left(\frac{n}{2}\right) + c$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + c$

$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + c$

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$$\boxed{1.\ T(n) < 3^{k}\, T\left(\frac{n}{2^k}\right) + 2n\left(\frac{3}{2}\right)^{k}}$$

put $T(1) = T\left(\frac{n}{2^k}\right)$

$$\therefore \frac{n}{2^k} = 1 \Longrightarrow n = 2^k \Longrightarrow k = \log_2 n$$

قاعدة رياضية $\quad X^{\log y} = y^{\log x}$

$$T(n) < 3^{k}\, T(1) + 2n\cdot\left(\frac{3}{2}\right)^{\log_2 n}$$

$$3^{\log_2 n} = 2^{\log_3 n}$$

$$T(n) < 3^{\log_2 n}\, T(1) + 2\cancel{(n)}\cdot\frac{3^{\log_2 n}}{\cancel{2^{\log_2 n}}}$$

$$2^{\log_2 n} = n$$

$$T(n) < 3^{\log_2 (n)}\left[T(1) + 2\right]$$

$$T(n) < n^{\log_2 3}\left[T(1) + 2\right]$$

$$O\left(n^{\log_2 3}\right) = O\left(n^{1.57}\right)$$

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الحل (12 تمرين)

التعويض

$$T(n) = 3T\left(\frac{n}{2}\right) + n \rightarrow (1)$$

$$T(n) = 3T\left(\frac{n}{2}\right) + n$$

$$T\left(\frac{n}{2}\right) = 3\cdot T\left(\frac{n}{2^2}\right) + \frac{n}{2}$$

$$T\left(\frac{n}{2^2}\right) = 3\cdot T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$$

$$T(n) = 3\left(3\cdot T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$$

$$\boxed{T(n) = 3^2\cdot T\left(\frac{n}{2^2}\right) + \left(\frac{3}{2}\right)n + n} \rightarrow (2)$$

$$T(n) = 3^2\cdot\left(3^1\cdot T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right) + \left(\frac{3}{2}\right)^1 n + n$$

$$T(n) = 3^3\cdot T\left(\frac{n}{2^3}\right) + \left(\frac{3}{2}\right)^2 n + \left(\frac{3}{2}\right)^1 n + \left(\frac{3}{2}\right)^0 n \rightarrow (3)$$

$$\left(\frac{3^2}{2^2}\right)^1$$

$\vdots$

$$T(n) = \underbrace{\boxed{3^k\, T\left(\frac{n}{2^k}\right)}}_{A} + \underbrace{\boxed{\left(\frac{3}{2}\right)^{k-1} n + \dots \left(\frac{3}{2}\right)^1 n + \left(\frac{3}{2}\right)^0 n}}_{B} \rightarrow (k)$$

$$B = n\left(\left(\frac{3}{2}\right)^0 + \left(\frac{3}{2}\right)^1 + \dots + \left(\frac{3}{2}\right)^{k-1}\right)$$

$$k-1+1$$

$$B = n\sum_{i=0}^{k-1}\left(\frac{3}{2}\right)^i = n\cdot\frac{\left(\frac{3}{2}\right)^k - 1}{\left(\frac{3}{2} - 1\right)}$$

$$B = \frac{n\cdot\left(\left(\frac{3}{2}\right)^k - 1\right)}{\frac{1}{2}} = 2n\left(\left(\frac{3}{2}\right)^k - 1\right)$$

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حل الدكتور

$$\boxed{x^{\log y} = y^{\log x}}$$

$$= 3^{\log n} T(1) + 2n\left(3/2\right)^{\log n}$$

$$= n^{\log 3} T(1) + 2n \cdot n^{\log 3/2}$$

$$= 2n \cdot n^{\log 3 - \log 2}$$

$$= 2\cancel{n} \cdot \frac{n^{\log 3}}{\cancel{n^{\log 2}}}$$

$$= n^{\log 3} T(1) + 2n^{\log 3}$$

$$= O\left(n^{\log 3}\right)$$

$$= O\left(n^{1.57}\right)$$

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$$= 3\left[3T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right] + n$$

$k=1$

$$= 3^2 T\left(\frac{n}{2^2}\right) + \frac{3n}{2} + n$$

$$= 3^2\left[3T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right] + \frac{3n}{2} + n$$

$k=2$

$$= 3^3 T\left(\frac{n}{2^3}\right) + \frac{3^2 n}{2^2} + \frac{3n}{2} + n$$

$k=3$

after k-steps

$$= 3^k T\left(\frac{n}{2^k}\right) + \boxed{\sum_{i=0}^{k-1} \left(\frac{3}{2}\right)^i n}$$

$$= n \sum_{i=0}^{k-1} \left(\frac{3}{2}\right)^i$$

$$= n\left[\frac{\left(\frac{3}{2}\right)^k - 1}{\frac{3}{2} - 1}\right] < 2n\left(\frac{3}{2}\right)^k$$

$$T(n) \le 3^k T\left(\boxed{\frac{n}{2^k}}\right) + 2n\left(\frac{3}{2}\right)^k$$

stop when

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k$

$\Rightarrow k = \log_2 n$

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$\boxed{2018}$

1

قوانين ال $\sum$

1) $\sum_{i=1}^{n} a = \underbrace{a+a+\cdots+a}_{n\text{-times}} = (n-1+1)\,a$

$= \boxed{na}$

2) $\sum_{i=1}^{n} 1 = \underbrace{1+1+\cdots+1}_{n\text{-times}} = (n-1+1)\cdot 1$

$= \boxed{n}$

3) $\sum_{i=1}^{n} i = 1+2+\cdots+n = \frac{n(n+1)}{2}$

4) $\sum_{i=m}^{n} i = m+(m+1)+\cdots+n = \frac{(n-m+1)(n+m)}{2}$

$= \frac{\text{عددهم}\,(\text{الأخير} + \text{الأول})}{2}$

5) $\boxed{\sum_{i=m}^{n} a = (n-m+1)\,a}$

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قاعدة سـ سـ ن

$$2^{\log_4 16} = 2^{2} = 4$$

$$\log_4 2 = \frac{1}{2}$$

$$16^{\log_4 2} = 16^{\frac{1}{2}} = 4$$

$$\therefore\ 2^{\log_4 16} = 16^{\log_4 2}$$

$$x^{\log_a y} = y^{\log_a x}$$

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5) (2)

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{m} a\right)$$

$$= \sum_{i=1}^{n} ma = n \cdot ma$$

6) $$\sum_{i=1}^{n}\left(\sum_{j=1}^{m} j\right) = \sum_{i=1}^{n} \frac{m \cdot (m+1)}{2}$$

$$= n \cdot \left(\frac{m \cdot (m+1)}{2}\right)$$

7) $$\sum_{i=1}^{n-1}\left(\sum_{j=1}^{m-2} j\right) = \sum_{i=1}^{n-1} \frac{(m-2)(m-1)}{2}$$

$$= \frac{(n-1) \cdot (m-2)(m-1)}{2}$$

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$$\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}$$

العام

$$\boxed{\sum_{i=m}^{n} r^i = \frac{r^{n+1}-r^m}{r-1}}$$

$r^0 + r^1 + r^2 + \dots + r^n$

Exaple : Find Time Function

for i=1 → n

for j=1 → n

c[i,j] ← ...

$\sum_{i=1}^{n} 1 + 1$

$\sum_{i=1}^{n} \sum_{j=1}^{n} 1 + 1$

$\sum_{i=1}^{n} \sum_{j=1}^{n} 1$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} 1 + 1 = n + 1$$

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{n} 1 + 1\right) = \sum_{i=1}^{n} (n+1) = n(n+1) = n^2 + n$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} 1 = \sum_{i=1}^{n} n = n^2$$

$$T(n) = 2n^2 + n + 1$$

$O(n^2)$

$\Theta(n^2)$

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Example : Find $\boxed{\text{Time Function}}$
or # steps

for $i = 1 \rightarrow n$

$c[i] = 7$

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for $i = 1 \rightarrow n$ → $\sum_{i=1}^{n} 1 + 1$

$c[i] = 7$ → $\sum_{i=1}^{n} 1$

chek

$$\text{Total} = \sum_{i=1}^{n} 1 + 1 + \sum_{i=1}^{n} 1$$

$$= n + 1 + n$$

$$\boxed{\text{Total} = 2n + 1}$$

Big O is $O(n)$

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Ex: $\sum_{j=1}^{n} j^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Note: $\sum_{j=1}^{n} j^2 \neq \left(\sum_{j=1}^{n} j \right)^2$

Ex: $\left(\sum_{j=1}^{n} j \right)^2 = \left(\frac{n(n+1)}{2} \right)^2$

Ex: $\sum_{i=1}^{n} \sum_{j=i}^{m} c = ??$

sol

$$\sum_{j=i}^{m} c = (m - i + 1)\, c$$

$$\sum_{i=1}^{n} \sum_{j=i}^{m} c = \sum_{i=1}^{n} (m - i + 1)\, c = \sum_{i=1}^{n} (mc - ic + c)$$

$$= \sum_{i=1}^{n} mc - \sum_{i=1}^{n} ic + \sum_{i=1}^{n} c$$

$$= nmc - \frac{c\, n(n+1)}{2} + nc$$

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Tutorial # 1

calculate $\sum_{i=m}^{n} a r^{2i}$

sol

$$\sum_{i=m}^{n} a r^{2i} = \sum_{i=m}^{n} a (r^2)^i$$

$$= \sum_{i=m}^{n} a (R)^i \quad , \quad R = r^2$$

$$= a \cdot \left(\frac{R^{n+1} - R^m}{R - 1} \right)$$

$$= a \cdot \left(\frac{(r^2)^{n+1} - (r^2)^m}{r^2 - 1} \right)$$

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Ex: $\sum_{i=1}^{n} \sum_{j=i+1}^{m} i = ??$

sol

$$\sum_{j=i+1}^{m} i = (m - (i+1) + 1)\, i = (m - i)\, i$$

$$= mi - i^2$$

$$\therefore \sum_{i=1}^{n} \sum_{j=i+1}^{m} i = \sum_{i=1}^{n} (mi - i^2)$$

$$= \sum_{i=1}^{n} mi - \sum_{i=1}^{n} i^2$$

$$= m \cdot \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$

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EX: ②

$T(n) = T(n-1) + n$, $T(1) = 2$

الحل

التجهيز

$T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + (n-1)$

$T(n-2) = T(n-3) + n-2$

$T(n-3) = T(n-4) + n-3$

$T(n) = T(n-1) + n \longrightarrow ①$

$T(n) = T(n-2) + (n-1) + n \longrightarrow ②$

$T(n) = T(n-3) + (n-2) + (n-1) + n \longrightarrow ③$

$T(n) = T(n-4) + (n-3) + (n-2) + (n-1) + n \longrightarrow ④$

$T(n) = T(n-k) + (n-(k-1)) + \cdots (n-1) + n \longrightarrow k$

put $n-k = 1 \Rightarrow k = n-1$

$\because T(1) = 2 \Rightarrow$

$T(n) = T(1) + (n-(k-2)) + \cdots + n$

$T(n) = 2 + (2 + 3 + 4 + \cdots + n)$

$= 2 + \frac{(n-1)(n+2)}{2}$

$O(n^2)$

(a)

geometric series

$a + ar^1 + ar^2 + \cdots + ar^n$

$$= \sum_{i=0}^{n} ar^i = \frac{a(r^{n+1} - 1)}{r - 1}$$

Arithmetic sequence Example

$$(a,\ a+1d,\ a+2d,\ a+3d,\ \cdots)$$

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ a_1 & a_2 & a_3 & a_4 \end{array}$$

$\vdots$

$$\boxed{a_n = a + (n-1)d}$$ closed formula

recurrence formula

$$\boxed{a_n = a_{n-1} + d}$$

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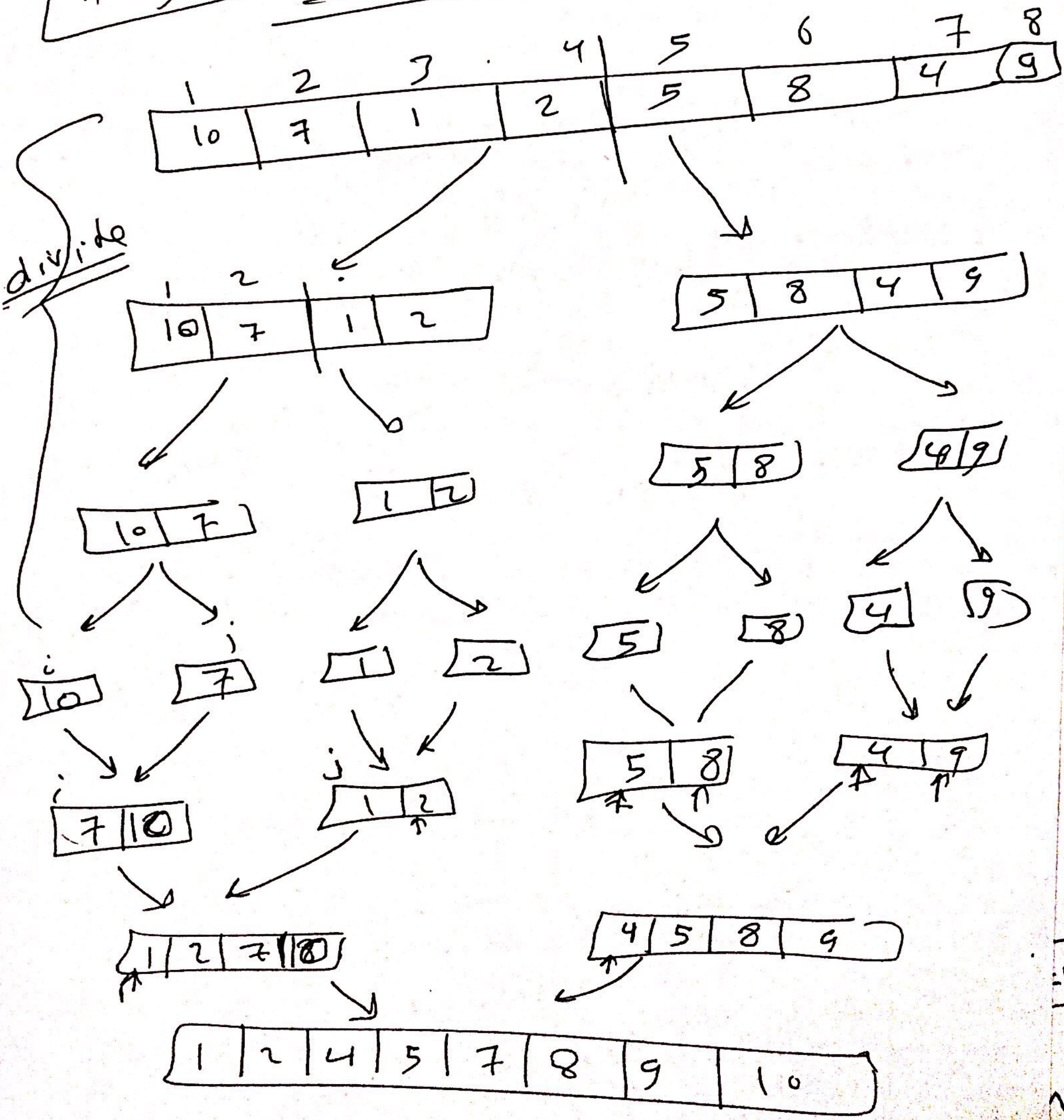
(1)

ch4: Devide and conquer

6

$\boxed{n \log n}$

Merge sort



الكود من ال slides

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(7)

solve : $T(n) = \begin{cases} 0 & \text{if } n=0 \\ T(n-1)+c , & n>0 \end{cases}$

الحل

خطوات التعويض

$T(n) = T(n-1) + c$

$T(n-1) = T(n-2) + c$

$T(n-2) = T(n-3) + c$

$T(n) = T(n-1) + c \rightarrow 1$

$T(n) = T(n-2) + \underbrace{c + c}_{} \rightarrow 2$

$T(n) = T(n-3) + \underbrace{c + c + c}_{3} \rightarrow 3$

$\vdots$

$T(n) = T(n-k) + \underbrace{c + c + \cdots + c}_{\text{عددهم } k} \rightarrow k$ $\quad \sum_{i=1}^{k} c$

$\boxed{T(n) = T(n-k) + kc} \rightarrow k$

put $n-k=0 \Rightarrow \boxed{k=n}$

$\because T(0) = 0 \Rightarrow$

$T(n) = T(0) + nc = \underline{nc}$

$O(n)$

②

قوانين هامة (1)

① $$\sum_{i=m}^{n} r^i = \frac{r^{n+1} - r^m}{r-1}$$

Ex: $2^0 + 2^1 + 2^2 + 2^3 + 2^4 \cdots + 2^n \Rightarrow \boxed{r=2}$

② if $-1<r<1$, $r \neq 0$ Ex: $(r = (\frac{1}{2}))$

$$\sum_{i=m}^{\infty} r^i = \frac{1}{1-r}$$

③ if $r>1$ or $r<-1$

$$\sum_{i=m}^{\infty} r^i = \pm\infty$$

④ if $-1<r<1$

$$\sum_{i=m}^{n} r^i \leq \sum_{i=m}^{\infty} r^i = \frac{1}{1-r}$$

⑤ $$a^{\log_b c} = c^{\log_b a}$$

$\boxed{\text{((لاحظ التبديل } a, c\text{))}}$

$4^{\log_2 8} \to 4^3 = 64$

$= 8^{\log_2 4}$

$\to 8^2 = 64$

⑥ $\log 1 = 0$

⑦ $\log 2 = 1$

$T(n)$ ④

Pseudocode

$\lfloor \frac{n}{2} \rfloor$ $\lceil \frac{n}{2} \rceil$

$T(n)$ MergeSort (*A, 1, n*) $T(n)$

m = *n* / *2*;

$T(\frac{n}{2})$ ← *A1* = MergeSort (*A, 1, m*); $T(n/2)$

$T(\frac{n}{2})$ ← *A2* = MergeSort (*A, m+1, n*); $T(n/2)$

$O(n)$ Merge (*A1, A2*); $O(n)$

$T(n) = 2T(\frac{n}{2}) + n$

Worst case time complexity:

$$T(n) = 2T(n/2) + O(n)$$

Master theorem

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