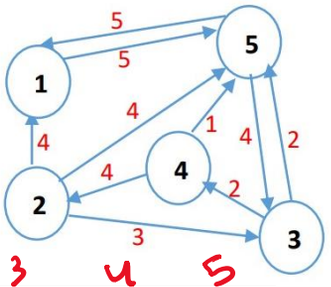
Q1) [BONUS] Given the sequences X = (7,9,10,3,4,1,2) and Y = (8,9,2,5,3,4) show the execution of the Dynamic Programming algorithm to find the Longest Common Subsequence (LCS), as explained in class. Fill out the dynamic programming table, and show all solution steps.

A. What is the length of the longest subsequence between X and Y? 3

B. What is the longest common subsequence between X and Y? (9 3 4)

	y	8	9	2	5	3	4
X	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0
9	0	0	1	1	1	1	1
10	0	0	1	1	1	1	1
3	0	0	1	1	1	2	2
4	0	0	1	1	1	2	3
1	0	0	1	1	1	2	3
2	0	0	1	2	2	2	3

Q2) [2.5] Solve the all-pairs shortest-path problem for the following graph:



0	∞	2	∞	5
4	0	3	∞	4
∞	∞	0	2	2
∞	4	∞	0	1
5	∞	3	∞	0

D_0

	1	2	3	4	5
1	0	∞	2	∞	5
2	4	0	3	∞	4
3	∞	∞	0	2	2
4	∞	4	∞	0	1
5	5	∞	3	∞	0

D_1

	1	2	3	4	5
1	0	∞	2	∞	5
2	4	0	3	∞	4
3	∞	∞	0	2	2
4	8	4	∞	0	2
5	5	∞	3	∞	0

D_2

	1	2	3	4	5
1	0	∞	2	∞	5
2	4	0	3	∞	4
3	∞	∞	0	2	2
4	8	4	7	0	1
5	5	∞	3	∞	0

D_3

	1	2	3	4	5
1	0	∞	2	4	4
2	4	0	3	5	4
3	∞	∞	0	2	2
4	8	4	7	0	1
5	5	∞	3	5	0

D_4

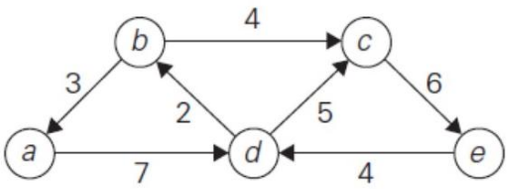
	1	2	3	4	5
1	6	8	2	4	4
2	4	0	3	5	4
3	10	6	0	2	2
4	8	4	7	6	1
5	5	9	3	5	0

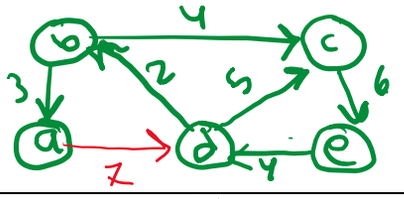
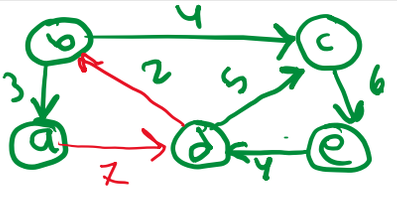
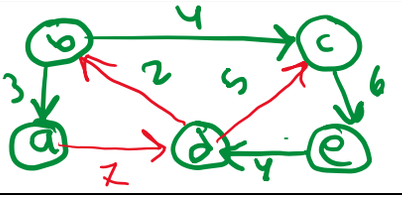
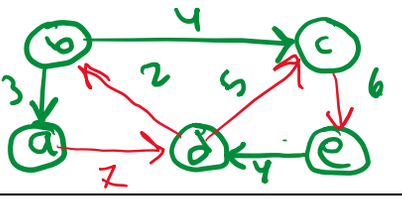
D_5

	1	2	3	4	5
1	0	8	2	4	4
2	4	0	3	5	4
3	7	6	0	2	2
4	6	4	4	0	1
5	5	9	3	5	0

II. Greedy Algorithms.

Q3) [2] Solve the following instances of the single-source shortest-paths problem with vertex *a* as the source (show all solution steps):



Tree vertices	Remaining vertices	Illustration
a (-,0)	b(-,infinite) c(-, infinite) d(a,7) e(- ,infinite)	
d(a,7)	b(d,7+2) c(d, 7+5) e(- ,infinite)	
b(d,9)	c(d, 12) e(- ,infinite)	
c(d, 12)	e(c ,12+6)	
e(c ,18)	--	--

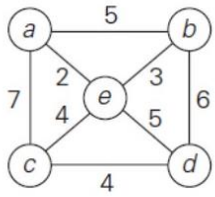
from a to d : a-d length 7

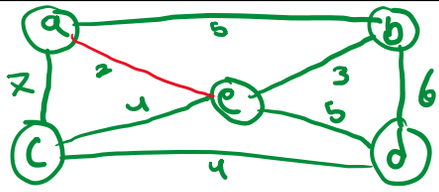
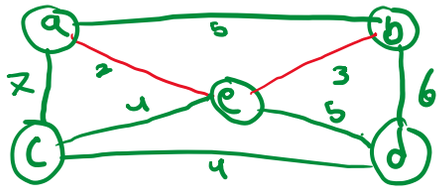
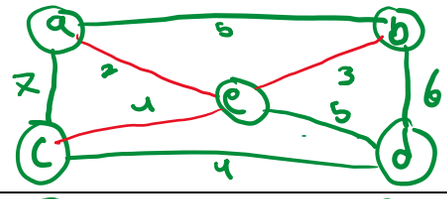
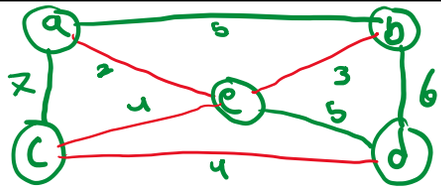
from a to b : a-d-b length 9

from a to c : a-d-c length 12

from a to e : a-d-b-c-e length 18

Q4) [1.5] Apply Prim's algorithm to the following graph. Include in the priority queue all the vertices not already in the tree.



Tree vertices	Remaining vertices	Illustration
a (-,-)	b(a,5) c(a, 7) d(a, infinite) e(a, 2)	
e(a, 2)	b(e,3) c(e, 4) d(e, 5)	
b(e,3)	c(e, 4) d(e, 5)	
c(e, 4)	d(c, 4)	
d(c, 4)	--	--

The minimum spanning tree is