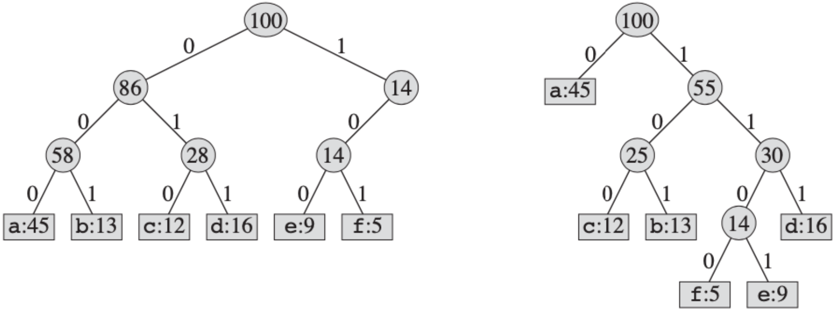
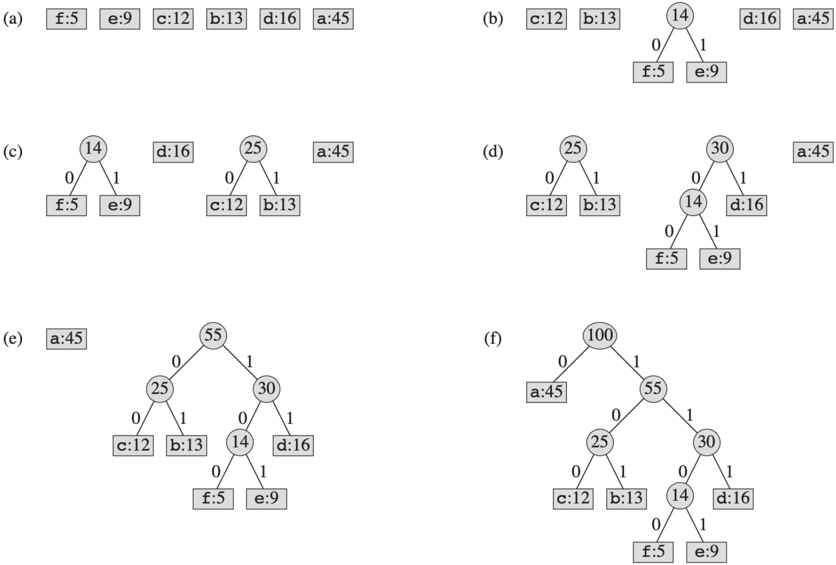
	a	b	c	d	e	f
Frequency (in thousands)	45	13	12	16	9	5
Fixed-length codeword	000	001	010	011	100	101
Variable-length codeword	0	101	100	111	1101	1100


100
0
1
86
14
0
1
0
58
28
14
0
1
0
1
0
1
a:45
b:13
c:12
d:16
e:9
f:5
100
0
1
a:45
55
0
1
25
30
0
1
0
1
c:12
b:13
14
d:16
0
1
f:5
e:9

(a) (b)

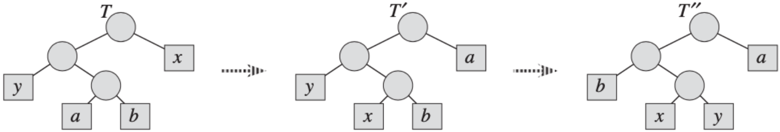
```
HUFFMAN(C)
n = |C|
Q = C
for i = 1 to n - 1
    allocate a new node z
    z.left = x = EXTRACT-MIN(Q)
    z.right = y = EXTRACT-MIN(Q)
    z.freq = x.freq + y.freq
    INSERT(Q, z)
return EXTRACT-MIN(Q)    // return the root of the tree
```



(a) f:5 e:9 c:12 b:13 d:16 a:45
(b) c:12 b:13 14 f:5 e:9 d:16 a:45
(c) 14 f:5 e:9 d:16 25 c:12 b:13 a:45
(d) 25 c:12 b:13 30 14 f:5 e:9 d:16 a:45
(e) a:45 55 25 c:12 b:13 30 14 f:5 e:9 d:16
(f) 100 a:45 55 25 c:12 b:13 30 14 f:5 e:9 d:16

Lemma 16.2

Let C be an alphabet in which each character $c \in C$ has frequency $c.freq$. Let x and y be two characters in C having the lowest frequencies. Then there exists an optimal prefix code for C in which the codewords for x and y have the same length and differ only in the last bit.


T
x
y
a
b
T′
a
y
x
b
T″
a
b
x
y

Lemma 16.3

Let C be a given alphabet with frequency $c.freq$ defined for each character $c \in C$. Let x and y be two characters in C with minimum frequency. Let C' be the alphabet C with the characters x and y removed and a new character z added, so that $C' = C - \{x, y\} \cup \{z\}$. Define f for C' as for C, except that $z.freq = x.freq + y.freq$. Let T' be any tree representing an optimal prefix code for the alphabet C'. Then the tree T, obtained from T' by replacing the leaf node for z with an internal node having x and y as children, represents an optimal prefix code for the alphabet C.