يوم الأحد

$$\sum_{i=1}^{n}\sum_{j=1}^{m} j = \sum_{i=1}^{n} \frac{m(m+1)}{2} = \frac{n\,m(m+1)}{2}$$

$$\sum_{i=1}^{n}\sum_{j=i}^{m} j = \sum_{i=1}^{n}\left(\sum_{j=1}^{m} j - \sum_{j=1}^{i-1}\right)$$

$$= \sum_{i=1}^{n}\left(\frac{m(m+1)}{2} - \frac{i(i-1)}{2}\right) = \frac{n\,m(m+1)}{2} -$$

$$= \frac{1}{2}\left[\sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i\right]$$

$$\sum_{i=0}^{n} a^i = 1 + a + a^2 + \dots + a^n = \frac{a^{n+1} - 1}{a-1}$$

$$S = \sum_{i=0}^{n} a^i = 1 + a + a^2 + a^3 + \dots + a^n$$

$$S * a = a + a^2 + a^3 + \dots + a^{n+1}$$

$$S * a - S = a^{n+1} - 1$$

$$S(a-1) = a^{n+1} - 1$$

$$S = \frac{a^{n+1} - 1}{a - 1}$$

Sum (A[ ], n) # operations

$i \leftarrow 1 \dots n \longrightarrow n+1$

{ sum $\leftarrow$ sum + A[i] } $\rightarrow 2*M$

return sum ——— 1

$$\boxed{2n + 2}$$

$n = 10000 \longrightarrow 0.3\,s$

$n = 75000 \longrightarrow ?$

first, we find the speed

$$\text{speed} = \frac{20002}{0.3} =$$

$$\text{time} = \frac{\cancel{75000}\ \ 2(75000)+2}{\text{speed}} = \frac{\cancel{[illegible]}}{} =$$

يوم ١ خميس

Method (A [ ], n)

```
for i ← 1 ... n-1
    for j ← i+1 ... n
        if (A[i] > A[j])
            A[i] ↔ A[j]
```

\# steps

i) $1 + \sum_{k=1}^{n-1} 1 = n$

ii) $\sum_{i=1}^{n-1} \left(1 + \sum_{k=i+1}^{n} 1 \right)$

iii) $\sum_{i=1}^{n-1} \sum_{k=i+1}^{n} 1$

$$\sum_{k=i+1}^{n} 1 = n - (i+1) + 1 = n - i - 1 + 1 = n - i$$

$$\# \text{steps}: \quad 1 + n - 1 + \left(\sum_{i=1}^{n-1} (n - i + 1) \right) + \sum_{i=1}^{n-1} (n - i)$$

$$= 1 + n - 1 + (n-1)n - \frac{(n-1)n}{2} + n - 1 + (n-1)n +$$

ii)

$f(n) = 5n + 12$

prove that $f(n)$ is $O(n)$.

$$5n + 12 \le 5n + 12n$$

$$\le 17n$$

$c = 17 \qquad n_0 = 1$

$$5n + 12 \le 6n$$

$c = 6$

$12 \le n \qquad n_0 = 12$

$$\underbrace{5n + 12}_{x} \le 17n^2$$

ناخذ أقل عدد أكبر من
المعادلة x بيكون
في هذه الحالة 1

يوم الخميس، ٢٦، سبتمبر

A, B

$C = \underset{m \times 1}{A} \times \underset{1 \times n}{B}$

with complexity

```
Method (A[m,1], B[1,n])
for i ← 1 ... m
for j ← 1 .. n
C[i][j] ← A[i][1] * B[1][j]
return c ;
```

complexity ⟶ m x n

```
x ← 1
  for i ← 1 .. n step 3
    x = x + 2
    print x
```

$\left\lfloor \frac{n}{3} \right\rfloor$

$$x = 1 + \sum_{i=1}^{\lfloor \frac{n}{3} \rfloor} 2 = 1 + \left\lfloor \frac{n}{3} \right\rfloor * 2$$

inputs: A $\langle a_1, a_2, \ldots, a_n \rangle$

value v

output: index i, v = A[i]

Method(A[], v)

for i → 0 .. n-1 {

if(A[i] = v)

return i }

return -1;

---

two n bits binary integers

A n element

\+ B n element

carry ← 0

for i ← n .. 1 {

$C[i+1] \leftarrow (A[i] + B[i] \cancel{\% 2} + carry) \% 2$

carry ← (A[i] + B[i]) / 2 }

C[1] ← carry

return C;

نوم ، أحسن

$$T(n) = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + \dots + a_k n^k \qquad a_i \in \mathbb{R}$$

$$\le |a_0| + |a_1| n + |a_2| n^2 + |a_3| n^3 + \dots + |a_k| n^k$$

$$\le \left(\underbrace{\frac{|a_0|}{n^k}}_{\le |a_0|} + \underbrace{\frac{|a_1|}{n^{k-1}}}_{\le |a_1|} + \underbrace{\frac{|a_2|}{n^{k-2}}}_{\le |a_2|} + \underbrace{\frac{|a_3|}{n^{k-3}}}_{|a_3|} + \dots + |a_k| \right) n^k$$

n تمثل ال inputs

$$T(n) \le \underbrace{\left(|a_0| + |a_1| + |a_2| + \dots + |a_k| \right)}_{C} n^k$$

$\boxed{\cancel{f_1(n) + f_2(n)}}$ $\quad f_1(n)$ is $O(g_1(n))$ I

$f_2(n)$ is $O(g_2(n))$ II

$[f_1(n) + f_2(n)]$ is $O(\cancel{g}\, \max(g_1(n), g_2(n)))$

I $\exists\, c_1, n_{01}$ st $f_1(n) \le c_1 g_1(n)$ $\forall n \ge n_{01}$

II $\exists\, c_2, n_{02}$ st $f_2(n) \le c_2 g_2(n)$ $\forall n \ge n_{02}$

$$[f_1(n) + f_2(n)] \le c_1 g_1(n) + c_2 g_2(n) \qquad \forall n \ge \max(n_{01}, n_{02})$$

$$f_1(n) + f_2(n) \le c_1 \max(g_1, g_2) + c_2 \max(g_1, g_2)$$

$$f_1(n) + f_2(n) \le \underbrace{(c_1 + c_2)}_{C} \max(g_1, g_2) \qquad \forall n \ge \max(n_{01}, n_{02})$$

$$f_1(n) \cancel{*} f_2(n) \longrightarrow O(g_1(n)\, g_2(n))$$

$$f_1 f_2 \leq \underbrace{c_1 \, c_2}_{c} (g_1 g_2)$$

$n_0 = \max(n_{01}, n_{02})$

$$f_1 f_2 \text{ is } O(g_1 g_2)$$

$$\boxed{\begin{matrix} c_1 \\ = \\ 3 \end{matrix}}\, n \leq 5n + 12 \leq \boxed{\begin{matrix} c_2 \\ = \\ 6 \end{matrix}}\, n$$

$3n \leq 5n + 12$

$2n \cancel{\geq} -12$

$n \cancel{\geq} -6$

$n_0 \geq 1$

$5n + 12 \leq 6n$

$12 \leq n$

$\underline{\underline{12}}_{n_0} \leq n$

$n_0 = \max(1, 12)$

$n_0 = 12$

يوم لثلاثاء

$T(n) = \log(3n+8) \qquad \leq c \log n$

$n \geq n_0$

$3n + 8 \leq 4n \qquad n \geq 8$

$\log(3n+8) \leq \log(4n)$

$\leq \boxed{\log 4} + \boxed{\log n}$ نفس بعض

$\leq \log n + \log n$

نقارن n في الأصل أكبر من 8 فسيكون أيضاً أكبر من 4

$T(n) \leq 2 \log n$

$3n + 8 \leq 11n \qquad n \geq 1$

$\leq \boxed{\log 11} + \boxed{\log n}$ نفس بعض

$\leq \log n + \log n \qquad n \geq 11$

يجب أن يكون n أكبر من 11

$\leq 2 \log(n)$

$\log(n!)$ is $O(n \log(n))$

~~log~~ $= \log 1 + \log 2 + \dots + \log n$

$\leq \log n + \log n + \dots + \log n$

$\leq n \log n$

$c = 1$

$n_0 = 1$

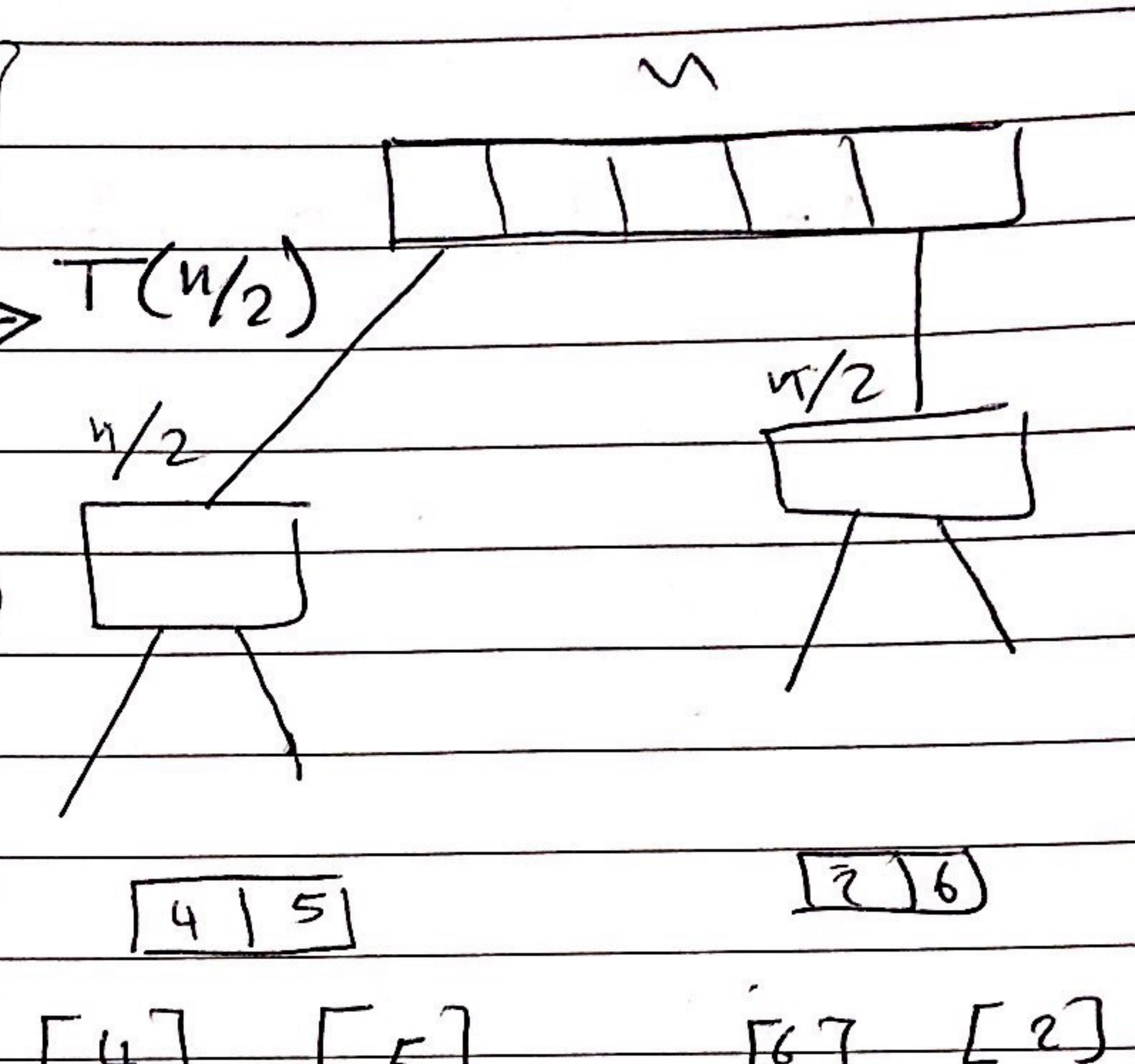
H.W

Prove that $\log(n!)$ is $\Omega(n \log n)$

insertion sort

يوم خميس

```
mergeSort(A[], L, R)
{
    mid = (L+R)/2              -> 1
    mergSort(A[], L, mid)      -> T(n/2)
T(n/2) mergSort(A[], mid+1, R)
    merge ~~verge~~ (A[], mid) -> n
```



2 ←	3 ←
5 ←	6 ←
7	10
9	12

2	3	5	6	7	9	10	12

$$T(n) = 2T(n/2) + n + 1$$

$$T(n/2) = 2T(n/4) + n/2 + 1$$

Recursive substitution:-

$$T(n) = 2T\left(\frac{n}{2}\right) + n + 1$$

$$= 2\left[2T(n/4) + n/2 + 1\right] + n + 1$$

$$= 4\left[2T(n/8) + n/4 + 1\right] + 2n + 3$$

$$= 8T(n/8) + 3n + 7$$

$$= 8\left[2T(n/16) + n/8 + 1\right] + 3n + 7$$

$$= 16T(n/16) + 4n + 15$$

$$T(n) = 2^k\, T\left(\boxed{n/2^k}\right) + kn + (2^k - 1)$$

$$n/2^k = 1 \iff n = 2^k \iff k = \log n$$

$$\Rightarrow T(n) = \cancel{n} \; nT(1) + n\log n + (n-1)$$

$$T(n) = 2n + n\log n - 1$$

$$\leq 2n\log n + n\log n + n\log n$$

$$\leq 4n\log n \qquad \underset{n_0}{n \geq 1}$$

بدل التقسيم على 2 يتم التقسيم على 3 ∴ الأساس للـ log يكون 3

يوم الخميس تمارين

$1^2 + 2^2 + 3^2 + \dots + n^2$ is $O(n^3)$

$\leq n^2 + n^2 + \dots + n^2$

$= (n^2)\, n$

$= n^3 \qquad c = 1, \; n_0 = 1$

$O(n^3)$

$\dfrac{3n - 8 - 4n^3}{2n - 1}$ is $O(n^2)$

$\leq \dfrac{3n^3 + 8n^3 + 4n^3}{2n - n} = \dfrac{15n^3}{n} = 15n^2$

$c = 15$

$n_0 = 1$

$1.2 + 2.3 + 3.4 + \dots + (n-1)n$ is $O(n^3)$

$\leq (n-1)n + (n-1)n + (n-1)n + \dots + (n-1)n$

$= \cancel{(n-1)[n^2 - n]} \; (n-1)^2 n$

$= \cancel{n^3 - n^2 - n^2 + n} \; (n^2 - 2n + 1)\, n$

$= \cancel{n^3 - 2n^2 + n} = n^3 - 2n^2 + n$

$$\sum_{j=1}^{n} (j^3 + j) \quad \text{is } O(n^4)$$

$$\leq \sum_{j=1}^{n} (n^3 + n)$$

$$\leq \sum_{j=1}^{n} (n^3 + n^3)$$

$$= \sum_{j=1}^{n} 2n^3$$

$$= 2n^4 \quad O(n^4)$$

$c = 2 \quad n_0 = 1$

$$f(x) = \underbrace{(x+2)}_{f_1} \cdot \underbrace{\log(x^2+1)}_{f_2} + \underbrace{\log(x^3+1)}_{f_3}$$

is $O(x \log x)$

$$\log(x^2+1) \leq \log(x^2 + x^2)$$

$$= \log 2x^2$$

$$= \log_2 + 2 \log x$$

$$\leq \log x + 2 \log x$$

$$= 3 \log x$$

$c = 3 \quad n = 2$

يوم الأحد

$\log(n!)$ is $\Omega(n \log n)$

$$n! = \boxed{1} \times \boxed{2} \times \boxed{3} \times \dots \times n$$

$$n! = \boxed{n} \times \boxed{(n-1)} \times \boxed{(n-2)} \times \dots \times 1$$

$$(n!)^2 = \prod_{k=1}^{n} \underbrace{k(n-k+1)}_{\geq n} \geq \prod_{k=1}^{n} n$$

$k(n-k+1) \geq n$

$kn - k^2 + k \geq n$

$kn - n \geq k^2 - k$

$n\cancel{(k-1)} \geq k\cancel{(k-1)}$

$n \geq k$

$(n!)^2 \geq \prod_{k=1}^{n} n$

$(n!)^2 \geq n^n$

$\log((n!)^2) \geq \log(n^n)$

$2\log(n!) \geq n \log n$

$\log(n!) \geq \frac{1}{2} n \log n \quad n \geq 1$

$c \qquad n_0$

MinMax (A[], l, r)

{

1 — mid = $\frac{l+r}{2}$

1 — if (l $\geq$ r)

~~break~~ return (A[r], A[r])

else

$T(n/2)$ $[x_1, y_1]$ $\leftarrow$ minMax(A[], l, mid)

$T(n/2)$ $[x_2, y_2]$ $\leftarrow$ minMax (A[], mid+1, r)

1 — return (min(x_1, x_2), max(y_1, y_2))

steps

$$T(n) = 2T(n/2) + 3$$

$$= 2\left[2T(n/4) + 3\right] + 3 = 4T(n/4) + \overset{2\times 3+3}{\cancel{\ }}$$

$$= 4\left[2T(n/8) + 3\right] + 9 = 8T(n/8) + \overbrace{\cancel{\ }\ \cancel{\ }}^{4\times 3 + 2\times 3 + 3}$$

$$= 2^k\left(\frac{n}{2^k}\right) + \frac{3}{2}\sum_{i=1}^{k} 2^i$$

$$\frac{n}{2^k} = 1 \longrightarrow n = 2^k \longrightarrow k = \log n$$

$$T(n) = nT(1) + \frac{3}{2}\ \frac{2^{k+1} - 1}{2-1}$$

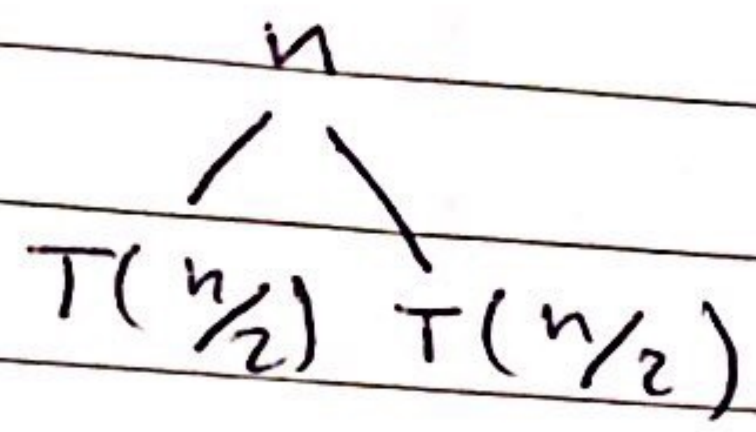
$$= n + \frac{3}{2} \times 2 \times n$$

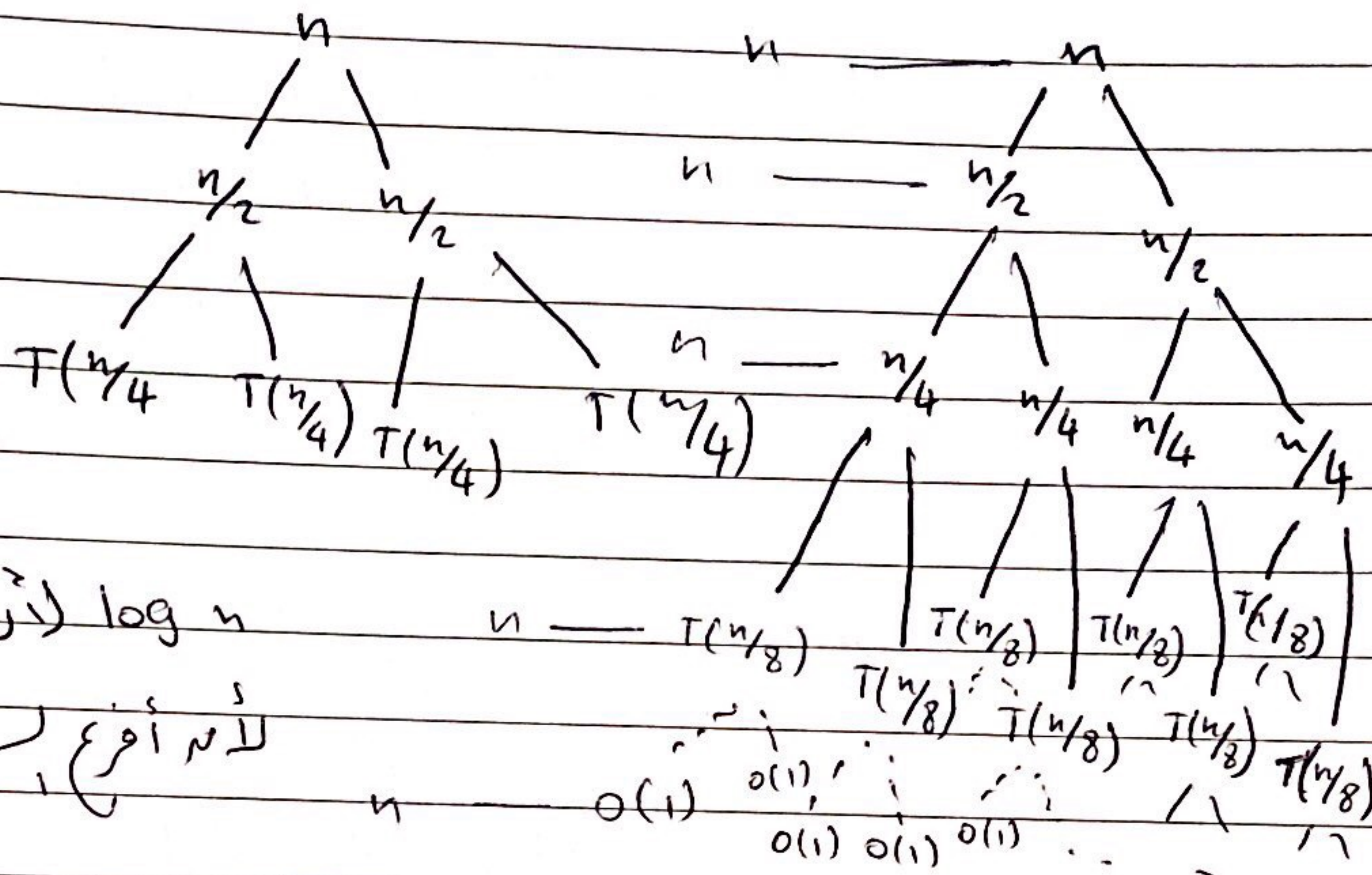
يوم الثلاثاء اختبار يوم

$$T(n) = T(n/3) + T(2n/3) + n$$

$$\left[T(n/9) + T\left(\frac{2n}{9}\right) + n/3\right] + \left[T(2n/9) + T(4n/9) + 2n/3\right] + n$$

طريقة ال recursive substitution لا تصلح لحل إعادات

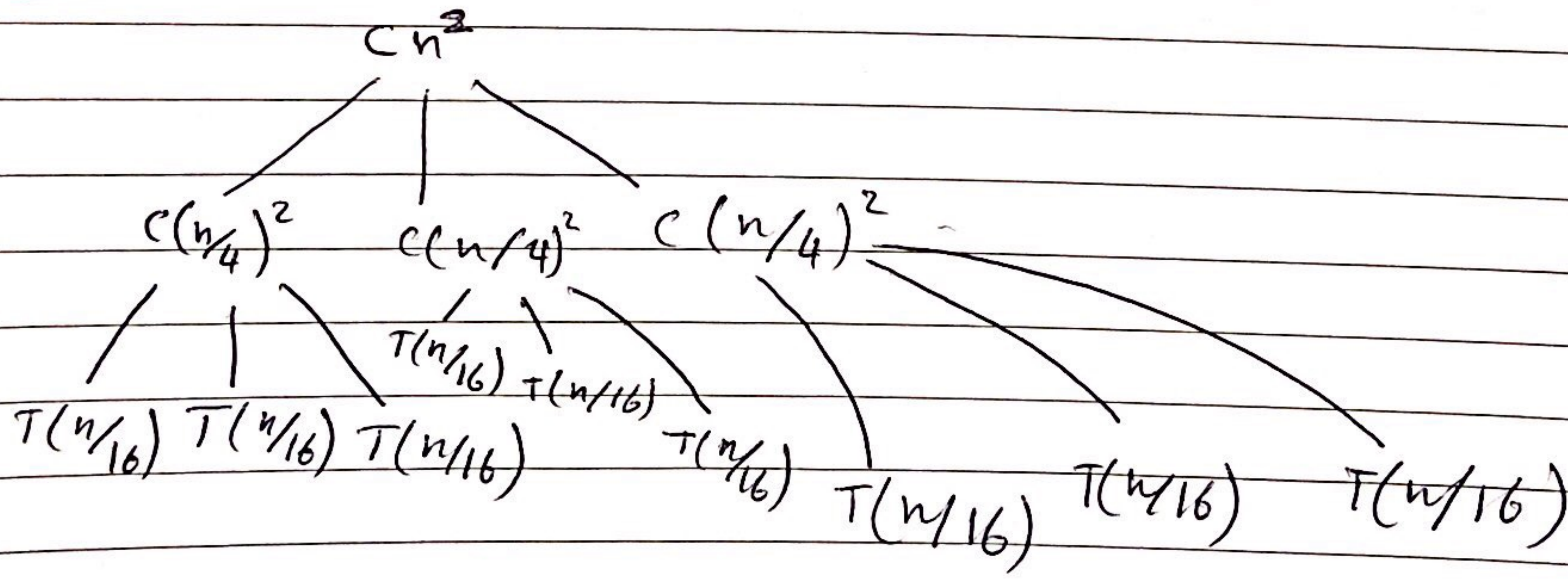




n log n لأن التقسيم على 2 وليس

لأنه فرع الشجرة 2

$$O(n \log n)$$



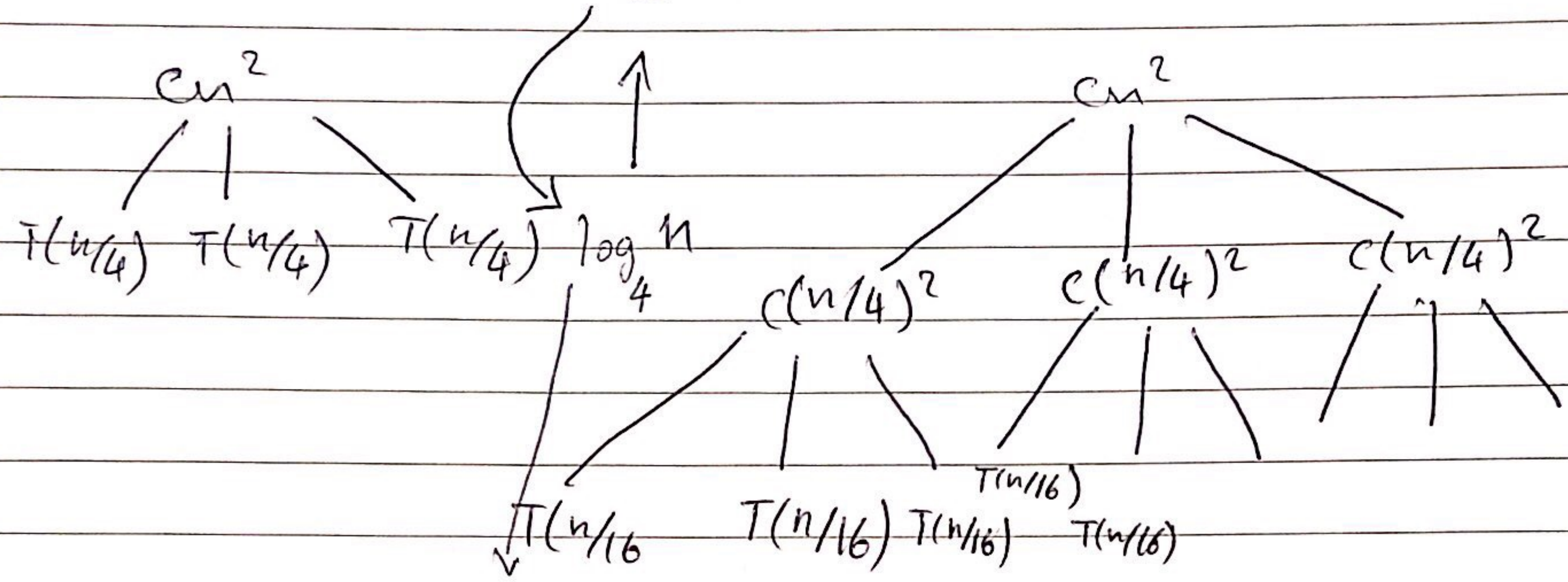
$$T(n) = T(n/3) + \underbrace{T(2n/3)}_{\text{الشجرة أكبر هنا}} + \_\_\_$$

unbalanced tree

طول الشجرة هو $\log_{\frac{3}{2}} n$

$$2^{\log_2 n} = n^{\log_2 2} = n$$

$$T(n) = 3T(\lfloor n/4 \rfloor) + O(n^2)$$



$\frac{1}{4}$ مقلوبها يكون الأساس
الـ log

طول الشجرة يكون $\log_4 n$

عدد الـ leafs تكون $3^{\log_4 n}$
طول الشجرة لـ التقسيم

$$T(n) \le dn^2$$

$$T(n) = 3T(n/4) + cn^2 \le dn^2$$

$$\le 3d(n/4)^2 + cn^2 \le dn^2$$

$$\left(\frac{3}{16}d + c\right)n^2 \le dn^2$$

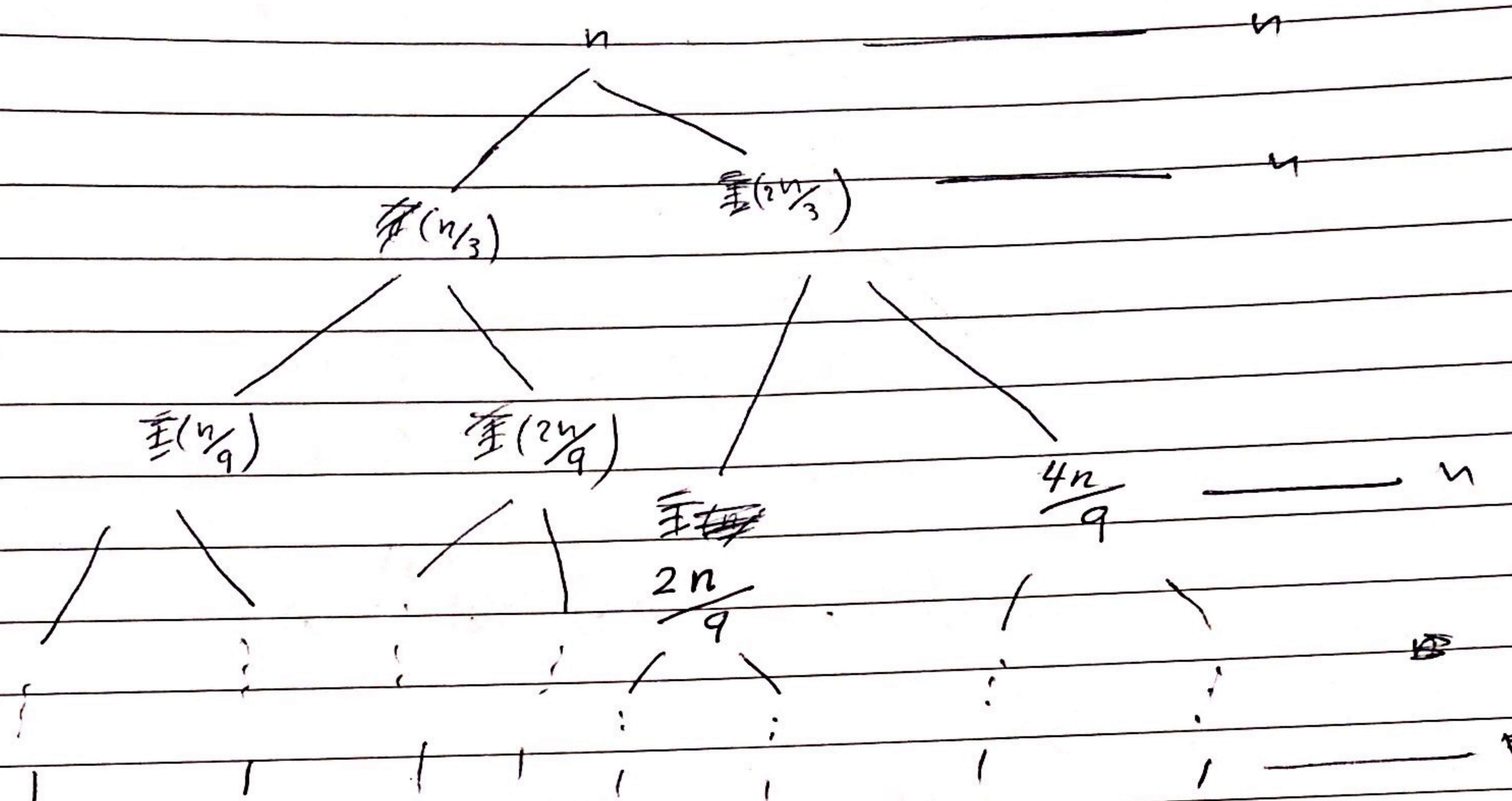
$$\frac{3}{16}d + c \le d$$

$$c \ge \frac{13}{16}d$$

$$\frac{16c}{13} \ge d$$

يوم الخميس

$$T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + n$$



2^{height} = عدد ال leaf هو $\log_{\frac{3}{2}} n$

$= 2^{\log_{\frac{3}{2}} n} = n^{\log_{3/2} 2}$

لانه $\frac{3}{2}$ أساس اللوغ $O(n \log n)$

$$T(n) = 2T(n/2) + n^2$$

$a = 2$

$b = 2$

$f(n) = n^2$

n^2 $\quad$ $n^{1+\varepsilon}$

$$T(n) = 2T(n/2) + 5$$

$a = 2$

$b = 2$

$f(n) = 5$

$f(n) = 5$ $\quad$ $O(n^{1-1})$ $\quad$ $(n^{\log_b a - \varepsilon})$

$O(1)$ $\quad$ $\Theta(1)$

$$T(n) = 2T(n/2) + n$$

$a = 2$ $b = 2$ $f(n) = n$

The master theorem case $\underline{\underline{2}}$

تمارين يوم الخميس

$T(n) = T(n-1) + 5$

$T(1) = 0$

$T(n) = T(n-1) + 5$

$= T(n-2) + 5 + 5$

$= T(n-3) + 5 + 5 + 5$

$= T(n-3) + 3 \times 5$

$= T(n-k) + 5k$

$n - k = 1$

$k = n - 1$

$= T(1) + 5(n-1)$

$= 0 + 5n - 5$

$T(n) = 3T(n-1) + 5 \quad n > 1$

$T(1) = 4$

$T(n) = 3T(n-1) + 5$

$= 3[3T(n-2) + 5] + 5$

$= 9T(n-2) + 5 + 3 \times 5$

$= 9[3T(n-3) + 5] + 5 + 3 \times 5$

$= 27T(n-3) + 5 + 3 \times 5 + 9 \times 5$

$$3^k T(n-k) + 5 \sum_{i=1}^{k-1} 3^{i}$$

$n - k = 1$

$k = n - 1$

$$3^{n+1} T(1) + 5 \frac{3^{k-1} - 1}{3 - 1} =$$

$$T(n) = T(n-1) + n$$

$$= T(n-2) + n + (n-1)$$

$$= T(n-3) + n + (n-1) + (n-2)$$

$$= T(n-k) + (n-(k-1)) + (n-(k-2)) + \dots + (n-0)$$

$$= T(n-k) + \sum_{i=0}^{k-1} (n-i)$$

$T(n) = T(n/3) + 1$ $\quad n > 1$

$T(1) = 1$ $\quad n = 3^{\ell}$

$T(3^{\ell}) = T(3^{\ell-1}) + 1$ $\quad \ell = \log_3 n$

$$= T(3^{\ell-2}) + 2$$

$$= T(3^{\ell-3}) + 3$$

$$= T(3^{\ell-k}) + k$$

يوم الأحد

1- $T(n) = 9T(n/3) + n$

2- $T(n) = T(2n/3) + 1$

3- $T(n) = 3T(n/4) + n\log n$

1- $a = 9$ $\quad b = 3$ $\quad f(n) = n$

$\Theta(n^{2-\varepsilon})$ case one

2- $a = 1$ $\quad b = \frac{3}{2}$ $\quad f(n) = 1$

$\cancel{\Theta}$

$n^{\log_b a} = n^{\log_{3/2} 1} = \Theta(n^0)$

$\log n$ case two

3- $a = 3$ $\quad b = 4$ $\quad f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = \tilde{\Theta}(n^{0.793})$

$af(n/b) \le cf(n)$ $\quad c < 1$

$\frac{3\cancel{n}}{4}\log\left(\frac{n}{4}\right) \le c\cancel{n}\log n$ $\quad \uparrow \frac{3}{4}$ case 3

$T(n) = 2T(n/2) + n\log n$

$a = 2, \; b = 2, \; f(n) = n\log n$

$\frac{\cancel{2}\cancel{n}}{\cancel{2}}\log\frac{n}{2} \le c\cancel{n}\log n$

والشرط هو أن تكون $c < 1$ وهنا تطلع مو واحد

يوم الثلاثاء

case 1 $f(n) <$

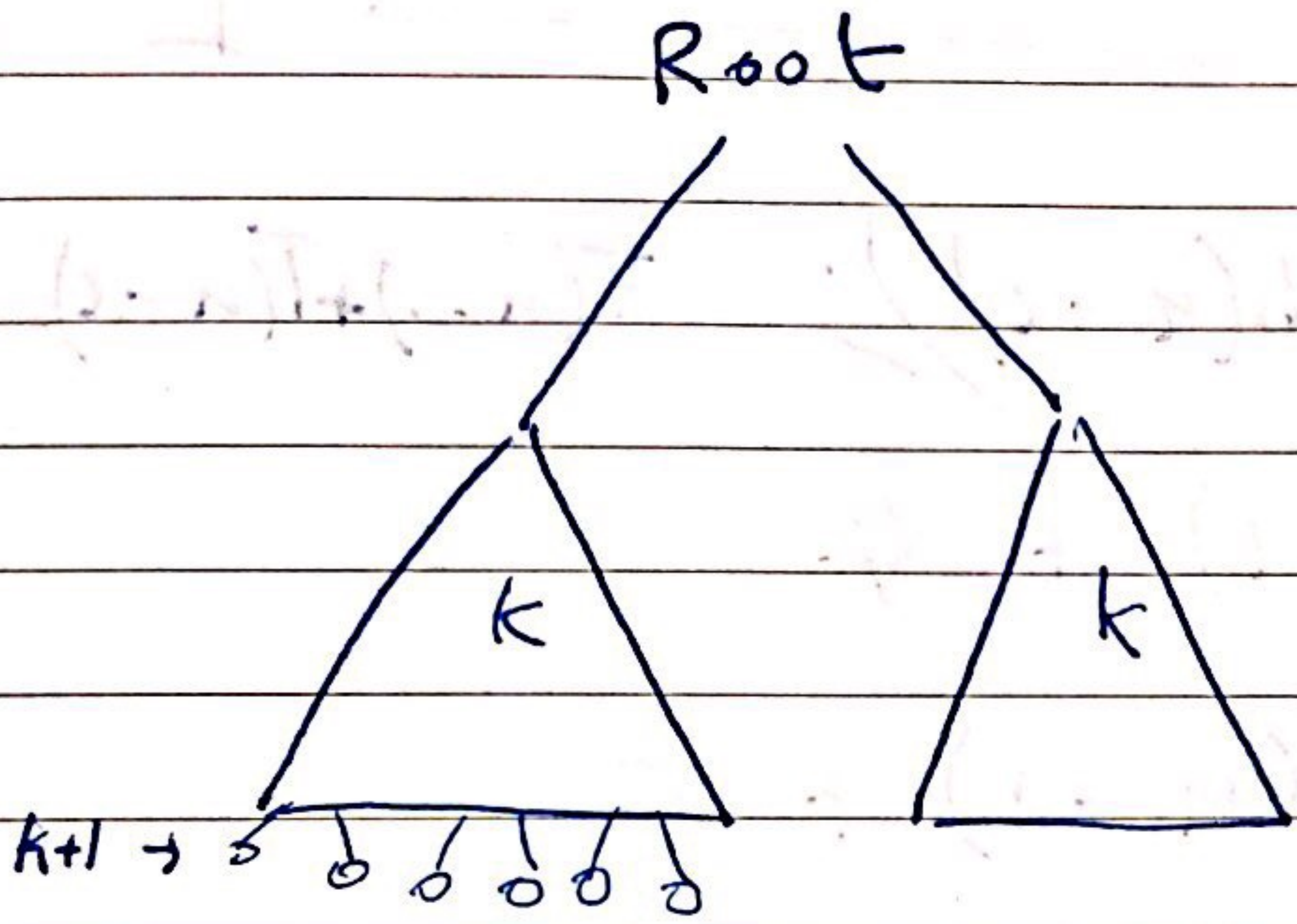
لتحديد الماستر ثيوريم إذا كانت أي case

case 2 $f(n) =$

case 3 $f(n) >$

ونحتاج لشرط إضافي

في ال heap لتحديد مكان الchilds يكون 2i و 2i+1



$L = 2k+1$

$R = k$

$T = 3k+2$

$$\frac{L}{T} = \frac{2k+1}{3k+2} = \frac{2+\cancel{1/k}}{3+} \quad \frac{1/k}{2/k}$$

$$T(n) = 5 + T\left(\frac{2n}{3}\right)$$

$a = 1$

$b = \frac{3}{2}$

$f(n) = O(n)$

$$n^{\log_{\frac{3}{2}} 1} = n^0 = O(1)$$

$\text{Case } (2) \longrightarrow \log n$

يوم الخميس

```
Fib(n)
  if(n==0)                  ———— 1
     return 0;              ———— 1
  if(n==1)                  ———— 1
     return 1;              ———— 1
  return (Fib(n-1)+Fib(n-2));    T(n-1)+T(n-2)
```

$$T(n) = T(n-1) + T(n-2) + 4$$

$$T(n) \leq T(n-1) + T(n-1) + c$$

$$\leq 2T(n-1) + c$$

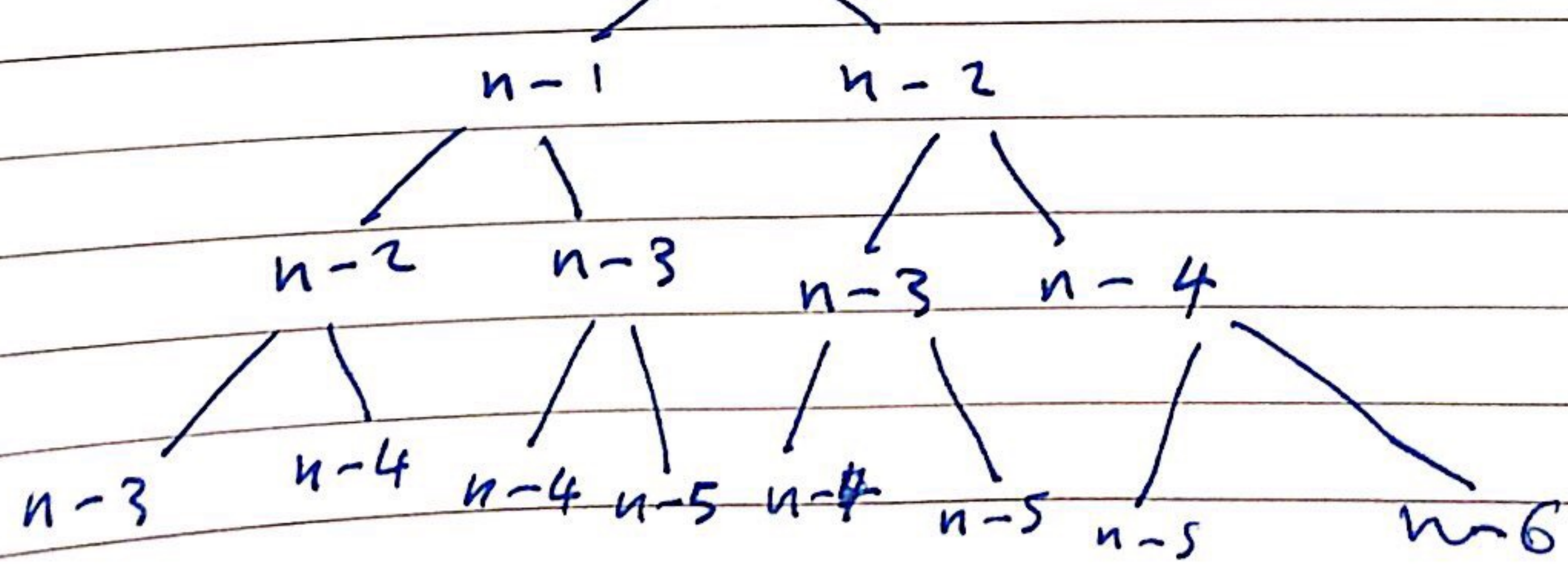
$$\leq 2[2T(n-2) + c] + c = 4T(n-2) + 3c$$

$$\leq 4[2T(n-3) + c] + 3c = 8T(n-3) + 7c$$

$$= 2^k T(\underbrace{n-k}) + (2^k - 1)c$$

↳ stop where $(n-k) = 0$

$$T(n) = 2^n T(0) + 2^n - 1$$

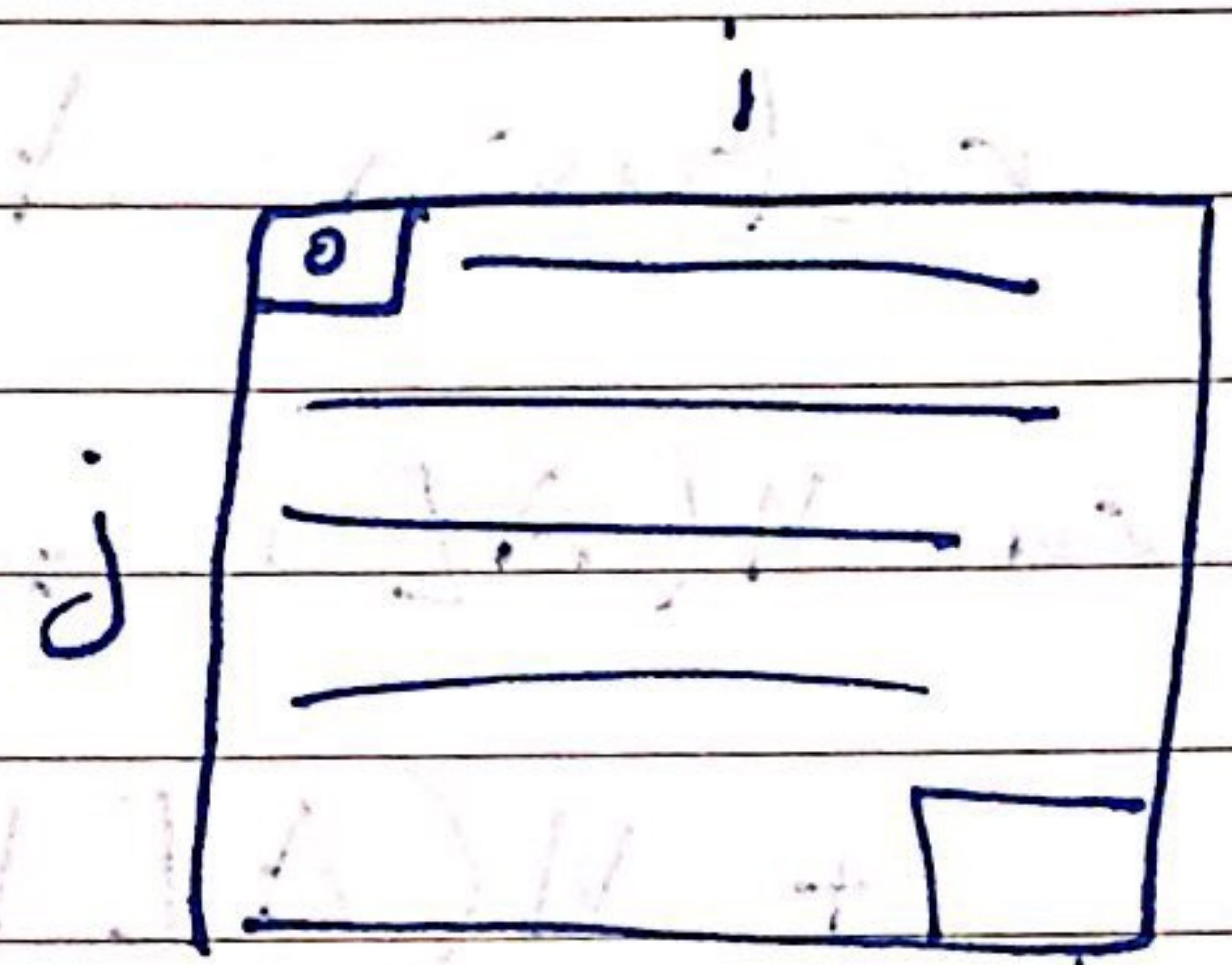


~~tog~~ longest common subsequence

LCS (X, y)

$X_i = \{ x_1, x_2, \ldots, x_i \}$

$y_j = \{ y_1, y_2, \ldots, y_j \}$



optimal solution

تمارين

يوم الخميس

Sum of n numbers

M (A)

if (n = 1)

return A[0]

return $M(A[0 \to \lfloor \frac{n}{2} \rfloor - 1)$

$+ M(A[\lfloor \frac{n}{2} \rfloor \longrightarrow n-1)$

$$T(n) = 2T(\frac{n}{2}) + 1$$

$a = 2$ $\quad n^{\log_b a} = n^{\log_2 2}$

$b = 2$

$f(n) = 1$

$$T(n) = 16\,T(n/4) + n^2$$

$a = 16 \quad b = 4 \quad f(n) = n^2$

$n^{\log_a b}$

$=$

$T(n) = 7T(n/2) + n^2$

$a = 7 \quad b = 2 \quad f(n) = n^2$

$n^{\log_2 7}$

$n^2 < n^{\log_2 7} < n^3$

$2 < \log_2 7 < 3$

$n^{\log_2 7} > n^2$

$T(n) = \Theta(n^{\log_2 7})$ case 1

$T(n) = 2T(n/4) + \sqrt{n}$

$a = 2 \quad b = 4 \quad f(n) = n^{1/2}$

$n^{\log_b a} = n^{\log_4 2} = n^{1/2} = f(n)$

case 2

$T(n) = \Theta(n^{1/2} \log n)$

$T(n) = 7T(n/3) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

$n^{\log_3 7} \qquad 1 < \log_3 7 < 2$

$a f(\frac{n}{b}) \leq c.f(n)$

case 3

$$7\left(\frac{n}{3}\right)^2 \leq c \cdot n^2$$

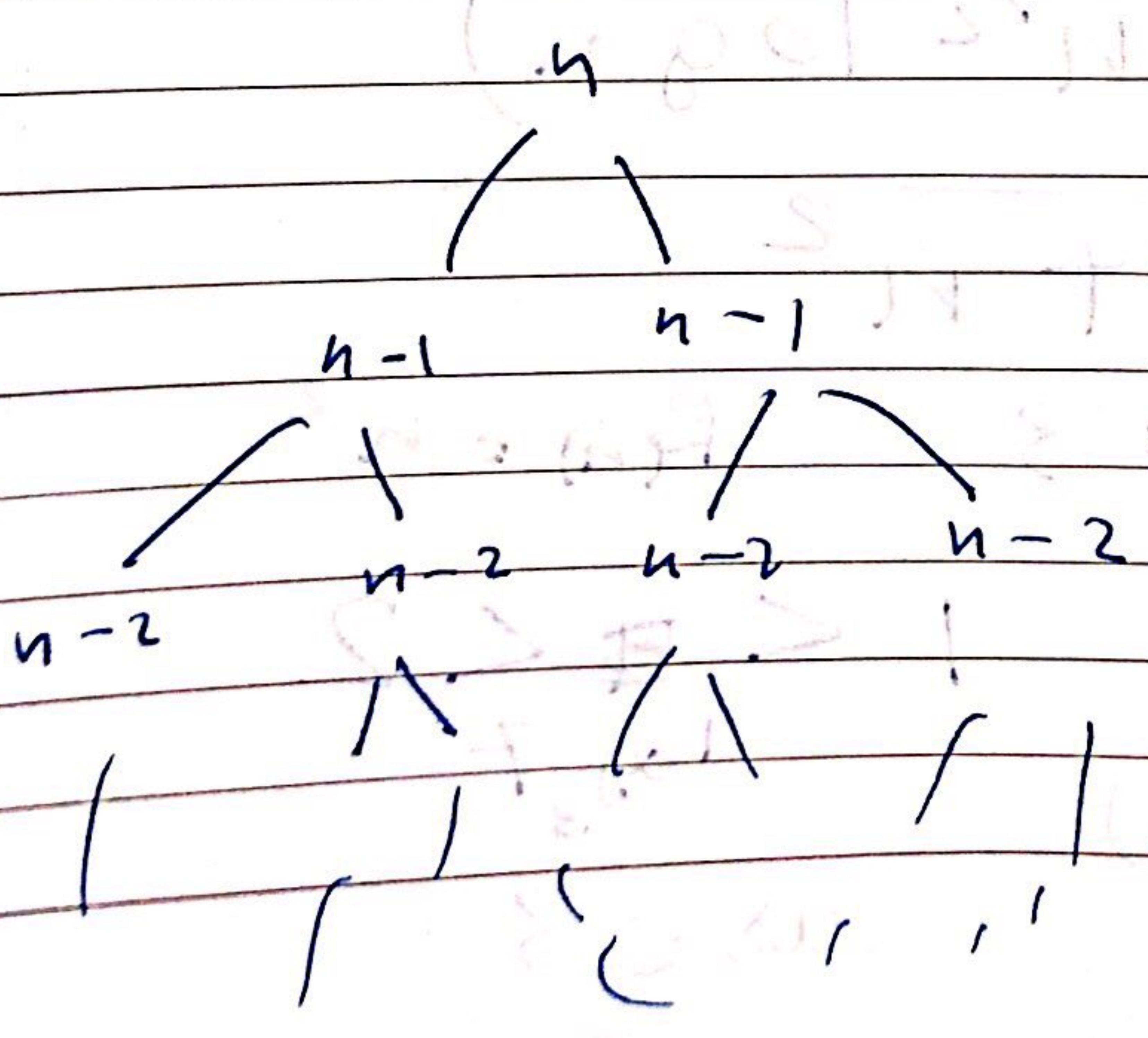
$$c = \frac{7}{9} < 1$$

$$2^n = 2^{n-1} + 2^{n-1} \qquad 2^0 = 1$$

$$T(n) = T(n-1) + T(n-1)$$

```
M(n)
if n = 0
    return 1
return M(n-1) + M(n-1)
```

$$T(n) = \begin{cases} 0 & n = 0 \\ 2T(n-1)+1 & n \geq 1 \end{cases}$$



$$\sum_{0}^{n} 2^i$$ عدد الركرسف

كول

يوم الأحد

$X = ABCAABAC$

$y = BAABBABAB$

X_i = sub string from begining until i

$LCS(x_i, j_i) = \cancel{LCS(x_{i-1}, y_{j-1}) + 1}$

$$C(x_i, y_i) = \begin{cases} LCS(x_{i-1}, y_{j-1}) + 1 & \text{if } (x[i] == y[j]) \\ Max(C(x_{i-1}, y_j), C(x_i, y_{j-1})) & \text{otherwise} \end{cases}$$

		1	2	3	4	5	6	7
	x	A	B	B	A	C	A	D
y	0	0	0	0	0	0	0	0
A	0	1	1	1	1	1	1	1
B	0	1	2	2	2	2	2	2
B	0	1	2	3	3	3	3	3
B	0	1	2	3	3	3	3	3
A	0	1	2	3	4	4	4	4

Max sum of contigous problems

Maximum sum Contiguous Subsequence

A

1	-2	10	4	2	1	-3	2

S

1							

$$S[i] = \max(S[i-1] + A[i], A[i])$$

hint : نضيف متغير آخر