CSC 311 – Winter 2022-2023
Design and Analysis of Algorithms

5. Brute force algorithms

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Outline

- Brute force algorithms
- Brute force sorting algorithm
- Brute force string matching
- Brute force polynomial evaluation
- Brute force matrix multiplication
- Brute force closest-pair problem
- Brute force convex hull
- Brute force strengths and weaknesses
- Exhaustive search

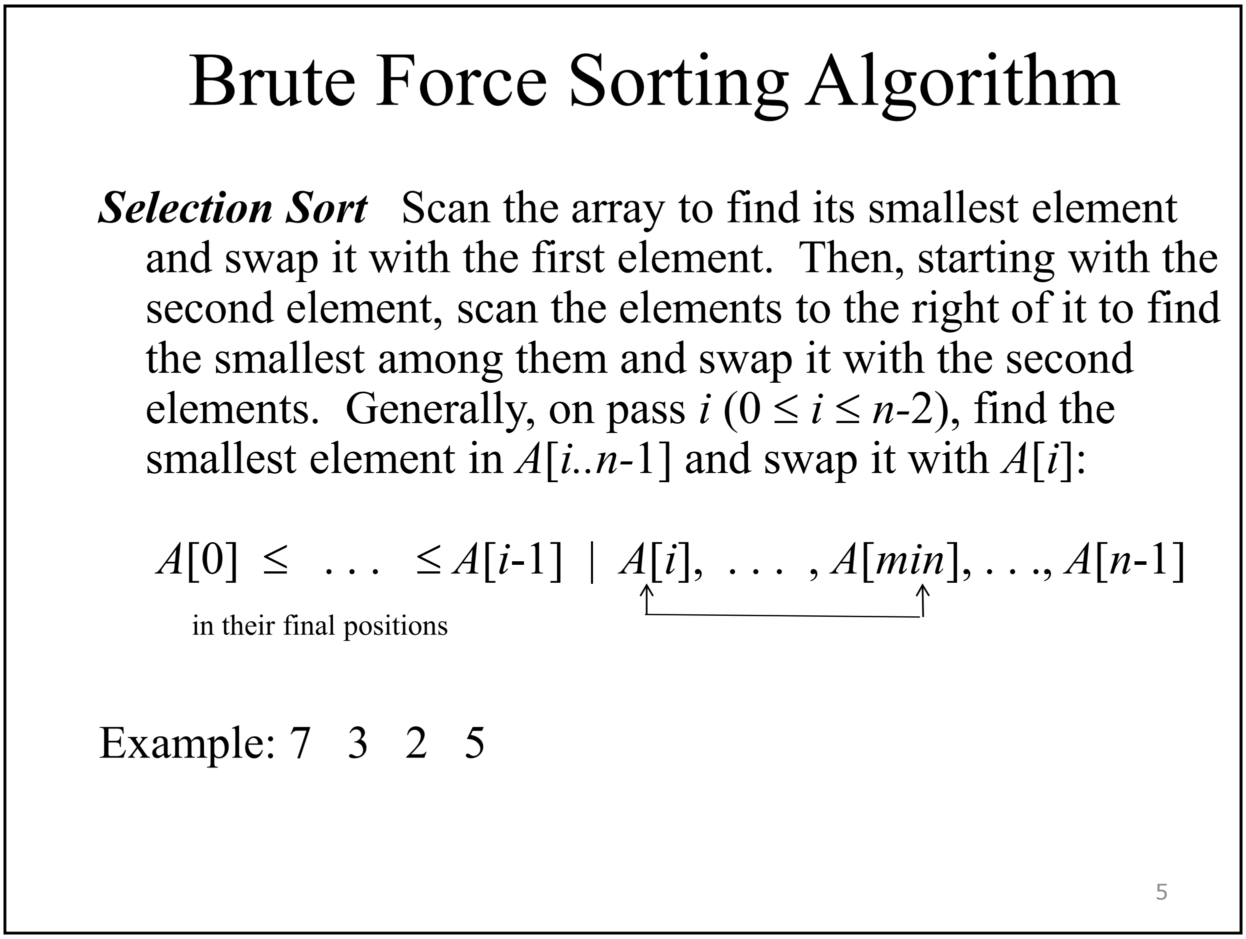
Brute Force Algorithms

- A straightforward approach, usually based directly on the problem's statement and definitions of the concepts involved without effort to make the algorithm smart.
  - "just do it" approach
  - Usually inefficient

Brute Force Algorithms

Examples:

1. Computing a^n ($a > 0$, n a nonnegative integer)
2. Computing $n!$
3. Multiplying two matrices
4. Searching for a key of a given value in a list



Brute Force Sorting Algorithm

Selection Sort Scan the array to find its smallest element and swap it with the first element. Then, starting with the second element, scan the elements to the right of it to find the smallest among them and swap it with the second elements. Generally, on pass i ($0 \le i \le n$-2), find the smallest element in $A[i..n\text{-}1]$ and swap it with $A[i]$:

$$A[0] \le \;\ldots\; \le A[i\text{-}1] \mid \underset{\uparrow}{A[i]},\;\ldots\;, \underset{\uparrow}{A[min]}, \ldots, A[n\text{-}1]$$

in their final positions

Example: 7 3 2 5

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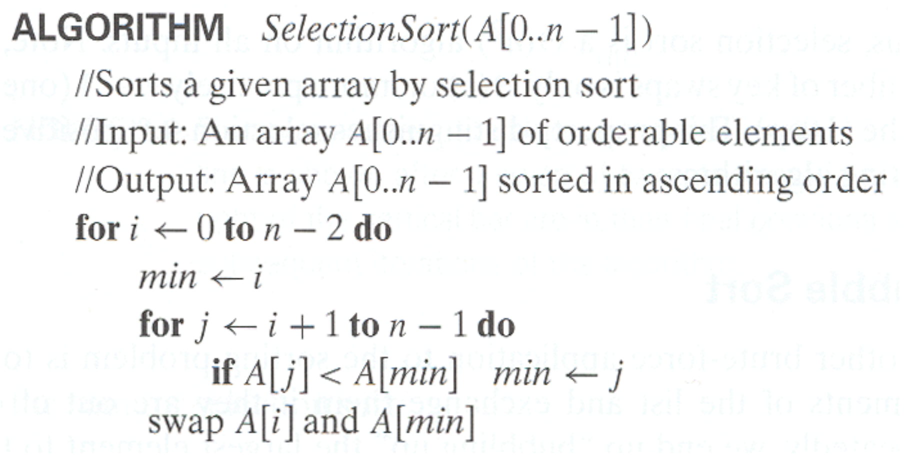
Analysis of Selection Sort



```
ALGORITHM  SelectionSort(A[0..n − 1])
    //Sorts a given array by selection sort
    //Input: An array A[0..n − 1] of orderable elements
    //Output: Array A[0..n − 1] sorted in ascending order
    for i ← 0 to n − 2 do
        min ← i
        for j ← i + 1 to n − 1 do
            if A[j] < A[min]   min ← j
        swap A[i] and A[min]
```

Time efficiency?

Space efficiency?

Brute Force String Matching

- *Pattern*: a string of *m* characters to search for
- *Text*: a (longer) string of *n* characters to search in
- problem: find a substring in the text that matches the pattern

Brute force algorithm

Step 1 Align pattern at beginning of text

Step 2 Moving from left to right, compare each character of pattern to the corresponding character in text until

- all characters are found to match (successful search); or
- a mismatch is detected

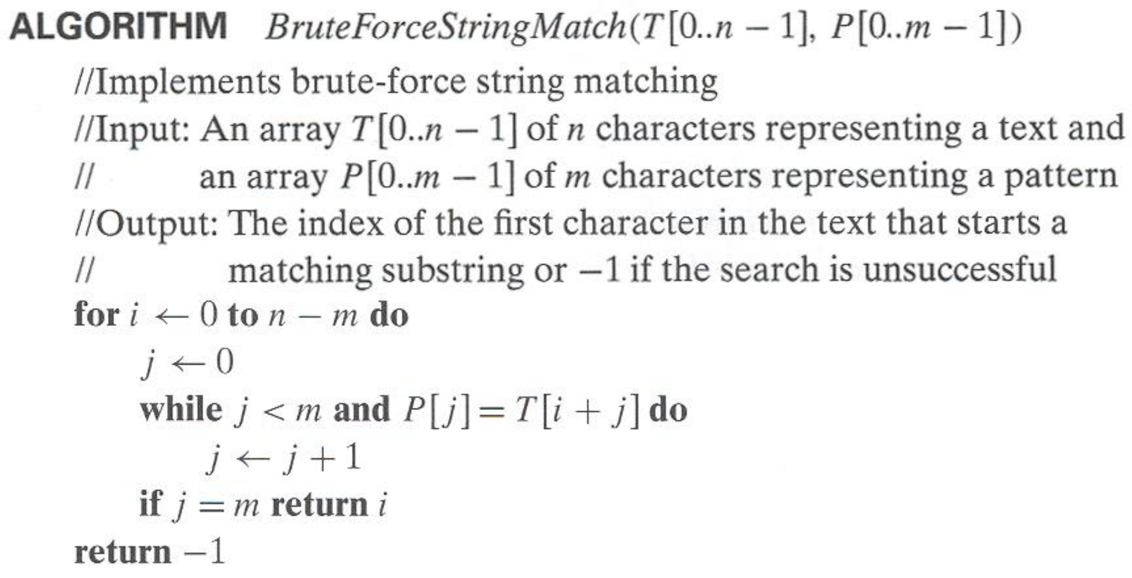
Step 3 While pattern is not found and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2

Examples of Brute Force String Matching

1. Pattern: 001011
   Text: 10010101101001100101111010

2. Pattern: happy
   Text: It is never too late to have a happy childhood.

Pseudocode and Efficiency



```
ALGORITHM  BruteForceStringMatch($T[0..n-1]$, $P[0..m-1]$)
    //Implements brute-force string matching
    //Input: An array $T[0..n-1]$ of $n$ characters representing a text and
    //       an array $P[0..m-1]$ of $m$ characters representing a pattern
    //Output: The index of the first character in the text that starts a
    //        matching substring or $-1$ if the search is unsuccessful
    for $i \leftarrow 0$ to $n-m$ do
        $j \leftarrow 0$
        while $j < m$ and $P[j] = T[i+j]$ do
            $j \leftarrow j+1$
        if $j = m$ return $i$
    return $-1$
```

Time efficiency?

Brute Force Polynomial Evaluation

Problem: Find the value of polynomial

$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$

at a point $x = x_0$

Brute force algorithm

```
$p \leftarrow 0.0$
for $i \leftarrow n$ downto 0 do
    $power \leftarrow 1$
        for $j \leftarrow 1$ to $i$ do        //compute $x^i$
            $power \leftarrow power * x$
        $p \leftarrow p + a[i] * power$
return $p$
```

Efficiency?

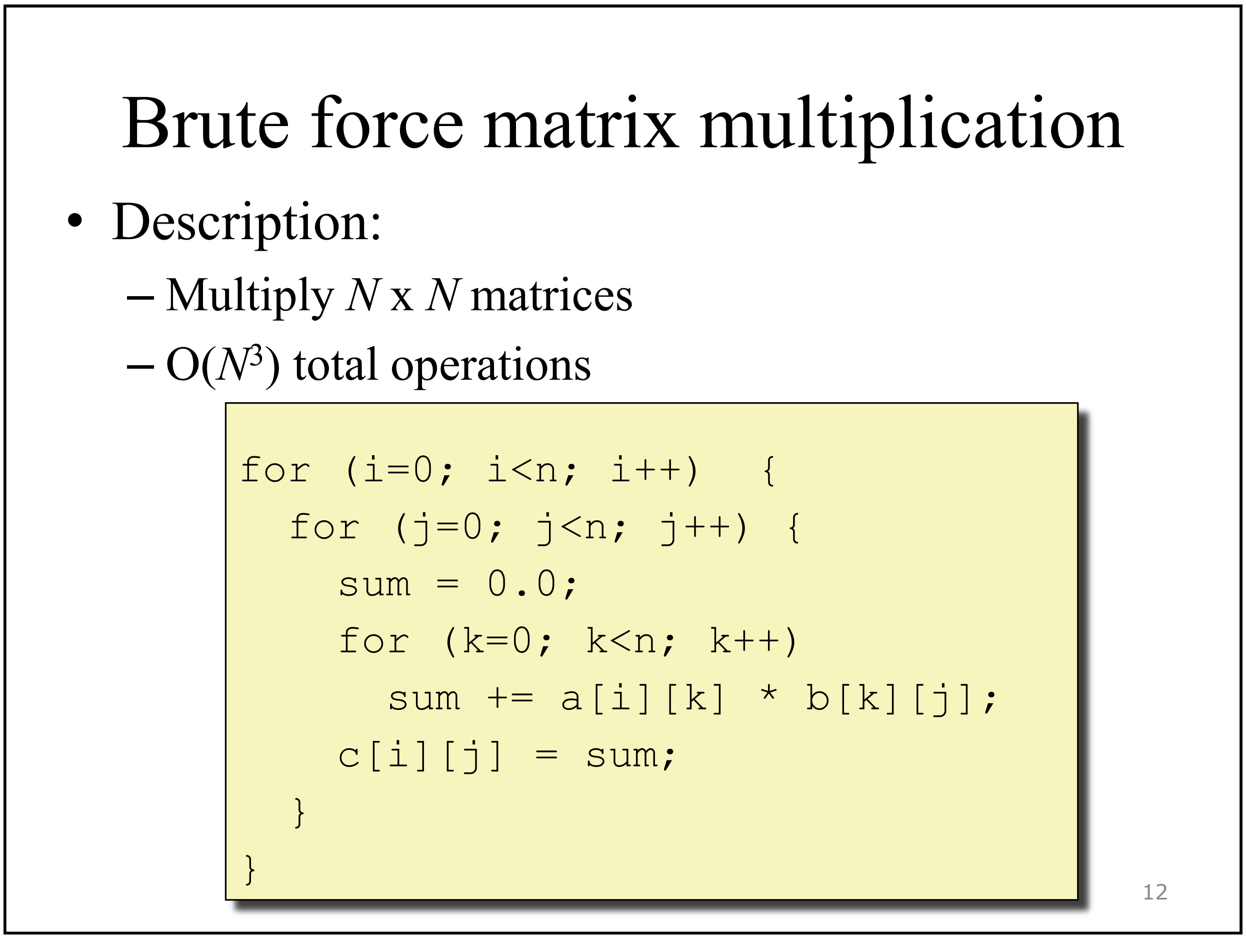
Polynomial Evaluation: Improvement

We can do better by evaluating from right to left:

Better brute force algorithm

```
$p \leftarrow a[0]$
$power \leftarrow 1$
for $i \leftarrow 1$ to $n$ do
    $power \leftarrow power * x$
    $p \leftarrow p + a[i] * power$
return $p$
```

Efficiency?



Brute force matrix multiplication

- Description:
  - Multiply *N* x *N* matrices
  - O(N^3) total operations

```
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

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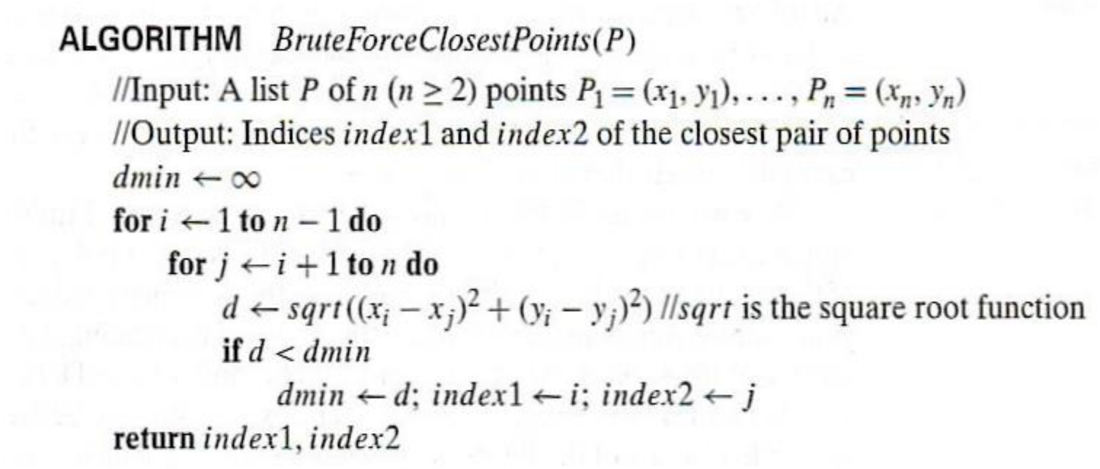
Brute Force Closest-Pair Problem

Find the two closest points in a set of *n* points (in the two-dimensional Cartesian plane).

Brute force algorithm

- Compute the distance between every pair of distinct points
- and return the indexes of the points for which the distance is the smallest.

Brute Force Closest-Pair Problem



```
ALGORITHM  BruteForceClosestPoints(P)
    //Input: A list P of $n$ ($n \geq 2$) points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
    //Output: Indices index1 and index2 of the closest pair of points
    $dmin \leftarrow \infty$
    for $i \leftarrow 1$ to $n - 1$ do
        for $j \leftarrow i + 1$ to $n$ do
            $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //sqrt is the square root function
            if $d < dmin$
                $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
    return index1, index2
```

Efficiency?

How to make it faster?

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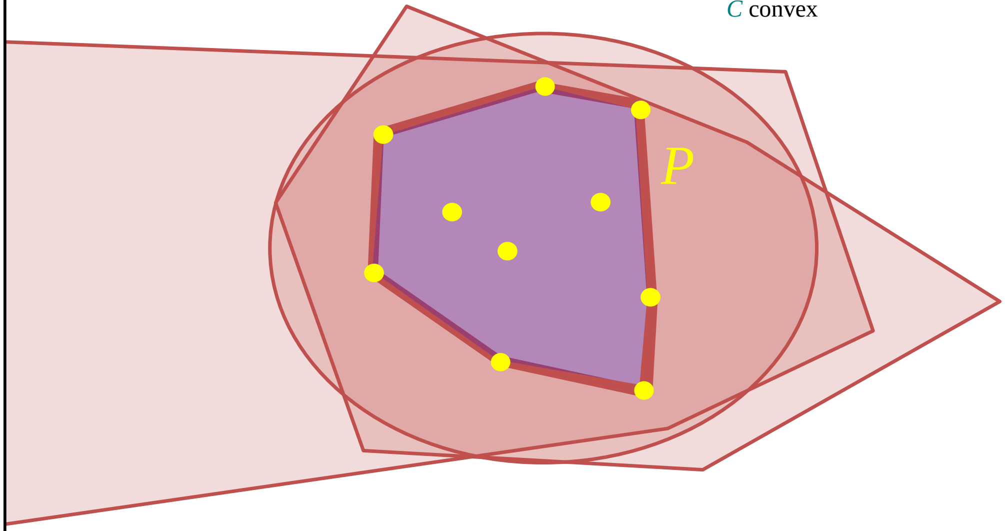
Brute force convex hull

- The convex hull *CH(P)* of a point set $P \subseteq \mathbf{R}^2$ is the smallest convex set $C \supseteq P$. In other words $\mathrm{CH(P)} = \bigcap_{\substack{C \supseteq P \\ C \text{ convex}}} \mathrm{C}$.



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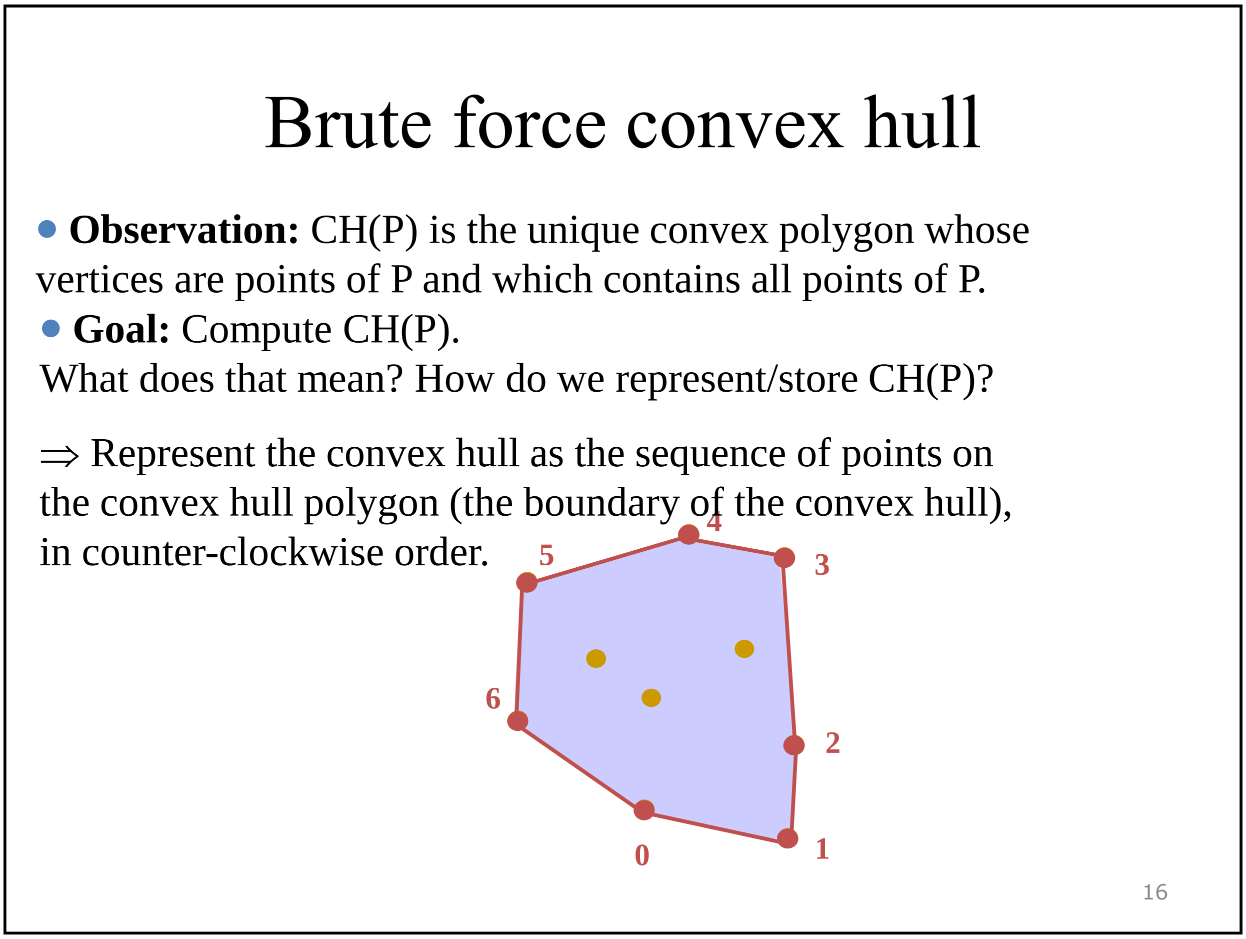
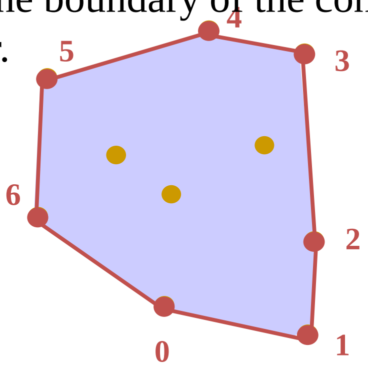


Brute force convex hull

- **Observation:** CH(P) is the unique convex polygon whose vertices are points of P and which contains all points of P.
- **Goal:** Compute CH(P).

What does that mean? How do we represent/store CH(P)?

⇒ Represent the convex hull as the sequence of points on the convex hull polygon (the boundary of the convex hull), in counter-clockwise order.



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Brute force convex hull

```
Algorithm Brute_Force_CH($P$):
/* CH(P) = Intersection of all half-planes that are defined by the directed line through
    ordered pairs of points in P and that have all remaining points of P on their left */
Input: Point set $P \subseteq \mathbf{R}^2$
Output: A list $L$ of vertices describing the CH($P$) in counter-clockwise order
$E := \emptyset$
for all $(p,q) \in P \times P$ with $p \neq q$   // ordered pair
    valid := true
    for all $r \in P$, $r \neq p$ and $r \neq q$
        if $r$ lies to the right of directed line through $p$ and $q$
            valid := false
    if valid then
        $E := E \cup \overrightarrow{pq}$
Construct from $E$ sorted list $L$ of vertices of CH($P$) in counter-clockwise order
```

- Runtime: $O(n^3)$, where $n = |P|$

Brute Force Strengths and Weaknesses

- Strengths
  - wide applicability
  - simplicity
  - yields reasonable algorithms for some important problems (e.g., matrix multiplication, sorting, searching, string matching)
- Weaknesses
  - rarely yields efficient algorithms
  - some brute-force algorithms are unacceptably slow
  - not as constructive as some other design techniques

Exhaustive Search

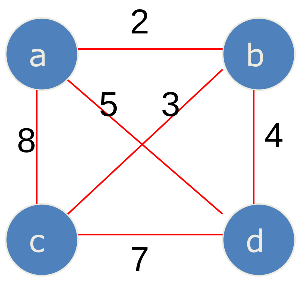
- A brute force solution to a problem involving search for an element with a special property, usually among combinatorial objects such as permutations, combinations, or subsets of a set.

Method:

- generate a list of all potential solutions to the problem in a systematic manner
- evaluate potential solutions one by one, disqualifying infeasible ones and, for an optimization problem, keeping track of the best one found so far
- when search ends, announce the solution(s) found

Example 1: Traveling Salesman Problem

- Given *n* cities with known distances between each pair, find the shortest tour that passes through all the cities exactly once before returning to the starting city
- Alternatively: Find shortest *Hamiltonian circuit* in a weighted connected graph
- Example:



TSP by Exhaustive Search

Tour	Cost
a→b→c→d→a	2+3+7+5 = 17
a→b→d→c→a	2+4+7+8 = 21
a→c→b→d→a	8+3+4+5 = 20
a→c→d→b→a	8+7+4+2 = 21
a→d→b→c→a	5+4+3+8 = 20
a→d→c→b→a	5+7+3+2 = 17

More tours?

Efficiency:

Example 2: Knapsack Problem

Given *n* items:

- weights: w_1 w_2 ... w_n
- values: v_1 v_2 ... v_n
- a knapsack of capacity W

Find most valuable subset of the items that fit into the Knapsack

Example: Knapsack capacity W=16

item	weight	value
1	2	$20
2	5	$30
3	10	$50
4	5	$10

Knapsack Problem by Exhaustive Search

Subset	Total weight	Total value
{1}	2	$20
{2}	5	$30
{3}	10	$50
{4}	5	$10
{1,2}	7	$50
{1,3}	12	$70
{1,4}	7	$30
{2,3}	15	$80
{2,4}	10	$40
{3,4}	15	$60
{1,2,3}	17	not feasible
{1,2,4}	12	$60
{1,3,4}	17	not feasible
{2,3,4}	20	not feasible
{1,2,3,4}	22	not feasible

Efficiency?

Example 3: The Assignment Problem

There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *i* to job *j* is C[*i*,*j*]. Find an assignment that minimizes the total cost.

	Job 1	Job 2	Job 3	Job 4
Person 1	9	2	7	8
Person 2	6	4	3	7
Person 3	5	8	1	8
Person 4	7	6	9	4

Algorithmic Plan:
- Generate all legitimate assignments,
- compute their costs, and
- select the cheapest one.

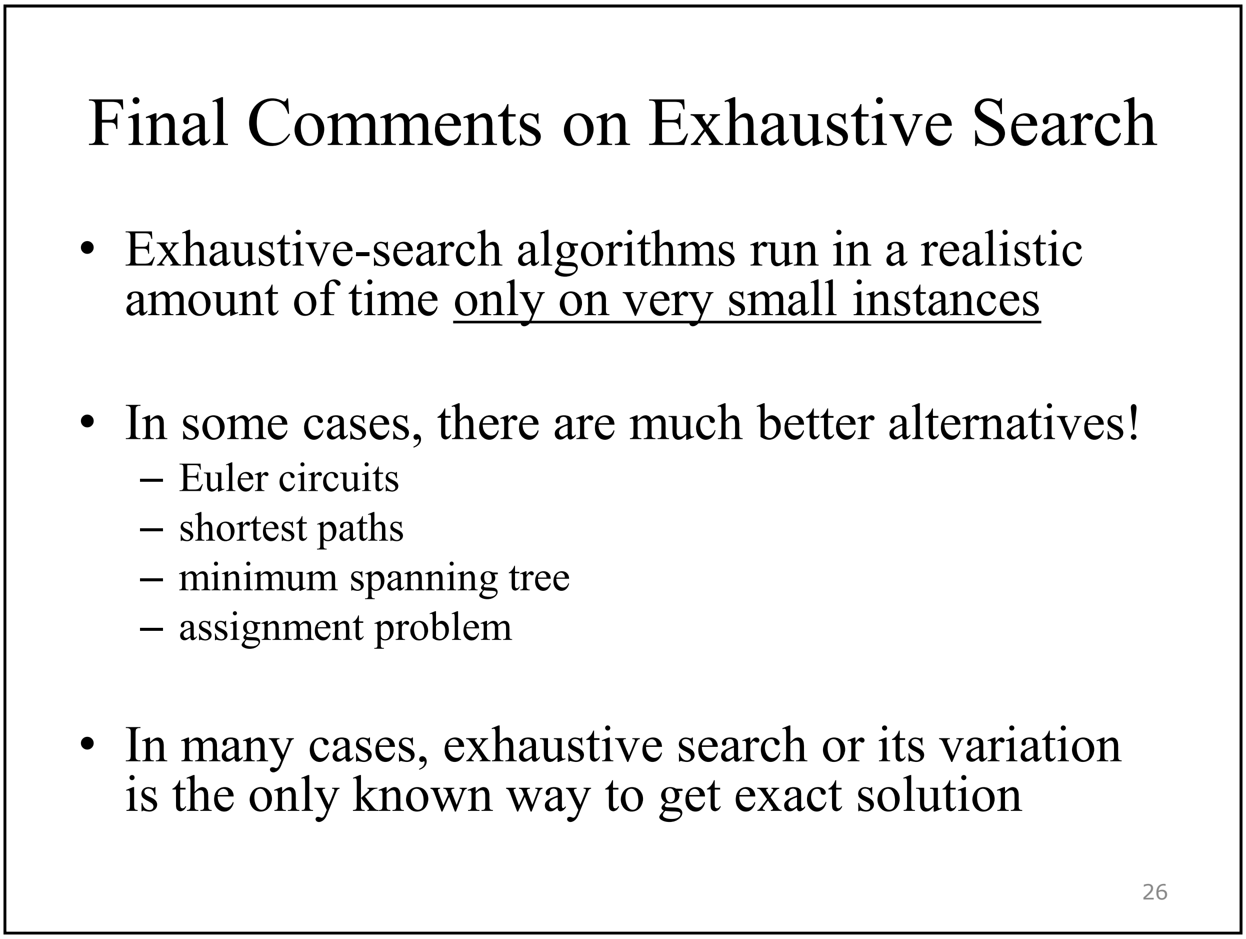
How many assignments are there?

Pose the problem as the one about a cost matrix.

Assignment Problem by Exhaustive Search

$$C = \begin{array}{cccc} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{array}$$

Assignment	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23
	etc.



Final Comments on Exhaustive Search

- Exhaustive-search algorithms run in a realistic amount of time only on very small instances

- In some cases, there are much better alternatives!
  - Euler circuits
  - shortest paths
  - minimum spanning tree
  - assignment problem

- In many cases, exhaustive search or its variation is the only known way to get exact solution

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Reading

Chapter 3

Anany Levitin, Introduction to the design and analysis of algorithms, 3rd Edition, Pearson, 2011.