

1. Given the matrices A and B of sizes  $m \times l$  and  $l \times n$  respectively.

- Write the pseudocode to compute the matrix  $C = A \times B$
- What is the complexity of the code that you wrote?

a)  $\text{for } i \leftarrow 0 \text{ to } m \text{ do}$   
      $\text{for } j \leftarrow 0 \text{ to } n \text{ do}$   
          $C[i, j] = A[i, 1] * B[1, j]$   
     return C

b) 1st Line  $m+1$   
     2nd Line  $m(n+1)$   
     3rd Line  $m(n)$   
     4th Line 1

$$\begin{aligned} T(n) &= m+1 + mh + m + mh + 1 \\ &= 2mh + 2m + 2 = 2(hh + h + 1) \\ &= mh + m + 1 \\ &O(mh) \end{aligned}$$

2. Consider the following code fragment,

```
x ← 1
for i ← 1 .. n step 3 do
  x ← x + 2
print x
```

$$x = 1 + \sum_{i=1}^{\lfloor \frac{n}{3} \rfloor} 2 = 1 + \lfloor \frac{n}{3} \rfloor \cdot 2$$

3. Consider the following code fragment,

```
x ← 5
i ← 1
While (2 i < N) do
  i ← i + 2;
  x ← x + 3;

print x
```

$$5 + \sum_{i=1}^{\frac{n}{2}} 2i + 3 = 5 + n^2 + n + \frac{3}{2}(3^n - 1)$$

4. Show that  $6n + 3n \log(n^5) = O(n \log n)$ . Find the appropriate values of C and  $n_0$ .

$$6n + 3n \log(n^5) \leq Cg(n) \quad : n \geq n_0$$

$$6n + 15n \log(n) \leq 21n \log n \quad n \geq 2$$

$$C = 21, n_0 = 2$$

5. Show that  $2n^3 - 10n^2 + 2 = O(n^3)$ . Find the appropriate values of C and  $n_0$ .

$$2n^3 - 10n^2 + 2 \leq 14n^3 \quad n \geq 1$$

$$C = 14 \quad n_0 = 1$$

6. Prove or disprove the statement,  $2^{n+2} = O(n^2)$ .

the statement is false

because:  $\forall n \geq 1 \rightarrow 2^{n+2}$  it will always be bigger than  $n^2$

$$2^n \cdot 2^2 \leq C n^2$$

$$\frac{2 \cdot 4}{n^2} \leq C$$

7. Prove that  $3^n = O(n!)$ . Find the appropriate values of C and  $n_0$ .

$$3^n \leq C(n)$$

$$\leq 5 (n \cdot n-1 \cdot \dots \cdot 2 \cdot 1) \quad n \geq 1$$

$$\leq 5 n! \quad n \geq 1$$

$$C = 5 \quad n_0 = 1$$

8. Compare the order of growth for  $3^{2n}$  and  $5^n$ .

$$3^{2n} \leq C 5^n \quad n \geq n_0$$

if we set  $C = 81$   $n_0 = 1$  then

$$3^{2n} = 81 (5^n) \quad n \geq 1$$