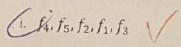
- 1. Choose the correct answer for the following questions:
- [/ points]

a- Consider the following five functions:

$$f_1 = 10^n, f_2 = n^{1/3}, f_3 = n^n, f_4 = \log(\log n), f_5 = 2^{\sqrt{\log n}}$$

Then, arrange the above functions in ascending order.



- ii.  $f_5, f_4, f_2, f_1, f_3$
- iii.  $f_4, f_5, f_1, f_2, f_3$
- iv.  $f_5, f_4, f_2, f_3, f_1$
- b- What is the average case time complexity of recursive insertion sort?
  - i. O(n)

  - iii. O(n2)
  - iv. O(log n)
- c- Which of the following sorting algorithm is in place?
  - (i. Recursive insertion sort
- ii. Merge sort
- iii. Radix sort
- iv. Counting sort
- d- What is the typical running time of a quick sort algorithm?
  - i. O(N)
  - ,ii. O(N log N)



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- iii. O(log N)
- vi. O(N2)
- e. Master's theorem is used for?
- (i. Solving recurrences.
- ii. Solving iterative relations
- iii. Analysing loops
- vi. Calculating the time complexity of any code
- f. Under what case of Master's theorem will the recurrence relation of merge sort fall?

a-2,6=2, ((n)=n

[6 points]

- vi. It cannot be solved using master's theorem.
- g. We can solve any recurrence by using Master's theorem.

i. true (ii. false)

2. Using the definition of  $\theta$ , find g(n),  $C_1$ ,  $C_2$ , and  $n_0$  in the following:

$$7n^6 - 3n^4 + 2n^3 + n^2 - 2 \in \theta(g(n))$$

 $\frac{2n^{4}-2n^{4}+2n^{2}+n^{2}-2}{2n^{4}-2n^{4}+2n^{2}+n^{2}-2} \leq 15n^{4}$   $C_{1}=15$   $C_{2}=15$ 

613 
$$\sqrt{n}$$
 $7n^{6} - 3n^{4} + 12n^{4} + n^{2} - 2 \ge 6n^{6}$ 
 $613 \sqrt{n}$ 
 $7n^{6} - 3n^{4} + 2n^{4} + n^{2} - 2 \ge 6n^{6}$ 
 $613 \sqrt{n}$ 
 $7n^{6} - 3n^{4} + 2n^{4} + n^{2} - 2 \ge 6n^{6}$ 
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 $7n^{6} - 3n^{4} + 2n^{4} + n^{2} - 2 \ge 6n^{6}$ 
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## roblem 2 [9 points]

a- Consider the following pseudo-code:

```
void SampleFct(int arr[], int n)
      if (n <= 1)
      SampleFct( arr, n-1 ); n-1
      int key = arr[n-1]; /
      int j = n-2;
       n-2 arr[j+1] = arr[j];
           j-; n-2
      arr[j+1] = key;
```

b- Which problem does this algorithm solve (What does it do)?

it will sort the array
ly comparing each array with arrain

Smaller at every recurrence until 1 sis two nozes

c- What is the recursive equation corresponding this pseudo-code.

$$T(n) = T(n+1) + 3n - 2$$

d- Prove the big-Oh performance of the SampleFct.

We Will stop warn

$$T(n) = T(1) + 3(n+(n-1)+1)-2$$

$$= 3(2n)+2-2^{n-1}$$

Problem 3 [9 points]

Consider the following recurrence relations. Solve each recurrence relation using recursive substitutions and find its asymptotic performance.

(a)

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$$T(n) = 4T(\frac{n}{5}) + 3n^{2}$$

$$T(n) = 4T(\frac{n}{5}) + 3n^{2}$$

$$T(n) = 4\left[4T(\frac{n}{25}) + 3\frac{n^{2}}{25}\right] + 3n^{2}$$

$$F(n) = 4\left[4T(\frac{n}{25}) + 3\frac{n^{2}}{25}\right] + 3n^{2}$$

$$F(n) = |4\left[4\frac{n}{25}\right] + |2\frac{n^{2}}{25}\right] + |2\frac{n^{2}}{25}| + |3\frac{n^{2}}{25}| + |2\frac{n^{2}}{25}| + |3\frac{n^{2}}{25}| +$$

$$T(n) = T(\frac{n}{3}) + 2n + 5$$

$$T(n) = T(\frac{n}{3}) + 2n + 5$$

$$T(n) = T(\frac{n}{3}) + 2n + 5$$

$$T(n) = \left[T(\frac{n}{3}) + 2\frac{n}{4} + 5\right] \xrightarrow{2n} + 2n + 5 + 5$$

$$= T(\frac{n}{3}) + 2\frac{n}{4} + 2\frac{n}{3} + 2n + 5 + 5$$

$$T(\frac{n}{3}) + 2\frac{n}{4} + 2\frac{n}{3} + 2n + 5 + 5$$

$$T(\frac{n}{3}) + \frac{2n}{3^{1/3}n - 1} + \frac{2n}{$$

7=1

Problem 4 [4 points]

Solve the following recurrence using the Master theorem by giving tight 0-notation bounds. Justify your answers.

(a)  $T(n) = 4T(n/2) + 3n \log(n)$   $\alpha = 9$ , b = 2,  $c = 3n \log(n)$  a = 9, b = 2,  $c = 3n \log(n)$  a = 9, b = 2,  $c = 3n \log(n)$  a = 9, b = 2,  $c = 3n \log(n)$ 

Problem 5 [15 points]

(a) Describe an in-place algorithm that takes as input an array of n integers and rearranges it so that all even integers come first and then all odd integers come next. You are allowed to use ONLY a constant amount of extra storage. [10 points]

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(b) What is the time complexity of your algorithm? You need to show the step count and Big\_Oh estimate. [5 points]