Computer Science Department
CCIS, King Saud University

CSC 311 — Midterm I. — March 09, 2021.

The Second Semester 2020/2021.

Duration: 90 Minutes — Points: 100

Q1:[40 points= 10+10+10+10]

(a) Using the definition of Ω, prove the following by finding c and n_0:
$3n^4 - 4n^2 \in \Omega(n^4)$.

(b) Using the definition of O, prove the following by finding c and n_0:
$\log(n!) \in O(n \log n)$.

(c) For two constants $a, b \geq 1, a \neq b$, do a^n and b^n have the same order of growth? why?

(d) Find a Θ estimate for the function $M(n)$, which is defined as follows:
$M(1) = 1, M(n) = M(\lfloor \frac{n}{2} \rfloor) + n^2$.

Q2:[30 points=6+12+12]
Consider the following algorithm, where A is an array of n integers and indexed from 1 to n. L and R are indexes of the array A. The first call to the algorithm will be $S(A[1..n])$.

```
Algorithm S(A[L..R])
if (L > R) then        — 1
   return 0;           — 1
else
   M:= 1;              — 1
   for i = 1 to R do   — R+1
      M:=M * A[R];     — R
   end
   return S(A[L..R − 1]) + M;   — T(n-1) + M
end
```

$T(n) = T(n-1) + \Theta(n)$

(a) What does this algorithm do? Give an example.

(b) Set up a recurrence relation for the basic operation count as a function of the input size n.

(c) What is the time complexity of this algorithm? Justify your answer.

Q3:[30 points=10+20]
A circular shift operation on an array moves each item to the next location and the last item is moved to the first location. For example a circular shift to the array $[1, 5, 9]$ would result in the array $[9, 1, 5]$. You are given an array of n distinct integers (where $n > 1$) and you are told that the array was initially sorted in an increasing order and then k circular shift operations were applied to the array $(0 < k < n)$.

(a) *Give the pseudocode of a brute force algorithm to find k.*

(b) *Give the pseudocode of an algorithm that finds k in $O(\log n)$ time.*

The Master Theorem:
Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be an asymptotically positive function, and let $M(n)$ be defined on the nonnegative integers by the recurrence:
$M(n) = aM(\frac{n}{b}) + f(n)$,
where we interpret $\frac{n}{b}$ to mean either $\lfloor \frac{n}{b} \rfloor$ or $\lceil \frac{n}{b} \rceil$. Then $M(n)$ can be bounded asymptotically as follows.

1. If $f(n) \in O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $M(n) \in \Theta(n^{\log_b a})$.
2. If $f(n) \in \Theta(n^{\log_b a} \log^k n)$, with $k \geq 0$, then $M(n) \in \Theta(n^{\log_b a} \log^{k+1} n)$.
3. If $f(n) \in \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$ AND $af(\frac{n}{b}) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $M(n) \in \Theta(f(n))$.