

Quiz 1

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Grade: [redacted]

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Question 1 | 2 / 2 Points

Use the formal definition of θ to prove that $2n^3 - 7n + 1 \in \theta(n^3)$. Provide appropriate c_1 , c_2 , and n_0 constants. Show all steps required.

$$c_1 n^3 \geq 2n^3 - 7n + 1 \geq c_2 n^3$$

prove big-O:

$$2n^3 - 7n + 1 \leq c_1 n^3 \quad \forall n \geq n_0$$

$$2 - \frac{7}{n^2} + \frac{1}{n^3} \leq c_1$$

ignore - term in (\leq)

$$2 + \frac{1}{n^3} \leq c_1$$

$$n_0 = 2, c_1 = 3 \rightarrow \forall n \geq 2$$

$$c_1 = 3, c_2 = 3 - \frac{7}{4} + \frac{1}{18}, n_0 = 3$$

prove Ω :

$$2n^3 - 7n + 1 \geq c_2 n^3$$

$$2 - \frac{7}{n^2} + \frac{1}{n^3} \geq c_2$$

$$n_0 = 3, c_2 = 3 - \frac{7}{4} + \frac{1}{18}$$

Question 2 | 1 / 1 Point

Use limits of rational functions to prove that $(n) = 7n^2 + 3n \log n + 5n + 100 \in \theta(n^2)$. Show all steps required.

$$\lim_{n \rightarrow \infty} \frac{7n^2 + 3n \log n + 5n + 100}{n^2} = \lim_{n \rightarrow \infty} \frac{7n^2}{n^2} = 7$$

constant

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Question 3 | 1.65/2 Point

For each blank, indicate whether A_i is in O or/and Ω of B_i . More than one space per row can be valid. No explanation is required.

Note: $\ln n = \log_e n$.

A	B	$A = O(B)$	$A = \Omega(B)$
n^3	n^2	No	yes ✓
$0.0000001 n^3$	$5000n(n+1)$	No	yes ✓
$\log^7 n$	$n^{0.7}$ $n^{0.7} \rightarrow \sqrt[n]{n}$	yes ✓	yes -0.1
$\ln n$	$\log n$	yes ✓	yes ✓
$\left(\frac{12}{13}\right)^n$	$\left(\frac{13}{12}\right)^n$	No ✗	yes -0.25
$\sqrt{n} \rightarrow 0.5$	$\log^2 n$	yes -0.1	yes ✓
$\log(n!)$	$\log(n^n)$	yes ✓	No