Frequency and Period

Frequency and period are the inverse of each other.

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$

Units of period and frequency

Unit	*Equivalent*	*Unit*	*Equivalent*
Seconds (s)	1 s	Hertz (Hz)	1 Hz
Milliseconds (ms)	10^{-3} s	Kilohertz (kHz)	10^{3} Hz
Microseconds (μs)	10^{-6} s	Megahertz (MHz)	10^{6} Hz
Nanoseconds (ns)	10^{-9} s	Gigahertz (GHz)	10^{9} Hz
Picoseconds (ps)	10^{-12} s	Terahertz (THz)	10^{12} Hz

Examples

A digital signal has 8 levels. How many bits are needed per level?

We calculate the number of bits from the formula

$$\text{Number of bits per level} = \log_2 8 = 3$$

Each signal level is represented by 3 bits.

A digital signal has 9 levels. How many bits are needed per level?

Each signal level is represented by 3.17 bits.
The number of bits sent per level needs to be an integer as well as a power of 2.
Hence, 4 bits can represent one level.

Examples

The power we use at home has a frequency of 60 Hz. What is the period of this sine wave ?

$$T = \frac{1}{f} = \frac{1}{60} = 0.0166\ \text{s} = 0.0166 \times 10^3\ \text{ms} = 16.6\ \text{ms}$$

The period of a signal is 100 ms. What is its frequency in kilohertz?

$$100\ \text{ms} = 100 \times 10^{-3}\ \text{s} = 10^{-1}\ \text{s}$$

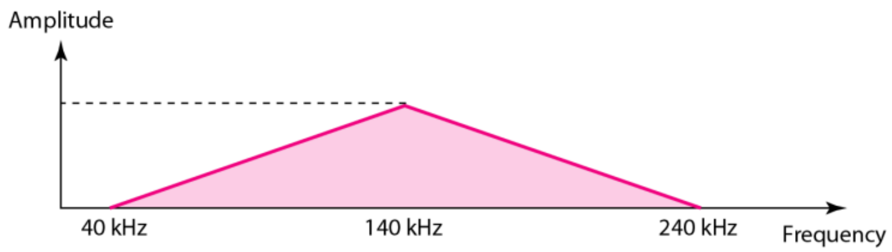
$$f = \frac{1}{T} = \frac{1}{10^{-1}}\ \text{Hz} = 10\ \text{Hz} = 10 \times 10^{-3}\ \text{kHz} = 10^{-2}\ \text{kHz}$$

Example

A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.

Solution

The lowest frequency must be at 40 kHz and the highest at 240 kHz.



Examples

Assume we need to download files at a rate of 100 pages per minute. A page is an average of 24 lines with 80 characters in each line where one character requires 8 bits. What is the required bit rate of the channel?

$$\boxed{100 \times 24 \times 80 \times 8 = 1{,}636{,}000 \text{ bps} = 1.636 \text{ Mbps}}$$

A digitized voice channel is made by digitizing a 4-kHz bandwidth analog voice signal. We need to sample the signal at twice the highest frequency (two samples per hertz). Assume that each sample requires 8 bits.
What is the required bit rate?

$$\boxed{2 \times 4000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}}$$

HDTV uses digital signals to broadcast high quality video signals. There are 1920 by 1080 pixels per screen, and the screen is renewed 30 times per second. Also, 24 bits represents one color pixel.

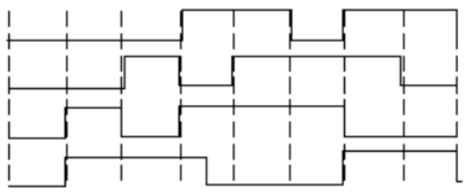
What is the bit rate for high-definition TV (HDTV)?

$$\boxed{1920 \times 1080 \times 30 \times 24 = 1{,}492{,}992{,}000 \text{ or } 1.5 \text{ Gbps}}$$

Code Division Multiplexing

A = (−1 −1 −1 +1 +1 −1 +1 +1)
B = (−1 −1 +1 −1 +1 +1 +1 −1)
C = (−1 +1 −1 +1 +1 +1 −1 −1)
D = (−1 +1 −1 −1 −1 −1 +1 −1)

(a)



(b)

(a) Chip sequences for four stations.
(b) Signals the sequences represent.

Example

Suppose a signal travels through a transmission medium and its power is reduced to one-half.

This means that P_2 is $(1/2)P_1$.

In this case, the attenuation (loss of power) can be calculated as

$$10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{0.5 P_1}{P_1} = 10 \log_{10} 0.5 = 10(-0.3) = -3 \text{ dB}$$

A loss of 3 dB (–3 dB) is equivalent to losing one-half the power.

Example

A signal travels through an amplifier, and its power is increased 10 times.

This means that $P_2 = 10P_1$.

What is the amplification (gain of power)?

$$\boxed{10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{10P_1}{P_1}}$$

$$\boxed{= 10 \log_{10} 10 = 10(1) = 10 \text{ dB}}$$

Example

The power of a signal is 10 mW and the power of the noise is 1 μW; what are the values of SNR and SNR_{dB} ?

Solution

The values of SNR and SNRdB can be calculated as follows:

$$\text{SNR} = \frac{10{,}000\ \mu\text{W}}{1\ \text{mW}} = 10{,}000$$

$$\text{SNR}_{\text{dB}} = 10 \log_{10} 10{,}000 = 10 \log_{10} 10^4 = 40$$

Example

The values of SNR and SNRdB for a noiseless channel are

$$\text{SNR} = \frac{\text{signal power}}{0} = \infty$$

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \infty = \infty$$

We can never achieve this ratio in real life; it is an ideal.

A very important consideration in data communications is how fast we can send data, in bits per second, over a channel.

Data rate depends on three factors:

1. The bandwidth available
2. The level of the signals we use
3. The quality of the channel (the level of noise)

Increasing the levels of a signal may reduce the reliability of the system.

Nyquist Theorem

For noiseless channel,

$$BitRate = 2 \times Bandwith \times \log_2 Levels$$

In baseband transmission, we said the bit rate is 2 times the bandwidth if we use only the first harmonic in the worst case.

However, the Nyquist formula is more general than what we derived intuitively; it can be applied to baseband transmission and modulation.

Also, it can be applied when we have two or more levels of signals.

Examples

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. What is the maximum bit rate?

$$\text{BitRate} = 2 \times 3000 \times \log_2 2 = 6000 \text{ bps}$$

Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). What is the maximum bit rate?

$$\text{BitRate} = 2 \times 3000 \times \log_2 4 = 12{,}000 \text{ bps}$$

Example

We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

Solution

We can use the Nyquist formula as

$$265{,}000 = 2 \times 20{,}000 \times \log_2 L$$
$$\log_2 L = 6.625 \qquad L = 2^{6.625} = 98.7 \text{ levels}$$

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate.

If we have 128 levels, the bit rate is 280 kbps.

If we have 64 levels, the bit rate is 240 kbps.

Shannon Capacity

In reality, we can not have a noisless channel

For noisy channel,

Capacity = Bandwith x $\log_2(1+SNR)$

The Shannon capacity gives us the upper limit;
the Nyquist formula tells us how many signal levels we need.

Example

Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero.

In other words, the noise is so strong that the signal is faint. What is the channel capacity?

Solution

$$C = B \log_2 (1 + \mathrm{SNR}) = B \log_2 (1 + 0) = B \log_2 1 = B \times 0 = 0$$

This means that the capacity of this channel is zero regardless of the bandwidth.

In other words, we cannot receive any data through this channel.

Example

Let's calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000. The signal-to-noise ratio is usually 3162.

What is the channel capacity?

Solution

$$C = B \log_2 (1 + \text{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \log_2 3163$$
$$= 3000 \times 11.62 = 34{,}860 \text{ bps}$$

This means that the highest bit rate for a telephone line is 34.860 kbps.

If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

Example

The signal-to-noise ratio is often given in decibels.

Assume that SNR_{dB} = 36 and the channel bandwidth is 2 MHz.

What is the theoretical channel capacity?

Solution

$$\text{SNR}_{\text{dB}} = 10 \log_{10} \text{SNR} \rightarrow \text{SNR} = 10^{\text{SNR}_{\text{dB}}/10} \rightarrow \text{SNR} = 10^{3.6} = 3981$$
$$C = B \log_2 (1 + \text{SNR}) = 2 \times 10^6 \times \log_2 3982 = 24 \text{ Mbps}$$

Example

For practical purposes, when the SNR is very high, we can assume that SNR + 1 is almost the same as SNR.
In these cases, the theoretical channel capacity can be simplifiedto

$$C = B \times \frac{SNR_{dB}}{3}$$

For example, we can calculate the theoretical capacity of the previous example as

$$C = 2\ MHz \times \frac{36}{3} = 24\ Mbps$$

Example

We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63.
What are the appropriate bit rate and signal level?

Solution
First, we use the Shannon formula to find the upper limit.

$$C = B \log_2 (1 + SNR) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 64 = 6\ Mbps$$

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example.
Then we use the Nyquist formula to find the number of signal levels.

$$4\ Mbps = 2 \times 1\ MHz \times \log_2 L \rightarrow L = 4$$

2-6 PERFORMANCE

One important issue in networking is the performance of the network—how good is it?

In networking, we use the term bandwidth in two contexts

The first, bandwidth in hertz, refers to the range of frequencies in a composite signal or the range of frequencies that a channel can pass.

The second, bandwidth in bits per second, refers to the speed of bit transmission in a channel or link.

Message Latency

- Latency is the delay to send a message over a link
  - Transmission delay: time to put M-bit message "on the wire"

    T-delay = M (bits) / Rate (bits/sec) = M/R seconds

  - Propagation delay: time for bits to propagate across the wire

    P-delay = Length / speed of signals = D seconds

  - Combining the two terms we have: Latency = M/R + D

Metric Units

- The main prefixes we use:

Prefix	Exp.	prefix	exp.
K(ilo)	10^3	m(illi)	10^{-3}
M(ega)	10^6	μ(micro)	10^{-6}
G(iga)	10^9	n(ano)	10^{-9}

- Use powers of 10 for rates, 2 for storage
  - 1 Mbps = 1,000,000 bps, 1 KB = 1024 bytes
- "B" is for bytes, "b" is for bits

Examples

The bandwidth of a subscriber line is 4 kHz for voice or data.
The bandwidth of this line for data transmission can be up to 56,000 bps using a sophisticated modem to change the digital signal to analog.

If the telephone company improves the quality of the line and increases the bandwidth to 8 kHz, we can send 112,000 bps.

Example

A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?

Solution

We can calculate the throughput as

$$\text{Throughput} = \frac{12{,}000 \times 10{,}000}{60} = 2 \text{ Mbps}$$

The throughput is almost one-fifth of the bandwidth in this case.

Example

What is the propagation time if the distance between the two points is 12,000 km? Assume the propagation speed to be 2.4 × 108 m/s in cable.

Solution

We can calculate the propagation time as

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

The example shows that a bit can go over the Atlantic Ocean in only 50 ms if there is a direct cable between the source and the destination.

Example

What are the propagation time and the transmission time for a 2.5-kbyte message if the bandwidth of the network is 1 Gbps? Assume that the distance is 12,000 km and that light travels at 2.4 × 108 m/s.

Solution

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{2500 \times 8}{10^9} = 0.020 \text{ ms}$$

Note that in this case, because the message is short and the bandwidth is high, the dominant factor is the propagation time, not the transmission time.
The transmission time can be ignored.

Example

What are the propagation time and the transmission time for a 5-Mbyte message if the bandwidth of the network is 1 Mbps? Assume that the distance is 12,000 km and that light travels at 2.4×10^8 m/s.

Solution

$$\text{Propagation time} = \frac{12{,}000 \times 1000}{2.4 \times 10^8} = 50 \text{ ms}$$

$$\text{Transmission time} = \frac{5{,}000{,}000 \times 8}{10^6} = 40 \text{ s}$$

Note that in this case, because the message is very long and the bandwidth is not very high, the dominant factor is the transmission time, not the propagation time.
The propagation time can be ignored.