...1: Identify regular and non-regular languages

Question 1.1 [2.5 pts]

Answer with TRUE or FALSE next to each of the following statements:

Statement	True	False	
1. There are some languages that can be designed by finite automata but not by regular expression.		✓	✓
2. If L is a regular language and F is a finite language, then $L \cup F$ is a regular language.	✓		✓
3. Define $EVEN(w)$, for a finite string w, to be the string consisting of the symbols of w in even-numbered positions. For example, $EVEN(1011010) = 011$. If L is a regular language, then $\{EVEN(w) : w \in L\}$ must be regular.		✓	T
4. If L is a regular language, then $\{ww^R : w \in L\}$ must be a regular language.		✓	✓
5. If $L_1 \cup L_2$ is a non-regular language, then both L_1, L_2 must be non-regular languages.	✓		F

✗ $\boxed{-0.5}$

Question 1.2 [2.5 pts]

Select the correct answer:

d 1. Which of the following languages are not regular:

A. $L = \{(01)^n 0^k \mid n > k, k \geq 0\}$ $(01)^* 0^*$

B. $L = \{c^n b^k a^{n+k} \mid n \geq 0, k \geq 0\}$ ✓

C. $L = \{0^n 1^k \mid n \neq k\}$ ✓

a) A and B only

b) B and C only ✗ $\boxed{-0.5}$

c) A and C only

d) A, B and C

c 2. Which of the following languages is regular:

a) $L = \{a^i b^i \mid i \geq 0\}$ ✗

b) $L = \{a^i b^i \mid i \geq 1\}$

c) $L = \{a^i b^i \mid 0 < i < 5\}$ ✗

d) None of the above. ✓ ✗ $\boxed{-0.5}$

c 3. The reverse of $(0 + 1)^*$ will be:

a) $\emptyset$ ✓

b) ε

c) $(0 + 1)^*$

d) $(0 + 1)$ ✗ $\boxed{-0.5}$

4. $L = \{wcx : w, x \in \{a, b\}^*$ and the number of a's in w is equal to the number of b's in $x\}$ is a non-regular language. For example, $w = \underline{ababab c bbb} \in L$. We use pumping lemma to prove that the language L is non-regular. Fill in blanks to complete the proof.

A. Pick a string $w \in L$ and length $|w| \geq m$, such that m is the critical length. $\overbrace{a^{m}}^{m} c\, \overbrace{b^{m}}^{m}$

$w =$ ....$a^m c\, b^m$.................. $= xyz, |xy| \leq m, |y| \geq 1$. $\underbrace{a, a}_{x}, \underbrace{a}_{y}, \ldots \underbrace{c\ b, b, b, \ldots}_{z}$

B. Y=..a^k.............................., $1 \leq k \leq m$.

C. From pumping lemma, $xy^iz \in L, i = 0,1,2, \ldots,$ we choose $i =$ 2.............Thus the string after pumping is........................$\in L$, which is a contrdiction.

$a^{m+k} c\, b^m \notin L$ $\overbrace{a, a, a, \ldots}^{m+k}$ x y z

Part 2: Produce computing-based solutions using regular expressions, and context free grammar

Question 2: [7 pts]

Question 2.1 Construct a context-free grammar that generates $L = \{w \in \{a,c\}^* \mid w \text{ contains at least 3 } c\text{'s}\}$

$S \to AcAcAcA$
$A \to aA \mid cA \mid \lambda$

Question 2.2 Construct a context-free grammar that generates $L = \{0^n 1^n 0^m 1^m \mid n, m \geq 0\}$ and $\Sigma = \{0,1\}$

$S \to 0S1A \mid \lambda$
$A \to 0A1 \mid \lambda$

Question 2.3 Which of the following context-free grammar productions generates words of balanced brackets. An example for the generated string is (())().

A. $P=\{S \to \lambda, S \to TS, T \to (T)\}$

B. $P=\{S \to \lambda, S \to SS, S \to ()\}$

C. $P=\{S \to \lambda, S \to (T)S, T \to ()\}$

$(T)S \to (())S \to (())$

D. $P=\{S \to \lambda, S \to (S)S\}$

Question 2.4 Construct regular expressions representing the following languages. [4 pts]

(1) The language over the alphabet { 0,1 } that doesn't contain the substring 110

$(0+10)^* 1^*$

(2) The language { $w \in \Sigma^*$ | w ends with a double letter } over the alphabet { a, b }. A double letter over this alphabet is aa or bb.

$(a+b)^* (aa+bb)$

(3) $L = \{x \in \{a,b\}^* \mid x \text{ ends with } a \text{ and does not contain the substring } bb\}$

$(b+\lambda)(a+ab)^* a$

(4) Find a regular expression corresponding to the language of all strings over the alphabet { a, b } that do not end with ab.

A. $(a+b)^*(a+bb)$ ✓

B. $(ab^* + bb)$

C. $(a+ab)^*(a+bb)$

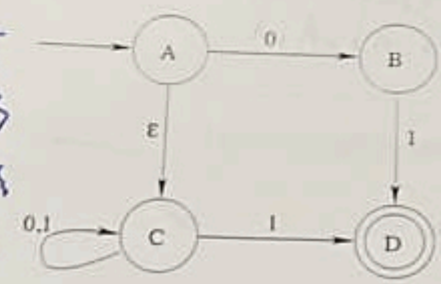
D. None of the above

Part 3: Design different machine models (DFA, NFA, PDA, TM)

Question 3.1: [5 pts]

Consider the following NFA.

State	0	1
{A, C}	{B, C} ✓	{D, C}
{B, C}	{C}	{D, C}
{D, C}	{C}	{D, C}
{C}	{C}	{D, C}



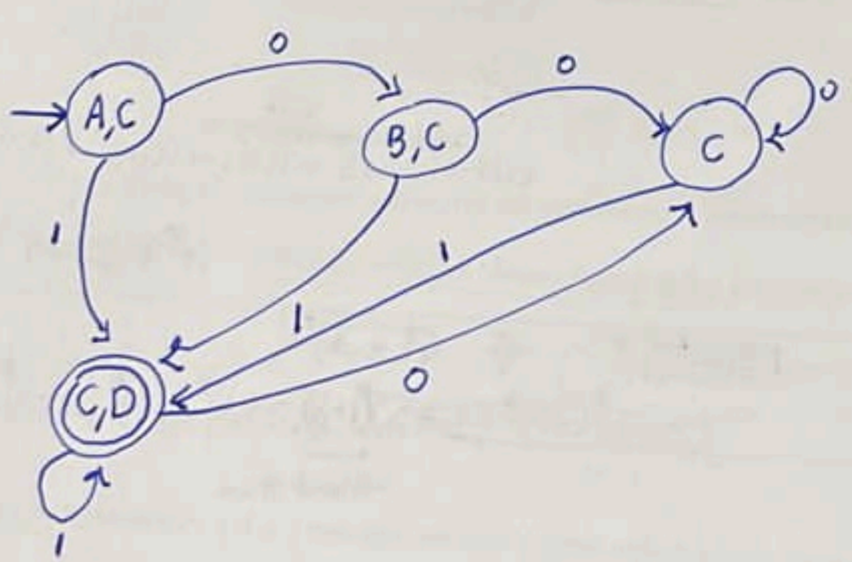
$\delta(A, \lambda) = \{A, C\}$

$\delta(B, \lambda) = \{B\}$

$\delta(C, \lambda) = \{C\}$

$\delta(D, \lambda) = \{D\}$

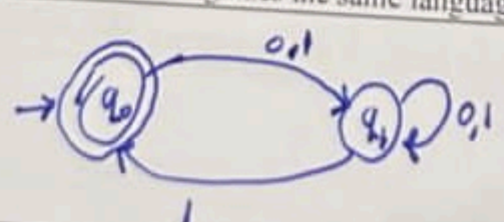
1. Convert this NFA into an equivalent DFA. **Your answer should be the state diagram of a DFA . [3 pts]**



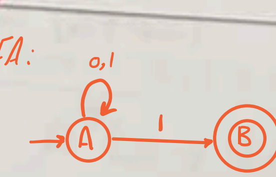
2. What is the language recognize by this DFA. Express your answer as a regular expression. [1 pt]

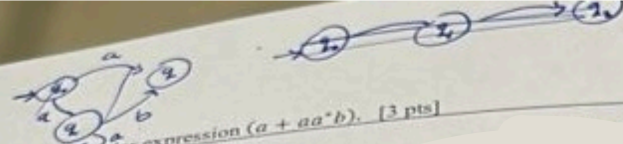
$(01 + (0+1)^*1)$

3.Construct an NFA with two states that recognizes the same language. [1 pt]

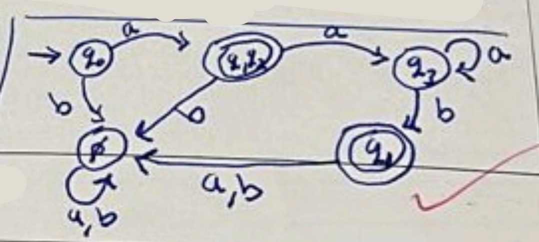


NFA:





Question 3.2: Construct a DFA that recognizes the regular expression $(a + aa^*b)$. [3 pts]



Part 4: Evaluate the language accepted by a machine, a regular expression, and a context free grammar

Question 4.1: [2 pts]
Describe the language generated by the following Grammar productions.

Grammar	Corresponding L(G)
(1) $S \rightarrow ABC$ $A \rightarrow 0A1 \mid \varepsilon$ $B \rightarrow 1B \mid 1$ $C \rightarrow 1C0 \mid \varepsilon$ $0^n A 1^n \; 1^m \; 1^n 0^n$	$L = \{0^n 1^n 1^z 1^n 0^n \mid n, n \geq 0 \text{ and } m \geq 1\}$
(2) $S \rightarrow 0Y1 \mid 1Y0$ $Y \rightarrow 0Y \mid 1Y \mid \varepsilon$ languge that statt and end with deffrent alpha bet	L = {~~start with 0~~ (every string start with 0 and ends with 1) or (every string start with 1 and ends with zero)

Question 4.3 : [3 pts]

aab

1. Is the grammar below ambiguous?
$S \rightarrow aS \mid aSbS \mid c$

A. Yes

B. No

2. Is the grammar below ambiguous?

$S \to T \mid U$

$T \to aTbT \mid c$

$U \to aS \mid aTbU$

A. Yes

B. No

3. Are CFGs provided in (1) and (2) equivalent?

A. Yes

B. No

End of Exam