

Tutorial: Context-free languages and grammar

- 1) Give CFG's for the language:
Determining if a string is an integer in the following format: An optional minus sign followed by at least one digit

Solution:

$$\begin{aligned} I &\rightarrow SN \\ S &\rightarrow - | \lambda \\ N &\rightarrow DN | D \\ D &\rightarrow 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 \end{aligned}$$

- 2) Give CFG for the language: strings in the following format: An optional minus sign followed by at least one digit or an optional minus sign followed by any number of digits, a decimal point and at least one digit.

Solution:

$$\begin{aligned} R &\rightarrow SN_1 | SNo \cdot N_1 \\ S &\rightarrow - | \lambda \\ No &\rightarrow DNo | \lambda \\ N_1 &\rightarrow DN_1 \\ D &\rightarrow 0 | 1 | 2 | \dots | 9 \end{aligned}$$

- 3) Give CFG's for the following languages:
- 3.1) The language of strings that contain the substring 001:

$$\begin{aligned} S &\rightarrow T001T \\ T &\rightarrow 0T | 1T | \lambda \end{aligned}$$

- 3.2) The language of strings that start and end with the same symbol:

$$\begin{aligned} S &\rightarrow 0T0 | 1T1 | 0 | 1 \\ T &\rightarrow 0T | 1T | \lambda \end{aligned}$$

4) Give a CFG for the language = $\{1^i \# 1^j \# 1^{i+j}\}$
 $\Sigma = \{1, \#\}$

Solution:
 $1^i \# 1^j \# 1^{i+j} = 1^i \# 1^j \# \underbrace{1^i 1^j}_S$

$S \rightarrow 1S1 \mid \#T$

$T \rightarrow 1T1 \mid \#$

5) Give CFG's for the following languages:
 $\Sigma = \{0, 1\}$

a) $A_1 = \{0^n 1 0^n \mid n \geq 0\}$

Solution: $S \rightarrow 0S0 \mid 1$

b) ~~$A_2 = \{w \mid ww^R\}$~~

Solution: $S \rightarrow 0S0 \mid 1S1 \mid \lambda$

c) $A_3 = \{0^n 1 2^n \mid n \geq 0\}$

Solution:
 $S \rightarrow 0S11 \mid \lambda$

d) $A_4 = \{0^i 1^j \mid i \leq j\}$

Solution:

$\begin{cases} S \rightarrow 0S1 \mid T \\ T \rightarrow T1 \mid \lambda \end{cases}$

or
 $\begin{cases} S \rightarrow ZS1 \mid \lambda \\ Z \rightarrow 0 \mid \lambda \end{cases}$