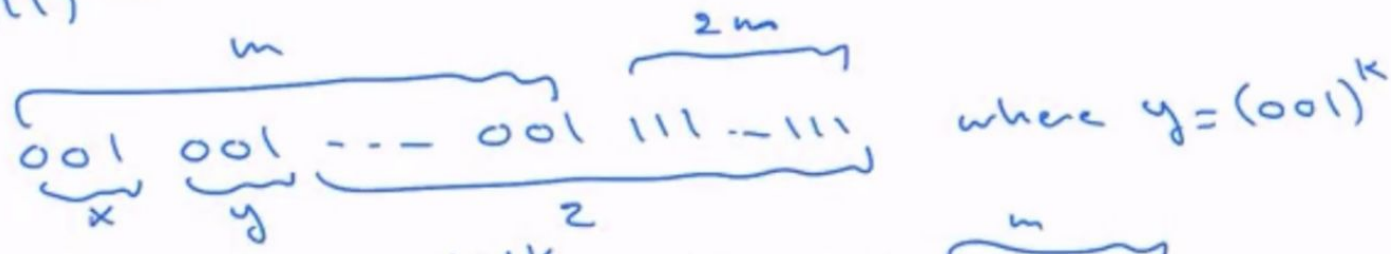
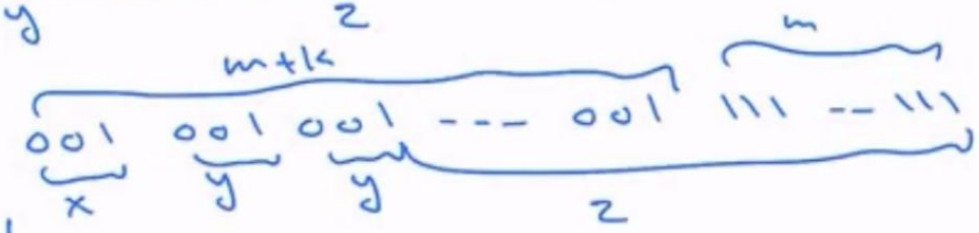
Part1: Identify regular and non-regular languages (K1)

Exercise 1.1 [2.5 pts]

Prove that the following language is non-regular:

$$L = \{(001)^n(111)^{2n}\}$$

1. Assume that L is regular lang. and let m be the critical length
2. let $w = (001)^m (111)^{2m}$
3. $w = xyz$:- 

4. let $i=2 \Rightarrow w' = xy^2z$ 

$w' = (001)^{m+1k} (111)^{2m} \notin L$
5. contradiction ∴ L is not regular

Exercise 2.1 [3 pts]

Construct context free grammars to accept the following languages:

1. $L(G) = \{w \mid w \text{ starts and ends witth the same symbol}\},\ \Sigma = \{0,1\}$ [1 pt]

2. $L(G) = \{w \mid |w| \text{ is odd}\},\ \Sigma = \{0,1\}$ [1 pt]

3. $L(G) = \{a^n b^m c^m d^{2n} \mid n \geq 0, m > 0\},\ \Sigma = \{a,b,c,d\}$ [1 pt]

Exercise 2.2 [3 pts]

Construct regular expressions representing the following languages:

1. The language over the alphabet $\{\$, \%, \#\}$, in which for every string w it holds that the number of $\$$'s in w is a multiple of 5. [1 pt]

2. The language over the alphabet $\{0,1\}$, consisting of even number of 0's followed by odd number of 1's. [1 pt]

3. The language over the alphabet $\{\&, @\}$, consisting of strings of $\&$'s and $@$'s of any length including the null string. [1 pt]

Exercise 2.2 [3 pts]

Construct regular expressions representing the following languages:

1. The language over the alphabet $\{\$, \%, \#\}$, in which for every string w it holds that the number of $\$$'s in w is a multiple of 5. [1 pt]

$(\%+\#)^*(\$(\%+\#)^*\$(\%+\#)^*\$(\%+\#)^*\$(\%+\#)^*\$(\%+\#)^*)^*$ OR

$(\%+\#)^*((\$(\%+\#)^5)^*$

2. The language over the alphabet $\{0,1\}$, consisting of even number of 0's followed by odd number of 1's. [1 pt]

$2x \quad 2x+1$

$(00)^*(11)^*1$

3. The language over the alphabet $\{\&, @\}$, consisting of strings of &'s and @'s of any length including the null string. [1 pt]

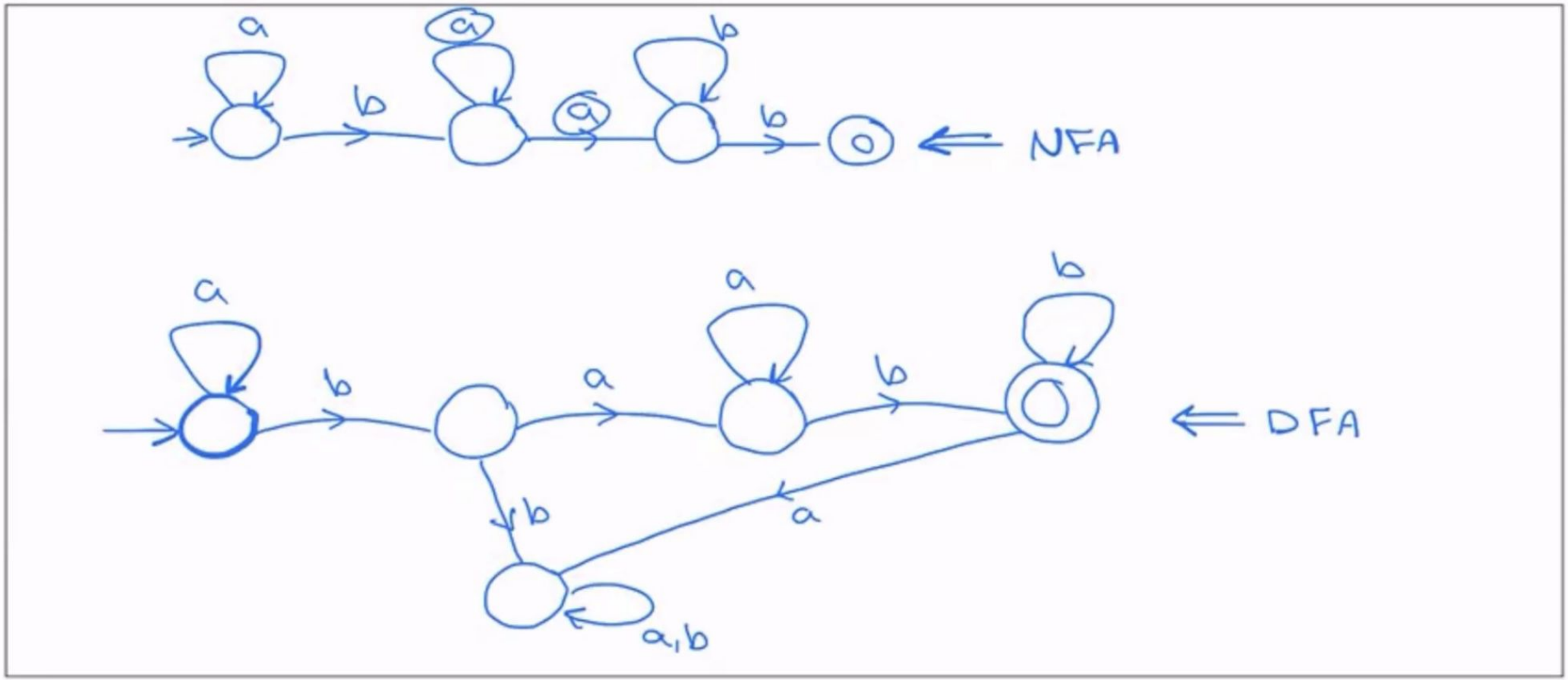
Σ^*

$(\&+@)^*$

Exercise 3.1 [3 pts]

Construct a **DFA** for the following regular expression:

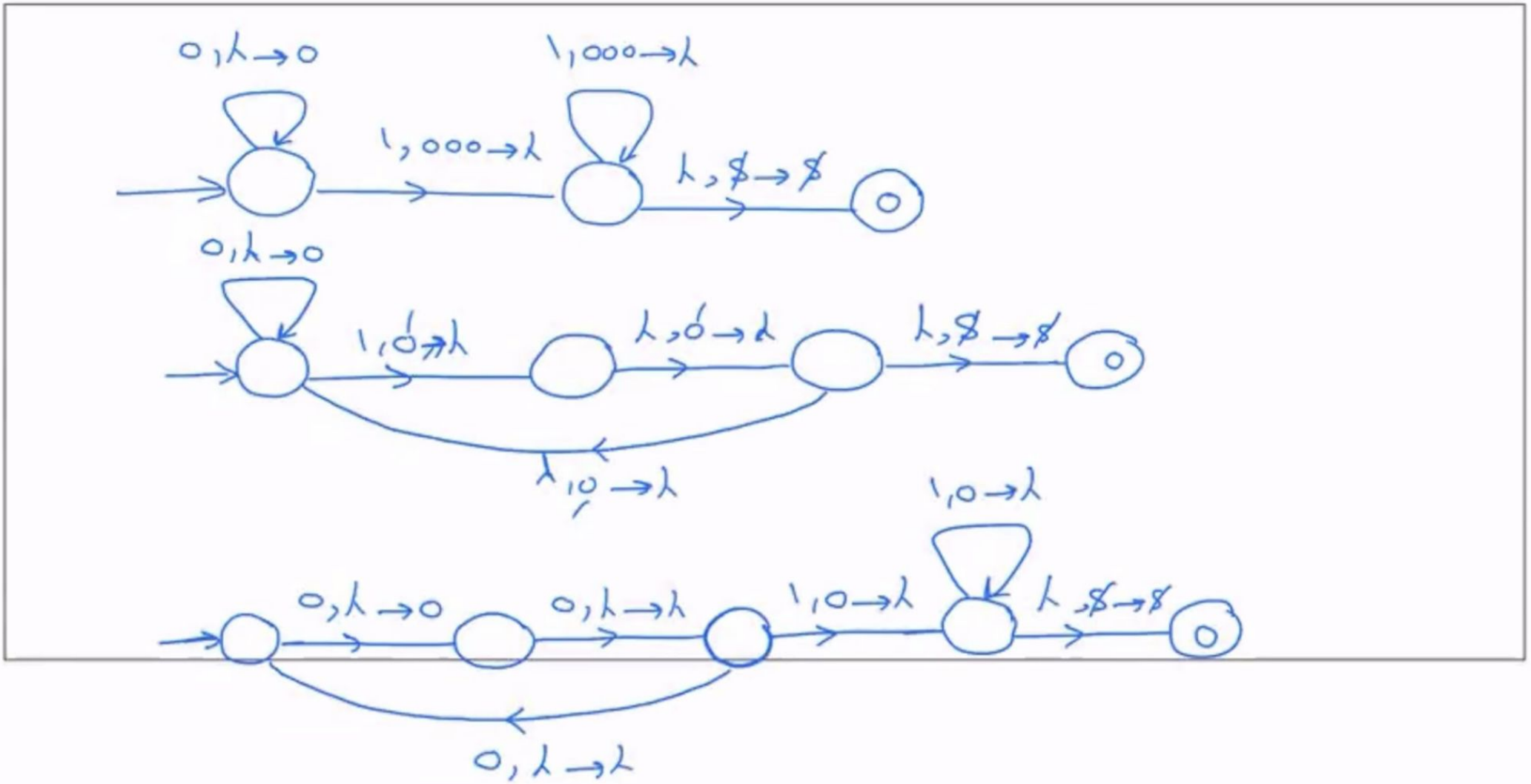
$$a^*ba^*ab^*b$$



Exercise 3.2 [3 pts]

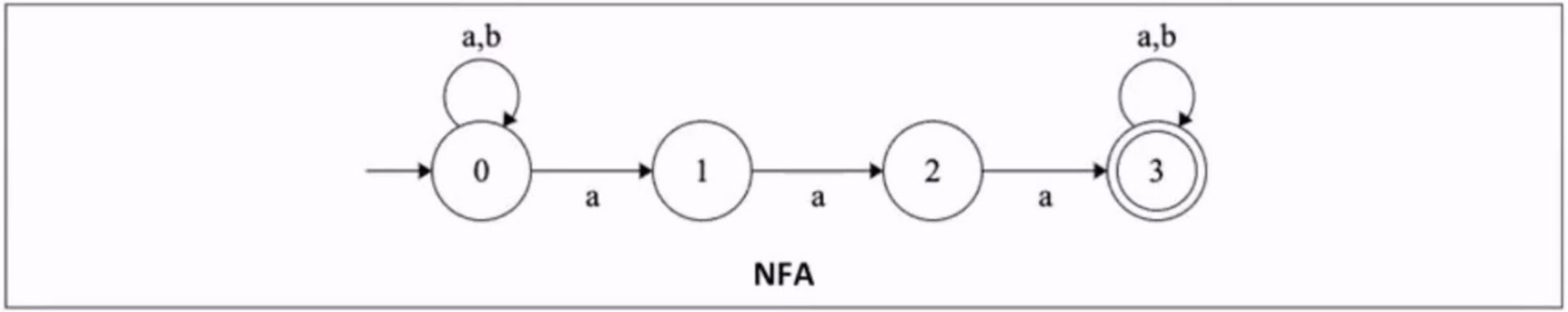
Construct a push down automata over $\{0,1\}$ that accepts the following language:

$$L = \{0^{3n}1^n \mid n > 0\}$$

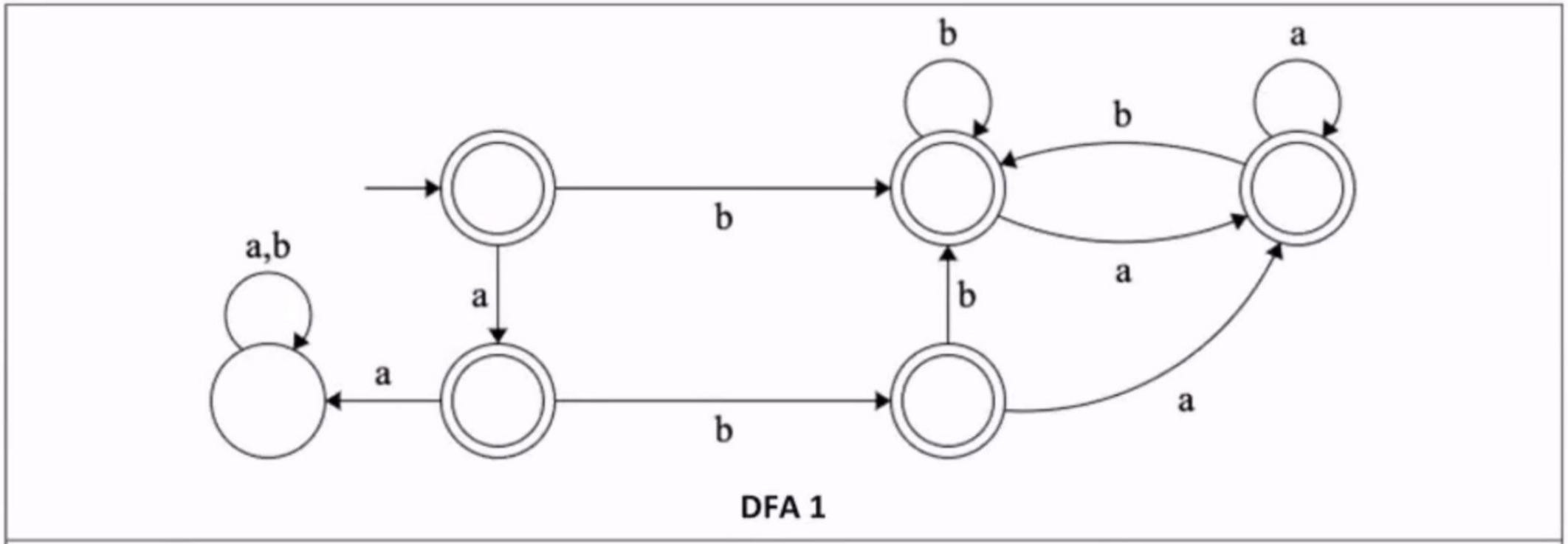


Exercise 3.3 [2 pts]

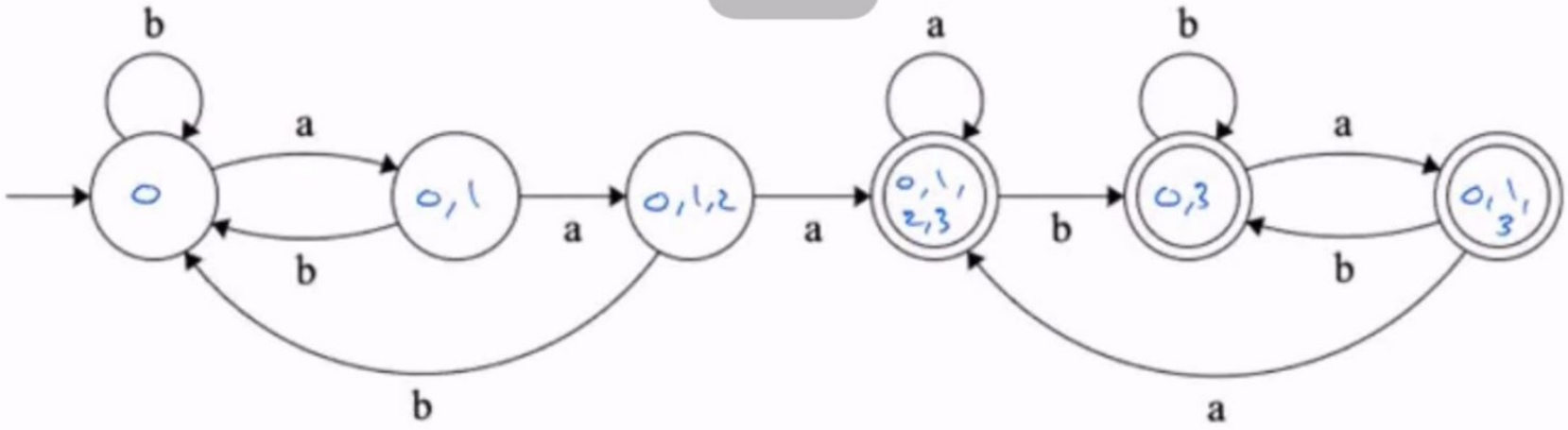
Consider the NFA and the two DFA (DFA 1 and DFA 2) represented below:



NFA



DFA 1



DFA 2

1. Which DFA (1 or 2) is equivalent to the above NFA? [1 pt]

2. Label the states of the considered DFA according to the conversion algorithm (on the provided figure). [1 pt]

	a	b
{0}	{0,1}	{0}
{0,1}	{0,1,2}	{0}
{0,1,2}	{0,1,2,3}	{0}
{0,1,2,3}	{0,1,2,3}	{0,3}
{0,3}	{0,1,3}	{0,3}
{0,1,3}	{0,1,2,3}	{0,3}

$\delta^*(0,\lambda) = \{0\}$

$\delta^*(1,\lambda) = \{1\}$

$\delta^*(2,\lambda) = \{2\}$

$\delta^*(3,\lambda) = \{3\}$

Exercise 4.1 [3 pts]

Which language is generated by the grammar G given by each of the following productions:

1. $S \rightarrow aSb \mid aSbb \mid \lambda$ [1 pt]

$L = \{$ $\}$

2. $S \rightarrow aSc \mid B$
   $B \rightarrow bBc \mid \lambda$ [1 pt]

$L = \{$ $\}$

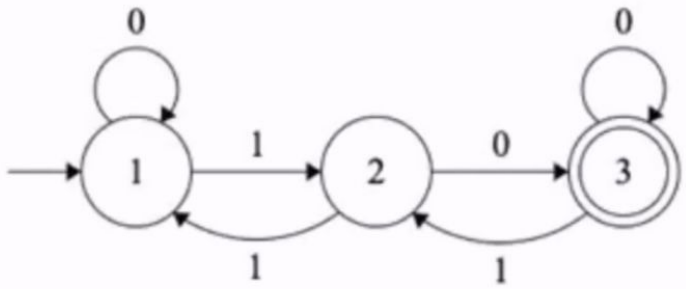
3. $S \rightarrow aaSaba \mid aaBaba$
   $B \rightarrow bB \mid b$ [1 pt]

aa b aba
aa aa bbb b aba aba

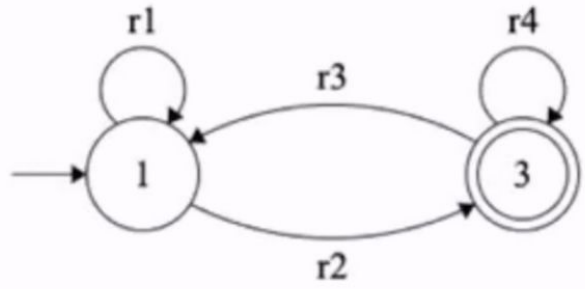
$L = \{ (aa)^n b^m (aba)^n \mid n > 0, m > 0$ $\}$

Exercise 4.2 [1 pt]

Given the following two automata (automata 1 and automata 2):



Automata 1

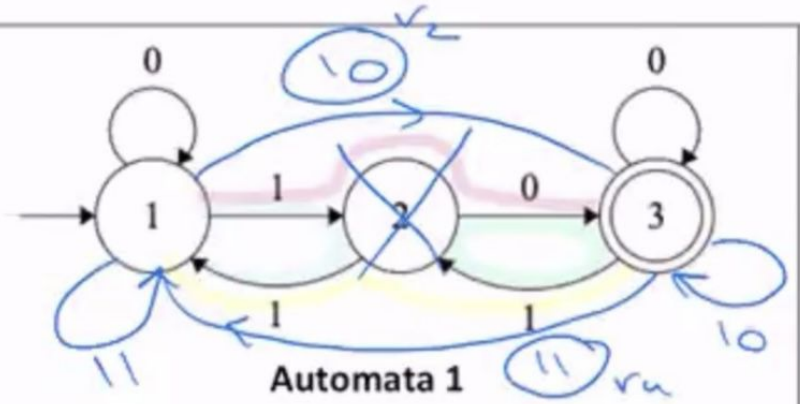


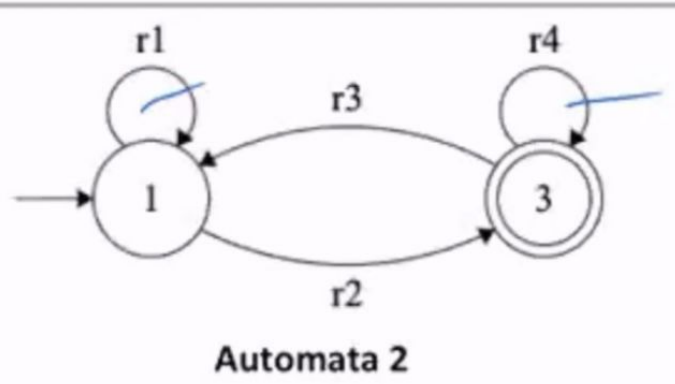
Automata 2

$L = \{ (aa)^n b^m (aba)^n \mid n > 0, m > 0 \}$

Exercise 4.2 [1 pt]

Given the following two automata (automata 1 and automata 2):



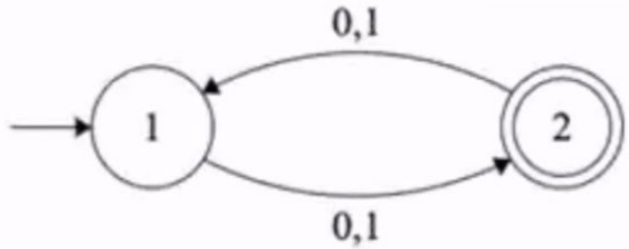


Give the expressions of $r1, r2, r3$ and $r4$ so that automata 2 is equivalent to automata 1.

$r1$	$0 + 11$
$r2$	10
$r3$	11
$r4$	$0 + 10$

Exercise 4.3 [1 pt]

Given the following automata, give the corresponding grammar.

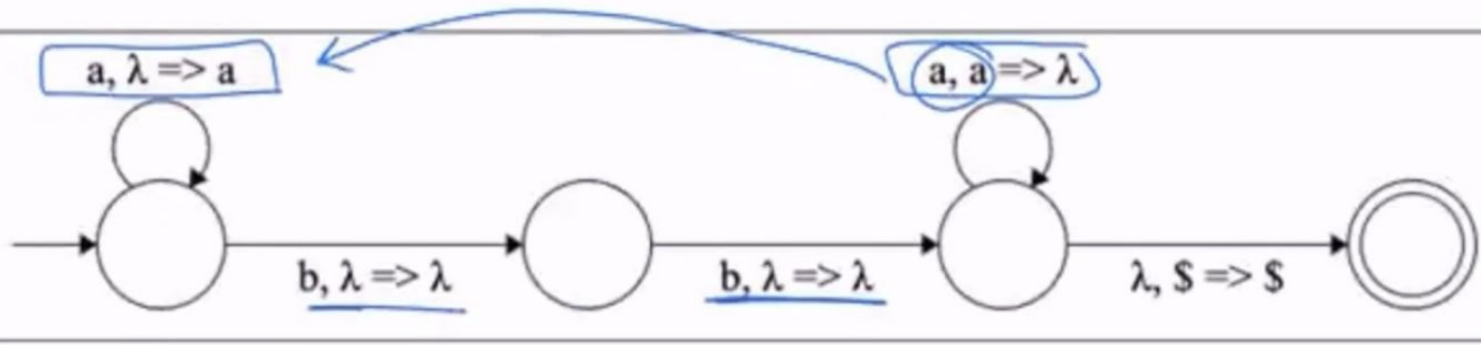


$S \rightarrow 0A \mid 1A$

$A \rightarrow 0S \mid 1S \mid \lambda$

Exercise 4.4 [1 pt]

What is the language covered by the following push down automata?



$L = \{ a^n b b a^n \mid n \geq 0 \}$