Show that the language $L = \{ww \mid w \in \{0, 1\}^*\}$ is not regular using pumping lemma theorem.

Correct word choice:

1. Let p be the critical length for L, pick w such that $w \in L$ and $|w| \geq p$: $w = 0^p10^p1$
2. From the pumping lemma, we can write $w = 0^p10^p1 = xyz$ with length $|xy| \leq p$ and $|y| \geq 1$:
   $w = xyz = 0^p10^p1 = \underbrace{0 \ldots 0}_{x}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 010 \ldots 01}_{z}$ which have the following length details $\overbrace{0 \ldots 00 \ldots 00 \ldots 01}^{p+1}\overbrace{0 \ldots 01}^{p+1}$. Thus, $y = 0^k; 1 \leq k \leq p$

3. From the pumping lemma, $xy^iz \in L$ $where$ $i = 0,1,2,3, \ldots$. Thus $xy^3z \in L$:
   $xy^3z = \underbrace{0 \ldots 0}_{x}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 010 \ldots 01}_{z}$ which have the following length details
   $\overbrace{0 \ldots 00 \ldots 00 . . 00 . . 00 \ldots 01}^{p+2k+1}\overbrace{0 \ldots 01}^{p+1}$.

4. After pumping, the resulted word is $w'' = 0^{p+2k}10^p1$, let us check if $w'' \in L$ or not:
   $\boldsymbol{w'' = 0^{p+2k}10^p1 \notin L}$
5. From step 4, there is contradiction, therefore our assumption that L is a regular language is not true. Thus, L is not a regular language.

Incorrect word choice:

1. Let p be the critical length for L, pick w such that $w \in L$ and $|w| \geq p$:
   $w = 0^p0^p$ ➔ Bad choice! Continue the proof to understand why.

2. From the pumping lemma, we can write $w = 0^p0^p = xyz$ with length $|xy| \leq p$ and $|y| \geq 1$:
   $w = xyz = 0^p0^p = \underbrace{0 \ldots 0}_{x}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 00 \ldots 0}_{z}$ which have the following length details
   $\overbrace{0 \ldots 00 \ldots 00 \ldots 0}^{p}\overbrace{0 \ldots 0}^{p}$. Thus, $y = 0^k; 1 \leq k \leq p$

3. From the pumping lemma, $xy^iz \in L$ $where$ $i = 0,1,2,3, \ldots$. Thus $xy^3z \in L$:
   $xy^3z = \underbrace{0 \ldots 0}_{x}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 0}_{y}\underbrace{0 \ldots 00 \ldots 0}_{z}$ which have the following length details
   $\overbrace{0 \ldots 00 \ldots 00 . . 00 . . 00 \ldots 0}^{p+2k}\overbrace{0 \ldots 0}^{p}$.

4. After pumping, the resulted word is $w'' = 0^{p+2k}0^p$, let us check if $w'' \in L$ or not:
   $\boldsymbol{w'' = 0^{p+2k}0^p = 0^{2p+2k} = 0^{p+k}0^{p+k} \in L}$
5. From step 4, there is **NO** contradiction! Which indicates that our choice for the word with the critical length is a bad choice.