Exercise 1 [4 pts]

1- What is the pumping lemma?

1- Given infinite Regular language L

2- There is exist integer m (critical length)

3- for any string $w \in L$ and $|w| \geq m$

4- write $w = xyz$

5- Such That $w' = xy^iz$ ($i = 0, 1, 2, \ldots$)

NO!!

if $w \in L \Rightarrow w' \in L$

-0,25

2- How the pumping Lemma is used to prove that a language is non regular?

1- assume The opposite L is Regular

2- The pumping lemma should hold for L

3- Using Pumping lemma To obtain Contradiction:

A- Let m be The critical length

B- for any string $w \in L$ and $|w| \geq m$
Can write $w = xyz$

C- From pumping lemma $w = xy^iz \in L$, $i = 0, 1, 2$

d- but we found $w' = xy^iz \notin L$, $i \neq 1$ $\Rightarrow$ we have Contradiction.

4- L is non-regular language.

Exercise 2 [2 pt]

Prove that the following language is non-regular

$$L = \{a^n b^{2n}\}$$

1- we assume L is Regular for contradiction
2- Since L is infinite Regular language, can apply pumping lemma
3- Let m be the critical length
4- Pick string $w \in L$ and $|w| \geq m$
we can write $w = xyz$, $|xy| \leq m$, $|y| \geq 1$
4- $w = xyz = a^m b^{2m} = \underbrace{\underbrace{a}_{x}\underbrace{a}_{y}\underbrace{\ldots a\, b b \ldots b}_{z}}$
5- From Pumping lemma $w = xy^i z \in L$
6- [illegible] $(i = 2)$ $w = xy^2z = \underbrace{a}_{x}\underbrace{a}_{y}\underbrace{a}_{y}\underbrace{\ldots b}_{z}$
7- From Pumping lemma $[a^{m+k} b^{2m}] \in L$
8- but $L = \{a^n b^{2n}\} \Rightarrow [a^{m+k} b^{2m}] \notin L$
∴ we have Contradiction
9- Therefore, our assumption [illegible]
10- Conclusion: L is non-Regular language
∴ End of proof

Exercise 3 [6 pts]

Which language generates the grammar G given by the productions

1-

$S \rightarrow aSaa \mid aBaa$
$B \rightarrow bB \mid b$

$$L(G) = \{a^n b^m a^{2n} : m, n > 0\}$$

$a^n b^n a^{2n}$

2-

$S \rightarrow aSb \mid aSbb \mid \lambda$

$$L(G) = \{a^n b^m \mid 0 \leq n \leq m \leq 2n\}$$

3-

$S \rightarrow aSc \mid B$

$B \rightarrow bBc \mid \lambda$

$$L(G) = \{a^n b^m c^k : k = n + m\}$$

Exercise 4 [6pts]

Construct context free grammars to accept the following languages.

a- {w | w starts and ends with the <u>same</u> symbol} $\Sigma = \{0, 1\}$

$S \to 0A0 \mid 1A1$

$A \to 0A \mid 1A \mid \lambda$

b- {w | |w| is odd} $\Sigma = \{0, 1\}$

$S \to 0A \mid 1A \mid 0 \mid 1$

$A \to 0S \mid 1S \mid \lambda$

c- $L(G) = \{ a^n b^m c^m d^{2n} \mid n \geq 0, m > 0\}$ $\Sigma = \{a, b$, c, d$\}$

$S \to aSdd \mid A$

$A \to bAc \mid bc$

Exercise 5 [2 pt]

Construct a regular expression representing a language, over the alphabet {a, b, c}, in which for every string w it holds that the number of a's in w is a multiple of 3.

$(b+c)^* \left(\left(a(b+c)^*\right)^3 \right)^*$

or

another solution

$\left((b+c)^* a (b+c)^* a (b+c)^* a (b+c)^*\right)^*$