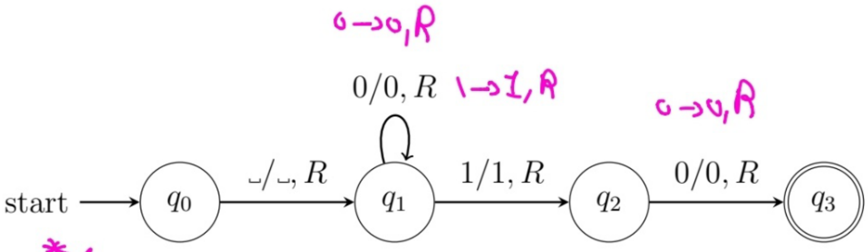
## True/False Questions [8 pts]

| | |
|---|---|
| 1. If $L$ is Turing-acceptable language; then $\bar{L}$ is not Turing-acceptable. | **True** |
| 2. Halting problem is decidable. | **False** |
| 3. Every two tapes Turing machine has an equivalent single tape Turing machine. | **True** |
| 4. Universal Turing Machine are re-programmable. | **True** |
| 5. All regular languages are Turing-decidable. | **True** |
| 6. NP-Complete class includes all problems that can be solved only by exponential time algorithms. | **False** |
| 7. The only way for a Turing machine to reject a string is to halt on a non-accepting state. | **False** |
| 8. Every non-deterministic Turing machine has an equivalent deterministic Turing machine. | **True** |

**Exercise 2:**

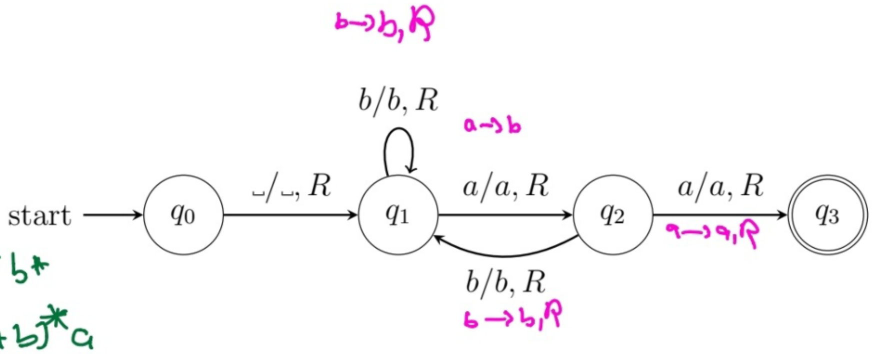
Describe what are the languages accepted by the following Turing machines.
Note: in figures below, "␣" means the blank symbol.

(a).




0*10

(b).




b* a* b*

(a+b)*a

**Solution:**

(a). $L(M) = 0^*10(1+0)^*$

(b). $L(M) = (a+b)^*aa(a+b)^*$

## Exercise 4:

Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

(a). Can a Turing machine ever write the blank symbol on its tape?

(b). Can the tape alphabet $\Gamma$ be the same as the input alphabet $\Sigma$?

(c). Can a Turing machine contain just a single state?

## Solution:

(a). Yes. The tape alphabet $\Gamma$ contains the blank symbol $\sqcup$.

(b). No. $\Sigma$ never contains $\sqcup$, but $\Gamma$ always contains $\sqcup$. So they cannot be equal.

(c). No. Any Turing machine must contain two distinct states $q_{\text{accept}}, q_{\text{reject}}$.



**Question 5: (Turing machine)** .............................................................. **20 points**

Consider the following formal definition of a Turing machine: $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$ where $Q = \{q_0, q_1, q_2, q_{acc}, q_{rej}\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \cancel{\sqcup}\}$ #, and $\delta$ is given as follows.

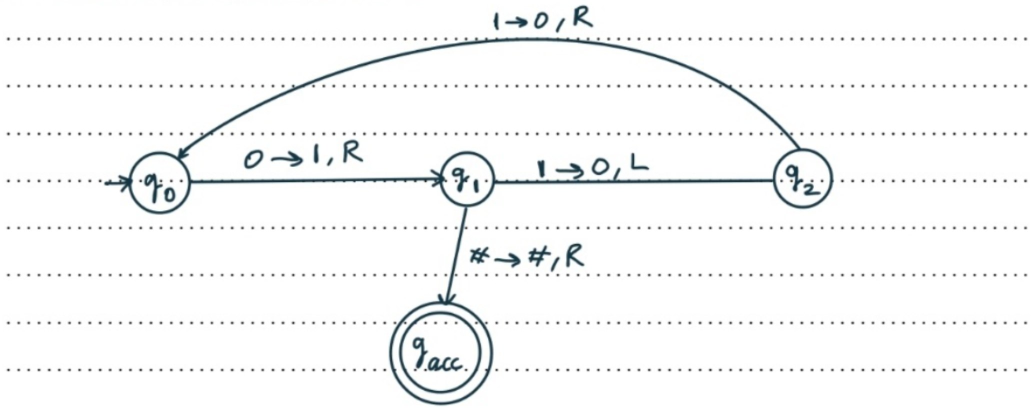
$$\delta(q_0, 0) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_2, 0, L)$$

$$\delta(q_1, \cancel{\sqcup}\#) = (q_{acc}, \cancel{\sqcup}\#, R)$$

$$\delta(q_2, 1) = (q_0, 0, R)$$

(a) Draw the corresponding Turing machine.




(b) Is the string 0111 accepted by M:

**(A) Yes** (B) No

(c) Is the string 0110 accepted by M:

(A) Yes **(B) No**

(d) The language accepted by $M$:

(A) cannot be described by a regular expression. ✗

✓ **(B) can be described by the regular expression $01^*$.**

(C) can be described by the regular expression $(0110)^+$.

(D) None

(i) If L is a language recognized by a non-deterministic Turing machine, L cannot be recognized by a deterministic turing machine.

(A) True **(B) False**

(j) A Turing machine with multiple track tapes has multiple heads.

(A) True **(B) False**



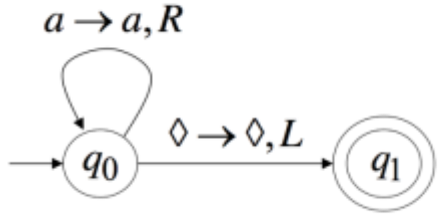
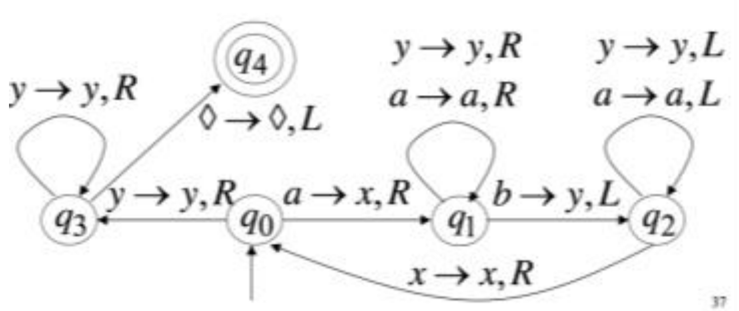
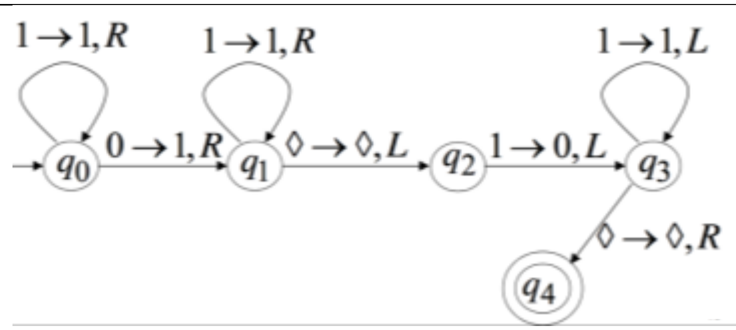
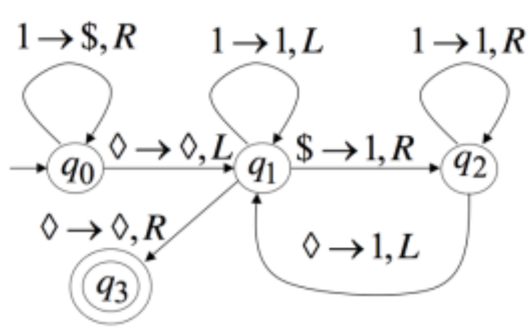
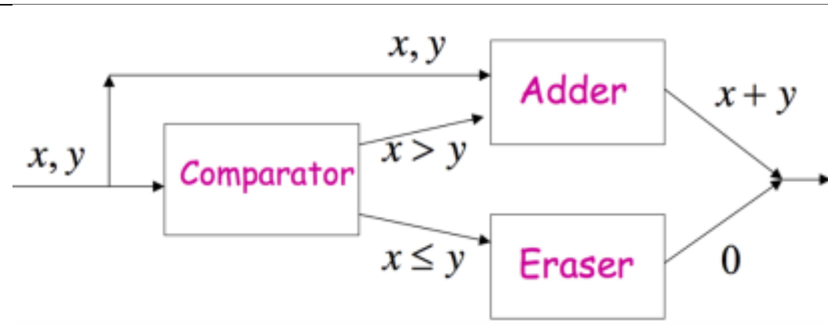


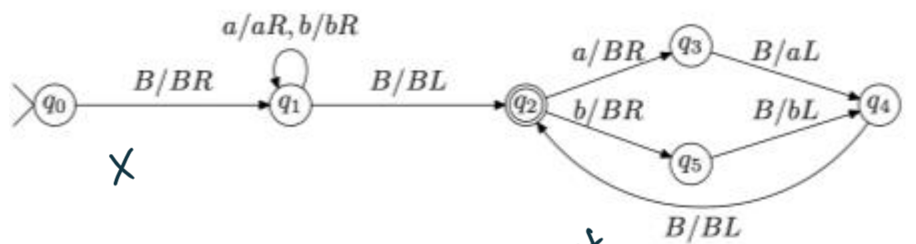
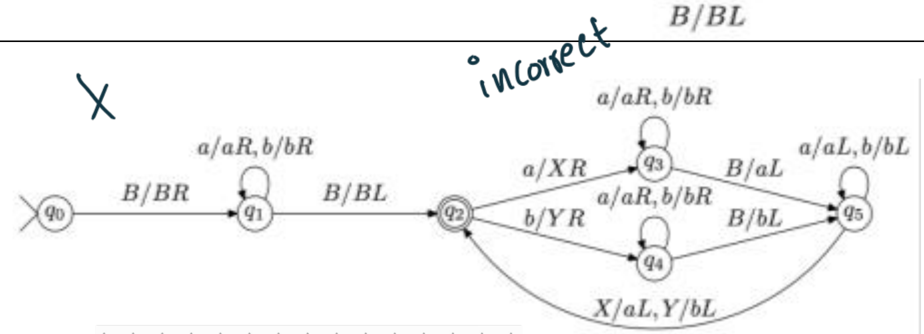
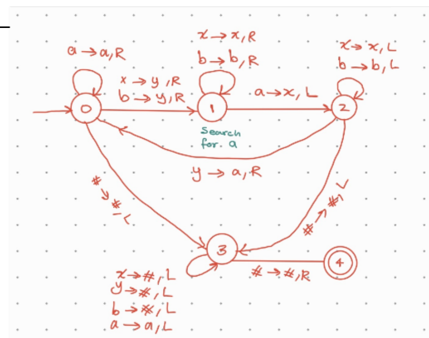
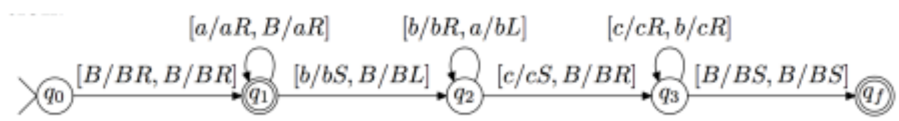
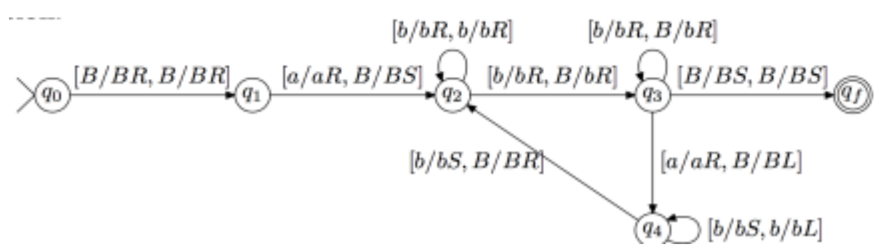
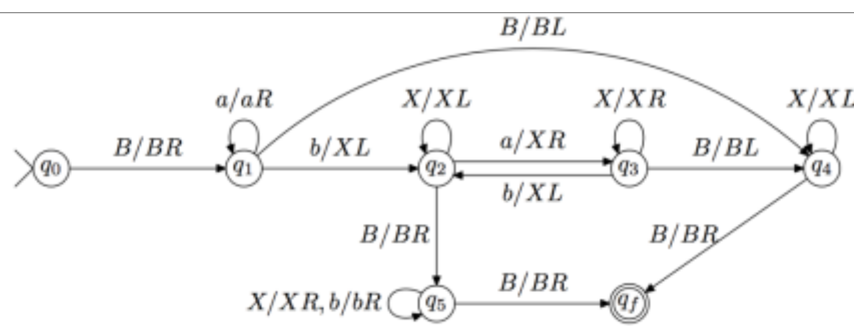
(k) Multidimensional Turing machine can be replaced by a deterministic Turing machine.

**(A) True** (B) False

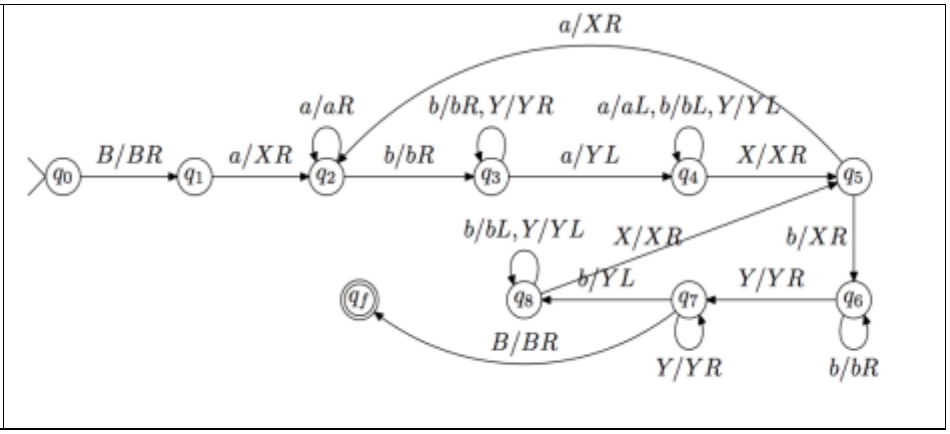
(l) In Turing machines, the accept state is always different from the reject state. **(A) True** (B) False

### *Turing Machines And Languages*

| Language Or Functions | Turing Machine |
|---|---|
| *L= a** |   |
| $L(M) = \{a^n b^n : n \geq 1\}$ |   |
| $f(x, y) = x + y$ |   |
| $f(x) = 2x$ |   |
| $f(x, y) = \begin{cases} x + y & \text{if } x > y \\ 0 & \text{if } x \leq y \end{cases}$ |   |

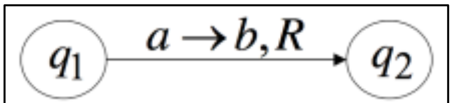
| | |
|---|---|
| *Shift input one space* |  |
| *Concatenate a copy of the reversed input string to the input* |  |
| *Erase b from the input stream* |  |
| *$L = \{a^i b^i c^i : i \geq 0\}$* <br> *( two-tape Turing machine)* |  |
| *Each a is followed by an increasing number of b's.* <br><br> *( two-tape Turing machine)* |  |
| *$L = \{a^i b^j \mid i \geq 0, j \geq i\}$* |  |

$L = \{a^i b^j a^i b^j \mid i,j>0\}$



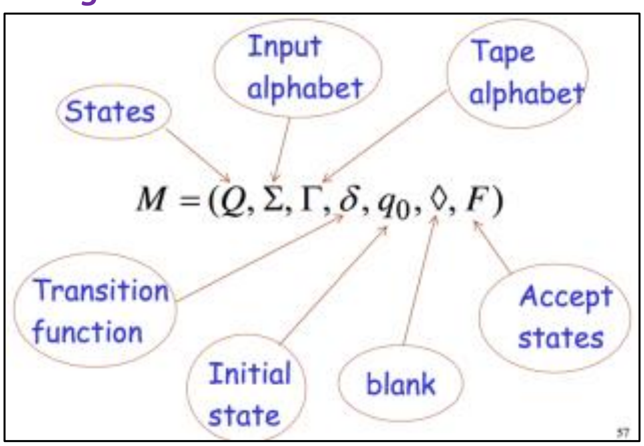

***Formal Definitions***

1. ***Transition function:***

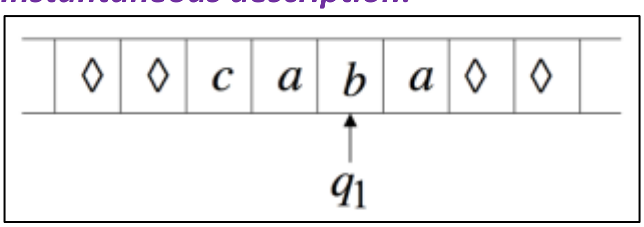



$$\delta(q_1, a) = (q_2, b, R)$$

2. ***Turing Machine:***




3. ***Instantaneous description:***




***ca $\underline{q_1}$ ba***

4. *Accepted Language by Turing Machine*

$$L(M) = \{w : \quad q_0\, w \overset{*}{\succ} x_1\, q_f\, x_2\}$$

5. *Computing Functions with Turing Machines*

$$q_0\, w \overset{*}{\succ} q_f\, f(w)$$

*for all w* $\in$ *D Domain*

*Turing's thesis (1930):*

Any computation carried out by mechanical means can be performed by a Turing Machine

*Algorithm:*

An algorithm for a problem is a Turing Machine which solves the problem

*Turing machines with:*

1 • Stay-Option
2 • Semi-Infinite Tape 3 • Off-Line
4 • Multitape
5 • Multidimensional
6 • Nondeterministic

Different Turing Machine *Classes*

*Same Power of two classes means:*

Both classes accept the same set of languages
for any machine M1 of first class there is a machine M2 of second class
such that: L(M1) = L(M2) and vice-versa