Tutorial 4

Use the pumping lemma to show that the following languages are not regular:

1) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

2) $A_2 = \{a^{2^n} \mid n \geq 0\}$, a^{2^n} is a string of 2^n a's.

1) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

- Assume that A_1 is regular.
- Let p be the pumping length given by the pumping lemma.
- Choose s the string $0^p 1^p 2^p$, $s \in A_1$ and $|s| \geq p$.
- The pumping lemma guarantees that s can be split into three parts: $s = xyz$ / $\forall i \geq 0$, $xy^iz \in A_1$, $|y| \geq 1$, $|xy| \leq p$
- Counterexample:

$$\begin{cases} x = (p-1)\ 0\text{'s} = 0^{p-1} \\ y = 0 \quad (|y| \geq 1) \\ z = (p)\ 1\text{'s and } (p)\ 2\text{'s} \\ \quad = 1^p 2^p \end{cases}$$

So, $s = xyz$

for $i=2$: $s' = xy^2z$
$= 0^{p-1} 0^2 1^p 2^p$
$= 0^{p+1} 1^p 2^p \notin A_1$

The same result is obtained with any other splitting that means
So, this is a contradiction
that A_1 is not regular.

2) $A_2 = \{ a^{2^n} \mid n \geq 0 \}$

- Assume that A_2 is regular.
- Let p be the pumping length given by the pumping lemma.
- Let s be a^{2^p}, $|s| > p$
  $s \in A_3$
- s can be split into: $xyz \,/\, \begin{cases} |xy| \leq p \\ |y| \geq 1 \end{cases}$

- $p < 2^p \Rightarrow |y| < 2^p$

Therefore
$s' = |xyyz| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$

$\underline{|y| \geq 1:}$
$2^p < |xyyz| < 2^{p+1} \Rightarrow$ The length of $xyyz$ cannot be a power of 2

$\Rightarrow s' = xyyz \notin A_2$
(Contradiction)

$\Rightarrow A_2$ is not regular