Closure Properties of Context Free Languages

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- **Properties of Context Free Languages**
  Union : If L1 and L2 are two context free languages, their union L1 ∪ L2 will also be context free.
- CFL are closed under Union.
- **Concatenation** : If L1 and If L2 are two context free languages, their concatenation L1.L2 will also be context free.
- CFL are closed under Concatenation.

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- **Kleene Closure** : If L1 is context free, its Kleene closure L1* will also be context free.
- CFL are closed under Kleen Closure.
- **Intersection and complementation** : If L1 and If L2 are two context free languages, their intersection L1 ∩ L2 need not be context free. For example,
  L1 = { $a^n b^n c^m$ | n >= 0 and m >= 0 } and
- L2 = ($a^m b^n c^n$ | n >= 0 and m >= 0 }
  L3 = L1 ∩ L2 = { $a^n b^n c^n$ | n >= 0 } need not be context free.

- L1 says number of a's should be equal to number of b's and L2 says number of b's should be equal to number of c's.
- Their intersection says both conditions need to be true, but push down automata can compare only two. So it cannot be accepted by pushdown automata, hence not context free.
  Similarly, complementation of context free language L1 which is $\Sigma^* - L1$, need not be context free.
- CFL are not closed under Intersection and Complementation.

Deterministic Context-free Languages

- Deterministic CFL are subset of CFL which can be recognized by Deterministic PDA.
- L1= { $a^n b^n c^m$ | m >= 0 and n >= 0} is a DCFL because for a's, we can push on stack and for b's we can pop. It can be recognized by Deterministic PDA.
- On the other hand, L3 = { $a^n b^n c^m \cup a^n b^m c^m$ | n >= 0, m >= 0 } cannot be recognized by DPDA
- it can only be implemented by NPDA. Thus, it is CFL but not DCFL.