	King Saud University College of Computer and Information Sciences Computer Science Department
CSC 339: Theory of Computation	**Quiz 3: Second Semester 2020**
Duration: 72 Hours (3 Days)	Due Date: April 10, 2020 at 8:00 AM

Name:		ID:		Section:	

Question 1 [3 pts]	Question 2 [3 pts]	Question 3 [2.5 pts]	Question 4 [3 pts]	Question 5 [3.5 pts]
Total [15 pts]				

Instructions

- This is a take-home and open-book quiz that **should** be solved **individually**.
- All questions **should** be answered on this examination paper.
- Write your **full information** (name, ID, and section) on the specified above table.
- You **should** follow these **submission instructions**:
  - Late submission is **NOT** acceptable.
  - Save your examination paper as **(.pdf)** file and name it **[Your Section]_[Your ID]_[Your Full Name in Arabic]**.
  - Through LMS, go to **\Quizzes\Quiz 3** and submit your paper.

Good Luck ☺

Question 1 [3 pts]

Answer with TRUE or FALSE next to each of the following statements:

1. There is -at least- one equivalent PDA for every finite automaton.	True
2. All context-free language can be generated by a regular expression. Take a counter example: let $L = \{a^n b^n \in \{a,b\}^* \mid n \geq 0\}$ be the context-free language, there is no regular expression can generate the language L.	False
3. To proof that a given language L belongs to the context-free family; it is sufficient to build PDA for that L.	True
4. If a language L fails to satisfy pumping lemma; then L is a context-free language. If a language L fails to satisfy pumping lemma; then L is a non-regular language.	False
5. All regular languages satisfy pumping lemma theorem.	True
6. If $L1$ is a regular language, $L2$ is a context-free language, and $L3 = L1 \circ L2$; then $L3$ will not satisfy the pumping lemma theorem. Take a counter example: let $L1 = \{a^m \in \{a,b\}^* \mid m \geq 1\}$ and $L2 = \{a^n b^n \in \{a,b\}^* \mid n \geq 0\}$; then $L3 = L1 \circ L2 = \{a^{n+m} b^n \in \{a,b\}^* \mid n \geq 0, m \geq 1\}$ which is a regular language that satisfies pumping lemma.	False

Question 2 [3 pts]

Choose the most correct answer:

1. Which of the following context-free grammar productions generates the language of all palindromes over $\{0,1\}$?
   a. $P = \{S \to 0S0, S \to 1S1, S \to \lambda\}$
   b. $P = \{S \to 0S0, S \to 1S1, S \to 0, S \to 1\}$
   c. $P = \{S \to 0S0, S \to 1S1, S \to 0, S \to 1, S \to \lambda\}$
   d. None

2. The context-free language that generated by the production $P = \{S \to AB|C, A \to xAy, A \to \lambda, B \to zB, B \to \lambda, C \to xCz, C \to D, D \to yD, D \to \lambda\}$ is:
   a. $L = \{x^i y^i z^i \in \{x,y,z\}^* \mid i \geq 0\}$
   b. $L = \{x^i y^j z^k \in \{x,y,z\}^* \mid i \geq 0 \wedge (i = j \vee i = k)\}$
   c. $L = \{x^i y^j z^k \in \{x,y,z\}^* \mid i \geq 0 \wedge j \geq 0 \wedge k \geq 0\}$
   d. $L = \{x^i y^j z^k \in \{x,y,z\}^* \mid i \geq 0 \wedge ((i = j \wedge i \neq k) \vee (i \neq j \wedge i = k))\}$

3. Which of the following PDAs transitions Δ accepts $L = \{u\#v \mid u \wedge v \in \{a,b\}^* \wedge |u| = |v|\}$?
   Note: each transition is written as:
   $$(\textbf{current state}, \textbf{read char}, \textbf{poped char})(\textbf{new state}, \textbf{pushed char})$$
   a. $\Delta = (q0, a, \lambda)(q0, X),\ (q0, b, \lambda)(q0, X),\ (q0, \#, \lambda)(q1, \lambda),\ (q1, a, X)(q1, \lambda),$ $(q1, b, X)(q1, \lambda),\ (q1, \lambda, \$)(q2, \$)$

b. $\Delta = (q0, a, \lambda)(q0, X),\ (q0, b, \lambda)(q0, \lambda),\ (q0, \#, \lambda)(q1, \lambda),\ (q1, a, X)(q1, \lambda),$ $(q1, b, \lambda)(q1, \lambda),\ (q1, \lambda, \$)(q2, \$)$

c. $\Delta = (q0, a, \lambda)(q0, X),\ (q0, b, \lambda)(q0, Y),\ (q0, \#, \lambda)(q1, \lambda),\ (q1, a, X)(q1, \lambda),$ $(q1, b, Y)(q1, \lambda),\ (q1, \lambda, \$)(q2, \$)$

d. None

4. The language $L = \{1^*0\}$ is:
   a. Regular language
   b. Context-free language
   c. Regular and context-free language
   d. None

5. Consider the language $L = \left\{1^i 0^j 1^k \in \{0,1\}^* \mid i > 0 \wedge j > 0 \wedge k = i * j\right\}$; to show that L is not a regular language using pumping lemma, the correct choice for the word is:
   a. 10011
   b. $1^p 0^p 1^p$
   c. $1^{2p} 0^{p^3} 1^{2p^4}$ ➔ This choice is the most correct one.
   d. $10^p 1^p$ ➔ Both of c and d are correct choices.

6. Which of the following context-free grammar productions generates words of balanced brackets?
   a. $P = \{S \to \lambda, S \to TS, T \to (\ T\ )\}$
   b. $P = \{S \to \lambda, S \to (\ S\ )S\}$
   c. $P = \{S \to \lambda, S \to SS, S \to (\ )\}$
   d. $P = \{S \to \lambda, S \to (T)S, T \to (\ )\}$

Question 3 [2.5 pts]

Consider the language $L = \{a^{3n}\#b^n \in \{a, b, \#\}^* \mid n \geq 1\}$; answer the following question to show that the language L is not regular using pumping lemma:

1. Pick a word $w \in L$ that satisfies the condition $|w| \geq p$, where p is the critical length.
   $w = a^{3p}\#b^p$

2. Complete the proof steps using the picked word in point 1.
   - From the pumping lemma, we can write $w = a^{3p}\#b^p = xyz$ with length $|xy| \leq p$ and $|y| \geq 1$:
     - $w = xyz = a^{3p}\#b^p = \underbrace{a \dots a}_{x}\,\underbrace{a \dots a}_{y}\,\underbrace{a \dots a\#b \dots b}_{z}$ which have the following length details $\overbrace{a \dots aa \dots a}^{p}\,\overbrace{a \dots a}^{2p}\#\,\overbrace{b \dots b}^{p}$. Thus, $y = a^k; 1 \leq k \leq p$
   - From the pumping lemma, $xy^iz \in L\ where\ i = 0,1,2,3, \dots$. Thus $xy^2z \in L$:

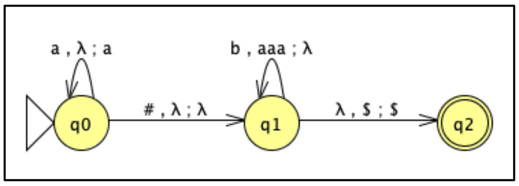
- $xy^2z = \underbrace{a \dots a}_{x}\underbrace{a \dots a}_{y}\underbrace{a \dots a}_{y}\underbrace{a \dots a\#b \dots b}_{z}$ which have the following length details $\overbrace{a \dots aa \dots aa \dots a}^{p+k}\overbrace{a \dots a}^{2p}\#\overbrace{b \dots b}^{p}$.

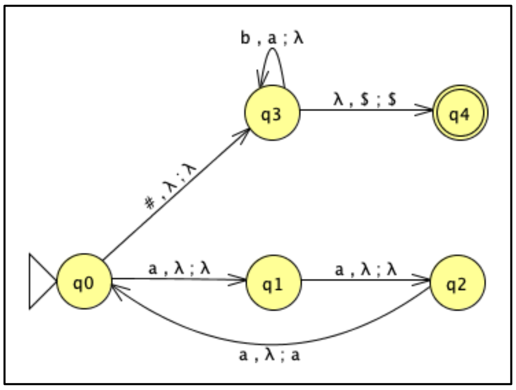
- After pumping, the resulted word is $w'' = a^{3p+k}\#b^p \notin L$
- Contradiction and the language L is not regular.

Question 4 [3 pts]

Construct a PDA for the non-regular language $L = \{a^{3n}\#b^n \in \{a, b, \#\}^* \mid n \geq 1\}$. The PDA **should** satisfy the following characteristics:

- Except the last transition which is $\boldsymbol{\lambda}, \$ \rightarrow \$$**,** another Lambda transition is **NOT** allowed. Note that, Lambda transition has this form $\boldsymbol{\lambda}, \boldsymbol{symbol} \rightarrow \boldsymbol{symbol}$
- Multiple transitions for the same symbol from the same state is **NOT** allowed.





Question 5 [3.5 pts]

Consider the grammar $G = (\{S, T\}, \{x, y\}, S, \{S \rightarrow TS, S \rightarrow \lambda, T \rightarrow Ty, T \rightarrow xTy, T \rightarrow xy\})$:

1. Give the **mathematical** definition of the language $L(G)$.

   $L = \{(x^i y^j)^* \mid 1 \geq i \geq j\}$

2. Give two left-most derivations for the terminal string $\boldsymbol{xxyyy}$.

   $S \Longrightarrow TS \Longrightarrow xTyS \Longrightarrow xTyyS \Longrightarrow xxyyyS \Longrightarrow xxyyy$

   $S \Longrightarrow TS \Longrightarrow TyS \Longrightarrow xTyyS \Longrightarrow xxyyyS \Longrightarrow xxyyy$

3. Is the grammar ambiguous? If so, change the grammar to remove the ambiguity.

   Yes, the grammar is ambiguous. The un-ambiguous grammar is:

   $P = \{S \rightarrow TS, S \rightarrow \lambda, T \rightarrow xTy, T \rightarrow M, M \rightarrow My, M \rightarrow xy\}$

   Or:

   $P = \{S \rightarrow TS, S \rightarrow \lambda, T \rightarrow Ty, T \rightarrow M, M \rightarrow xMy, M \rightarrow xy\}$