

Tutorial # 4

Context Free Grammar (CFG)

Exercise 1

- Which language is generated by the grammar G given by each of the following productions:

$$S \rightarrow aSa | aBa$$

$$B \rightarrow bB | b$$

$$L(G) = \{a^n b^m a^n : n, m > 0\}$$

$$S \rightarrow abScB | \lambda$$

$$B \rightarrow bB | b$$

$$\cancel{L(G) = \{(ab)^n (c(b)^m)^n : n \geq 0, m > 0\}}$$

$$\{(ab)^n cb^{m_1} cb^{m_2} \dots cb^{m_n} \mid n \geq 0, m_1, m_2, \dots, m_n \geq 1 \text{ number of } c\text{'s equal to } n\}$$

Exercise 2

- Find a CFG that generates each of the following languages over $\Sigma = \{a, b, c, d\}$:

1. $L(G) = \{a^n b^m c^m d^{2n} \mid n \geq 0, m > 0\}$

$S \rightarrow aSdd \mid A$

$A \rightarrow bAc \mid bc$

Exercise 2

- Find a CFG that generates each of the following languages over $\Sigma = \{a, b, c, d\}$:

1. $L(G) = \{a^n b^m \mid 0 \leq n \leq m \leq 2n\}$

$n=0, m=0 \rightarrow \lambda$

$n=1, m \in [1, 2] \rightarrow ab, abb$

$n=2, m \in [2, 4] \rightarrow aabb, aabbbb, aabbbbbb$

$S \rightarrow aSb \mid aSbb \mid \lambda$

Exercise 2

- Find a CFG that generates each of the following languages over $\Sigma = \{a, b, c, d\}$:
1. $L(G) = \{a^n b^m c^k \mid k = n + m\}$

$S \rightarrow aSc \mid B$

$B \rightarrow bBc \mid \lambda$

Exercise 3

- Construct a CFG to generate the following languages over $\Sigma = \{0,1\}$:
- $L(G) = \{w \mid w \text{ starts and ends with the same symbol}\}$

$S \rightarrow 0A0 \mid 1A1$

$A \rightarrow 0A \mid 1A \mid \lambda$

- $L(G) = \{w : |w| \text{ is odd}\}$

$S \rightarrow 0A \mid 1A$

$A \rightarrow 0S \mid 1S \mid \lambda$

Exercise 4

- Explain why the grammar below is ambiguous:

$$S \rightarrow 0A \mid 1B$$

$$A \rightarrow 0AA \mid 1S \mid 1$$

$$B \rightarrow 1BB \mid 0S \mid 0$$

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$$S \rightarrow 0A \mid 1B$$

$$A \rightarrow 0AA \mid 1S \mid 1$$

$$B \rightarrow 1BB \mid 0S \mid 0$$

001101

$S \rightarrow 0A \rightarrow \rightarrow 00AA \rightarrow 001A \rightarrow 0011S \rightarrow 00110A \rightarrow 001101$

$S \rightarrow 0A \rightarrow \rightarrow 00AA \rightarrow 001SA \rightarrow 0011BA \rightarrow 00110A \rightarrow 001101$

Exercise 5

- Given the following ambiguous CFG:

$$S \rightarrow Ab|aaB$$

$$A \rightarrow a|Aa$$

$$B \rightarrow b$$

- Find the string s generated by the grammar that has two leftmost derivations and show them.
- Show the two derivation trees for the string s .
- Find an equivalent un-ambiguous CFG.
- Give the unique leftmost derivation and derivation tree for the string s generated from the un-ambiguous grammar above.

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- $s = aab$

- $S \rightarrow Ab \rightarrow Aab \rightarrow aab$

- $S \rightarrow aaB \rightarrow aab$

Exercise 5

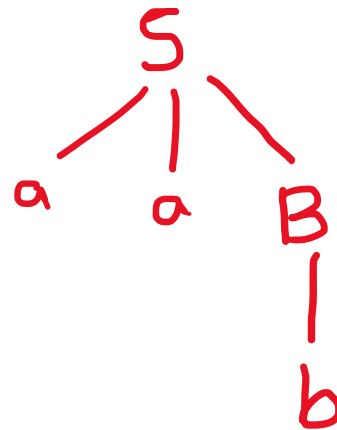
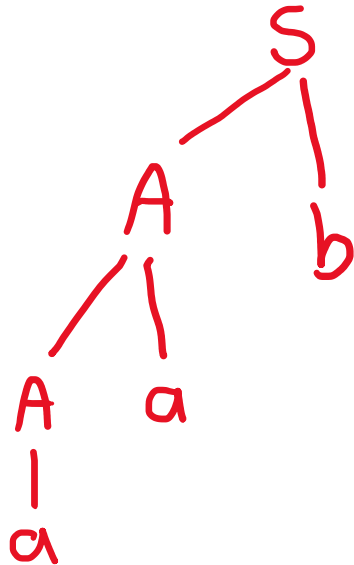
- Given the following ambiguous CFG:

$$S \rightarrow Ab|aaB$$

$$A \rightarrow a|Aa$$

$$B \rightarrow b$$

- Show the two derivation trees for the string s . {there is derivation tree for each left most derivation}



Exercise 5

- Given the following ambiguous CFG:
 $S \rightarrow Ab|aaB$
 $A \rightarrow a|Aa$
 $B \rightarrow b$
- Find an equivalent un-ambiguous CFG.
- Give the unique leftmost derivation and derivation tree for the string s generated from the un-ambiguous grammar above.

$S \rightarrow Ab$

$A \rightarrow a|Aa$

$S \rightarrow Ab \rightarrow Aab \rightarrow aab$

Exercise 6

- Convert the following ambiguous grammar into un-ambiguous grammar
 $\text{bexp} \rightarrow \text{bexp or bexp} \mid \text{bexp and bexp} \mid \text{not bexp} \mid \text{T} \mid \text{F}$
- where bexp represents Boolean expression, T represents True and F represents False.
- **To convert the given grammar into its corresponding unambiguous grammar, we implement the precedence and associativity constraints. We have-**
- **Given grammar consists of the following operators-**
or , and , not
- **Given grammar consists of the following operands-**
T , F
- **The priority order is-**
(T , F) > not > and > or

bexp \rightarrow bexp or M | M

M \rightarrow M and N | N

N \rightarrow not N | T | F