Turing's Thesis

Turing's thesis (1930):

Any computation carried out
by mechanical means
can be performed by a Turing Machine

Algorithm:

An algorithm for a problem is a
Turing Machine which solves the problem

The algorithm describes the steps of
the mechanical means

This is easily translated to computation steps
of a Turing machine

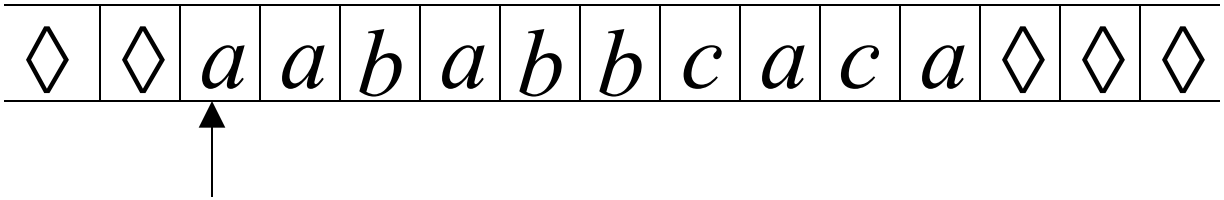
When we say: There exists an algorithm

We mean: There exists a Turing Machine that executes the algorithm

Variations of the Turing Machine

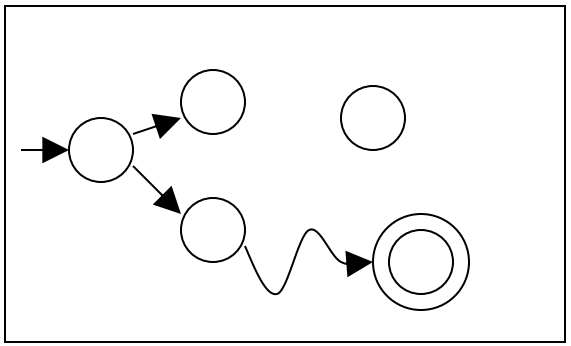
The Standard Model

Infinite Tape



Read-Write Head (Left or Right)

Control Unit



Deterministic

Variations of the Standard Model

Turing machines with:

- Stay-Option
- Semi-Infinite Tape
- Off-Line
- Multitape
- Multidimensional
- Nondeterministic

Different Turing Machine **Classes**

Same Power of two machine classes:

both classes accept the same set of languages

Same Power of two classes means:

for any machine M_1 of first class

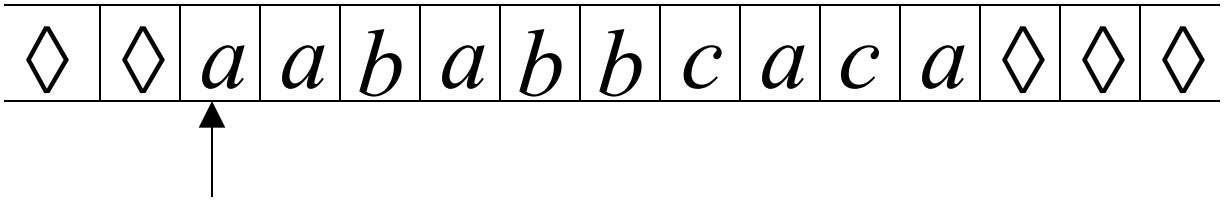
there is a machine M_2 of second class

such that: $L(M_1) = L(M_2)$

and vice-versa

Turing Machines with Stay-Option

The head can stay in the same position

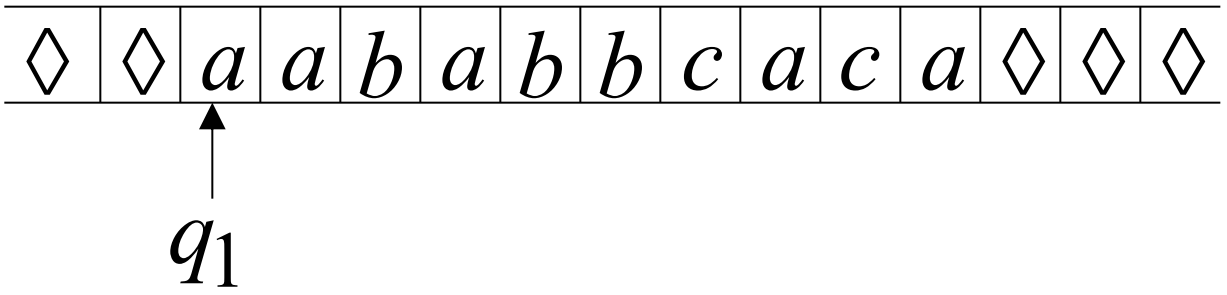


Left, Right, Stay

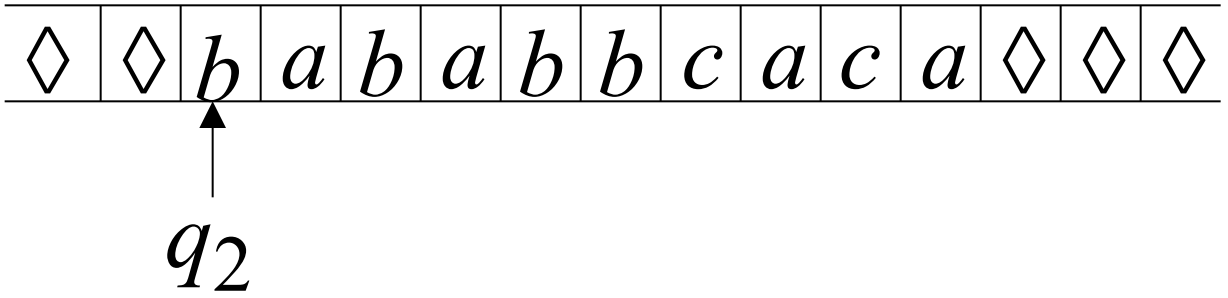
L,R,S: possible head moves

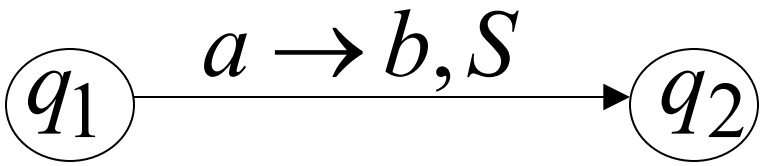
Example:

Time 1



Time 2

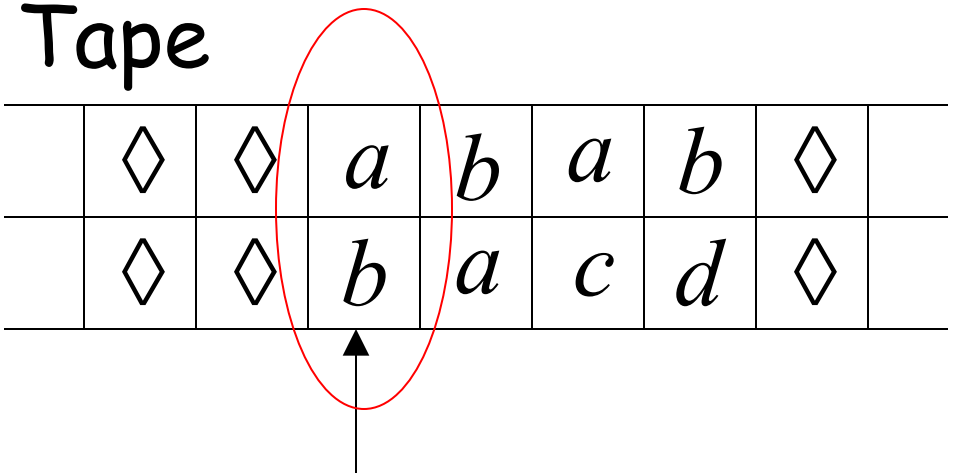




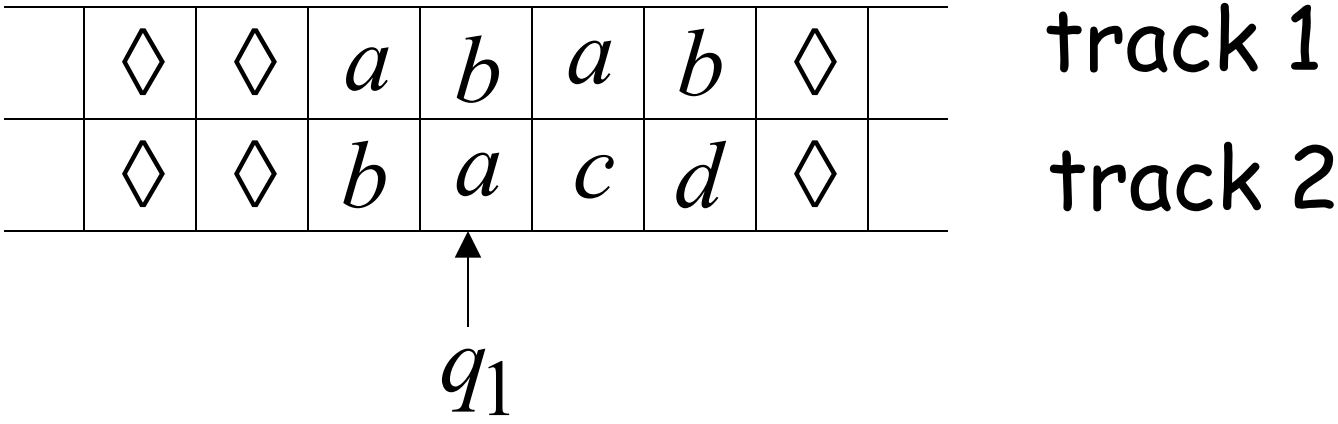
Multiple Track Tape

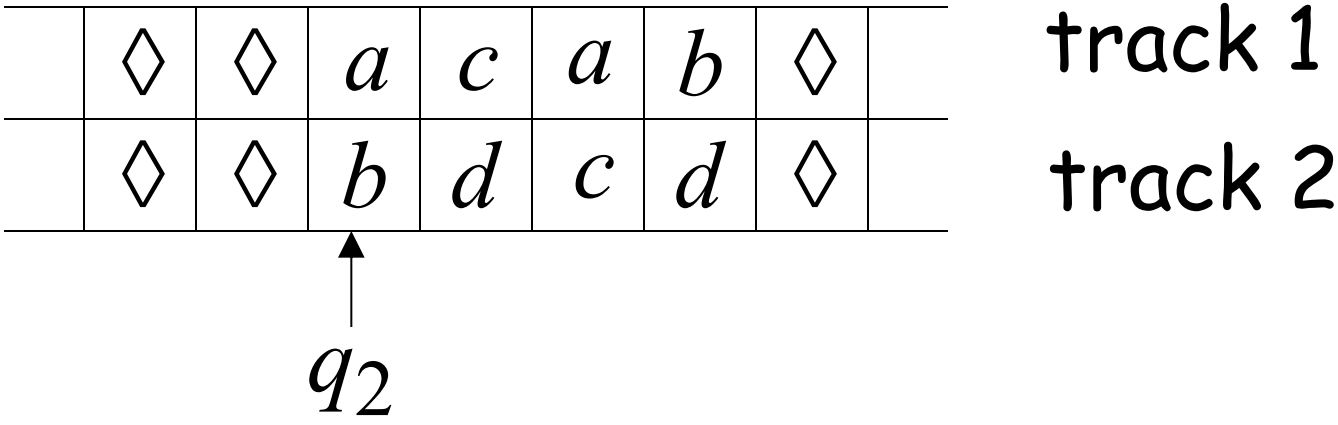
A useful trick to perform more complicated simulations

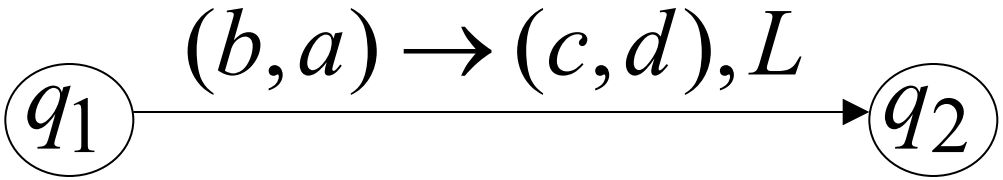
One Tape



One symbol (a,b)

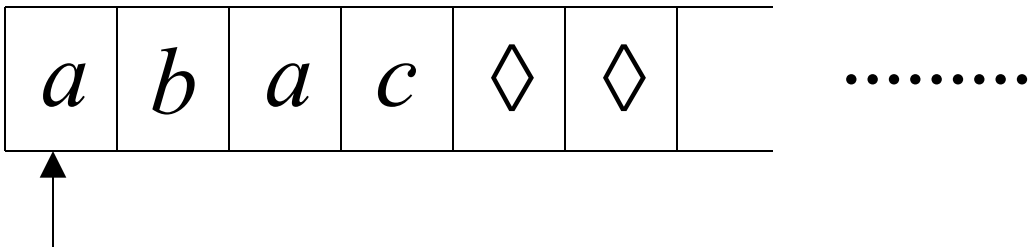

◊ ◊ a b a b ◊
◊ ◊ b a c d ◊
track 1
track 2
q_1


◊ ◊ a c a b ◊
◊ ◊ b d c d ◊
track 1
track 2
q_2


$(b,a) \rightarrow (c,d), L$
q_1
q_2

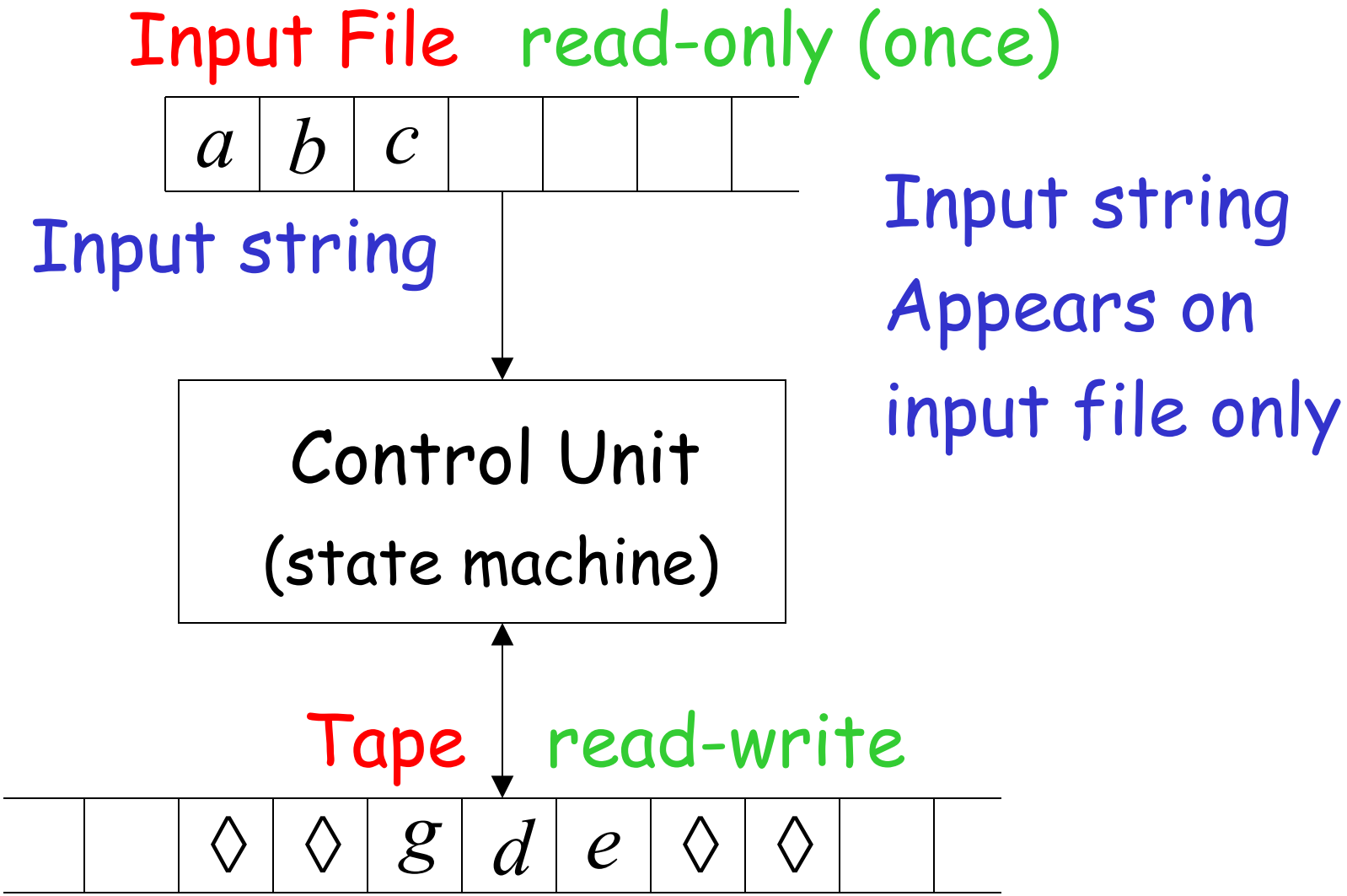
Semi-Infinite Tape

The head extends infinitely only to the right

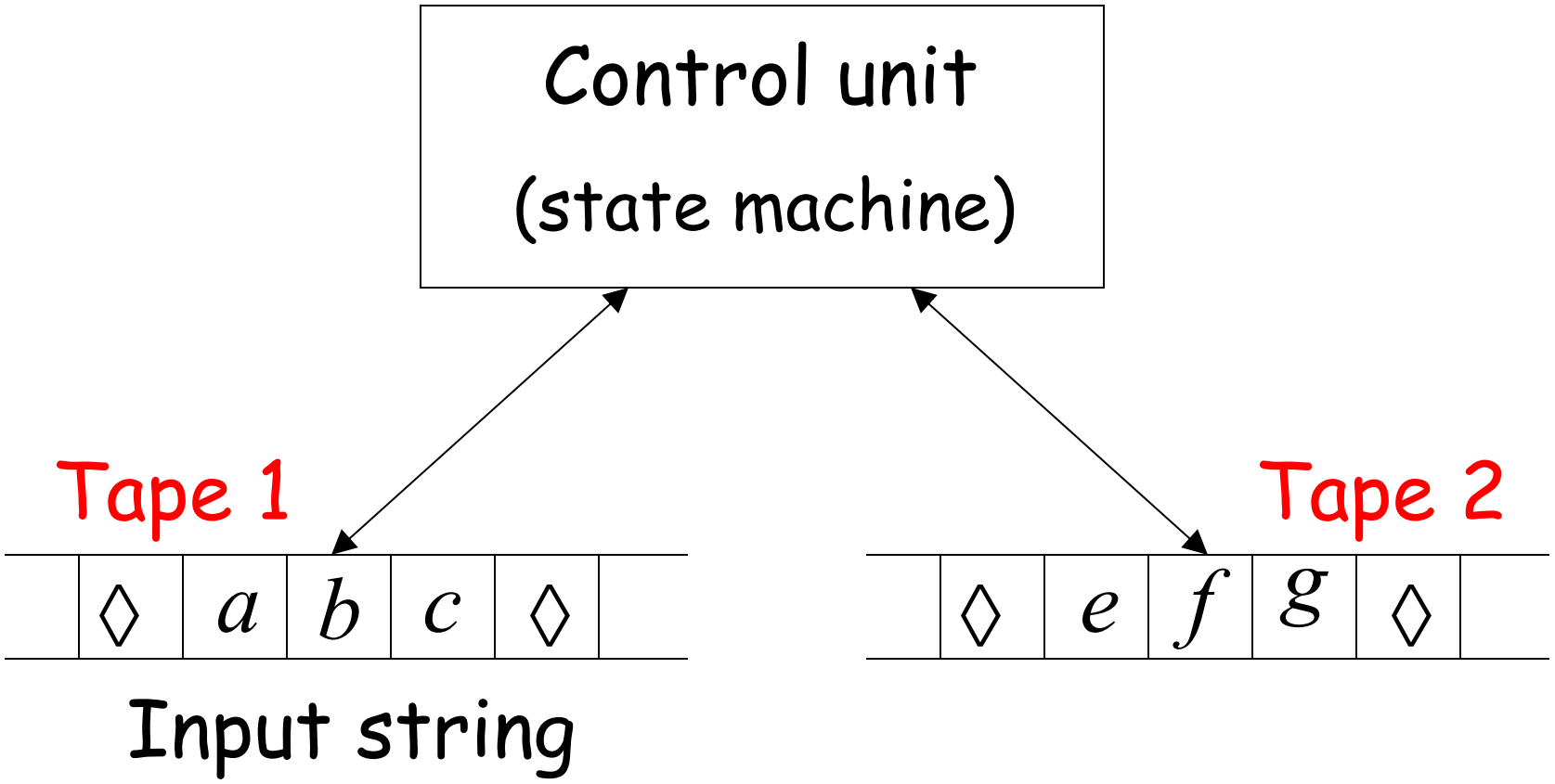


- Initial position is the leftmost cell
- When the head moves left from the border, it returns to the same position

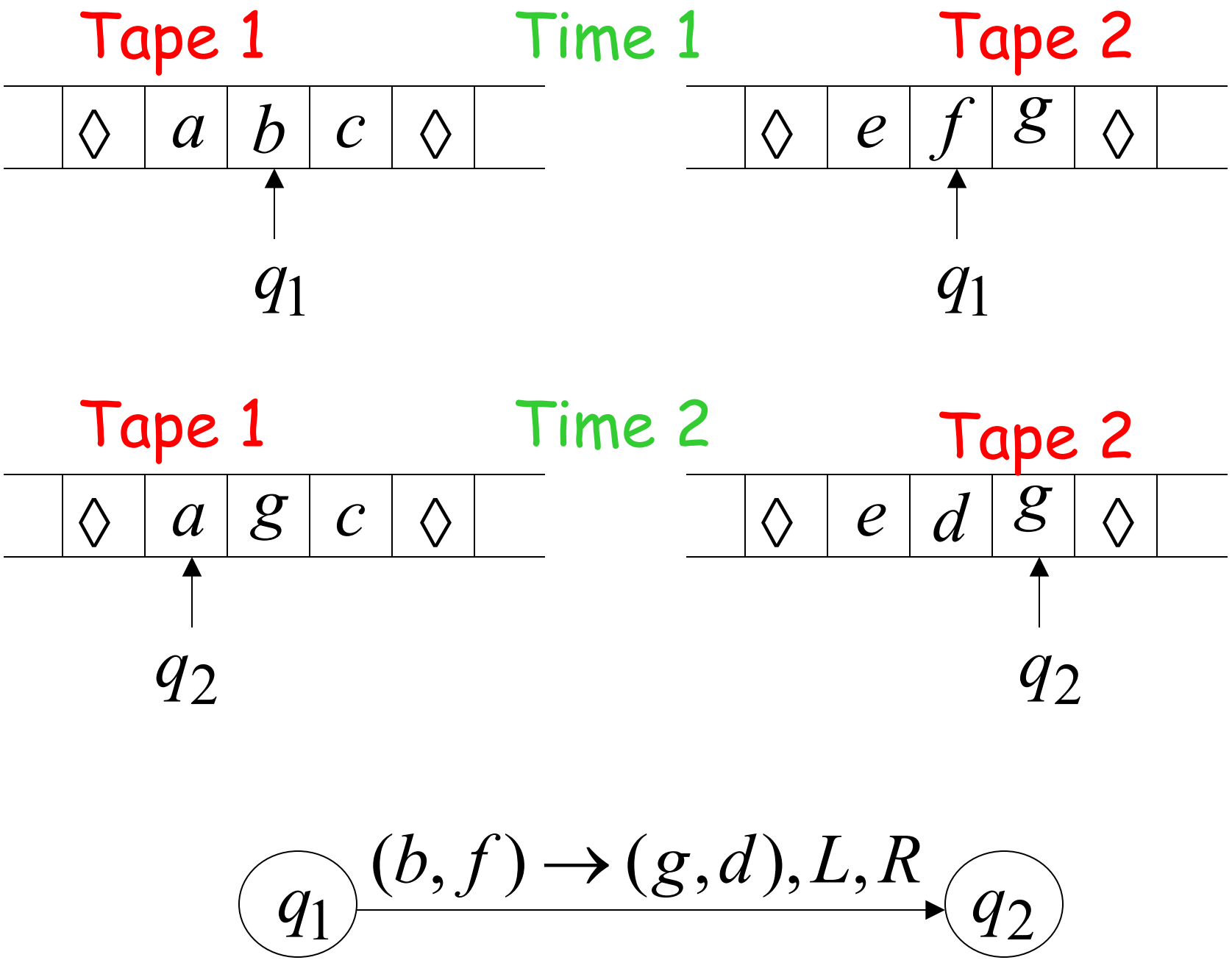
The Off-Line Machine



Multi-tape Turing Machines

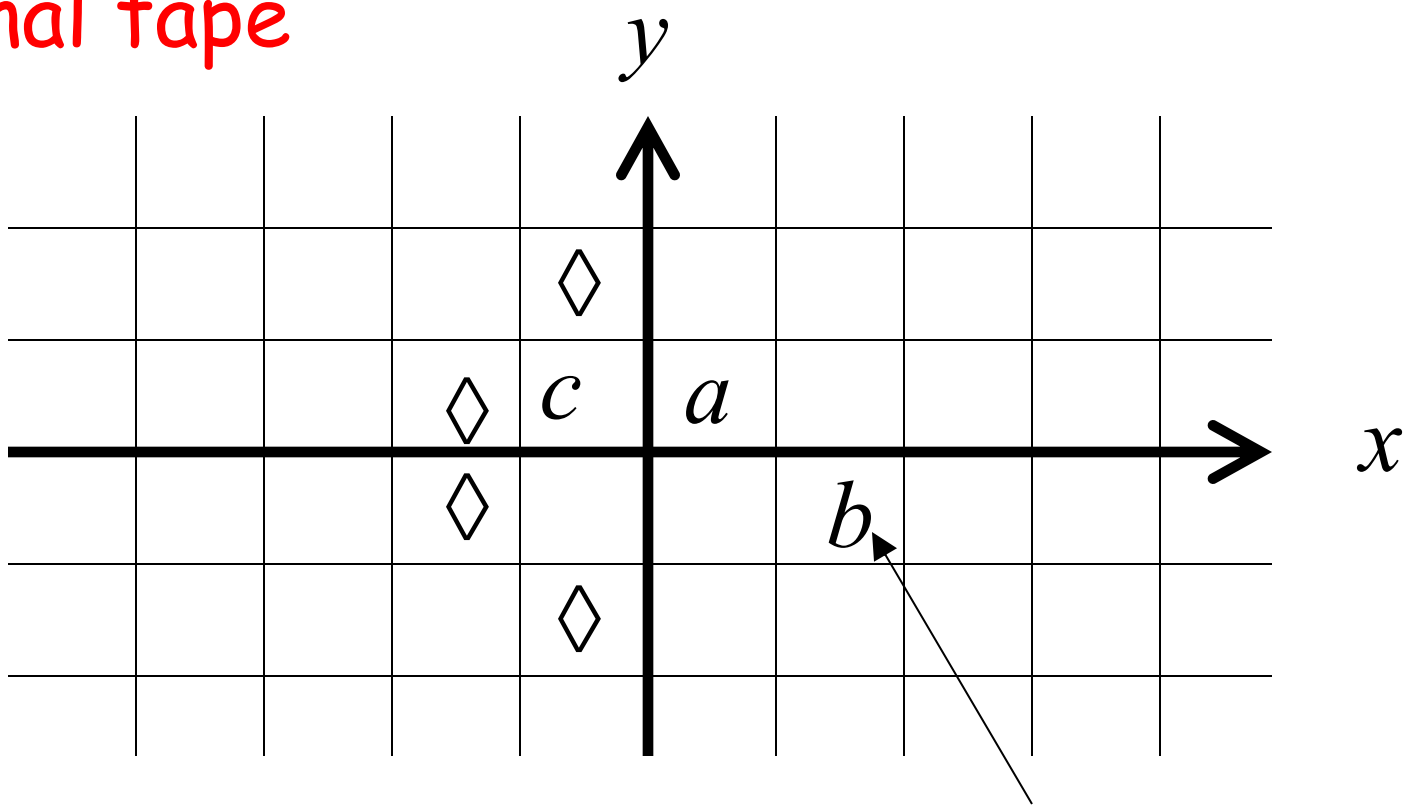


Input string appears on Tape 1


Tape 1
Time 1
Tape 2
◊ a b c ◊
◊ e f g ◊
q_1
q_1
Tape 1
Time 2
Tape 2
◊ a g c ◊
◊ e d g ◊
q_2
q_2
q_1
$(b, f) \rightarrow (g, d), L, R$
q_2

Multidimensional Turing Machines

2-dimensional tape



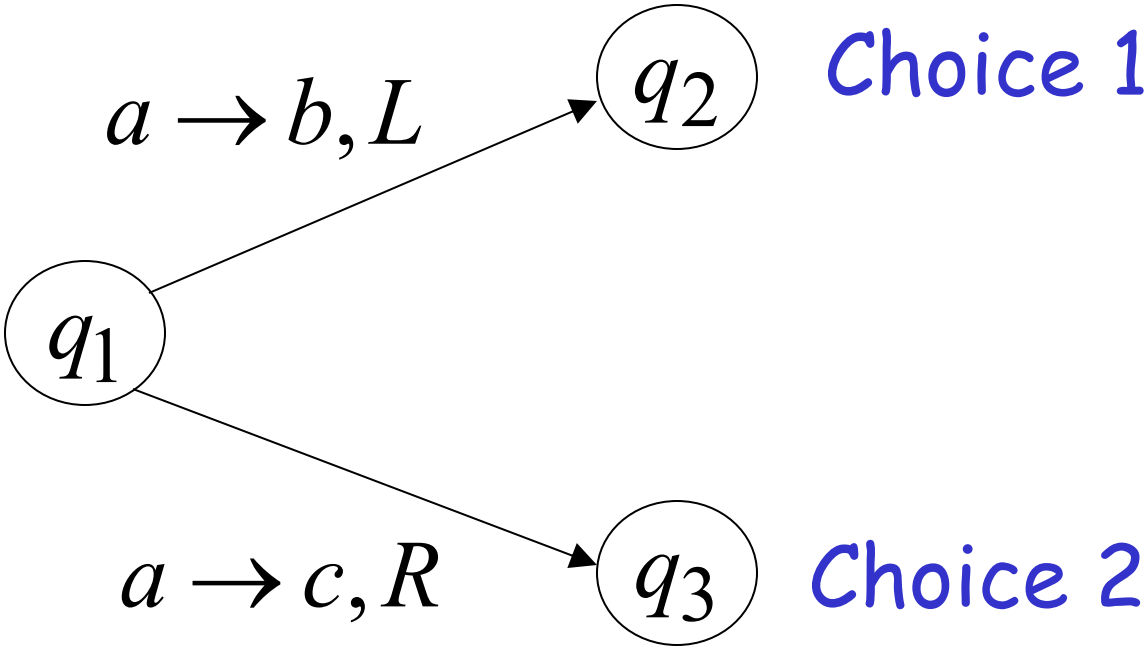
MOVES: L,R,U,D

U: up D: down

HEAD

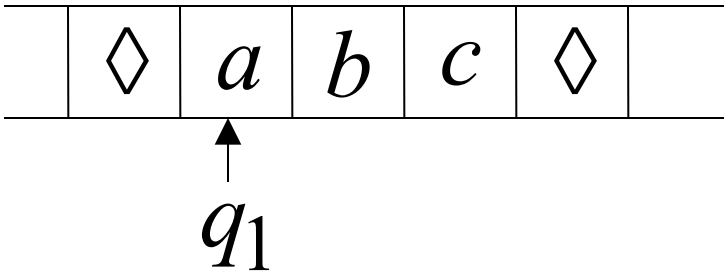
Position: +2, -1

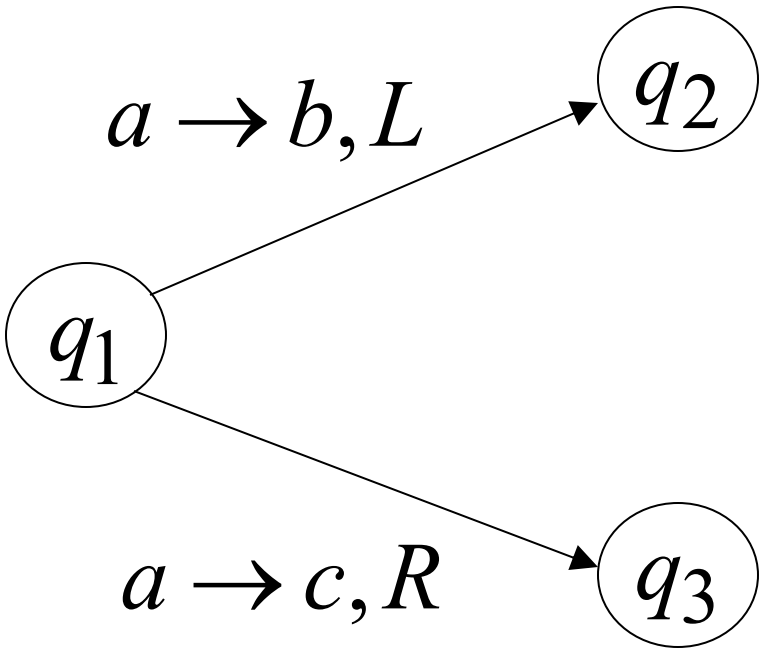
Nondeterministic Turing Machines



Allows Non Deterministic Choices

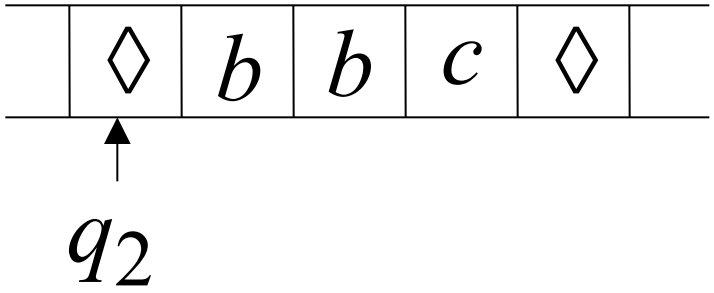
Time 0



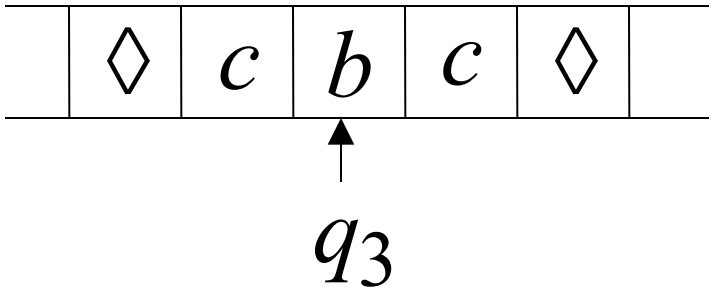


Time 1

Choice 1



Choice 2



Input string w is accepted if there is a computation:

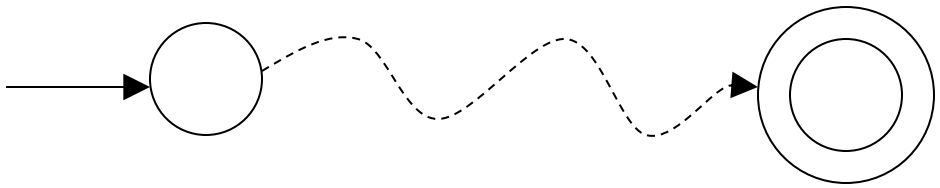
$$q_0 w \overset{*}{\succ} x\, q_f\, y$$

Initial configuration

Any accept state

Final Configuration

There is a computation:



All possible computation paths