King Saud University, department of Computer Science

Theory of Computation - CSC 339

Midterm exam – 25/2/2021

Name: ____________________

Student ID: ____________________

Section ID (or time): ____________________

Question 1..6 points

For each of the following statements, circle either True or False. (No explanation needed.)

a) If S1 = {a,b,c} and S2 = {S1}. Then, S1 and S2 are equivalent. .... True **False**

b) If L_1 is regular and L_2 is regular. Then, $L_1 \cup L_2$ is regular. .... **True** False

c) If L_1 is regular and L_2 is regular. Then, $L_1 \circ L_2$ is not regular. .... True **False**

d) The class of regular languages is closed under the (kleene) star operation. .... **True** False

e) Every NFA can be converted into an equivalent DFA. .... **True** False

f) There exists some regular expression that cannot be converted into DFA. .... True **False**

g) Any regular expression can be converted into an equivalent NFA, and vice versa. .... **True** False

h) Any context-free grammar (CFG) can be converted into an equivalent NFA. .... True **False**

i) Let $L = \{a^ib^k \mid i \neq k\}$. L is a regular language. .... True **False**

j) If string $w \in B$ passes the three conditions of the pumping lemma, then B must be regular. .... True **False**

k) All regular languages are context-free. .... **True** False

l) A CFG is said to be ambiguous if there are different parse trees for different strings. .... True **False**

Question 2..5 points

For each of the following questions, select exactly one option. Assume the alphabet Σ is {0,1} unless otherwise noted.

1) Which of the following languages is regular?

(a) $\{0^i1^i \mid i \geq 0\}$ **(b) $\{0^i1^i \mid i = 2\}$** (c) $\{0^i1^i0^i \mid i \geq 0\}$ (d) None of the mentioned.

2) Which of the following regular expressions represent the language $L = \{w \mid w$ of an even length and starts and ends with 0$\}$

(a) $0\ \Sigma^*\ \Sigma^*0^*$ (b) $0\ \Sigma\ \Sigma^*0$ **(c) $0(\Sigma\Sigma)^*0$** (d) All of the mentioned.

3) Consider a language ***L*** represented by the regular expression Σ*011*. Which of the following strings belongs to this language?

(a) 0001 (b) 000111 (c) 1101 **(d) All mentioned strings ∈ *L***

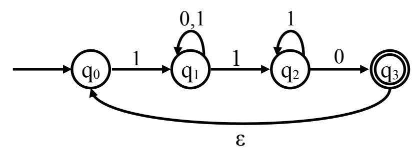
4) When using the pumping lemma to prove non-regularity, we select a string *w*, and fragment it into three parts such that *w* = xyz. Which of these parts cannot be an empty string?

(a) x **(b) y** (c) z (d) All of the mentioned

5) What regular expression is equivalent to the NFA on the right?

(a) 1*(0 ∪ 1)*110 (b) 1 (0 ∪ 1)*11*0 **(c) (1(0 ∪ 1)*11*0)+**

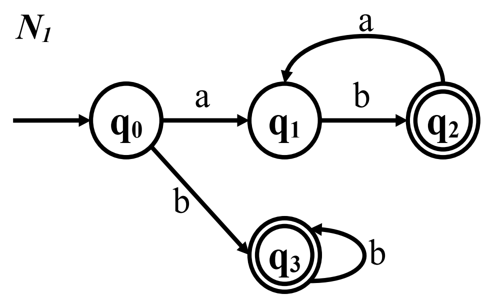
(d) (1(0 ∪ 1)*11*0)*



Question 3..7 points

Answer each of the following questions about NFA N_1.

N_1



a) (*1.5 point*) Assume Σ = {a,b}. What are Q, q_0, and F for N_1?

$Q = \{q_0, q_1, q_2, q_3\}$

$q_0 = q_0$

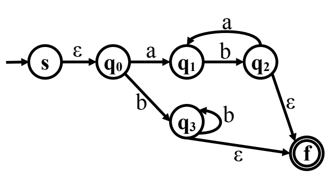
$F = \{q_2, q_3\}$

b) (*2 point*) Provide 2 example strings (of any length) that N_1 will accept, and provide 2 other example strings that N_1 will not accept.

Accepted	*Not accepted*
w1 = ab	*w3* = aa
w2 = b	*w4* = ba

c) (*3.5 points*) Convert N_1 into an equivalent regular expression. ***Show your work step by step****, and write the final expression inside the designated box. You may use the back of this page for extra space.*

1



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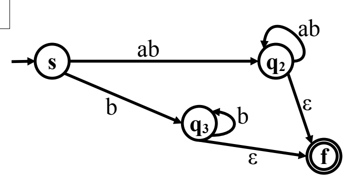
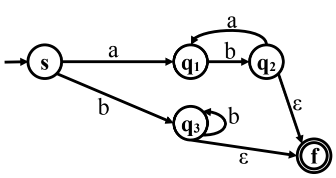


R = bb* ∪ ab(ab)*

2



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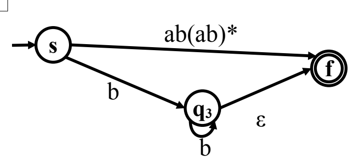

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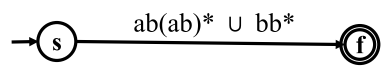

Question 4……………………………………………………………………………………………3 points

Answer the following questions about language $L_1 = \{0^n 1^{2n} \mid n \geq 0\}$.

a) (*1 point*) Informally describe the language L_1 (*i.e.,* what type of strings does it consist of?).

L_1 consists of all strings that have a number of 0's followed by twice the number of 1's. The empty string is also a member of L_1.

b) (*2 points*) List down the three conditions of the pumping lemma. And use the pumping lemma to prove that L_1 is not regular, and clearly state the length of the string you choose.

The three conditions of the pumping lemma are:
(1) $xy^iz \in L_1$ for each $i \geq 0$
(2) $|y| > 0$, and
(3) $|xy| \leq$ p.
Proof:
We assume L_1 is regular. Let $s = 0^p 1^{2p}$, where p is the pumping length. Then, $|s|$ = 3p. Condition (3) of the pumping lemma tells us that $|xy| \leq p$. By condition (2), we know that y can consist of only 0's. Let $|y| = p$. Now, if we pump the string such that $|y| = p+1$, then the new pumped string $s1 = 0^{p+1}1^{2p}$. Condition (1) tells us the new (pumped) string should $\in L_1$, but $s1$ is not in the right form, and therefore $s1 \notin L_1$. We have a contradiction; thus, L_1 is not regular.

Question 5……………………………………………………………………………………………4 points

a) (*3 points*) Design a CFG G_1 that generates the same language that NFA N_1 (in question 3) recognizes.

$A \rightarrow aB \mid bD$
$B \rightarrow bC$
$C \rightarrow aB \mid \varepsilon$
$D \rightarrow bD \mid \varepsilon$

b) (*1 point*) Provide the formal definition for G_1 (without the rules set). (hint: it should consist of: V, Σ, and S)

$V = \{A, B, C, D\}$
$\Sigma = \{a, b, \varepsilon\}$
$S = A$

c) (*2* ***bonus*** *points*) Choose 1 string that is accepted by N_1 (from your answer to question 3-b), and use grammar G_1 to draw the parse tree for that string.

```
    A
   / \
  a   B
     / \
    b   C
        |
        ε
```