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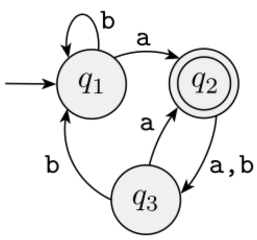
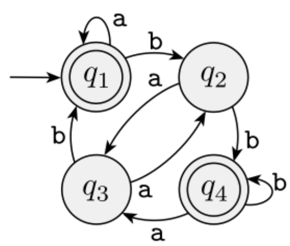
[A]**1.1** The following are the state diagrams of two DFAs, M_1 and M_2. Answer the following questions about each of these machines.



M_1



M_2

- **a.** What is the start state?
- **b.** What is the set of accept states?
- **c.** What sequence of states does the machine go through on input aabb?
- **d.** Does the machine accept the string aabb?
- **e.** Does the machine accept the string ε?

[A]**1.2** Give the formal description of the machines M_1 and M_2 pictured in Exercise 1.1.

1.3 The formal description of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{\mathtt{u}, \mathtt{d}\}, \delta, q_3, \{q_3\})$, where δ is given by the following table. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

1.4 Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote 3 (page 46) to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

a. $\{w \mid w$ has at least three a's and at least two b's$\}$

A**b.** $\{w \mid w$ has exactly two a's and at least two b's$\}$

c. $\{w|\ w$ has an even number of `a`'s and one or two `b`'s$\}$

[A]**d.** $\{w|\ w$ has an even number of `a`'s and each `a` is followed by at least one `b`$\}$

e. $\{w|\ w$ starts with an `a` and has at most one `b`$\}$

f. $\{w \mid w$ has an odd number of a's and ends with a b$\}$

g. $\{w \mid w$ has even length and an odd number of a's$\}$

1.5 Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

[A]**a.** $\{w \mid w$ does not contain the substring ab$\}$

[A]**b.** $\{w \mid w$ does not contain the substring baba$\}$

c. $\{w \mid w$ contains neither the substrings $\mathtt{ab}$ nor $\mathtt{ba}\}$

d. $\{w \mid w$ is any string not in $\mathtt{a}^*\mathtt{b}^*\}$

e. $\{w \mid w$ is any string not in $(\mathtt{ab}^+)^*\}$

f. $\{w \mid w$ is any string not in $\mathtt{a}^* \cup \mathtt{b}^*\}$

g. $\{w \mid w$ is any string that doesn't contain exactly two `a`'s$\}$

h. $\{w \mid w$ is any string except `a` and `b`$\}$

1.6 Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0,1\}$.

a. $\{w \mid w$ begins with a 1 and ends with a 0$\}$

b. $\{w \mid w$ contains at least three 1s$\}$

c. $\{w \mid w$ contains the substring `0101` (i.e., $w = x\texttt{0101}y$ for some x and y)$\}$

d. $\{w \mid w$ has length at least 3 and its third symbol is a `0`$\}$

e. $\{w \mid w$ starts with `0` and has odd length, or starts with `1` and has even length$\}$

f. $\{w \mid w$ doesn't contain the substring 110$\}$

g. $\{w \mid$ the length of w is at most $5\}$

h. $\{w \mid w$ is any string except 11 and 111$\}$

i. $\{w \mid$ every odd position of w is a 1$\}$

j. $\{w \mid w$ contains at least two $\texttt{0}$s and at most one $\texttt{1}\}$

k. $\{\varepsilon, \texttt{0}\}$

l. $\{w \mid w$ contains an even number of $\texttt{0}$s, or contains exactly two $\texttt{1}$s$\}$

m. The empty set

n. All strings except the empty string

1.7 Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{0,1\}$.

[A]**a.** The language $\{w \mid w \text{ ends with } 00\}$ with three states

b. The language of Exercise 1.6c with five states

c. The language of Exercise 1.6l with six states

d. The language $\{0\}$ with two states

e. The language $0^*1^*0^+$ with three states

[A]**f.** The language $1^*(001^+)^*$ with three states

g. The language $\{\varepsilon\}$ with one state

h. The language 0^* with one state

1.8 Use the construction in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages described in

a. Exercises 1.6a and 1.6b.

b. Exercises 1.6c and 1.6f.

1.9 Use the construction in the proof of Theorem 1.47 to give the state diagrams of NFAs recognizing the concatenation of the languages described in

a. Exercises 1.6g and 1.6i.

b. Exercises 1.6b and 1.6m.

1.10 Use the construction in the proof of Theorem 1.49 to give the state diagrams of NFAs recognizing the star of the languages described in

a. Exercise 1.6b.

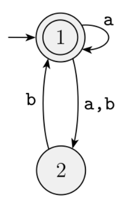
b. Exercise 1.6j.

c. Exercise 1.6m.

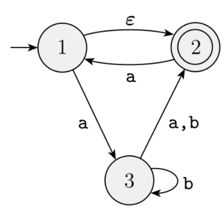
1.12 Let $D = \{w \mid w$ contains an even number of a's and an odd number of b's and does not contain the substring ab$\}$. Give a DFA with five states that recognizes D and a regular expression that generates D. (Suggestion: Describe D more simply.)

1.13 Let F be the language of all strings over $\{0,1\}$ that do not contain a pair of 1s that are separated by an odd number of symbols. Give the state diagram of a DFA with five states that recognizes F. (You may find it helpful first to find a 4-state NFA for the complement of F.)

1.16 Use the construction given in Theorem 1.39 to convert the following two nondeterministic finite automata to equivalent deterministic finite automata.



(a)



(b)

1.17 **a.** Give an NFA recognizing the language $(\texttt{01} \cup \texttt{001} \cup \texttt{010})^*$.

b. Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.

1.18 Give regular expressions generating the languages of Exercise 1.6.

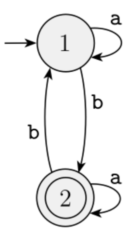
1.19 Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

a. $(0 \cup 1)^*000(0 \cup 1)^*$

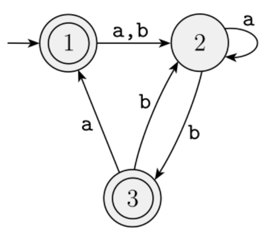
b. $(((00)^*(11)) \cup 01)^*$

c. $\emptyset^*$

1.21 Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.



(a)



(b)

1.28 Convert the following regular expressions to NFAs using the procedure given in Theorem 1.54. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

a. $\mathtt{a(abb)^* \cup b}$

b. $\mathtt{a^+ \cup (ab)^+}$

c. $\mathtt{(a \cup b^+)a^+b^+}$

1.29 Use the pumping lemma to show that the following languages are not regular.

[A]**a.** $A_1 = \{\mathtt{0}^n\mathtt{1}^n\mathtt{2}^n \mid n \geq 0\}$

b. $A_2 = \{www \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$

[A]**c.** $A_3 = \{\mathtt{a}^{2^n} \mid n \geq 0\}$ (Here, $\mathtt{a}^{2^n}$ means a string of 2^n a's.)

2.1 Recall the CFG G_4 that we gave in Example 2.4. For convenience, let's rename its variables with single letters as follows.

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T \times F \mid F \\ F &\rightarrow (E) \mid \mathtt{a} \end{aligned}$$

Give parse trees and derivations for each string.

a. `a`

b. `a+a`

c. `a+a+a`

d. `((a))`

[A]**2.3** Answer each part for the following context-free grammar G.

$$\begin{aligned} R &\to XRX \mid S \\ S &\to \mathtt{a}T\mathtt{b} \mid \mathtt{b}T\mathtt{a} \\ T &\to XTX \mid X \mid \varepsilon \\ X &\to \mathtt{a} \mid \mathtt{b} \end{aligned}$$

a. What are the variables of G?
b. What are the terminals of G?
c. Which is the start variable of G?
d. Give three strings in $L(G)$.
e. Give three strings *not* in $L(G)$.
f. True or False: $T \Rightarrow \mathtt{aba}$.
g. True or False: $T \overset{*}{\Rightarrow} \mathtt{aba}$.
h. True or False: $T \Rightarrow T$.
i. True or False: $T \overset{*}{\Rightarrow} T$.
j. True or False: $XXX \overset{*}{\Rightarrow} \mathtt{aba}$.
k. True or False: $X \overset{*}{\Rightarrow} \mathtt{aba}$.
l. True or False: $T \overset{*}{\Rightarrow} XX$.
m. True or False: $T \overset{*}{\Rightarrow} XXX$.
n. True or False: $S \overset{*}{\Rightarrow} \varepsilon$.
o. Give a description in English of $L(G)$.

2.4 Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{\mathtt{0},\mathtt{1}\}$.

[A]**a.** $\{w \mid w \text{ contains at least three } \mathtt{1}\text{s}\}$
b. $\{w \mid w \text{ starts and ends with the same symbol}\}$
c. $\{w \mid \text{the length of } w \text{ is odd}\}$
[A]**d.** $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is a } \mathtt{0}\}$
e. $\{w \mid w = w^{\mathcal{R}}, \text{ that is, } w \text{ is a palindrome}\}$
f. The empty set

2.5 Give informal descriptions and state diagrams of pushdown automata for the languages in Exercise 2.4.

2.6 Give context-free grammars generating the following languages.

[A]**a.** The set of strings over the alphabet $\{\mathtt{a},\mathtt{b}\}$ with more a's than b's

b. The complement of the language $\{\mathtt{a}^n\mathtt{b}^n \mid n \geq 0\}$

[A]**c.** $\{w\#x \mid w^{\mathcal{R}} \text{ is a substring of } x \text{ for } w, x \in \{\mathtt{0},\mathtt{1}\}^*\}$

d. $\{x_1\#x_2\# \cdots \#x_k \mid k \geq 1, \text{ each } x_i \in \{\mathtt{a}, \mathtt{b}\}^*, \text{ and for some } i \text{ and } j,\ x_i = x_j^{\mathcal{R}}\}$

[A]**2.7** Give informal English descriptions of PDAs for the languages in Exercise 2.6.

2.9 Give a context-free grammar that generates the language

$$A = \{\mathtt{a}^i\mathtt{b}^j\mathtt{c}^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}.$$

Is your grammar ambiguous? Why or why not?

2.10 Give an informal description of a pushdown automaton that recognizes the language A in Exercise 2.9.

2.11 Convert the CFG G_4 given in Exercise 2.1 to an equivalent PDA, using the procedure given in Theorem 2.20.

2.12 Convert the CFG G given in Exercise 2.3 to an equivalent PDA, using the procedure given in Theorem 2.20.

2.14 Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \to BAB \mid B \mid \varepsilon$$
$$B \to 00 \mid \varepsilon$$

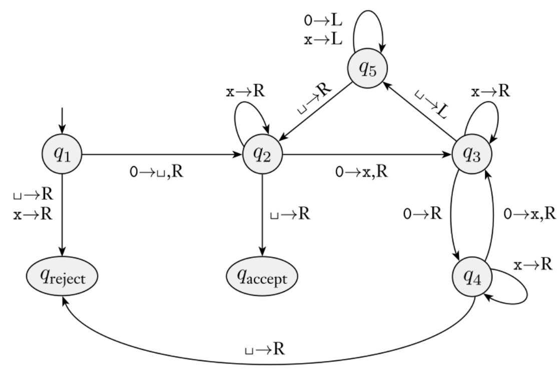
3.1 This exercise concerns TM M_2, whose desc
ample 3.7. In each of the parts, give the sec
when started on the indicated input string.

a. 0.

A**b.** 00.

c. 000.

d. 000000.



3.2 This exercise concerns TM M_1, whose description and state diagram appear in Example 3.9. In each of the parts, give the sequence of configurations that M_1 enters when started on the indicated input string.

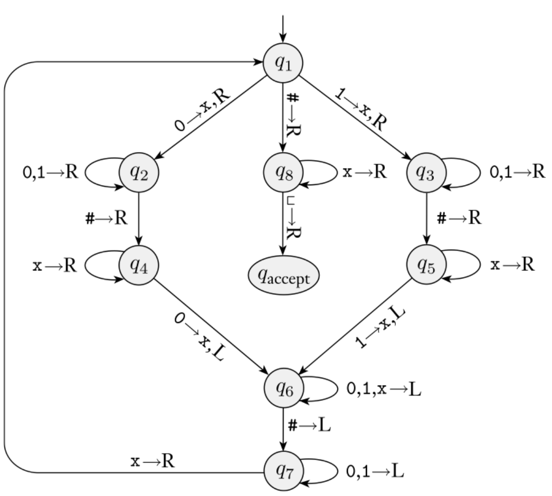
A**a.** 11.

b. 1#1.

c. 1##1.

d. 10#11.

e. 10#10.



3.5 Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

a. Can a Turing machine ever write the blank symbol ␣ on its tape?

b. Can the tape alphabet Γ be the same as the input alphabet Σ?

c. Can a Turing machine's head *ever* be in the same location in two successive steps?

d. Can a Turing machine contain just a single state?

3.8 Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet $\{0,1\}$.

[A]**a.** $\{w \mid w \text{ contains an equal number of 0s and 1s}\}$

b. $\{w \mid w \text{ contains twice as many 0s as 1s}\}$

c. $\{w \mid w \text{ does not contain twice as many 0s as 1s}\}$