Polynomial Space (PS)

- The emphasis on the computation resources is usually on the time used to perform the computation process (especially when measuring a performance issue or complexity).
- However, the amount of space required is often just as important (called the space requirement).

Polynomial Space (PS)

In any Turing machine:

- the time used is the number of steps taken before halting or entering a final state.
- The space required is defined as the number of distinct tape squares (cells) “visited” by the read/write head.

Polynomial Space (PS)

Proof

Since the time taken to visit all cells < = the time used, then the number of cells visited < = the time used.

→ the number of cells visited cannot be more than the steps of computation

This implies:

Any Polynomial Solvable in polynomial time, It’s solvable in polynomial space.

Polynomial Space (PS)

Although all problems solvable in polynomial time can be solved in polynomial space, it is still a controversial and unresolved question whether there exist problems solvable in polynomial space which cannot be solved in polynomial time.

- This is so difficult to conclude since all problems in NP, including P and NP-C problems are solved in both polynomial space and polynomial time.

Polynomial Space (PS)

- PS

  is the class of all languages recognizable by polynomial space bounded DTM that halts on all inputs.

  i.e. All and Only the languages that are L_M for some polynomial space bounded DTM M.

Nondeterministic Polynomial Space (NPS)

- NPS

  is the class of all languages recognizable by polynomial space bounded NDTM that halts on at least one of the possible input structures.

  i.e. All and Only the languages that are L_M for some polynomial space bounded NDTM M.

PS and NPS

Theorem

If M is a polynomial-space bounded TM (deterministic or nondeterministic), and p(n) is its polynomial space bound, then there is a constant c such that if M accepts its input w of length n, it does so within $c^{1 + p(n)}$ moves

PS and NPS

Evidently, PS є NPS, since every DTM is technically NDTM.

However, The surprising result is that:

PS = NPS

PS and NPS

Savitch's Theorem

PS = NPS

PS and NPS

To be more accurate

Savitch's Theorem states that any $p(n)$-space NDTM can be converted to a $p^2(n)$-space DTM

one of the earliest results on space complexity

PS and NPS

At the End, we conclude that:

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PS} = \mathbf{NPS} \subseteq \mathbf{EXPTIME}$$

PS and NPS