Tutorial #3

Regular Languages and Regular Expression

Exercise 1.1

- Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them in one DFA simulates in parallel M1 and M2 In all parts, $\Sigma = \{a, b\}$.

- {w| w has even length and an odd number of a’s}

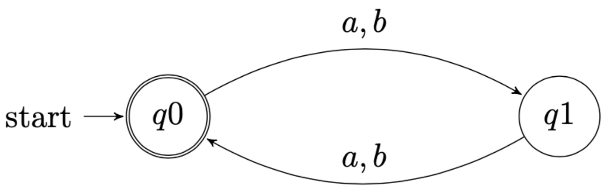


Figure 1: L1: w:w has even length

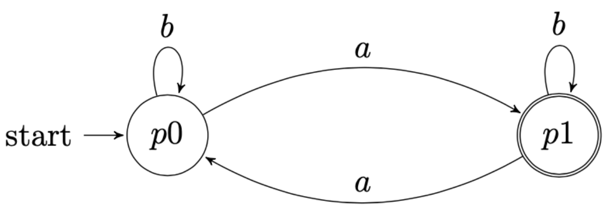


Figure 2: L2: w:w has odd number of a's

Solution

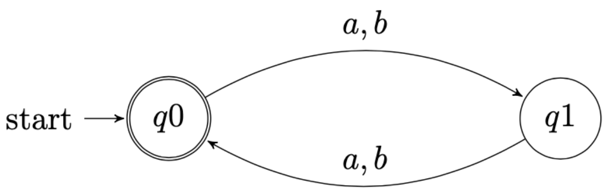


Figure 1: L1: w:w has even length

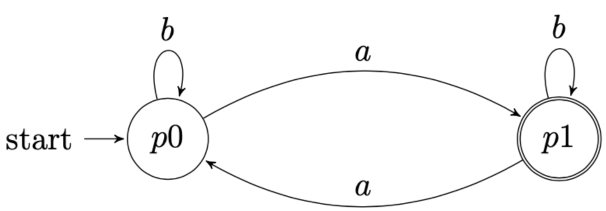


Figure 2: L2: w:w has odd number of a's

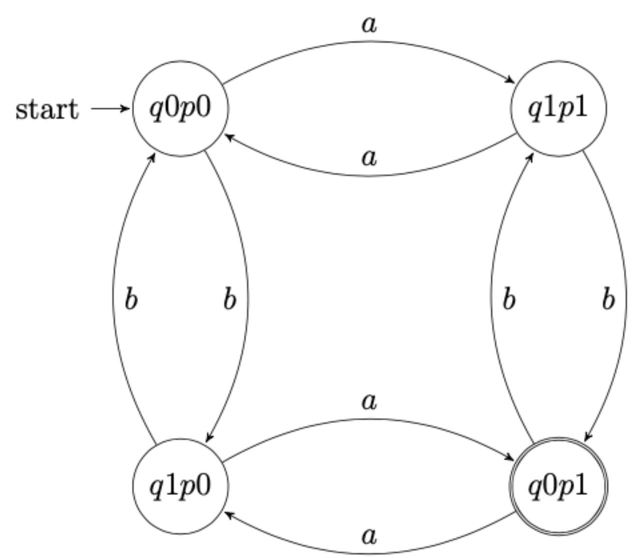


Figure 3: L: w:w has odd number of a's and has even length

Exercise 2.1

- Construct a regular expression representing the following languages:

1. Over the alphabet $\{a, b, c\}$, in which for every string w it holds that the number of a's is even.

2. Over the alphabet {0, 1}, in which w consists of alternating zeroes and ones.

- Over the alphabet $\{a, b, c\}$, in which for every string w it holds that the number of a's is even.

$$(b+c)^*(a(b+c)^*\ a\ (b+c)^*)^*$$

- Over the alphabet {0, 1}, in which *w* consists of alternating zeroes and ones.

$$(1+\lambda)(01)^{*}(0+\lambda)$$

Exercise 2.2

- Consider the following regular expression:
- $(\mathbf{0}(\mathbf{23})^{*}\mathbf{1})^{*}$
- 1. Find a string over $\{0,1,2,3\}^4$ which matches the expression.

  0101

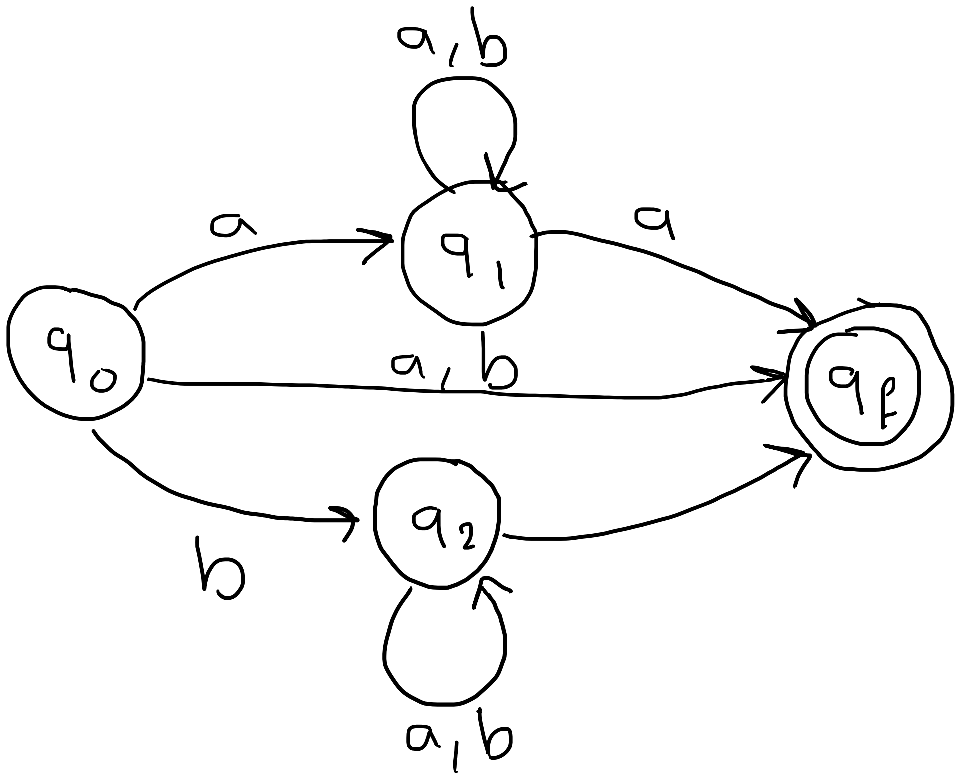
- 2. Find a string over $\{0,1,2,3\}^4$ which does not match the expression.

0123

Exercise 3

- Construct a finite automaton (deterministic or non-deterministic) for the following regular expression:

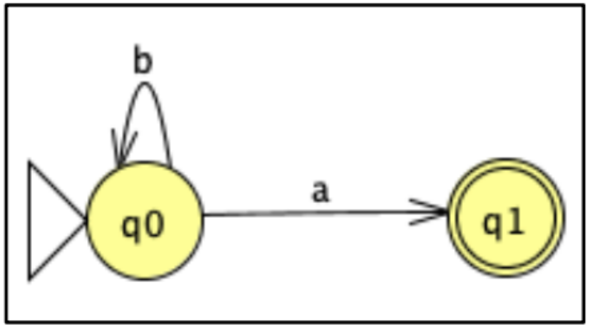
$$\boldsymbol{a}(\boldsymbol{a} \cup \boldsymbol{b})^{*}\boldsymbol{a} \cup \boldsymbol{b}(\boldsymbol{a} \cup \boldsymbol{b})^{*}\boldsymbol{b} \cup \boldsymbol{a} \cup \boldsymbol{b}$$


a, b
a
q_1
a
q_0
a, b
q_f
q_2
b
a, b

Exercise 4

- Find out which regular expressions describes the following automata's languages:

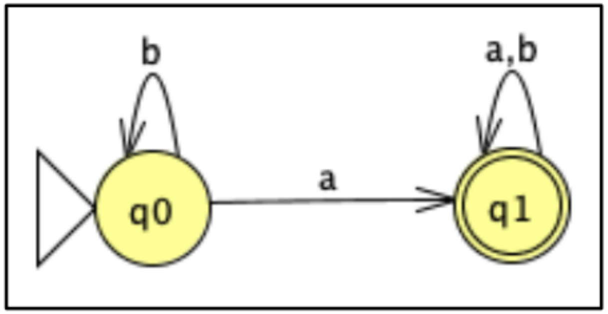
b*a



Exercise 4

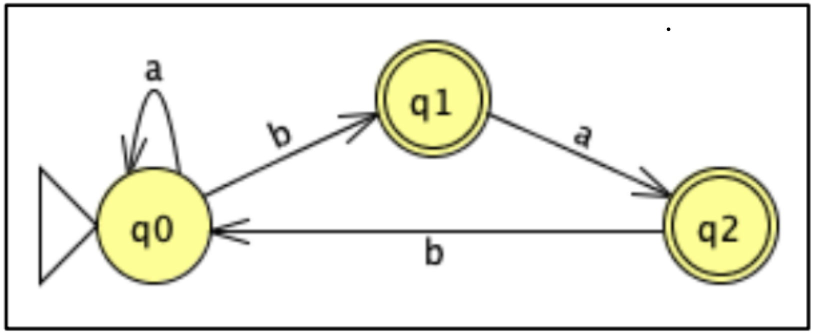
- Find out which regular expressions describes the following automata's languages:

$b^* a (a+b)^*$



Exercise 4

- Find out which regular expressions describes the following automata's languages:



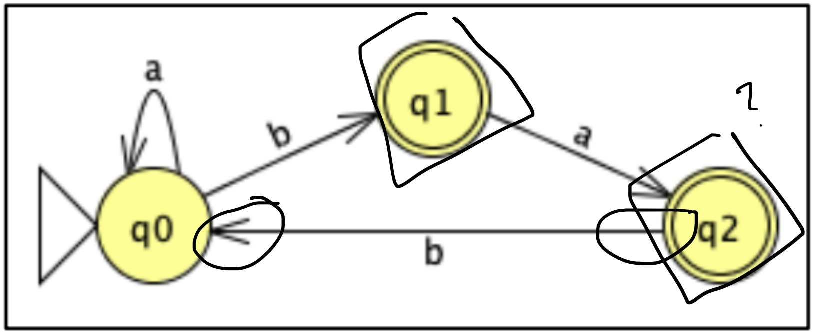
① get rid of trap state → DFA

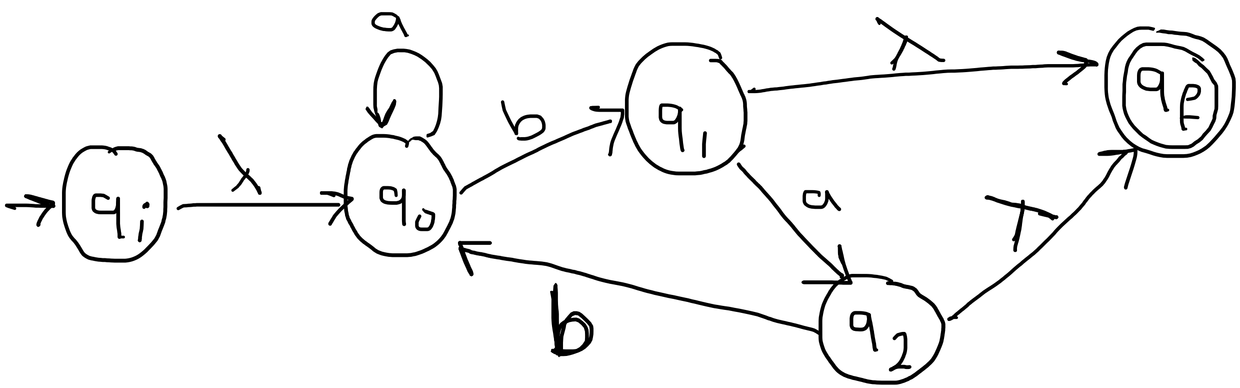
② only one final state

③ to make it easy

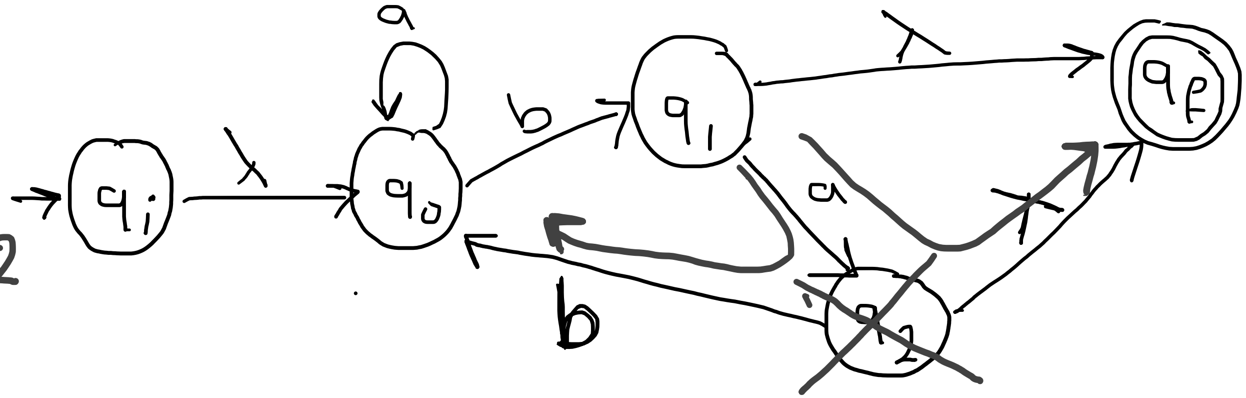
→ make sure that no incoming edge to initial state

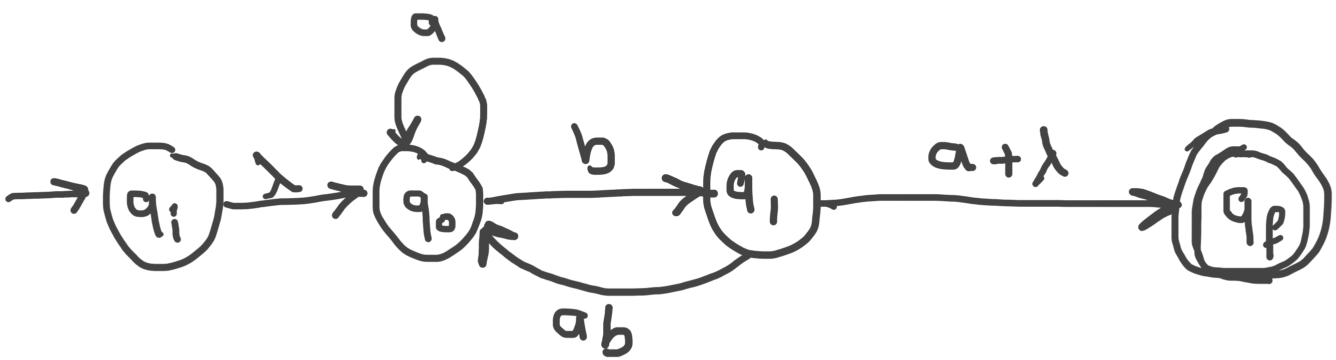
→ " " " no outgoing edge from final state


a
b
q1
a
q0
b
q2

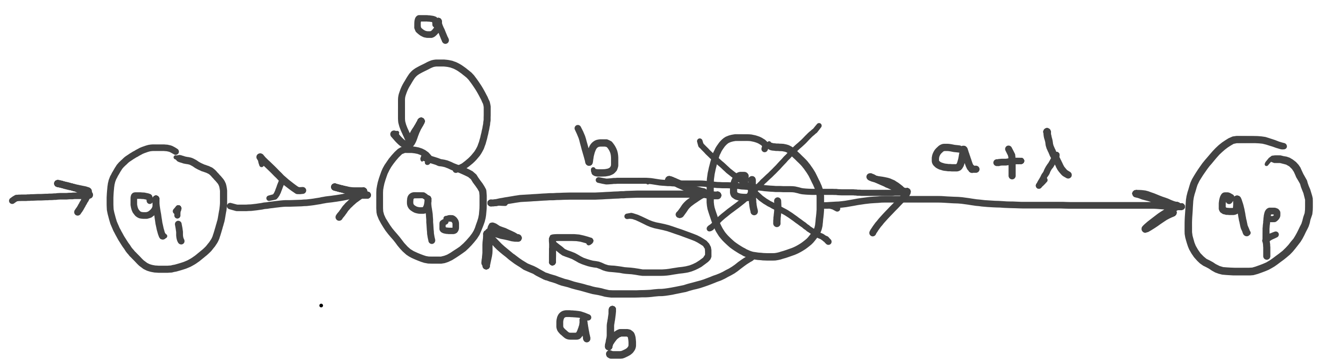

a
λ
b
λ
q_i
q_0
q_1
q_f
a
λ
b
q_2

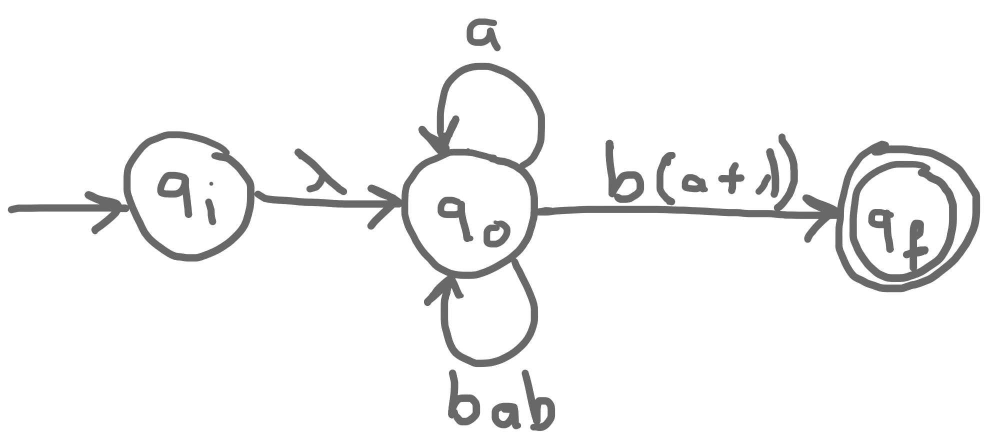
remove state 2



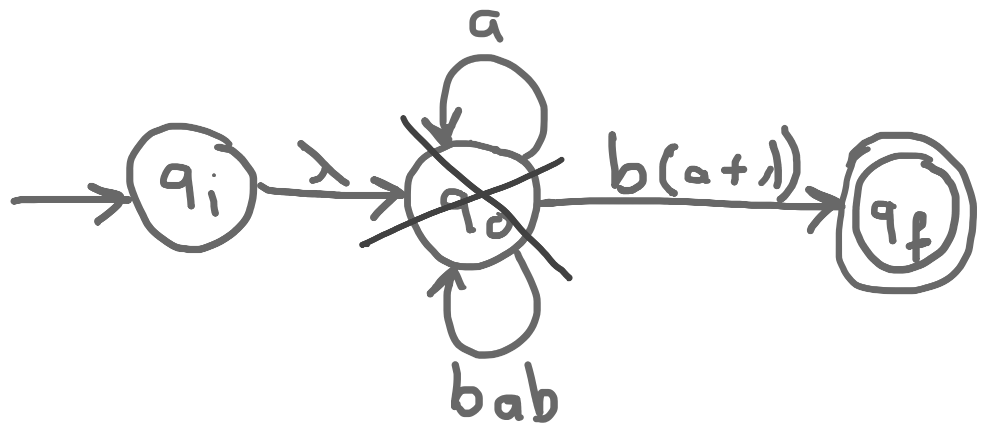


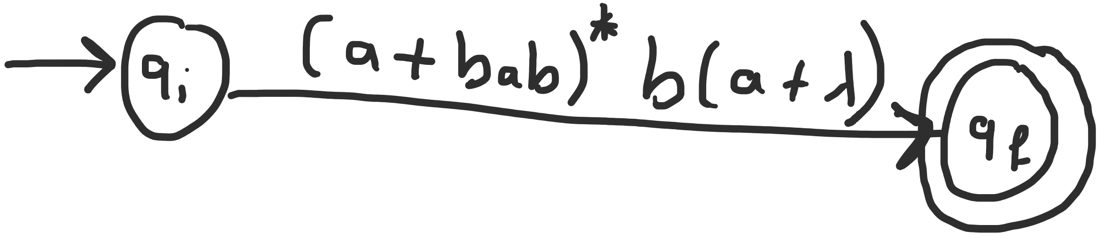
remove
State 1





remove
state 0





Exercise 5

- Consider the alphabet Σ = {a, b], give a RE for each of the following languages:

$L = \{w \in \Sigma * \mid w\ contains\ the\ substring\ aaa\}$

(a+b)*aaa(a+b)*

$L = \{w \in \Sigma * \mid w\ contains\ the\ substring\ aaa\ as\ a\ prefix\}$

aaa(a+b)*

$L = \{w \in \Sigma * \mid w\ contains\ the\ substring\ aaa\ as\ a\ suffix\}$

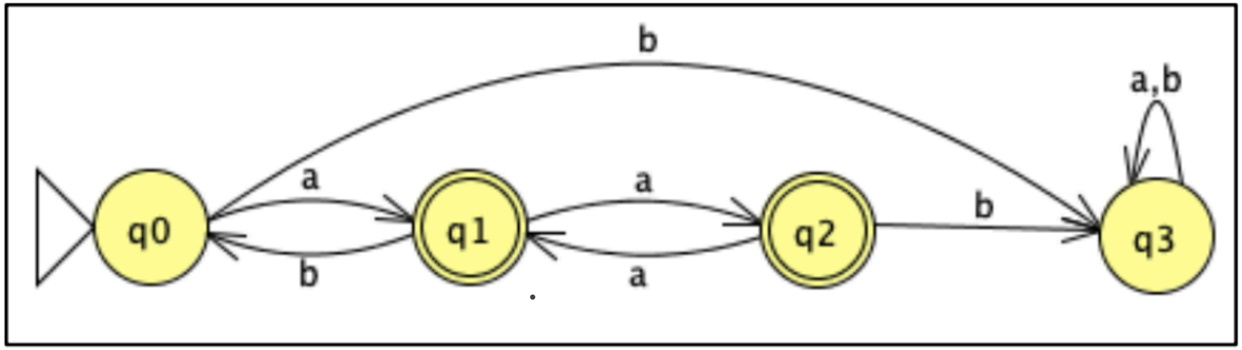
(a+b)*aaa

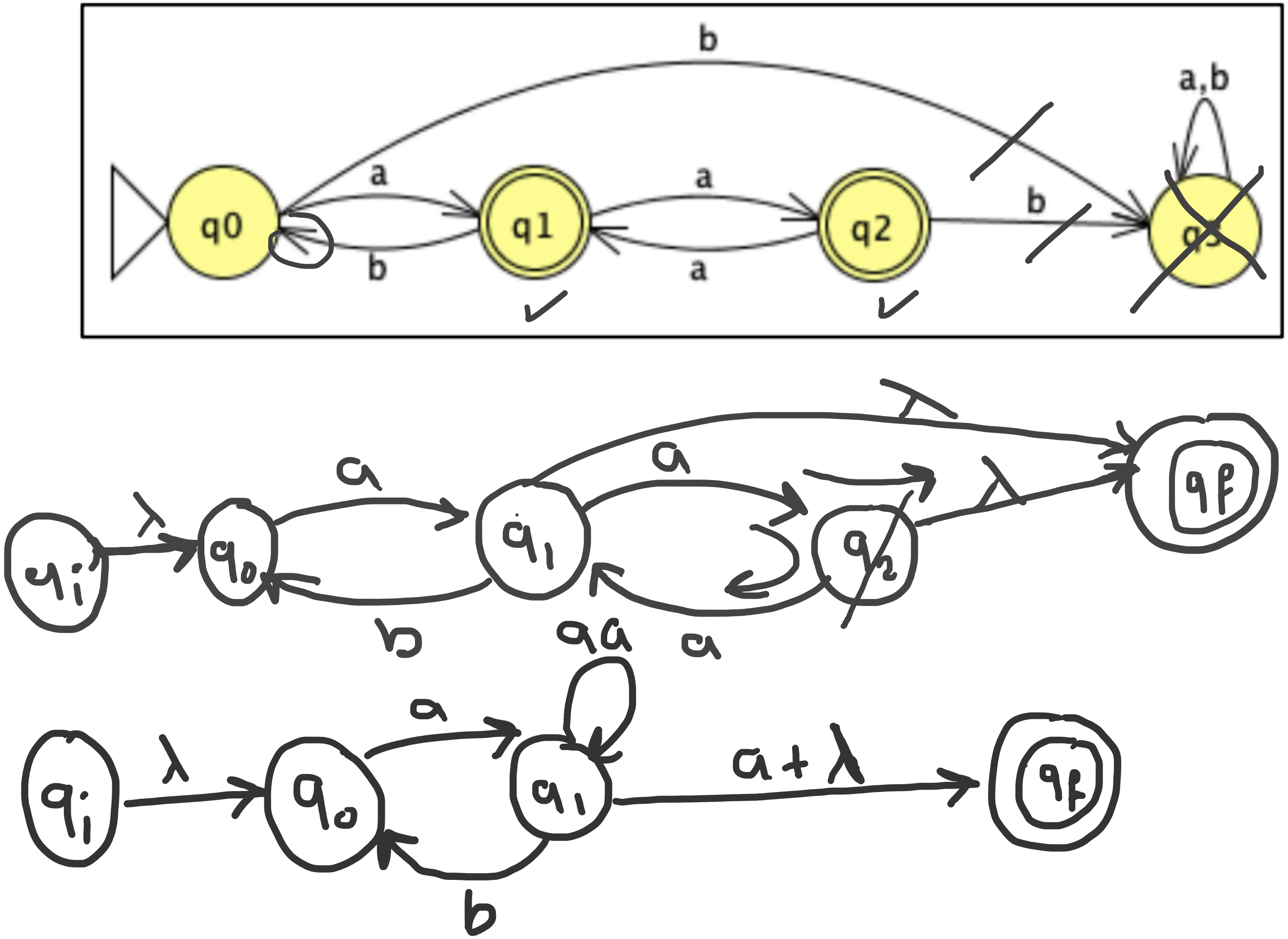
$L = \{w \in \Sigma * \mid w\ does\ not\ contains\ the\ substring\ aaa\}$

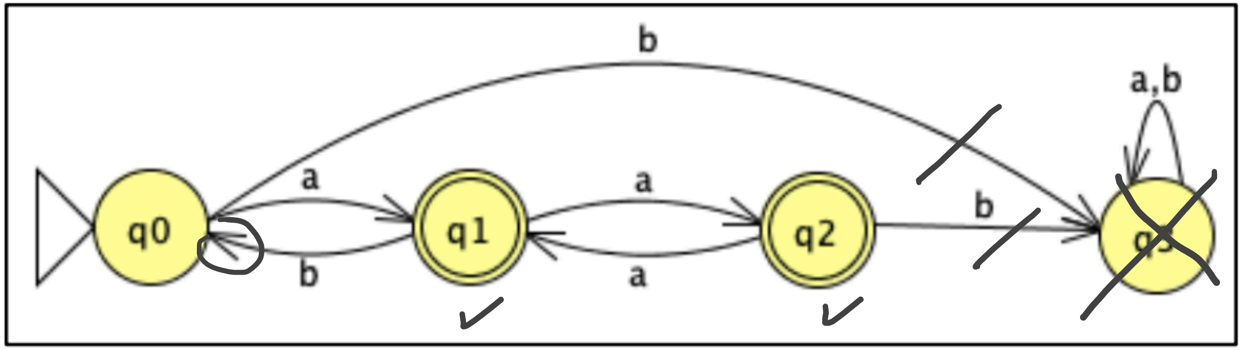
(b+ab+aab)* (aa+a+ λ)

Exercise 6

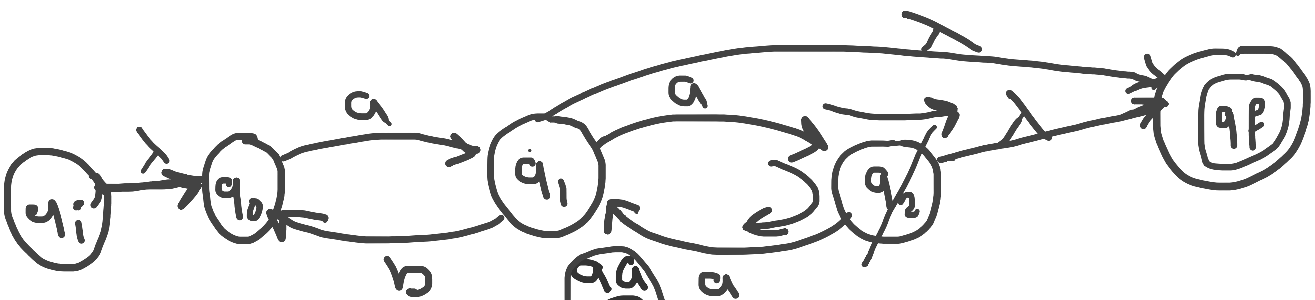
- Construct a regular expression which is equivalent to the following (deterministic) finite automata:



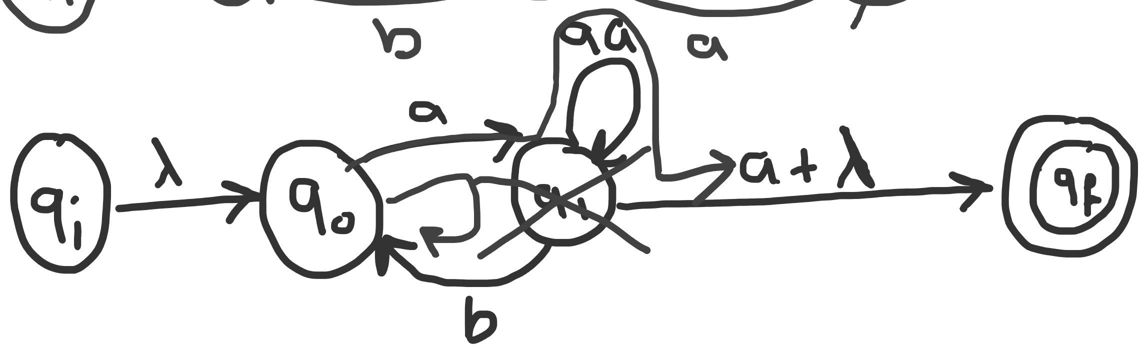

b
a,b
a
a
b
q0
q1
q2
q3
b
a
λ
a
a
λ
qi
q0
q1
q2
qf
b
aa
a
a
λ
a + λ
qi
q0
q1
qf
b



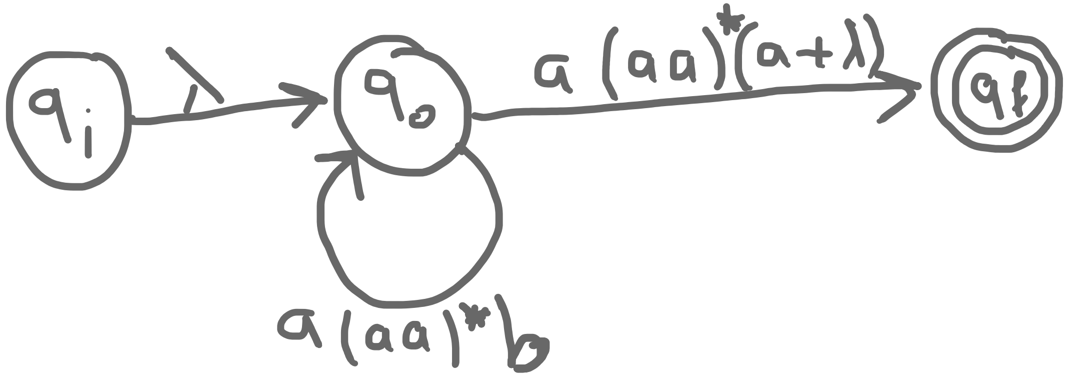
b
a,b
a
a
b
q0
q1
q2
q3
b
a



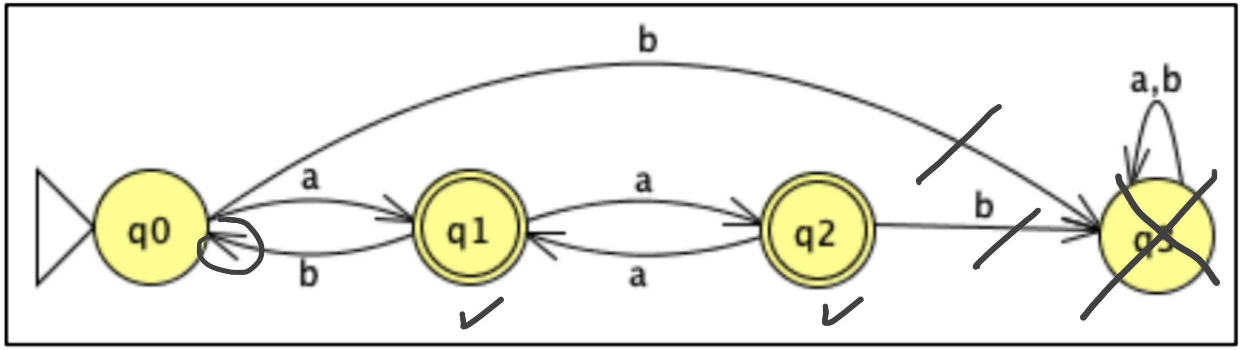
λ
a
a
λ
λ
q_i
q_0
q_1
q_2
q_f
b
a

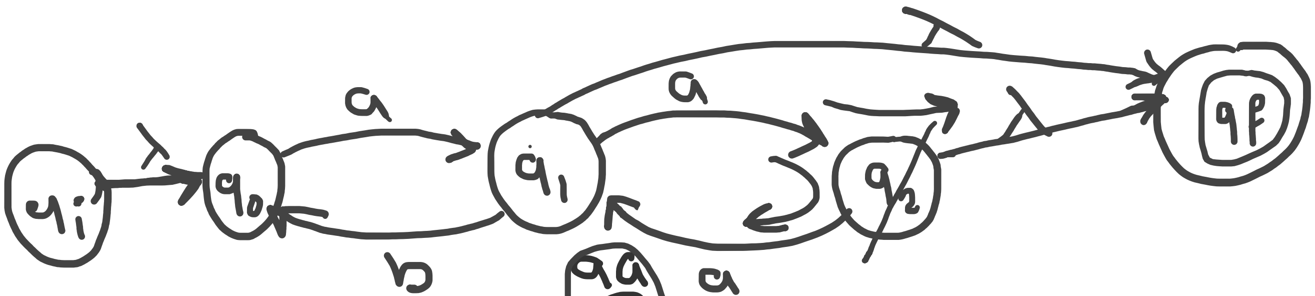


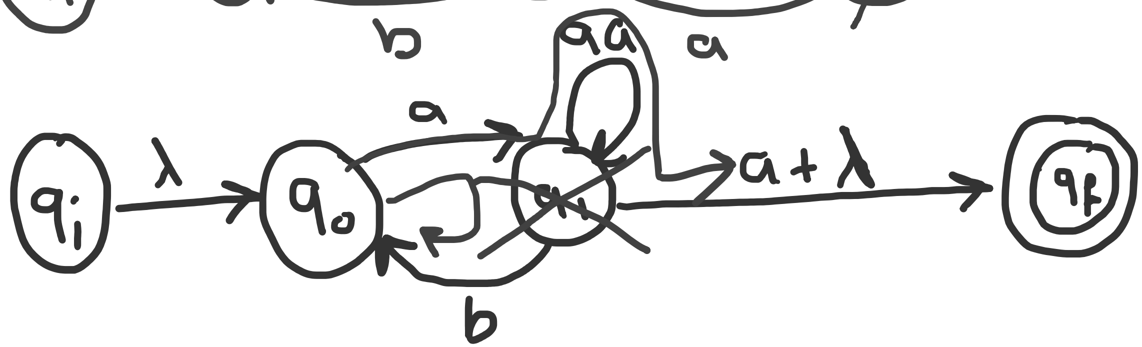
aa
a
λ
a + λ
q_i
q_0
q_1
q_f
b

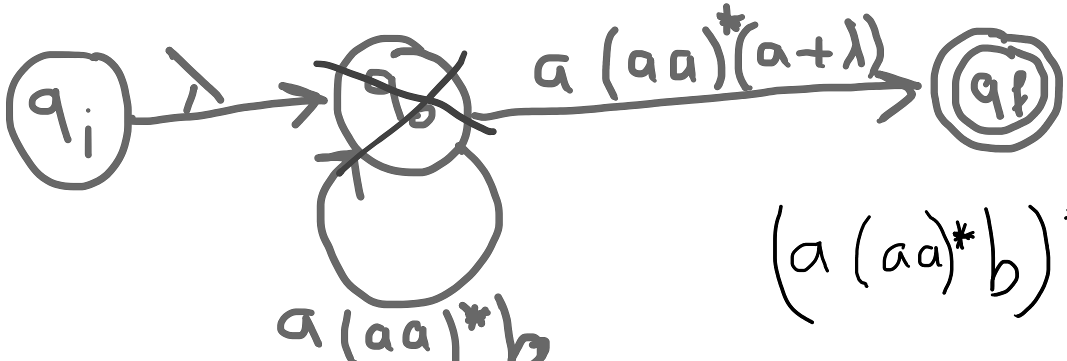


λ
a (aa)*(a+λ)
q_i
q_0
q_f
a (aa)*b









$(a(aa)^*b)^*a(aa)^*(a+\lambda)$