KING SAUD UNIVERSITY
COLLEGE OF COMPUTER AND INFORMATION SCIENCES
DEPARTMENT OF COMPUTER SCIENCE

Theory of Computation (CSC 339) – Fall 2023
Instructor: Prof. M.B. Menai

Tutorial 4: Pumping Lemma

1. Use the pumping lemma to show that the following languages are not regular.
   (a) $A_1 = \{0^n 1^n 2^n | n \geq 0\}$.
   (b) $A_2 = \{a^{2^n} | n \geq 0\}$, a^{2^n} is a string of 2^n a's.
2. For each of the following languages, give the minimum pumping length and justify your answer.
   (a) 0001^*
   (b) 0^*1^*
   (c) $0^*1^+0^+1^* + 10^*1$
3. Prove that the following language is not regular: $\{0^m 1^n | m \neq n\}$.

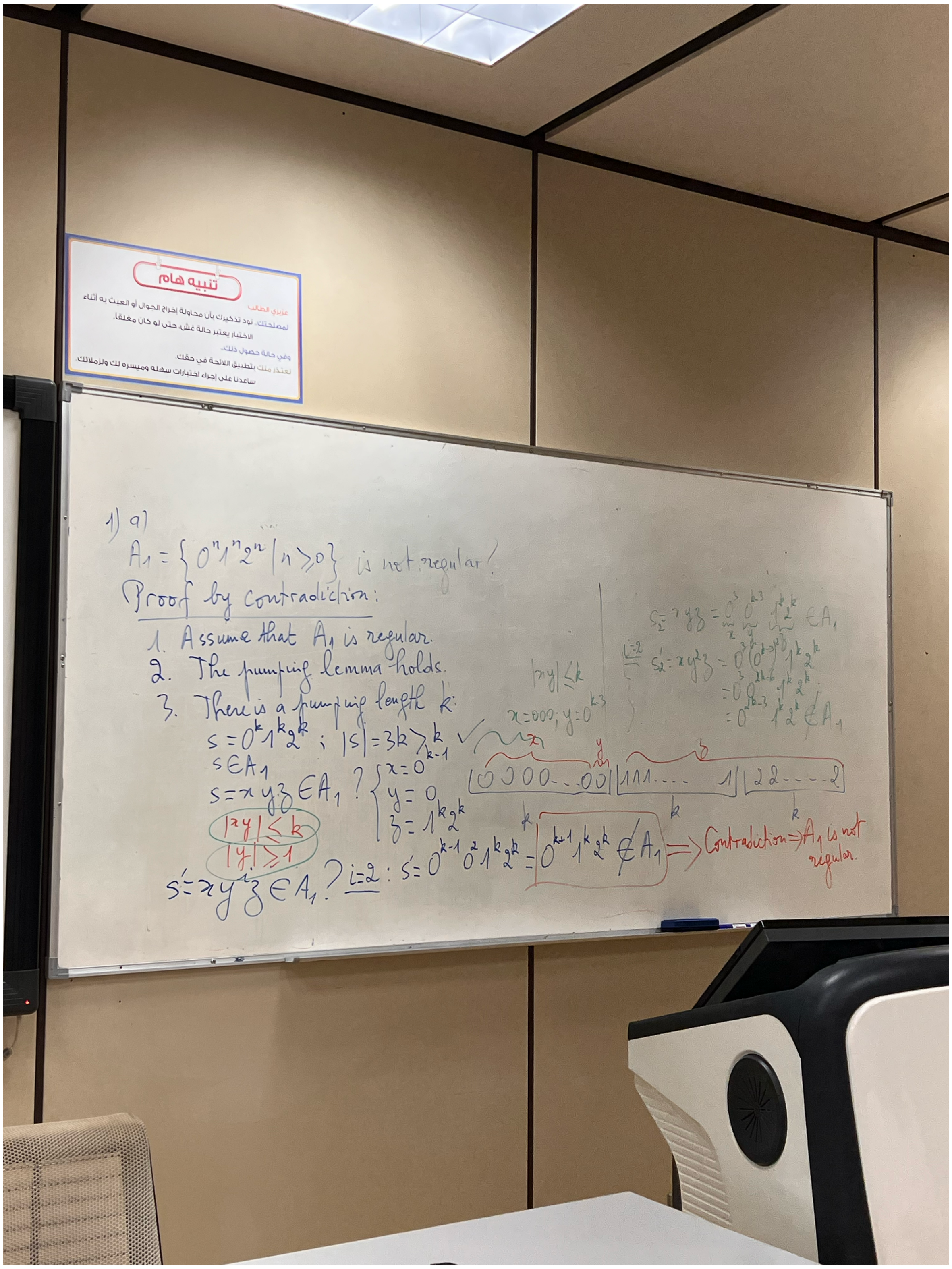
pumping lemma:

1) pumping length = $K > 0$

2) $|s| \geq K$: $s = xyz$ $\left\{ \begin{array}{l} |xy| \leq K \\ |y| \geq 1 \end{array} \right.$
$s \in L$

$\Rightarrow s' = xy^iz \in L$
$(i \geq 0)$

1a)


1) a)
A1 = {0^n 1^n 2^n | n ≥ 0} is not regular
Proof by contradiction:
1. Assume that A1 is regular.
2. The pumping lemma holds.
3. There is a pumping length k.
s = 0^k 1^k 2^k ; |s| = 3k ≥ k
s ∈ A1
s = xyz ∈ A1 ?
x = 0^(k-1)
y = 0
z = 1^k 2^k
|xy| ≤ k
|y| ≥ 1
s' = xy^i z ∈ A1 ? i=2 : s' = 0^(k-1) 0^2 1^k 2^k = 0^(k+1) 1^k 2^k ∉ A1
Contradiction ⇒ A1 is not regular.
|xy| ≤ k
x = 000; y = 0^(k-3)

2) pumping length:

a) minimum substring we can repeat to obtain other strings

$L = \{000, 0001, 00011, 00011, 000111, \ldots\}$

0001 : هي الي دايم تكرر

the minimum pumping:

length = 4

b)

$L = \{0^* 1^*\}$ $L = \{0, 1, 01, 000, 111, 000111$

مانقدر نقول ε عشان εεεε كونك تضيفها مع ε

الناتج دايم ε (ما تنتج شي جديد)

لذا minimum length هو = 1 عشان يديك تكرر 0,

أو 1

c) $L = \{ 0^* 1^+ 0^+ 1^* + 1 0^* 1 \}$

min = 2

min = 3

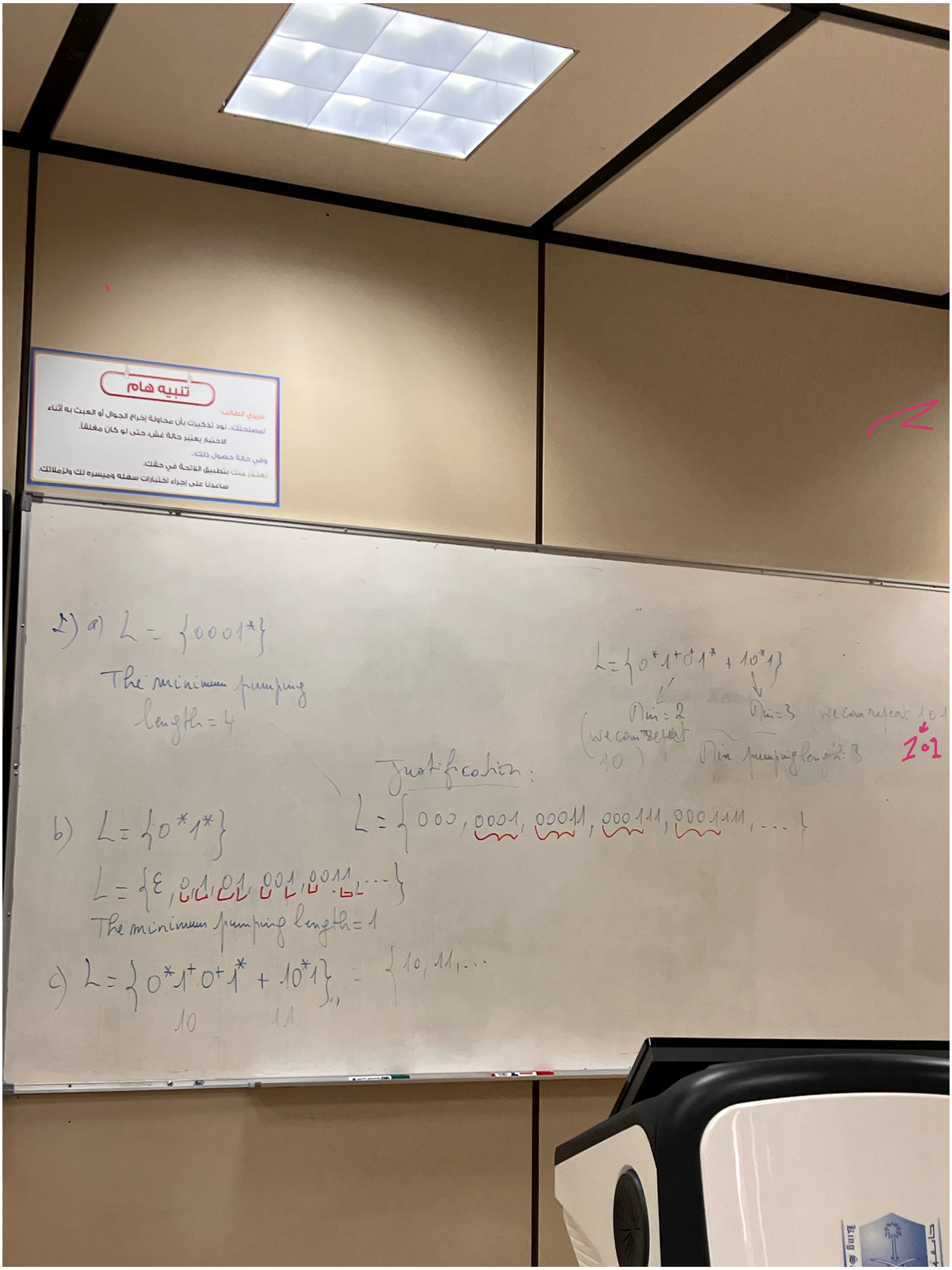
min pumping length = 3

$L = \{ 10, 1100, 1111000, \ldots \}$

we can repeat 10

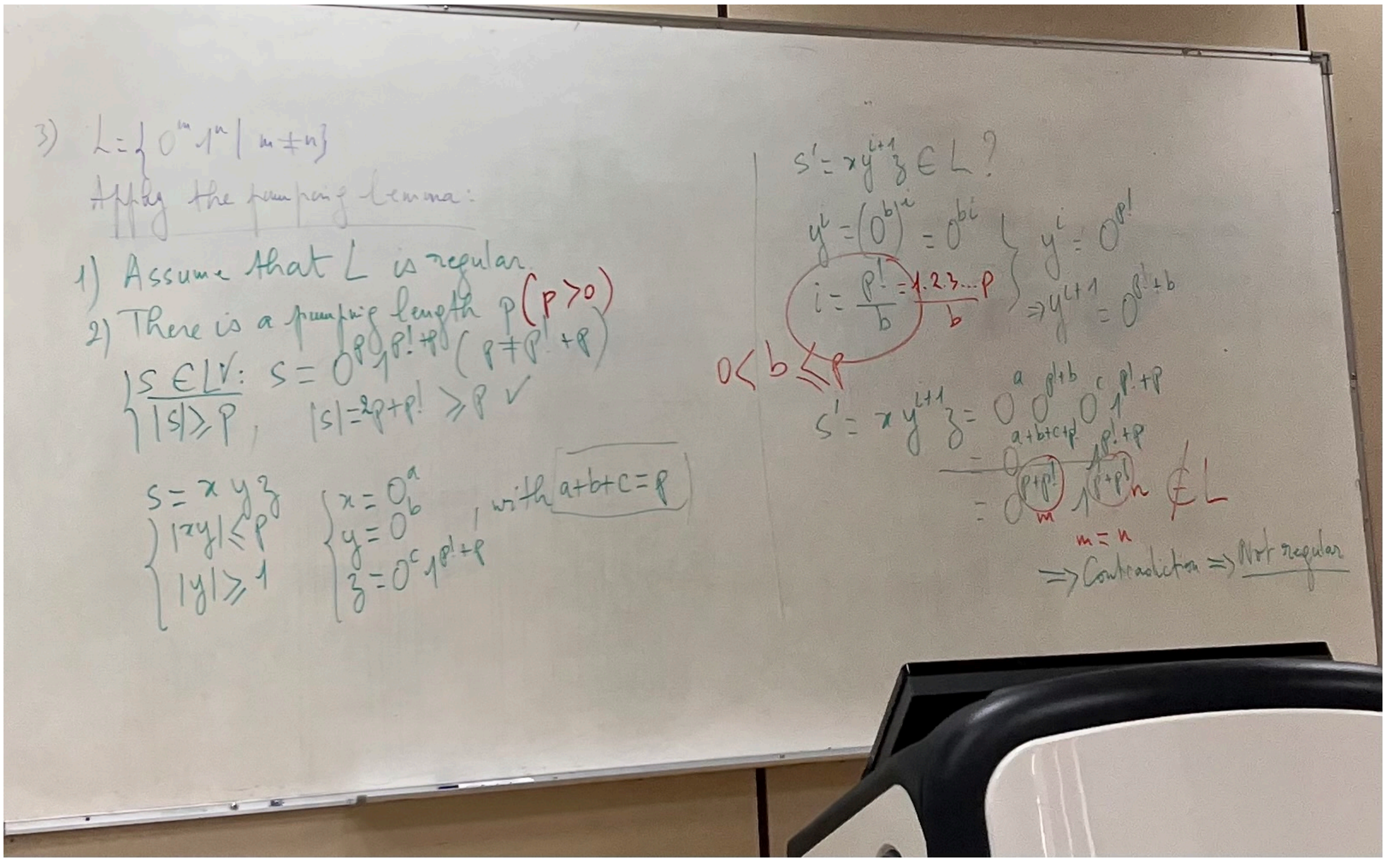
$L = \{ 11, 101, 1001, 10001 \}$

we can repeat 101



تنبيه هام
عزيزي الطالب
لمصلحتك، نود تذكيرك بأن محاولة إخراج الجوال أو العبث به أثناء
الاختبار يعتبر حالة غش، حتى لو كان مغلقاً.
وفي حالة حصول ذلك،
نعتذر منك بتطبيق اللائحة في حقك.
ساعدنا على إجراء اختبارات سهلة وميسره لك ولزملائك.
2) a) $L = \{0001^*\}$
The minimum pumping length = 4
Justification:
$L = \{000, 0001, 00011, 000111, 0001111, \dots\}$
b) $L = \{0^*1^*\}$
$L = \{\varepsilon, 0, 1, 01, 001, 0011, \dots\}$
The minimum pumping length = 1
c) $L = \{0^*1^+0^+1^* + 10^*1\}$ = $\{10, 11, \dots$
10 11
$L = \{0^*1^+0^+1^* + 10^*1\}$
Min = 2 Min = 3
(we can repeat 10)
we can repeat 101
Min pumping length = 3
101

3)



3) $L = \{0^m 1^n \mid m \neq n\}$
Apply the pumping lemma:
1) Assume that L is regular
2) There is a pumping length p (p>0)
$s \in L$: $s = 0^p 1^{p!+p}$ ($p \neq p!+p$)
$|s| \geq p$, $|s| = 2p+p! \geq p$ ✓
$s = xyz$
$|xy| \leq p$
$|y| \geq 1$
$x = 0^a$
$y = 0^b$
$z = 0^c 1^{p!+p}$
with $\boxed{a+b+c=p}$
$s' = xy^{i+1}z \in L$?
$y^i = (0^b)^i = 0^{bi}$
$i = \frac{p!}{b} = \frac{1.2.3...p}{b}$
$0 < b \leq p$
$y^i = 0^{p!}$
$\Rightarrow y^{i+1} = 0^{p!+b}$
$s' = xy^{i+1}z = 0^a 0^{p!+b} 0^c 1^{p!+p}$
$= 0^{a+b+c+p!} 1^{p!+p}$
$= 0^{p+p!} 1^{p+p!} \notin L$
m = n
$\Rightarrow$ Contradiction $\Rightarrow$ Not regular

$\{0^n 1^n ; n \geq 0\}$

1- Assume A is Regular language

2- PL holds

3- there is pumping length K

$S = 0^K 1^K$; $|S| = 2K > K$

$S \in A$

$|xy| \leq K$ | $x = 0^{K-2}$
$|y| \geq 1$ | $y = 00$
 | $z = 1^K$

$S' = xy^iz \in A$? $i=2$; $S' = 0^{K-2} (00)^2 1^K$
$= 0^{K-2}\, 0000\, 1^K$
$= 0^{K+2} 1^K \notin A$

contradiction

$L = \{ 0^{2n} 1^{n} : n \geq 0 \}$

1 - Assume L is regular

2 - $P\,L$ holds

3 - there is $P\,L$ "K" $K > 0$

$s = 0^{2K} 1^{K}$: $|s| = 3K > K$

$s \in L$

$|xy| \leq K$

$|y| \geq 1$

$x = 0$

$y = 0$

$z = 0^{2K-2} 1^{K}$

$s' = xy^{i}z \in L$? $i = 2$

$s' = 0\,(0)^{2}\,0^{2K-2}\,1^{K}$

$= 0^{3}\,0^{2K-2}\,1^{K}$

$= 0^{2K+1}\,1^{K} \notin L$

contradiction L is not regular