CSC 339 – Theory of Computation
Fall 2023

11. Turing Machine Variations

Outline

- Turing's thesis
- Variations of the Turing Machine

Turing's thesis (1930):

Any computation carried out by mechanical means can be performed by a Turing Machine.

Algorithm:

An algorithm for a problem is a Turing Machine which solves the problem.

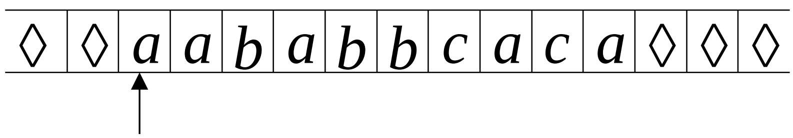
The algorithm describes the steps of the mechanical means. This is easily translated to computation steps of a Turing machine.

There exists an algorithm to solve a problem, means that there exists a Turing Machine that executes the algorithm.

Variations of the Turing Machine

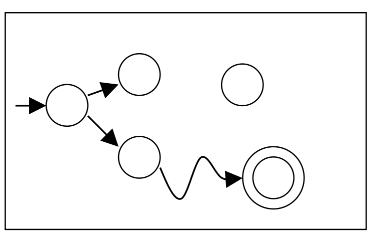
The Standard Model

Infinite Tape



Read-Write Head (Left or Right)

Control Unit



Deterministic

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Variations of the Standard Model

Turing machines with:
- Stay-Option
- Semi-Infinite Tape
- Off-Line
- Multitape
- Multidimensional
- Nondeterministic

Different Turing Machine **Classes**

Same Power of two machine classes:

Both classes accept the same set of languages.

Theorem:

Each new class has the same power with Standard Turing Machine (accept Turing-Recognizable Languages).

Simulation:

A technique to prove same power. Simulate the machine of one class with a machine of the other Class.

First Class
Original Machine

M_1

Second Class
Simulation Machine

M_2

M_1

M_2 simulates M_1

Configurations in the Original Machine M_1
have corresponding configurations in the
Simulation Machine M_2.

$$\text{Original Machine:} \quad d_0 \overset{M_1}{\succ} d_1 \succ \cdots \succ d_n$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$\text{Simulation Machine:} \quad d'_0 \overset{*}{\succ} d'_1 \overset{*}{\succ} \cdots \overset{*}{\succ} d'_n$$

$$M_2$$

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Accepting Configuration

Original Machine: d_f

$\updownarrow$

Simulation Machine: d'_f

The Simulation Machine
and the Original Machine
accept the same strings

$$L(M_1) = L(M_2)$$

Turing Machines with Stay-Option

The head can stay in the same position

◊	◊	a	a	b	a	b	b	c	a	c	a	◊	◊	◊

Left, Right, Stay

L,R,S: possible head moves

Example:

Time 1

$\Diamond$	$\Diamond$	a	a	b	a	b	b	c	a	c	a	$\Diamond$	$\Diamond$	$\Diamond$

q_1

Time 2

$\Diamond$	$\Diamond$	b	a	b	a	b	b	c	a	c	a	$\Diamond$	$\Diamond$	$\Diamond$

q_2

$$q_1 \xrightarrow{a \rightarrow b, S} q_2$$

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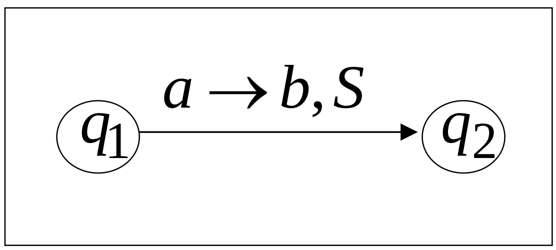
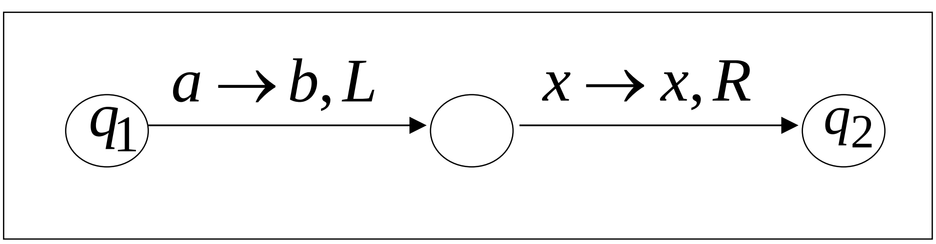
Stay-Option Machine



Simulation in Standard Machine



For every possible tape symbol x

For other transitions nothing changes

Stay-Option Machine

$$q_1 \xrightarrow{a \rightarrow b, L} q_2$$

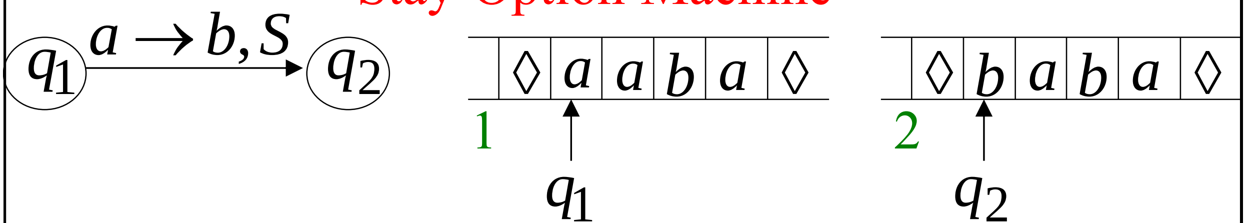
Simulation in Standard Machine

$$q_1 \xrightarrow{a \rightarrow b, L} q_2$$

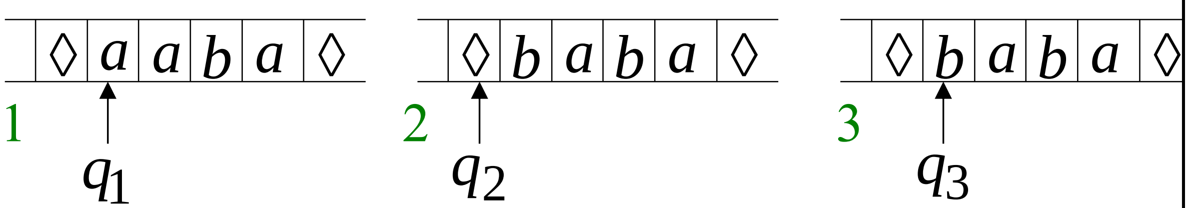
Similar for Right moves

Example of simulation

Stay-Option Machine



Simulation in Standard Machine



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Theorem:

Stay-Option machines have the same power with Standard Turing machines.

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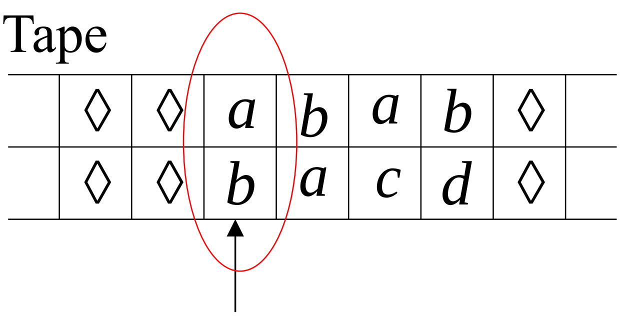
Multiple Track Tape TM

A useful trick to perform more complicated simulations

One Tape



track 1

track 2

One head

One symbol (a, b)

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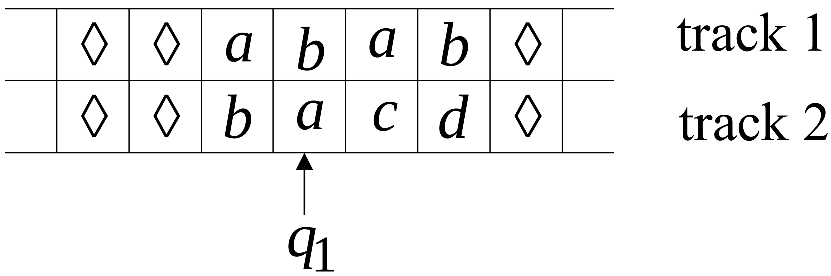
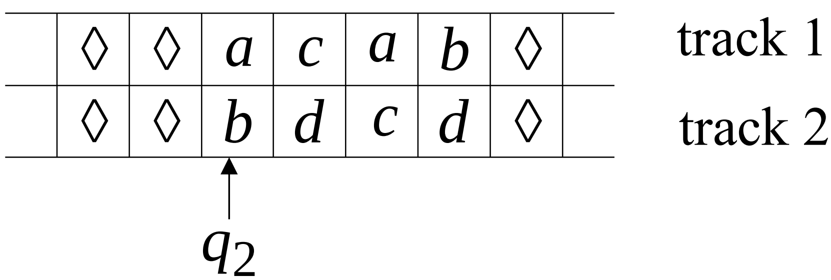
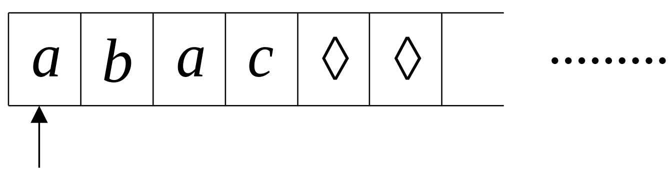




$$q_1 \xrightarrow{(b,a)\rightarrow(c,d),L} q_2$$

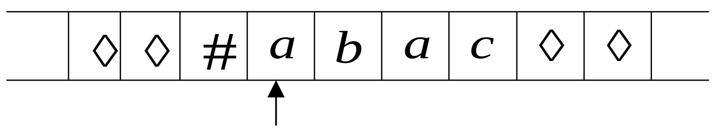
Semi-Infinite Tape

The head extends infinitely only to the right.



- Initial position is the leftmost cell.
- When the head moves left from the border, it returns to the same position.

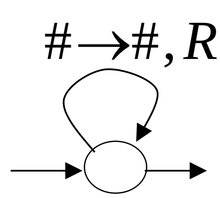
Standard Turing machines simulate Semi-Infinite machines:



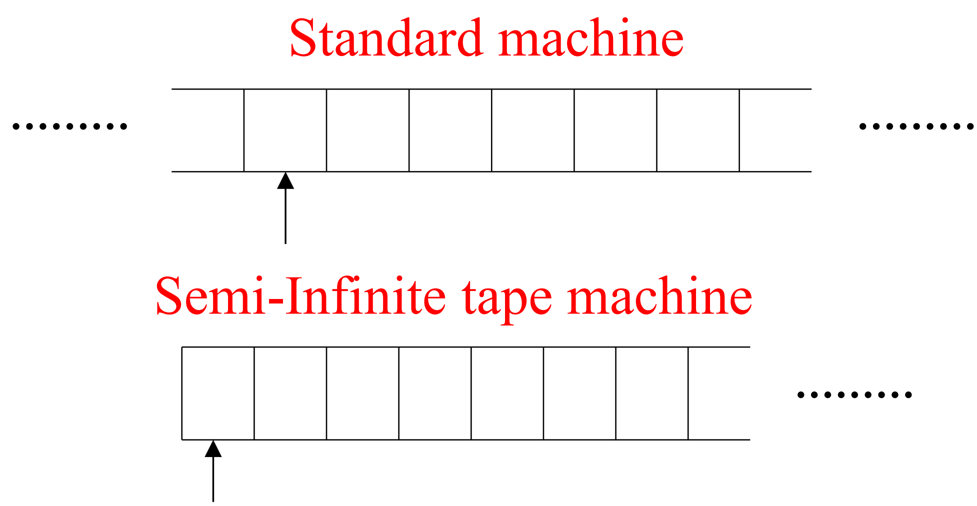
Standard Turing Machine

a. Insert special symbol #
at left of input string

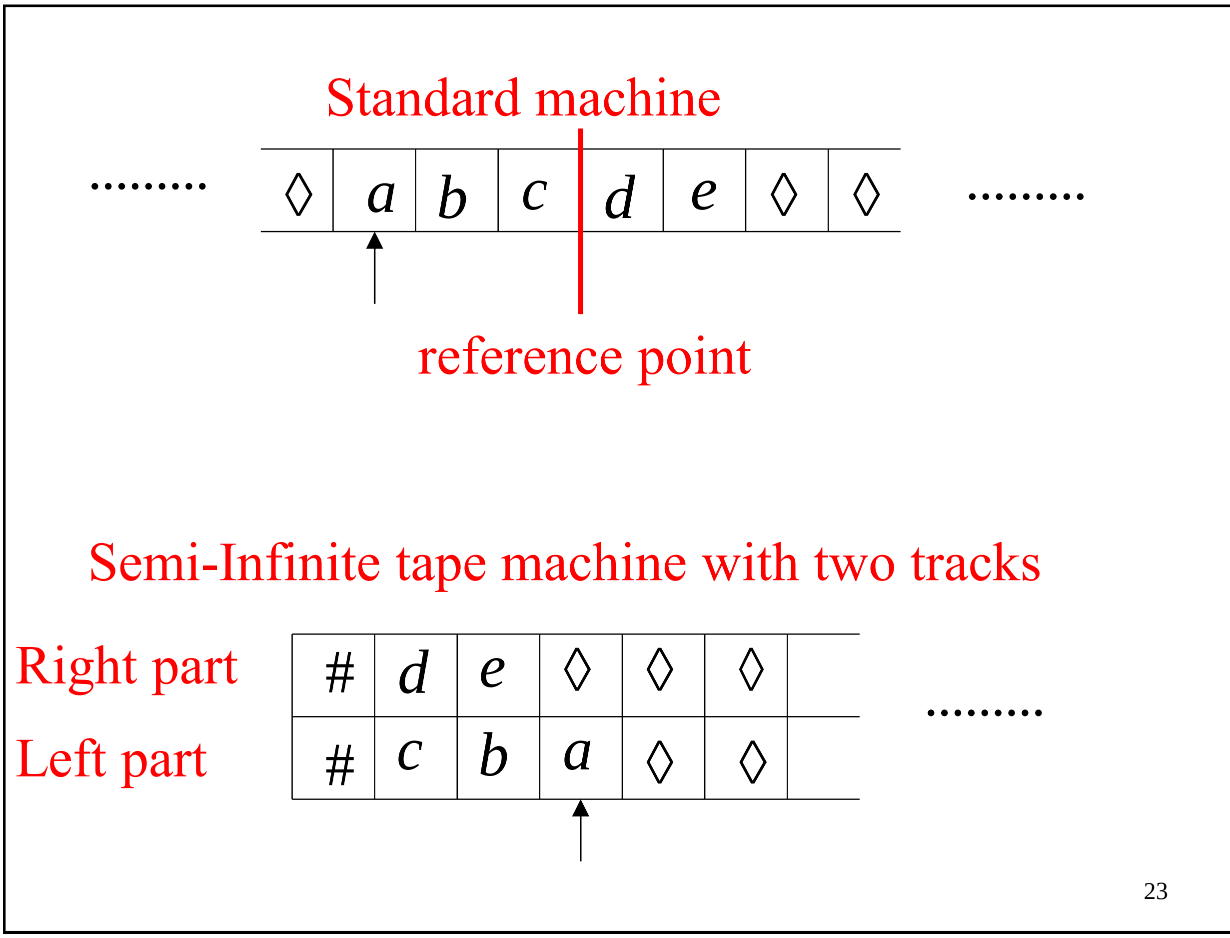
b. Add a self-loop
to every state
(except states with no
outgoing transitions)



Standard Turing machines simulate Semi-Infinite machines:



Squeeze infinity of both directions in one direction.


Standard machine
....
◊
a
b
c
d
e
◊
◊
....
reference point
Semi-Infinite tape machine with two tracks
Right part
Left part
#
d
e
◊
◊
◊
#
c
b
a
◊
◊
....
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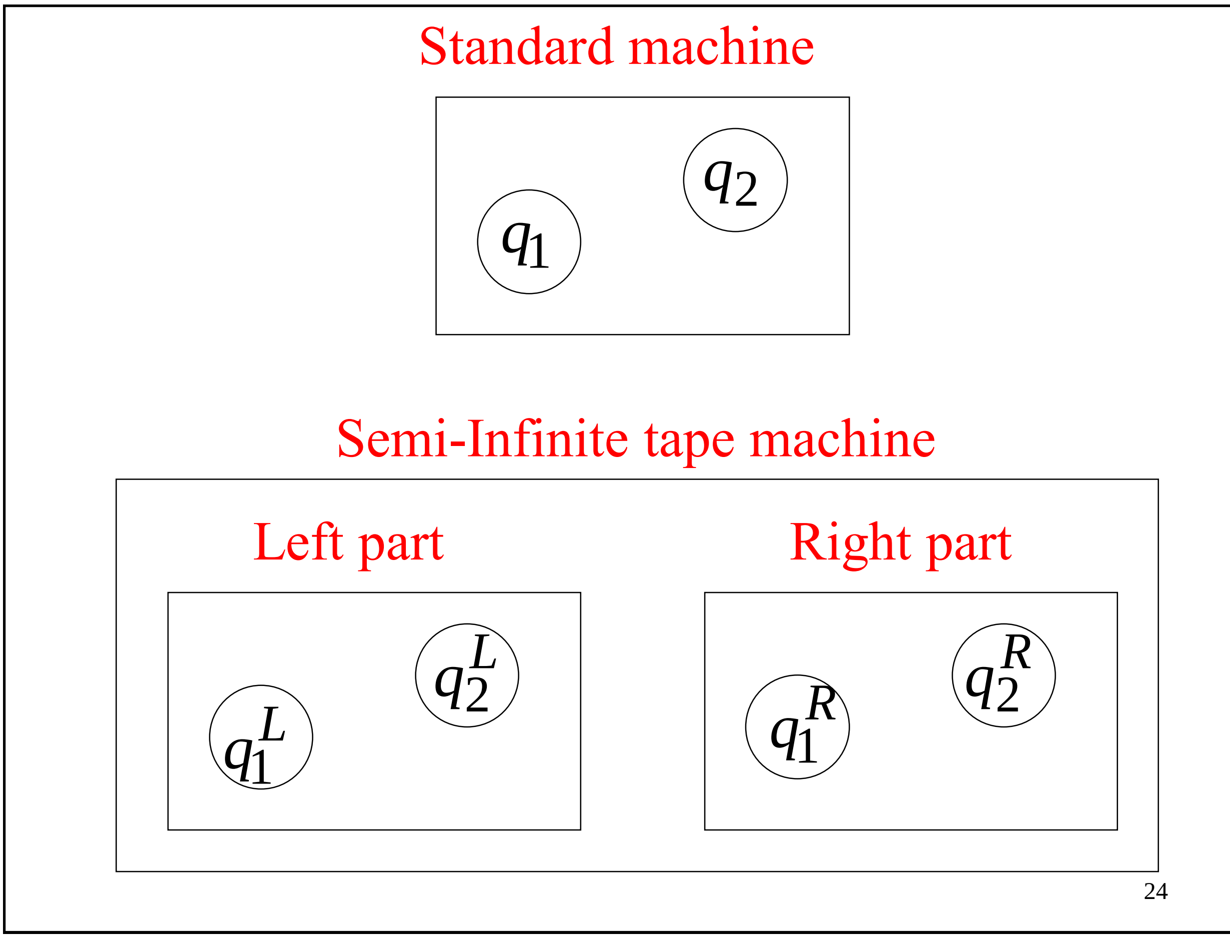
Standard machine
q_1
q_2
Semi-Infinite tape machine
Left part
q_1^L
q_2^L
Right part
q_1^R
q_2^R
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Standard machine

$$q_1 \xrightarrow{a \rightarrow g, R} q_2$$

Semi-Infinite tape machine

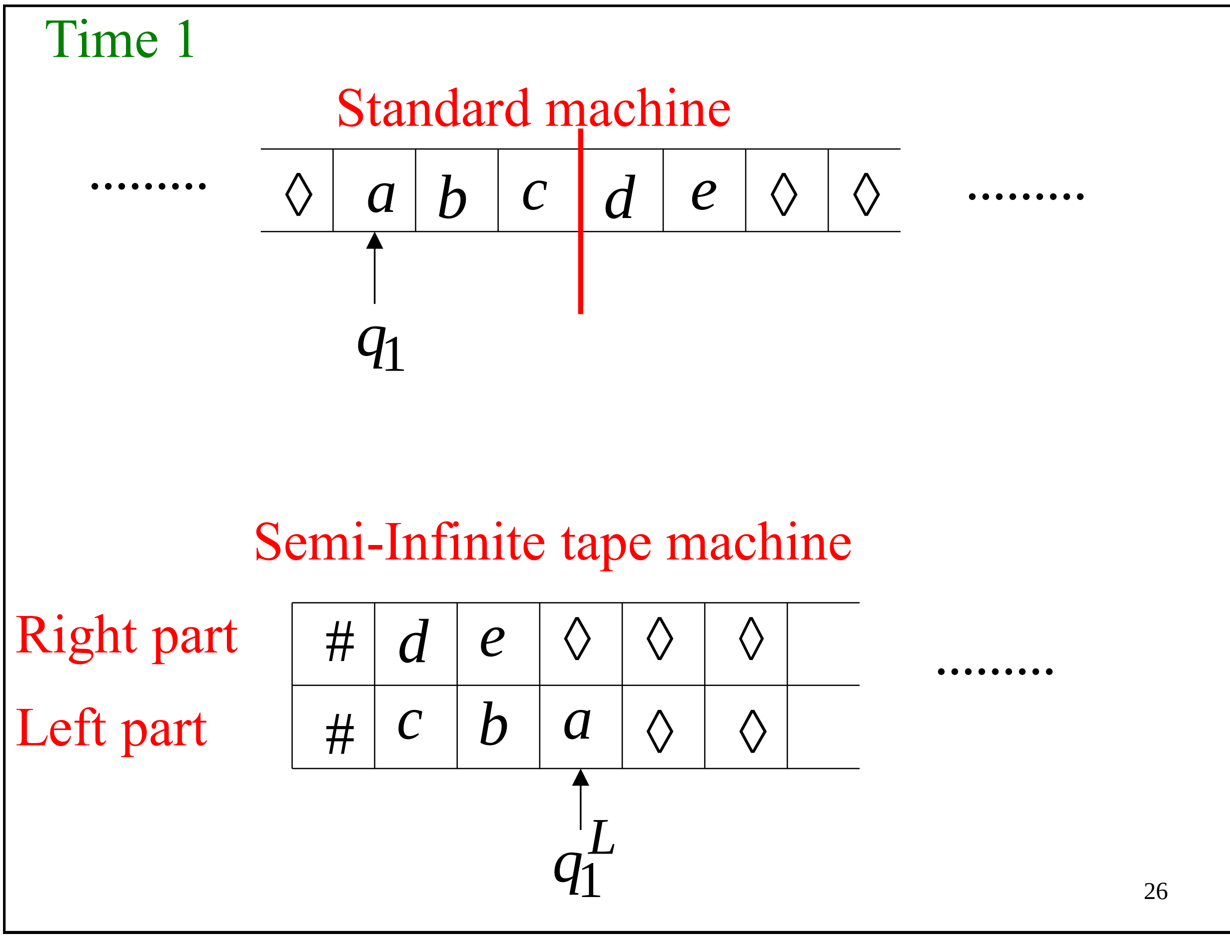
Right part

$$q_1^R \xrightarrow{(a, x) \rightarrow (g, x), R} q_2^R$$

Left part

$$q_1^L \xrightarrow{(x, a) \rightarrow (x, g), L} q_2^L$$

For all tape symbols x


Time 1
Standard machine
....
◊
a
b
c
d
e
◊
◊
....
q_1
Semi-Infinite tape machine
Right part
Left part
#
d
e
◊
◊
◊
#
c
b
a
◊
◊
....
q_1^L
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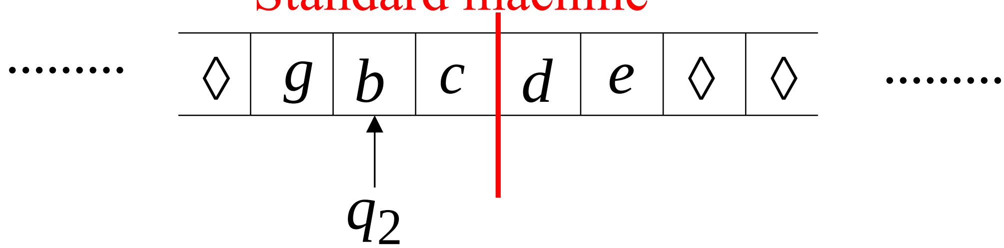
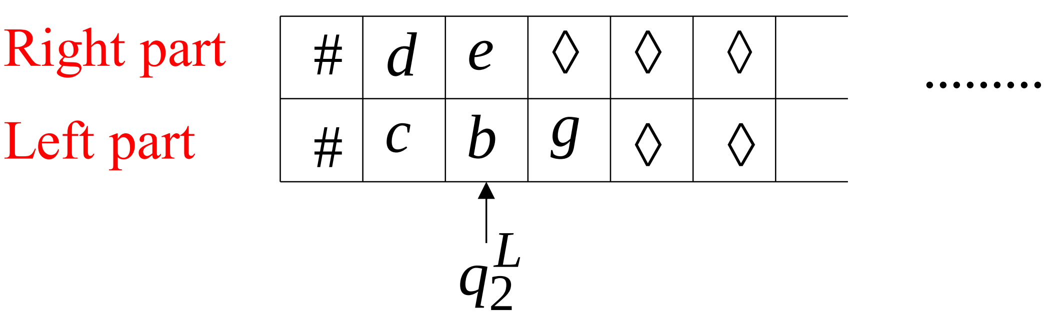
Time 2

Standard machine



Semi-Infinite tape machine



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At the border:

Semi-Infinite tape machine

Right part

$$q_1^R \xrightarrow{(\#,\#) \rightarrow (\#,\#), R} q_1^L$$

Left part

$$q_1^L \xrightarrow{(\#,\#) \rightarrow (\#,\#), R} q_1^R$$

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Semi-Infinite tape machine

Time 1

Right part	#	d	e	◊	◊	◊	....
Left part	#	c	b	g	◊	◊	

q_1^L

Time 2

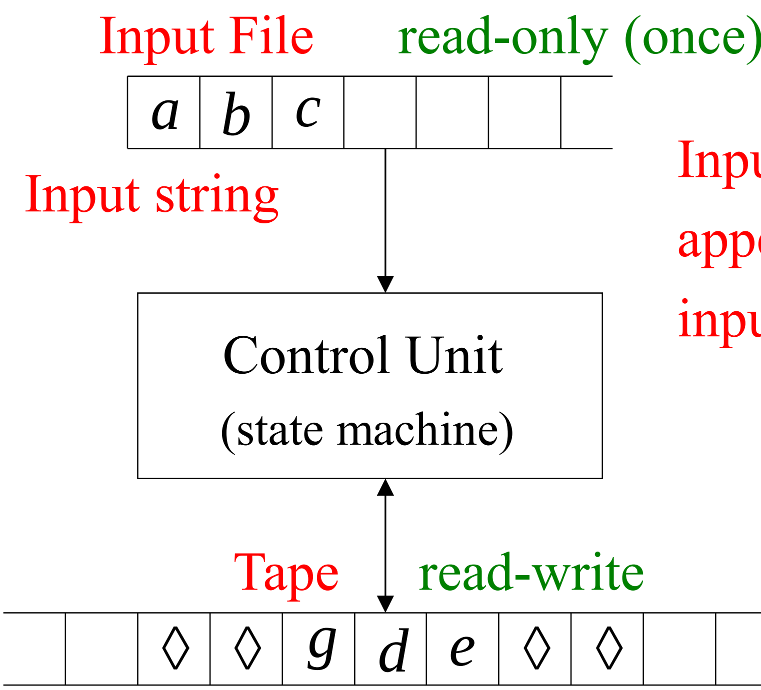
Right part	#	d	e	◊	◊	◊	....
Left part	#	c	b	g	◊	◊	

q_1^R

Theorem:

Semi-Infinite machines have the same power with Standard Turing machines.

The Off-Line Machine



Input string appears on input file only.

Theorem:

Off-Line machines have the same power with Standard Turing machines.

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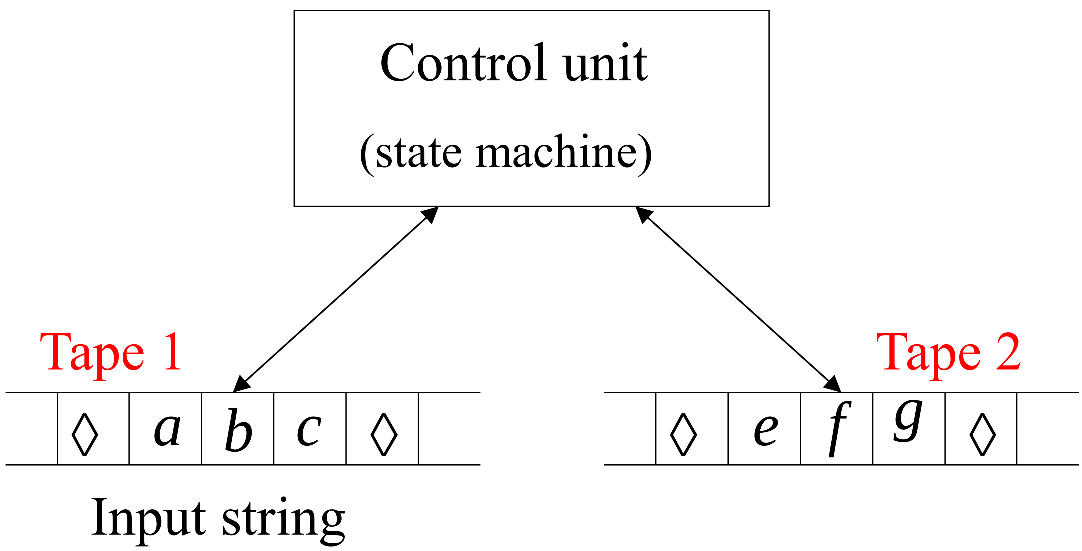
Multi-tape Turing Machines



Input string appears on Tape 1

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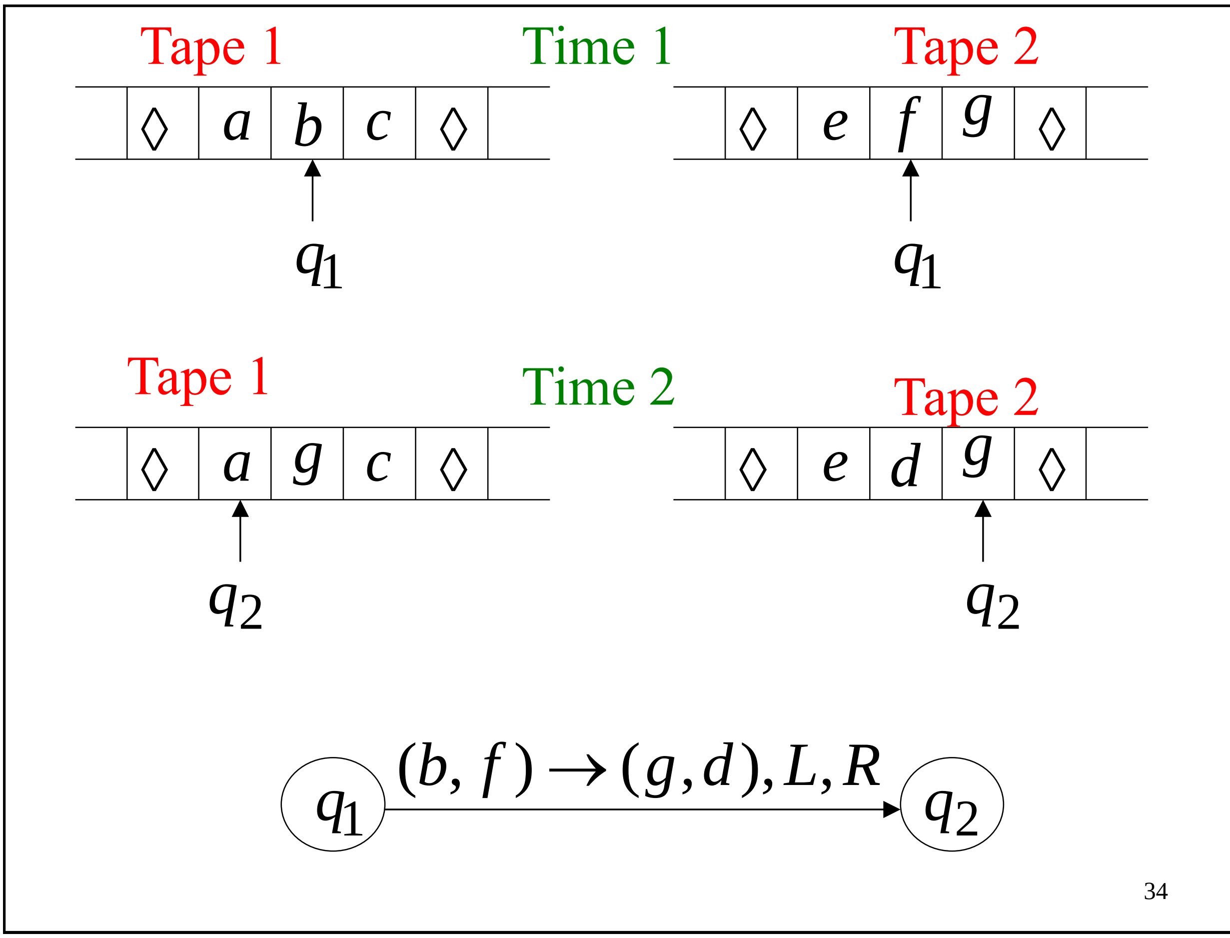
Tape 1
Time 1
Tape 2
◊ a b c ◊
◊ e f g ◊
q1
q1
Tape 1
Time 2
Tape 2
◊ a g c ◊
◊ e d g ◊
q2
q2
q1
(b, f) → (g, d), L, R
q2
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Same power doesn't imply same speed:

$$L = \{a^n b^n\}$$

Standard Turing machine: $O(n^2)$ time

Go back and forth $O(n^2)$ times
to match the a's with the b's.

2-tape machine: $O(n)$ time

1. Copy b^n to tape 2. ($O(n)$ steps)

2. Compare a^n on tape 1
and b^n on tape 2. ($O(n)$ steps)

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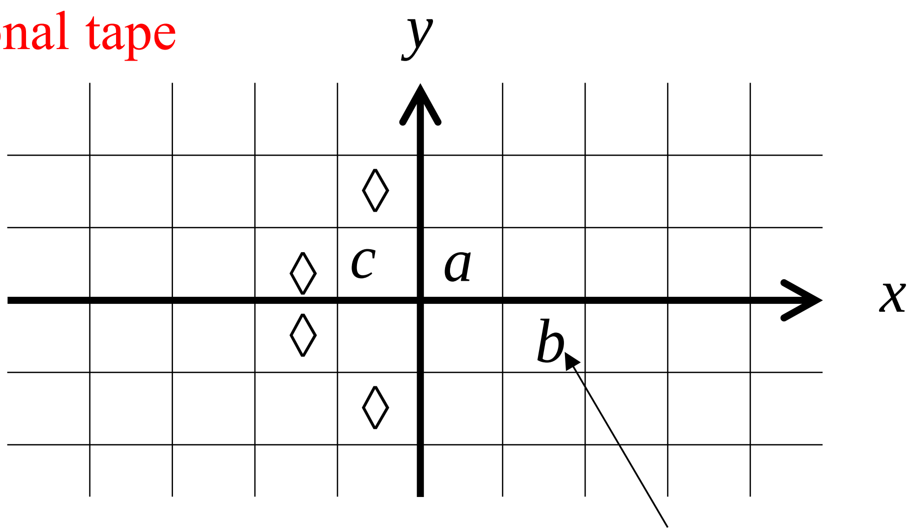
Theorem:

Multi-tape Turing machines have the same power with Standard Turing machines.

Multidimensional Turing Machines

2-dimensional tape



MOVES: L,R,U,D

U: up; D: down

HEAD

Position: +2, -1

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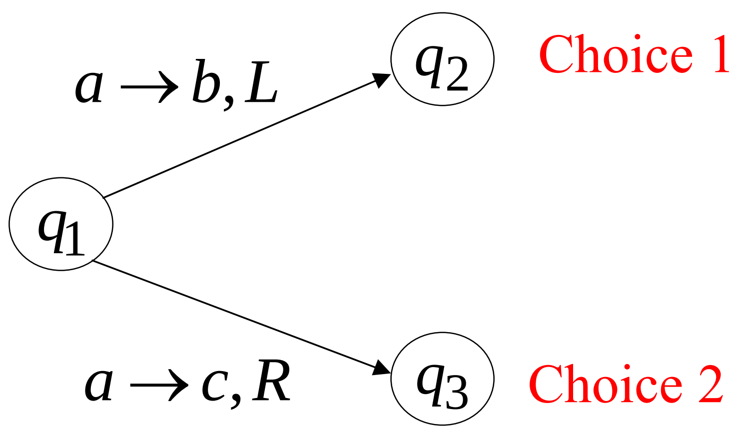
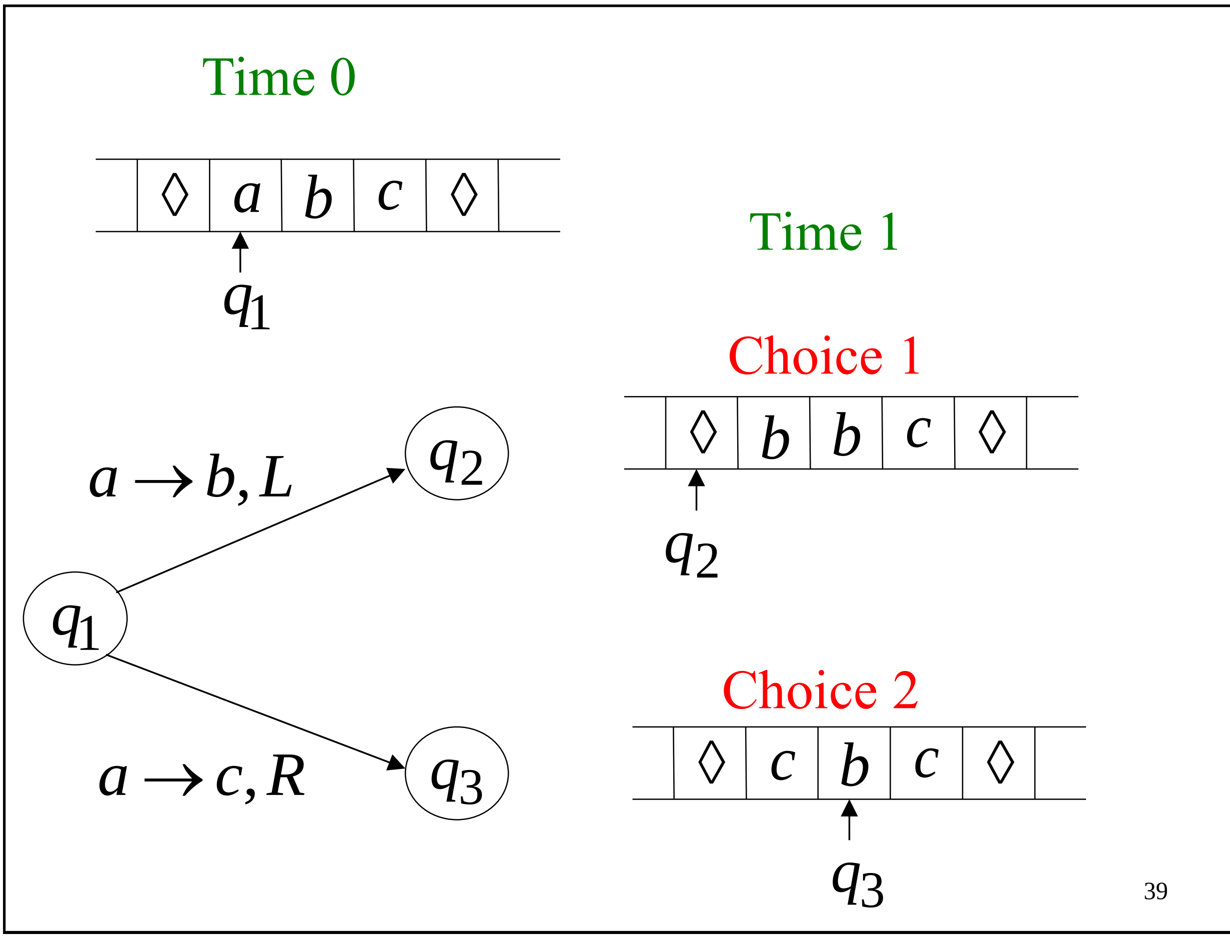
Nondeterministic Turing Machines



Allows Non Deterministic Choices


Time 0
◊ a b c ◊
q_1
Time 1
Choice 1
◊ b b c ◊
q_2
$a \rightarrow b, L$
q_1
q_2
q_3
$a \rightarrow c, R$
Choice 2
◊ c b c ◊
q_3
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Input string w is accepted if
there is a computation:

$$q_0 w \overset{*}{\succ} x \, q_f \, y$$

Initial configuration

Final Configuration

Any accept state

There is a computation: