Question 1 . . . . *6 points*

Give regular expressions describing the following languages. $\Sigma = \{0, 1\}$.

(a) [2 points] $\{w : w$ represents a binary number dividable by $2\}$.

$RE = (0+1)^*0$

(b) [2 points] $\{w : w$ contains at least two 1s and at most one $0\}$.

$RE = 1^*(0+\varepsilon)111^* + 1^*1(0+\varepsilon)11^* + 1^*11(0+\varepsilon)1^*$

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$$RE = 1^*(0+\varepsilon)11 1^* + 1^*1(0+\varepsilon)11^* + 1^*11(0+\varepsilon)1^*$$

(c) [2 points] $\{w : w$ contains an even number of 1s or an odd number of 0s$\}$.

$$RE = (0^*10^*10^*)^* + 1^*0(1^*01^*01^*)^*1^*$$

(a) [2 points] $L_1 = \{a^n b^m c^{n+m} : n \ge 0, m \ge 0\}$.

1. Assume L_1 regular $\Rightarrow$ the pumping lemma holds.

2. $\exists s \in L_1 \,/\, \begin{cases} s = xyz \\ |s| \geq p \\ |y| \geq 1 \\ |xy| \leq p \end{cases}$ (p: the pumping length)

so that $s' = xy^iz \in L_1$

3. Consider $s = a^p b^{2p} c^{3p}$ $(n = p,\ m = 2p)$

$s \in L_1$

$s = xyz \begin{cases} x = \varepsilon \\ y = a^p \\ z = b^{2p} c^{3p} \end{cases}$

$\exists i = 2 \,/\, s' = xy^2z = \varepsilon (a^p)^2 b^{2p} c^{3p} = a^{2p} b^{2p} c^{3p} \notin L_1$

4. This shows a contradiction which means that [illegible] (1) is false.

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4. This shows a contradiction which means that the assumption (1) is false.

5. Consequently, L_1 is not regular.

(b) [2 points] $L_2 = \{ww | w \in \{a, b\}^*\}$.

- Assume to the contrary that L_2 is regular.
- Let p be the pumping length given by the pumping lemma.
- Let s be the string $s = a^p b a^p b$, $s \in L_2$ and $|s| > p$
- The pumping lemma guarantees that s can be split into three pieces; $s = xyz$ satisfying the three conditions of the lemma:

$|xy| \leq p \Rightarrow \begin{cases} x = a^m \\ y = a^n \end{cases}$ with $n + m = p$

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- Let s be the string $s = a^p b a^p b$, $s \in L_2$ and $|s| > p$
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$$|xy| \le p \Rightarrow \begin{cases} x = a^m \\ y = a^n \text{ with } n+m = p \\ z = b a^p b \end{cases}$$

- $$xy^iz = a^m \cdot (a^n)^i \cdot b a^p b$$
$$= a^{n+m} a^{m(i-1)} b a^p b$$
$$a^p a^{m(i-1)} b a^p b \notin L_2$$

Question 3. .... 6 points

(a) [2 points] Give a CFG to generate the following language: $L = \{ww^R \mid w \in \{a, b\}^*\}$.

$S \to aSa \mid bSb \mid \varepsilon$

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(b) [2 points] Give a CFG to generate the following language: The set of strings over the alphabet $\{a, b\}$ with more b's than a's.

$$S \to TbT$$

$$T \to TT \mid aTb \mid bTa \mid b \mid \varepsilon$$

$$Tb \to TTb \to T\ bTab$$

$$\to T\ bb\ aab$$

(c) [2 points] Convert the following CFG into an equivalent CFG in Chomsky normal form.

$A \to BAB \mid B \mid \epsilon$

$B \to 00 \mid \epsilon$

$A \to BV_1 \mid T_0T_0 \mid AB \mid BA \mid \varepsilon$

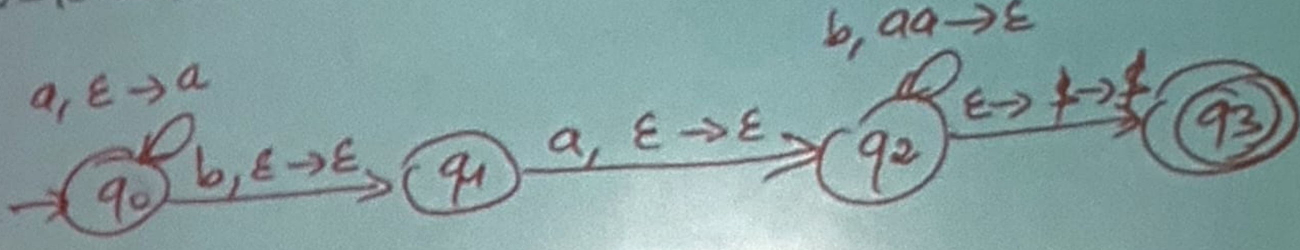
$B \to T_0T_0$

$T_0 \to 0$

$V_1 \to AB$

Question 4. . . . . 4 points

(a) [3 points] Consider the alphabet $\Sigma = \{a, b\}$. Give a PDA for the following language: $L = \{a^{2n}bab^{n} \mid n \geq 0\}$.



(b) [1 point] Convert the following CFG into a PDA.

$A \to BAB|B|\epsilon$

$B \to 00|\epsilon$

$0, 0 \to \varepsilon$

$\varepsilon, A \to BAB$

$\varepsilon, A \to B$

$\varepsilon, A \to \varepsilon$

$\varepsilon, B \to 00$

$\varepsilon, B \to \varepsilon$