

Answer Section A and (either section B OR Section C (15 marks)).

Section A: 25 marks    Section B: 15 Marks    Section C: 15 Marks

### Section A

A1: a) Answer each part as True or False (5 marks)

1. Every algorithm that decides any CFG is a member of P.
2. Every multitape Turing machine (TM) has an equivalent single-tape Turing machine.
3. The complement of a recursively enumerable language is always not TM decidable.
4. The collection of decidable languages is closed under the union operation.
5. The language  $A = \{0^n 1^n \mid n \geq 0\}$  is a member of  $SPACE(\log n)$ .

|                       |   |   |   |   |   |
|-----------------------|---|---|---|---|---|
| Q                     | 1 | 2 | 3 | 4 | 5 |
| [1/5] T=True, F=False | T | F | T | T | F |

b) Give an implementation-level description of a Turing machine that decides the language

$\{0^n 1^n 0^n \mid n \geq 0\}$  over the alphabet  $\{0, 1\}$  (5 marks)

$$L = \{0^n 1^n 0^n \mid n \geq 0\}$$

let  $p$  is  $xy^iz$ ,  $i \geq 0$

Turing machine is a 7-tuple  
 $\{Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}\}$   
 $Q \rightarrow$  set of state  
 $\Sigma \rightarrow$  input alphabet  
 $\Gamma \rightarrow$  transition function  
 $q_0 \rightarrow$  start state  
 $q_{\text{accept}} \rightarrow$  if the end accept  
 $q_{\text{reject}} \rightarrow$  if the end reject

A2 a. Let  $A_{reg} = \{ \langle R, w \rangle \mid R \text{ is a regular expression that generates string } w \}$ .

Prove that  $A_{reg}$  is decidable (5 marks)

1. convert RE  $R$  to DFA  $A$
2. Run  $M$  on the theorem  $A_{DFA}$  on input  $\langle A, w \rangle$
3. If  $M$  accepts, accept otherwise reject.

theorem  $A_{DFA}$  Prove:

$M$  on input  $\langle A, w \rangle$   $A$  is a DFA and  $w$  is a string

1. simulate  $A$
2. If the simulation ends accepts, accept.  
If it ends nonaccepts, reject.

b. Analyze the space complexity of the SAT problem. (5 marks)

for space complexity, SATIVITYCH THEOREM shows that any non deterministic  $TM$  that uses  $f(n)$  and can convert to deterministic that uses only  $f^2(n)$

Q2 Give a high level description of a Turing machine that adds two binary numbers. (5 marks)

~~Turing machine is a type~~

~~of automata~~

high level description of a Turing machine  
and the Turing machine have 7-tuple

$\{Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}\}$

$Q \rightarrow$  set of state

$\Sigma \rightarrow$  input alphabet

$\delta \rightarrow$  transition function

$q_0 \rightarrow$  start of state

$q_{\text{accept}} \rightarrow$  if the end accept

$q_{\text{reject}} \rightarrow$  if the end reject

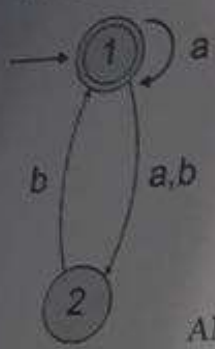


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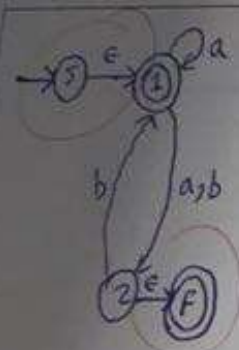
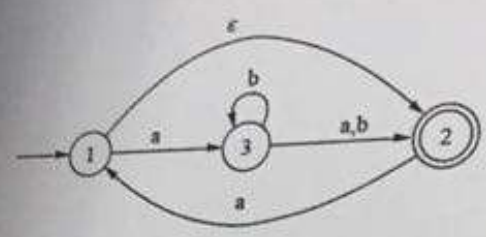
Section II

1. Find the language recognized by the union of the following two finite automata (6 Marks)

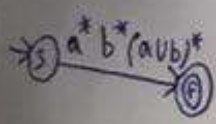
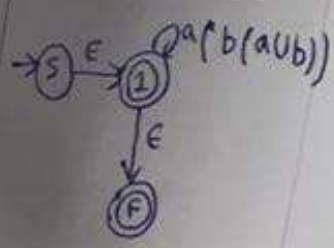
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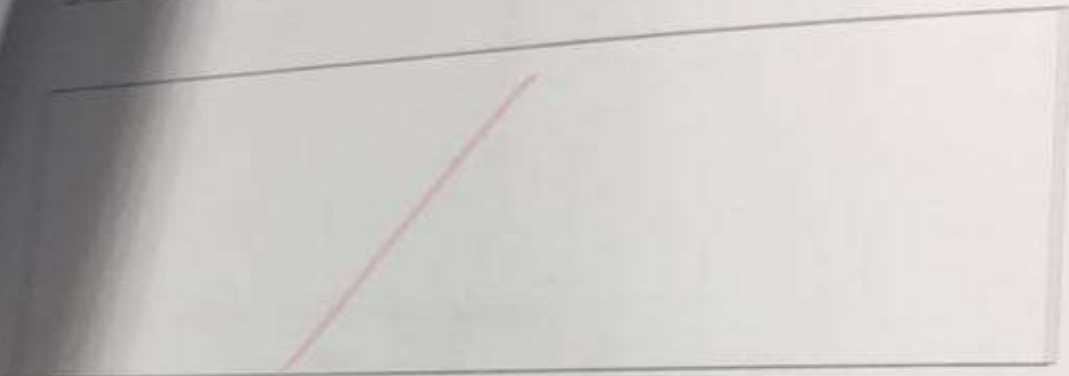
AND



~



11. Assuming that  $\Sigma = \{0,1\}$ , construct a language for  $\Sigma^+ - \{0\}$  (10 Marks) \*



12. Convert the following NFA into its equivalent DFA (10 Marks) \*



|   | a   | b | $\epsilon$ |
|---|-----|---|------------|
| 0 | 0   | 2 | 0,2        |
| 1 | 0,2 | 1 | 1          |
| 2 | 1   | 0 | 2          |

|     | a   | b |
|-----|-----|---|
| 0   | 1   | 2 |
| 1   | 0   | 2 |
| 2   | 0,2 | 1 |
| 0,2 | 0,2 | 2 |
| 1   | 1   | 0 |



Use the pumping lemma to show that the language  $C = \{w \mid w \text{ has an equal number of 0s and 1s}\}$  is not regular.

$$S \rightarrow 0S1 \mid \epsilon$$

0011



and prove of pumping lemma:

$$\text{let } n = F(\epsilon)$$

$$w = L(\epsilon) \text{ and } |w| \geq n$$

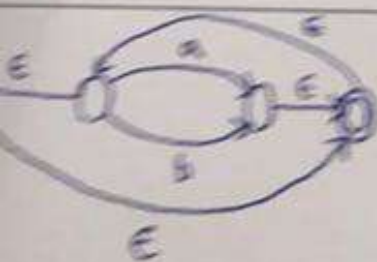
Let  $T$  is the pars of tree, the smallest hight

Let  $P$  is be path of length  $n$ ,  $n$  is the hight of  $T$

Since  $|w| \geq F(n)^{n+1}$  and the length of  $P \geq |v|+1$

and number of nodes in  $P$  is at least  $|v|+2$

Convert the regular expression  $(a \cup b)^*$  to an NFA. (6 Marks)



3. Find a DFA that can accept the language  $\{(10)^n \mid n \geq 0\}$  (3 Marks)

language  $\{1^n 0^n \mid n \geq 0\}$

$S \rightarrow 150 \mid \epsilon$

1100



10 ✓

1100 ✓

111000 ✓

