Artificial Intelligence
CSC 361

Tutorial
FOL

Q1:Translate the following first-order sentences into English:

1.$\forall x, bird(x) \Rightarrow flies(x)$

2.$\forall x, \exists y, person(x) \Rightarrow mother(y,x)$
3. $\exists x, \forall y, person(x) \wedge mother(x, y)$

4.$\forall x,y, mother(x,y) \Longrightarrow \neg mother(y,x)$

where:

- bird(x) means x is a bird,
- flies(x) means x flies,
- person(x) means x is a person,
- mother(x, y) means x is the mother of y.

1.$\forall x, bird(x) \Rightarrow flies(x)$

All birds fly.

2.$\forall x, \exists y, person(x) \Rightarrow mother(y,x)$

Every person has a mother.

3. $\exists x, \forall y, person(x) \wedge mother(x, y)$

All persons have at least one common mother.

4.$\forall x,y, mother(x,y) \Longrightarrow \neg mother(y,x)$

A person is not the mother of his/her mother.

Q2:Translate the following natural language statements into first order logic:

1. There is exactly one student that received a grade A in CSC 411.
2. Only one CS class is named ”Artificial Intelligence”.
3. All red apples are delicious.
4. Some apples are not red, yet they are delicious.
5. The enemy of the enemy of a person is his friend.

1. There is exactly **one** student that received a grade A in CSC 411.

$\exists x,\ \text{Student}(x) \wedge [\text{Grade}(x, \text{CSC411}) = A] \wedge (\forall y, \text{Student}(y) \wedge [\text{Grade}(y, \text{C411}) = A]) \Rightarrow y = x$

2.Only one CS class is named ”Artificial Intelligence”.

$\exists x, \text{Course}(x, \text{CS}) \wedge [\text{Name}(x) = \text{”AI”}] \wedge \forall y, \text{Course}(y, \text{CS}) \wedge [\text{Name}(y) = \text{”AI”}] \Rightarrow y = x$

3. All red apples are delicious.

$\forall x, \text{Apple}(x) \wedge \text{Red}(x) \Rightarrow \text{Delicious}(x)$

4.Some apples are not red, yet they are delicious.

$\exists x, \text{Apple}(x) \wedge \neg \text{Red}(x) \wedge \text{Delicious}(x)$

5. The enemy of the enemy of a person is his friend.

$\forall x, y, z, \text{Person}(x) \wedge \text{Enemy}(y, x) \wedge \text{Enemy}(z, y) \Rightarrow \text{Friend}(z, x)$

Table 1: Some tips for translating into FOL. P (x) can be any predicate.

At least one x verifies P (x) At most one x verifies P (x) Exactly one x verifies P (x)	$\exists x, \ P \ (x)$ $\forall x,y, \ P(x) \wedge P(y) \Rightarrow \ x=y$ $\exists x, \ P(x) \wedge \forall y, \ P(y) \Rightarrow \ x=y$

Q3:Convert the following KB into propositional logic:

- ∀x,WonPrize(x) ⇒ Happy(x).
- ∀y,PlayGame(y)∧Lucky(y) ⇒ WonPrize(y).
- PlayGame(Ali).
- PlayGame(Mona).
- Lucky(Ali).
- Happy(Mona).

Can you deduce who won the prize and whether (s)he is happy?

First we apply universal instantiation to the first and second rules. We have two values in our domain: Ali and Mona.

A1:WonPrize (Ali) ⇒ Happy (Ali), Replace x with Ali

A2: WonPrize(Mona) ⇒ Happy(Mona), Replace x with Mona

A3: PlayGame (Ali) ∧ Lucky (Ali) ⇒ WonPrize (Ali), Replace y with Ali

A4:PlayGame (Mona) ∧ Lucky (Mona) ⇒ WonPrize (Mona)

A5:PlayGame(Ali).

A6:PlayGame(Mona).

A7:Lucky(Ali).

A8:Happy(Mona)

Now we apply the inference rules of propositional logic.

A9: PlayGame (Ali) ∧ Lucky (Ali), A5 & A7, And introduction

A10: WonPrize(Ali), A3 & A9, Modus Ponens
A11: Happy (Ali), A1 & A10, Modus Ponens

So Ali won the prize and he is happy. Note that we can not know whether Mona won the prize or not.

Q4: Given the following:

1.A1:$\forall x,y,On(x,y) \Rightarrow Above(x,y)$.

2.A2:$\forall x,y,z,On(x,y) \wedge Above(y,z) \Rightarrow Above(x,z)$.

3. A3:$\forall x, On(A, x)$.
4. A4:$\forall x, On(x, C)$.

Prove: Above(A,C).

We use inference rules of FOL.

- A5: Above (x, c), A1 & A4, Generalized Modus Ponens, $\theta = \{y/C\}$.
- A6: On (A, x) $\wedge$ Above (x, c), A3 & A5, And introduction, $\theta = \{\}$
- A7: Above (A, C),
  A2 & A6, Generalized Modus Ponens, first rename x by w in A6, then apply $\theta = \{x/A, y/w, z/C\}$

Q5

The American law says that it is a crime for an American to sell weapons to hostile nations. The country Nano, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American. Prove using forward and backward chaining that West is a criminal.

FOL

A1: American(x)∧Weapon(y)∧Sells(x,y,z)∧Hostile(z) ⇒ Criminal(x)

A2:Owns(Nano,M1)
A3:Missile(M1)
A4: Missile(x)∧Owns(Nano,x) ⇒ Sells(West,x,Nano)

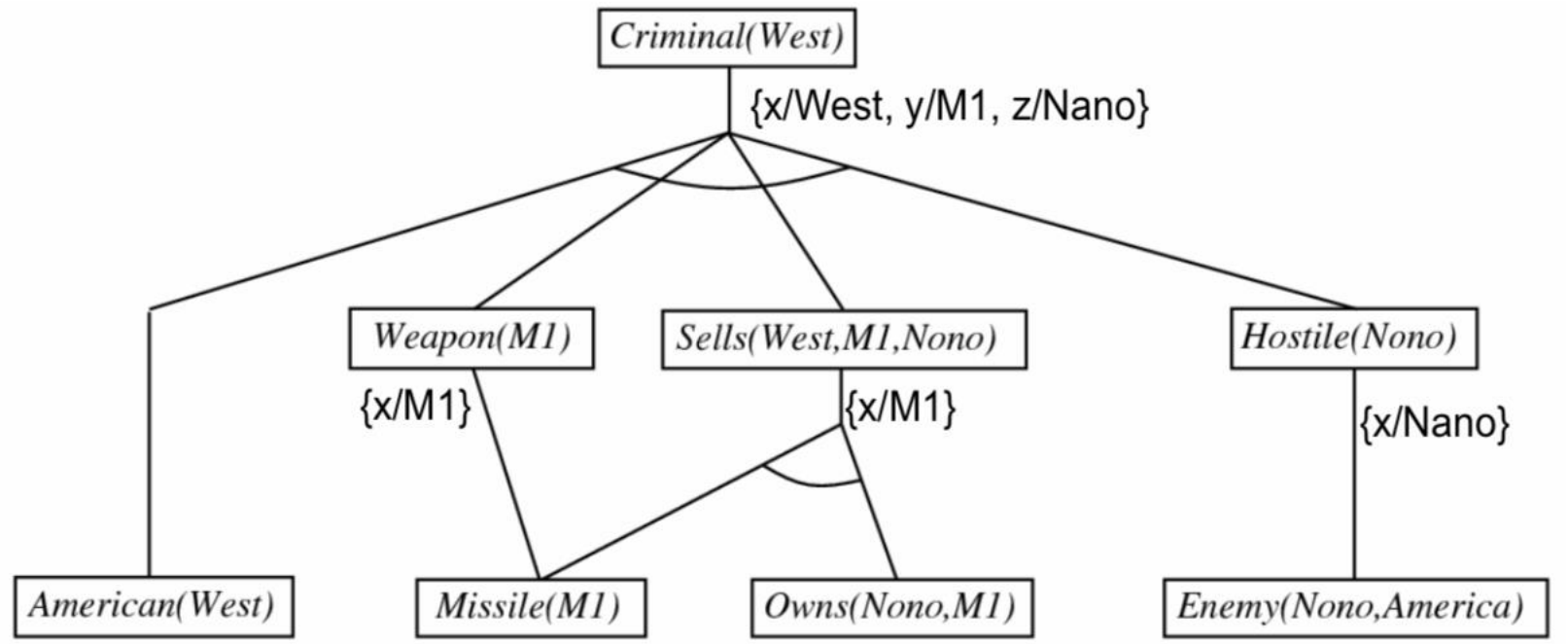
A5:Missile(x) ⇒ Weapon(x)

A6: Enemy(x,America) ⇒ Hostile(x)

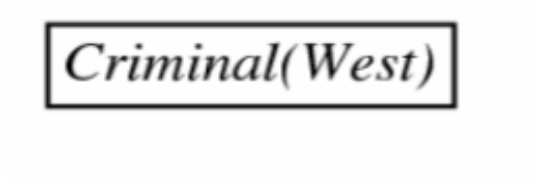
A7:American(West)

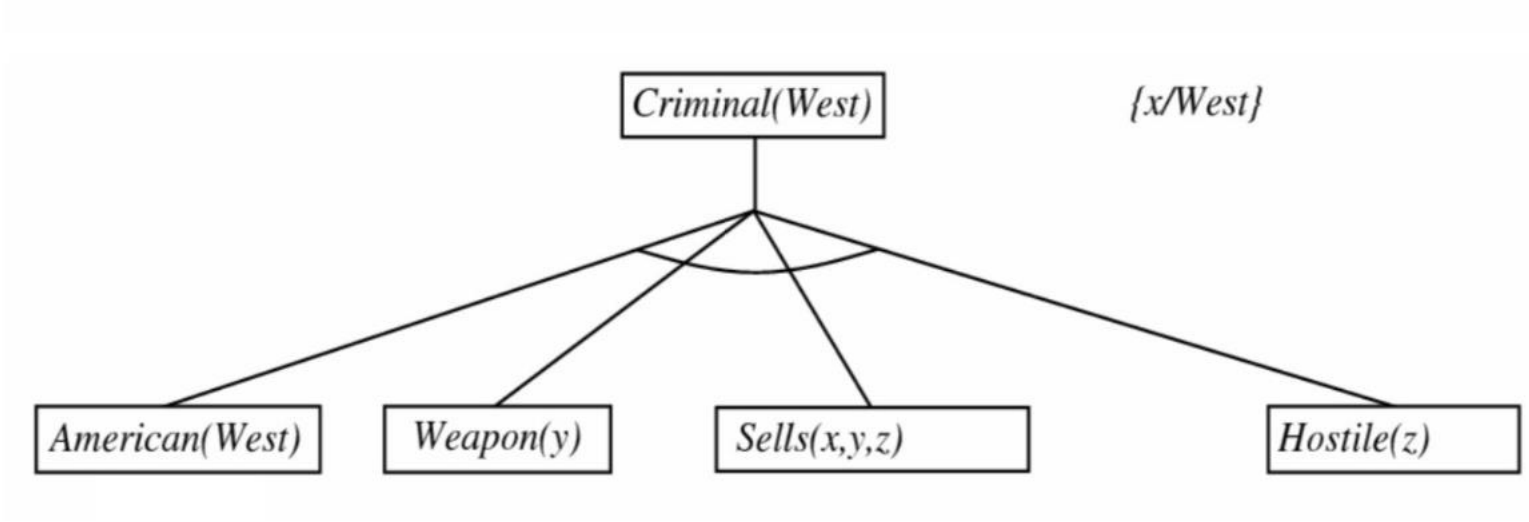
A8: Enemy(Nano,America)

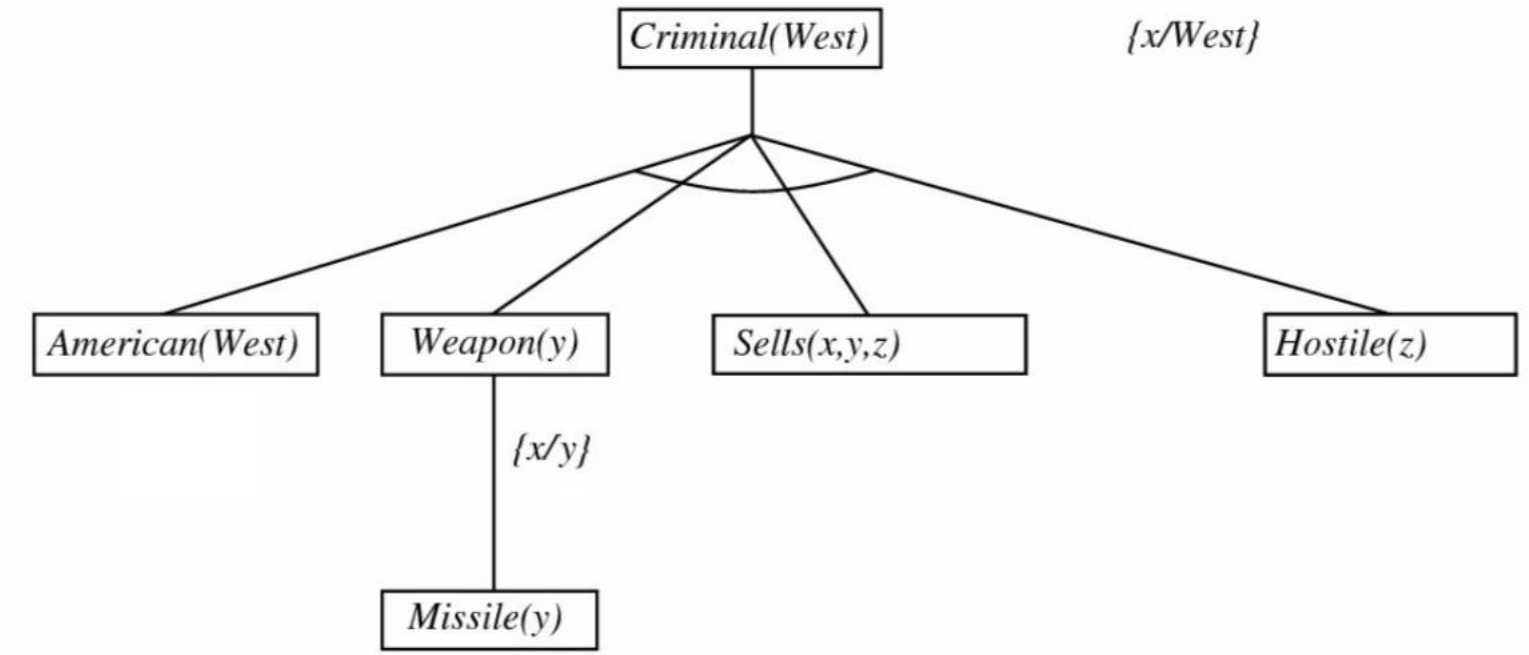
using forward that West is a criminal.

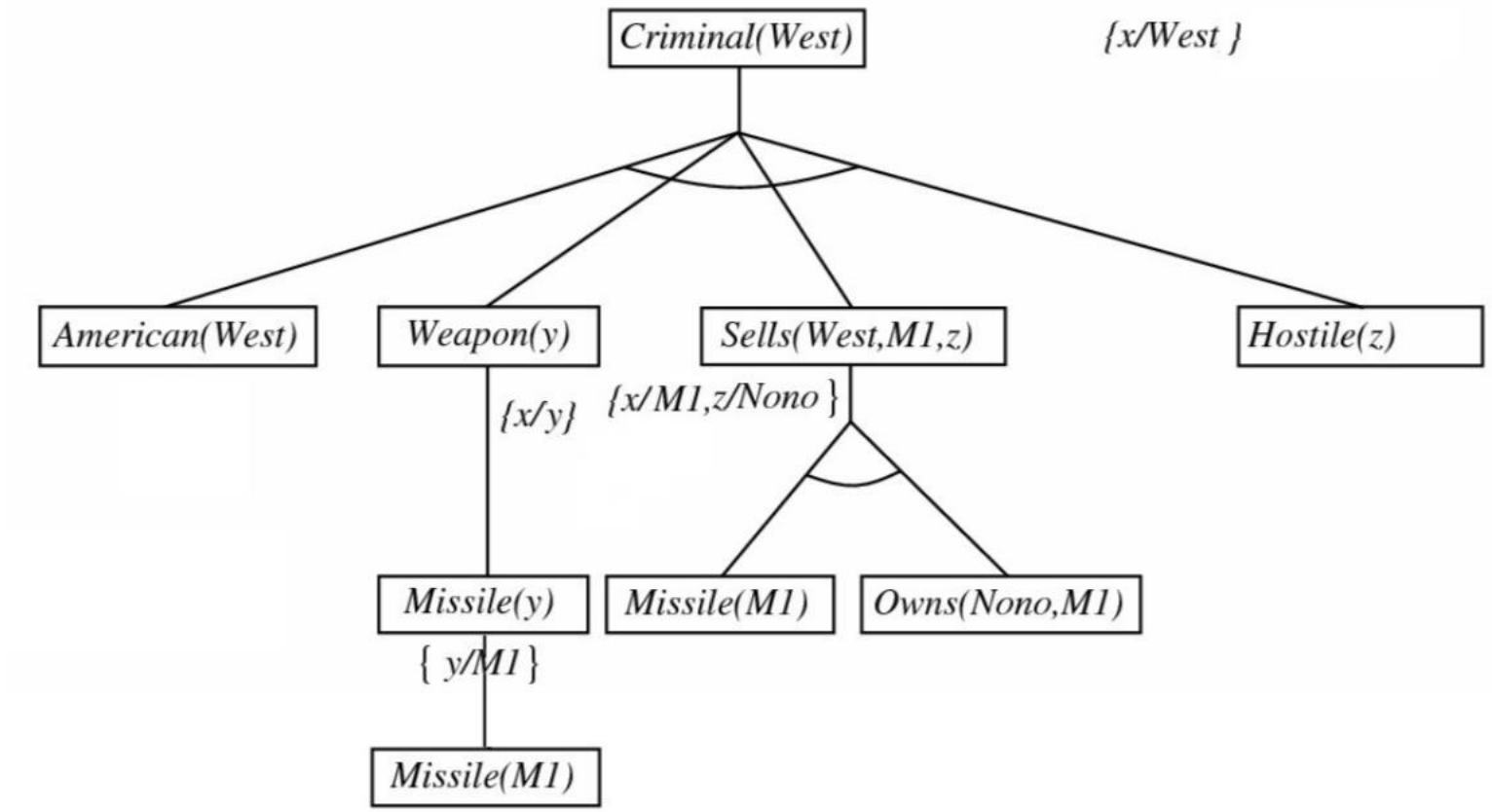


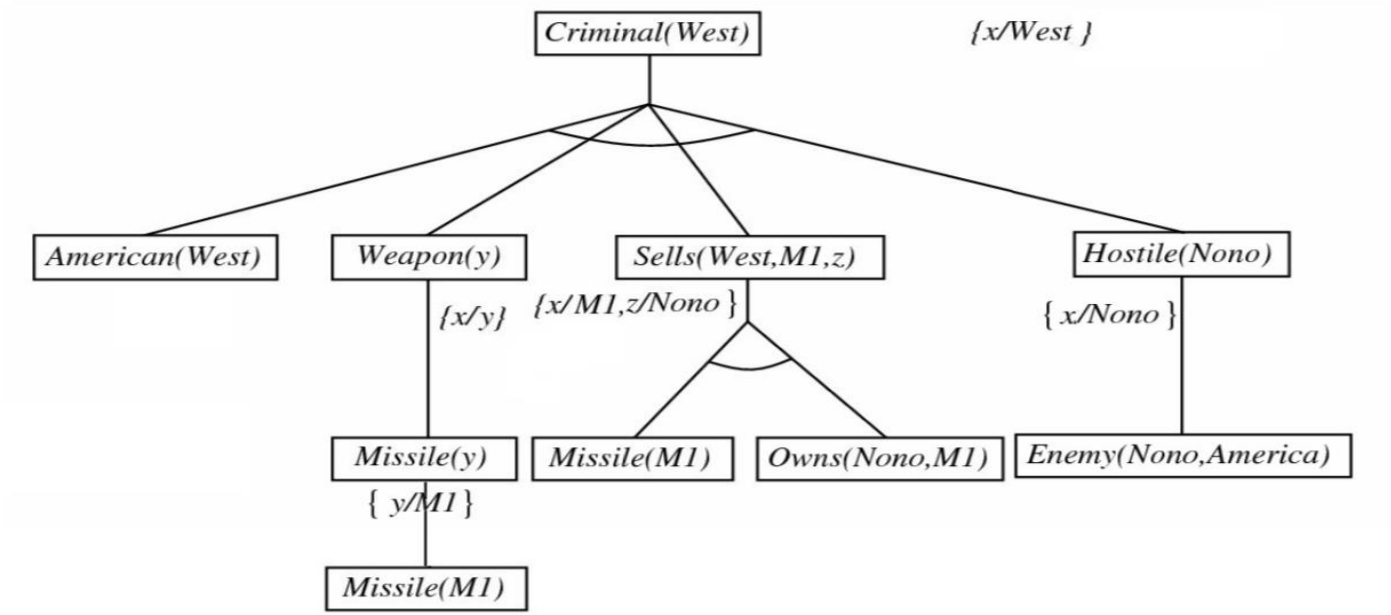
Using Backward chaining




Criminal(West)
{x/West}
American(West)
Weapon(y)
Sells(x,y,z)
Hostile(z)


Criminal(West)
{x/West}
American(West)
Weapon(y)
Sells(x,y,z)
Hostile(z)
{x/y}
Missile(y)


Criminal(West)
{x/West }
American(West)
Weapon(y)
Sells(West,M1,z)
Hostile(z)
{x/y}
{x/M1,z/Nono }
Missile(y)
Missile(M1)
Owns(Nono,M1)
{ y/M1 }
Missile(M1)


Criminal(West)
{x/West }
American(West)
Weapon(y)
Sells(West,M1,z)
Hostile(Nono)
{x/y}
{x/M1,z/Nono }
{ x/Nono }
Missile(y)
Missile(M1)
Owns(Nono,M1)
Enemy(Nono,America)
{ y/M1 }
Missile(M1)