SHEET #7

Knowledge representation and reasoning

Q1: Convert ($KB \wedge \neg\alpha$) into Conjunctive Normal

- KB: $(B_{11} \Leftrightarrow P_{12} \vee P_{21}) \wedge \neg B_{11}$,   α: $\neg P_{12}$

- Sentence: $(B_{11} \Leftrightarrow P_{12} \vee P_{21}) \wedge \neg B_{11} \wedge \neg\neg P_{12}$
- Solution:
  1. Eliminate Bidirectional
  2. Eliminate Implication
  3. De Morgan (Not)
  4. Distribution of V over Λ
  5. Double negation elimination

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

- Conclusion (CNF):

$(\neg B_{11} \vee P_{12} \vee P_{21}) \wedge (\neg P_{12} \vee B_{11}) \wedge (\neg P_{21} \vee B_{11}) \wedge \neg B_{11} \wedge P_{12}$

Q2: Use the PL inference rules to prove P

- R_1: $R \lor T$,
- R_2: $Q \land U$,
- R_3: $(S \lor T) \Rightarrow (Q \Rightarrow P)$,
- R_4: $\neg R \lor S$

- **Solution:**
  - R_1, R_4 Resolution Rule
    - R_5: $S \lor T$
  - R_3, R_5 modus ponens
    - R_6: $Q \Rightarrow P$
  - R_2 and-elimination
    - F_1: Q
  - F_1, R_6 modus ponens
    - Conclusion: p

Q3: Translate facts into prepositional logic and prove.

- A is on the right of B: R_{AB}
- B is on the left of A: L_{BA}
- C is on the top of B: T_{CB}
- C is on the top of A: T_{CA}

- If A is on the right of B, then B is on the left of A:

$$R_1: R_{AB} \Rightarrow L_{BA}$$

- If C is on the top of B which is on the left of A, then it is not on the top of A.

$$R_2: T_{CB} \wedge L_{BA} \Rightarrow \neg T_{CA}$$

Q3: Translate facts into prepositional logic and prove, Cont…

- **Rules**
  - R_1: $R_{AB} \Rightarrow L_{BA}$, R_2:$T_{CB} \wedge L_{BA} \Rightarrow \rceil T_{CA}$
- **Fact (knowing that R_{AB} and T_{CB})**
  - F_1: R_{AB} ,F_2: T_{CB}
- **Prove ($\rceil T_{CA}$)**

F1 & R1 modus ponens gives F3: L_{BA}

F2 & F3 and-introduction gives F4:$T_{CB} \wedge L_{BA}$

F4 & R2 modus ponens gives conclusion $\rceil T_{CA}$

Q4 : deduce who is the prize winner and if he is happy or not?

- Rules
  - R_1: $\forall x$ Won_Prize(x) $\Rightarrow$Happy(x)
  - R_2: $\forall y$ Play_Game(y) ^ Lucky(y) $\Rightarrow$ Won_Prize(y)
- Facts
  - F_1:Play_Game(Ali), F_2:Play_Game(Mona), F_3:Lucky(Ali), F_4:Happy(Mona)

- Solution
- F_1 & F_3 and-introduction gives F_5: Play_Game(Ali) $\wedge$ Lucky(Ali)
- F_5 & R_2 modus ponens gives *F_6: Play_Game(Ali) $\wedge$ Lucky(Ali) $\Rightarrow$ Won_Prize(Ali)
- F_6 & R_1 modus ponens gives **conclusion
- Conclusion
  - Ali won and Ali is happy

* We concluded using MGU to Subst{y/Ali}

** We concluded using MGU to Subst{x/Ali}

Q5: Represent statements using first order logic.

- Predicates: Bigger(x,y), Apple(x), Red(x), Delicious(x)

- All red apples are delicious.
  - $\forall$ x, Apple(x) $\land$ Red(x) $\Rightarrow$ Delicious(x)

- Every delicious apple is bigger than some red apple.
  - $\forall$ x, $\exists$ y: Apple(x) $\land$ Delicious(x) $\Rightarrow$ Apple(y) $\land$ Red(y) $\land$ Bigger(x,y)
  - $\forall$ x, $\exists$ y: Apple(x) $\land$ Delicious(x) $\land$ Apple(y) $\land$ Red(y) $\Rightarrow$ Bigger(x,y)

Q6: Prove Above (A,C).

- **Rules**
  - R1: $\forall$ x, y On(x, y) $\Rightarrow$ Above(x, y)
  - R2: $\forall$ x, y, z On(x, y) $\wedge$ Above(y, z) $\Rightarrow$ Above(x, z)
- **Fact**
  - F1: On(A, X) , F2: On(X, C)

- **Solution**

  F2 & R1 modus ponens gives *F3: Above(X, C)

  F1 & F3 and-introduction gives **F5: On(A, X) $\wedge$ Above(X, C)

  F5 & R2 modus ponens gives conclusion

- **Conclusion**
  - Above (A,C)

* We concluded using MGU to Subst{y/C, x/X}

** We concluded using MGU to Subst{x/A, y/X, z/C}

Extra Q: $KB \models \alpha$?

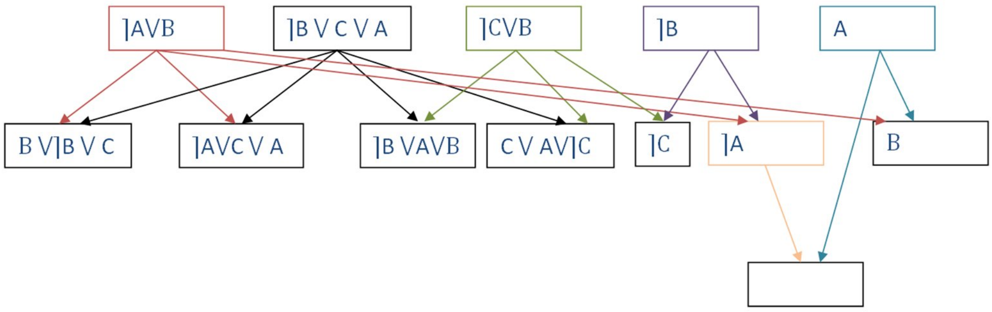
- Rules
  - R_1: $A \Rightarrow B$
  - R_2: $(B \Rightarrow C) \vee A$
  - R_3: $C \Rightarrow B$
  - R_4: $\rceil B$
  - α: $\rceil A$

- $KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

1. Put $KB \wedge \neg\alpha$ into CNF
   - $(\rceil A \vee B) \wedge (\rceil B \vee C \vee A) \wedge (\rceil C \vee B) \wedge (\rceil B) \wedge A$

Extra Q: *KB* ⊨ α ?

2. **Resolution of *KB* ∧ ¬α**



3. **An empty clause appear, then (*KB* ∧ ¬α) is unsatisfaiable**
   - *KB* ⊨ α