Artificial Intelligence
CSC 361

Tutorial#7

Q1

Consider a world composed of three objects A, B and C. The rules of this world are the following:

1. If A is on the right of B, then B is on the left of A
2. If C is on the top of B which is on the left of A, then it is not on the top of A.

- Translate these rules into propositional logic.
- If you know that in such world, A is on the right of B and C on the top of B, Can you deduce from the previous facts that C is not on the top of A.

Answer

Rules

R1: $Right_{AB} \Rightarrow Left_{BA}$

R2: $Top_{CB} \wedge Lleft_{BA} \Rightarrow \neg Top_{CA}$

The facts are:

– F1: $Right_{AB}$

– F2: Top_{CB}.

- Prove $\neg Top_{CA}$:

– F3: $Left_{BA}$, F1 & R1 Modus Ponens
– F4: $Top_{CB} \wedge Left_{BA}$, F2 & F3 And-introduction

– F5: $\neg Top_{CA}$ R2 & F4 Modus Ponens

Use propositional logic inference rule to show that

An inference rule: Modus Ponens

$\alpha \Rightarrow \beta, \quad \alpha$ Premise

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β Conclusion

An inference rule: AND elimination

$\alpha 1 \wedge \alpha 2 \wedge \ldots \alpha n$ Premise

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αi Conclusion

An inference rule: Resolution

l1 V l2, ¬l2 V l3

l1 V l3

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$
$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$
$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$
$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$
$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$
$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$
$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$
$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan}$$
$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan}$$
$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$
$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$

Use propositional logic inference rule to show that

1

KB:

- A1: $(R \Rightarrow \neg S) \wedge (T \Rightarrow \neg U)$
- A2: $(V \Rightarrow \neg W) \wedge (X \Rightarrow \neg Y)$
- A3: $(T \Rightarrow W) \wedge (U \Rightarrow \neg Y)$
- A4: $(V \vee R)$

entails $(\neg T \vee \neg U)$.

A5: $T \Rightarrow \neg U$, A1, And elimination

A6: $\neg T \vee \neg U$, A5, implication elimination

Use propositional logic inference rule to show that

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KB:
- A1: $R \vee T$
- A2: $Q \wedge U$
- A3: $(S \vee T) \Rightarrow (Q \Rightarrow P)$
- A4: $\neg R \vee S$

entails P.

A5: $S \vee T$ A1,A4, Resolution

A6: $(Q \Rightarrow P)$ A3,A5, Modus Ponens

A7: Q, A2, and elimination

A8: P, A7 & A6, Modus Ponens

Use propositional logic inference rule to show that

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KB:

- A1: $S_{1,1} \Leftrightarrow W_{1,2} \vee W_{2,1}$
- A2: $\neg S_{1,2}$
- A3: $S_{1,1}$
- A4: $\neg W_{2,2}$
- A5: $\neg W_{1,2}$

entails $W_{2,1}$.

A6: $(S_{1,1} \Rightarrow W_{1,2} \vee W_{2,1}) \wedge (W_{1,2} \vee W_{2,1} \Rightarrow S_{1,1})$, Biconditional elimination

A7: $S_{1,1} \Rightarrow W_{1,2} \vee W_{2,1}$, A6, And elimination

A8: $W_{1,2} \vee W_{2,1}$, A7 & A3, Modus Ponens

A9: W2,1, A8 & A5, Resolution

Q3. Use the resolution rule to show that KB |= α, KB :

KB:

- **A1: $S_{1,1} \Leftrightarrow W_{1,2} \vee W_{2,1}$**
- **A2: $\neg S_{1,2}$**
- **A3: $S_{1,1}$**
- **A4: $\neg W_{2,2}$**
- **A5: $\neg W_{1,2}$**

$\alpha = W_{2,1}$.

To prove KB |= α, we need to show that KB ∧ ¬α is not satisfiable.

First we convert KB ∧ ¬α to CNF (conjunctive normal form).

Very important: When you convert to CNF, use only logical equivalences. Do not use and elimination, Modus Ponens or resolution, because the new sentence will not be equivalent to the original (you loose information).

A6: $(S_{1,1} \Rightarrow W_{1,2} \lor W_{2,1}) \land (W_{1,2} \lor W_{2,1} \Rightarrow S_{1,1})$, A1, Biconditional elimination

A7: $(\neg S_{1,1} \lor W_{1,2} \lor W_{2,1}) \land (\neg (W_{1,2} \lor W_{2,1}) \lor S_{1,1})$, A6, Implication elimination

A8: $(\neg S_{1,1} \lor W_{1,2} \lor W_{2,1}) \land ((\neg W1,2 \land \neg W2,1) \lor S1,1)$, A7, De Morgan

A9: $(\neg S1,1 \lor W1,2 \lor W2,1) \land (\neg W1,2 \lor S1,1) \land (\neg W2,1 \lor S1,1)$ A8, Distribuitivity of $\lor$ over $\land$.

Now KB ∧ ¬α looks like:

- B1: $\neg S_{1,1} \vee W_{1,2} \vee W_{2,1}$

- B2: $\neg W_{1,2} \vee S_{1,1}$
- B3: $\neg W_{2,1} \vee S_{1,1}$
- B4: $\neg S_{1,2}$
- B5: $S_{1,1}$
- B6: $\neg W_{2,2}$
- B7: $\neg W_{1,2}$
- B8: $\neg W_{2,1}$ (this is ¬α)

Now, we can apply resolution:

- B9: $W_{1,2} \vee W_{2,1}$, B1 & B5
- B10: $W_{2,1}$, B9 & B7
- B11: Empty clause, B10 & B8

Since we derived the empty clause (which means a contradiction),

we deduce that KB |= α.

$W_{2,1}$

Q4: Consider the following KB:

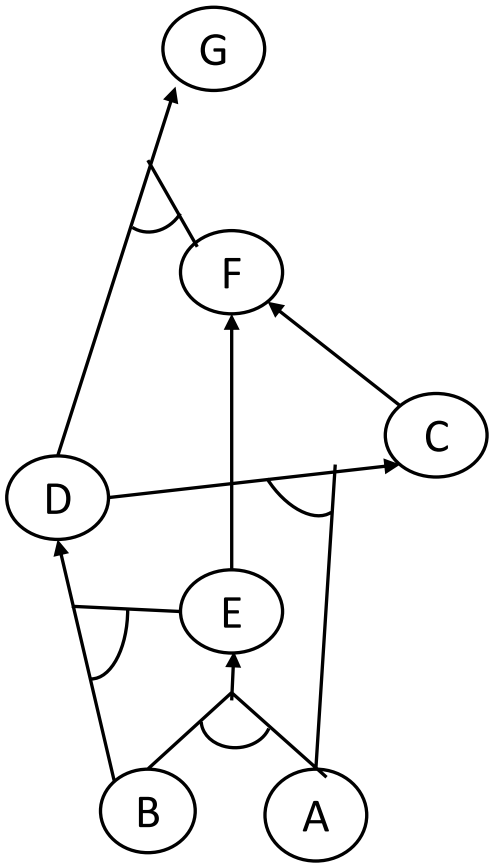
- $A \wedge B \Rightarrow E$
- $A \wedge D \Rightarrow C$
- $E \Rightarrow F$
- $B \wedge E \Rightarrow D$
- $C \Rightarrow F$
- $D \wedge F \Rightarrow G$
- A
- B

1. What is the standard form in which the KB is written?

The KB is in Horn form.

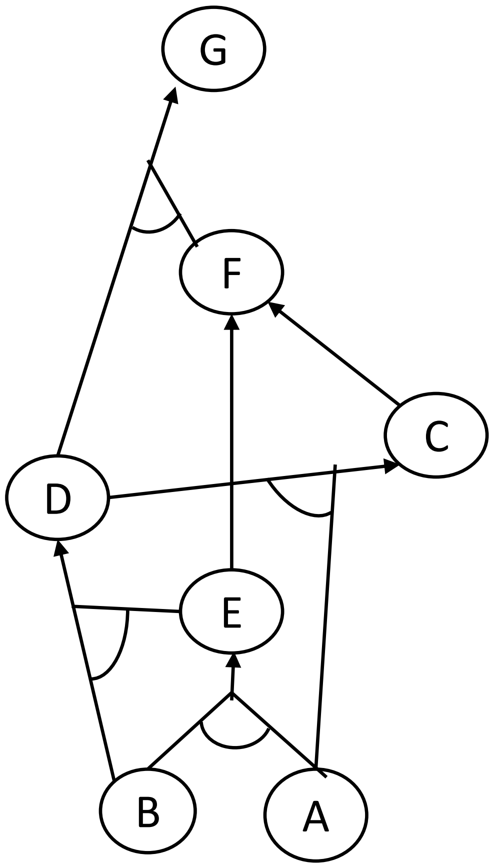
2. Draw the And-Or graph that represents the KB.

- $A \land B \Rightarrow E$
- $A \land D \Rightarrow C$
- $E \Rightarrow F$
- $B \land E \Rightarrow D$
- $C \Rightarrow F$
- $D \land F \Rightarrow G$



3. Use forward and backward chaining to show that KB |= G.

Facts	Clauses
A, B A, B , E A, B, E, F A, B, E, F, D A, B, E, F, D, C A, B, E, F, D, C, G	A ∧ B ⇒ E E ⇒ F B ∧ E ⇒ D A ∧ D ⇒ C D ∧ F ⇒ G



Facts	Goals	Clauses
A, B A, B A, B A, B, E A, B, E, D A, B, E, D, F A, B, E, D, F, G	G G, F, D G, F, D, E G, F, D G, F G	$D \wedge F \Rightarrow G$ $B \wedge E \Rightarrow D$ $A \wedge B \Rightarrow E$ $B \wedge E \Rightarrow D$ $E \Rightarrow F$ $D \wedge F \Rightarrow G$