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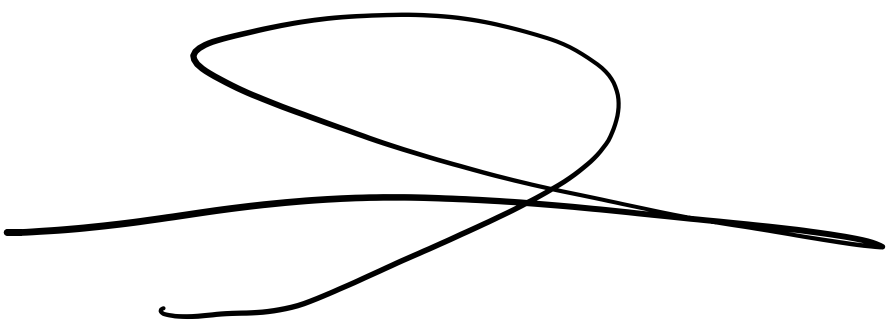
PM 1:30 [illegible]

الأحد



Q1: $A = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$

let $u = A$, $v = B$

$$\langle u, v \rangle = 2(-3) + 4(1) + -1(4) + 3(2)$$

$$= \cancel{-6} + \cancel{4} - \cancel{4} + \cancel{6} = 0$$

$$\|u\| = \sqrt{\underbrace{4+16}_{20} + \underbrace{1+9}_{10}} = \sqrt{30}$$

$$\|v\| = \sqrt{\underbrace{9+1}_{10} + \underbrace{16+4}_{20}} = \sqrt{30}$$

we know $\cos\theta = \dfrac{\langle u, v\rangle}{\|u\|\,\|v\|}$

$$\theta = \cos^{-1}\left(\frac{\langle u, v\rangle}{\|u\|\,\|v\|}\right)$$

$$\cos\theta = \left(\frac{0}{\sqrt{30}\,\sqrt{30}}\right) = 0 \implies \theta = \cos^{-1}(0)$$

$$= 90$$

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Q3 : $u_1 = (1,1,1)$, $u_2 = (0,1,1)$, $u_3 = (0,0,1)$

$e_1 = u_1 = (1,1,1)$

$$e_2 = u_2 - \frac{(u_2, e_1)}{\|e_1\|^2} e_1$$

$\|e_1\|^2 = 3$

$\langle u_2, e_1 \rangle = (0,1,1)(1,1,1) = 2$

$$(0,1,1) - \frac{2}{3}(1,1,1)$$

$$\left(0,\frac{3}{3},\frac{3}{3}\right)-\left(\frac{2}{3},\frac{2}{3},\frac{2}{3}\right)$$

$$e_2=\left(-\frac{2}{3},\frac{1}{3},\frac{1}{3}\right)$$

$$e_3=v_3-\frac{(e_1,v_3)}{\|e_1\|^2}e_1-\frac{(v_3,e_2)}{\|e_2\|^2}e_2$$

$$\|e_1\|^2=3$$

$$\|e_2\|^2=\left(-\frac{3}{3}\right)^2,\left(\frac{1}{3}\right)^2,\left(\frac{1}{3}\right)^2$$

$$= \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

$$\langle e_1, u_3 \rangle = (1,1,1)(0,0,1)$$

$$= 1$$

$$(0,0,1) \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right) = \frac{1}{3}$$

$$(0,0,1) - \frac{1}{3}(1,1,1)$$

$$- \left(\frac{\frac{1}{3}}{\frac{2}{3}} \right) \frac{3}{6} = \frac{1}{2}$$

$$\left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$= (0, 0, 1) - \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$- \left(-\frac{2}{6}, \frac{1}{6}, \frac{1}{6}\right)$$

$$\left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}\right) + \left(+\frac{2}{6}, -\frac{1}{6}, -\frac{1}{6}\right)$$

$$\left(-\frac{2}{6}, -\frac{2}{6}, \frac{4}{6}\right) + \left(\frac{2}{6}, -\frac{1}{6}, -\frac{1}{6}\right)$$

$$\left(0, -\frac{1}{2}, \frac{1}{2}\right)$$

$$e1 = (1, 1, 1), \quad e2 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right),$$

$$V_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$V_2 = \left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$V_3 = \left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$v_1 = \frac{v_1}{||v_1||}$$

$$v_2 = \frac{v_2}{||v_2||}$$

$$v_3 = \frac{v_3}{||v_3||}$$

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Q2: by counter example $u = (5, -3, 4)$

$\langle u, u \rangle \geq 0$

$u_1 u_2 + u_2 u_1$

$5(-3) + 5(-3)$

$-15 - 15 = -30$

so is not inner product