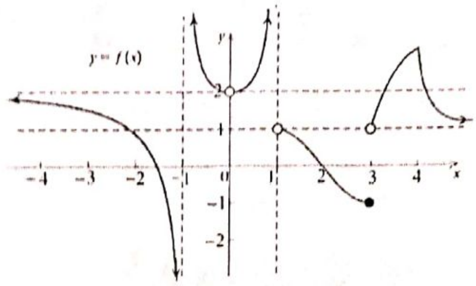
KING SAUD UNIVERSITY
DEANSHIP OF COMMON FIRST YEAR
BASIC SCIENCES DEPARTMENT

MATH 101 | HW # 2 / FIRST SEMESTER 1441 | Date: 24/10/2019

Question 1 — 6 Marks (1 each)

Use the graph of $f(x)$ to answer the following questions (if any)



1. $\lim_{x\to 3} f(x)$
2. $\lim_{x\to 0} f(x)$
3. $2f(-2) + 3\lim_{x\to 2} \frac{f(x)}{5x+1}$
4. Find all vertical asymptotes of f (if any).
5. Find all horizontal asymptotes of f (if any).
6. Discuss the continuity of f on its domain.

أبو عمر حجازي
0546236003

① $\lim_{x\to 3^+} F(X) = 1$, $\lim_{x\to 3^-} F(X) = -1$, so, $\lim_{x\to 3} F(X)$ D.N.E

② $\lim_{x\to 0^+} F(X) = 2$, $\lim_{x\to 0^-} F(X) = 2$, so, $\lim_{x\to 0} F(X) = 2$

③ $\lim_{x\to 2^+} F(X) = 0$, $\lim_{x\to 2^-} F(X) = 0$, $\lim_{x\to 2} F(X) = 0$

So, $2f(-2) + 3\left(\frac{\lim_{x\to 2} F(X)}{\lim_{x\to 2}(5X+1)}\right)$

$= 2(1) + 3\left(\frac{0}{11}\right) = 2 + 0 = 2$

④ $\lim_{x\to 1^-} f(X) = \infty$, $\lim_{x\to -1^+} f(X) = \infty$ } so, F(X) have V.A at $X=1, X=-1$

⑤ $\lim_{x\to \infty} f(X) = 1$, $\lim_{x\to -\infty} f(X) = 2$ } so, f(X) have H.A at $y=1, y=2$

⑥ $D_f = R - \{-1, 0, 1\}$, $f(3) = -1$

$\lim_{x\to 3^+} F(X) = 1$, $\lim_{x\to 3^-} F(X) = -1$, $\lim_{x\to 3} F(X)$ D.N.E

F(X) is dis continuous at $X=3$ on its domain

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Question 2

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4 Marks (2 each)

Use the definition of limit to show the following:

1. $\lim_{x\to -2}(1-2x)=5$

2. $\lim_{x\to \frac{3}{2}^+}\sqrt{2x-3}=0$

① For any $\varepsilon>0$ there exist $\delta>0$

such that : $0<|x+2|<\delta$

then : $|1-2x-5|<\varepsilon$

$|-2x-4|<\varepsilon$

$|-2(x+2)|<\varepsilon$

$2|x+2|<\varepsilon$

$|x+2|<\frac{\varepsilon}{2}$

we choose $\delta=\frac{\varepsilon}{2}$

So, $\lim_{x\to -2}(1-2x)=5$

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② For any $\varepsilon>0$ ther is $\delta>0$

such that : $\frac{3}{2}<x<\frac{3}{2}+\delta$

$0<x-\frac{3}{2}<\delta$

$0<2x-3<2\delta$

then: $|\sqrt{2x-3}-0|<\varepsilon$

$|\sqrt{2x-3}|<\varepsilon$

$2x-3<\varepsilon^2$

we choose $2\delta=\varepsilon^2$

$\delta=\frac{\varepsilon^2}{2}$

So, $\lim_{x\to 2^+}\sqrt{2x-3}=0$

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Question 3 0546236003 أبو عمر حجازي

A. Find all horizontal asymptotes for the following functions (if any) **8 Marks (2 each)**

1. $f(x) = \dfrac{2x-1}{\sqrt{9x^2+4x}-x}$ 2. $f(x) = \dfrac{x}{\sqrt{9-x^2}}$

B. Find all vertical asymptotes for the following functions (if any)

1. $f(x) = \dfrac{\sin x}{x}$ 2. $f(x) = \dfrac{x|x|-4}{x^2-2x}$

A ① : $F(x) = \dfrac{2x-1}{\sqrt{9x^2+4x}-x}$

$$\lim_{x\to\infty} \frac{2x-1}{\sqrt{x^2(9+\frac{4}{x})}-x} = \lim_{x\to\infty} \frac{2x-1}{|x|\sqrt{9+\frac{4}{x}}-x} = \lim_{x\to\infty} \frac{2x-1}{x\sqrt{9+\frac{4}{x}}-x} = \lim_{x\to\infty} \frac{2-\frac{1}{x}}{\sqrt{9+\frac{4}{x}}-1} = \frac{2-0}{\sqrt{9}-1} = \frac{2}{2} = 1$$

$$\lim_{x\to-\infty} \frac{2x-1}{\sqrt{x^2(9+\frac{4}{x})}-x} = \lim_{x\to-\infty} \frac{2x-1}{|x|\sqrt{9+\frac{4}{x}}-x} = \lim_{x\to-\infty} \frac{2x-1}{-x\sqrt{9+\frac{4}{x}}-x} = \lim_{x\to-\infty} \frac{2-\frac{1}{x}}{-\sqrt{9+\frac{4}{x}}-1} = \frac{2-0}{-\sqrt{9}-1} = \frac{2}{-4} = \frac{-1}{2}$$

So, $y=1$, $y=\frac{-1}{2}$ are H.A

A ② : $f(x) = \dfrac{x}{\sqrt{9-x^2}}$

$9-x^2>0$

$x^2<9$

$|x|<9$

$-3<x<3$

$D_f = (-3,3)$

$\lim_{x\to\infty} f(x)$ D.N.E , $\lim_{x\to-\infty} f(x)$ D.N.A

So, $f(x)$ has No H.A

$\boxed{Q_3}$ (B) ① $F(x) = \dfrac{\sin x}{x}$

Zeroes of denominator is $x = 0$

$$\lim_{x \to 0^+} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0^-} \frac{\sin x}{x} = 1$$

So, F(x) has No V.A

(B) ② $F(x) = \dfrac{x|x| - 4}{x^2 - 2x}$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$x = 0$ | $x - 2 = 0$

$x = 2$

$x = 0$, $x = 2$ are zeroes of denominator

at $x = 0$

$$\lim_{x \to 0^+} f(x) = \infty$$

at $x = 0$, f(x) has V.A

$$\lim_{x \to 0^-} f(x) = -\infty$$

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at $x = 2$

$$\lim_{x \to 2^+} \frac{x|x| - 4}{x^2 - 2x} = \lim_{x \to 2^+} \frac{x^2 - 4}{x^2 - 2x} = \lim_{x \to 2^+} \frac{\cancel{(x-2)}(x+2)}{x\cancel{(x-2)}} = \frac{4}{2} = 2$$

$$\lim_{x \to 2^-} \frac{x|x| - 4}{x^2 - 2x} = \lim_{x \to 2^-} \frac{x^2 - 4}{x^2 - 2x} = \lim_{x \to 2^-} \frac{\cancel{(x-2)}(x+2)}{x\cancel{(x-2)}} = \frac{4}{2} = 2$$

at $x = 2$ f(x) has No V.A

So, F(x) has V.A at $x = 0$

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Question 4

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3 Marks

Find the value of a and b such that:

$$\lim_{x\to 0}\frac{a-\cos(bx)}{x^2}=8$$

the type of indeterminate is form $\left(\frac{0}{0}\right)$

so, $a-\cos(0)=0$

$a-1=0$

$a=1$

$$\lim_{x\to 0}\frac{1-\cos(bx)}{x^2}\cdot\frac{1+\cos(bx)}{1+\cos(bx)}=8$$

$$\lim_{x\to 0}\frac{1-\cos^2(bx)}{x^2(1+\cos(bx))}=8$$

$$\lim_{x\to 0}\frac{\sin^2(bx)}{x^2}\cdot\frac{1}{1+\cos(bx)}=8$$

$$\lim_{x\to 0}\left(\frac{\sin(bx)}{x}\right)^2\cdot\lim_{x\to 0}\frac{1}{1+\cos(bx)}=8$$

$$b^2\cdot\frac{1}{2}=8$$

$$b^2=16$$

$$b=\pm 4$$

Question 5

20 Marks (2 each)

Find the following limits (if exists)

1. $\lim\limits_{x\to -1}\frac{x^2-1}{2x+1}$
2. $\lim\limits_{x\to 0}\frac{(x+2)^3-8}{x}$
3. $\lim\limits_{x\to\infty}(\sqrt{x^2+2}-x)$
4. $\lim\limits_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$
5. $\lim\limits_{x\to 3}\frac{x-3}{|x-3|}$
6. $\lim\limits_{x\to\infty}\frac{2x+\sin x}{4x+1}$
7. $\lim\limits_{x\to 0^+}\frac{\sqrt{1-\sin^2(\frac{\pi}{2}-x)}}{3x}$
8. $\lim\limits_{x\to 0}\left[\frac{1}{x}\left(\frac{1}{\sqrt{1+x}}-1\right)\right]$
9. $\lim\limits_{x\to 2}\cos\left(\frac{x^2-4}{x+1}\right)$
10. $\lim\limits_{x\to 0}\frac{x}{\tan(2x)+\sin(3x)}$

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P5 ① $\lim_{x\to -1} \frac{x^2-1}{2x+1} = \frac{(-1)^2-1}{2(-1)+1} = \frac{0}{-1} = 0$

② $\lim_{x\to 0} \frac{(x+2)^3-8}{x} \quad \left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{(x+2-2)\left((x+2)^2+2(x+2)+4\right)}{x}$

$= \lim_{x\to 0} \frac{\cancel{x}\left((x+2)^2+2(x+2)+4\right)}{\cancel{x}}$

$= \lim_{x\to 0} \left((x+2)^2+2(x+2)+4\right) = 4+4+4 = 12$

③ $\lim_{x\to\infty} \left(\sqrt{x^2+2}-x\right) \quad (\infty-\infty)$

$= \lim_{x\to\infty} \frac{\sqrt{x^2+2}-x}{1} \cdot \frac{\sqrt{x^2+2}+x}{\sqrt{x^2+2}+x}$

$= \lim_{x\to\infty} \frac{\cancel{x^2}+2-\cancel{x^2}}{\sqrt{x^2+2}+x}$

$= \lim_{x\to\infty} \frac{2}{\sqrt{x^2+2}+x} = \frac{2}{\infty} = 0$

④ $\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1}\right) = \lim_{x\to 1} \left(\frac{1}{x-1} - \frac{2}{(x-1)(x+1)}\right)$

$= \lim_{x\to 1} \left(\frac{x+1-2}{(x-1)(x+1)}\right) = \lim_{x\to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} \left(\frac{0}{0}\right)$

$= \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2}$

⑤ $\lim_{x\to 3} \frac{x-3}{|x-3|} \quad \left(\frac{0}{0}\right)$

$\lim_{x\to 3^+} \frac{x-3}{|x-3|} = \lim_{x\to 3^+} \frac{x-3}{x-3} = 1$

$\lim_{x\to 3^-} \frac{x-3}{|x-3|} = \lim_{x\to 3^-} \frac{x-3}{-(x-3)} = -1$

so, $\lim_{x\to 3} \frac{x-3}{|x-3|}$ D.N.E

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⑥ $\lim\limits_{x\to\infty} \dfrac{2x+\sin x}{4x+1} \quad \left(\dfrac{\infty}{\infty}\right)$ 0546236003 أبوعمر حجازي

$= \lim\limits_{x\to\infty} \dfrac{2+\dfrac{\sin x}{x}}{4+\dfrac{1}{x}}$

$= \dfrac{2+0}{4+0} = \dfrac{2}{4} = \dfrac{1}{2}$

$-1 \le \sin x \le 1$

$\dfrac{-1}{x} \le \dfrac{\sin x}{x} \le \dfrac{1}{x}$

$\lim\limits_{x\to\infty} \dfrac{-1}{x} = 0$

$, \lim\limits_{x\to\infty} \dfrac{1}{x} = 0$

so, $\lim\limits_{x\to\infty} \dfrac{\sin x}{x} = 0$

⑦ $\lim\limits_{x\to 0^-} \dfrac{\sqrt{1-\sin^2(\frac{\pi}{2}-x)}}{3x} \quad \left(\dfrac{0}{0}\right)$

$= \lim\limits_{x\to 0^-} \dfrac{\sqrt{\cos^2(\frac{\pi}{2}-x)}}{3x}$

$= \lim\limits_{x\to 0^-} \dfrac{\cos(\frac{\pi}{2}-x)}{3x} = \lim\limits_{x\to 0^-} \dfrac{\sin x}{3x} = \dfrac{1}{3}$

⑧ $\lim\limits_{x\to 0} \left[\dfrac{1}{x}\left(\dfrac{1}{\sqrt{1+x}} - \dfrac{1}{1}\right)\right] \quad (\infty \cdot 0)$

$= \lim\limits_{x\to 0} \left[\dfrac{1}{x}\left(\dfrac{1-\sqrt{1+x}}{\sqrt{1+x}}\right)\right]$

$= \lim\limits_{x\to 0} \dfrac{1-\sqrt{1+x}}{x\sqrt{1+x}} \cdot \dfrac{1+\sqrt{1+x}}{1+\sqrt{1+x}}$

$= \lim\limits_{x\to 0} \dfrac{\cancel{1}-\cancel{1}-x}{x\sqrt{1+x}\,(1+\sqrt{1+x})} \quad \left(\dfrac{0}{0}\right)$

$= \lim\limits_{x\to 0} \dfrac{\cancel{-x}^{\,-1}}{\cancel{x}\sqrt{1+x}\,(1+\sqrt{1+x})} = \lim\limits_{x\to 0} \dfrac{-1}{\sqrt{1+x}\,(1+\sqrt{1+x})} = \dfrac{-1}{2}$

⑨ $\lim\limits_{x\to 2} \cos\left(\dfrac{x^2-4}{x+1}\right) = \cos\left(\lim\limits_{x\to 2} \dfrac{x^2-1}{x+1}\right) = \cos(0) = 1$

⑩ $\lim\limits_{x\to 0} \dfrac{x}{\tan(2x)+\sin(3x)} \quad \left(\dfrac{0}{0}\right)$

$= \lim\limits_{x\to 0} \dfrac{\frac{x}{x}}{\frac{\tan(2x)}{x} + \frac{\sin(3x)}{x}} = \dfrac{1}{2+3} = \dfrac{1}{5}$

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Question 6

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A. Discuss the continuity of the function $f(x) = \cos(x^2+1)$.

9 Marks (3 each)

B. Use the Intermediate Value Theorem to prove that the equation $\frac{x^5+1}{x+3} = 3$ has at least a real solution.

C. Find the constants a and b such that the function

$$f(x) = \begin{cases} \sqrt{\dfrac{x+4}{x+b}}, & x > 0 \\ a+b, & x = 0 \\ \dfrac{\sin(2x)}{3x}, & x < 0 \end{cases}$$

is continuous on $\mathbb{R}$.

(A) Note that;

$F(X) = h(g(X))$ such that $g(X) = X^2+1$

, $h(X) = \cos(X)$

both of $g(X)$ and $h(X)$ are countinuious on R

So, $F(X)$ is countinuous on R

(B) $\frac{X^5+1}{X+3} - 3 = 0$

$\frac{X^5+1-3X-9}{X+3} = 0$

$F(X) = \frac{X^5-3X-8}{X+3}$

$f(X)$ is rational function so, $f(X)$ is countinuous on R exept the zeroes of denominator.

$X+3=0$

$X=-3$

So, $f(X)$ is countinuous on $R - \{-3\}$

$f(0) = \frac{-8}{3}$, $f(2) = \frac{18}{5}$

$[0,2] \subset D_F$, $\frac{-8}{3} < 0 < \frac{18}{5}$

by I.V.t there exist $c \in [0,2]$ such that $f(c) = 0$

So, the equation has at least areal solutio.

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(c) $f(x)$ is continuous on R

$f(x)$ is continuous at $x=0$

$$f(0) = \lim_{x \to 0^-} \frac{\sin(2x)}{3x}$$

$$\boxed{a + b = \frac{2}{3}} \rightarrow (1)$$

$$\lim_{x \to 0^+} \sqrt{\frac{x+4}{x+b}} = \lim_{x \to 0^-} \frac{\sin(2x)}{3x}$$

$$\frac{2}{\sqrt{b}} = \frac{2}{3}$$

$$2\sqrt{b} = 6$$

$$\sqrt{b} = 3$$

$$\boxed{b = 9}$$

From equation (1)

$$a + 9 = \frac{2}{3}$$

$$a = \frac{2}{3} - 9$$

$$\boxed{a = \frac{-25}{3}}$$

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