



محاضرات في ريف ١٠١ جامعة الملك سعود
مع الحفني جوال: ٠٥٨٣٤٢٢٢٠٠

حل الميف الثاني ٣٩/٤٠ math (101)



كورس ريف ١٠١ عبدالله الحفني ٠٥٨٣٤٢٢٢٠٠

عبدالله الحفني جوال ٠٥٨٣٤٢٢٢٠٠

نظم اقوي مراجعات ريف ١٠١ +A مع ليلة الامتحان تشمل علي جميع الاسئلة المتوقعة لهذا العام



نحياتي لكم جميعا عبدالله الحفني

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Question(1)

A). Use the graph below of a function f to answer the following:

1 $\lim_{x \rightarrow 3} f(x)$

خطوات الحل

$\lim_{x \rightarrow 3^+} f(x) = 2$

$\lim_{x \rightarrow 3} f(x) = 2$

$\lim_{x \rightarrow 3^-} f(x) = 2$

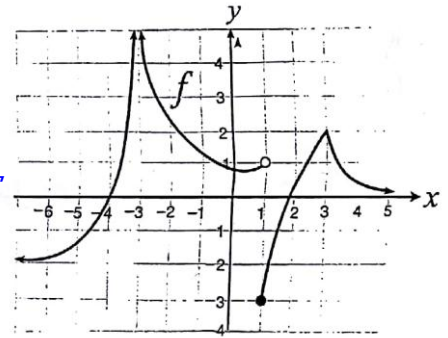
2 $\lim_{x \rightarrow 1} f(x)$

خطوات الحل

$\lim_{x \rightarrow 1^+} f(x) = -3$

$\lim_{x \rightarrow 1} f(x) = 1$

$\neq D \cdot N \cdot E$



3 Find the vertical asymptotes and horizontal asymptotes of f

خطوات الحل

(i) V.A at : $x = -3$ $\left\{ \begin{array}{l} \lim_{x \rightarrow -3^+} f(x) = \infty \\ \lim_{x \rightarrow -3^-} f(x) = \infty \end{array} \right.$

(ii) H.A at $\left\{ \begin{array}{l} y = -2 \Rightarrow \lim_{x \rightarrow -\infty} f(x) = -2 \\ y = 0 \Rightarrow \lim_{x \rightarrow \infty} f(x) = 0 \end{array} \right.$

4 Determine the x - coordinats (s) in domain of f
at which the function is not differentiable.

خطوات الحل

$D_f = (-\infty, -3) \cup (-3, 1] \cup [1, \infty)$

not Diff at: $x = 1$
at: $x = 3$ because cusp

(B). Use the definition of the limit to prove that: $\lim_{x \rightarrow 1} (3x + 2) = 5$

خطوات الحل

Assume $\varepsilon > 0$ we are going to find $\delta > 0$

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let $\varepsilon > 0, \delta > 0$

$$0 < |x - 1| < \delta$$

$$|x - 1| < \delta$$

$$|3x + 2 - 5| < \varepsilon$$

$$|3x - 3| < \varepsilon$$

$$3|x - 1| < \varepsilon$$

$$|x - 1| < \frac{\varepsilon}{3}$$

$$\text{take } \delta = \frac{\varepsilon}{3}$$

نحدد

$$a = 1$$

$$f(x) = 3x + 2$$

$$L = 5$$

Question(2)

Evaluate the following limits (if exists):

$$1 \lim_{x \rightarrow 0} (4x + 5)^2$$

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خطوات الحل

$$\left[\lim_{x \rightarrow 0} 4x + 5 \right]^2 = (5)^2 = 25$$

$$2 \lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4} \left(\frac{0}{0} \right)$$

خطوات الحل

$$\lim_{x \rightarrow 4} \frac{\sqrt{x+5} - 3}{x - 4} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3}$$

$$= \lim_{x \rightarrow 4} \frac{(x+5) - 9}{(x-4)\sqrt{x+5} + 3}$$

$$= \lim_{x \rightarrow 4} \frac{(x-4)}{(x-4)\sqrt{x+5} + 3}$$

$$= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x+5} + 3} = \frac{1}{6}$$

$$3 \lim_{x \rightarrow 0} \frac{\sin(2x) + \tan(4x)}{x}$$

خطوات الحل

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{x} + \lim_{x \rightarrow 0} \frac{\tan 4x}{x}$$

$$= 2 + 4 = 6$$

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$$4 \lim_{x \rightarrow \infty} \frac{\sin^2 x}{x^2 + 1}$$

خطوات الحل

using $S \cdot T$.

$$0 \leq \sin^2 x \leq 1 \quad \boxed{(\div) x^2 + 1}$$

$$\frac{0}{x^2 + 1} \leq \frac{\sin^2 x}{x^2 + 1} \leq \frac{1}{x^2 + 1}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x^2 + 1} = 0 = \lim_{x \rightarrow \infty} \frac{0}{x^2 + 1} = 0$$

$$\lim_{x \rightarrow \infty} \frac{\sin^2 x}{x^2 + 1} = 0$$

Question(3)**A). Use the intermediate value theorem to show that function:**

$$f(x) = \frac{-4x+2}{x+3} \text{ Has a zero in } [-1,1].$$

خطوات الحل

$$f(x) = \frac{-4x+2}{x+3}; x \neq -3 \notin [-1,1] \quad f(-1) = \frac{-4(-1)+2}{-1+3} = 3 > 0$$

$$f(x) \text{ is cont } \in [-1,1]$$

$$f(1) = \frac{-4(1)+2}{1+3} = -\frac{1}{2} < 0$$

$$-\frac{1}{2} < 0 < 3$$

Using **I.V.T** then exist at least $c \in [-1,1] \Rightarrow f(c) = 0$

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B). Use the definition of the derivative to find: f' of $f(x) = x^2$.

خطوات الحل

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} = 2x$$

Note

$$(a+b)^2 = a^2 + 2ab + b^2$$

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C). Find an equation for the tangent line $f(x) = x + 3\cos x$ at $x = \frac{\pi}{2}$.

خطوات الحل

$$f'(x) = 1 - 3\sin x$$

$$m = f'\left(\frac{\pi}{2}\right) = 1 - 3\sin \frac{\pi}{2} = -2$$

$$y = m(x - a) + f(a)$$

$$= -2\left(x - \frac{\pi}{2}\right) + \frac{\pi}{2}$$

$$= -2x + \pi + \frac{\pi}{2}$$

$$y = -2x + \frac{3\pi}{2}$$

$$a = \frac{\pi}{2}$$

$$f(a) = f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$m = -2$$

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Question(4)

A). Find the derivative of each of the following :

$$[1] f(x) = x^3 + 4x^2 + 5x + 2$$

خطوات الحل

$$f'(x) = 3x^2 + 8x + 5$$

$$[2] f(x) = \frac{x^2 - 7}{x + 5}$$

خطوات الحل

$$\begin{aligned} f'(x) &= \frac{2x(x+5) - (x^2 - 7)}{(x+5)^2} \\ &= \frac{2x^2 + 10x - x^2 + 7}{(x+5)^2} \\ &= \frac{x^2 + 10x + 7}{(x+5)^2} \end{aligned}$$

$$[3] f(x) = \cos x \sin x$$

خطوات الحل

$$\begin{aligned} f(x) &= \frac{1}{2} \sin 2x \\ f'(x) &= \frac{1}{2} \cdot 2 \cos 2x \\ &= \cos 2x \end{aligned}$$

B). Find the values of a, b and c such that $f(x) = \begin{cases} ax^2 + bx + 3c & , x > 1 \\ 5ax^2 + bx + 3c - 2 & , x = 1 \\ 5ax^3 + bx^2 + c & , x < 1 \end{cases}$

Is differentiable at $x = 1$.

خطوات الحل

F is diff at $x = 1$

Cont at $x = 1$

$$f(1) = \lim_{x \rightarrow 1^-} f(1) \dots [1]$$

$$5a + b + 3c - 2 = a + b + 3c$$

$$4a = 2 \Rightarrow a = \frac{1}{2}$$

$$f(1) = \lim_{x \rightarrow 1^+} f(1) \dots [2]$$

$$15a + 2b = 2a + b$$

$$5a + b + 3c - 2 = 5a + b + c$$

$$2c = 2, c = 1$$

$$f'_+(1) = f'_-(1)$$

$$15a + 2b = 2a + b$$

$$2b - b = 2a - 15a$$

$$b = -13a$$

$$b = -13\left(\frac{1}{2}\right)$$

$$b = \frac{-13}{2}$$

$$\begin{array}{c} 5ax^2 + bx + 3c - 2 \\ \leftarrow f : 5ax^3 + bx^2 + c \quad ax^2 + bx + 3c \rightarrow \\ \bar{1} \quad \quad \quad 1 \end{array}$$

$$\begin{array}{c} 10ax + b \\ \leftarrow f' : 15ax^2 + 2bx \quad 2ax + b \rightarrow \\ (\bar{1})' \quad \quad \quad (1)' \\ 1 \end{array}$$



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