In this section, we are concerned with testing hypotheses concerning population variances or standard deviations. Let us first consider the problem of testing the null hypothesis H_0 that the population variance σ^2 equals a specified value σ_0^2 against one of the usual alternatives $\sigma^2 < \sigma_0^2$, $\sigma^2 > \sigma_0^2$, or $\sigma^2 \neq \sigma_0^2$. If we assume that the distribution of the population being sampled is normal, the chi-squared value for testing $\sigma^2 = \sigma_0^2$ is given by

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

where n is the sample size, s^2 is the sample variance, and σ_0^2 is the value of σ^2 given by the null hypothesis. If H_0 is true, χ^2 is a value of the chi-squared distribution with $\nu = n - 1$ degrees of freedom.

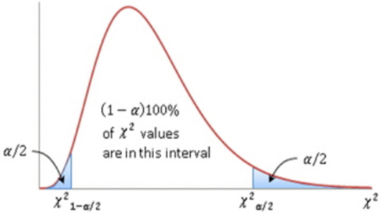

Figure 5-2: Figure 5.2

Hence, for a two-tailed test at the α-level of significance, the critical region is $\chi^2 < \chi^2_{1-\alpha/2}$ or $\chi^2 > \chi^2_{\alpha/2}$ (see figure 5.2). For the one-sided alternative $\sigma^2 < \sigma_0^2$, the critical region is $\chi^2 < \chi^2_{1-\alpha}$, and for the one-sided alternative $\sigma^2 > \sigma_0^2$, the critical region is $\chi^2 > \chi^2_{\alpha}$.

Example

A manufacturer of car batteries claims that the life of the company's batteries is approximately normally distributed with a standard deviation equal to 0.9 year. If a random sample of 10 of these batteries has a standard deviation of 1.2 years, do you think that $\sigma > 0.9$ year? Use a 0.05 level of significance.

Solution

1)

$$\begin{cases} H_0 : \sigma^2 = 0.81, \\ H_1 : \sigma^2 > 0.81. \end{cases}$$

2) $\alpha = 0.05$.

3) Critical region: The null hypothesis is rejected when $\chi^2 > 16.919$, where $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$, with $\nu = 9$ degrees of freedom.

4) Computations: $s^2 = 1.44$, $n = 10$, and

$$\chi^2 = \frac{(9)(1.44)}{0.81} = 16.0, \ P \approx 0.07.$$

5) Decision: The χ^2-statistic is not significant at the 0.05 level. However, based on the P-value 0.07, there is evidence that $\sigma > 0.9$.

Now let us consider the problem of testing the equality of the variances σ_1^2 and σ_2^2 of two populations. That is, we shall test the null hypothesis H_0 that $\sigma_1^2 = \sigma_2^2$ against one of the usual alternatives $\sigma_1^2 < \sigma_2^2$, $\sigma_1^2 > \sigma_2^2$, or $\sigma_1^2 \neq \sigma_2^2$. For independent random samples of sizes n_1 and n_2, respectively, from the two populations, the f-value for testing $\sigma_1^2 = \sigma_2^2$ is the ratio

$$f = \frac{s_1^2}{s_2^2}$$

where s_1^2 and s_2^2 are the variances computed from the two samples. If the two populations are approximately normally distributed and the null hypothesis is true, then the ratio $f = s_1^2/s_2^2$ is a value of the F-distribution with $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$ degrees of freedom. Therefore, the critical regions of size α corresponding to the one-sided alternatives $\sigma_1^2 < \sigma_2^2$ and $\sigma_1^2 > \sigma_2^2$ are, respectively, $f < f_{1-\alpha}(\nu 1, \nu 2)$ and $f > f_{\alpha}(\nu 1, \nu 2)$. For the two-sided alternative $\sigma_1^2 \neq \sigma_2^2$, the critical region is $f < f_{1-\alpha/2}(\nu 1, \nu 2)$ or $f > f_{\alpha/2}(\nu 1, \nu 2)$.

Example

In testing for the difference in the abrasive wear of the two materials in Example 166, we assumed that the two unknown population variances were equal. Were we justified in making this assumption? Use a 0.10 level of significance.

Solution

Let σ_1^2 and σ_2^2 be the population variances for the abrasive wear of material 1 and material 2, respectively.

1)

$$\begin{cases} H_0 : \sigma_1^2 = \sigma_2^2, \\ H_1 : \sigma_1^2 \neq \sigma_2^2. \end{cases}$$

2) $\alpha = 0.10$.

3) Critical region: We have $f_{0.05}(11, 9) = 3.11$, and, by using Theorem 99, we find $f_{0.95}(11, 9) = \frac{1}{f0.05(9,11)} = 0.34$. Therefore, the null hypothesis is rejected when $f < 0.34$ or $f > 3.11$, where $f = s_1^2/s_2^2$ with $\nu_1 = 11$ and $\nu_2 = 9$ degrees of freedom.

4) Computations: $s_1^2 = 16$, $s_2^2 = 25$, hence $f = 16/25 = 0.64$.

5) Decision: Do not reject H_0. Conclude that there is insufficient evidence that the variances differ.