KING SAUD UNIVERSITY
COMMON FIRST YEAR
BASIC SCIENCES DEPARTMENT
Math 101 Final Exam 1439/1440 H.
Second Semester
Time Allowed - 3 Hours

السنة الأولى المشتركة

St. Name: _____ St. ID: _____ Section: _____

ملاحظات :

1- اكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة (الإجابة على ورقة الأسئلة غير معتمدة).
علما بأن عدد الأسئلة (5)، وعدد الصفحات (2).

2- لا يسمح بالكتابة إلا بالقلم الأزرق فقط.

3- لا يسمح بتدوال الآلة الحاسبة بين الطلاب.

4- لا يسمح باستخدام آلة حاسبة قابلة للبرمجة أوآلة حاسبة ترسم دوال.

Question 1: (13 Marks)

A) Solve the following inequality, and write your answer in an interval notation

$$4x + 2 > 3x + 5 .$$

B) Use the definition of the limit to prove that $\lim_{x \to 3}(5 - 2x) = -1$.

C) Evaluate each of the following limits (if exist):

1) $\lim_{x \to 3} \left(x^2 + x + 1 \right)$

2) $\lim_{x \to 4} \dfrac{x^2 - 2x - 8}{x - 4}$

3) $\lim_{x \to 0} \dfrac{x - 2\sin(3x)}{4x + \tan x}$

4) $\lim_{x \to 3} \dfrac{\sqrt{x + 1} - 2}{x - 3}$

5) $\lim_{x \to \infty} \left(1 + \cos\left(\dfrac{\pi}{2x + 1} \right) \right)$

D) Suppose the inequality $\dfrac{1}{2} - \dfrac{x^2}{24} \le \dfrac{1 - \cos x}{x^2} \le \dfrac{1}{2}$ holds for values of x close to 0 .

Find $\lim_{x \to 0} \dfrac{1 - \cos x}{x^2}$.

Question 2: (6 Marks)

A) Prove that, if a function f is differentiable at a , then f is continuous at a .

B) Discuss the continuity of $f(x) = \begin{cases} \dfrac{2x^2 - 6x}{x - 3} & , \quad x \ne 3 \\ \\ 9 & , \quad x = 3 \end{cases}$ at $x = 3$.

C) The position of a particle is given by the equation $s(t) = 3t^3 - 4t^2 + 5$, where s in meters and t in seconds.

1- What is the velocity of the particle after 1 seconds?

2- When the acceleration is positive?

Question 3: (9 Marks)

Find the derivative $\dfrac{dy}{dx}$ for each of the following, in the simplest form:

A) $y = x^6 - 4x^3 + 2x - 4$

B) $y = \sqrt[3]{x^3 + 3x}$

C) $y = \dfrac{x^4 - 4}{x + 2}$

D) $y = \cos x \sec x$

E) $y = \cos^3\left(\sin^{-1}(x)\right)$

F) $x^3 y + \tan y = x$

Question 4: (7 Marks)

A) Show that the function $f(x) = x^4 - 8x^2 + 2$ satisfies the conditions of Rolle's Theorem on the interval $[-2, 2]$, then find the number c such that $f'(c) = 0$.
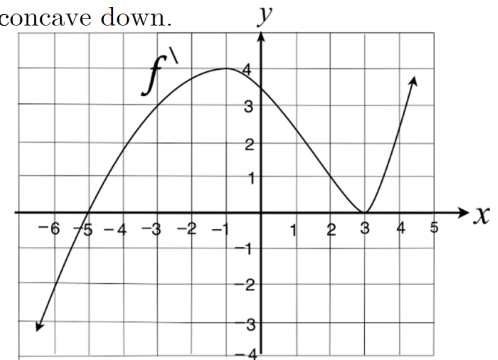
B) Let $f(x) = \dfrac{x^2 + 4x + 4}{x^3 + 2x^2}$. Find the horizontal asymptote(s), if exist.

C) Use the graph below of f' to find:

1) x-coordinate of the critical point(s).

2) The interval(s) in which the function f is increasing or decreasing.

3) The interval(s) in which the function f is concave up or concave down.

Question 5: (5 Marks)

For the function $f(x) = x^3 - 3x + 3$, find the following (if any):

A) The critical numbers of f.

B) The interval(s) on which f is increasing and decreasing.

C) The local extrema of f.

D) The interval(s) on which f is concave upward or downward.

E) Sketch the graph of f.

Good Luck