BASIC SCIENCES

King Saud University

Math 101 Final Exam 1441 H.

First Semester

Time Allowed - 3 Hours

سنة الأولى المشتركة

St. Name: ____________ St. ID: ____________ Section: ____________

ملاحظات :

1- اكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة (الاجابة على ورقة الأسئلة غير معتمدة).
علمًا بأن عدد الأسئلة (5)، وعدد الصفحات (2).

2- لا يسمح بالكتابة إلا بالقلم الأزرق فقط.

3- لا يسمح بتداول الآلة الحاسبة بين الطلاب.

4- لا يسمح باستخدام آلة حاسبة قابلة للبرمجة أو آلة حاسبة ترسم دوال.

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**Question 1:** (13 Marks)

A) Find the domain of

$$f(x)=\frac{1}{x-2}.$$

B) Use the definition of the limit to prove that $\lim\limits_{x\to 1}(2x+4)=6$.

C) Evaluate each of the following limits (if exist):

1) $\lim\limits_{x\to 3}(x^2-x+1)$

2) $\lim\limits_{x\to 4}\dfrac{\sqrt{x+5}-3}{x-4}$

3) $\lim\limits_{x\to 3}\dfrac{x^2+x-12}{x-3}$

4) $\lim\limits_{x\to \infty}\dfrac{2x^3+4x^2+5}{5x^3+7}$

5) $\lim\limits_{x\to 0} x^4\cos\left(\dfrac{2}{x^4}\right)$

D) Let $f(x)=\begin{cases}\dfrac{\sin(3x)}{kx}, & x<0\\ k(1-x)+2, & x\ge 0\end{cases}$.

Find the value(s) of $k$ such that $\lim\limits_{x\to 0} f(x)$ exists.

**Question 3:** **(9 Marks)**

Find the derivative $\frac{dy}{dx}$ for each of the following. Write your answer in the simplest form:

A) $y = 2x^8 + 5x^4 + 3x^2 + 10$

B) $y = (2x+7)^{40}$

C) $y = \frac{x^3}{x^2+1}$

D) $y = \sin^4 x + \pi^2$

E) $y = \tan^{-1}(2x)$

F) $x \tan y = x + y$

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**Question 4:** **(7 Marks)**

A) Show that the function $f(x) = x^2 - 3x + 1$ satisfies the conditions of the Mean Value Theorem [1, 3]. Find a number $c$ that satisfies the conclusion of the theorem.

B) Let $f(x) = \frac{1}{x^2 - 25}$. Find the vertical asymptote(s) of $f$.

C) Find the value of $k$ so that $f(x) = x^2 + \frac{x}{k}$ has a critical number at $x = 3$.

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**uestion 5:** **(5 M**

For the function $f(x) = x^3 - 6x^2$, find the following (if any):

A) The critical numbers of $f$.

B) The interval(s) on which $f$ is increasing and decreasing.

C) The local extrema of $f$.

D) The interval(s) on which $f$ is concave upward or downward.

E) Sketch the graph of $f$.

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***Good Luck***

حل

# Math 101 Final exam
## First semester

Q1

A) $R-\{2\}$

B) $\forall \varepsilon > 0 \ \exists \delta > 0$ S.T if $|x-1| < \delta$ then $|2x+4-6| < \varepsilon$

$|2x+4-6| < \varepsilon$

$|2x-2| < \varepsilon$

$|2(x-1)| < \varepsilon \Rightarrow \frac{\cancel{2}}{2} |x-1| < \boxed{\frac{\varepsilon}{2} = \delta}$

$\forall \varepsilon > 0 \ \exists \delta = \frac{\varepsilon}{2} > 0$ S.T if $|x-1| < \delta$ then $|2x+4-6| < \varepsilon$

c)

1) $\lim_{x\to 3} (x^2 - x + 1)$

$(3)^2 - (3) + 1 = 7$

2) $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \frac{0}{0}$ I.F

$\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$\lim_{x\to 4} \frac{(x+5)-9}{x-4(\sqrt{x+5}+3)}$

$\lim_{x\to 4} \frac{\cancel{x-4}}{\cancel{x-4}(\sqrt{x+5}+3)}$

$\lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{4+5}+3} = \frac{1}{6}$

3) $\lim_{x\to 3} \frac{x^2+x-12}{x-3} = \frac{0}{0}$ I.F

$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+4)}{\cancel{x-3}} = 3+4 = 7$

4) $\lim_{x\to\infty} \frac{2x^3+4x^2+5}{5x^3+7}$

طريقة أولى

$\lim_{x\to\infty} \frac{\frac{2x^3}{x^3} + \frac{4x^2}{x^3} + \frac{5}{x^3}}{\frac{5x^3}{x^3} + \frac{7}{x^3}}$

$\lim_{x\to\infty} \frac{2 + \frac{4}{x} + \frac{5}{x^3}}{5 + \frac{7}{x^3}} = \frac{2+0+0}{5+0}$

$= \frac{2}{5}$

طريقة ثانية

بما أن درجة البسط = درجة المقام

$= \frac{2}{5}$

5) $\lim_{x\to 0} x^4 \cos\left(\frac{2}{x^4}\right)$

by sandwich theorem

$-1 \leq \cos\left(\frac{2}{x^4}\right) \leq 1$

$-x^4 \leq x^4 \cos\left(\frac{2}{x^4}\right) \leq x^4$

$\lim_{x\to 0} -x^4 = 0$

$\lim_{x\to 0} x^4 = 0$

$\therefore \lim_{x\to 0} x^4 \cos\left(\frac{2}{x^4}\right) = 0$

$Q_1$

D) $f(x) = \begin{cases} \frac{\sin(3x)}{kx} & x<0 \\ k(1-x)+2 & x \geq 0 \end{cases}$ $\lim_{x\to 0} f(x)$ exists

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$Q_1$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

A; $\lim_{x\to 0} \frac{\sin(3x)}{kx} = \lim_{x\to 0} k(1-x)+2$

B) $\frac{3}{k} = \lim_{x\to 0} k - xk + 2 = k+2$

$\frac{3}{k} = k+2$

$k^2+2k = 3$

$k^2+2k-3=0$

$(k+3)(k-1)=0$

$k=-3, k=1$

c)

$Q_2$

A) $f(3)=4$

$\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{0}{0}$ I.f

$\lim_{x\to 3} \frac{(x-3)(x+3)}{\cancel{(x-3)}} = 3+3=6$

B)

1) $V(t) = S'(t) = 10t^4 - 3$

$V(2) = 10(2)^4 - 3 = 157$ m/s

2) $A(t) = V'(t) = 40t^3$

$A(2) = 40(2)^3 = 320$ m/$s^2$

$Q_2$ c) $y = \frac{1}{x}$ at $x=4$

$y = m(x-4) + f(4)$

$m = f'(4) = \frac{-1}{x^2} = \frac{-1}{(4)^2} = \frac{-1}{16}$

$f(4) = \frac{1}{4}$

$y = \frac{-1}{16}(x-4) + \frac{1}{4}$

$y = \frac{-x}{16} + \frac{1}{4} + \frac{1}{4}$

$y = \frac{-x}{16} + \frac{1}{2}$

$Q_3$

A) $y = 2x^8 + 5x^4 + 3x^2 + 10$

$y' = 16x^7 + 20x^3 + 6x$

B) $y = (2x+7)^{40}$

$y' = 40(2x+7)^{39}(2)$

$y' = 80(2x+7)^{39}$

c) $y = \frac{x^3}{x^2+1}$

$y' = \frac{3x^2(x^2+1) - (2x)(x^3)}{(x^2+1)^2}$

$y' = \frac{3x^4 + 3x^2 - 2x^4}{(x^2+1)^2}$

$y' = \frac{x^4+3x^2}{(x^2+1)^2}$

D) $y = \sin^4 x + \pi^2$

$y' = 4\sin^3 x(\cos x) + 0$

$y' = 4\sin^3 x \cos x$

E) $y = \tan^{-1}(2x)$

$y' = \frac{1}{1+(2x)^2} \cdot 2$

$y' = \frac{2}{1+4x^2}$

F) $x \tan y = x + y$

$x(\sec^2 y\, y') + (1)\tan y = 1 + y'$

$x\sec^2 y\, y' + \tan y = 1 + y'$

$x\sec^2 y\, y' - y' = 1 - \tan y$

$\frac{y'(x\sec^2 y - 1)}{x\sec^2 y - 1} = \frac{1 - \tan y}{x\sec^2 y - 1}$

$y' = \frac{1-\tan y}{x\sec^2 y - 1}$

$Q_4$

A) $f(x) = x^2 - 3x + 1 \quad [1, 3]$

$f(x)$ is polynomial funktion So $f(x)$ is cont on $[1,3]$

$f(x)$ is polynomial function So $f(x)$ is diff on $(1,3)$

$$f'(c) = \frac{f(3) - f(1)}{3 - 1} = \frac{1 - (-1)}{3 - 1} = \frac{2}{2} = 1$$

$f'(x) = 2x - 3$

$f'(c) = 2c - 3$

$f(3) = (3)^2 - 3(3) + 1 = 1$

$f(1) = (1)^2 - 3(1) + 1 = -1$

$2c - 3 = 1$

$2c = 4$

$\boxed{c = 2 \in (1, 3)}$

B) $f(x) = \frac{1}{x^2 - 25}$

$x^2 - 25 = 0$

$x = \pm 5$

$\lim_{x \to 5^+} \frac{1}{0^+} = +\infty$

$\lim_{x \to 5^-} \frac{1}{0^-} = -\infty$

$x = 5$ is V.A

$\lim_{x \to -5^+} \frac{1}{0^-} = -\infty$

$\lim_{x \to -5^-} \frac{1}{0^+} = +\infty$

$x = -5$ is V.A

c) $f(x) = x^2 + \frac{x}{k}$ at $x = 3$ has c.m

$f'(3) = 0 \qquad f'(3)$ D.N.E

$f'(x) = 2x + \frac{1}{k}$

$f'(3) = 0$

$2(3) + \frac{1}{k} = 0$

$6 + \frac{1}{k} = 0$

$\frac{1}{k} = -6$

$-6k = 1$

$k = \frac{1}{-6}$

Scanned with CamScanner

Q5 $f(x) = x^3 - 6x^2$.

1) $D_f = \mathbb{R}$

2) x-int $y=0$

$x^3 - 6x^2 = 0$

$x^2(x-6) = 0$

$x=0 \quad x=6$

$\boxed{(0,0)}, \boxed{(6,0)}$

y-int $x=0$

$y = (0)^3 - 6(0)^2$

$y = 0$

$\boxed{(0,0)}$

3) Symmetry

$f(-x) = -x^3 - 6x^2$ not even

$-f(x) = -x^3 + 6x^2$ not odd

f(x) is neither, no symmetry

4) $f'(x) = 3x^2 - 12x$

$f'(x) = 0$

$3x^2 - 12x = 0$

$3x(x-4) = 0$

$3x = 0 \quad x - 4 = 0$

$x = 0 \in D_f \quad x = 4 \in D_f$

$x = 0, 4$ are c.p

$\boxed{(0,0)}, \boxed{(4,-32)}$

$f'(x)$ D.N.E

No Sol.

| | $(-\infty,0)$ | $(0,4)$ | $(4,\infty)$ |
|---|---|---|---|
| $k$ | -1 | 1 | 5 |
| $f'(k)$ | 15 | -9 | 15 |
| | + ↗ | - ↘ | + ↗ |

inc: $(-\infty,0], [4,\infty)$

dec: $[0,4]$

5) $f''(x) = 6x - 12$

$f''(x) = 0$

$6x - 12 = 0$

$\frac{6x}{6} = \frac{12}{6}$

$x = 2 \in D_f$

$\boxed{(2,-16)}$

concave up: $(2,\infty)$

concave down: $(-\infty,2)$

$x = 2$ is I.P

$f''(x)$ D.N.E

No Sol.

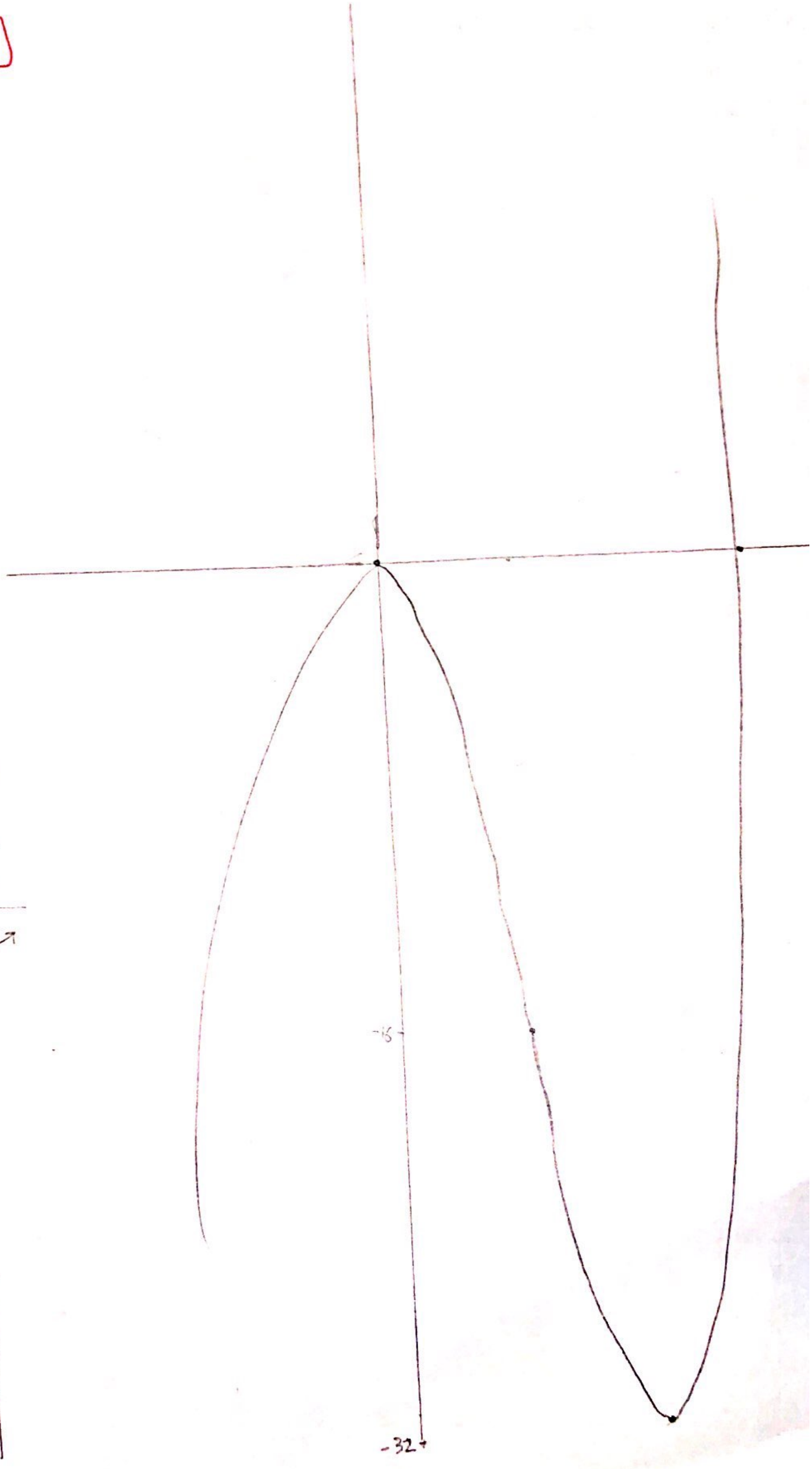
| | $(-\infty,2)$ | $(2,\infty)$ |
|---|---|---|
| $k$ | 1 | 3 |
| $f''(k)$ | -6 | 6 |
| | ⌒ | ⌣ |

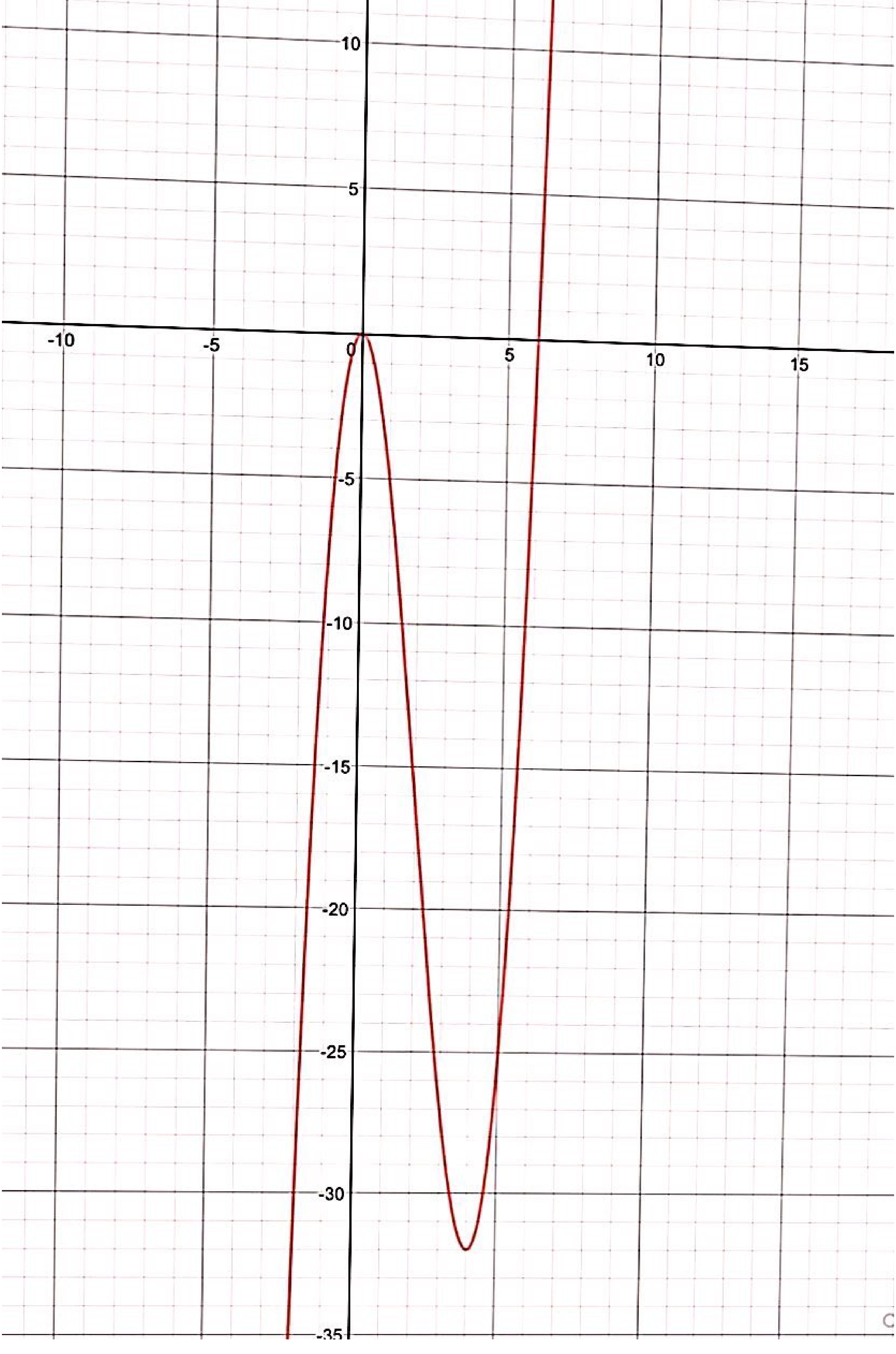
6) H.A

$\lim_{x \to \pm\infty} x^3 - 6x^2 \neq L$ No H.A

V.A

$\lim_{x \to c} x^3 - 6x^2 \neq \pm\infty$ No V.A

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