Q1A

$$\lim_{X \to -2} (-5X - 11) = -1$$

If $0 < |X - a| < \delta$ then $|f(x) - L| < \varepsilon$

if $0 < |X + 2| < \delta$ then $|-5X - 11 + 1| < \varepsilon$

$$|-5X - 10| < \varepsilon$$
$$|-5(X+2)| < \varepsilon$$
$$|-5||X+2| < \varepsilon$$
$$|X+2| < \frac{\varepsilon}{5}$$

for $\varepsilon > 0$ we Choose $\delta = \frac{\varepsilon}{5}$

there exsist $\delta = \frac{\varepsilon}{5}$ therefor by defination of a limit

$$\lim_{X \to -2} (-5X - 11) = -1$$

Q1B

$$f(x) = \frac{4 - X^2}{X^2 - 4|X| + 4}$$

$$f(x) = \begin{cases} \frac{4 - x^2}{(x-2)^2}, & x \geq 0 \\ \frac{4 - x^2}{(x+2)^2}, & x < 0 \end{cases}$$

Zero of denominator at $X = 2, -2$

$$\lim_{X \to 2^+} \frac{4 - x^2}{(x-2)^2} = \frac{4 - (2^+)^2}{(2^+ - 2)^2} = \frac{-}{+}(\infty) = -\infty$$

$$\lim_{X \to 2^-} \frac{4 - x^2}{(x-2)^2} = \frac{4 - (2^-)^2}{(2^- - 2)^2} = \frac{+}{+}(\infty) = \infty$$

vertical at $X = 2$

$$\lim_{X \to -2^+} \frac{4 - x^2}{(x+2)^2} = \frac{4 - (-2^+)^2}{(2 + (-2^+))^2} = \frac{+}{+}(\infty) = \infty$$

$$\lim_{X \to -2^-} \frac{4 - x^2}{(x+2)^2} = \frac{4 - (-2^-)^2}{(2 + (-2^-))^2} = \frac{-}{+}(\infty) = -\infty$$

vertical at $X = -2$

Q1C

$$\lim_{x\to\infty}\left(\frac{2x^2+1}{x+1}-ax+b\right)=5$$

$$\lim_{x\to\infty}\left(\frac{2x^2+1+(x+1)(-ax+b)}{(x+1)}\right)=5$$

$$\lim_{x\to\infty}\left(\frac{2x^2+1-ax^2+bx-ax+b}{x+1}\right)=5$$

$$\lim_{x\to\infty}\left(\frac{x^2(2-a)+x(b-a)+b+1}{x+1}\right)=5$$

$2-a=0 \Rightarrow -a=-2$

$b-a=5 \Rightarrow b-2=5$

$a=2$

$b=7$

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Q2 ①

$$\lim_{x\to 2}\frac{x-1}{x^2-1}=\frac{2-1}{4-1}=\frac{1}{3}$$

Q2 ②

$$\lim_{x\to 9}\frac{x+\sqrt{x}-12}{x-9}=\frac{0}{0}$$

$$=\lim_{x\to 9}\frac{\cancel{(\sqrt{x}-3)}(\sqrt{x}+4)}{\cancel{(\sqrt{x}-3)}(\sqrt{x}+3)}=\lim_{x\to 9}\frac{\sqrt{x}+4}{\sqrt{x}+3}=\frac{7}{6}$$

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Q2 (3)

الحل

$$\lim_{x \to 1^+} \frac{x^2-1}{x^2-2x+1}$$

$$= \lim_{x \to 1^+} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-1)} = \lim_{x \to 1^+} \frac{x+1}{x-1} = \frac{1^+ + 1}{1^+ - 1} = \frac{+}{+}(\infty) = \infty$$

Q2 (4)

الحل

$$\lim_{x \to 0} \frac{\sin^2 x + \sin(2x)}{3x}$$

$$\lim_{x \to 0} \underbrace{\sin x}_{\to 0} \cdot \lim_{x \to 0} \underbrace{\frac{\sin x}{3x}}_{\frac{1}{3}} + \lim_{x \to 0} \underbrace{\frac{\sin 2x}{3x}}_{\frac{2}{3}} = 0\left(\tfrac{1}{3}\right) + \frac{2}{3} = \frac{2}{3}$$

Q2 (5)

الحل

$$\lim_{x \to 1} \left(\frac{2}{x^2-1} - \frac{1}{x-1} \right) = \frac{0}{0}$$

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$$\lim_{x \to 1} \left(\frac{2(x-1) - 1(x^2-1)}{(x^2-1)(x-1)} \right)$$

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$$\lim_{x \to 1} \left(\frac{2x - 2 - x^2 + 1}{(x^2-1)(x-1)} \right) = \lim_{x \to 1} \left(\frac{-(x^2-2x+1)}{(x^2-1)(x-1)} \right)$$

$$= \lim_{x \to 1} \frac{-\cancel{(x-1)}\cancel{(x-1)}}{\cancel{(x-1)}(x+1)\cancel{(x-1)}} = \lim_{x \to 1} \frac{-1}{x+1} = \frac{-1}{2}$$

Q2 (6)

الحل

$$\lim_{x \to \infty} \frac{\tan^{-1}(\sin x)}{x}$$

$$-1 \leq \sin x \leq 1$$

$$\tan^{-1}(-1) \leq \tan^{-1}(\sin x) \leq \tan^{-1}(1)$$

$$\frac{-\pi}{4} \leq \tan^{-1}(\sin x) \leq \frac{\pi}{4}$$

$$\frac{-\pi}{4x} \leq \frac{\tan^{-1}(\sin x)}{x} \leq \frac{\pi}{4x}$$

$$\lim_{x \to \infty} \frac{-\pi}{4x} = \lim_{x \to \infty} \frac{\pi}{4x} = 0$$

from squeeze therom

$$\lim_{x \to \infty} \frac{\tan^{-1}(\sin x)}{x} = 0$$

Q2 (7)

الحل

$$\lim_{x\to\infty} \frac{\sin^2 x}{x^2+1}$$

$$0 \leqslant \sin^2 x \leqslant 1$$

$$\frac{0}{x^2+1} \leqslant \frac{\sin^2 x}{x^2+1} \leqslant \frac{1}{x^2+1}$$

$$0 \leqslant \frac{\sin^2 x}{x^2+1} \leqslant \frac{1}{x^2+1}$$

$$\lim_{x\to\infty}(0) = \lim_{x\to\infty} \frac{1}{x^2+1} = 0$$

from squeeze therom

$$\lim_{x\to\infty} \frac{\sin^2 x}{x^2+1} = 0$$

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Q2 (8)

الحل

$$\lim_{x\to\infty}\left(x^2\left(1-\cos\frac{1}{x}\right)\right)$$

Let $\frac{1}{x} = y \Rightarrow x = \frac{1}{y}$ at $x\to\infty$ then $y\to 0$

$$\lim_{x\to\infty}\left(x^2\left(1-\cos\frac{1}{x}\right)\right) = \lim_{y\to 0}\left(\frac{1}{y^2}\left(1-\cos y\right)\right)$$

$$= \lim_{y\to 0}\left(\frac{1-\cos y}{y^2}\right) = \lim_{y\to 0}\left(\frac{1-\cos y}{y^2}\cdot\frac{1+\cos y}{1+\cos y}\right)$$

$$= \lim_{y\to 0}\left(\frac{1-\cos^2 y}{y^2(1+\cos y)}\right) = \lim_{y\to 0}\left(\frac{\sin^2 y}{y^2(1+\cos y)}\right)$$

$$= \lim_{y\to 0}\left(\frac{\sin y}{y}\right)^2 \cdot \lim_{y\to 0}\frac{1}{1+\cos y} = (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

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Q3 Intermediate th. $\cos x = x$ $[0, \frac{\pi}{2}]$

الحل $\cos x - x = 0$

Let $f(x) = \cos x - x$, $g(x) = \cos x$ and $h(x) = x$

* g and h are continuous on $[0, \frac{\pi}{2}]$

then $f(x)$ continuous on $[0, \frac{\pi}{2}]$

* $f(0) = \cos(0) - 0 = 1$ * $f(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) - \frac{\pi}{2} = \frac{-\pi}{2}$

* $f(\frac{\pi}{2}) < 0 < f(0)$ then from intermediate th.

there is $c \in [0, \frac{\pi}{2}]$ such that $f(c) = 0$

f has a zero in the $[0, \frac{\pi}{2}]$

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Q4

$$f(x) = \begin{cases} cx^2 + d & x > 1 \\ 6 & x = 1 \\ 2cx - d & x < 1 \end{cases}$$ f continuous on $\mathbb{R}$

$$\lim_{x \to 1^-} 2cx - d = f(1) = 6 = \lim_{x \to 1^+} cx^2 + d$$

$$2c - d = 6 = c + d$$

$2c - d = 6$

$c + d = 6$

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$3c = 12$ $c = 4$ $\Rightarrow$ $8 - d = 6$ $-d = -2$ $d = 2$

Q5 $f(x) = x|x|$ at $x = 0$

الحل

$$f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$ $f'(0) = \lim_{h \to 0} \left[f(0+h) - f(0) / h \right]$

$$\lim_{h \to 0^-} \frac{-h^2 + (0)^2}{h} = 0 , \quad \lim_{h \to 0^+} \frac{h^2 - (0)^2}{h} = 0$$

$\Rightarrow$ f diffrentiable at $x = 0$

Q6 (A)

at $x=1 \rightarrow f$ is not differentiable
and $\lim_{x \to 1^-} f(x) = \infty \Rightarrow$ vertical at $x=1$.

at $x=-1 \rightarrow f$ is not differentiable
and $\lim_{x \to -1^-} f(x) = -\infty$, $\lim_{x \to -1^+} f(x) = \infty$
$\Rightarrow$ vertical at $x=-1$

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* $\lim_{x \to \infty} f(x) = 1$ and $\lim_{x \to -\infty} f(x) = 2$
then horizontal at $y = 1, 2$

Q7 (B)

$$\lim_{x \to 0} f(x) = \begin{cases} \lim_{x \to 0^+} f(x) = 2 \\ \lim_{x \to 0^-} f(x) = 2 \end{cases} \Rightarrow \lim_{x \to 3} f(x) = 2$$

Q7 (C)

$$\lim_{x \to 1} f(x) = \begin{cases} \lim_{x \to 1^+} f(x) = 1 \\ \lim_{x \to 1^-} f(x) = \infty \end{cases} \Rightarrow \lim_{x \to 1} f(x) = DNE$$

Q7 (D) حل بنفسك

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