First Semester
1439/1440

Q1) $2x-4 \le 6$

(A)

$2x \le 10$

$x \le 5$

5

$S.S = (-\infty, 5]$

(B) $\lim_{x\to 2} (x+4) = 6$

for any $\epsilon > 0 \ \exists \ \delta > 0 \ \ni$

$0 < |x-2| < \delta$ then $|x+4-6| < \epsilon$

$|x-2| < \epsilon$

choose $\delta = \epsilon$

Q1) (C)

(1) $\lim_{x\to 1} (3x+5) = 3(1)+5 = \boxed{8}$

(2) $\lim_{x\to 0} \frac{\sin(4x)}{3x}$ $\frac{0}{0}$

$= \boxed{\frac{4}{3}}$

(3) $\lim_{x\to 5} \frac{x^2-8x+15}{x-5}$ $\frac{0}{0}$

$= \lim_{x\to 5} \frac{\cancel{(x-5)}(x-3)}{\cancel{(x-5)}}$

$= \lim_{x\to 5} x-3 = 5-3 = \boxed{2}$

(4) $\lim_{x\to 2} \frac{x-2}{\sqrt{x+2}-2}$ $\frac{0}{0}$

$= \lim_{x\to 2} \frac{x-2}{\sqrt{x+2}-2} \cdot \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$

$= \lim_{x\to 2} \frac{(x-2)(\sqrt{x+2}+2)}{x+2-4}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(\sqrt{x+2}+2)}{\cancel{x-2}}$

$= \lim_{x\to 2} \sqrt{x+2} + 2$

$= \sqrt{2+2} + 2 = \sqrt{4} + 2$

$= 2 + 2 = \boxed{4}$

(5) $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$ $0 \cdot \infty$

$-1 \le \cos\left(\frac{1}{x^2}\right) \le 1$

$-x^2 \le x^2 \cos\left(\frac{1}{x^2}\right) \le x^2$

$\lim_{x\to 0} -x^2 = 0$

$\lim_{x\to 0} x^2 = 0$

$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

(6) $\lim_{x\to\infty} \sin\left(\frac{\pi x+2}{2x+1}\right)$

$\lim_{x\to\infty} \frac{\pi x+2}{2x+1}$ $\frac{\infty}{\infty}$

since degree $(2x+1) = 1 =$ degree $(\pi x+2)$

So $\lim_{x\to\infty} \frac{\pi x+2}{2x+1} = \frac{\pi}{2}$

domain $\sin x$ is $\mathbb{R}$

So $\sin x$ is continuous at $x = \frac{\pi}{2}$

so $\lim_{x\to\infty} \sin\left(\frac{\pi x+2}{2x+1}\right) = \sin\left(\lim_{x\to\infty} \frac{\pi x+2}{2x+1}\right)$

$= \sin\left(\frac{\pi}{2}\right) = \boxed{1}$

Q2 (A) $f(x) = mx+c$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{m(x+h)+c-(mx+c)}{h}$ $\frac{0}{0}$

$= \lim_{h\to 0} \frac{\cancel{mx}+mh\cancel{+c}-\cancel{mx}-\cancel{c}}{h}$

$= \lim_{h\to 0} \frac{m\cancel{h}}{\cancel{h}} = \lim_{h\to 0} m = m$

So $f'(x) = m$

(B) at $x=2$

(0) $f(2) = -2$

(1) $\lim_{x\to 2} \frac{x^2-4}{x-2}$ $\frac{0}{0}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2} x+2 = 2+2 = 4$

(3) $\lim_{x\to 2} f(x) \ne f(2)$

$4 \ne -2$

So f is discontinuous at $x=2$

Q 2 | Ⓒ $s(t)=3t^3+2t^2+7$

①
$v(t)=s'(t)=9t^2+4t$

$v(3)=9(3)^2+4(3)=93$ m/sec

②
$a(t)=v'(t)=18t+4$

$a(5)=18(5)+4=94$ m/sec^2

Q3 | Ⓐ $y=x^5+x^3+x^2+7$

$$\frac{dy}{dx}=5x^4+3x^2+2x$$

Ⓑ $y=(x^2+x)^{25}$

$$\frac{dy}{dx}=25(x^2+x)^{24}(2x+1)$$
$$=(50x+25)(x^2+x)^{24}$$

Ⓒ $y=\dfrac{\sin x}{x+1}$

$$\frac{dy}{dx}=\frac{\cos x\,(x+1)-\sin x}{(x+1)^2}$$
$$=\frac{x\cos x+\cos x-\sin x}{(x+1)^2}$$

Ⓓ $y=x^2\tan^{-1}(3x)$

$$\frac{dy}{dx}=2x\tan^{-1}(3x)+x^2\left(3\left(\frac{1}{1+(3x)^2}\right)\right)$$
$$=2x\tan^{-1}(3x)+\frac{3x^2}{1+9x^2}$$

Ⓔ $y=\cos^3x^2+\tan\left(\frac{\pi}{3}\right)$

$y=(\cos(x^2))^3+\tan\left(\frac{\pi}{3}\right)$

$$\frac{dy}{dx}=3(\cos(x^2))^2(2x(-\sin(x^2)))+0$$
$$=-6x\cos^2(x^2)\sin(x^2)$$

Ⓕ $x^2+y^2=\sin(xy)$

$2x+2yy'=(y+xy')\cos(xy)$

$2yy'=y\cos xy+xy'\cos(xy)-2x$

$2yy'-xy'\cos(xy)=y\cos xy-2x$

$$y'=\frac{y\cos xy-2x}{2y-x\cos(xy)}$$

Q 4 | Ⓐ $f(x)=x^2-4x+5$

① since f is poly so f is cont on R
then f is continuous on $[0,2]$

② $f'(x)=2x-4$, $D_{f'}=R$

So f is diff on $(0,2)$

f satisfy the condition of M.V.T

③ $f'(c)=\dfrac{f(2)-f(0)}{2-0}$

$2c-4=\dfrac{1-5}{2}$

$2c-4=\dfrac{-4}{2}$

$2c-4=-2$

$2c=2$

$c=1\in(0,2)$

Ⓑ $f(x)=x^2$ $[3,5]$

Since $D_f=R$

So f is continuous on $[3,5]$

$f'(x)=2x$

$2x=0$

$x=0\notin(3,5)$

So no critical numb

$f(3)=9$

$f(5)=25$

- absolute max at $x=5$ and equal $f(5)=25$
- absolute min at $x=3$ and equal $f(3)=9$

Ⓒ

$2x+2y=100$

$x+y=50$

$\boxed{y=50-x}$

$A=x\cdot y$

$0\le 2x\le 100$

$0\le x\le 50$

$A=x(50-x)$

$A=50x-x^2$

$\dfrac{dA}{dx}=50-2x$

$50-2x=0$

$2x=50$

$x=25\in(0,50)$

$A(25)=625$ cm^2

$A(0)=0$

$A(50)=0$

- maximum area equal 625 cm^2 at $x=25$ cm and $y=25$ cm

Q4 | (D) $f(x) = \frac{x^2}{x^2-9}$

V.A.

$D_f = \mathbb{R} - \{-3, 3\}$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

$\lim_{x \to 3^+} \frac{x^2}{x^2-9} = \infty$ $\quad \lim_{x \to -3^+} \frac{x^2}{x^2-9} = -\infty$

$\lim_{x \to 3^-} \frac{x^2}{x^2-9} = -\infty$ $\quad \lim_{x \to -3^-} \frac{x^2}{x^2-9} = \infty$

so vertical asymptotes at $x = -3, 3$

Q5 | $f(x) = x^4 - 4x^3 + 10$

$D_f = \mathbb{R}$ since f is Poly

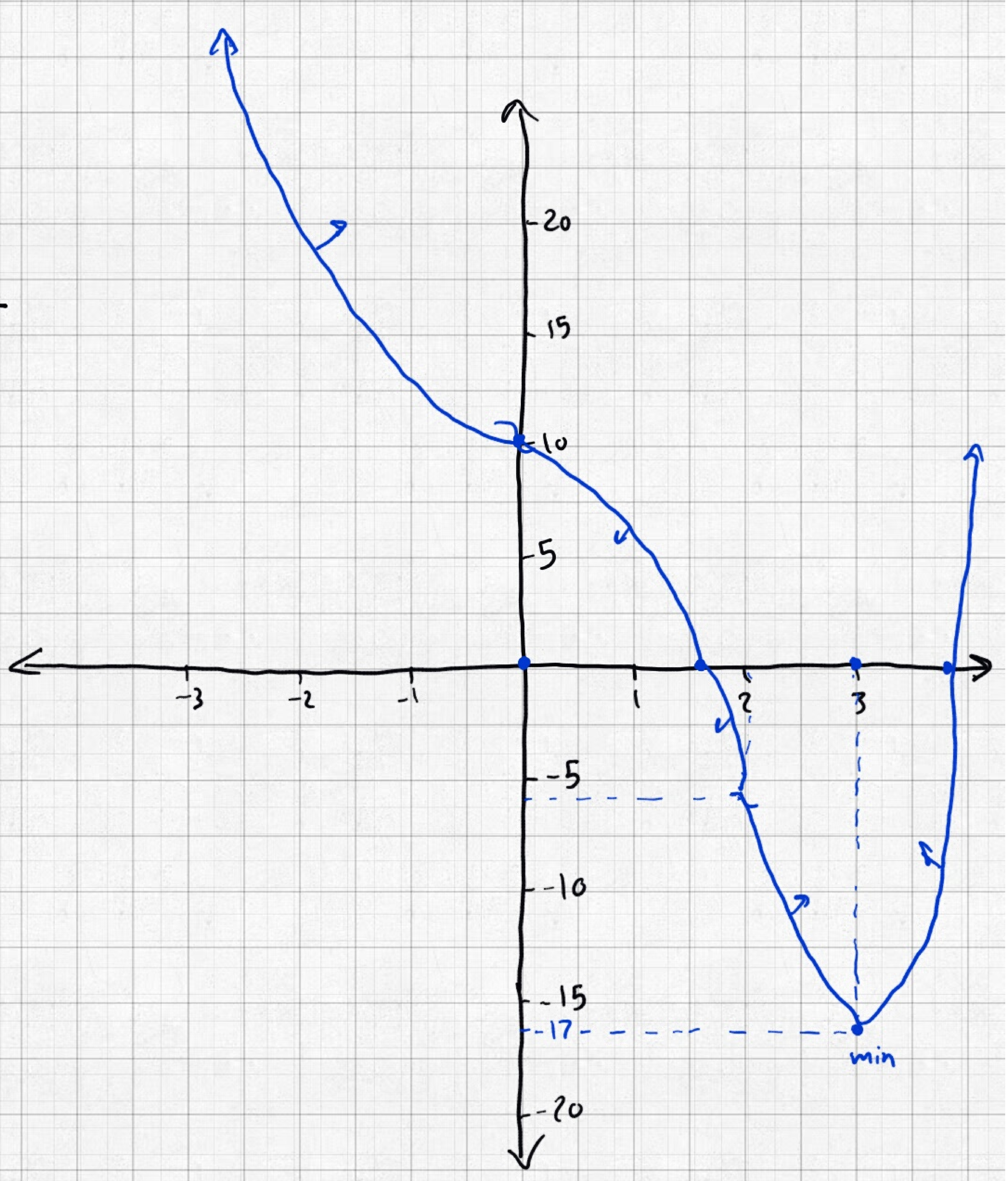
x-int

$x^4 - 4x^3 + 10 = 0$

$\boxed{x \approx 3.82 \quad x \approx 1.61}$

y-int

$\boxed{y = 10}$



(A) $f'(x) = 4x^3 - 12x^2$

$4x^3 - 12x^2 = 0$

$4x^2(x - 3) = 0$

$4x^2 = 0$ or $x - 3 = 0$

$x = 0 \in D_f$ | $x = 3 \in D_f$

Critical at $x = 0$, $x = 3$

(B) $-\infty$ – – 0 – – 3 + + ∞

$f'(-1) = -16 \quad f'(1) = -8 \quad f'(4) = 64$

f increasing on $(3, \infty)$

f decreasing on $(-\infty, 3)$

(C) f has local min at $x = 3$ and equal $f(3) = -17$

no local max

(D) $f''(x) = 12x^2 - 24x$

$12x^2 - 24x = 0$

$12x(x - 2) = 0$

$12x = 0$ | $x - 2 = 0$

$x = 0 \in D_f$ | $x = 2 \in D_f$

$-\infty$ + + 0 – – – 2 + + ∞

$f''(-1) = 36 \quad f''(1) = -12 \quad f''(3) = 36$

f concave up on $(-\infty, 0)$ and $(2, \infty)$

f concave down on $(0, 2)$

inflection points

$(0, f(0)) = (0, 10)$

$(2, f(2)) = (2, -6)$