COMMON FIRST YEAR
BASIC SCIENCES DEPARTMENT
Math 101 Final Exam 1441 H.
First Semester
Time Allowed - 3 Hours

جامعة الملك سعود
King Saud University

السنة الأولى المشتركة

St. Name: _____ St. ID: _____ Section: _____

ملاحظات:

1- اكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة (الإجابة على ورقة الأسئلة غير معتمدة).
علمًا بأن عدد الأسئلة (5)، وعدد الصفحات (2).
2- لا يسمح بالكتابة إلا بالقلم الأزرق فقط.
3- لا يسمح بتداول الآلة الحاسبة بين الطلاب.
4- لا يسمح باستخدام آلة حاسبة قابلة للبرمجة أو آلة حاسبة ترسم دوال.

Question 1: **(13 Marks)**

A) Find the domain of

$$f(x) = \frac{1}{x-2}.$$

B) Use the definition of the limit to prove that $\lim_{x\to 1}(2x+4) = 6$.

C) Evaluate each of the following limits (if exist):

1) $\lim_{x\to 3}(x^2 - x + 1)$

2) $\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}$

3) $\lim_{x\to 3}\frac{x^2+x-12}{x-3}$

4) $\lim_{x\to\infty}\frac{2x^3+4x^2+5}{5x^3+7}$

5) $\lim_{x\to 0} x^4 \cos\left(\frac{2}{x^4}\right)$

D) Let $f(x) = \begin{cases} \frac{\sin(3x)}{kx} & , \quad x < 0 \\ k(1-x)+2 & , \quad x \geq 0 \end{cases}$.

Find the value(s) of k such that $\lim_{x\to 0} f(x)$ exists.

(6 Marks)

Question 2:

A) Discuss the continuity of $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 4, & x = 3 \end{cases}$ at $x = 3$.

B) The position of a particle is given by the equation $s(t) = 2t^5 - 3t + 10$, where s seconds.

1- What is the velocity of the particle after 2 seconds?
2- What is the acceleration of the particle after 2 seconds?

C) Find the equation of tangent line of $y = \frac{1}{x}$ at $x = 4$.

Question 3: **(9 Marks)**

Find the derivative $\frac{dy}{dx}$ for each of the following. Write your answer in the simplest form:

A) $y = 2x^8 + 5x^4 + 3x^2 + 10$

B) $y = (2x+7)^{40}$

C) $y = \frac{x^3}{x^2+1}$

D) $y = \sin^4 x + \pi^2$

E) $y = \tan^{-1}(2x)$

F) $x \tan y = x + y$

Question 4: **(7 Marks)**

A) Show that the function $f(x) = x^2 - 3x + 1$ satisfies the conditions of the Mean Value Theorem on $[1,3]$. Find a number c that satisfies the conclusion of the theorem.

B) Let $f(x) = \frac{1}{x^2 - 25}$. Find the vertical asymptote(s) of f.

C) Find the value of k so that $f(x) = x^2 + \frac{x}{k}$ has a critical number at $x = 3$.

uestion 5: **(5 M**

For the function $f(x) = x^3 - 6x^2$, find the following (if any):

A) The critical numbers of f.

B) The interval(s) on which f is increasing and decreasing.

C) The local extrema of f.

D) The interval(s) on which f is concave upward or downward.

E) Sketch the graph of f.

Good Luck