KING SAUD UNIVERSITY
COMMON FIRST YEAR
BASIC SCIENCES DEPARTMENT
Math 101 Final Exam 1442 H
First Semester

Time Allowed : 3 Hours

St. Name: _____ St. ID: _____ Section: _____

يجب قراءة هذه التعليمات قبل البدء في حل الاختبار

تعليمات:

1- اكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة (الإجابة على ورقة الأسئلة غير معتمدة).
علما بان عدد الأسئلة (5) وعدد الصفحات (2).
2- لا يسمح بالكتابة إلا بالقلم الأزرق فقط.
3- لا يسمح بتداول أي أدوات بين الطلاب.
4- لا تستخدم الة حاسبة قابلة للبرمجة أو الة حاسبة لرسم دوال

Question 1: (16 Marks)

A) Determine whether the functions

$$f(x)=\frac{x^3+x-6}{x-2}, \text{ and } g(x)=x+3$$

are the same or not, and Why?

B) Evaluate each of the following limits (If exist):

1) $\lim_{x\to 1}(3x^2+5)$

2) $\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{x-3}$

3) $\lim_{x\to 0}\frac{x^2-5x}{x-5}$

4) $\lim_{x\to\infty}(2x^4-3x^3+5)$

C) Use Sandwich Theorem to find $\lim_{x\to 1} f(x)$, where $3x^3-1\le f(x)\le x^2+1$.

D) If $\lim_{x\to 1}\frac{f(x)-8}{x-1}=10$, find the value of $\lim_{x\to 1} f(x)$.

Question 2: (11 Marks)

A) Discuss the continuity of $f(x)=\begin{cases}\frac{\sin(4x)}{2x}, & x\neq 0\\ 2, & x=0\end{cases}$ at $x=0$.

B) The position of a particle is given by the equation $s(t)=3t^3-4t+5$, where s in meters and t in seconds.
1- What is the velocity of the particle after 3 seconds?
2- What is the acceleration of the particle after 4 seconds?

C) Find the equation of tangent line of $y=\sin x$ at $x=0$.

D) Use the definition of derivative to find f', where $f(x)=2x^2+1$.

Question 3: **(12 Marks)**

Find the derivative $\frac{dy}{dx}$ for each of the following. Write your answer in the simplest form:

A) $y = 3x^3 - 6x^2 - 10x + \pi^2$

B) $y = \sqrt{5x^3 + x}$

C) $y = \frac{x+1}{2x-3}$

D) $y = \tan(3x) - \cos(6x)$

E) $y = \sin^{-1}(x^2)$

F) $2x^3 - 4xy^3 = 2y$

Question 4: **(6 Marks)**

A) Use the Intermediate Value Theorem to show that $f(x) = x^4 - x^3 - 1$ has a zero in the interval $[1,2]$.

B) Show that the function $f(x) = x^4 - 8x^2 + 2$ satisfies the conditions of the Rolle's Theorem on $[-2,2]$. Find a number c that satisfies the conclusion of the theorem.

C) Find the absolute extrema of $f(x) = x^2$ on $[-2,5]$.

Question 5: **(5 Marks)**

For the function $f(x) = 2x^3 - 9x^2 + 12x$, find the following (if any):

A) The critical numbers of f.

B) The interval(s) on which f is increasing and decreasing.

C) The local extrema of f.

D) The interval(s) on which f is concave upward or downward.

E) Sketch the graph of f.

Good Luck