قروبات القمة في الرياضيات أبداع ليس له حدود

شرح الواجب الاول ٤٢ math(101)

قروبات القمة تتقدم بكل الشكر للبشمهندس أسامة المسند

عبدالله الحفني جوال ٠٥٨٣٤٢٢٢٠٠

المحاضرة تتكلم عن:- (منـاقـشـة الـواجـب الاول)

Question(1)

A). Classify the following numbers into rational and irrational

$$\left\{(1.\bar{5})^2\,,\,\frac{3.14}{6}\,,\,\sin\pi\,,\,\sqrt[7]{2^7}\,,\,\sqrt{\sqrt{9}+6}\,,\,\frac{2}{3\pi}\,,\,\frac{22}{7}\,,\,\sqrt{\frac{16}{64}}\right\}$$

SOLUTION STEPS

rational is $\left\{(1.\bar{5})^2\,,\,\frac{3.14}{6}\,,\,\sin\pi\,,\,\sqrt[7]{2^7}\,,\,\sqrt{\sqrt{9}+6}\,,\,\frac{22}{7}\,,\,\sqrt{\frac{16}{64}}\right\}$

irrational is $\left\{\frac{2}{3\pi}\right\}$

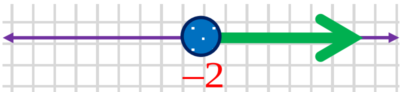
B). Solve the Following inequalities and write the solution in interval notation:

$\boxed{1}$ $3x+5\geq x+1$

SOLUTION STEPS

$$3x+5\geq x+1$$
$$-x-5 \quad -x-5$$
$$\Rightarrow 2x\geq -4 \quad \div 2$$
$$\Rightarrow \frac{2x}{2}\geq -\frac{4}{2}$$
$$\Rightarrow x\geq -2$$

$s.s=[-2\,,\infty)$

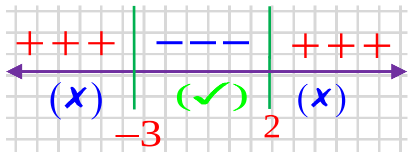


$\boxed{2}$ $x^2+x-6\leq 0$

SOLUTION STEPS

$$(x-2)(x+3)\leq 0$$
$$x=2 \quad or \quad x=-3$$

$s.s=[-3\,,2]$



$\boxed{3}$ $\frac{2|x+1|-3}{-3}<1$ multipy by(-3)

SOLUTION STEPS

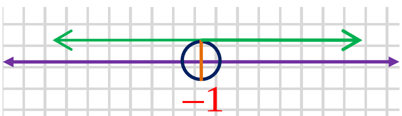
$$\Rightarrow \frac{2|x+1|-3}{\cancel{-3}}(\cancel{-3})>1(-3)$$
$$\Rightarrow 2|x+1|-3>-3$$
$$\Rightarrow 2|x+1|>0$$

$$|x+1|>0$$

or

$x+1>0$ | $x+1<0$

$x>-1$ | $x<-1$



$s.s=(-\infty\,,-1)\cup(-1\,,\infty)=\mathbb{R}-\{-1\}$

$\boxed{4}$ $5|3x+1|-8>2+3|3x+1|$

SOLUTION STEPS

$$\Rightarrow 5|3x+1|-\cancel{8}>2+3\cancel{|3x+1|}$$
$$\Rightarrow -3|3x+1|+\cancel{8} \qquad 8-3\cancel{|3x+1|}$$
$$\Rightarrow 2|3x+1|>10 \quad \div 2$$
$$\Rightarrow \frac{\cancel{2}|3x+1|}{\cancel{2}}>\frac{10}{2}\Rightarrow|3x+1|>5$$

or

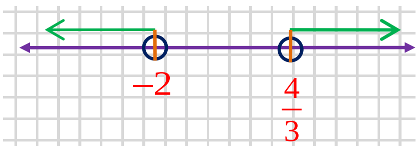
$3x+1<-5$ | $3x+1>5$

$3x<-6$ | $3x>4$

$x<-\frac{6}{3}$ | $x>\frac{4}{3}$

$x<-2$

$s.s=(-\infty\,,-2)\cup\left(\frac{4}{3}\,,\infty\right)$



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$\boxed{5}$ $\dfrac{3x-6}{x^2-3x-18}<0 \iff \dfrac{3(x-2)}{x^2-3x-18}<0$

SOLUTION STEPS

$3x-6=0$

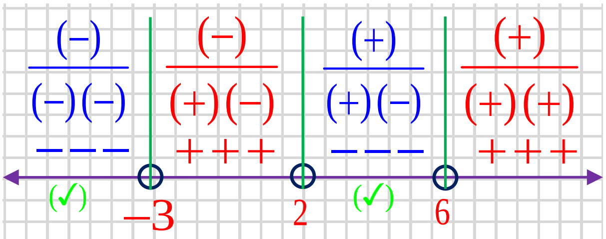
$3x=6$

$x=2$

$x^2-3x-18=0$

$\Rightarrow (x+3)(x-6)=0$

$\Rightarrow x=-3 \quad or \quad x=6$



$$s.s=(-\infty,-3)\cup(2,6)$$

$\boxed{6}$ $|2x-5|-|3x-2|<0$

SOLUTION STEPS

$|2x-5|<|3x-2|$ to ^{2}

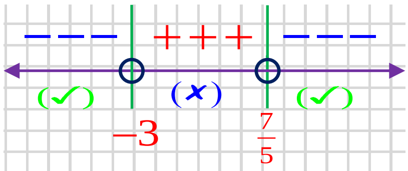
$(2x-5)^2<(3x-2)^2 \iff (2x-5)^2-(3x-2)^2<0$

$\left[(2x-5)-(3x-2)\right]\left[(2x-5+3x-2)\right]<0$

$(-x-3)(5x-7)<0$

$x=-3$ or $x=\dfrac{7}{5}$

$s.s=(-\infty,-3)\cup\left(\frac{7}{5},\infty\right)$



Another technique

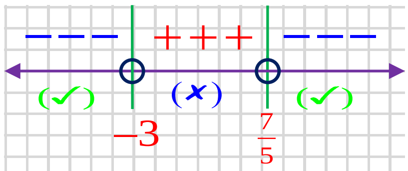
by using rule $|x|<|y| \iff (x-y)(x+y)<0$

$(2x-5+3x+2)(2x-5+3x-2)<0$

$(-x-3)(5x-7)<0$

$x=-3$ or $x=\dfrac{7}{5}$

$s.s=(-\infty,-3)\cup\left(\frac{7}{5},\infty\right)$



$\boxed{7}$ If $f(x)=1-x-x^2$ and $g(x)=3-x$, then solve

$$(f\circ g)(x)+x<1$$

SOLUTION STEPS

to find:

$\Rightarrow (f\circ g)(x)=f\left(g(x)\right)=1-(3-x)-(3-x)^2$

$\Rightarrow 1-3+x-9+6x-x^2=-x^2+7x-11$

now, sub. in $(f\circ g)x+x<1$

$-x^2+7x-11+x<1$ add (-1)

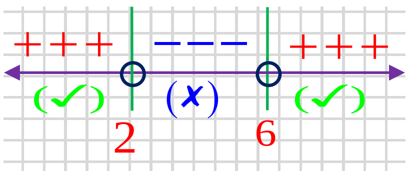
$-x^2+8x-11-1<0$

$-x^2+8x-12<0 \quad \times(-1)$

$x^2-8x+12>0$

$(x-2)(x-6)=0$

$x=2$ or $x=6$



$s.s=(-\infty,2)\cup(6,\infty)$

Question(2) Find the domain of the following function :-

1 $f(x) = 2x^3 + 5x - 3$

SOLUTION STEPS

Since $f(x)$ is Poly

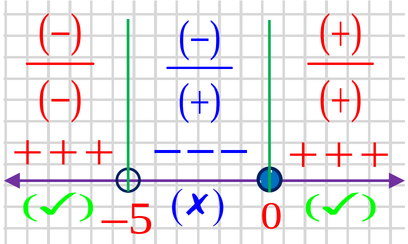
then; $D_f = \mathbb{R}$

2 $f(x) = \sqrt{\dfrac{x}{x+5}}$

SOLUTION STEPS

$$\frac{x}{x+5} \geq 0 \Rightarrow$$

$x = 0$ | $x + 5 = 0$, $x = -5$



then ; $D_f = (-\infty, -5) \cup [0, \infty)$

3 $f(x) = \dfrac{|2-x|+1}{x^2 - 3x - 18}$

SOLUTION STEPS

$$f(x) = \frac{|2-x|+1}{(x-6)(x+3)}$$

$D(|2-x|+1)$ is $\mathbb{R}$ | $\Rightarrow (x-6)(x+3) = 0$

$\Rightarrow x = 6$ or $x = -3$

then ; $D_f = \mathbb{R} - \{6, -3\}$

4 $f(x) = \sqrt[3]{\dfrac{2x+2}{x+4}}$

SOLUTION STEPS

$\Rightarrow x + 4 = 0$

$\Rightarrow x = -4$

then ; $D_f = \mathbb{R} - \{-4\}$

5 $f(x) = \dfrac{5}{\sec(3x)}$

SOLUTION STEPS

$\sec 3x = 0 \Leftrightarrow \dfrac{1}{\cos 3x} = 0$

$\Rightarrow \cos 3x = 0$

$\Rightarrow 3x = \dfrac{\pi}{2} + \pi n \Rightarrow x = \dfrac{\pi}{6} + \dfrac{\pi}{3} n \; ; \forall n \in \mathbb{Z}$

then ; $D_f = \mathbb{R} - \left\{ \dfrac{\pi}{6} + \dfrac{\pi}{3} n \; ; \forall n \in \mathbb{Z} \right\}$

6 $f(x) = \sqrt{|x+2|}$

SOLUTION STEPS

$$|x+2| \geq 0$$

or

$x + 2 \geq 0$, $x \geq -2$ | $x + 2 \leq 0$, $x \leq -2$

then ; $D_f = (-\infty, -2] \cup [-2, \infty) = \mathbb{R}$

Question(3)

Determine whther the functions

$$f(x)=1-\cos^2 x \quad \text{and} \quad g(x)=\frac{\sin x}{\csc x}$$

are the same or not

SOLUTION STEPS

$f(x)=1-\cos^2 x$

$D_f=(-\infty,\infty)=\mathbb{R}$

because

$\forall x\in(-\infty,\infty)$

$then, 1-\cos^2 x$ is definition.

$D_{g(x)}$

$\csc x=0 \Leftrightarrow \dfrac{1}{\sin x}=0 \Rightarrow \sin x=0$

$x=n\pi \quad \forall n\in\mathbb{Z}$

$D_{g(x)}=\mathbb{R}-\{n\pi ; n\in\mathbb{Z}\}$

since $D_f\neq D_g$; then f and g are not the same.

Question(4)

Let $f(x)=\dfrac{x-1}{x+2}$

$\boxed{1}$ Find D_f.

$\boxed{2}$ Show that f is one-to-one

$\boxed{3}$ Find f^{-1}.

$\boxed{4}$ Find the range of f.

SOLUTION STEPS

$\boxed{1}$ $D_f : x+2=0 \Rightarrow x=-2$

then $D_f=\mathbb{R}-\{-2\}$

$\boxed{2}$ Suppose that $f(x_1)=f(x_2) \quad \forall x_1, x_2\in D_f=\mathbb{R}-\{-2\}$

$\Rightarrow \dfrac{x_1-1}{x_1+2}=\dfrac{x_2-1}{x_2+2} \Rightarrow \dfrac{x_1+2-3}{x_1+2}=\dfrac{x_2+2-3}{x_2+2}$

$\Rightarrow \dfrac{\cancel{(x_1+2)}}{\cancel{x_1+2}}-\dfrac{3}{x_1+2}=\dfrac{\cancel{x_2+2}}{\cancel{x_2+2}}-\dfrac{3}{x_2+2}$

$\Rightarrow \cancel{1}-\dfrac{3}{x_1+2}=\cancel{1}-\dfrac{3}{x_2+2}$

$\Rightarrow -\dfrac{3}{x_1+2}=-\dfrac{3}{x_2+2} \Rightarrow x_1+\cancel{2}=x_2+\cancel{2}$

$\Rightarrow x_1=x_2$

then ; $f(x)$ is (1-1)

$\boxed{3}$ since $f(x)$ is (1-1)

$y=\dfrac{x-1}{x+2}$

$x=\dfrac{y-1}{y+2} \Rightarrow xy+2x=y-1$

$xy-y=-2x-1 \Rightarrow (x-1)y=-2x-1$

$y=\dfrac{-2x-1}{x-1} \Rightarrow f^{-1}(x)=\dfrac{-2x-1}{x-1}$

$\boxed{4}$ since $R_f=D_{f^{-1}}$

from $f^{-1}(x)=\dfrac{-2x-1}{x-1}$

$D_{f^{-1}} : x-1=0 \Rightarrow x=1$

$R_f=D_{f^{-1}}=\mathbb{R}-\{1\}$

لايجاد مدي $f(x)$

(1) نستخدم الدالة العكسية

(2) $R_f=D_{f^{-1}}$

Question(5)

Let $f(x)=\dfrac{2}{x-1}$, $g(x)=x+1$

$\boxed{1}$ Find $f\cdot g$ and its domain. $\boxed{2}$ Find $\dfrac{f}{g}$ and its domain

$\boxed{3}$ Find $(f\circ g)(3)$ and $(g\circ f)(5)$.

SOLUTION STEPS

$\boxed{1}$ $f\cdot g=\dfrac{2}{x-1}\cdot x+1=\dfrac{2x+2}{x-1}$

$D_f : x-1=0\Rightarrow x=1$

$D_f=\mathbb{R}-\{1\}$

$D_g=\mathbb{R}$ $g(x)$ is Poly

$D_{f\cdot g}=D_f\cap D_g=\mathbb{R}-\{1\}\cap\mathbb{R}=\mathbb{R}-\{1\}$

$\boxed{2}$ $\dfrac{f}{g}(x)=\dfrac{\dfrac{2}{x-2}}{x+1}=\dfrac{2}{(x-2)(x+1)}$

$D_{\left(\frac{f}{g}\right)}=D_f\cap D_g-\{g(x)=0\}$

$=\mathbb{R}-\{1\}-\{-1\}=\mathbb{R}-\{1,-1\}$

$\boxed{3}$ $(f\circ g)(3)=f\left(g(3)\right)=f\left(3+1\right)=f(4)=\dfrac{2}{4-1}=\dfrac{2}{3}$

$,(g\circ f)(5)=g\left(f(5)\right)=g\left(\dfrac{2}{5-1}\right)=g\left(\dfrac{1}{2}\right)=\dfrac{1}{2}+1=\dfrac{3}{2}$.

Question(6)

Find the exact value of the following :-

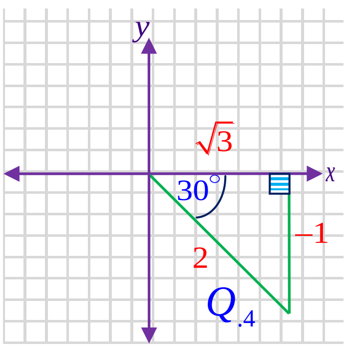
$\boxed{1}$ $\cos(330^\circ)$.

SOLUTION STEPS

(1) put $\theta=330^\circ$

(2) θ lies in $Q_{.4}$

(3) $\theta'=360^\circ-330^\circ=30^\circ$



(4) using figure as $\cos 330^\circ=\cos 30^\circ=\dfrac{\sqrt{3}}{2}$

Another technique

$\cos 330^\circ=\cos(360^\circ-330^\circ)=\cos 30^\circ$

$(*)$ using table $\cos 30^\circ=\dfrac{\sqrt{3}}{2}$

Then , $\cos 330^\circ=\dfrac{\sqrt{3}}{2}$

$\boxed{2}$ $\cos(15^\circ)$.

SOLUTION STEPS

$= \cos(45^\circ - 30^\circ)$ using rule $\cos(A - B)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

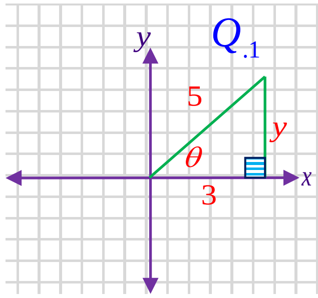
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$\boxed{3}$ $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$.

SOLUTION STEPS

Let $\cos^{-1}\left(\frac{3}{5}\right) = \theta \Rightarrow \cos\theta = \frac{3}{5}$

$y = \sqrt{5^2 - 3^2} = 4$



$\sin\theta = \frac{4}{5}$

Then , $\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$

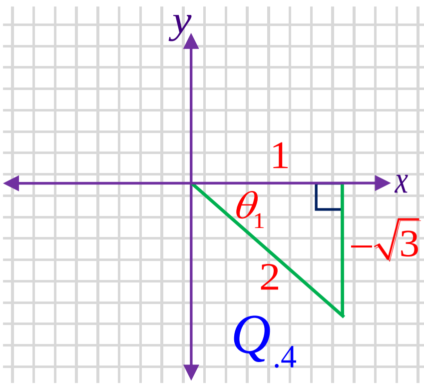
$\boxed{4}$ $\sin\left(\tan^{-1}\left(-\sqrt{3}\right) + \cos^{-1}\left(\frac{5}{13}\right)\right)$

SOLUTION STEPS

let $\theta_1 = \tan^{-1}\left(-\sqrt{3}\right)$ lies in $Q_{.4}$

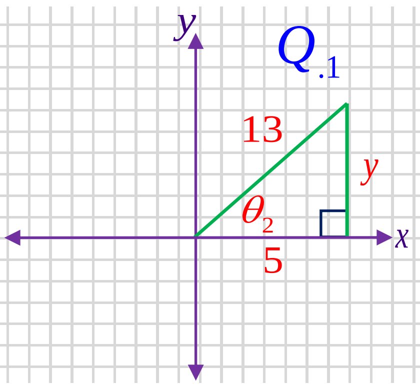
$\tan\theta_1 = -\sqrt{3}$

$\sin\theta_1 = \frac{-\sqrt{3}}{2}$ $\quad \cos\theta_1 = \frac{1}{2}$



let $\theta_2 = \cos^{-1}\left(\frac{5}{13}\right)$ lies in $Q_{.1}$

$\cos\theta_2 = \frac{5}{13}$ $\quad \sin\theta_2 = \frac{12}{13}$



$$\sin(\theta_1 + \theta_2) = \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2$$

$$= \frac{-\sqrt{3}}{2} \cdot \frac{5}{13} + \frac{1}{2} \cdot \frac{12}{13} = \frac{-5\sqrt{3}+12}{26}$$

Question(7) Solve the equation : $(\sin x+2)^2=1,\ x\in[0,2\pi]$.

SOLUTION STEPS

$$|\sin x+2|=1 \Leftrightarrow \sin x+2=\pm 1$$

or

$$\sin x+2=-1$$
$$\sin x=-3 \quad \text{no sol}$$
because
$$-1\le \sin x\le 1$$

$$\sin x+2=1$$
$$\sin x=-1$$
$$\sin x=\sin\frac{3\pi}{2}$$
$$x=\frac{3\pi}{2}$$

$$s.s=\left\{\frac{3\pi}{2}\right\}$$

Question(8) Verify each of the following :-

$\boxed{1}$ $2\sin^2(2x)+\cos(4x)=1.$

SOLUTION STEPS

by using rule : $\boxed{\cos 2x=\cos^2 x-\sin^2 x}$

$$\Rightarrow \cos 4x=\cos^2 2x-\sin^2 2x$$

$$\text{L.H.S}: 2\sin^2 2x+\cos^2 2x-\sin^2 2x$$
$$\sin^2 2x+\cos^2 2x=1(\text{THEOREM}) \quad \text{R.H.S}$$

$\boxed{3}$ $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x}=2\sec^2 x.$

SOLUTION STEPS

$$\text{L.H.S}: \frac{1+\cancel{\sin x}+1-\cancel{\sin x}}{(1-\sin x)(1+\sin x)}=\frac{2}{1-\sin^2 x}$$
$$=\frac{2}{\cos^2 x}=2\sec^2 x \quad \text{R.H.S}$$

$\boxed{2}$ $\dfrac{\cot^2 x}{\csc^2 x}+\sin^2 x=1.$

SOLUTION STEPS

$$\text{L.H.S} \quad \left[\frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}}\right]+\sin^2 x=\frac{\cos^2 x}{\cancel{\sin^2 x}}\bullet\cancel{\sin^2 x}+\sin^2 x$$
$$=\cos^2 x+\sin^2 x=1 \quad (\text{THEOREM})\ \text{R.H.S}$$