

St. Name: \_\_\_\_\_

ملاحظات :

- 1- اكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة (الإجابة على ورقة الأسئلة غير معتمدة).
- علمًا بأن عدد الأسئلة (6)، وعدد الصفحات (2).
- 2- لا يسمح بالكتابة إلا بالقلم الأزرق فقط.
- 3- لا يسمح بتداول الآلة الحاسبة بين الطلاب.
- 4- لا تستخدم آلة حاسبة قابلة للبرمجة أو آلة حاسبة ترسم دوال.

Question 1:

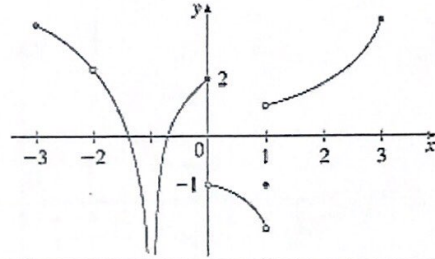
(5 Marks)

A) Solve the following inequality, and write your answer in interval notation

$$|2x - 5| + 1 \leq 3$$

B) Use the graph of  $y = f(x)$  to find the following:

1.  $\lim_{x \rightarrow 0^+} f(x)$
2. The vertical asymptote(s) for the graph of  $f(x)$ .
3. The  $x$ -value(s) at which  $f(x)$  is not continuous.



Question 2: Evaluate each of the following limits (if exist):

(12 Marks)

A)  $\lim_{x \rightarrow 2} \frac{3x - 1}{x + 3}$

B)  $\lim_{x \rightarrow 1} \frac{x^2 + 7x - 8}{1 - x}$

C)  $\lim_{x \rightarrow 0} \frac{\sin(3x) + 4x^2}{5x - \tan(2x)}$

D)  $\lim_{x \rightarrow 0} \frac{\sqrt{x + 4} - 2}{x}$

E)  $\lim_{x \rightarrow \infty} \sin\left(\frac{\pi x + 2}{3x + 1}\right)$

F)  $\lim_{x \rightarrow 1} \frac{x + 3}{x - 1}$

Question 3:

(6 Marks)

1. Use the Intermediate Value Theorem to show that  $f(x) = x^5 - 4x^3 + 1$  has a zero in the interval  $[0, 1]$ .

2. Discuss the continuity of  $f(x) = \begin{cases} 6x^2 - 5 & , \quad x < 1 \\ 2x - 1 & , \quad x > 1 \end{cases}$  at  $x = 1$ .

3. The position of a particle is given by the equation  $s(t) = \frac{t}{t+1}$ , where  $s$  is measured in meters and  $t$  in seconds. What is the velocity of the particle after 3 seconds?
- 

Question 4: Find the derivative  $\frac{dy}{dx}$  for each of the following (12 Marks)

A)  $y = x^{-3} + \tan x + \cos(\pi)$

B)  $y = \sqrt[3]{(2x^2 - 5)^7}$

C)  $y = \sin^{-1}(x^2)$

D)  $y = x^2 \sec x$

E)  $y = \cos^2(x^3 + 5)$

F)  $y \tan(x^2) + y^2 = 5x$

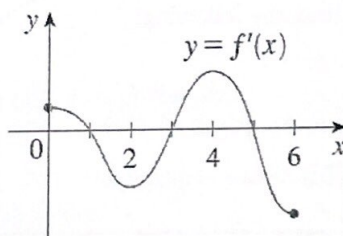
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Question 5: (5 Marks)

A) Let  $f(x) = \sqrt{x^2 + 4x - 1}$ , find the equation of the tangent line to the graph at  $x = 1$ .

B) The figure shows the graph of  $f'(x)$ . Determine:

1. The  $x$ -coordinate(s) at which  $f$  has local minimum or local maximum.
2. The  $x$ -coordinate(s) at which  $f$  has inflection point.



Question 6: (10 Marks)

For the function  $f(x) = 3x^2 - x^3$ , find the following (if any):

- A) The critical numbers of  $f$ .
  - B) The interval(s) on which  $f$  is increasing or decreasing.
  - C) The local extrema of  $f$ .
  - D) The interval(s) on which  $f$  is concave upward or downward.
  - E) Sketch the graph of  $f$ .
- 

Good Luck

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Question 1:

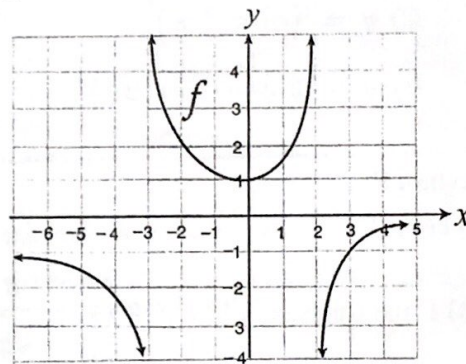
(4 Marks)

A) Solve the following inequality, and write your answer in interval notation

$$|3x - 5| \geq 4$$

B) Use the graph of the function to determine the following (if exists):

- 1) The vertical asymptote(s).
- 2) The horizontal asymptote(s).



Question 2:

(12 Marks)

Evaluate each of the following limits (if exist):

A)  $\lim_{x \rightarrow 2} \sqrt{2x + 5}$

B)  $\lim_{x \rightarrow 1} \frac{x^2 - x}{3 - 3x}$

C)  $\lim_{x \rightarrow 0} \frac{1 + \sin(3x) - \cos x}{6x}$

D)  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

E)  $\lim_{x \rightarrow \infty} \cos\left(\frac{\pi x^2 + 2}{2x^2}\right)$

F)  $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$



**Question 3:****(7Marks)**

- A) Prove that  $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$ .
- B) Discuss the continuity of  $f(x) = \begin{cases} \sin x & , \quad x \leq 0 \\ x^2 - 1 & , \quad x > 0 \end{cases}$  at  $x = 0$ .
- C) The position of a particle is given by the equation  
 $s(t) = t^5 - 10t^2 + 1$ ,  
where  $s$  in meters and  $t$  in seconds. Find the velocity of the particle when its acceleration is zero.

**Question 4:****(12 Marks)**

Find the derivative  $\frac{dy}{dx}$  for each of the following:

A)  $y = x^3 + \sin x + 4$

B)  $y = x \sqrt[3]{(2x^2 - 5)^2}$

C)  $y = (\cos^{-1} x)^4$

D)  $y = \frac{x}{x+1}$

E)  $y = 3 \cos(\sin(2x))$

F)  $x^2 + 3xy + y^2 = 2$

**Question 5:****(5 Marks)**

- A) Let  $f(x) = 3x^4 - 2x + 2$ , find the equation of the tangent line to the graph at  $x = 1$ .
- B) If the function  $f(x) = a\sqrt{x} + \frac{b}{\sqrt{x}}$  has a critical point at  $(1, 4)$ , find the values of  $a$  and  $b$ .

**Question 6:****(10 Marks)**

For the function  $f(x) = x^3 - 3x + 2$ , find the following (if any):

- A) The critical numbers of  $f$ .
- B) The interval(s) on which  $f$  is increasing and decreasing.
- C) The local extrema of  $f$ .
- D) The interval(s) on which  $f$  is concave upward or downward.
- E) Sketch the graph of  $f$ .

**Good Luck**

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علما بأن عدد الأسئلة (٦)، وعدد الصفحات (٢).

**QUESTION 1**

(14 Marks)

A. Using definition of limit, show that  $\lim_{x \rightarrow 2} (3x - 2) = 4$

B. Find each of the following limits (if exist).

1)  $\lim_{x \rightarrow -2} (x^2 + 6x + 3)$

2)  $\lim_{x \rightarrow 0} \frac{x^2 + \sin(4x)}{x + \tan(7x)}$

3)  $\lim_{x \rightarrow \infty} \cos\left(\frac{1}{x}\right)$

4)  $\lim_{x \rightarrow 0} 3x^2 \sin\left(\frac{1}{x}\right)$

5)  $\lim_{x \rightarrow 1} \frac{\frac{4}{x-5} + 1}{1-x^2}$

6)  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$

**QUESTION 2:**

(5 Marks)

A) Find the values of  $a$  and  $b$  such that the function

$$f(x) = \begin{cases} ax + b, & x > 1 \\ 5x + 2a, & x < 1 \\ \frac{1}{4}, & x = 1 \end{cases} \text{ is continuous at } x = 1.$$

B) Prove that  $\frac{d}{dx}(\sin x) = \cos x$ , by using definition of the derivative.

**QUESTION 3:**

(4 Marks)

A) Show that the function  $f(x) = x^3 - 4x$  satisfies the conditions of the Mean Value Theorem on  $[-2, 1]$ . Find a number  $C$  that satisfy the conclusion of the theorem.

B) Find  $\frac{d^2}{dx^2} \left[ \left( 5x^3(x^2 + 1) \right) \frac{d}{dx} (\tan^{-1} x) \right]$

**QUESTION 4:****(5 Marks)**

A. Suppose  $f$  and  $g$  are differentiable functions at  $x = 5$  such that:

$$f(5) = 4, \quad f'(5) = 3, \quad g(5) = 1 \quad \text{and} \quad g'(5) = -2. \quad \text{Find}$$

$$1) \left. \frac{d}{dx} (g(x) + f(x)) \right|_{x=5}$$

$$2) \left. \frac{d}{dx} (g(x))^2 \right|_{x=5}$$

$$3) \left. \frac{d}{dx} \left( \frac{1}{f(x)} \right) \right|_{x=5}$$

B. Given that  $f(x) = 2x^3 + x - 3$ . Find an equation of the line tangent to the graph of  $f(x)$  at  $(1, 0)$ .

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**QUESTION 5:****(10 Marks)**

A. Find the derivative  $\frac{dy}{dx}$  for each of the following functions:

$$1) y = x^3 + 5x^2 + 5$$

$$2) y = 3 \cos(\sin(2x))$$

$$3) y = \sin^{-1}(\sqrt{2x}) + \sin(\pi)$$

$$4) y^2 = xy + 2y$$

B. Find  $k$  given that  $f(x) = kx^2 + \frac{1}{x}$  has  $(1, f(1))$  as an inflection point.

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**QUESTION 6:****(12 Marks)**

For the function  $f(x) = \frac{1-x}{x^2}$ , find the following (if any):

- 1) The horizontal and vertical asymptotes of  $f$ .
  - 2) The critical numbers of  $f$ .
  - 3) The interval(s) on which  $f$  is increasing and decreasing.
  - 4) The local extrema of  $f$ .
  - 5) The interval(s) on which  $f$  is concave upward or downward.
  - 6) Sketch the graph of  $f$ .
- 

*Good Luck*



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Question 1:

(4 Marks)

A) Solve the following inequality, and write your answer in interval notation

$$|2x - 6| \leq 4$$

B) Use definition of limit to show that  $\lim_{x \rightarrow 1} (2x + 3) = 5$ .

Question 2:

(12 Marks)

Evaluate each of the following limits (if exist):

1)  $\lim_{x \rightarrow 1} (2x + 4)^2$

2)  $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$

3)  $\lim_{x \rightarrow 0} \frac{\sqrt{x + 9} - 3}{x}$

4)  $\lim_{x \rightarrow 0} \frac{x^2 - 2x}{x}$

5)  $\lim_{x \rightarrow \infty} \left[ 1 + \cos \left( \frac{3}{2x + 1} \right) \right]$

6)  $\lim_{x \rightarrow 2} \frac{4x + 3}{x - 2}$

Question 3:

(6 Marks)

A) Prove that if a function  $f$  is differentiable at  $a$ , then  $f$  is continuous at  $a$ .

B) Discuss the continuity of  $f(x) = \begin{cases} x + 3 & , \quad x \leq 0 \\ \frac{\sin(6x)}{2x} & , \quad x > 0 \end{cases}$  at  $x = 0$ .

C) The position of a particle is given by the equation  $s(t) = \frac{t - 1}{t + 1}$ , where  $s$  is measured in meters and  $t$  in seconds. What is the acceleration of the particle after 3 seconds?

**Question 4:****(12 Marks)**

Find the derivative  $\frac{dy}{dx}$  for each of the following functions:

1)  $y = \sin x + 2 \cos x$

2)  $y = (3x^2 + 5x + 2)^{30}$

3)  $y = \sqrt{5x^2 + 7}$

4)  $y = x^2 \tan(3x)$

5)  $y = \tan^{-1}(4x)$

6)  $\frac{x^2 + y^2}{\sec x} = 1$

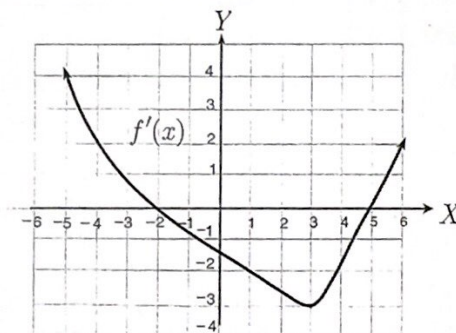
**Question 5:****(6 Marks)**

A) Given that  $g(x) = 3x^2 + 5x + 1$ , find the equation of the tangent line to the graph of  $g(x)$  at  $(1, 9)$ .

B) Show that the function  $f(x) = x^2 + x$  satisfies the conditions of the Mean Value Theorem on  $[-4, 6]$ . Then find a number  $c$  that satisfies the conclusion of the theorem.

C) The figure shows the graph of  $f'(x)$ . Determine the local minimum

and local maximum of the function  $f(x)$ .

**Question 6:****(10 Marks)**

For the function  $f(x) = x^4 - 4x^2$ , find the following (if any):

- 1) The critical numbers of  $f$ .
- 2) The interval(s) on which  $f$  is increasing and decreasing.
- 3) The local extrema of  $f$ .
- 4) The interval(s) on which  $f$  is concave upward or downward.
- 5) Sketch the graph of  $f$ .

**Good Luck**