

St. Name: _____ St. ID: _____ Section: _____

ملاحظات :

- 1- اكتب خطوات الحل بالتفصيل لجميع الأسئلة داخل دفتر الإجابة (الإجابة على ورقة الأسئلة غير معتمدة).
- 2- علمًا بأن عدد الأسئلة (٥)، وعدد الصفحات (٢).
- 3- لا يسمح بالكتابة إلا بالقلم الأزرق فقط.
- 4- لا يسمح باستخدام آلة حاسبة قابلة للبرمجة أو آلة حاسبة ترسم دوال.

Question 1: (13 Marks)

A) Solve the following inequality, and write your answer in an interval notation

$$2x - 4 \leq 6.$$

B) Use the definition of the limit to prove that $\lim_{x \rightarrow 2} (x + 4) = 6$.

C) Evaluate each of the following limits (if exist):

1) $\lim_{x \rightarrow 1} (3x + 5)$

2) $\lim_{x \rightarrow 0} \frac{\sin(4x)}{3x}$

3) $\lim_{x \rightarrow 5} \frac{x^2 - 8x + 15}{x - 5}$

4) $\lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x + 2} - 2}$

5) $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x^2}\right)$

6) $\lim_{x \rightarrow \infty} \sin\left(\frac{\pi x + 2}{2x + 1}\right)$

Question 2: (6 Marks)

A) Prove that, if $f(x) = mx + c$ where m and c are real numbers, then

$$f'(x) = m.$$

B) Discuss the continuity of $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & , \quad x \neq 2 \\ -2 & , \quad x = 2 \end{cases}$ at $x = 2$.

C) The position of a particle is given by the equation $s(t) = 3t^3 + 2t^2 + 7$, where s in meters and t in seconds.

- 1- What is the velocity of the particle after 3 seconds?
- 2- What is the acceleration of the particle after 5 seconds?

Question 3:

(9 Marks)

Find the derivative $\frac{dy}{dx}$ for each of the following:

A) $y = x^5 + x^3 + x^2 + 7$

B) $y = (x^2 + x)^{25}$

C) $y = \frac{\sin x}{x + 1}$

D) $y = x^2 \tan^{-1}(3x)$

E) $y = \cos^3 x^2 + \tan\left(\frac{\pi}{3}\right)$

F) $x^2 + y^2 = \sin(xy)$

Question 4:

(7 Marks)

A) Show that the function $f(x) = x^2 - 4x + 5$ satisfies the conditions of the mean value theorem on the interval $[0, 2]$, then find the number c that satisfies the conclusion of the theorem.

B) Let $f(x) = x^2$. Find the absolute extrema of f in $[3, 5]$.

C) What are the dimensions of a rectangle with perimeter equals 100 cm which has maximum area?

D) Let $f(x) = \frac{x^2}{x^2 - 9}$. Find the vertical asymptote(s), if exist.

Question 5:

(5 Marks)

For the function $f(x) = x^4 - 4x^3 + 10$, find the following (if any):

A) The critical numbers of f .

B) The interval(s) on which f is increasing and decreasing.

C) The local extrema of f .

D) The interval(s) on which f is concave upward or downward.

E) Sketch the graph of f .

Good Luck

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- 4- لا يسمح باستخدام آلة حاسبة قابلة للبرمجة أو آلة حاسبة ترسم دوال.

Question 1:

(13 Marks)

A) Solve the following inequality, and write your answer in an interval notation

$$|3x + 2| > 9.$$

B) Use the definition of the limit to prove that $\lim_{x \rightarrow -3} (-x - 8) = -5$.

C) Evaluate each of the following limits (if exist):

$$1) \lim_{x \rightarrow 4} \left(\frac{2x + 1}{2x} \right)$$

$$2) \lim_{x \rightarrow 0} \left(\frac{\sin(x)}{4x} + \frac{\sin(x)}{5} \right)$$

$$3) \lim_{x \rightarrow \infty} \tan \left(\frac{\pi x + 2}{x + 1} \right)$$

$$4) \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{2x^2}$$

$$5) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

$$6) \lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x - 3}$$

Question 2:

(6 Marks)

A) Prove that, $\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$, $-1 < x < 1$.

B) Discuss the continuity of $f(x) = \begin{cases} \frac{x^3 - 3x^2}{x - 3}, & x < 3 \\ 6, & x > 3 \end{cases}$ at $x = 3$.

C) The position of a particle is given by the equation $s(t) = 2t^3 - t^2 + 3$, where s in meters and t in seconds, what are the velocity and acceleration of the particle after 4 seconds?

Question 3:**(9 Marks)**

A- Find the derivative $\frac{dy}{dx}$ for each of the following:

A) $y = x^6 + x^5 + 2$

B) $y = x^4 \sin^{-1}(x^3) + \cos\left(\frac{\pi}{3}\right)$

C) $y = (\cos^2(x^5) + \sin^2(x^2))^5$

D) $xy + y^2 = \sin(xy + 4)$

B- Suppose that f and g are differentiable functions at $x = 2$ and that

$$f(2) = -3, f'(2) = 5, g(2) = 1, \text{ and } g'(2) = -4$$

Find $\frac{d}{dx} \left(\frac{x + g(x)}{x^2 - f(x)} \right) \Big|_{x=2}$.

Question 4:**(7 Marks)**

A) Show that the function $f(x) = x^3 - 3x^2 + x$ satisfies the conditions of the mean value theorem on the interval $[0, 2]$, then find the number c that satisfies the conclusion of the theorem.

B) Let $f(x) = x^3$. Find the absolute extrema of f in $[-2, 4]$.

C) What are the dimensions of a rectangle with perimeter equals 120 cm which has maximum area?

D) Let $f(x) = \frac{x^2}{\sqrt{x^2 - 9}}$. Find the horizontal asymptote(s), if exist.

Question 5:**(5 Marks)**

For the function $f(x) = x^5 - 5x$, find the following (if any):

A) The critical numbers of f .

B) The interval(s) on which f is increasing and decreasing.

C) The local extrema of f .

D) The interval(s) on which f is concave upward or downward.

E) Sketch the graph of f .

Good Luck

KING SAUD UNIVERSITY
COMMON FIRST YEAR
BASIC SCIENCES DEPARTMENT
Math 101 Final Exam alternative 1439/1440 H.

First Semester

Time Allowed - 3 Hours

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3) $\lim_{x \rightarrow \infty} \tan \left(\frac{\pi x + 2}{x + 1} \right)$

4) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{2x^2}$

5) $\lim_{x \rightarrow 0} \frac{\sqrt{1 + x} - 1}{x}$

6) $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x - 3}$

Question 2:

(6 Marks)

A) Prove that, $\frac{d}{dx} (\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$, $-1 < x < 1$.

B) Discuss the continuity of $f(x) = \begin{cases} \frac{x^3 - 3x^2}{x - 3} & , \quad x < 3 \\ 6 & , \quad x > 3 \end{cases}$ at $x = 3$.

C) The position of a particle is given by the equation $s(t) = 2t^3 - t^2 + 3$, where s in meters and t in seconds, what are the velocity and acceleration of the particle after 4 seconds?

Question 3:**(9 Marks)**

A- Find the derivative $\frac{dy}{dx}$ for each of the following:

A) $y = x^6 + x^5 + 2$

B) $y = x^4 \sin^{-1}(x^3) + \cos\left(\frac{\pi}{3}\right)$

C) $y = (\cos^2(x^5) + \sin^2(x^2))^5$

D) $xy + y^2 = \sin(xy + 4)$

B- Suppose that f and g are differentiable functions at $x = 2$ and that

$$f(2) = -3, f'(2) = 5, g(2) = 1, \text{ and } g'(2) = -4$$

Find $\frac{d}{dx} \left(\frac{x + g(x)}{x^2 - f(x)} \right) \Big|_{x=2}$.

Question 4:**(7 Marks)**

A) Show that the function $f(x) = x^3 - 3x^2 + x$ satisfies the conditions of the mean value theorem on the interval $[0, 2]$, then find the number c that satisfies the conclusion of the theorem.

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C) The local extrema of f .

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E) Sketch the graph of f .

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Question 1: (4 Marks)

A) Solve the following inequality, and write your answer in interval notation

$$|2x - 6| \leq 4$$

B) Use definition of limit to show that $\lim_{x \rightarrow 1} (2x + 3) = 5$.

Question 2: (12 Marks)

Evaluate each of the following limits (if exist):

1) $\lim_{x \rightarrow 1} (2x + 4)^2$

2) $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x - 2}$

3) $\lim_{x \rightarrow 0} \frac{\sqrt{x + 9} - 3}{x}$

4) $\lim_{x \rightarrow 0} \frac{x^2 - 2x}{x}$

5) $\lim_{x \rightarrow \infty} \left[1 + \cos \left(\frac{3}{2x + 1} \right) \right]$

6) $\lim_{x \rightarrow 2} \frac{4x + 3}{x - 2}$

Question 3: (6 Marks)

A) Prove that if a function f is differentiable at a , then f is continuous at a .

B) Discuss the continuity of $f(x) = \begin{cases} x + 3 & , \quad x \leq 0 \\ \frac{\sin(6x)}{2x} & , \quad x > 0 \end{cases}$ at $x = 0$.

C) The position of a particle is given by the equation $s(t) = \frac{t - 1}{t + 1}$, where s is measured in meters and t in seconds. What is the acceleration of the particle after 3 seconds?

Question 4:**(12 Marks)**

Find the derivative $\frac{dy}{dx}$ for each of the following functions:

1) $y = \sin x + 2 \cos x$

2) $y = (3x^2 + 5x + 2)^{30}$

3) $y = \sqrt{5x^2 + 7}$

4) $y = x^2 \tan(3x)$

5) $y = \tan^{-1}(4x)$

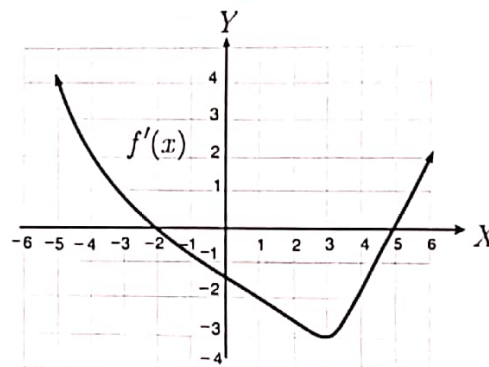
6) $\frac{x^2 + y^2}{\sec x} = 1$

Question 5:**(6 Marks)**

A) Given that $g(x) = 3x^2 + 5x + 1$, find the equation of the tangent line to the graph of $g(x)$ at $(1, 9)$.

B) Show that the function $f(x) = x^2 + x$ satisfies the conditions of the Mean Value Theorem on $[-4, 6]$. Then find a number c that satisfies the conclusion of the theorem.

C) The figure shows the graph of $f'(x)$. Determine the local minimum and local maximum of the function $f(x)$.

**Question 6:****(10 Marks)**

For the function $f(x) = x^4 - 4x^2$, find the following (if any):

- 1) The critical numbers of f .
- 2) The interval(s) on which f is increasing and decreasing.
- 3) The local extrema of f .
- 4) The interval(s) on which f is concave upward or downward.
- 5) Sketch the graph of f .

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Question 1:

(5 Marks)

- A) Determine algebraically whether the function $f(x) = \left| \frac{2x^4 + x^2}{\sin x} \right|$ is even, odd, or neither.
- B) Solve $1 - 2|2x - 3| \geq -6$

Question 2:

(6 Marks)

A) Let $f(x) = \frac{3}{\sqrt{x-4}}$, $g(x) = x^2 + 4$. Find:

- 1) $(f \circ g)(x)$.
- 2) D_f , D_g , and $D_{f \circ g}$.

B) Show that $f(x) = x^2 - 4x - 5$, $x > 2$ is a one-to-one function.

C) Find the exact value of $\cos\left(2\cos^{-1}\left(\frac{4}{5}\right)\right)$, without using calculator.

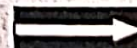
Question 3:

(4 Marks)

A) Use definition of limit to show that $\lim_{x \rightarrow 0} (3x + 4) = 4$.

B) Use the Intermediate Value Theorem to show that $f(x) = x^2 - \frac{9}{x} + 1$ has a zero in the interval $[1, 3]$.

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Question 4:**(9 Marks)**

C) Evaluate each of the following limits (if exist):

1) $\lim_{x \rightarrow 1} \frac{x^2 + 4}{2x + 2}$

2) $\lim_{x \rightarrow 0} x^6 \sin\left(\frac{1}{x^2}\right)$

3) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x^2 - 9}$

4) $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x^2 - 4x + 3}$

5) $\lim_{x \rightarrow \infty} \tan\left(\frac{\pi x - x}{x^2 + 5x}\right)$

6) $\lim_{x \rightarrow \infty} \frac{x^2}{\sqrt{x-1}}$

Question 5:**(6 Marks)**A) Let $f(x) = 3x^2 - 2$, then use the definition of derivative to find $f'(x)$.B) Find all vertical and horizontal asymptotes (if any) for $f(x) = \frac{2x-5}{x+3}$ C) Find the values of a and b such that the function $f(x) = \begin{cases} 2ax + 4b, & x > 1 \\ 3x + 2a, & x < 1 \\ 4, & x = 1 \end{cases}$

is continuous at every real number.

Good Luck**Page 2 of 2**

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Question 1:

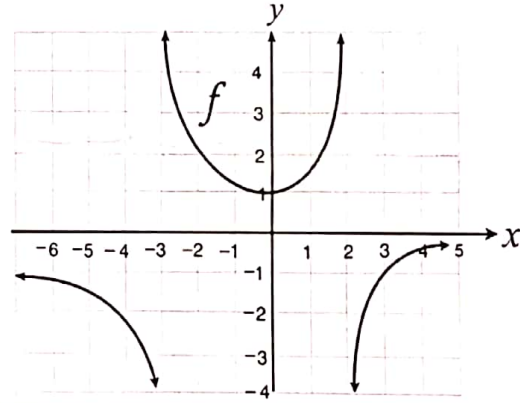
(4 Marks)

A) Solve the following inequality, and write your answer in interval notation

$$|3x - 5| \geq 4$$

B) Use the graph of the function to determine the following (if exists):

- 1) The vertical asymptote(s).
- 2) The horizontal asymptote(s).



Question 2:

(12 Marks)

Evaluate each of the following limits (if exist):

A) $\lim_{x \rightarrow 2} \sqrt{2x + 5}$

B) $\lim_{x \rightarrow 1} \frac{x^2 - x}{3 - 3x}$

C) $\lim_{x \rightarrow 0} \frac{1 + \sin(3x) - \cos x}{6x}$

D) $\lim_{x \rightarrow 3} \frac{\sqrt{x + 1} - 2}{x - 3}$

E) $\lim_{x \rightarrow \infty} \cos \left(\frac{\pi x^2 + 2}{2x^2} \right)$

F) $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$

Question 3:**(7Marks)**

A) Prove that $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$.

B) Discuss the continuity of $f(x) = \begin{cases} \sin x & , \quad x \leq 0 \\ x^2 - 1 & , \quad x > 0 \end{cases}$ at $x = 0$.

C) The position of a particle is given by the equation

$$s(t) = t^5 - 10t^2 + 1,$$

where s in meters and t in seconds. Find the velocity of the particle when its acceleration is zero.

Question 4:**(12 Marks)**

Find the derivative $\frac{dy}{dx}$ for each of the following:

A) $y = x^3 + \sin x + 4$

B) $y = x \sqrt[3]{(2x^2 - 5)^2}$

C) $y = (\cos^{-1} x)^4$

D) $y = \frac{x}{x+1}$

E) $y = 3 \cos(\sin(2x))$

F) $x^2 + 3xy + y^2 = 2$

Question 5:**(5 Marks)**

A) Let $f(x) = 3x^4 - 2x + 2$, find the equation of the tangent line to the graph at $x = 1$.

B) If the function $f(x) = a\sqrt{x} + \frac{b}{\sqrt{x}}$ has a critical point at $(1, 4)$, find the values of a and b .

Question 6:**(10 Marks)**

For the function $f(x) = x^3 - 3x + 2$, find the following (if any):

A) The critical numbers of f .

B) The interval(s) on which f is increasing and decreasing.

C) The local extrema of f .

D) The interval(s) on which f is concave upward or downward.

E) Sketch the graph of f .

Good Luck**Page 2 of 2**