KING SAUD UNIVERSITY

DEANSHIP OF COMMON FIRST YEAR

BASIC SCIENCES DEPARTMENT

MATH 101 | HW # 1 / FIRST SEMESTER 1442 | Date: 24/09/2020

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Question 1

A. Classify the following numbers into rational and irrationals. (2 Marks)

$$\left\{(1.\bar{5})^2,\ \frac{3.14}{6},\ \sin\pi,\ \sqrt[7]{2^7},\ \sqrt{\sqrt{9}+6},\ \frac{2}{3\pi},\ \frac{22}{7},\ \sqrt{\frac{16}{64}}\right\}$$

Solution:

$$(1.\bar{5})^2=\left(\frac{14}{9}\right)^2=\frac{196}{81}\in Q$$

$$\frac{3.14}{6}=0.52\bar{3}\in Q$$

$$\sin\pi=0\in Q$$

$$\sqrt[7]{2^7}=2\in Q$$

$$\sqrt{\sqrt{9}+6}=3\in Q$$

$$\frac{2}{3\pi}\in I$$

$$\frac{22}{7}\in Q$$

$$\sqrt{\frac{16}{64}}=\frac{1}{2}\in Q$$

B. Solve the following inequalities and write the solution in interval notation. (6x2 + 3=15) Marl

1. $3x+5\geq x+1$
2. $x^2+x-6\leq 0$
3. $\frac{2|x+1|-3}{-3}<1$
4. $5|3x+1|-8>2+3|3x+1|$
5. $\frac{3x-6}{x^2-3x-18}<0$
6. $|2x-5|-|3x-2|<0$
7. **If** $f(x)=1-x-x^2$ **and** $g(x)=3-x$, **then Solve**
$$(f\circ g)(x)+x<1$$

Solution

1) $3x + 5 \geq x + 1$

$-x \qquad -x$

$2x + 5 \geq +1$

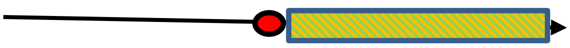
$-5 \qquad -5$

$2x \geq -4$

$\frac{2x}{2} \geq \frac{-4}{2}$

$x \geq -2$

-2



S.S : $[-2, \infty)$

2) $x^2 + x - 6 \leq 0$

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0$

$x - 2 = 0 \rightarrow x = 2$

$x + 3 = 0 \rightarrow x = -3$

$+ + \; -3 \; - - - - - \; 2 \; + + + +$

S.S : $[-3, 2 \;]$

3) $\frac{2|x+1|-3}{-3} < 1$

$$-3 \times \frac{2|x+1|-3}{-3} < -3 \times 1$$

$$2|x+1| - 3 > -3$$

$$+3 \qquad +3$$

$$2|x+1| > 0$$

$$\frac{2|x+1|}{2} > \frac{0}{2}$$

$$|x+1| > 0$$

$$|x+1| = 0 \rightarrow x + 1 = 0 \rightarrow x = -1$$

$$SS; R - \{-1\}$$

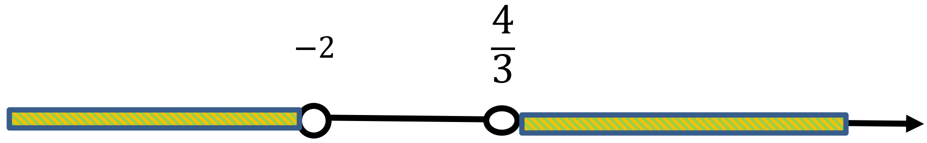
4) $5\,|3x+1| - 8 > 2 + 3\,|3x+1|$

Solution

$$5\,|3x+1| - 8 > 2 + 3\,|3x+1|$$
$$-3\,|3x+1| \qquad\qquad -3\,|3x+1|$$
$$2\,|3x+1| - 8 > 2$$
$$+8 \qquad +8$$
$$2\,|3x+1| > 10$$
$$\frac{2\,|3x+1|}{2} > \frac{10}{2}$$
$$|3x+1| > 5$$

$$3x+1 > 5$$
$$-1 \quad -1$$
$$3x > 4$$
$$\frac{3x}{3} > \frac{4}{3}$$
$$x > \frac{4}{3}$$

$$3x+1 < -5$$
$$-1 \quad -1$$
$$3x < -6$$
$$\frac{3x}{3} < \frac{-6}{3}$$
$$x < -2$$



S.S : $\left((-\infty, -2) \cup \left(\frac{4}{3}, \infty\right)\right)$

5) $\frac{3x-6}{x^2-3x-18} < 0$

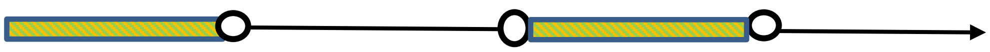
$$\frac{3x-6}{(x+3)(x-6)} < 0$$

$3x - 6 = 0 \rightarrow 3x = 6 \rightarrow x = 2$

$(x+3)(x-6) = 0 \rightarrow \begin{cases} x+3=0 \rightarrow x=-3 \\ x-6=0 \rightarrow x=6 \end{cases}$

x	-3	2	6	
$3x-6$	$---$	$-----$	$++++$	$++++$
$x+3$	$---$	$+++++$	$+++++$	$+++++$
$x-6$	$---$	$-----$	$-----$	$+++++$
$\frac{3x-6}{(x+3)(x-6)}$	$---$	$+++++$	$-----$	$+++++$

$---- \; -3 \; +++++ \; 2 \; ----- \; 6 \; +++++$



S.S : $(-\infty, -3) \cup (2, 6)$

6) $|2x-5|-|3x-2|<0$

$$|2x-5|<|3x-2|$$

$$(\ |2x-5|)^2<\ (|3x-2|)^2$$

$$(\ 2x-5)^2<\ (3x-2)^2$$

$$(\ 2x-5)^2-\ (3x-2)^2<0$$

$$[(2x-5)+(3x-2)][(2x-5)-(3x-2)]<0$$

$$\big((5x-7)(-x-3)\big)<0$$

$5x-7=0\rightarrow 5x=7\rightarrow x=\dfrac{7}{5}$

$-x-3=0\rightarrow x=-3$

$$------\ -3+++++\ \frac{7}{5}\ -------$$

S.S : $\left(-\infty,-3\right)\cup\left(\frac{7}{5}\ ,\infty\ \right)$

7) $(fog)(x) + x < 1$

$$f\big(g(x)\big) + x < 1$$

$$f(3 - x) + x < 1$$

$$1 - (3 - x) - (3 - x)^2 + x < 1$$

$$1 - 3 + x - (9 - 6x + x^2) + x < 1$$

$$1 - 3 + x - 9 + 6x - x^2 + x - 1 < 0$$

$$-x^2 + 8x - 12 < 0$$

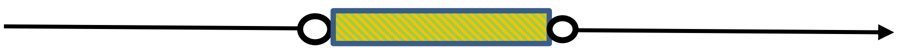
$$x^2 - 8x + 12 > 0$$

$$(x - 6)\,(x - 2) > 0$$

$x - 6 = 0 \rightarrow x = 6$

$x - 2 = 0 \rightarrow x = 2$

$$+ + + + + \quad 2 - - - - - - 6 + + + + +$$



S.S : $(-\infty\,, 2) \cup (6\,, \infty)$

Question 2

Find the domain of the following functions (6x3=18 Marks)

1. $f(x) = 2x^3 + 5x - 3$
2. $f(x) = \sqrt{\dfrac{x}{x+5}}$
3. $f(x) = \dfrac{|2-x|+1}{x^2 - 3x - 18}$
4. $f(x) = \sqrt[3]{\dfrac{2x+2}{x+4}}$
5. $f(x) = \dfrac{5}{\sec(3x)}$
6. $f(x) = \sqrt{|x+2|}$

1) $f(x) = 2x^3 + 5x - 3$

$$f(x)\ \ is\ \ poly\ \rightarrow D_f = R$$

2) $f(x) = \sqrt{\frac{x}{x+5}}$

Solution

$$\frac{x}{x+5} \geq 0$$

$x = 0$

$x + 5 = 0 \rightarrow x = -5$

x	-5		0
x	$- - - - -$	$- - - - -$	$+ + + + +$
$x+5$	$- - - - -$	$+ + + + +$	$+ + + + +$
$\frac{x}{x+5}$	$+ + + + +$	$- - - - -$	$+ + + + +$

$+ + + + + + \ -5 - - - - - - \ 0 \ \ + + + + + +$

$D_f = (-\infty\,, -5) \cup [0\,, \infty)$

3) $f(x) = \frac{|2-x|+1}{x^2-3x-18}$

$$x^2 - 3x - 18 = 0$$

$$(x+3)(x-6) = 0$$

$x + 3 = 0 \rightarrow x = -3$

$x - 6 = 0 \rightarrow x = 6$

$$D_f = R - \{-3\,, 6\,\}$$

4) $f(x) = \sqrt[3]{\frac{2x+2}{x+4}}$

$$x + 4 = 0 \rightarrow x = -4$$

$$D_f = R - \{-4\}$$

5) $f(x) = \frac{5}{\sec(3x)}$

Domain $\sec(x)$ *is* $R - \left\{\frac{\pi}{2} + \pi k\right\}$

$$\text{Domain} \quad \sec(3x) \quad is \quad R - \left\{\frac{\frac{\pi}{2}}{3} + \frac{\pi k}{3}\right\}$$

$$D_f = R - \left\{\frac{\pi}{6} + \frac{\pi k}{3}\right\}$$

6) $f(x) = \sqrt{|x+2|}$

$$|x+2| \geq 0$$

$$\boxed{|a| \geq 0}$$

$$D_f = R$$

Question 3

Determine whether the functions (3 Marks)

$$f(x) = 1 - \cos^2 x \text{, and } g(x) = \frac{\sin x}{\csc x}$$

are the same or not.

Solution

Domaine $f(x) = 1 - \cos^2 x$

$$D_f = R$$

$$g(x) = \frac{\sin x}{\csc x}$$

Domain $\sin x$ *is* R

Domain $\csc x$ *is* $R - \{\pi k\}$

$$D_g = R - \{\pi k\}$$

$$D_f \neq D_g$$

.f and g are not the same

Question 4

Let $f(x) = \dfrac{x-1}{x+2}$. (2x4=8 Marks)

1. Find D_f.
2. Show that f is one-to-one.
3. Find f^{-1}.
4. Find the range of f.

Solution

Domaine $f(x) = \frac{x-1}{x+2}$ $\qquad x + 2 = 0 \ \rightarrow x = -2$

$$D_f = R - \{-2\}$$

Let $x_1, x_2 \in D_f$ *such that* $f(x_1) = f(x_2)$ *then*

$$\frac{x_1 - 1}{x_1 + 2} = \frac{x_2 - 1}{x_2 + 2}$$
$$(x_1 - 1)(x_2 + 2) = (x_2 - 1)(x_1 + 2)$$
$$x_1x_2 + 2x_1 - x_2 - 2 = x_1x_2 + 2x_2 - x_1 - 2$$
$$2x_1 - x_2 = 2x_2 - x_1$$
$$2x_1 + x_1 = 2x_2 + x_2$$
$$3x_1 = 3x_2$$
$$\frac{3x_1}{3} = \frac{3x_2}{3}$$
$$x_1 = x_2$$

$f(x)$ is a one -to – one

$$f(x) = \frac{x-1}{x+2}$$
$$y = \frac{x-1}{x+2}$$

$$y(x+2) = x-1$$
$$yx + 2y = x - 1$$
$$yx - x = -2y - 1$$

$$x(y-1) = -2y - 1$$

$$\frac{x(y-1)}{y-1} = \frac{-2y-1}{y-1}$$

$$x = \frac{-2y-1}{y-1}$$

$$f^{-1}(y) = \frac{-2y-1}{y-1}$$

$$f^{-1}(x) = \frac{-2x-1}{x-1}$$
$$D_{f^{-1}} = R - \{1\}$$
range of $\boldsymbol{f(x)}$**:** $\mathrm{R}_f = D_{f^{-1}} = R - \{1\}$

Question 5

Let $f(x)=\dfrac{2}{x-1}$, $\quad g(x)=x+1$. (3x2=6 Marks)

1. Find $f\cdot g$ and its domain.
2. Find $\dfrac{f}{g}$ and its domain.
3. Find $(f\circ g)(3)$ and $(g\circ f)(5)$.

Solution

$$(f.g)(x)=f(x).g(x)$$

$$(f.g)(x)=\left(\frac{2}{x-1}\right).(x+1)$$

$$D_f=D_f=R-\{1\}$$
$$D_g=R$$

$$D_{f.g}=D_f\cap D_g=R-\{1\}$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$$

$$\left(\frac{f}{g}\right)(x)=\frac{\frac{2}{x-1}}{x+1}$$

$$D_f=R-\{1\}$$

$$D_g=R$$

$$g(x)=0\to x+1=0\to x=-1$$

$$D_{\frac{f}{g}}=(D_f\cap D_g)-\{g(x)=0\}=R-\{-1,1\}$$

$$(fog)(3) = f\big(g(3)\big)$$

$$= f(3+1)$$

$$= f(4) = \frac{2}{4-1} = \frac{2}{3}$$

$$(gof)(5) = g\big(f(5)\big)$$

$$= g\left(\frac{2}{5-1}\right) = g\left(\frac{2}{4}\right)$$

$$= g\left(\frac{1}{2}\right) = \frac{1}{2} + 1 = \frac{3}{2}$$

Question 6

Find the exact value of the following: (4x2=8) Marks

1. $\cos(330^\circ)$
2. $\cos(15^\circ)$
3. $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$.
4. $\sin\left(\tan^{-1}\left(-\sqrt{3}\right) + \cos^{-1}\left(\frac{5}{13}\right)\right)$.

1) $\cos(330^\circ) = \cos(360^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

2) $\cos(15^\circ) = \cos(60^\circ - 45^\circ)$

$$= \cos 60^\circ . \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$

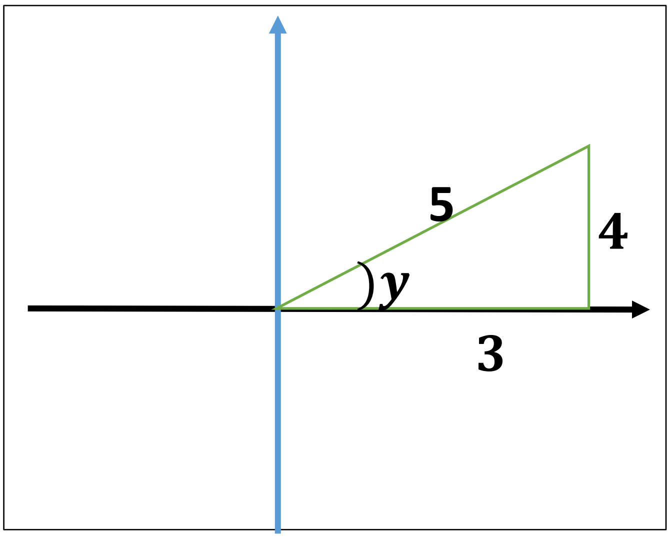
$$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

3) $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$

Let : $y = \cos^{-1}\left(\frac{3}{5}\right) \rightarrow \cos y = \frac{3}{5} \quad 0 \le y \le \pi$

$$opp = \sqrt{5^2 - 3^2} = 4$$

$$\sin y = \frac{4}{5}$$



$$\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right) = \sin 2y$$

$$\sin 2y = 2\sin y\cos y$$

$$\sin 2y = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$$

4) $\sin\left(\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(\frac{5}{13}\right)\right)$

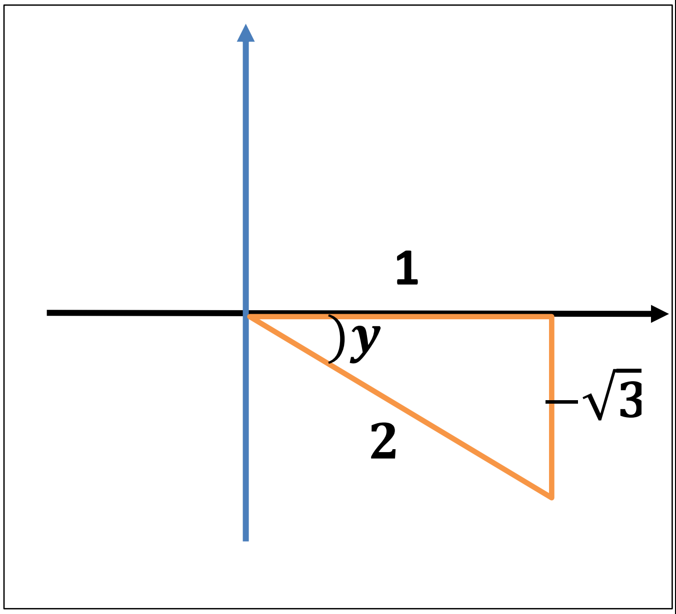
Let $y = \tan^{-1}(-\sqrt{3}) \to \tan y = \frac{-\sqrt{3}}{1}$ $\quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

$$hyb = \sqrt{(-\sqrt{3})^2 + (1)^2}$$

$$hyb = 2$$

$$\cos y = \frac{1}{2}$$

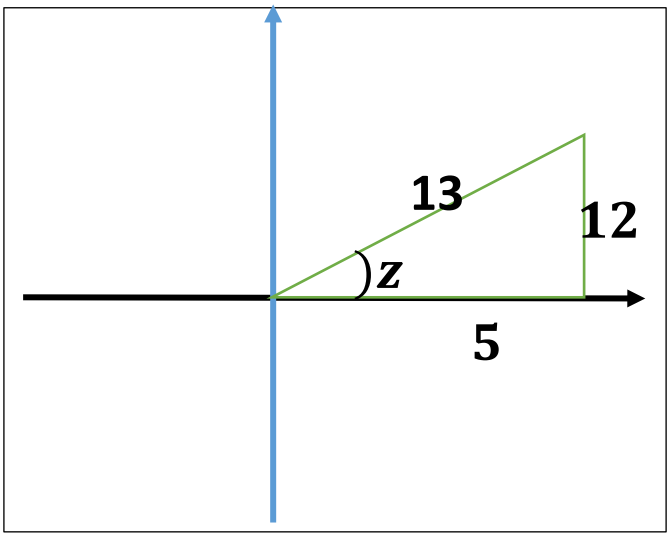
$$\sin y = \frac{-\sqrt{3}}{2}$$



Let : $z = \cos^{-1}\left(\frac{5}{13}\right) \to \cos z = \frac{5}{13}$ $\quad 0 \le z \le \pi$

$$opp = \sqrt{13^2 - 5^2} = 12$$

$$\sin z = \frac{12}{13}$$



$$\sin\left(\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(\frac{5}{13}\right)\right)$$

$$\sin(y+z) = \sin y \cos z + \cos y \sin z$$

$$\sin(y+z) = \frac{-\sqrt{3}}{2} \cdot \frac{5}{13} + \frac{1}{2} \cdot \frac{12}{13}$$

$$\sin(y+z) = \frac{-5\sqrt{3}}{26} + \frac{12}{26}$$

$$\sin(y+z) = \frac{-5\sqrt{3}+12}{26}$$

Question 7

Solve the equation (4 Marks)

$$(\sin x + 2)^2 = 1 \quad , \quad x \in [0, 2\pi]$$

$$(\sin x + 2)^2 = 1$$

$$\sin^2 x + 4\sin x + 4 = 1$$

$$\sin^2 x + 4\sin x + 4 - 1 = 0$$

$$\sin^2 x + 4\sin x + 3 = 0$$

$$(\sin x + 1)(\sin x + 3) = 0$$

$\sin x + 1 = 0$ $\sin x = -1$ $\sin x = \sin\left(-\frac{\pi}{2}\right)$ $x = -\frac{\pi}{2} \notin [0, 2\pi]$ $x = \pi - \left(-\frac{\pi}{2}\right) = \frac{3\pi}{2} \in [0, 2\pi]$	$\sin x + 3 = 0$ $\sin x = -3$ *no sol* $-1 \le \sin x \le 1$

$$S.S = \left\{\frac{3\pi}{2}\right\}$$

Question 8

Verify each of the following: (3x2=6) Marks

1) $2\sin^2(2x) + \cos(4x) = 1$

2) $\dfrac{\cot^2 x}{\csc^2 x} + \sin^2 x = 1$

3) $\dfrac{1}{1-\sin x} + \dfrac{1}{1+\sin x} = 2\sec^2 x$

1) $2\sin^2(2x) + \cos(4x) = 1$

L.H.S= $2\sin^2(2x) + \cos(4x) =$

$$= 2\sin^2(2x) + \cos(2(2x))$$

$$= 2\sin^2(2x) + 1 - 2\sin^2(2x) = 1 = R.H.S$$

2) $\dfrac{\cot^2 x}{\csc^2 x} + \sin^2 x = 1$

L.H.S= $\dfrac{\cot^2 x}{\csc^2 x} + \sin^2 x = \dfrac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}} + \sin^2 x$

$$= \frac{\cos^2 x}{\cancel{\sin^2 x}} * \frac{\cancel{\sin^2 x}}{1} + \sin^2 x = \cos^2 x + \sin^2 x = 1 = R.H.S$$

3) $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x} = 2\sec^2 x$

L.H.S= $\dfrac{1}{1-\sin x}+\dfrac{1}{1+\sin x} = \dfrac{1+\sin x+1-\sin x}{(1-\sin x)(1+\sin x)}+$

$= \dfrac{2}{1-\sin^2 x} = \dfrac{2}{\cos^2 x} = 2\dfrac{1}{\cos^2 x} = 2\sec^2 x = R.H.S$