EXERCISES

EXERCISES

1. COMBINATIONS

$\binom{n}{r}$ =The number of combinations of n distinct objects taken r at a time (r objects in each combination)

= The number of different selections of r objects from n distinct objects.

= The number of different ways to select r objects from n distinct objects.

= The number of different ways to divide a set of n distinct objects into 2 subsets; one subset contains r objects and the other subset contains the rest.

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

$$0! = 1$$

Q1. Compute:

(a) $\binom{6}{2}$ (b) $\binom{6}{4}$.

Q2. Compute:

(a) $\binom{n}{0}$, (b) $\binom{n}{1}$, (c) $\binom{n}{n}$

EXERCISES

DISCRETE UNIFORM DISTRIBUTION:

Q1. Let the random variable X have a discrete uniform with parameter k=3 and with values 0,1, and 2. Then:

(a) P(X=1) is

(A) 1.0	(B) 1/3	(C) 0.3	(D) 0.1	(E) None

(b) The mean of X is:

(A) 1.0	(B) 2.0	(C) 1.5	(D) 0.0	(E) None

(c) The variance of X is:

(A) 0/3=0.0	(B) 3/3=1.0	(C) 2/3=0.67	(D) 4/3=1.33	(E) None

EXERCISES

BINOMIAL DISTRIBUTION:

Q1. Suppose that 4 out of 12 buildings in a certain city violate the building code. A building engineer randomly inspects a sample of 3 new buildings in the city.

(a) Find the probability distribution function of the random variable X representing the number of buildings that violate the building code in the sample.

(b) Find the probability that:

(i) none of the buildings in the sample violating the building code.

(ii) one building in the sample violating the building code.

(iii) at lease one building in the sample violating the building code.

(c) Find the expected number of buildings in the sample that violate the building code (E(X)).

(d) Find σ^2=Var(X).

Q2. A missile detection system has a probability of 0.90 of detecting a missile attack. If 4 detection systems are installed in the same area and operate independently, then

(a) The probability that at least two systems detect an attack is

(A) 0.9963	(B) 0.9477	(C) 0.0037	(D) 0.0523	(E) 0.5477

(b) The average (mean) number of systems detect an attack is

(A) 3.6	(B) 2.0	(C) 0.36	(D) 2.5	(E) 4.0

Q3. Suppose that the probability that a person dies when he or she contracts a certain disease is 0.4. A sample of 10 persons who contracted this disease is randomly chosen.

(1) What is the expected number of persons who will die in this sample?

(2) What is the variance of the number of persons who will die in this sample?

(3) What is the probability that exactly 4 persons will die among this sample?

(4) What is the probability that less than 3 persons will die among this sample?

(5) What is the probability that more than 8 persons will die among this sample?

Q4. Suppose that the percentage of females in a certain population is 50%. A sample of 3 people is selected randomly from this population.

(a) The probability that no females are selected is

(A) 0.000 (B) 0.500 (C) 0.375 (D) 0.125

(b) The probability that at most two females are selected is

(A) 0.000 (B) 0.500 (C) 0.875 (D) 0.125

(c) The expected number of females in the sample is

(A) 3.0 (B) 1.5 (C) 0.0 (D) 0.50

(d) The variance of the number of females in the sample is

(A) 3.75 (B) 2.75 (C) 1.75 (D) 0.75

Q5. 20% of the trainees in a certain program fail to complete the program. If 5 trainees of this program are selected randomly,

(i) Find the probability distribution function of the random variable X, where:

X = number of the trainees who fail to complete the program.

(ii) Find the probability that all trainees fail to complete the program.

(iii) Find the probability that at least one trainee will fail to complete the program.

(iv) How many trainees are expected to fail completing the program?

(v) Find the variance of the number of trainees who fail completing the program.

EXERCISES

Q6. In a certain industrial factory, there are 7 workers working independently. The probability of accruing accidents for any worker on a given day is 0.2, and accidents are independent from worker to worker.

(a) The probability that at most two workers will have accidents during the day is

(A) 0.7865 (B) 0.4233 (C) 0.5767 (D) 0.6647

(b) The probability that at least three workers will have accidents during the day is:

(A) 0.7865 (B) 0.2135 (C) 0.5767 (D) 0.1039

(c) The expected number workers who will have accidents during the day is

(A) 1.4 (B) 0.2135 (C) 2.57 (D) 0. 59

Q7. From a box containing 4 black balls and 2 green balls, 3 balls are drawn independently in succession, each ball being replaced in the box before the next draw is made. The probability of drawing 2 green balls and 1 black ball is:

(A) 6/27 (B) 2/27 (C) 12/27 (D) 4/27

Q8. The probability that a lab specimen is contaminated is 0.10. Three independent samples are checked.

1) the probability that none is contaminated is:

(A) 0.0475 (B) 0.001 (C) 0.729 (D) 0. 3

2) the probability that exactly one sample is contaminated is:

(A) 0.243 (B) 0.081 (C) 0.757 (D) 0. 3

Q9. If X~Binomial(n,p), E(X)=1, and Var(X)=0.75, find P(X=1).

Q10. Suppose that X~Binomial(3,0.2). Find the cumulative distribution function (CDF) of X.

Q11. A traffic control engineer reports that 75% of the cars passing through a checkpoint are from Riyadh city. If at this checkpoint, five cars are selected at random.

(1) The probability that none of them is from Riyadh city equals to:

(A) 0.00098 (B) 0.9990 (C) 0.2373 (D) 0.7627

(2) The probability that four of them are from Riyadh city equals to:

(A) 0.3955 (B) 0.6045 (C) 0 (D) 0.1249

(3) The probability that at least four of them are from Riyadh city equals to:

(A) 0.3627 (B) 0.6328 (C) 0.3955 (D) 0.2763

(4) The expected number of cars that are from Riyadh city equals to:

(A) 1 (B) 3.75 (C) 3 (D) 0

EXERCISES

HYPERGEOMETRIC DISTRIBUTION:

Q1. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets.

(i) Find the probability distribution function of the random variable X representing the number of defective sets purchased by the hotel.

(ii) Find the probability that the hotel purchased no defective television sets.

(iii) What is the expected number of defective television sets purchased by the hotel?

(iv) Find the variance of X.

Q2. Suppose that a family has 5 children, 3 of them are girls and the rest are boys. A sample of 2 children is selected randomly and without replacement.

(a) The probability that no girls are selected is

(A) 0.0 (B) 0.3 (C) 0.6 (D) 0.1

(b) The probability that at most one girls are selected is

(A) 0.7 (B) 0.3 (C) 0.6 (D) 0.1

(c) The expected number of girls in the sample is

(A) 2.2 (B) 1.2 (C) 0.2 (D) 3.2

(d) The variance of the number of girls in the sample is

(A) 36.0 (B) 3.6 (C) 0.36 (D) 0.63

Q3. A random committee of size 4 is selected from 2 chemical engineers and 8 industrial engineers.

(i) Write a formula for the probability distribution function of the random variable X representing the number of chemical engineers in the committee.

(ii) Find the probability that there will be no chemical engineers in the committee.

(iii) Find the probability that there will be at least one chemical engineer in the committee.

(iv) What is the expected number of chemical engineers in the committee?

(v) What is the variance of the number of chemical engineers in the committee?

Q4. A box contains 2 red balls and 4 black balls. Suppose that a sample of 3 balls were selected randomly and without replacement. Find,

1. The probability that there will be 2 red balls in the sample.
2. The probability that there will be 3 red balls in the sample.
3. The expected number of the red balls in the sample.

Q5. From a lot of 8 missiles, 3 are selected at random and fired. The lot contains 2 defective missiles that will not fire. Let X be a random variable giving the number of defective missiles selected.

1. Find the probability distribution function of X.
2. What is the probability that at most one missile will not fire?
3. Find $\mu = E(X)$ and $\sigma^2 = Var(X)$.

Q6. A particular industrial product is shipped in lots of 20 items. Testing to determine whether an item is defective is costly; hence, the manufacturer samples production rather than using 100% inspection plan. A sampling plan constructed to minimize the number of defectives shipped to consumers calls for sampling 5 items from each lot and rejecting the lot if more than one defective is observed. (If the lot is rejected, each item in the lot is then tested.) If a lot contains 4 defectives, what is the probability that it will be accepted.

EXERCISES

Q7. Suppose that X~h(x;100,2,60); i.e., X has a hypergeometric distribution with parameters N=100, n=2, and K=60. Calculate the probabilities P(X=0), P(X=1), and P(X=2) as follows:

(a) exact probabilities using hypergeometric distribution.

(b) approximated probabilities using binomial distribution.

Q8. A particular industrial product is shipped in lots of 1000 items. Testing to determine whether an item is defective is costly; hence, the manufacturer samples production rather than using 100% inspection plan. A sampling plan constructed to minimize the number of defectives shipped to consumers calls for sampling 5 items from each lot and rejecting the lot if more than one defective is observed. (If the lot is rejected, each item in the lot is then tested.) If a lot contains 100 defectives, calculate the probability that the lot will be accepted using:

(a) hypergeometric distribution (exact probability.)

(b) binomial distribution (approximated probability.)

Q9. A shipment of 20 digital voice recorders contains 5 that are defective. If 10 of them are randomly chosen (without replacement) for inspection, then:

(1) The probability that 2 will be defective is:

(A) 0.2140 (B) 0.9314 (C) 0.6517 (D) 0.3483

(2) The probability that at most 1 will be defective is:

(A) 0.9998 (B) 0.2614 (C) 0.8483 (D) 0.1517

(3) The expected number of defective recorders in the sample is:

(A) 1 (B) 2 (C) 3.5 (D) 2.5

(4) The variance of the number of defective recorders in the sample is:

(A) 0.9868 (B) 2.5 (C) 0.1875 (D) 1.875

EXERCISES

Q10. A box contains 4 red balls and 6 green balls. The experiment is to select 3 balls at random. Find the probability that all balls are red for the following cases:

(1) If selection is without replacement

(A) 0.216 (B) 0.1667 (C) 0.6671 (D) 0.0333

(2) If selection is with replacement

(A) 0.4600 (B) 0.2000 (C) 0.4000 (D) 0.0640

EXERCISES

POISSON DISTRIBUTION:

Q1. On average, a certain intersection results in 3 traffic accidents per day. Assuming Poisson distribution,

(i) what is the probability that at this intersection:

(1) no accidents will occur in a given day?

(2) More than 3 accidents will occur in a given day?

(3) Exactly 5 accidents will occur in a period of two days?

(ii) what is the average number of traffic accidents in a period of 4 days?

Q2. At a checkout counter, customers arrive at an average of 1.5 per minute. Assuming Poisson distribution, then

(1) The probability of no arrival in two minutes is

(A) 0.0 (B) 0.2231 (C) 0.4463 (D) 0.0498 (E) 0.2498

(2) The variance of the number of arrivals in two minutes is

(A) 1.5 (B) 2.25 (C) 3.0 (D) 9.0 (E) 4.5

Q3. Suppose that the number of telephone calls received per day has a Poisson distribution with mean of 4 calls per day.

(a) The probability that 2 calls will be received in a given day is

(A) 0.546525 (B) 0.646525 (C) 0.146525 (D) 0.746525

(b) The expected number of telephone calls received in a given week is

(A) 4 (B) 7 (C) 28 (D) 14

(c) The probability that at least 2 calls will be received in a period of 12 hours is

(A) 0.59399 (B) 0.19399 (C) 0.09399 (D) 0.29399

Q4. The average number of car accidents at a specific traffic signal is 2 per a week. Assuming Poisson distribution, find the probability that:

(i) there will be no accident in a given week.

(ii) there will be at least two accidents in a period of two weeks.

Q5. The average number of airplane accidents at an airport is two per a year. Assuming Poisson distribution, find

1. the probability that there will be no accident in a year.
2. the average number of airplane accidents at this airport in a period of two years.
3. the probability that there will be at least two accidents in a period of 18 months.

Q6. Suppose that X~Binomial(1000,0.002). By using Poisson approximation, P(X=3) is approximately equal to (choose the nearest number to your answer):

(A) 0.62511 (B) 0.72511 (C) 0.82511 (D) 0.92511 (E) 0.18045

Q7. The probability that a person dies when he or she contracts a certain disease is 0.005. A sample of 1000 persons who contracted this disease is randomly chosen.

(1) What is the expected number of persons who will die in this sample?

(2) What is the probability that exactly 4 persons will die among this sample?

Q8. The number of faults in a fiber optic cable follows a Poisson distribution with an average of 0.6 per 100 feet.

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(1) The probability of 2 faults per 100 feet of such cable is:

(A) 0.0988 (B) 0.9012 (C) 0.3210 (D) 0.5

(2) The probability of less than 2 faults per 100 feet of such cable is:

(A) 0.2351 (B) 0.9769 (C) 0.8781 (D) 0.8601

(3) The probability of 4 faults per 200 feet of such cable is:

(A) 0.02602 (B) 0.1976 (C) 0.8024 (D) 0.9739

EXERCISES

NORMAL DISTRIBUTION:

Q1. (A) Suppose that Z is distributed according to the standard normal distribution.

1) the area under the curve to the left of $z = 1.43$ is:

(A) 0.0764 (B) 0.9236 (C) 0 (D) 0.8133

2) the area under the curve to the left of z = 1.39 is:

(A) 0.7268 (B) 0.9177 (C) .2732 (D) 0.0832

3) the area under the curve to the right of $z = -0.89$ is:

(A) 0. 7815 (B) 0.8133 (C) 0.1867 (D) 0.0154

4) the area under the curve between $z = -2.16$ and $z = -0.65$ is:

(A) 0.7576 (B) 0.8665 (C) 0.0154 (D) 0.2424

5) the value of k such that $P(0.93 < Z < k) = 0.0427$ is:

(A) 0.8665 (B) −1.11 (C) 1.11 (D) 1.00

(B) Suppose that Z is distributed according to the standard normal distribution. Find:

1) P(Z< −3.9)

2) P(Z> 4.5)

1) P(Z< 3.7)

2) P(Z> −4.1)

Q2. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Then,

1) the proportion of rings that will have inside diameter less than 12.05 centimeters is:

(A) 0.0475 (B) 0.9525 (C) 0.7257 (D) 0.8413

2) the proportion of rings that will have inside diameter exceeding 11.97 centimeters is:

(A) 0.0475 (B) 0.8413 (C) 0.1587 (D) 0.4514

3) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:

(A) 0.905 (B) –0.905 (C) 0.4514 (D) 0.7257

Q3. The average life of a certain type of small motor is 10 years with a standard deviation of 2 years. Assume the live of the motor is normally distributed. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 1.5% of the motors that fail, then he should give a guarantee of :

(A) 10.03 years (B) 8 years (C) 5.66 years (D) 3 years

Q4. A machine makes bolts (that are used in the construction of an electric transformer). It produces bolts with diameters (X) following a normal distribution with a mean of 0.060 inches and a standard deviation of 0.001 inches. Any bolt with diameter less than 0.058 inches or greater than 0.062 inches must be scrapped. Then

(1) The proportion of bolts that must be scrapped is equal to

(A) 0.0456 (B) 0.0228 (C) 0.9772 (D) 0.3333 (E) 0.1667

(2) If $P(X>a)= 0.1949$, then a equals to:

(A) 0.0629 (B) 0.0659 (C) 0.0649 (D) 0.0669 (E) 0.0609

EXERCISES

Q5. The diameters of ball bearings manufactured by an industrial process are normally distributed with a mean μ = 3.0 cm and a standard deviation σ = 0.005 cm. All ball bearings with diameters not within the specifications $\mu \pm d$ cm ($d > 0$) will be scrapped.

(1) Determine the value of d such that 90% of ball bearings manufactured by this process will not be scrapped.

(2) If d = 0.005, what is the percentage of manufactured ball bearings that will be scraped?

Q6. The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

(1) The percentage of fat persons with weights at most 110 kg is

(A) 0.09 % (B) 90.3 % (C) 99.82 % (D) 2.28 %

(2) The percentage of fat persons with weights more than 149 kg is

(A) 0.09 % (B) 0.99 % (C) 9.7 % (D) 99.82 %

(3) The weight x above which 86% of those persons will be

(A) 118.28 (B) 128.28 (C) 154.82 (D) 81.28

(4) The weight x below which 50% of those persons will be

(A) 101.18 (B) 128 (C) 154.82 (D) 81

Q7. The random variable X, representing the lifespan of a certain electronic device, is normally distributed with a mean of 40 months and a standard deviation of 2 months. Find

1. $P(X<38)$. (0.1587)
2. $P(38<X<40)$. (0.3413)
3. $P(X=38)$. (0.0000)
4. The value of x such that $P(X<x)=0.7324$. (41.24)

EXERCISES

Q8. If the random variable X has a normal distribution with the mean μ and the variance σ^2, then $P(X<\mu+2\sigma)$ equals to

(A) 0.8772 (B) 0.4772 (C) 0.5772 (D) 0.7772 (E) 0.9772

Q9. If the random variable X has a normal distribution with the mean μ and the variance 1, and if $P(X<3)=0.877$, then μ equals to

(A) 3.84 (B) 2.84 (C) 1.84 (D) 4.84 (E) 8.84

Q10. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 70 and the variance 25. If it is known that 33% of the student failed the exam, then the passing mark x is

(A) 67.8 (B) 60.8 (C) 57.8 (D) 50.8 (E) 70.8

Q11. If the random variable X has a normal distribution with the mean 10 and the variance 36, then

1. The value of X above which an area of 0.2296 lie is

   (A) 14.44 (B) 16.44 (C) 10.44 (D) 18.44 (E) 11.44

2. The probability that the value of X is greater than 16 is

   (A) 0.9587 (B) 0.1587 (C) 0.7587 (D) 0.0587 (E) 0.5587

Q12. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 65 and the variance 16. A student fails the exam if he obtains a mark less than 60. Then the percentage of students who fail the exam is

(A) 20.56% (B) 90.56% (C) 50.56% (D) 10.56% (E)40.56%

EXERCISES

Q13. The average rainfall in a certain city for the month of March is 9.22 centimeters. Assuming a normal distribution with a standard deviation of 2.83 centimeters, then the probability that next March, this city will receive:

(1) less than 11.84 centimeters of rain is:

(A) 0.8238 (B) 0.1762 (C) 0.5 (D) 0.2018

(2) more than 5 centimeters but less than 7 centimeters of rain is:

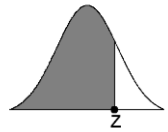
(A) 0.8504 (B) 0.1496 (C) 0.6502 (D) 0.34221

(3) more than 13.8 centimeters of rain is:

(A) 0.0526 (B) 0.9474 (C) 0.3101 (D) 0.4053

EXERCISES

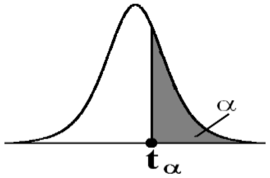
Areas under the Standard Normal Curve



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

EXERCISES

Percentage Points of the t Distribution; $t_{\nu,\alpha}$ $\{P(T > t_{\nu,\alpha}) = \alpha\}$



	α													
ν	0.40	0.30	0.20	0.15	0.10	0.05	0.025	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960	2.054	2.170	2.326	2.432	2.576	2.807	3.291

EXERCISES

Summary of Confidence Interval Procedures

Problem Type	Point Estimate	Two-Sided 100(1-α)% Confidence Interval
Mean μ variance σ^2 known, normal distribution, or any distribution with n>30	$\overline{X}$	$\overline{X} - Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$ or $\overline{X} \pm Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
Mean μ normal distribution, variance σ^2 unknown	$\overline{X}$	$\overline{X} - t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}} < \mu < \overline{X} + t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$ or $\overline{X} \pm t_{\frac{\alpha}{2}}\frac{S}{\sqrt{n}}$ (df: $\nu = n-1$)
Difference in two means μ_1 and μ_2 variances σ_1^2 and σ_2^2 are known, normal distributions, or any distributions with n_1, n_2 >30	$\overline{X}_1 - \overline{X}_2$	$(\overline{X}_1 - \overline{X}_2) - Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\overline{X}_1 - \overline{X}_2) + Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ or $(\overline{X}_1 - \overline{X}_2) \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
Difference in means μ_1 and μ_2 normal distributions, variances $\sigma_1^2 = \sigma_2^2$ and unknown	$\overline{X}_1 - \overline{X}_2$	$(\overline{X}_1 - \overline{X}_2) - t_{\frac{\alpha}{2}} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\overline{X}_1 - \overline{X}_2) + t_{\frac{\alpha}{2}} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ or $(\overline{X}_1 - \overline{X}_2) \pm t_{\frac{\alpha}{2}} S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$; (df: $\nu = n_1+n_2-2$)
Proportion p (or parameter of a binomial distribution)	$\hat{p}$	$\hat{p} - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ or $\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}$; $\hat{q} = 1 - \hat{p}$
Difference in two proportions $p_1 - p_2$ (or difference in two binomial parameters)	$\hat{p}_1 - \hat{p}_2$	$(\hat{p}_1 - \hat{p}_2) - Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$ or $(\hat{p}_1 - \hat{p}_2) \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

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EXERCISES

Summary of Hypotheses Testing Procedures

Null Hypothesis	Test Statistic	Alternative Hypothesis	Critical Region (Rejection Region)
H_o: $\mu = \mu_o$ variance σ^2 is known, Normal distribution, or any distribution with n>30	$Z = \dfrac{\overline{X} - \mu_o}{\sigma / \sqrt{n}}$	H_1: $\mu \neq \mu_o$	$\lvert Z\rvert > Z_{\alpha/2}$
		H_1: $\mu > \mu_o$	$Z > Z_{\alpha}$
		H_1: $\mu < \mu_o$	$Z < -Z_{\alpha}$
H_o: $\mu = \mu_o$ Normal distribution, variance σ^2 is unknown	$T = \dfrac{\overline{X} - \mu_o}{S / \sqrt{n}}$; df: $\nu = n-1$	H_1: $\mu \neq \mu_o$	$\lvert T\rvert > t_{\alpha/2}$
		H_1: $\mu > \mu_o$	$T > t_{\alpha}$
		H_1: $\mu < \mu_o$	$T < -t_{\alpha}$
H_o: $\mu_1 = \mu_2$ Variances σ_1^2 and σ_2^2 are known, Normal distributions, or any distributions with n_1, n_2 >30	$Z = \dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	H_1: $\mu_1 \neq \mu_2$	$\lvert Z\rvert > Z_{\alpha/2}$
		H_1: $\mu_1 > \mu_2$	$Z > Z_{\alpha}$
		H_1: $\mu_1 < \mu_2$	$Z < -Z_{\alpha}$
H_o: $\mu_1 = \mu_2$ Normal distributions, variances $\sigma_1^2 = \sigma_2^2$ and unknown	$T = \dfrac{\overline{X}_1 - \overline{X}_2}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$; df: $\nu = n_1 + n_2 - 2$ $S_p^2 = [(n_1-1)S_1^2 + (n_2-1)S_2^2]/(n_1+n_2-2)$	H_1: $\mu_1 \neq \mu_2$	$\lvert T\rvert > t_{\alpha/2}$
		H_1: $\mu_1 > \mu_2$	$T > t_{\alpha}$
		H_1: $\mu_1 < \mu_2$	$T < -t_{\alpha}$
H_o: $p = p_o$ Proportion or parameter of a binomial distribution p ($q = 1 - p$)	$Z = \dfrac{\hat{p} - p_o}{\sqrt{\dfrac{p_o q_o}{n}}} = \dfrac{X - np_o}{\sqrt{np_o q_o}}$	H_1: $p \neq p_o$	$\lvert Z\rvert > Z_{\alpha/2}$
		H_1: $p > p_o$	$Z > Z_{\alpha}$
		H_1: $p < p_o$	$Z < -Z_{\alpha}$
H_o: $p_1 = p_2$ Difference in two proportions or two binomial parameters $p_1 - p_2$	$Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$ $\hat{p} = \dfrac{X_1 + X_2}{n_1 + n_2} = \dfrac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$	H_1: $p_1 \neq p_2$	$\lvert Z\rvert > Z_{\alpha/2}$
		H_1: $p_1 > p_2$	$Z > Z_{\alpha}$
		H_1: $p_1 < p_2$	$Z < -Z_{\alpha}$