King Saud University
Department of Mathematics

Final Exam in Math 151
1st Semester, 1441 H.
(Duration: 3 Hours)

Calculators are not allowed

Q1. (a) Without using truth tables, show that $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$. (2 pts)

Sol

L.H.S $\equiv p \longleftrightarrow q$

$\equiv (p \to q) \wedge (q \to p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ (1)

R.H.S $\equiv \neg p \longleftrightarrow \neg q$

$\equiv (\neg p \to \neg q) \wedge (\neg q \to \neg p)$

$\equiv (p \vee \neg q) \wedge (q \vee \neg p)$

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ (2)

from (1), (2) $\equiv$

(b) Show that $3+\frac{3}{4}+\frac{3}{4^2}+\cdots+\frac{3}{4^n}=\frac{4^{n+1}-1}{4^n}$ for all integers $n \geq 0$. (4 pts)

$$P(n): 3+\frac{3}{4}+\frac{3}{4^2}+\cdots+\frac{3}{4^n}=\frac{4^{n+1}-1}{4^n}$$

$n \geq 0$

B.S prove $P(n=0): 3=\frac{4^{0+1}-1}{4^0}=\frac{3}{1}=3$

I.S Hyp: $P(n=k): \boxed{3+\frac{3}{4}+\frac{3}{4^2}+\cdots+\frac{3}{4^k}}=\boxed{\frac{4^{k+1}-1}{4^k}}$

Prove: $P(n=k+1): 3+\frac{3}{4}+\frac{3}{4^2}+\cdots+\frac{3}{4^{k+1}} \stackrel{?}{=} \frac{4^{k+2}-1}{4^{k+1}}$

Proof

$$L.H.S = \boxed{3+\frac{3}{4}+\frac{3}{4^2}+\cdots+\frac{3}{4^k}}+\frac{3}{4^{k+1}} \quad \text{from Hyp}$$

$$=\frac{\cdot 4}{\cdot 4}\,\frac{4^{k+1}-1}{4^k}+\frac{3}{4^{k+1}}$$

$$=\frac{4^{k+2}-4}{4^{k+1}}+\frac{3}{4^{k+1}}=\frac{4^{k+2}-4+3}{4^{k+1}}$$

$$=\frac{4^{k+2}-1}{4^{k+1}}=R.H.S$$

$x^a \cdot x^b = x^{a+b}$

(c) Let $R = \{(x,x), (y,x), (y,y), (y,z), (z,y)\}$ be a relation on $A = \{x, y, z\}$. Determine whether R is reflexive, symmetric, antisymmetric, transitive. (4 pts)

1) Ref R is Not Ref

since $(z,z) \notin R$

2) Sym R is Not Sym

since $(y,x) \in R$

but $(x,y) \notin R$

3) Tra R is Tra

since $R^2 = R$

$R^2 = R \circ R$

4) Anti R is Not Anti

since $(y,z) \in R$

" $(z,y) \in R$

Q2. (a) Find the CSP and CPS forms of $f(x,y,z) = \overline{x + \bar{x}\bar{y}z}$. (2+2 pts

Sol

نقطة

$$f = \bar{x} \cdot (x + y + \bar{z})$$

$$f = \cancel{\bar{x}x} + \bar{x}y + \bar{x}\bar{z}$$

$$f = \bar{x}y(z + \bar{z}) + \bar{x}(y + \bar{y})\bar{z}$$

$$f = \bar{x}yz + \bar{x}y\bar{z} + \cancel{\bar{x}y\bar{z}} + \bar{x}\bar{y}\bar{z}$$

$$CSP(f) = \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$$

CPS

$\bar{f} = x\,\bar{y}\,\bar{z} + \bar{x}\,\bar{y}\,z$

$\bar{f} = x(y+\bar{y})(z+\bar{z}) + \bar{x}\,\bar{y}\,z$

$= (xy + x\bar{y})(z+\bar{z}) + \bar{x}\,\bar{y}\,z$

$csp(\bar{f}) = xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}z$

$Cps(f) = \left(\quad csp(\bar{f}) \quad \right)'$

$Cps(f) = (\bar{x}+\bar{y}+\bar{z})\cdot(\bar{x}+\bar{y}+z)\cdot(\bar{x}+y+\bar{z})\cdot(\bar{x}+y+z)$

$\cdot\,(x+y+\bar{z})$

(b) Let g be the Boolean function represented by the K-map below.

(i) Write g in MSP form. (2 pts)

(ii) Write g in MPS form. (2 pts)

(iii) Construct a minimal *"AND-OR"* circuit for g. (1 pt)

(iv) Construct a circuit for g using ***NAND*** gates only. (1 pt)

(v) Construct a circuit for g using ***NOR*** gates only. (1 pt)

8
4
2
1

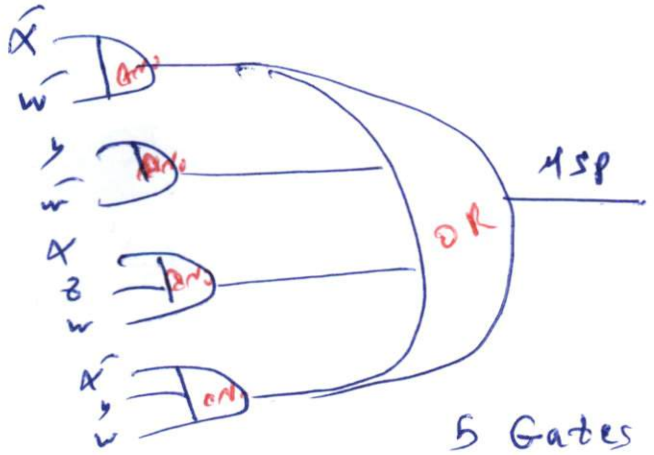
	zw	$z\bar{w}$	$\bar{z}\bar{w}$	$\bar{z}w$
xy	1	1	1	0
$x\bar{y}$	1	0	0	1
$\bar{x}\bar{y}$	0	1	1	0
$\bar{x}y$	0	1	1	0

$MSP = \bar{x}\bar{w} + y\bar{w} + xzw + x\bar{y}w$ 2 2 4 4 $\cancel{8}$

$\bar{f} = \bar{x}w + x\bar{y}\bar{w} + y\bar{z}w$ 2 2 4 $\cancel{8}$

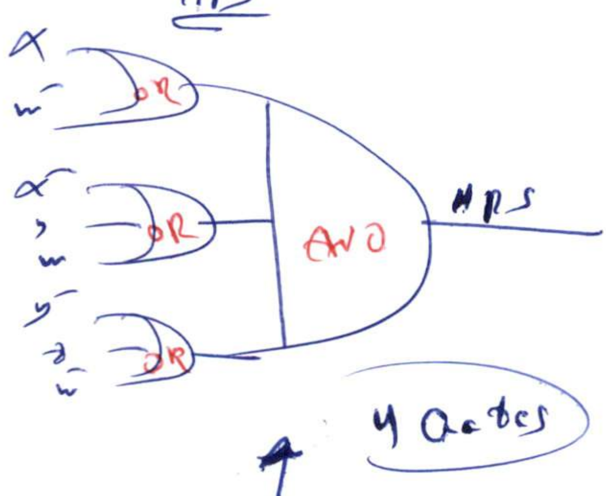
$$MPS = (\bar{f})' = (x+\bar{w})\cdot(\bar{x}+y+w)\cdot(\bar{y}+z+\bar{w})$$

MSP



5 Gates

MPS



4 Gates

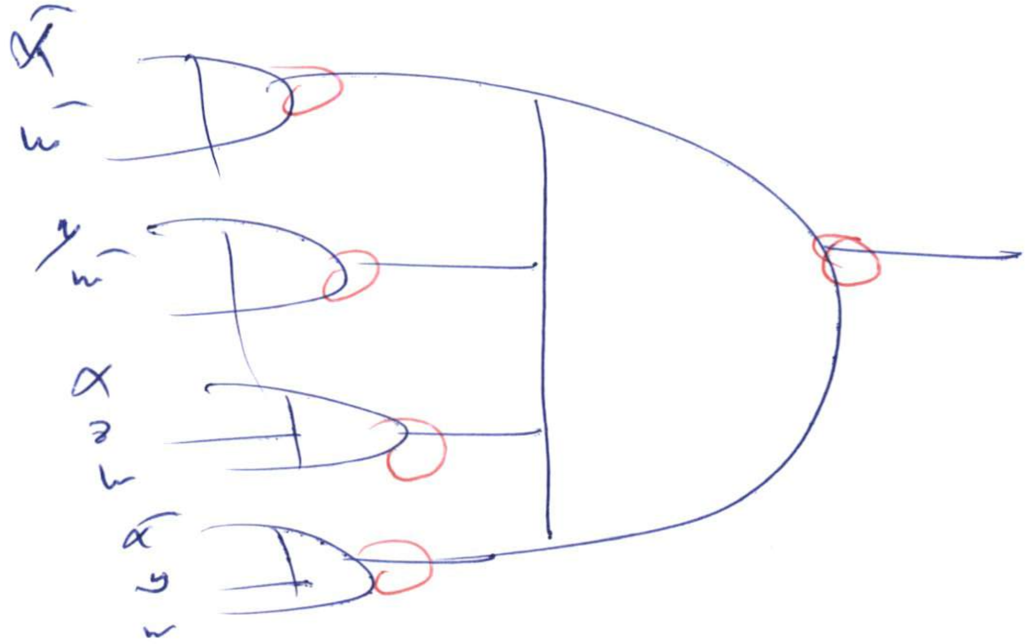
AND & OR ، فقط

minimal

N. AND

$$MSP = \left[\overline{\overline{\left[\bar{x}\bar{w} + y\bar{w} + xzw + \bar{x}yw\right]}}\right]$$

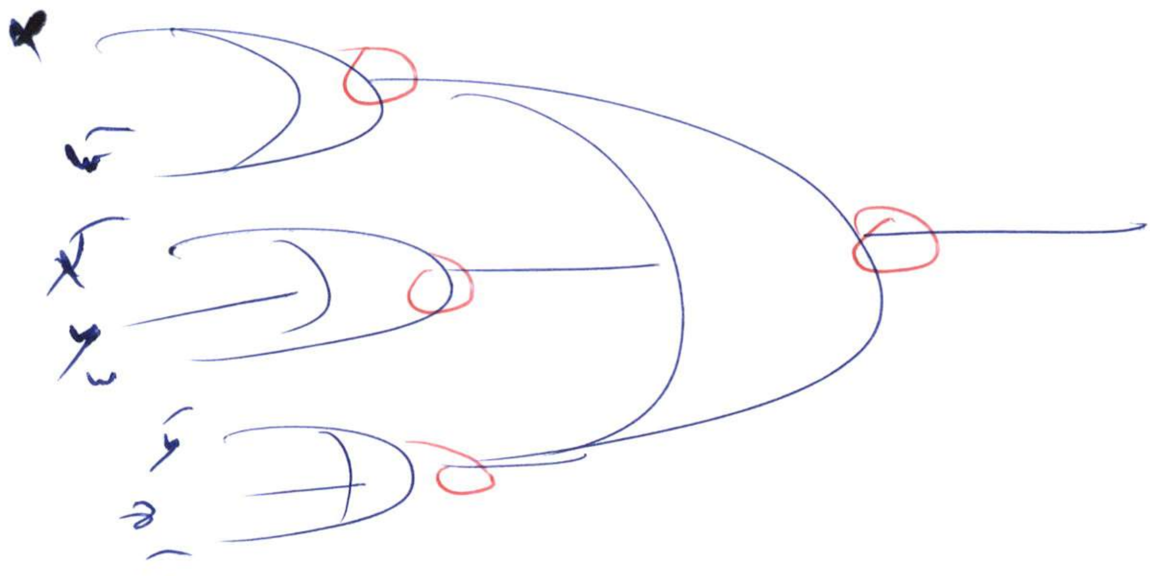
$$= \left[\overline{(\overline{\bar{x}\bar{w}})\cdot(\overline{y\bar{w}})\cdot(\overline{xzw})\cdot(\overline{\bar{x}yw})}\right]$$



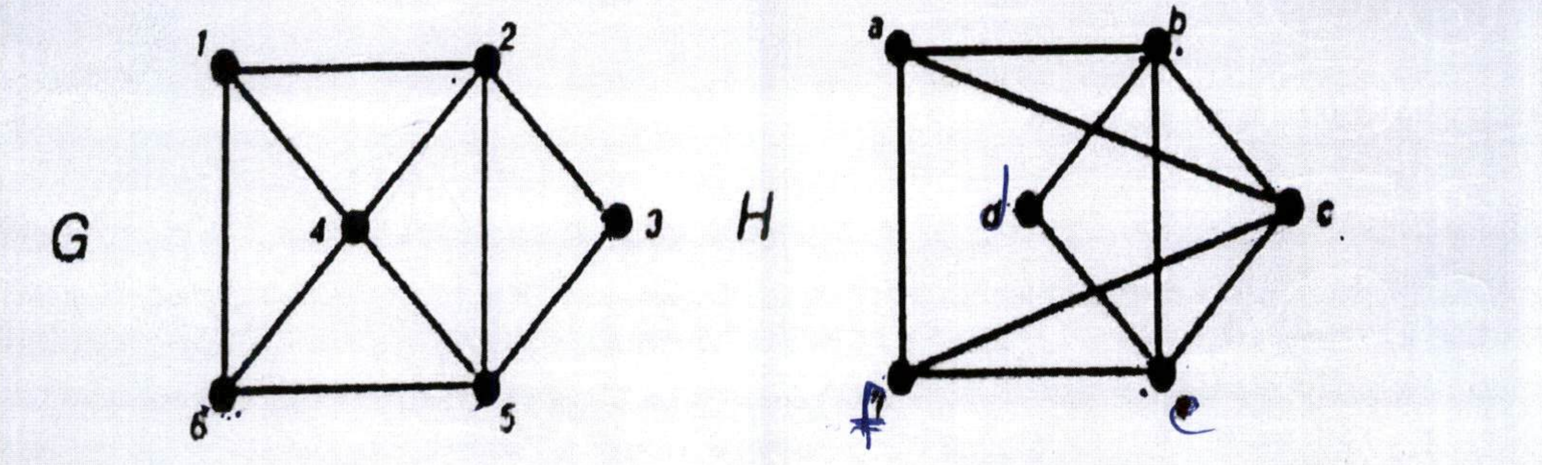
N OR

$$MPS = \overline{\left[\overline{\left\{ (x+\bar{w})\cdot(\bar{x}+y+w)\cdot(\bar{y}+z+\bar{w}) \right\}} \right]}$$

$$= \overline{\left[\overline{(x+\bar{w})} + \overline{(\bar{x}+y+w)} + \overline{(\bar{y}+z+\bar{w})} \right]}$$



Q3. (a) Determine whether the following graphs G and H are isomorphic. (2 pts)



V ✓

E ✓

D ✓

cycl ✓

$G \cong H$

G	1	2	3	5	6	4
H	a	b	d	e	f	c

(b) Let J be the graph represented by the following adjacency matrix.

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 1 \\ b & 1 & 0 & 0 & 0 \\ c & 1 & 0 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

(i) Determine whether J is bipartite. (1 pt)

(ii) Determine whether J is a tree. (1 pt)

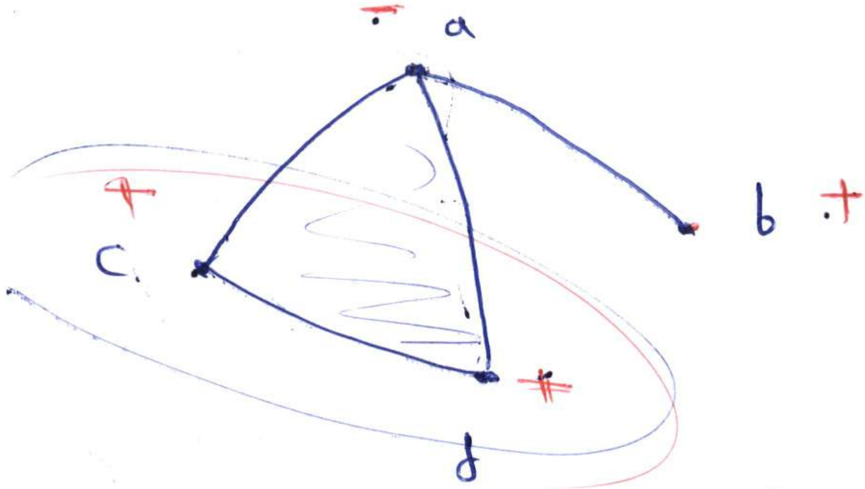
Sol

نحول المصفوفة لرسم

رؤوس

adj [ ]

الأضلاع



(i) J is not Bipartite Graph since $\boxed{J \text{ has odd cycle}}$

(ii) J is Not Tree since has a cycle

(c) Is there a tree with v vertices and e edges such that $3v = 5e$? (Justify your answer) (2 pts)

$\boxed{3V = 5e}$

$|E| = e$

$|V| = V$

$|E| = |V| - 1$

$e = V - 1$

$3V = 5(V-1)$

$3V = 5V - 5$

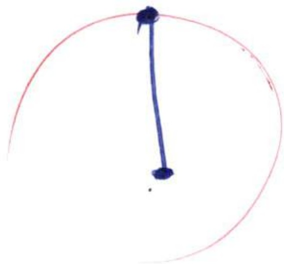
$5 = 5V - 3V$

$5 = 2V \longrightarrow V = \frac{5}{2} = 2.5$ غلط

No

(d) Give an example of a graph K which is complete, complete bipartite and a tree. (1 pt)

مثال



Complete Graph

Complete Bipartite

Tree

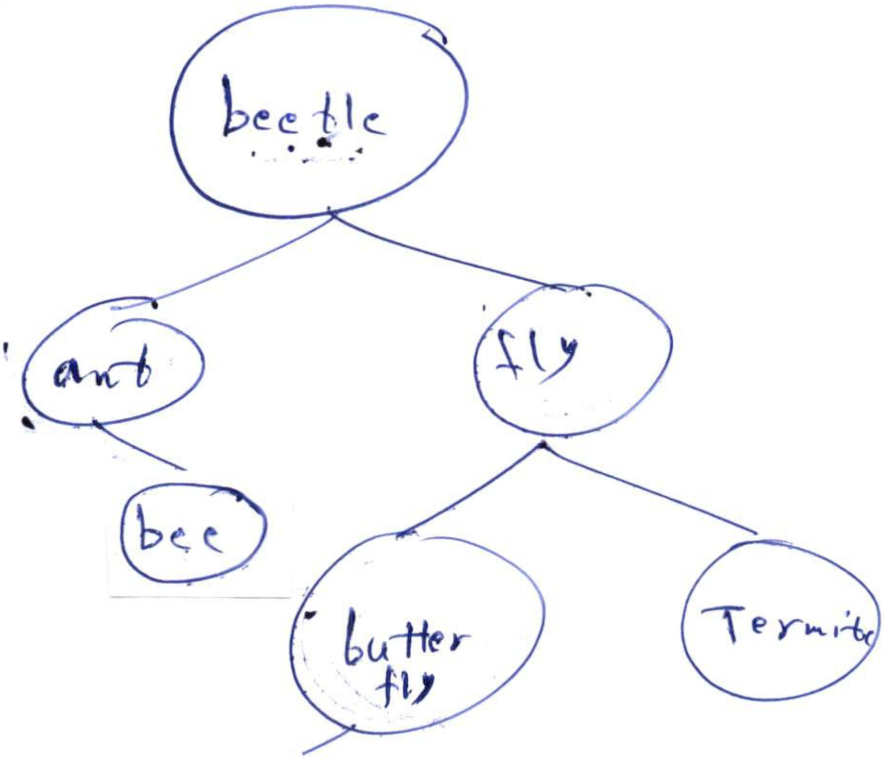
Q4. (a) Form a binary search tree for the words: *beetle, fly, ant, butterfly, bee, termite* (using alphabetical order). (2 pts)

beetle fly ant

butterfly bee termite

abcd

sol

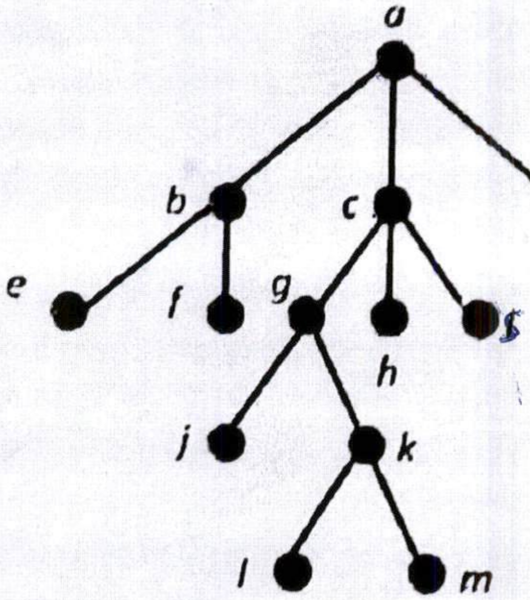


(b) Let T be the ordered rooted tree below.

(i) Find the *inorder* traversal of T. (2 pts)

(ii) Find the *postorder* traversal of T. (2 pts)

T



a → root
b → Left
c → Right (1)
d → Right (2)

(i) in L - Ro - R

| e | b | f | a | j | g | l | k | m | c | h | s | d |

(ii) post L - R - Ro

| e | f | b | j | l | m | k | g | h | s | c | d | a |

(c) Let E be the arithmetic expression $((4+y)*x)/((y-3)\uparrow 4)$

(i) Represent E by an ordered rooted tree. **(2 pts)**

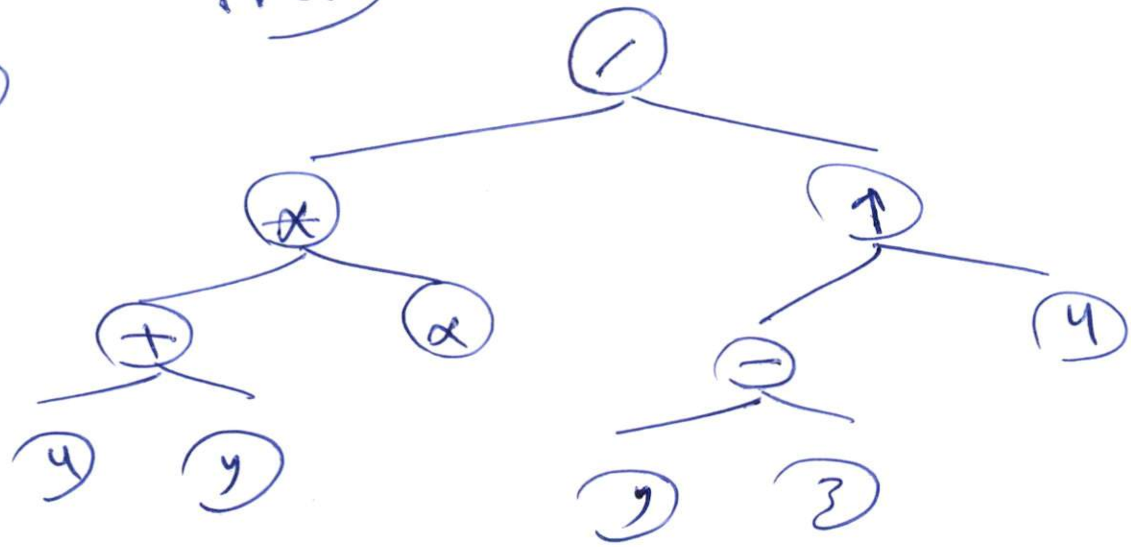
(ii) Write E in *prefix* notation. **(1 pt)**

(ii) Write E in *postfix* notation. **(1 pt)**

Sol

Tree

(i)



(ii) Pre Ro − L − R

| / | * | + | 4 | y | x | ↑ | − | y | 3 | 4 |

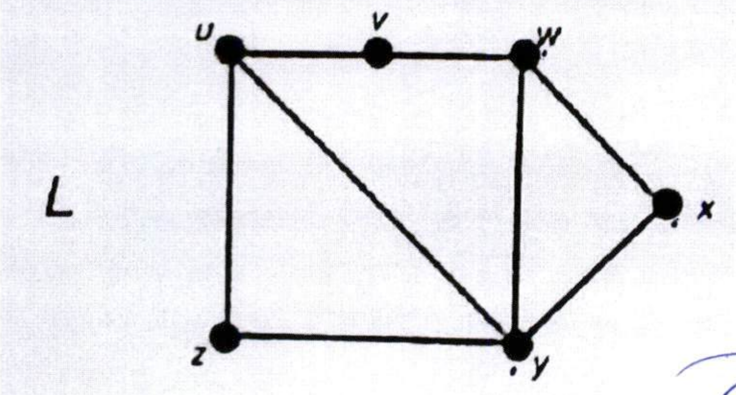
(iii) Post L − R − Ro

| 4 | y | + | x | * | y | 3 | − | 4 | ↑ | / |

(d) For the graph L below, find a spanning tree with root v,

(i) using *depth-first* search; (1 pt)

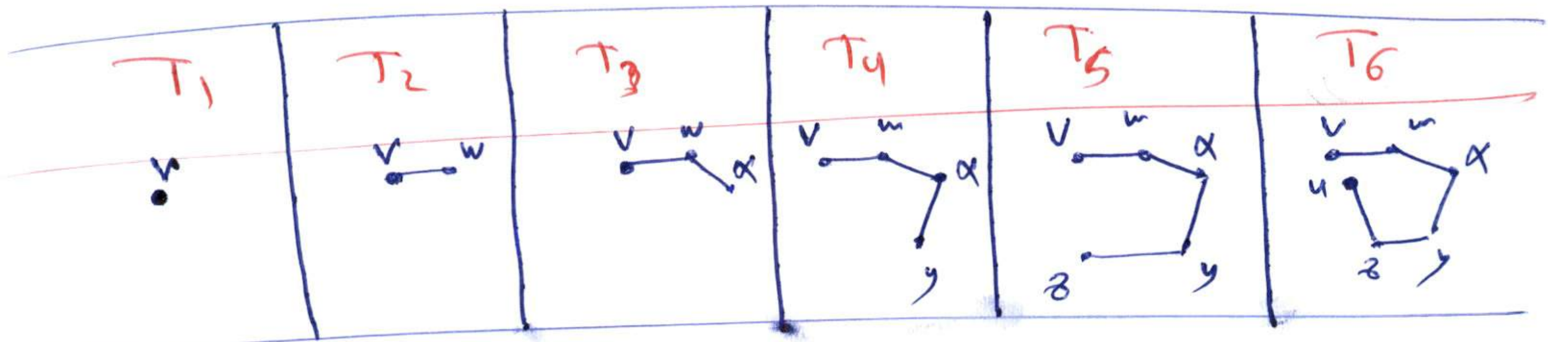
(ii) using *breadth-first* search. (1 pt)



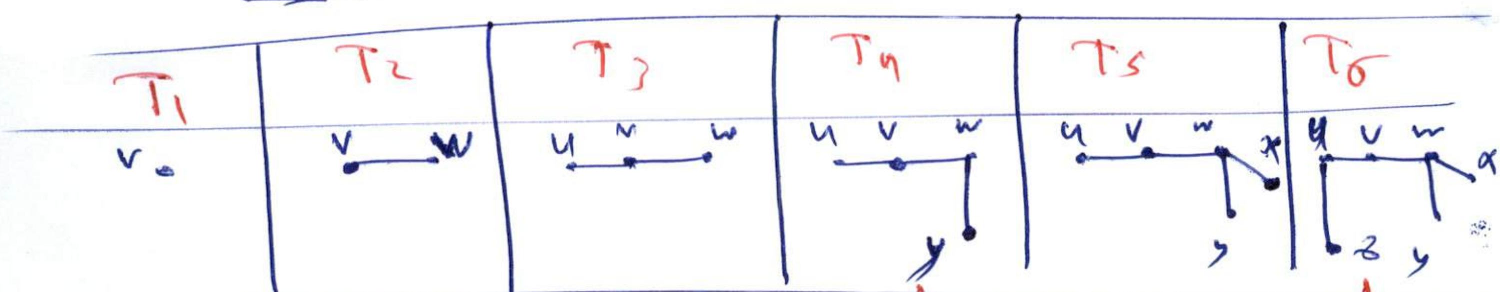
عدد خطوط 2

الحل = n = 6

Depth



Breadth



مضطر أرجع ثم ابحث "الأفقي"