

8. Suppose  $T$  is a relation defined on the integers set  $\mathbb{Z}$

$$m, n \in \mathbb{Z}, \quad m T n \Leftrightarrow m + n \geq 2$$

Decide whether the relation  $T$  is reflexive, symmetric, antisymmetric, and/or transitive.

Solution:

①  $T$  is not refl. Example:  $(-1) \not T (-1); -1 + (-1) = -2 \not\geq 2$

②  $m, n \in \mathbb{Z} : m T n \Rightarrow m + n \geq 2$

$$\Rightarrow n + m \geq 2 \Rightarrow n T m$$

$\therefore T$  is symm.

③  $T$  is not antisymm.

$$\left. \begin{array}{l} 3 T 4 : 3 + 4 = 7 \geq 2 \\ 4 T 3 : 4 + 3 = 7 \geq 2 \end{array} \right\} \text{but } 3 \neq 4$$

④  $T$  is not transitive.

$$\left. \begin{array}{l} -1 T 5 : -1 + 5 = 4 \geq 2 \\ 5 T -2 : 5 + (-2) = 3 \geq 2 \end{array} \right\} \begin{array}{l} \text{but } (-1) \not T (-2) \\ -1 + (-2) = -3 \not\geq 2 \end{array}$$

6. Let  $R$  be a relation defined on the set  $A = \{0,1,2,3\}$

$$a, b \in A, a R b \Leftrightarrow a \leq 2b$$

- (i) List all the ordered pairs of  $R$ .
- (ii) Represent  $R$  in a diagram.
- (iii) Decide whether  $R$  is reflexive, symmetric, antisymmetric, transitive. Why?

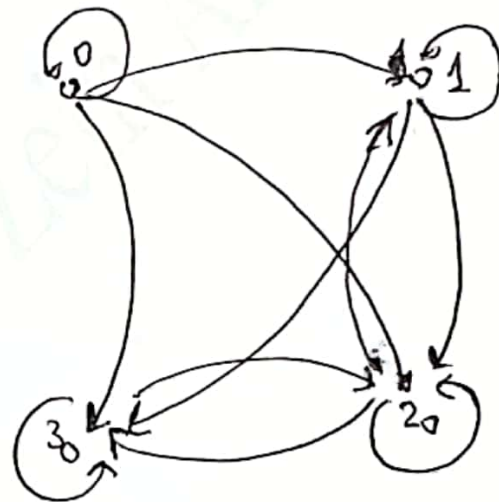
$$R = \{(0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$$

①  $(0,0), (1,1), (2,2), (3,3) \in R$

$\therefore R$  is reflexive.

②  $\because (0,1) \in R$ , but  $(1,0) \notin R$

$\therefore R$  is not symmetric.



③  $\because (1,2)$  and  $(2,1) \in R$ , but  $1 \neq 2$

$\therefore R$  is not antisymmetric.

④  $\because \left. \begin{array}{l} (3,2) \in R \\ (2,1) \in R \end{array} \right\} \text{ but } (3,1) \notin R$

$\Downarrow$   
 $R$  is not transitive.

15. Let  $R$  be a relation defined on the set  $A = \{-2, -1, 0, 1, 2\}$

$$a, b \in A, \quad a R b \Leftrightarrow a \cdot b < 0$$

- (i) List all the ordered pairs of the relation  $R$  ?
- (ii) Draw the directed graph (diagraph) that represents  $R$  ?
- (iii) Determine whether the relation  $R$  is reflexive, symmetric, antisymmetric, and/or transitive.

Solution:

$$(i) R = \{(-2, 1), (-2, 2), (-1, 1), (-1, 2), (1, -2), (1, -1), (2, -2), (2, -1)\}$$

①

$$(-2, -2) \notin R \Rightarrow R \text{ is not refl.}$$

②  $(2, -2), (-2, 2) \in R$

$(1, -2), (-2, 1) \in R$

$\Rightarrow R$  is Symm.



or  $a, b \in A : a R b \Rightarrow a \cdot b < 0$

$(-a) \cdot (-b) \Rightarrow b \cdot a < 0 \Rightarrow b R a.$

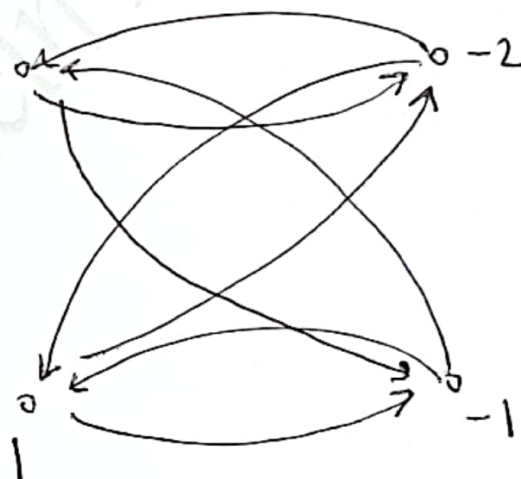
$\therefore R$  is Symm.

③  $\because (1, -1) \text{ and } (-1, 1) \in R$  but  $1 \neq -1$ .

$R$  is not antisymm.

④  $R$  is not transitive.  $(-1, 2) \text{ and } (2, -2) \in R$  but  $(-1, -2) \notin R$ .

(ii)



13. Let  $R$  be a relation defined on the set  $A = \{2,3,4,5,6\}$

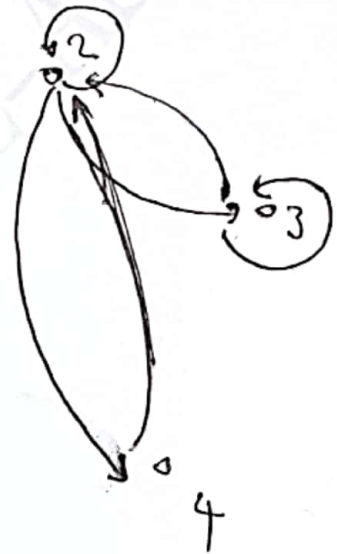
$$a, b \in A, \quad a R b \Leftrightarrow a \cdot b < 10$$

- (i) List all the ordered pairs of the relation  $R$ ?
- (ii) Draw the directed graph (diagraph) that represents  $R$ ?
- (iii) Determine whether the relation  $R$  is reflexive, symmetric, antisymmetric, and/or transitive.

Solution:

(i)  $R = \{ (2,2), (2,3), (2,4), (3,2), (3,3), (4,2) \}$ .

(ii)



(iii)

①  $R$  not ref1.  $(6,6) \notin R$ .

②  $a, b \in A: a R b \Rightarrow ab < 10$

$$\text{Symm. } a R b \Rightarrow ab < 10 \Rightarrow ba < 10 \Rightarrow b R a$$

$\therefore R$  is Symm.

③  $(2,4)$  and  $(4,2) \in R$ , but  $2 \neq 4$   
 $\therefore R$  is not antisymm.

④  $(4,2)$  and  $(3,3) \in R$ , but  $(4,3) \notin R$ .  
 $\therefore R$  is not transitive.



7. Let  $R$  be a relation defined on the set  $\mathbb{Z}^+ = \{1, 2, 3, \dots\} = \mathbb{N}$

$$m, n \in \mathbb{Z}^+, m R n \Leftrightarrow 6 \mid mn \Rightarrow mn = 6h : h \in \mathbb{N}.$$

Decide whether  $R$  is reflexive, symmetric, antisymmetric, transitive. Why?

①  $R$  is not refl.  $6 \nmid 4 = (2)(2) \Rightarrow 2 \not R 2.$

②  $m, n \in \mathbb{N} : m R n \Rightarrow 6 \mid mn$   
 Comm.  $\Rightarrow 6 \mid n.m \Rightarrow n R m$

$\therefore R$  is Symm.

③  $R$  is not antisymm.

$3 R 4$   $\left\{ \begin{array}{l} (3, 4) \in R : 6 \mid 12 = (3)(4) \\ \neq \\ (4, 3) \in R : 6 \mid 12 = (4)(3) \end{array} \right\}$  but  $3 \neq 4$

④  $\left\{ \begin{array}{l} (3, 6) \in R \Rightarrow 6 \mid 18 = (3)(6) \\ \neq \\ (6, 3) \in R \Rightarrow 6 \mid 18 = (6)(3) \end{array} \right\}$  but  $(3, 3) \notin R$   
 $6 \nmid 9 = (3)(3)$

$\therefore R$  is not transitive.

12. Let  $R$  be a relation defined on the set  $A = \{0,1,2,3\}$

$$a, b \in A, \quad a R b \Leftrightarrow a + b = 4$$

- (i) List all the ordered pairs of the relation  $R$  ?
- (ii) Draw the directed graph ( diagraph ) that represents  $R$  ?
- (iii) Determine whether the relation  $R$  is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

*Solution:*

10. Let  $R$  be a relation defined on the set  $A = \{1,2,3,4,5\}$

$$x, y \in A, \quad x R y \Leftrightarrow xy \leq 9$$

- (i) List all the ordered pairs of the relation  $R$  ?
- (ii) Draw the directed graph ( diagraph ) that represents  $R$  ?

*Solution:*