King Saud University

College of Science

Department of Mathematics

1 2 3 4 5 6 7 8 11 15 24 26

A B C D

151 Math Exercises

(5.2)

Graph Terminology and Special Types of Graphs

Malek Zein AL-Abidin

1440هـ

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Ex. 5.2

#1

x_i	a	b	c	d	e	f
$\deg x_i$	2 even	4 even	1 odd	0 even Isolated	2 even	3 odd

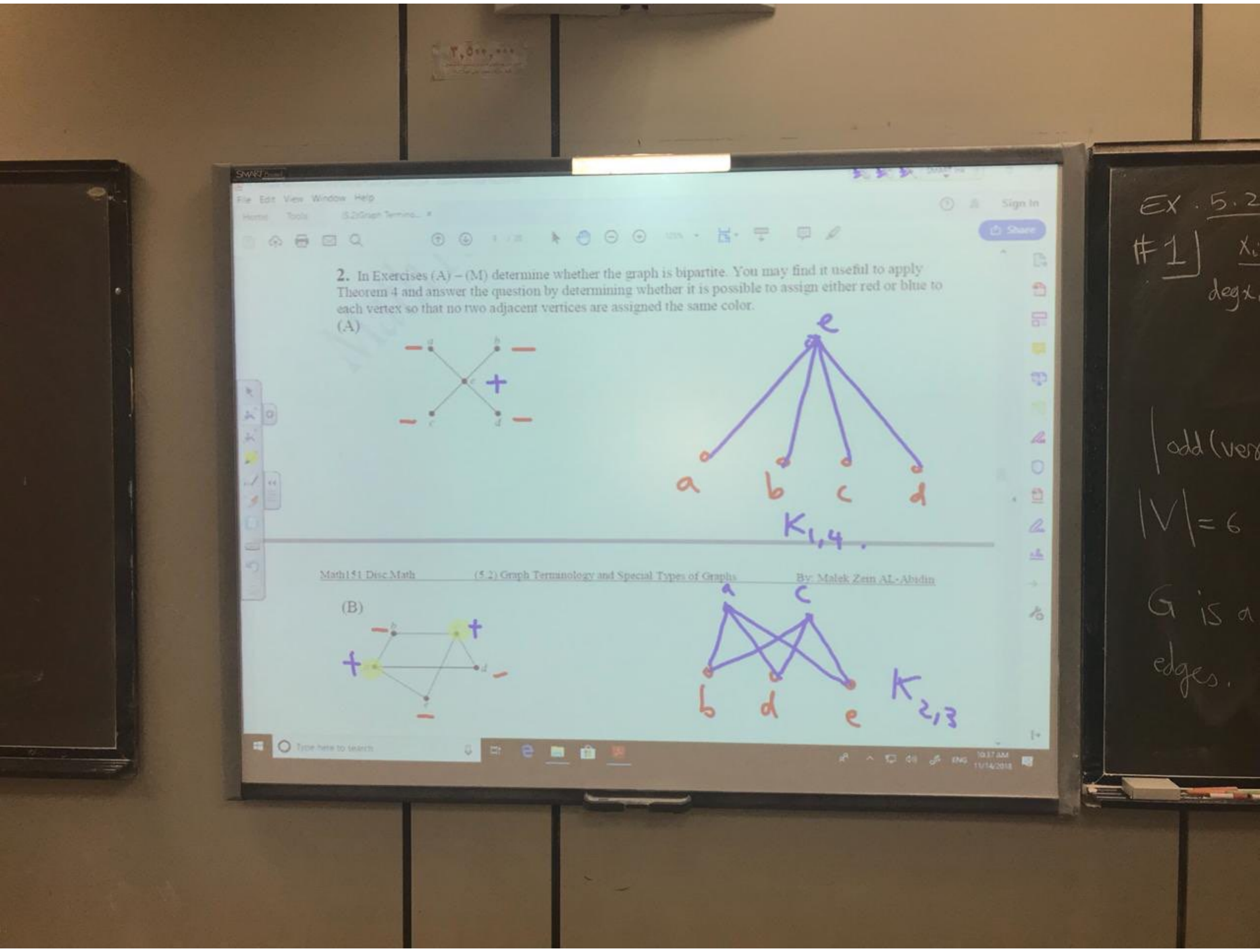
odd vertices

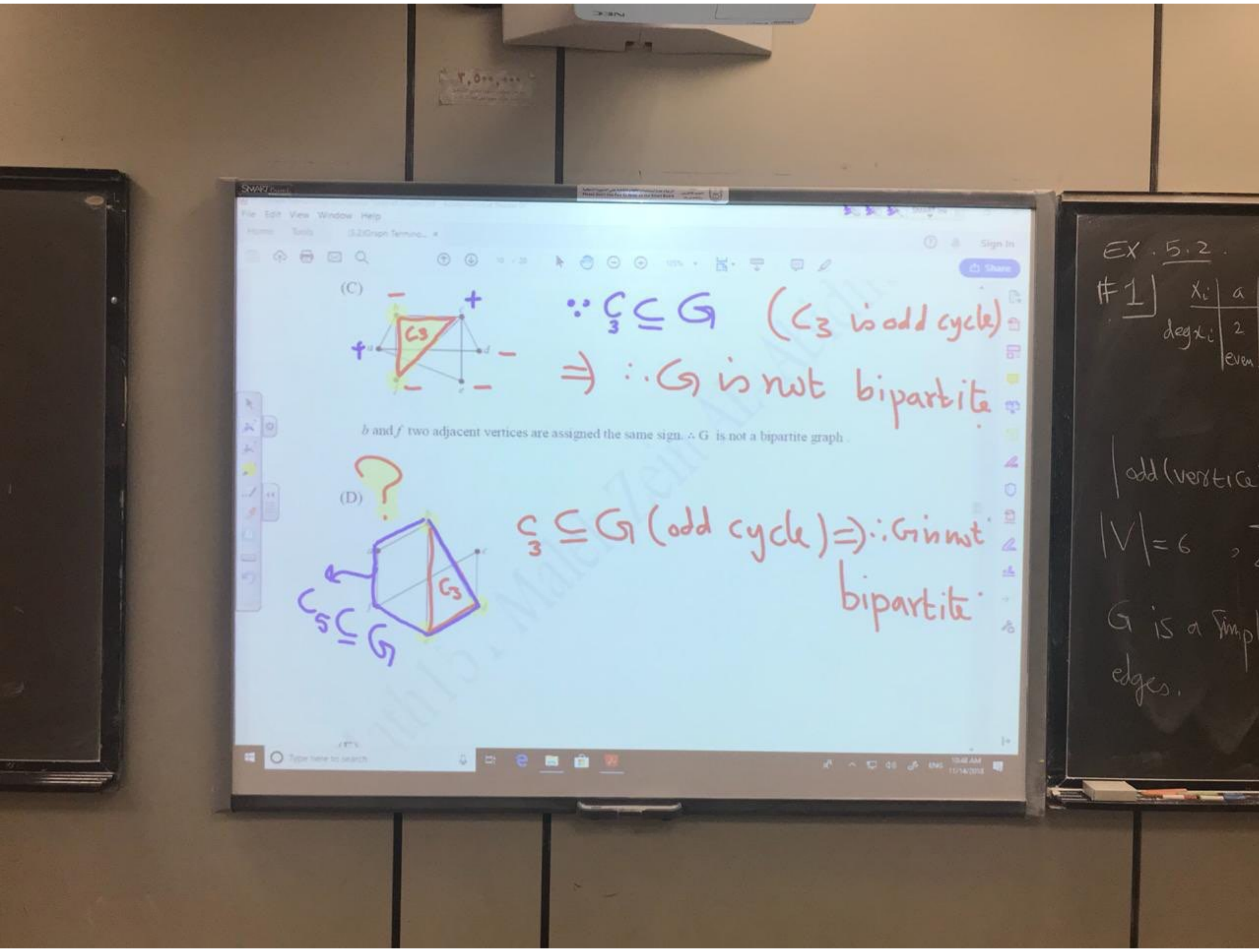
$|\text{odd (vertice)}| = \text{even}$

$|V| = 6$, $\sum_{i=1}^{6} \deg v_i = \underbrace{2+4+1+0+2+3}_{} = 2|E|$

$12 = 2|E| \Rightarrow \boxed{|E| = 6}$

G is a simple graph, cause has no loopes or multiple edges.


2. In Exercises (A) – (M) determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.
(A)
e
a
b
c
d
K1,4 .
Math151 Disc Math
(5.2) Graph Terminology and Special Types of Graphs
By: Malek Zein AL-Abdin
(B)
a
c
b
d
e
K2,3
Ex . 5.2
#1
|V| = 6
G is a
edges.


(C)
$\because C_3 \subseteq G$ (C_3 is odd cycle)
$\Rightarrow \therefore G$ is not bipartite
b and f two adjacent vertices are assigned the same sign. ∴ G is not a bipartite graph
(D)
$C_3 \subseteq G$ (odd cycle) $\Rightarrow \therefore G$ is not bipartite.
$C_5 \subseteq G$
Ex. 5.2
#1
$\deg x_i$
odd (vertice
$|V| = 6$
G is a simpl
edges.

3. Let G be a graph have 6 edges and the given degree sequence 1 , 3 , x , x . find the value of x ?

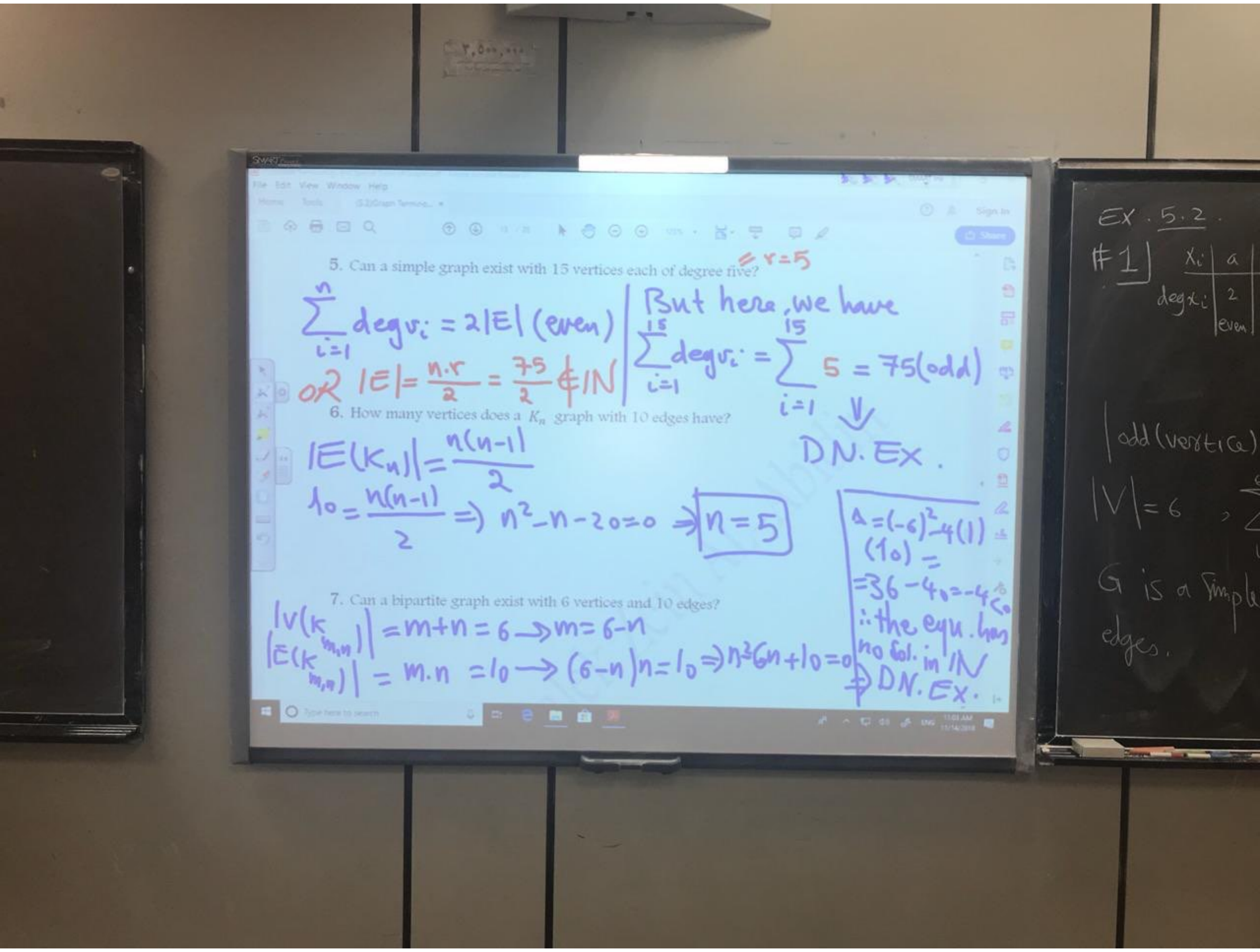
$$\sum_{i=1}^{n} \deg v_i = 2|E| \Rightarrow 1+3+x+x=12 .$$

$$2x=8 \Rightarrow \boxed{x=4}$$

4. How many vertices does a regular graph of degree four with 10 edges have?

$r=4$

$$|E| = \frac{nr}{2} \Rightarrow n = \frac{2|E|}{r} = \frac{2(10)}{4} = \boxed{5} .$$


5. Can a simple graph exist with 15 vertices each of degree five?
r=5
∑ deg v_i = 2|E| (even)
i=1
OR |E| = n.r/2 = 75/2 ∉ IN
But here, we have
15
∑ deg v_i = ∑ 5 = 75(odd)
i=1
DN. Ex.
6. How many vertices does a K_n graph with 10 edges have?
|E(K_n)| = n(n-1)/2
10 = n(n-1)/2 ⇒ n²-n-20=0 ⇒ n=5
7. Can a bipartite graph exist with 6 vertices and 10 edges?
|V(K_m,n)| = m+n = 6 → m = 6-n
|E(K_m,n)| = m.n = 10 → (6-n)n = 10 ⇒ n²-6n+10=0
Δ=(-6)²-4(1)(10) = 36-40 = -4 < 0
∴ the equ. has no sol. in IN
⇒ DN. Ex.
Ex. 5.2.
#1
x_i
a
deg x_i
2
even
odd (vertices)
|V| = 6
G is a simple
edges.

8. Show that $K_{m,n}$ is regular if and only if $m = n$?

Sol. (i) $K_{m,n}$ is regular $\Rightarrow m = n$?

$K_{m,n} = (V_1 \cup V_2 , E)$: $|V_1| = n$, $|V_2| = m$.

$\underset{x \in V_1}{\deg x} = |V_2| = m$ & $\underset{y \in V_2}{\deg y} = |V_1| = n$.

$\because K_{m,n}$ is regular $\Rightarrow \underset{x \in V_1}{\deg x} = \underset{y \in V_2}{\deg y} \Rightarrow m = n$

(ii) $m = n \Rightarrow K_{m,n}$ is regular ?

9. If G is a simple regular graph of degree k with n vertices, show that k is even or n is even.

11. If $K_{3,n}$ have the same number of edges of K_n . find the value of n ?

$$\left.\begin{array}{l} |E(K_{3,n})| = 3n \\ |E(K_n)| = \dfrac{n(n-1)}{2} \end{array}\right\} \quad 3\cancel{n} = \frac{\cancel{n}(n-1)}{2} \Rightarrow n-1=6 \quad \boxed{n=7}$$

12. Let G be a graph have 5 edges and the given degree sequence 2 , 2 , 2 , 2 , x
Decide whether G is a regular ?

13. How many vertices does a regular graph of degree 3 with 10 edges have?

15. Let G be a graph have 10 edges with two vertices of 4 degrees each, and the degree of each vertex is equal to 3. Find the number of vertices

$$\sum_{i=1}^{n} \deg v_i = 2|E| \Rightarrow 2(4) + (n-2)(3) = 20$$

$$\boxed{n = 6}$$

16. Let G be a graph have 11 edges and the given degree sequence $n, n, n, n, 2n, 2n, 3n$. find the value of n ?

17. If K_{m,m^2} have 42 vertices, find the number of edges ?

Ex. 5.2

#1 x_i

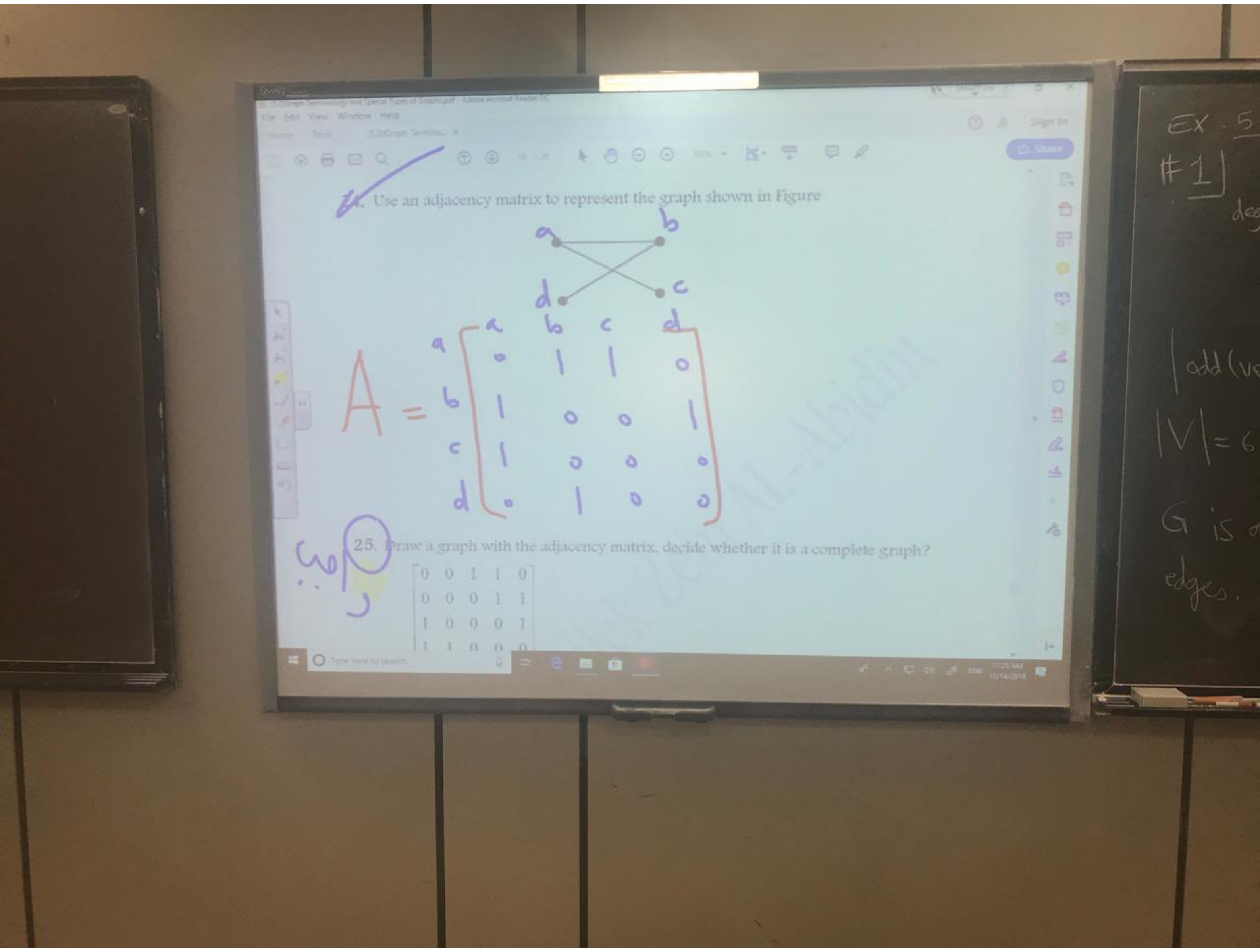
$\deg x_i$

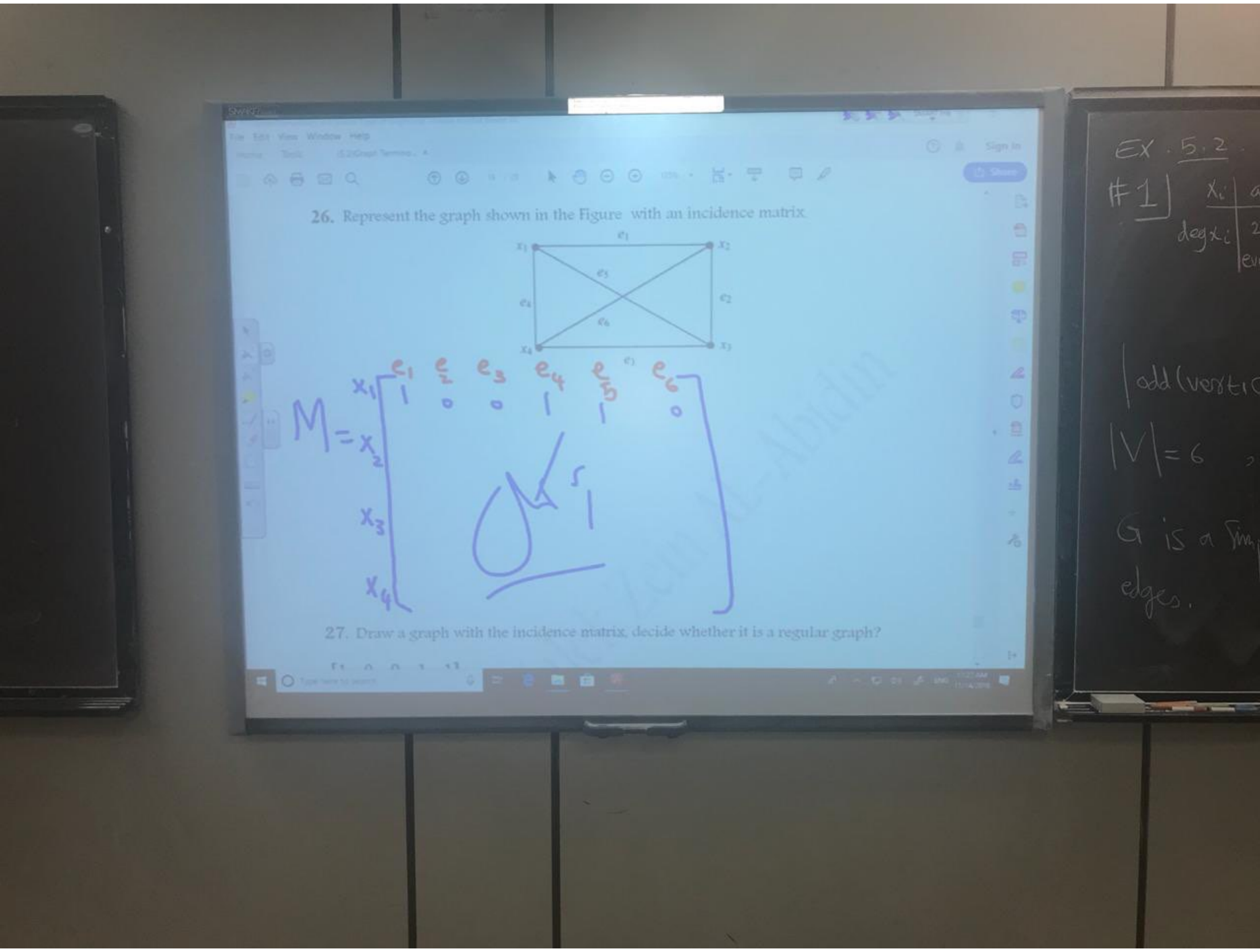
odd (vert

$|V| = 6$

G is a

edges.


24. Use an adjacency matrix to represent the graph shown in Figure
a b c d
A =
a 0 1 1 0
b 1 0 0 1
c 1 0 0 0
d 0 1 0 0
25. Draw a graph with the adjacency matrix, decide whether it is a complete graph?
0 0 1 1 0
0 0 0 1 1
1 0 0 0 1
EX. 5
#1
|V|=6
G is
edges.


26. Represent the graph shown in the Figure with an incidence matrix.
x_1
x_2
x_3
x_4
e_1
e_2
e_3
e_4
e_5
e_6
M =
x_1 1 0 0 1 1 0
x_2
x_3
x_4
27. Draw a graph with the incidence matrix, decide whether it is a regular graph?
EX. 5.2
#1
$|V| = 6$
edges.

كلية العلــــوم
قسم الرياضيـــات

2 3 4

س 2 3 5 6 7 9 10

تمارين 151 ريض

نظرية الرسومات

GRAPH THEORY

(~~4-2~~)

(5, 3)

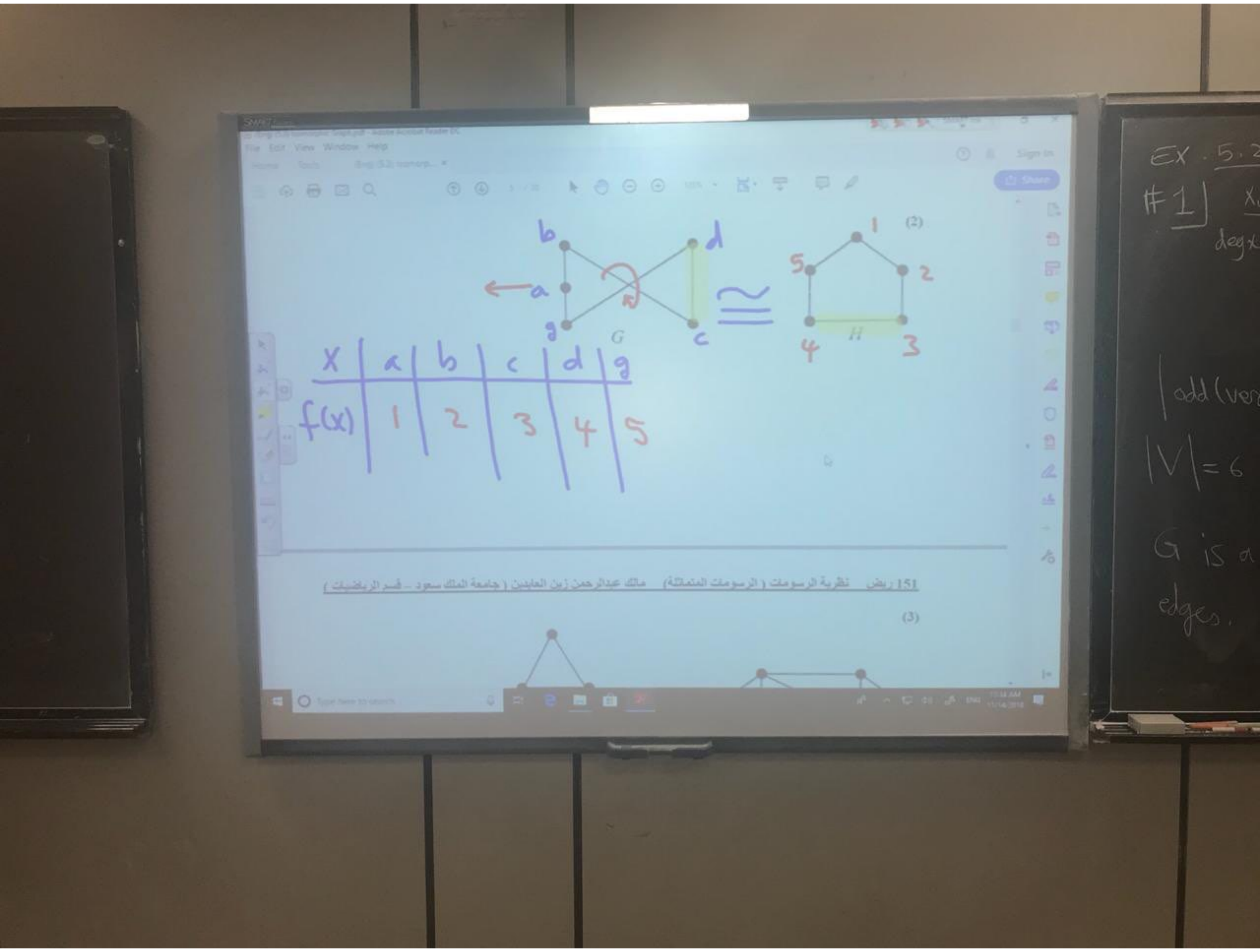
(الرسومات المتماثلة)

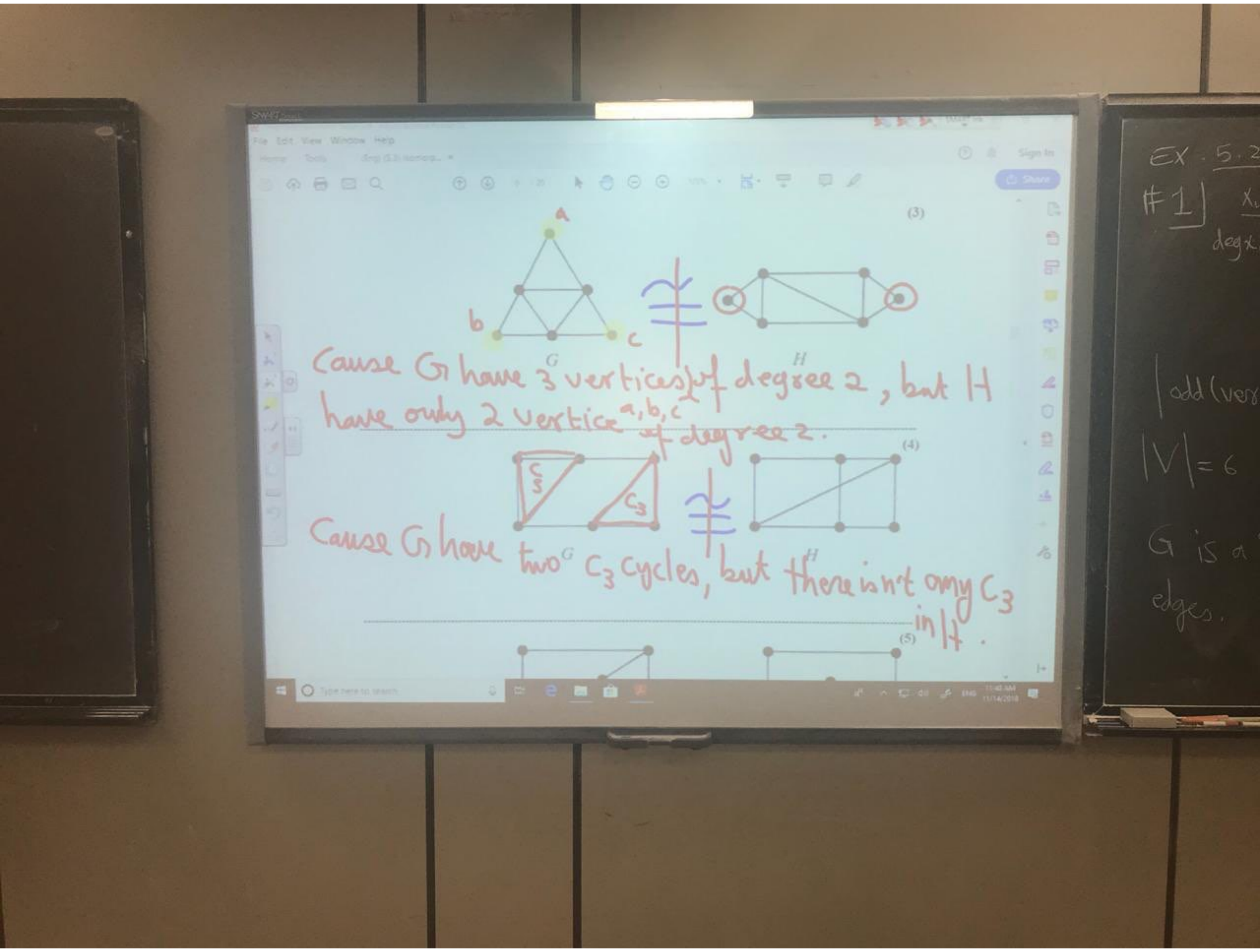
ISOMORPHIC GRAPHS

إعداد : مالك عبدالرحمن زين العابدين

1439 هـ

2018


(2)
b
d
a
g
c
G
1
2
3
4
5
H
x | a | b | c | d | g
f(x) | 1 | 2 | 3 | 4 | 5
(3)
Ex. 5.2
#1
|V| = 6
G is a
edges.


(3)
a
b
c
G
H
Cause G have 3 vertices of degree 2, but H
have only 2 vertice a,b,c of degree 2.
(4)
C_3
C_3
G
H
Cause G have two C_3 cycles, but there isn't any C_3 in H.
(5)
EX . 5.2
#1
|V| = 6
G is a
edges.

151 ريض نظرية الرسومات (الرسومات المتماثلة) مالك عبدالرحمن زين العابدين (جامعة الملك سعود - قسم الرياضيات)

س2 : عين جميع الرسومات الثنائية التجزئة التامة غير المتماثلة و التي عدد رؤوس كل منها 7 ؟

Q. List all ***nonisomorphic*** complete bipartite graphs with 7 total vertices ?

$$|V(K_{m,n})| = m+n = 7$$

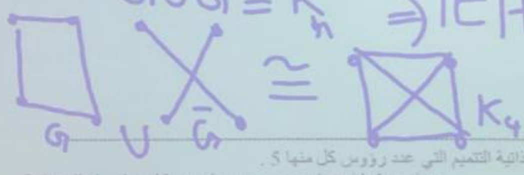
$$1+6 = 7 \Rightarrow K_{1,6} \ncong$$

$$2+5 = 7 \Rightarrow K_{2,5} \ncong$$

$$3+4 = 7 \Rightarrow K_{3,4}$$

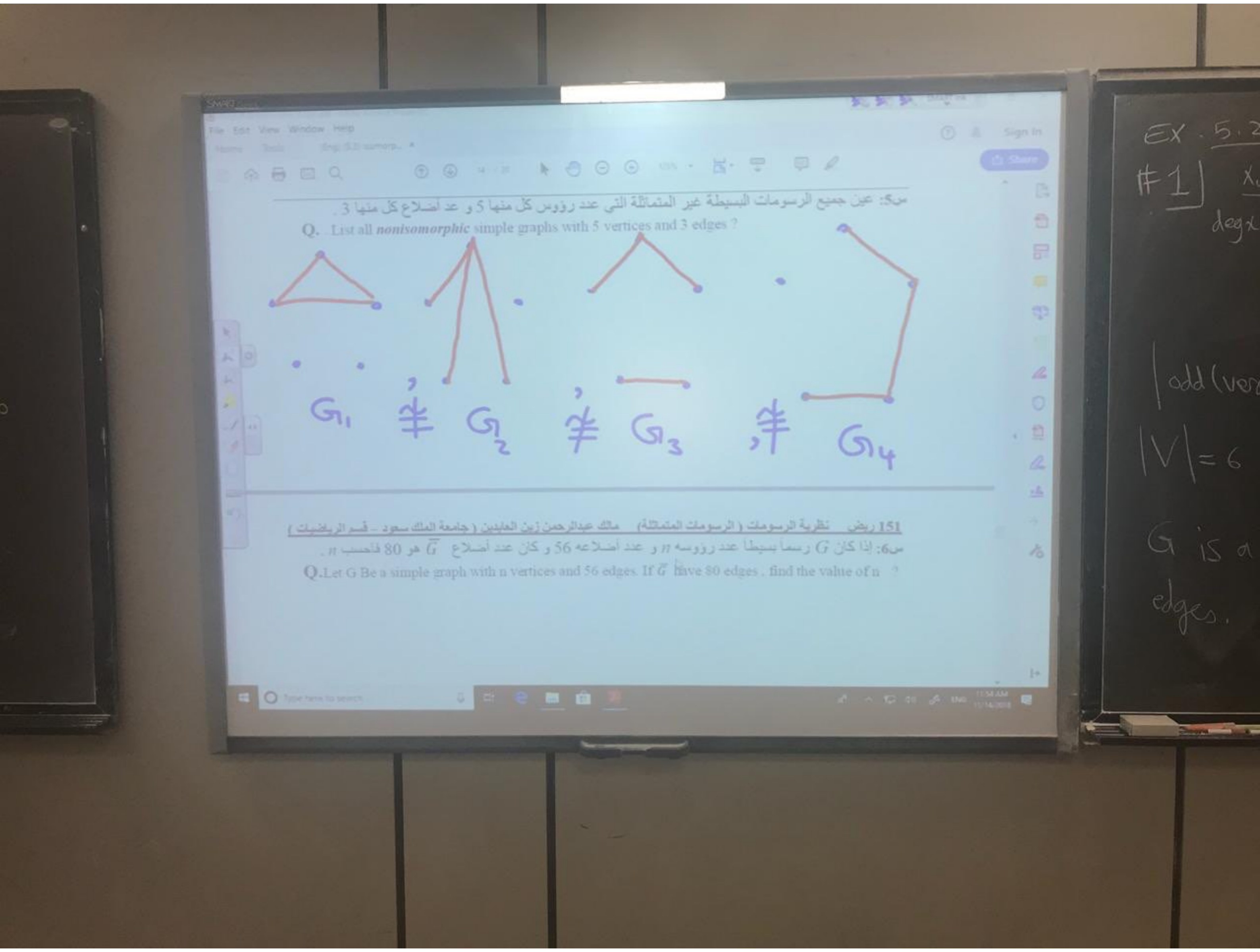
س3 : إذا كان G رسماً بسيطاً عدد رؤوسه n فاثبت أن $|E| + |\bar{E}| = \frac{n(n-1)}{2}$

Sol. $\because G \cup \bar{G} \cong K_n \Rightarrow |E| + |\bar{E}| = |E(K_n)| = \frac{n(n-1)}{2}$



س4: عين جميع الرسومات البسيطة ذاتية التتميم التي عدد رؤوس كل منها 5 .

Q.Set all simple self complementary graphs with 5 vertices



س5: عين جميع الرسومات البسيطة غير المتماثلة التي عدد رؤوس كل منها 5 و عد أضلاع كل منها 3 .
Q. List all nonisomorphic simple graphs with 5 vertices and 3 edges ?
G1 ≇ G2 ≇ G3 ≇ G4
151 ريض نظرية الرسومات (الرسومات المتماثلة) مالك عبدالرحمن زين العابدين (جامعة الملك سعود - قسم الرياضيات)
س6: إذا كان G رسما بسيطا عدد رؤوسه n و عدد أضلاعه 56 و كان عدد أضلاع G̅ هو 80 فاحسب n .
Q.Let G Be a simple graph with n vertices and 56 edges. If G̅ have 80 edges , find the value of n ?

س6: إذا كان G رسما بسيطا عدد رؤوسه n و عدد أضلاعه 56 و كان عدد أضلاع $\overline{G}$ هو 80 فاحسب n .

Q.Let G Be a simple graph with n vertices and 56 edges. If $\overline{G}$ have 80 edges , find the value of n ?

$$|E| + |\overline{E}| = \frac{n(n-1)}{2} \Rightarrow 56 + 80 = \frac{n(n-1)}{2}$$

$$n^2 - n - 272 = 0$$

$$(n-17)(n+16) = 0$$

$$\therefore n = 17 .$$

س7: جد مع التعليل عدد أضلاع الرسم المتمم للرسم $K_{10,14}$.

Q. Find the number of edges for the complementary graph of $K_{10,14}$. Explain the answer?

Q. Find the number of edges for the complementary graph of $K_{10,14}$. Explain the answer?

$$|E|+|\bar{E}|=\frac{|V|(|V|-1)}{2}$$

$$|E(K_{10,14})|+|E(\overline{K}_{10,14})|=\frac{(10+14)(10+14-1)}{2}$$

$$(10)(14)+|E(\overline{K}_{10,14})|=(12)(23)\Rightarrow|E(\overline{K}_{10,14})|=276-140=\underline{136}$$

س8: عين جميع الرسومات البسيطة غير المتماثلة التي عدد رؤوس كل منها 4 و عدد أضلاع كل منها 3 .

Q. List all ***nonisomorphic*** simple graphs with 4 vertices and 3 edges ?

Ex. 5.2

#1

$|V|=6$

G is a

edges.

س9: هل يوجد رسم G له 7 أضلاع و يحقق $G \cong \bar{G}$ ؟ وضح إجابتك .

Q.Is there exist a graph G with 7 edges satisfies $\bar{G} \cong G$? *explain the answer.*

Sol. $\because G \cong \bar{G}$ (self complemetary) $\Rightarrow$

$|E| = |\bar{E}| = 7 \Rightarrow |E| + |\bar{E}| = 7 + 7 = \frac{n(n-1)}{2}$.

$n^2 - n - 28 = 0 \quad \Delta = (-1)^2 - 4(1)(-28) = 1 + 112 = 113$

$n_{1,2} = \frac{1 \pm \sqrt{113}}{2} \notin \mathbb{N}$

D.N.Ex.

✱ س10: إذا كان G رسما بسيطا درجات رؤوسه 2,2,2,3,3,4 فأوجد عدد أضلاع $\bar{G}$ (الرسم المتمم) .

Q.Let G be a graph with the degree sequence 2,2,2,3,3,4.Find the number of edges of $\bar{G}$?

Sol. $|E| + |\bar{E}| = \frac{n(n-1)}{2}$

$\frac{2+2+2+3+3+4}{2} + |\bar{E}| = \frac{6(5)}{2} = 15 \Rightarrow |\bar{E}| = 7$

س11: إذا كان G رسما بسيطا عدد رؤوسه n و عدد أضلاعه 5، فجد ضعف عدد أضلاع الرسم المتمم $\bar{G}$.

Q. Let G be a graph with n vertices and 5 edges. find the double edges of $\bar{G}$?