King Saud University
Department of Mathematics

Final Exam in Math 151
1st Semester, 1439/40 H.
(Duration: 3 Hours)

Calculators are not allowed

Q1. (a) Without using truth tables, show that $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$. (2 pts)

(b) Use induction to show that the number $5n^2 - 3n$ is **even** for all integers $n \geq 0$. (3 pts)

Q2. (a) Let R be the relation on $\mathbb{Z}$ such that $x R y$ if and only if 4 divides $x + 3y$.

(i) Show that R is an equivalence relation. (3 pts)

(ii) Determine whether $-2 \in [6]$. (1 pt)

(b) Let $P = \{(1,1), (1,3), (2,1), (2,2), (2,3), (3,3), (4,1), (4,3), (4,4)\}$ be a partial order on the set $A = \{1,2,3,4\}$.

(i) Draw the Hasse diagram of P. (2 pts)

(ii) Determine whether P is a total order. (1 pt)

Q3. (a) (i) Write the Boolean function $f(x, y, z) = (x + y)(\bar{y} + z)$ in **CSP** form. (2 pts)

(ii) Write the Boolean function $g(x, y, z) = x\bar{y} + \bar{z}$ in **CPS** form. (2 pts)

(b) Let $h(x, y, z) = x\bar{z} + \bar{x}\bar{y} + \bar{x}z + \bar{y}\bar{z}$ be a Boolean function.

(i) Draw the Karnaugh map of h. (1 pt)

(ii) Write h in **MSP** form. (2 pts)

(iii) Write h in **MPS** form. (2 pts)

(iv) Construct a minimal "***AND-OR***" circuit for h. (1 pt)

(v) Construct a circuit for h using ***NAND*** gates only. (1 pt)

(vi) Construct a circuit for h using ***NOR*** gates only. (1 pt)

Turn the page.....

Q4. (a) Let T be a tree with n vertices $v_1, v_2, \ldots, v_n$, where $n > 2$. Find $\deg(v_n)$ if you know that $\deg(v_1) = \deg(v_2) = \cdots = \deg(v_{n-1}) = 1$. (2 pts)

(b) Form a binary search tree for the words *fox, tiger, dog, lion, wolf, cat* (using alphabetical order). (2 pts)

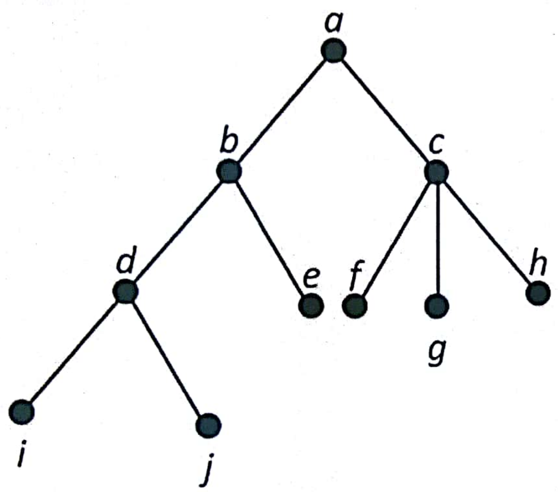
(c) Let T be the ordered rooted tree below.

(i) Find the *preorder* traversal of T. (2 pts)

(i) Find the *inorder* traversal of T. (2 pts)

(iii) Find the *postorder* traversal of T. (2 pts)

T



Q5. (a) Let E be the arithmetic expression $(x * y) - ((y \uparrow 3) + x)$.

(i) Represent E by an ordered rooted tree. (2 pts)

(ii) Write E in *prefix* notation. (1 pt)

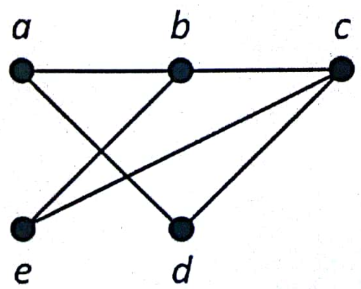
(ii) Write E in *postfix* notation. (1 pt)

(b) For the graph G below, find a spanning tree with root a,

(i) using *depth-first* search; (1 pt)

(ii) using *breadth-first* search. (1 pt)

G



First Semester 1439/1440

Answer Sheet Final exam Math 151

Dr Borhen

Q1 5 Marks

(a) $(p \to q) \vee (p \to r) \equiv (\neg p \vee q) \vee (\neg p \vee r)$

1 $\equiv (\neg p \vee \neg p) \vee (q \vee r)$

1 $\equiv \neg p \vee (q \vee r) \equiv p \to (q \vee r)$

(b) Put $P(n)$: $(5n^2 - 3n)$ is even

0,5 1st step : $n=0$; $5 \times 0^2 - 3 \times 0 = 0$ is even so $P(0)$ is true.

Inductive step let $k \geq 1$, we assume that $P(k)$ is true

0,5 (then $(5k^2 - 3k) = 2L, L \in \mathbb{Z}$). Now we prove

0,5 that $P(k+1)$ remains true. $5(k+1)^2 - 3(k+1)$ is even?

$$5(k+1)^2 - 3(k+1) = 5(k^2 + 2k + 1) - 3k - 3$$
$$= (5k^2 - 3k) + 10k + 5 - 3.$$
1,5
$$= 2L + 10k + 2$$
$$= 2(\underbrace{L + 5k + 1}) = 2M$$

we deduce that for $n \geq 0$, $(5n^2 - 3n)$ is even.

Q2 (7 Marks)

(a) $x R y \iff 4 \mid (x + 3y)$

- R is reflexive on $\mathbb{Z}$, because if $x \in \mathbb{Z}$ $4 \mid 4x = x + 3x$

1 So $x R x$

- R is Symmetric on $\mathbb{Z}$ because if $x R y$ then $4 \mid x + 3y$

$\iff x + 3y = 4L \iff 3x + 9y = 12L$

1 $\iff 3x + y = 12L - 8y = 4(3L - 2y) = 4M$

So $4 \mid y + 3x \iff y R x$.

- R is transitive on $\mathbb{Z}$ because if $x R y$ and $y R z$

then $4 \mid x + 3y \iff x + 3y = 4L$ and $4 \mid y + 3z \iff y + 3z = 4K$

1 by addition $x + 3y + y + 3z = 4L + 4K$

$x + 3z = 4L + 4K - 4y = 4(L + K - y)$

So $4 \mid x + 3z \iff x R z$

As R is reflexive, Symmetric and transitive

then R is an equivalence relation on $\mathbb{Z}$.

(i) $4 \mid f(2) + 3\times 6 = 16$ yes $-2\ R\ 6$

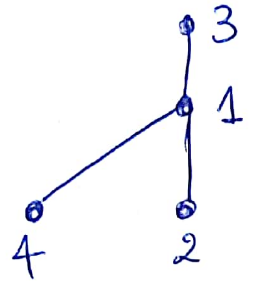
1

So $-2 \in [6]$.

(b)

(i)

2



1 (ii) Not total order because $2 \nmid 4$ & $4 \nmid 2$

(12 Marks)

Q3 (a) (i) $f = (x+y)(\bar{y}+z)$

$= x\bar{y} + xz + y\bar{y} + yz$ $\qquad y\bar{y} = 0$

$= x\bar{y}(z+\bar{z}) + x(y+\bar{y})z + (x+\bar{x})yz$

$f = x\bar{y}z + x\bar{y}\bar{z} + xyz + x\bar{y}z + xyz + \bar{x}yz$.

2 $\boxed{CSP(f) = x\bar{y}z + x\bar{y}\bar{z} + xyz + \bar{x}yz}$

(ii) $g = x\bar{y} + \bar{z}$

$\bar{g} = g' = \overline{(x\bar{y}+\bar{z})} = (\bar{x}+y)\cdot z = \bar{x}z + yz$

$\bar{g} = \bar{x}(y+\bar{y})z + (x+\bar{x})yz$

$= \bar{x}yz + \bar{x}\bar{y}z + xyz + \bar{x}yz$

$CSP(\bar{g}) = \bar{x}yz + \bar{x}\bar{y}z + xyz$

2 So $\boxed{CPS(g) = \overline{(CSP(\bar{g}))} = (x+\bar{y}+\bar{z})(x+y+\bar{z})(\bar{x}+\bar{y}+\bar{z})}$

(b) $h = x\bar{z} + \bar{x}\bar{y} + \bar{x}z + \bar{y}\bar{z}$

$= x(y+\bar{y})\bar{z} + \bar{x}\bar{y}(z+\bar{z}) + \bar{x}(y+\bar{y})z + (x+\bar{x})\bar{y}\bar{z}$

$= xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}$

$\boxed{CSP(h) = xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + \bar{x}yz}$

(i) (1)

	yz	$\bar{y}z$	$\bar{y}\bar{z}$	$y\bar{z}$
x	0	0	1	1
$\bar{x}$	1	1	1	0

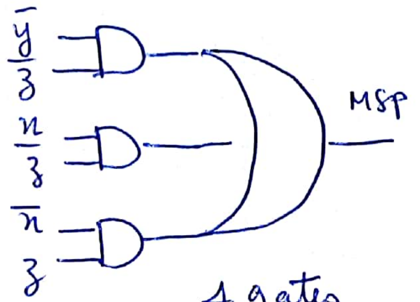
(2) (ii) $\boxed{MSP(h) = \bar{y}\bar{z} + x\bar{z} + \bar{x}z}$

(iii) $MSP(\bar{h}) = xz + \bar{x}y\bar{z}$

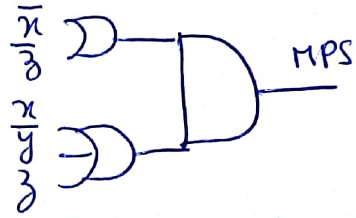
$MPS(h) = \overline{(MSP(\bar{h}))} = \overline{(xz + \bar{x}y\bar{z})}$

(2) $\boxed{MPS(h) = (\bar{x}+\bar{z})\cdot(x+\bar{y}+z)}$

(iv) (1)



4 gates



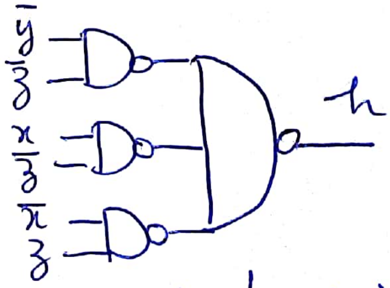
3 gates

So it a minimal "And-OR" circuit for h

(v) (1)

$MSP(h) = \left[\overline{\overline{(\bar{y}\bar{z} + x\bar{z} + \bar{x}z)}}\right]'$

$= \left[\overline{(\bar{y}\bar{z})}\cdot\overline{(x\bar{z})}\cdot\overline{(\bar{x}z)}\right]'$

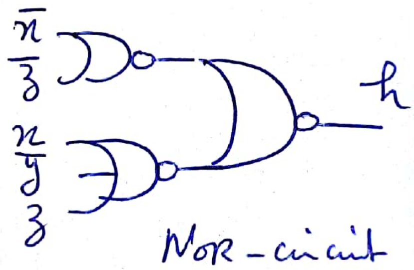


Nand-circuit

(vi) (1)

$MPS(h) = \left[\overline{\overline{(\bar{x}+\bar{z})\cdot(x+\bar{y}+z)}}\right]'$

$= \left[\overline{(\bar{x}+\bar{z})} + \overline{(x+\bar{y}+z)}\right]'$



NOR-circuit

Q4 (10 Marks)

(a) T is a tree $<$ n vertices, (n-1) edges

We know $\sum_{i=1}^{n} deg(v_i) = 2|E|$

$\underbrace{1+\cdots+1}_{(n-1)} + deg(v_n) = 2(n-1)$

(2) So $\boxed{deg\ v_n = n-1}$

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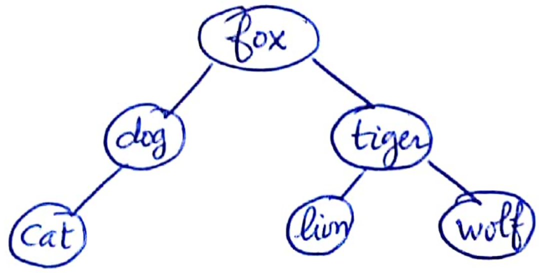
(b)

2



Binary search tree

(c)

2 (i) Preorder: a/b/d/i/j/e/c/f/g/h

2 (ii) Inorder: i/d/j/b/e/a/f/c/g/h

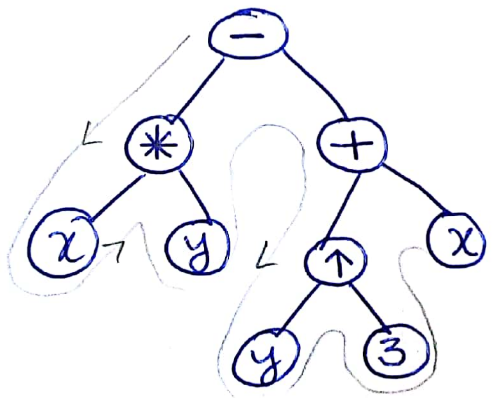
2 (iii) Posórder: i/j/d/e/b/f/g/h/c/a

Q5 (6marks)

(a)

(i)

2



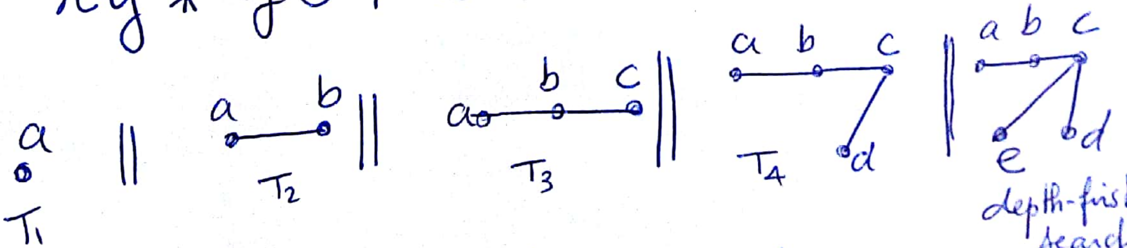
1 (ii) − * x y + ↑ y 3 x ← prefix

1 (iii) x y * y 3 ↑ x + − ← postfix

(b)

(i)

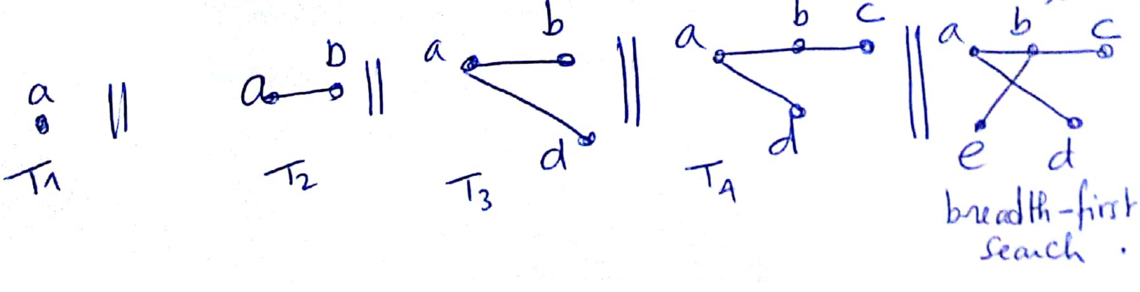
1



depth-first search

(ii)

1



breadth-first search.

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