8. Suppose T is a relation defined on the integers set $\mathbb{Z}$

$$m, n \in \mathbb{Z}, \quad m\,T\,n \Leftrightarrow m+n \geq 2$$

Decide whether the relation T is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

Solution:

① T is not refl. Example: $(-1)\not T(-1)$; $-1+(-1)=-2 \not\geq 2$

② $m,n\in\mathbb{Z}$: $mTn \Rightarrow m+n \geq 2$

$\Rightarrow n+m \geq 2 \Rightarrow nTm$

$\therefore$ T is symm.

③ T is not antisymm.

$3T4$: $3+4=7 \geq 2$

&

$4T3$: $4+3=7 \geq 2$

} but $3 \neq 4$

④ T is not transitive.

$-1\,T\,5$: $-1+5=4 \geq 2$.

&

$5\,T\,-2$: $5+(-2)=3 \geq 2$.

} but $(-1)\not T(-2)$ $-1+(-2)=-3 \not\geq 2$

6. Let R be a relation defined on the set $A = \{0,1,2,3\}$

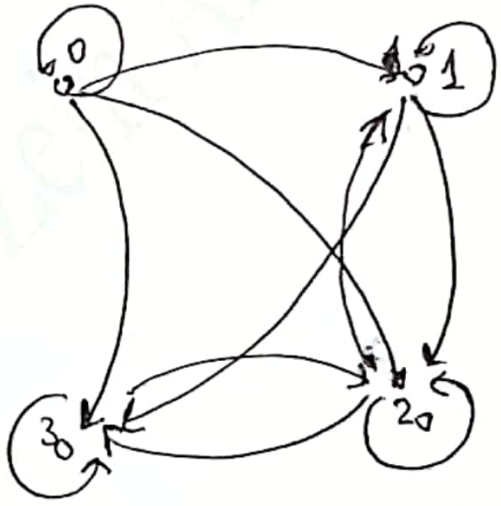
$$a, b \in A \ , \ a R b \Leftrightarrow a \leq 2b$$

(i) List all the ordered pairs of R.

(ii) Represent R in a diagram.

(iii) Decide whether R is reflexive, symmetric, antisymmetric, transitive. Why?

$R = \{(0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3)$
$(2,1), (2,2), (2,3), (3,2), (3,3)\}$.



① $(0,0), (1,1), (2,2), (3,3) \in R$
$\therefore$ R is ref l.

② $\because (0,1) \in R$, but $(1,0) \notin R$
$\therefore$ R is not symm.

③ $\because (1,2)$ and $(2,1) \in R$, but $1 \neq 2$
$\therefore$ R is not antisymm.

④ $\because (3,2) \in R$ & $(2,1) \in R$ } but $(3,1) \notin R$
$\Downarrow$
R is not transitive.

15. Let R be a relation defined on the set $A = \{-2,-1,0,1,2\}$

$$a,b \in A, \quad a\,R\,b \Leftrightarrow a.b < 0$$

(i) List all the ordered pairs of the relation R ?

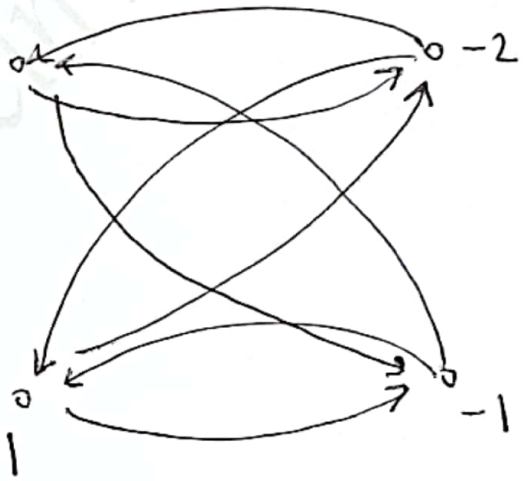
(ii) Draw the directed graph (diagraph) that represents R ?

(iii) Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

Solution:

(i) $R = \{(-2,1),(-2,2),(-1,1),(-1,2),(1,-2),(1,-1),(2,-2),(2,-1)\}$

(ii)



① $(-2,-2) \notin R \Rightarrow$ R is not refl.

② $(2,-2),(-2,2) \in R$

$(1,-2),(-2,1) \in R$

$\Rightarrow R$ is symm.

II

or $a,b \in A : aRb \Rightarrow ab < 0$

(تبادلي) $\Rightarrow ba < 0 \Rightarrow bRa$.

$\therefore R$ is symm.

③ $\because (1,-1)$ and $(-1,1) \in R$ but $1 \neq -1$.

R is not antisymm.

④ R is not transitive. $(-1,2)$ and $(2,-2) \in R$

but $(-1,-2) \notin R$.

13. Let R be a relation defined on the set $A = \{2,3,4,5,6\}$

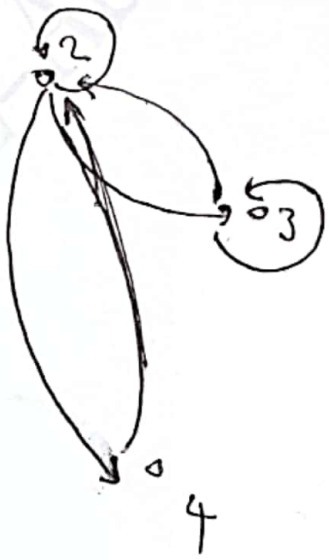
$$a, b \in A, \quad a\,R\,b \Leftrightarrow a.b < 10$$

(i) List all the ordered pairs of the relation R ?
(ii) Draw the directed graph (diagraph) that represents R ?
(iii) Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

Solution:

(i) $R = \{(2,2),(2,3),(2,4),(3,2),(3,3),(4,2)\}$.

(ii)



(iii)

① R not refl. $(6,6) \notin R$.

② $a,b \in A$: $aRb \Rightarrow ab < 10$

Comm. $\forall a,b \Rightarrow ba < 10 \Rightarrow bRa$

$\therefore R$ is Symm.

③ $\because (2,4)$ and $(4,2) \in R$, but $2 \neq 4$

$\therefore R$ is not antisymm.

④ $(4,2)$ and $(3,3) \in R$, but $(4,3) \notin R$.

$\therefore R$ is not transitive.

7. Let R be a relation defined on the set $\mathbb{Z}^+ = \{1,2,3,\ldots\} = \mathbb{N}$

$m,n \in \mathbb{Z}^+$, $m\,R\,n \Leftrightarrow 6 \mid mn \Rightarrow mn = 6h : h \in \mathbb{N}$.

Decide whether R is reflexive , symmetric , antisymmetric , transitive . Why?

① R is not refl. $6 \nmid 4 = (2)(2) \Rightarrow 2 \not R 2$.

② $m,n \in \mathbb{N} : mRn \Rightarrow 6 \mid mn$

Comm. $\Rightarrow 6 \mid n.m \Rightarrow nRm$

$\therefore$ R is symm.

③ R is not antisymm.

$3R4$ ← $(3,4) \in R : 6 \mid 12 = (3)(4)$ } but $3 \neq 4$

$\Downarrow$

$(4,3) \in R : 6 \mid 12 = (4)(3)$ }

④ $(3,6) \in R \Rightarrow 6 \mid 18 = (3)(6)$ } but $(3,3) \notin R$

$\Downarrow$

$(6,3) \in R \Rightarrow 6 \mid 18 = (6)(3)$ $6 \nmid 9 = (3)(3)$

$\therefore$ R is not transitive.

12. Let R be a relation defined on the set $A = \{0,1,2,3\}$

$$a, b \in A, \qquad a\,R\,b \Leftrightarrow a + b = 4$$

(i) List all the ordered pairs of the relation R ?

(ii) Draw the directed graph (diagraph) that represents R ?

(iii) Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

Solution:

10. Let R be a relation defined on the set $A = \{1,2,3,4,5\}$

$$x, y \in A, \qquad x R y \Leftrightarrow xy \leq 9$$

(i) List all the ordered pairs of the relation R ?
(ii) Draw the directed graph (diagraph) that represents R ?

Solution: