King Saud University

College of Science

Department of Mathematics

151 MATH EXERCISES

(6)

BOOLEAN ALGEBRAS

&

LOGIC GATES

&

MINIMIZATION OF CIRCUITS

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ﻫ1440

2018

Q2. Let B is a Boolean Algebra and $x, y \in B$. Show that $x + y = xy + xy' + x'y$ is valid.

x	y	x'	y'	xy	xy'	$x'y$	$xy + xy' + x'y$	$x+y$
1	1	0	0					
1	0	0	1					
0	1	1	0					
0	0	1	1					

Q5. Let $f(x,y,z) = (x'+z)(x+y)$. Find $CSP(f)$ (sum-of-products expansion) and $CPS(f)$ (product-of-sums expansion) ?

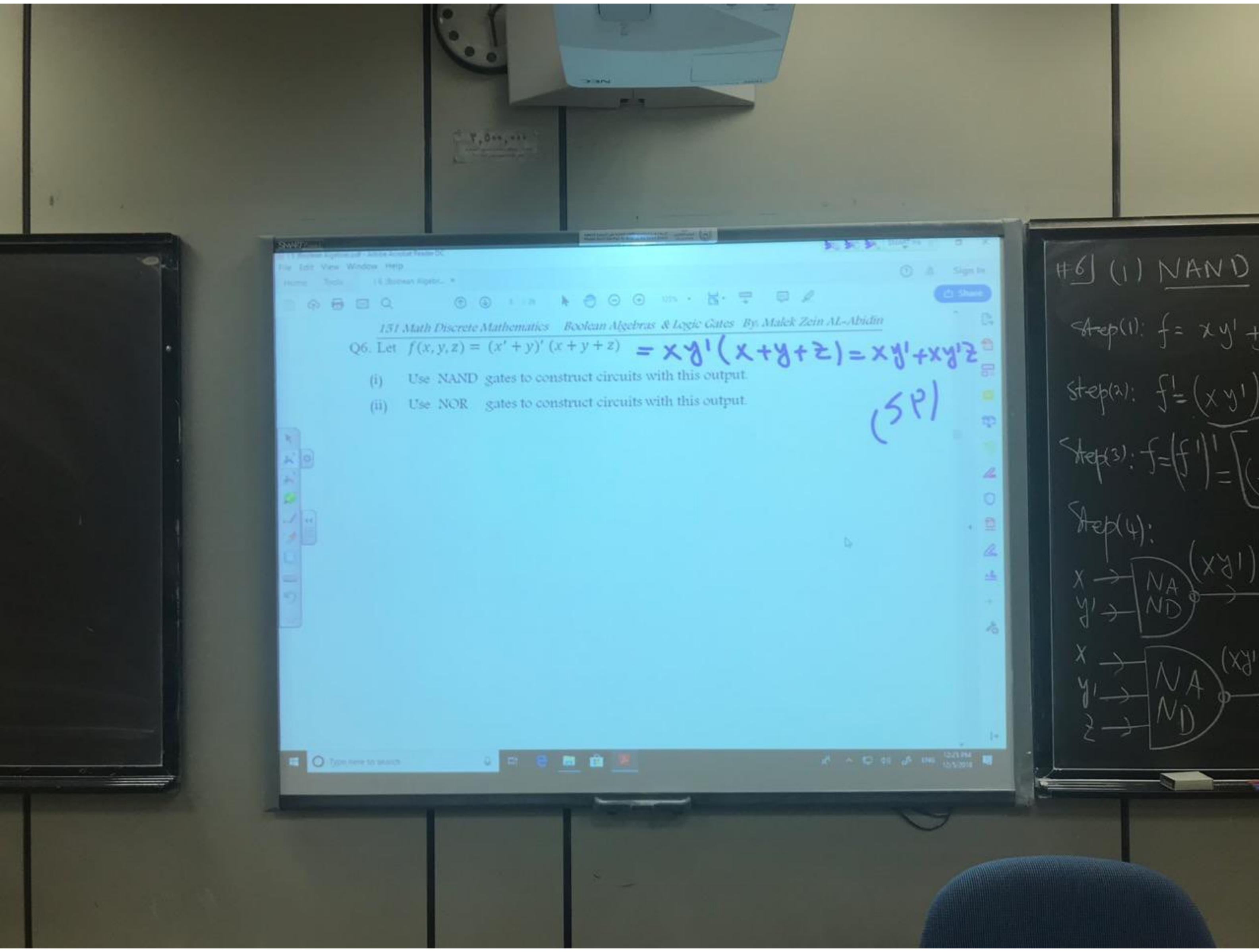
$f = x'y + xz + yz$. (Sp)

$csp(f) = xyz + xy'z + x'yz + x'yz'$.

	yz	yz'	$y'z'$	$y'z$
x	1	0	0	1
x'	1	1	0	0

$f' = xyz' + xy'z' + x'y'z' + x'y'z$.

$cps(f) = (f')' = (x'+y'+z).(x'+y+z).(x+y+z).(x+y+z')$.


151 Math Discrete Mathematics Boolean Algebras & Logic Gates By: Malek Zein AL-Abidin
Q6. Let $f(x,y,z) = (x'+y)'(x+y+z)$ $= xy'(x+y+z) = xy'+xy'z$
(i) Use NAND gates to construct circuits with this output.
(ii) Use NOR gates to construct circuits with this output.
(5P)
#6 (i) NAND
Step(1): $f = xy' +$
Step(2): $f' = (xy')$
Step(3): $f = (f')' =$
Step(4):
(xy')
NAND
$(xy'$
NAND

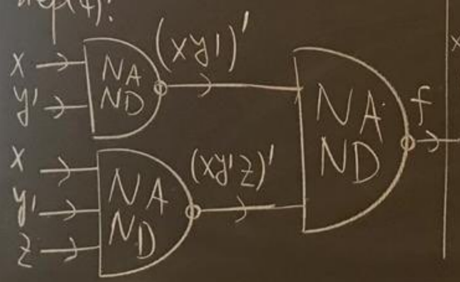
#6] (i) NAND

Step(1): $f = xy' + xy'z$ (SP)

Step(2): $f' = (xy')' \cdot (xy'z)'$ (*)

Step(3): $f = (f')' = \left[(xy')' \cdot (xy'z)'\right]'$

Step(4):



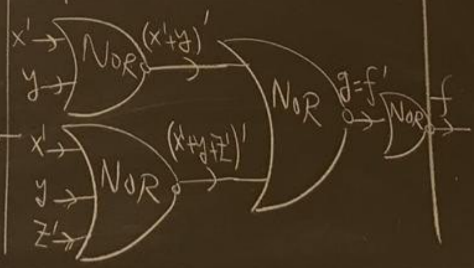
(ii) NOR

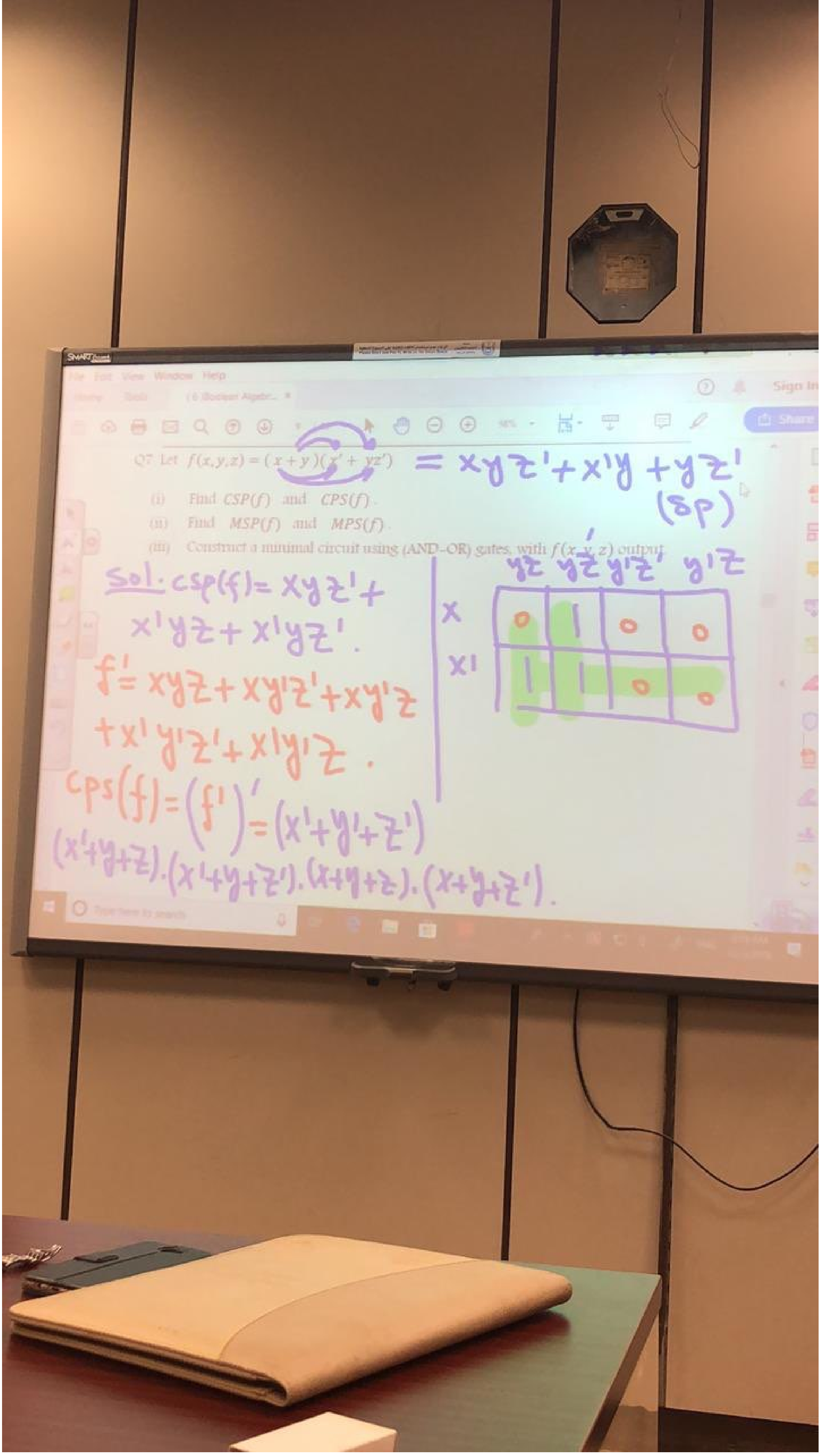
Step(1): (*) ⇒ $f' = (x'+y)\cdot(x'+y+z') = g(x,y,z)$ (PS)

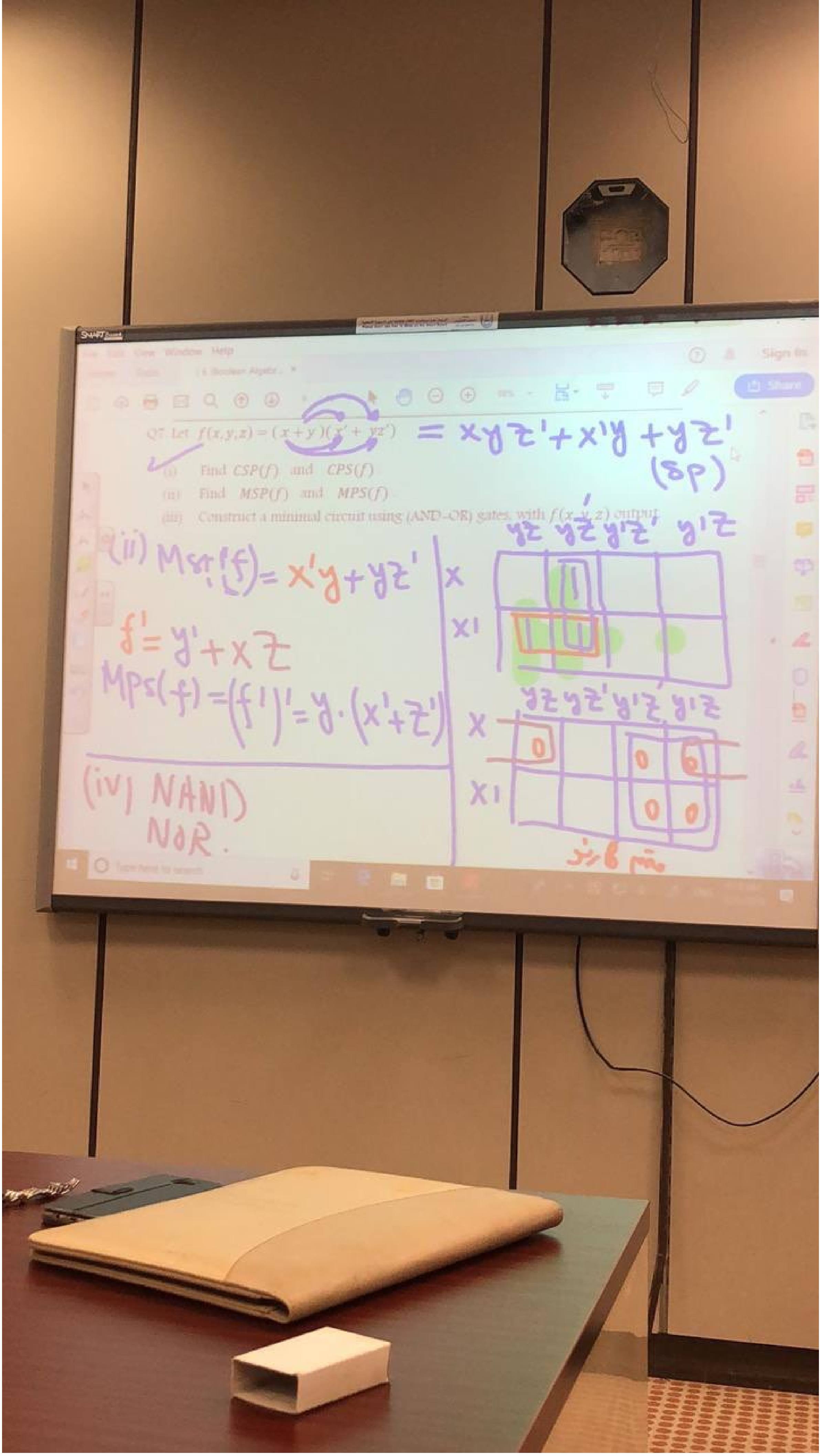
Step(2): $g' = (x'+y)' + (x'+y+z')'$

Step(3): $g = (g')' = \left[(x'+y)' + (x'+y+z')'\right]'$

Step(4):




Q7 Let f(x,y,z) = (x + y)(x' + yz')
= xyz' + x'y + yz'
(SP)
(i) Find CSP(f) and CPS(f).
(ii) Find MSP(f) and MPS(f).
(iii) Construct a minimal circuit using (AND–OR) gates, with f(x,y',z) output
Sol. CSP(f) = xyz' + x'yz + x'yz'.
f' = xyz + xy'z' + xy'z + x'y'z' + x'y'z.
CPS(f) = (f')' = (x'+y'+z')
(x'+y+z).(x'+y+z').(x+y+z).(x+y+z').
yz yz' y'z' y'z
x 0 1 0 0
x' 1 1 0 0


Q7. Let f(x,y,z) = (x+y)(x'+yz')
= xyz'+x'y+yz'
(SP)
(i) Find CSP(f) and CPS(f)
(ii) Find MSP(f) and MPS(f)
(iii) Construct a minimal circuit using (AND-OR) gates, with f(x,y,z) output
(ii) MSP(f) = x'y+yz'
f' = y'+xz
MPS(f) = (f')' = y·(x'+z')
(iv) NAND
NOR
yz yz' y'z' y'z
x
x'
x
x'

#7)

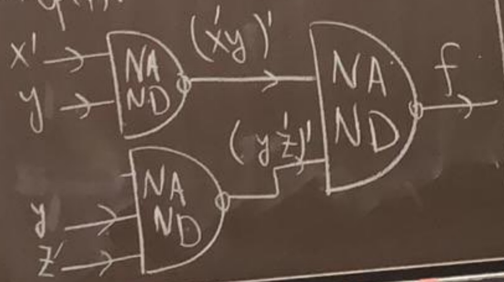
(ii) (a) NAND

Step(1): $Msp(f) = x'y + yz'$

Step(2): $f' = (x'y)' \cdot (yz')'$

Step(3): $f = (f')' = [(x'y)'(yz')']'$

Step(4):



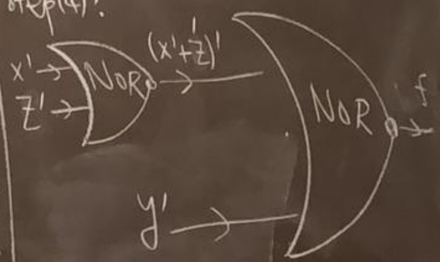
(b) NOR.

Step(1): $Mps(f) = y \cdot (x' + z')$

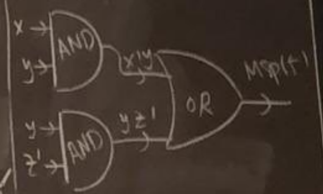
Step(2): $f' = y' + (x' + z')'$

Step(3): $f = (f')' = [y' + (x' + z')']'$

Step(4):

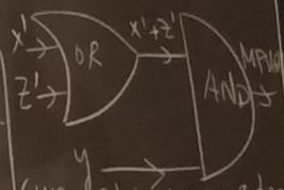


(iii) (a)

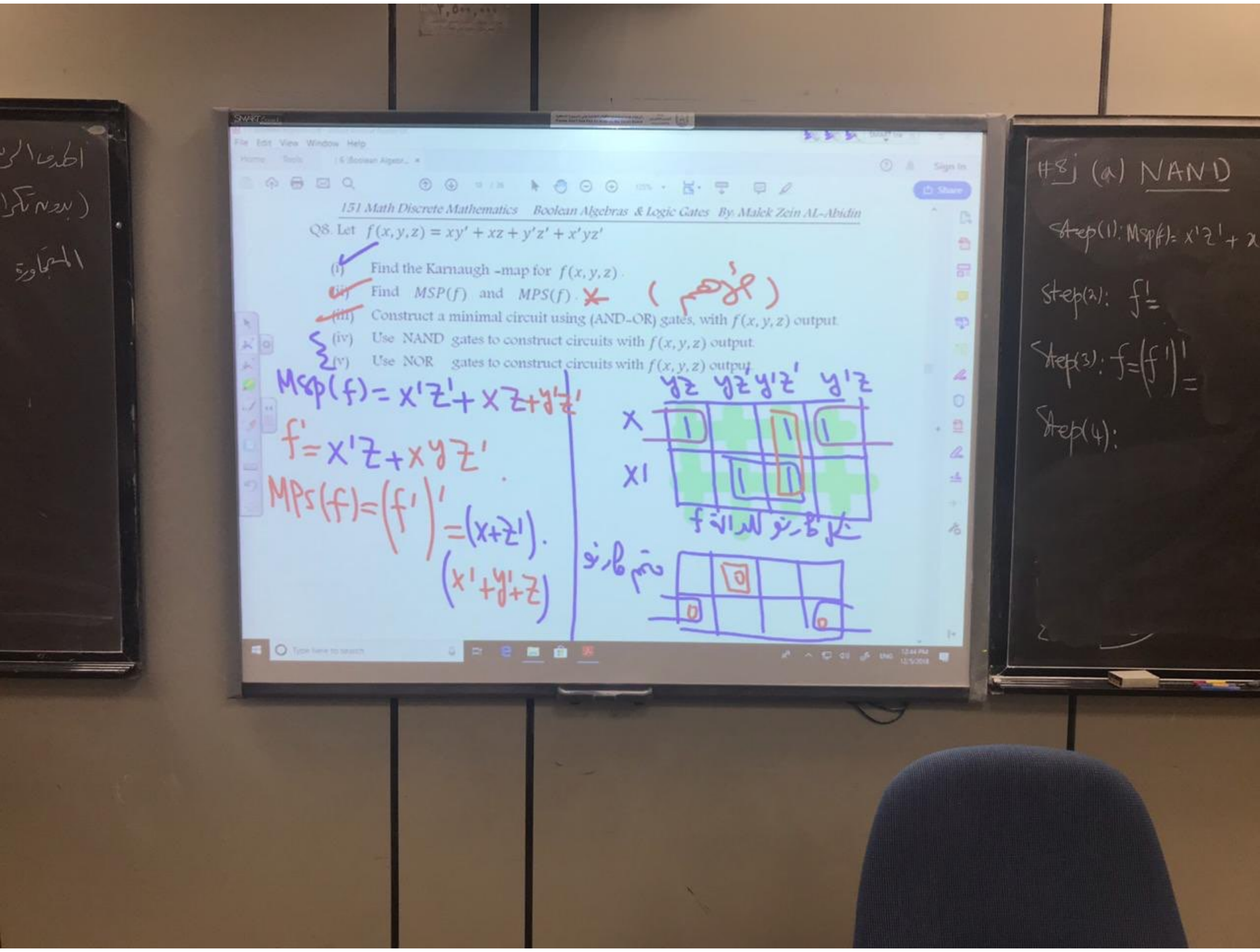


(AND-OR) circuit with 3 gates

(b)



(AND-OR) circuit with 2 gates (is the minimal).



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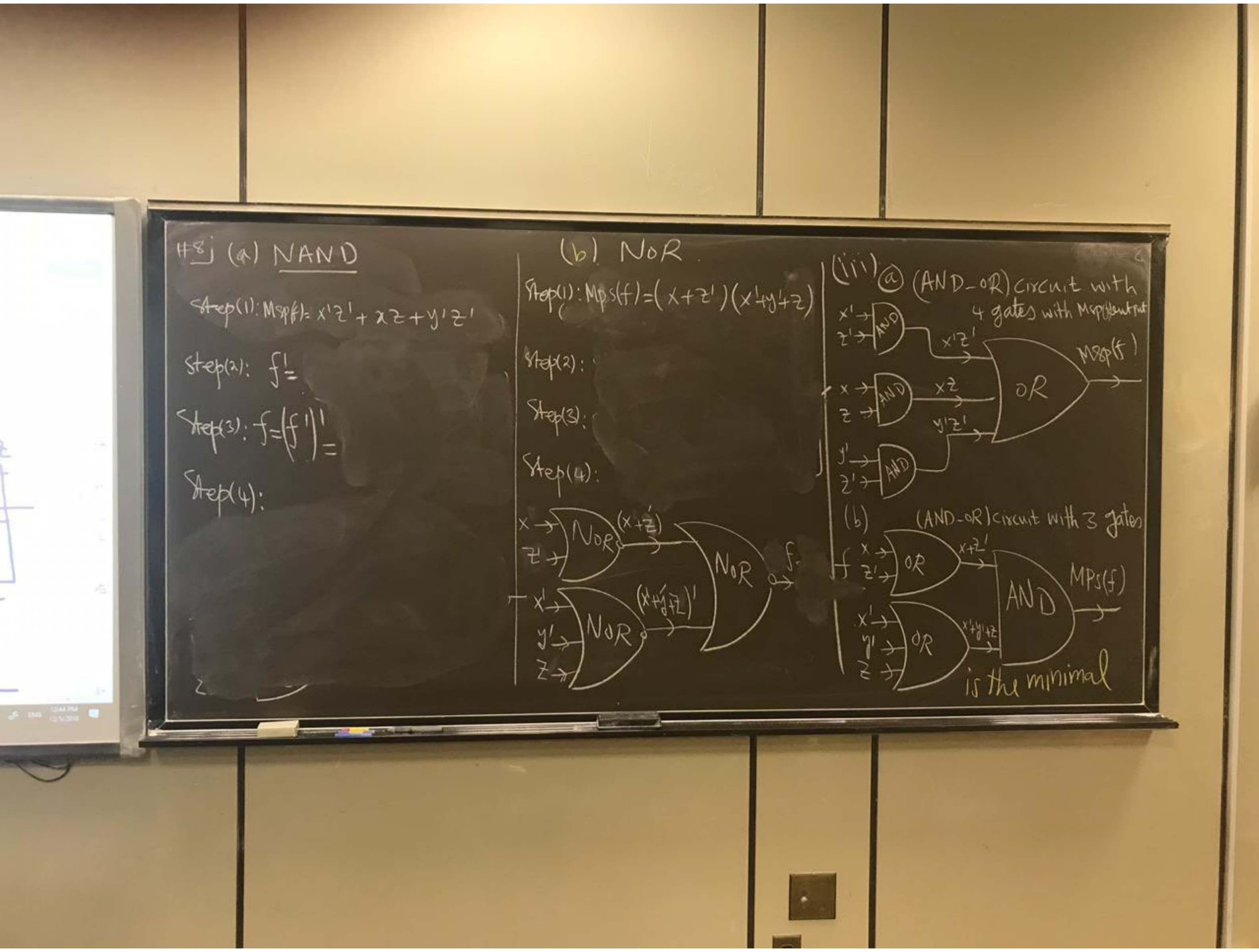
Q8. Let $f(x,y,z) = xy' + xz + y'z' + x'yz'$
(i) Find the Karnaugh -map for $f(x,y,z)$.
(ii) Find $MSP(f)$ and $MPS(f)$.
(iii) Construct a minimal circuit using (AND-OR) gates, with $f(x,y,z)$ output.
(iv) Use NAND gates to construct circuits with $f(x,y,z)$ output.
(v) Use NOR gates to construct circuits with $f(x,y,z)$ output.
$MSP(f) = x'z' + xz + y'z'$
$f' = x'z + xyz'$
$MPS(f) = (f')' = (x+z')\cdot(x'+y'+z)$
yz yz' y'z' y'z
x
x'
#8j (a) NAND
Step(1): $MSP(f) = x'z' + x$
Step(2): $f' =$
Step(3): $f = (f')' =$
Step(4):


#8j (a) NAND
Step(1): Msp(f) = x'z' + xz + y'z'
Step(2): f' =
Step(3): f = (f')' =
Step(4):
(b) NOR
Step(1): Mps(f) = (x+z')(x'+y'+z)
Step(2):
Step(3):
Step(4):
x
z'
NOR
(x+z')'
x'
y'
z
NOR
(x'+y'+z)'
NOR
f
(iii) (a) (AND-OR) circuit with
4 gates with Msp(f)
x'
z'
AND
x'z'
x
z
AND
xz
OR
Msp(f)
y'z'
y'
z'
AND
(b) (AND-OR) circuit with 3 gates
f
x
z'
OR
x+z'
x'
y'
z
OR
x'+y'+z
AND
MPs(f)
is the minimal

الهدف الرئيسي هو ايجاد أقل عدد ممكن من المستطيلات الأصلية (بدون تكرار) والتي تحتوي على أكبر عدد ممكن من الخلايا المتجاورة أفقياً أو عمودياً والتي عددها:

عدد الخلايا

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$ (خلية منفردة)

Q12. Let

#

	zw	zw'	$z'w'$	$z'w$
xy	0	1	0	1
xy'	1	1	1	1
$x'y'$	1	0	1	0
$x'y$	0	0	0	0

مقم طولها لـ f'

Be the Karnaugh -map for $f(x, y, z)$.

(i)Find $\underline{MSP(f)}$ and $\underline{MPS(f)}$

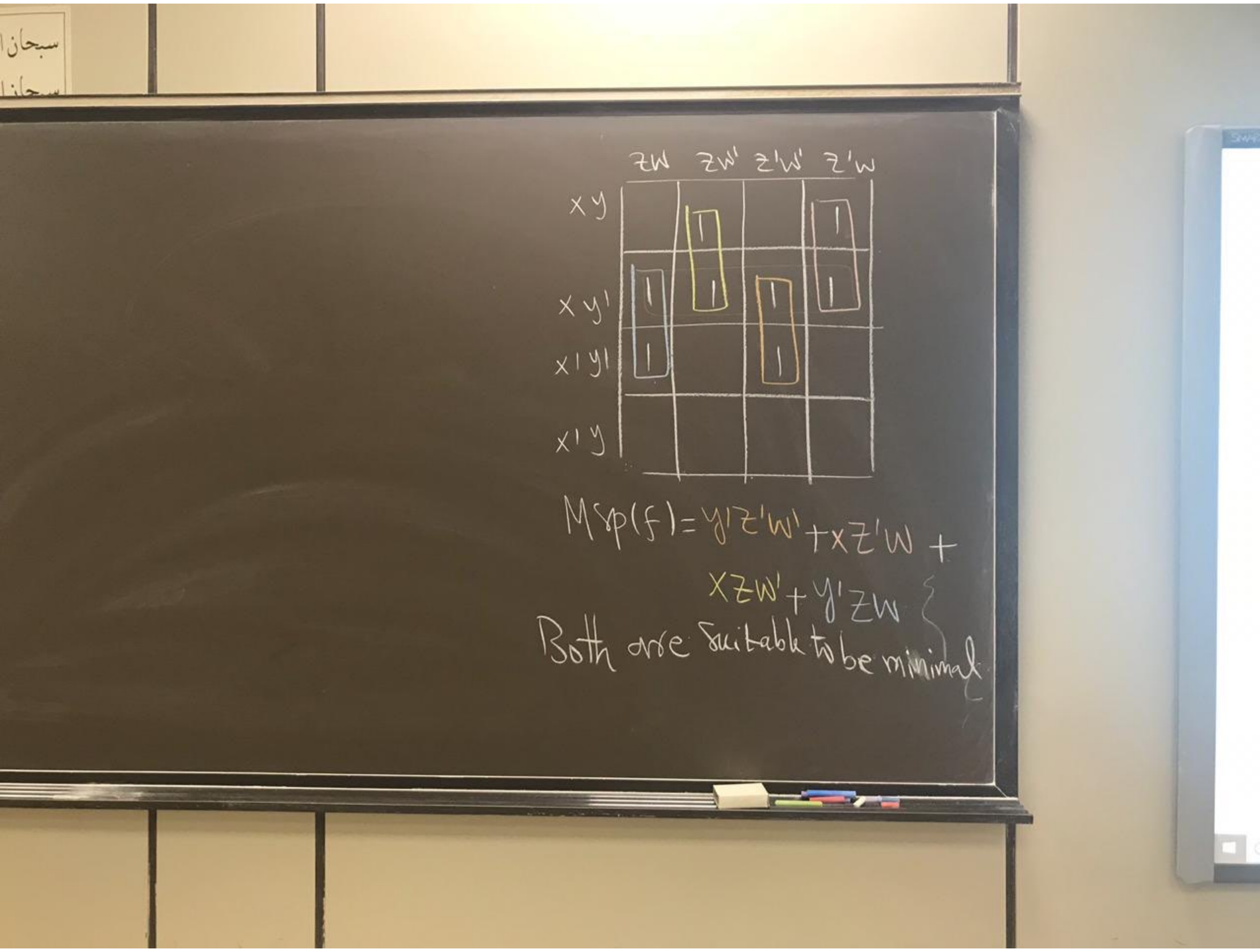
(ii)Construct a minimal circuit using (AND-OR) gates, with $f(x, y, z)$ output.

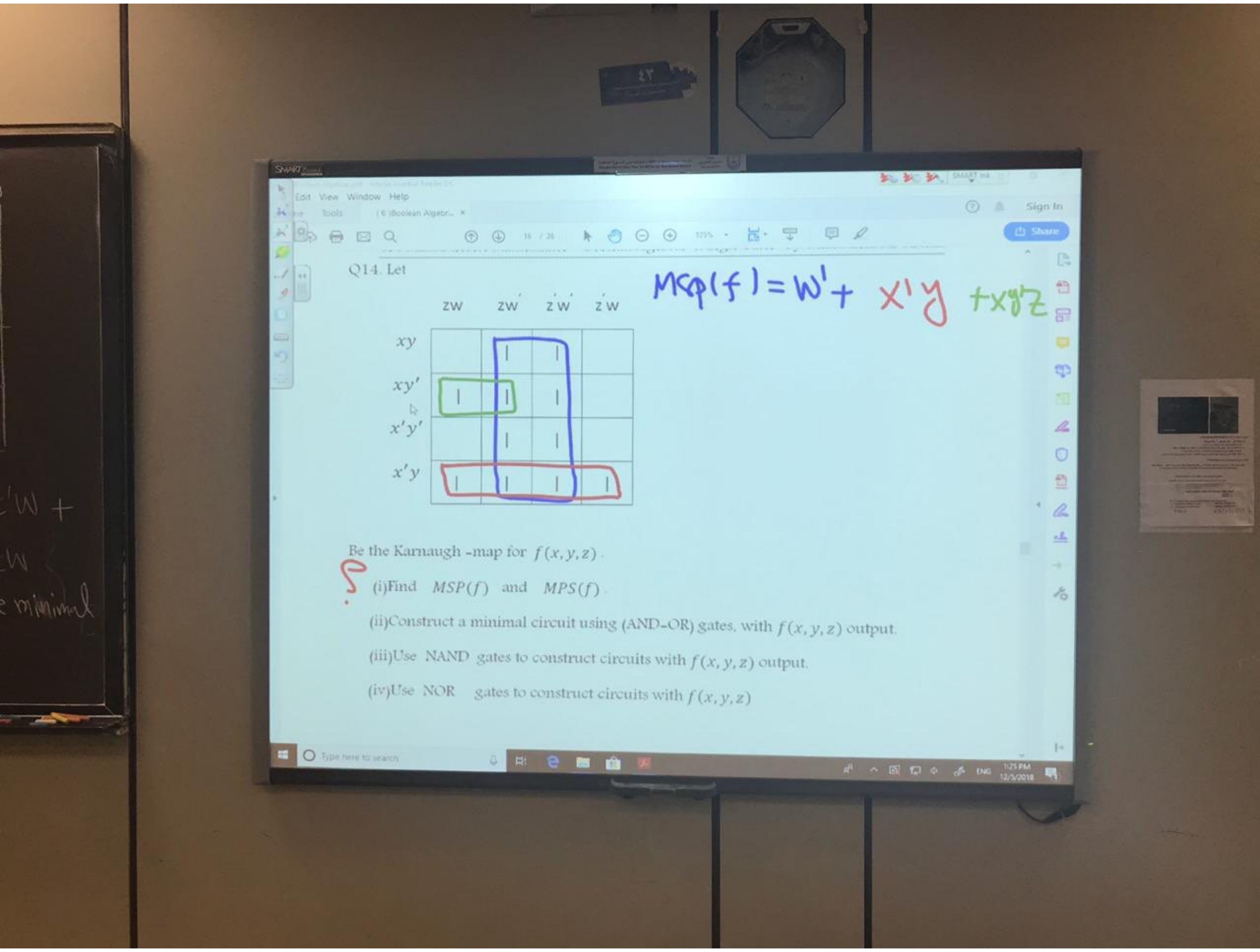
(iii)Use NAND gates to construct circuits with $f(x, y, z)$ output.

(iv)Use NOR gates to construct circuits with $f(x, y, z)$ output.

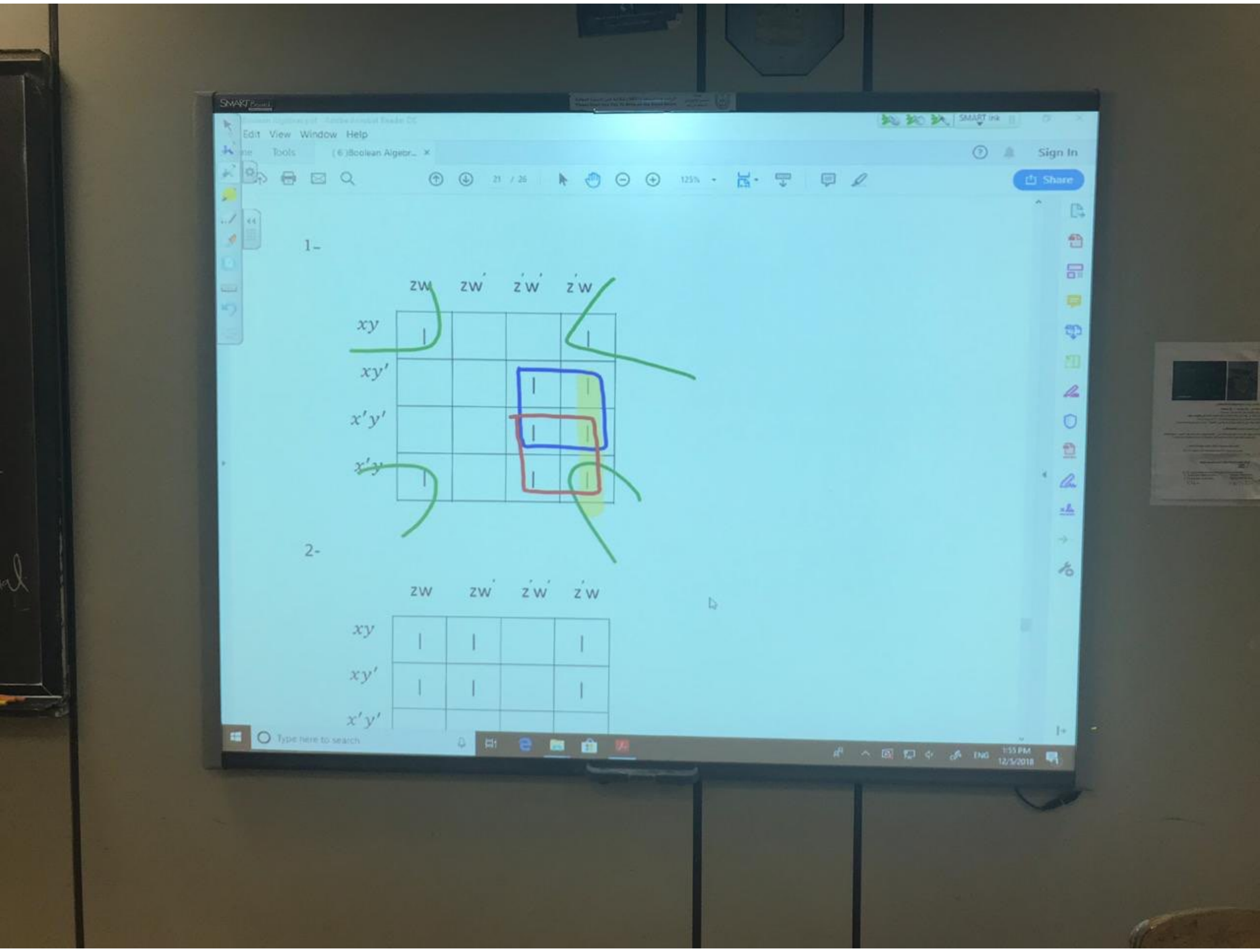
$$f' = x'z'w + yz'w' + yzw + x'zw'.$$

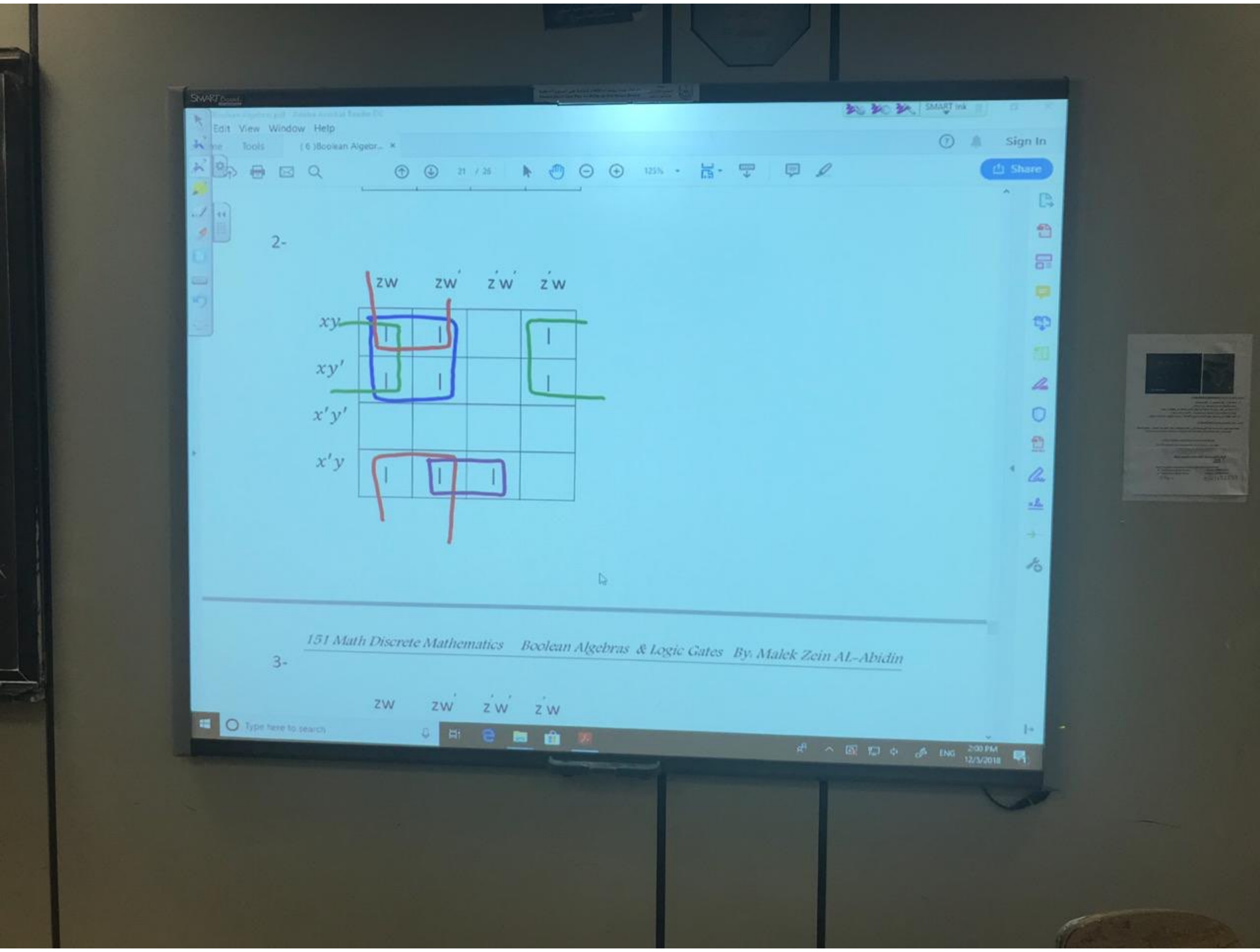
$$Mps(f) = (f')' = (x+z+w')\cdot(y'+z+w)(y'+z'+w')(x+z'+w)$$


zw zw' z'w' z'w
xy
xy'
x'y'
x'y
Msp(f) = y'z'w' + xz'w +
xzw' + y'zw
Both are suitable to be minimal



Q14. Let
zw zw' z'w' z'w
xy
xy'
x'y'
x'y
MSP(f) = W' + x'y +xy'z
Be the Karnaugh -map for f(x,y,z).
(i)Find MSP(f) and MPS(f).
(ii)Construct a minimal circuit using (AND-OR) gates, with f(x,y,z) output.
(iii)Use NAND gates to construct circuits with f(x,y,z) output.
(iv)Use NOR gates to construct circuits with f(x,y,z)


Edit View Window Help
Tools
Sign In
Share
1-
zw zw' z'w' z'w
xy
xy'
x'y'
x'y
2-
zw zw' z'w' z'w
xy
xy'
x'y'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Tools
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2-
zw zw' z'w' z'w
xy
xy'
x'y'
x'y
1 1 1
1 1 1
1 1 1
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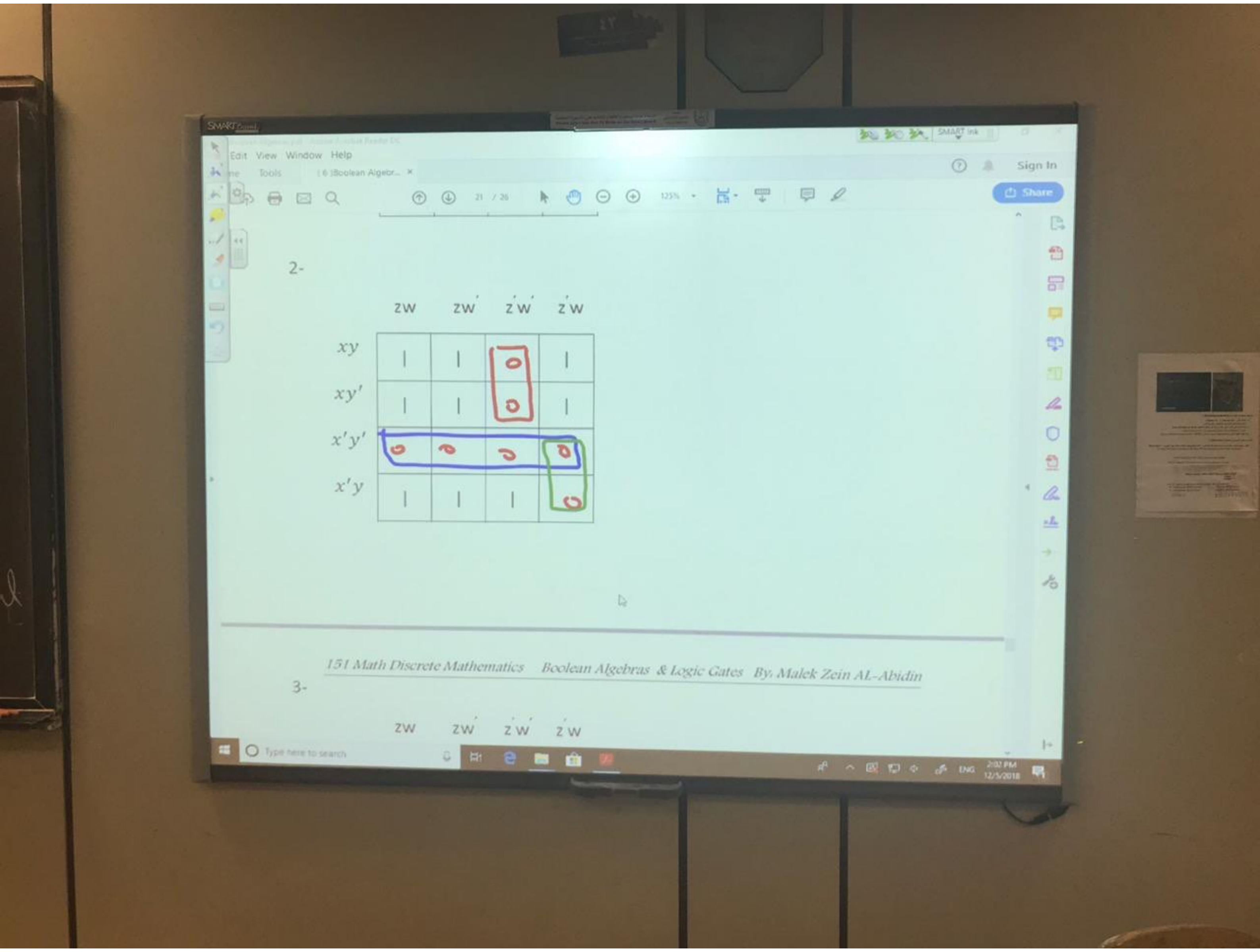
3-
zw zw' z'w' z'w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2-
zw zw' z'w' z'w
xy 1 1 0 1
xy' 1 1 0 1
x'y' 0 0 0 0
x'y 1 1 1 0
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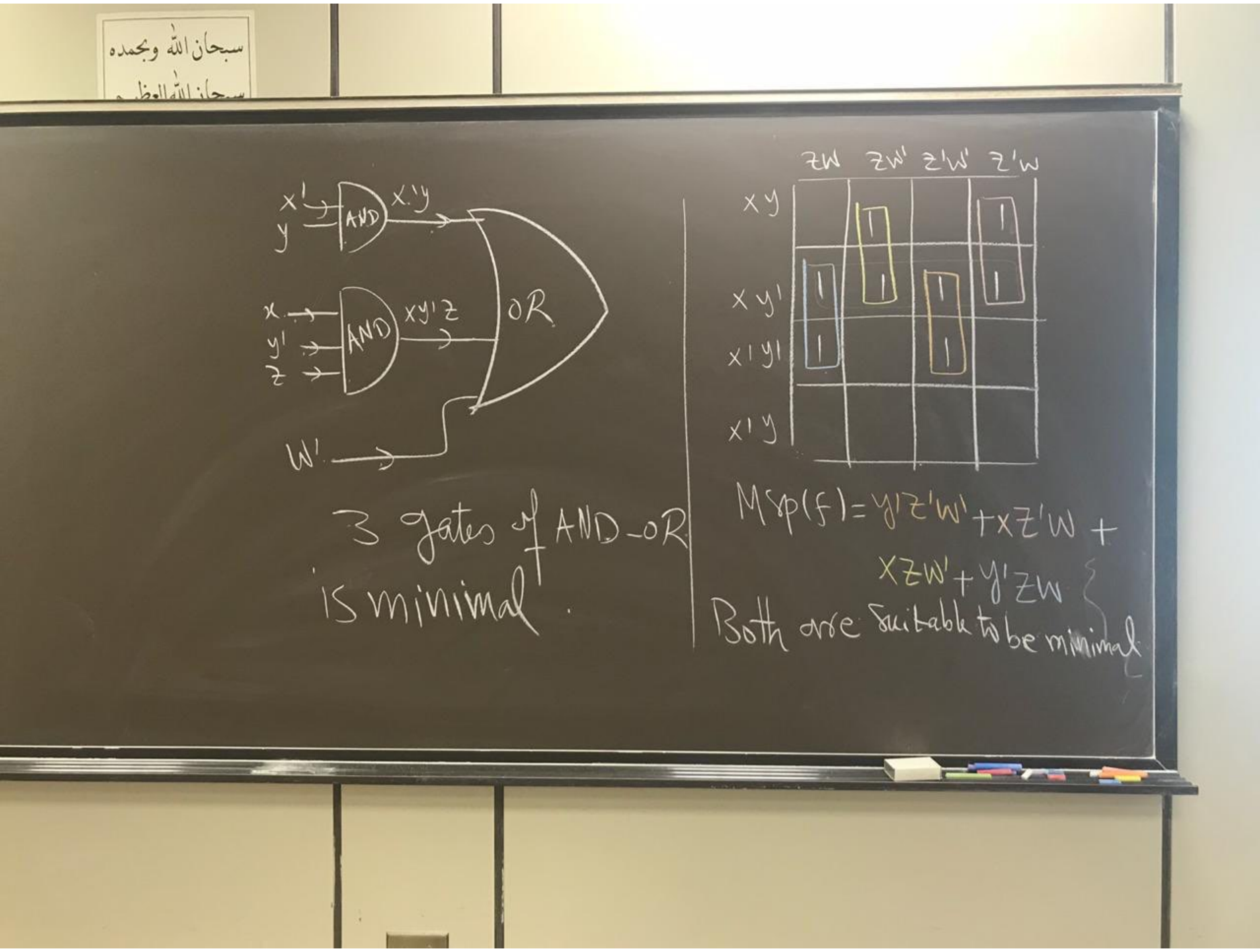
3-
zw zw' z'w' z'w
Type here to search



سبحان الله وبحمده
x'
y
AND
x'.y
x
y'
z
AND
xy'z
OR
W'
3 gates of AND-OR
is minimal.
zw z w' z'w' z'w
xy
xy'
x'y'
x'y
Msp(f) = y'z'w' + xz'w +
xzw' + y'zw
Both are suitable to be minimal

Q14. Let

	zw	zw'	$z'w'$	$z'w$
xy	0	1	1	0
xy'	1	1	1	0
$x'y'$	0	1	1	0
$x'y$	1	1	1	1

$MSP(f) = w' + x'y + xy'z$

$f' = x'y'w + y'z'w + xyw$

$MPS(f) = (f')' = (x+y+w')\cdot(y+z+w')\cdot(x'+y'+w')$

Be the Karnaugh -map for $f(x,y,z)$.

(i)Find $MSP(f)$ and $MPS(f)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $f(x,y,z)$ output.

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