1. Assume $\{a_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$a_1 = 3\,,\ \ a_2 = 6\ \ ,\ a_n = a_{n-1} + a_{n-2}\ :\forall n \geq 3$$

Prove that $3 | a_n$ for all integers $n \geq 1$.

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2. Assume $\{a_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$a_1 = 8\,,\ a_2 = 4\ ,\ a_n = a_{n-1} + a_{n-2}\ :\forall n \geq 3$$

Prove that a_n is even for all integers $n \geq 1$.

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3. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 9\,,\ a_1 = 15\ ,\ a_n = \frac{a_{n-1}\,a_{n-2}}{3} + 6\ :\forall n \geq 2$$

Prove that $3|a_n$ for all integers $n \geq 0$

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4. Assume $\{u_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$u_1 = 1\,,\ u_2 = 2\ ,\ u_n = 2u_{n-1} - u_{n-2}\ :\forall n \geq 3$$

Prove that $u_n = n$ for all positive integers n.

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5. Assume $\{a_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$a_1 = -1\,,\ a_2 = -\frac{1}{2}\,,\ a_3 = -\sqrt{10}\ ,\ a_{n+1} = a_n . a_{n-1} . a_{n-2}\ :\forall n \geq 3$$

Prove that $a_n \leq 0$ for all positive integers n.

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6. Assume $\{a_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$a_1 = 1\,,\ a_2 = 5\ ,\ a_{n+1} = 2a_n + 3\,a_{n-1}\ :\forall n \geq 2$$

Prove that $3^n \leq a_{n+1} \leq 2\,.\,3^n$ for all positive integers n.

7. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 1 \ , \quad a_n = 2a_{n-1} + 1 \quad : \forall n \geq 1$$

Prove that $a_n = 2^{n+1} - 1$ for all integers $n \geq 0$

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8. Assume $\{a_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$a_1 = 3 \,, \quad a_2 = 9 \,, \quad a_3 = 15 \ , \quad a_{n+1} = a_n + a_{n-1} + a_{n-2} \quad : \forall n \geq 3$$

Show that a_n is an integer divisible by 3, for all integers $n \geq 1$

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9. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 1 \,, \quad a_1 = 1 \ , \quad a_{n+1} = 2a_n + a_{n-1} \quad : \forall n \geq 1$$

Show that a_n is odd for all integers $n \geq 0$

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10. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 0 \,, \quad a_1 = 4 \ , \quad a_{n+1} = -2a_n + 3\, a_{n-1} \quad : \forall n \geq 1$$

Show that $a_n = 1 - (-3)^{n}$ for all integers $n \geq 0$

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11. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 0 \ , \quad a_1 = 2 \ , \quad a_{n+1} = 4a_n - 3a_{n-1} \quad : \forall n \geq 1$$

Show that $a_n = 3^n - 1$ for all integers $n \geq 0$

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12. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 9 \,, \quad a_1 = 15 \ , a_2 = 3 \qquad a_n = \frac{a_{n-1}\, a_{n-2}\, a_{n-3}}{9} + 6 \quad : \forall n \geq 3$$

Show that $3 | a_n$ for all integers $n \geq 0$

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13. Assume $\{u_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$u_1 = 1\,, u_2 = 2\,, u_3 = 3\,,\ u_n = 3u_{n-1} - u_{n-2} - u_{n-3} - 2 : \forall n \geq 4$$

Show that $u_n = n$ for all integers $n \geq 1$.

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14. Assume $\{a_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$a_1 = 1\,,\ a_2 = 2\,, a_3 = 3\ ,\ \ a_n = \frac{a_{n-1} + a_{n-2} + a_{n-3}}{3} : \forall n \geq 4$$

Show that $1 \leq a_n \leq 3$ for all integers $n \geq 1$

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15. Assume $\{u_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$u_1 = \frac{3}{4}\,,\ u_2 = \frac{8}{13}\ ,\ \ u_n = \frac{3\,u_{n-1} + 2\,u_{n-2} - 3}{3} : \forall n \geq 3$$

Show that $u_n < 1$ for all integers $n \geq 1$

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16. Assume $\{u_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$u_1 = 2\,,\ u_2 = 4\ ,\ \ u_n = \frac{2u_{n-1} + u_{n-2} + 8}{3} : \forall n \geq 3$$

Show that $u_n = 2n$ for all positive integers n.

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17. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 1\,,\ \ a_1 = 2\ ,\ a_2 = 3\ ,\ \ a_n = a_{n-1} + a_{n-2} + 2a_{n-3} : \forall n \geq 3$$

Show that $a_n \leq 3^n$ for all integers $n \geq 0$

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18. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 1\,,\ \ a_1 = 2\ ,\ \ a_n = 4a_{n-1} - 4a_{n-2} : \forall n \geq 2$$

Prove that $a_n = 2^n$ for all nonnegative integers n.

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19. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 1\,,\quad a_1 = 1\quad,\quad a_n = 4a_{n-1} - 4a_{n-2}\quad : \forall n \geq 2$$

Show that $a_n = 2^n - n2^{n-1}$ for all integers $n \geq 0$

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20. Assume $\{a_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$a_1 = 1\,,\ a_2 = 3\quad,\quad a_n = a_{n-1} + a_{n-2}\quad : \forall n \geq 3$$

Prove that $a_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$ for all positive integers *n*.

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21. Assume $\{a_n\}_{n=1}^{\infty}$ is a "Fibonacci" sequence defined as:

$$a_1 = 1\,,\ a_2 = 1\quad,\quad a_n = a_{n-1} + a_{n-2}\quad : \forall n \geq 3$$

Prove that $a_n \leq \left(\frac{1+\sqrt{5}}{2}\right)^n$ for all positive integers *n*.

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22. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 1\,,\ a_1 = 1\ ,a_2 = 3\ ,\quad a_n = a_{n-1} + a_{n-2} + a_{n-3}\quad : \forall n \geq 3$$

Show that $a_n < 3^n$ for all integers $n \geq 0$

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23. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 2\,,\ a_1 = 4\ ,a_2 = 6\ ,\quad a_n = 5a_{n-3}\quad : n = 3,4,5,\ldots$$

Show that $2|a_n$ for all integers $n \geq 0$

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24. Assume $\{a_n\}_{n=1}^{\infty}$ is a "Fibonacci" sequence defined as:

$$a_1 = 1\,,\ a_2 = 2\quad,\quad a_n = 2a_{n-1} + a_{n-2}\quad : \forall n \geq 2$$

Prove that $a_n \leq \left(\frac{5}{2}\right)^n$ for all positive integers *n*.

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25. Assume $\{u_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$u_0 = 2\,,\ u_1 = 3\quad,\quad u_{n+1} = 3u_n - 2u_{n-1} - 1\ : \forall n \geq 1$$

Show that $u_n = n + 2$ for all integers $n \geq 0$

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26. Assume $\{u_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$u_1 = 0\,,\ u_2 = 1 \quad , \quad u_{n+1} = 3u_n - 2u_{n-1} - 1 \ \ for \ \ n = 2,3,4,\ldots$$

Show that $u_n = n - 1$ for all integers $n \geq 1$

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27. Assume $\{u_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$u_0 = 12\,,\ u_1 = 21 \quad , \quad u_{n+1} = \frac{(u_n)^2\ u_{n-1}}{9} \quad for \ \ n = 1,2,3,\ldots$$

Show that u_n is an integer divisible by 3, for all integers $n \geq 0$

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28. Assume $\{u_n\}_{n=1}^{\infty}$ is a sequence defined as:

$$u_1 = 2\,,\ u_2 = 5 \quad , \quad u_{n+1} = 2u_n - u_{n-1} + 2 \quad for \ \ n = 2,3,4,\ldots$$

Show that $u_n = n^2 + 1$ for all integers $n \geq 1$

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29. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 2 \ , \quad a_1 = 4 \ , \ a_{n+1} = 4a_n - 3a_{n-1} \ : \forall n \geq 1$$

Show that $a_n = 1 + 3^n$ for all integers $n \geq 0$

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30. Assume $\{a_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$a_0 = 2 \ , \quad a_1 = 5 \ , \ a_{n+1} = 5a_n - 6a_{n-1} \ : \forall n \geq 1$$

Show that $a_n = 2^n + 3^n$ for all integers $n \geq 0$

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31. Assume $\{u_n\}_{n=0}^{\infty}$ is a sequence defined as:

$$u_0 = 2 \ , \quad u_1 = 6 \ , \ u_{n+1} = 3u_n + 10u_{n-1} - 12 \ : \forall n \geq 1$$

Show that $u_n = 5^n + 1$ for all integers $n \geq 0$