12. Let T be the relation defined on the set $A = \{0,1,2,3,4\}$, such that:

$a,b \in A$, $aTb \Leftrightarrow 3|(2a+b)$, 3 devides $(2a+b)$

✓ (i) *Show that T is an equivalence relation.*

(ii) *Find the equivalence classes* $[0],[1]$

(iii) *Find the number of equivalence classes of the relation* T.

Solution : $aTb \Rightarrow 3|(2a+b) \Rightarrow 2a+b=3h : h\in\mathbb{Z}$

(i) ① $\forall a\in A$, $3|(2a+a) \Rightarrow 3|3a \Rightarrow \therefore aTa \Rightarrow T$ is refl.

② $a,b\in A$: $aTb \Rightarrow 2a+b=3h$

الطرف

$bTa \Rightarrow 2b+a=3K$?

$b=3h-2a \overset{\times 2}{\Rightarrow} 2b=6h-4a \overset{+a}{\Rightarrow} 2b+a=6h-3a$

$\Rightarrow 2b+a=3(\underbrace{2h-a}_{K\in\mathbb{Z}})=3K \Rightarrow \therefore bTa \Rightarrow T$ is symmetric.

③ $a,b,c\in A$: $aTb \Rightarrow 2a+b=3h_1$ & $bTc \Rightarrow 2b+c=3h_2$ $: h_1, h_2\in\mathbb{Z}$

الطرف

aTc

$2a+c=3h$?

$(+)$ $2a+3b+c=3h_1+3h_2$

$\Rightarrow 2a+c=3(\underbrace{h_1+h_2-b}_{h\in\mathbb{Z}})=3h. \Rightarrow \therefore aTc \Rightarrow T$ is transt.

(Equiv.) T → refl., → symm., → transt.

(ii) $a\in A$

$[a]=\{x\in A : aRx\}=\{x\in A : 2a+x=3h : h\in\mathbb{Z}\}$.

$x=-2a+3h$

$[0]=\{x\in A : x=2(0)+3h : h\in\mathbb{Z}\}=\{x\in A : x=3h : h\in\mathbb{Z}\}$

$=\{0,3\}$

$[1]=\{x\in A : x=-2(1)+3h : h\in\mathbb{Z}\}=\{x\in A : x=-2+3h : h\in\mathbb{Z}\}=\{1,4\}$

$[2]=\{2\}$ $\Rightarrow$ The number of equiv. classes $=\boxed{3}$

$J(A)=\{\underset{A_1}{\{0,3\}},\underset{A_2}{\{1,4\}},\underset{A_3}{\{2\}}\} : \bigcup_{i=1}^{3}[a_i]=A$