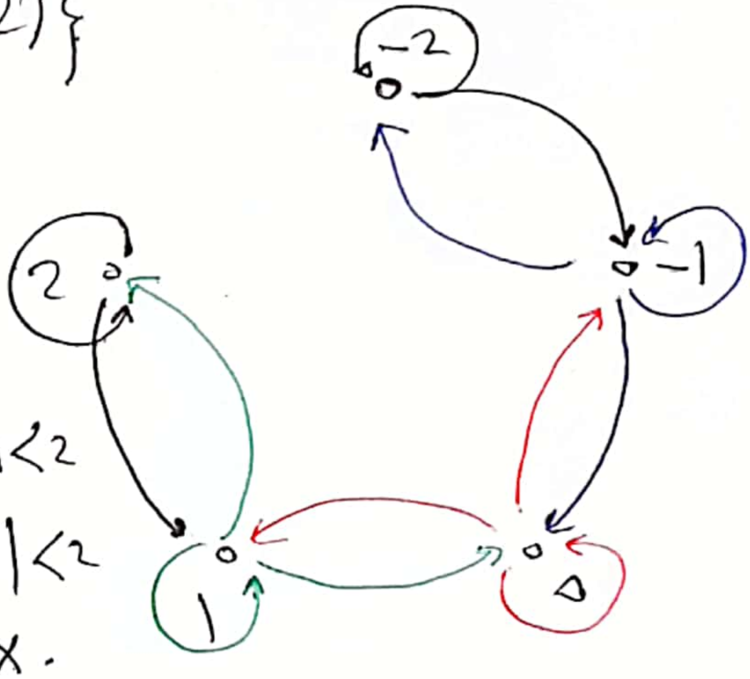
Let R be a relation defined on the set,
$A = \{-2, -1, 0, 1, 2\} \subset \mathbb{Z}$

$$x, y \in A, \quad xRy \iff |x-y| < 2.$$

Sol $R = \{(-2,-2), (-2,-1), (-1,-2), (-1,-1), (-1,0)$
$(0,-1), (0,0), (0,1), (1,0), (1,1), (1,2)$
$(2,1), (2,2)\}$



① $\forall x \in A, |x-x| = 0 < 2$
$\therefore xRx \Rightarrow R$ is refl.

② $x, y \in A : xRy \Rightarrow |x-y| < 2$
$\Rightarrow |-1(y-x)| = |-1|\,|y-x| < 2$
$\Rightarrow |y-x| < 2 \Rightarrow yRx.$
$\therefore R$ is symm.

③ R is not antisymm.
$\Downarrow$
$(1,2) \wedge (2,1) \in R$, but $1 \neq 2$.

④ R is not transitive
$\Downarrow$
$(0,1) \wedge (1,2) \in R$, but $(0,2) \notin R$.

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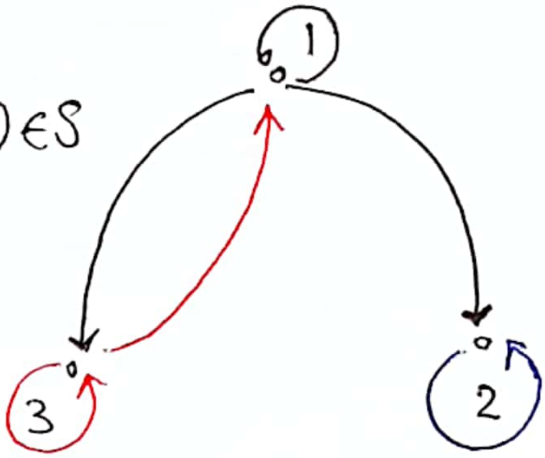
? Same # 2

Q3. Let $S = \{(1,1), (1,2), (1,3), (2,2), (3,1), (3,3)\}$ be a relation on the set $B = \{1,2,3\}$

(i) Find S^2

(ii) Determine whether S is reflexive, symmetric, antisymmetric, transitive

(i) $S^2 = \{(1,1), (1,2), (1,3), (2,2), (3,1), (3,2), (3,3)\}$



(ii)

① S is refl. : $(1,1), (2,2)$ and $(3,3) \in S$

② S is not symm

$\Downarrow$

$(1,2) \in S$, but $(2,1) \notin S$

③ S is not antisymm.

$\Downarrow$

$(1,3) \wedge (3,1) \in S$, but $1 \neq 3$.

④ S is not transitive:

$\Downarrow$

$(3,1) \wedge (1,2) \in S$, but $(3,2) \notin S$

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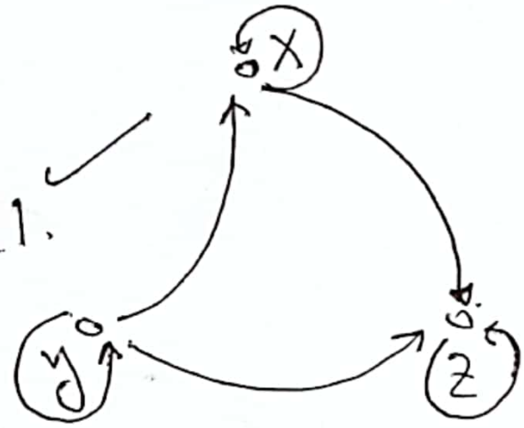
Q2. Let $T = \{(x,x),(x,z),(y,x),(y,y),(y,z),(z,z)\}$ be a relation on the set $B = \{x,y,z\}$

(i) Find T^2

(ii) Determine whether T is reflexive, symmetric , antisymmetric , transitive

(i) $T^2 = T \circ T = \{(x,x),(x,z),(y,x),(y,z),(y,y),(z,z)\}$

(ii)



① $\because (x,x),(y,y)$ and $(z,z) \in T \Rightarrow T$ is refl.

② $\because (y,z) \in T$, but $(z,y) \notin T \Rightarrow \therefore T$ is not symm.

③ $(x,z) \in T$, but $(z,x) \notin T$
$(y,z) \in T$, but $(z,y) \notin T$
$(y,x) \in T$, but $(x,y) \notin T$
$\Rightarrow T$ is antisymm.

$x T z$ but $z \not T x$

$\left.\begin{matrix} xRy \\ y \hat{R} x \end{matrix}\right\} \Rightarrow x = y$

or xRy but $y \not R x$
$\Downarrow$
R antisym

④ $\because (y,x) \in T \wedge (x,z) \in T$ and $(y,z) \in T \Rightarrow \therefore T$ is transitive.

Q2 (i) T: (x,x) (x,z) (y,x) (y,y) (y,z) (z,z)

T: (x,x) (x,z) (y,x) (y,y) (y,z) (z,z)

$T \circ T = T^2 = \{(x,x),(x,z),(y,x),(y,z),(y,y),(z,z)\}$

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Q1. **(a)** Let R be the relation on $A = \{1,2,3\}$ such that $m\ R\ n$ if and only if $m^2 \geq 2n$.

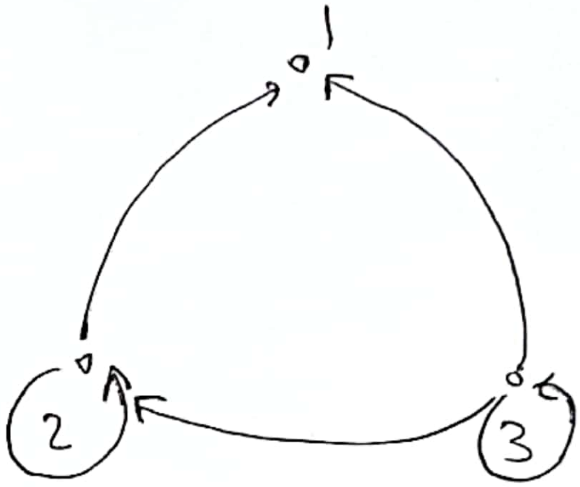
✓ (i) List all ordered pairs of the relation R. (2 pts)

✓ (i) Represent the relation R by a digraph. (1 pt)

(ii) Determine whether the relation R is reflexive, symmetric, antisymmetric, transitive.

(i) $R = \{(2,1), (2,2), (3,1), (3,2), (3,3)\}$

(ii)



(iii)

① $\because (1,1) \notin R$ or $1 \not R 1 \Rightarrow R$ is not refl.

② $\because (2,1) \in R$ but $(1,2) \notin R \Rightarrow R$ is not symm.

③ $\because (2,1) \in R$, but $(1,2) \notin R$
and $(3,2) \in R$, but $(2,3) \notin R$
and $(3,1) \in R$, but $(1,3) \notin R$
$\therefore R$ is antisymm. ✓

④ $(3,2) \wedge (2,1) \in R$ and $(3,1) \in R \Rightarrow$
R is transite ✓

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14. Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2, 3, 4\}$

$$a, b \in A, \qquad a\,R\,b \Leftrightarrow a^2 = b^2$$

(i) List all the ordered pairs of the relation R ?

(ii) Determine whether the relation R is *reflexive*, *symmetric*, ***antisymmetric***, and/or *transitive*.

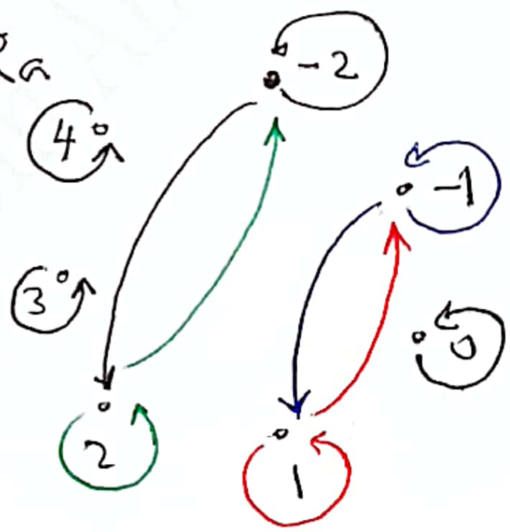
and/or *transitive*.

Solution:

(i) $R = \{(-2,-2), (-2,2), (-1,-1), (-1,1), (0,0), (1,-1),$
$(1,1), (2,-2), (2,2), (3,3), (4,4)\}$

(ii) ① $\forall a \in A,\ a^2 = a^2 \Rightarrow \therefore aRa$

$\therefore R$ is refl.



② $a, b \in A : aRb \Rightarrow a^2 = b^2$

$\Rightarrow b^2 = a^2$

$\Rightarrow bRa$

$\therefore R$ is symm.

③ R is not antisymm.

$(-2,2) \wedge (2,-2) \in R$. but $-2 \neq 2$.

④ $a, b, c \in A$: $\left.\begin{array}{l} aRb \Rightarrow a^2 = b^2 \\ \wedge \\ bRc \Rightarrow b^2 = c^2 \end{array}\right\} \Rightarrow a^2 = c^2$

$\Rightarrow \therefore aRc$

R is transitive.

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20. Let T be a relation defined on the set $\mathbb{N} = \{1, 2, 3, \ldots\}$,

$$m, n \in \mathbb{N}, \quad m\,T\,n \Leftrightarrow m + n > 3$$

Determine whether the relation R is *reflexive*, *symmetric*, *antisymmetric*, and/or *transitive*.

Solution:

① $1+1 = 2 \not> 3 \Rightarrow \therefore 1 \not{T} 1 \Rightarrow \therefore T$ is not refl.

② $m, n \in \mathbb{N}$; $mTn \Rightarrow m+n > 3$

الإبدالية (Comm.) $\Rightarrow n+m > 3 \Rightarrow nTm$.

$\therefore T$ is symm.

③ T is not antisymm.

$\Downarrow$

$4T5 : 4+5 = 9 > 3$, but $4 \neq 5$.

$\wedge$

$5T4 : 5+4 = 9 > 3$

④ T is not transitive:

$\Downarrow$

$1T4 : 1+4 = 5 > 3$ $\Rightarrow$ but $1 \not{T} 1 : 1+1 = 2 \not> 3$

$\wedge$

$4T1 : 4+1 = 5 > 3$

$\therefore T$ is only symm.

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Q. $m \in \mathbb{Z} : \overset{p}{3 \nmid m} \Rightarrow \overset{q}{3 \nmid (m+1)^2+2m^2+5}$.

Sol. Assume $3 \mid (m+1)^2+2m^2+5 \equiv \neg q$

$(m+1)^2+2m^2+5 = 3h : h \in \mathbb{Z}$.

$m^2+2m+1+2m^2+5 = 3h$.

$2m = 3h-3m^2-6 = 3(\underbrace{h-m^2-2}_{K \in \mathbb{Z}}) = 3K$

$\Rightarrow 3 \mid 2m \Rightarrow \because 3 \nmid 2 \Rightarrow \therefore 3 \mid m \equiv \neg p$

Gauss.

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if $a \mid b.c \Rightarrow a \mid b \vee a \mid c$.

Gauss

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