King Saud University

College of Science

Department of Mathematics

151 Math Exercises

(4.1)

Relations and Their Operations

Malek Zein AL-Abidin

1442هـ

2021

EXERCISES

1. Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2, 3, 4\}$

$$m, n \in A \quad , \quad m \, R \, n \Leftrightarrow n = m^2$$

(i) List all ordered pairs of R ?
(ii) Find the domain and the image of R ?
(iii) Represent R by the directed graph (diagraph) ?

Solution:

2. Let R be a relation defined on the set $A = \{1,2,3,4,5\}$

$$x, y \in A, \qquad x\,R\,y \Leftrightarrow xy \leq 9$$

(i) List all ordered pairs of R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?
(iv) Represent R with a matrix ?

Solution:

3. Let R be a relation defined on the set $A = \{-4,-3,-2,-1,0,1,2,3,4\}$

$$x, y \in A, \qquad x\,R\,y \Leftrightarrow y = 2x$$

(i) List all ordered pairs of R ?

(ii) Find the domain and the image of R ?

Solution:

4. Let R be a relation defined from the set $A = \{1,2,3,4\}$ to the set $B = \{2,3,4,5\}$

$$a\ R\ b \Leftrightarrow a + b = 5$$

(i) List all ordered pairs of R ?

(ii) Find the domain and the image of R ?

(iii) Represent R with a matrix ?

Solution:

5. Let R be a relation defined on the set $A = \{0,1,2,3\}$

$$a, b \in A, \qquad a\ R\ b \Leftrightarrow a + b = 4$$

(i) List all ordered pairs of R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?
(iv) Represent R with a matrix ?

Solution:

6. Let R be a relation defined on the set $A = \{2,3,4,5,6\}$

$$a, b \in A, \qquad a\ R\ b \Leftrightarrow a.b < 10$$

(i) List all ordered pairs of R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?
(iv) Find $\mathbf{M}_R$.

Solution:

7. Let *R* be a relation defined on the set $A = \{-2, -1, 0, 1, 2, 3, 4\}$

$$a, b \in A, \qquad a\ R\ b \Leftrightarrow a^2 = b^2$$

(i) List all ordered pairs of R ?
(ii) Find the domain and the image of *R* ?
(iii) Draw the directed graph (diagraph) that represents *R* ?

Solution:

8. Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2\}$

$$a, b \in A, \qquad a\ R\ b \Leftrightarrow a.b < 0$$

(i) List all ordered pairs of R ?
(ii) Find the domain and the image of R ?
(iii) Draw the directed graph (diagraph) that represents R ?
(iv) Find R^2 .

Solution:

9. Let R be a relation defined on the set $A = \{-2, -1, 0, 1, 2\}$

$$a, b \in A, \qquad a\ R\ b \Leftrightarrow a.b \geq 2$$

(i) List all ordered pairs of R ?

(ii) Find the domain and the image of R ?

(iii) Draw the directed graph (diagraph) that represents R ?

(iv) Find R^2 .

Solution:

10. Let $S = \{(1,1),(1,2),(1,3),(2,2),(3,1),(3,3)\}$ be a relation on the set $B = \{1, 2, 3\}$

(i) Draw the directed graph (diagraph) that represents S ?

(ii) Find S^2 , S^{-1} , $\bar{S}$, $S o \bar{S}$, $\bar{S} o S$, $\bar{S} - S^{-1}$, $S \, o \, S^{-1}$, $S^{-1} \, o \, S$, S^3 , $S \cap S^{-1}$.

(iii) Find $\mathbf{M}_S$

Solution:

11. Let $S = \{(a, c), (b, a), (c, b)\}$ be a relation on the set $B = \{a, b, c\}$.

(i) Find $\mathbf{M}_S$?

(ii) Find $\bar{S} - S^{-1}$

(iii) Find S^2 , S^3

Solution:

12. Let $S = \{(a,b),(b,c),(c,d),(d,a)\}$ be a relation on the set $B = \{a,b,c,d\}$.

(i) Find $\mathbf{M}_S$?

(ii) Find S^2

(iii) Find $S^{-1} \circ S$

Solution:

13. Let $S = \{(1,v),(1,w),(2,u),(2,v),(3,w)\}$ and
$T = \{(1,u),(1,w),(2,v),(2,w),(3,u),(3,v)\}$
are relations from the set $A = \{1,2,3\}$ to the set $B = \{u,v,w\}$.

(i) Find $\bar{S}$, $\bar{S} \cap T$, $T - \bar{S}$, $\overline{S \cup T}$

(ii) Find $T^{-1} \circ S$

(iii) Find $S^{-1} \circ T$

Solution:

14. Let $R = \{(a,c),(a,b),(b,b)\}$ and

$S = \{(a,a),(a,c),(b,c),(c,a)\}$

are relations on the set $A = \{a\ ,b\ ,c\ \}$

(i) Find $(\ R\ o\ S\ ) \cap R^{-1}$

(ii) Find $S^{-1} \circ R$

(iii) Find $\mathbf{M}_R$, $\mathbf{M}_S$, $\mathbf{M}_{R\cup S}$, $\mathbf{M}_{R\cap S}$, $\mathbf{M}_{R\circ S}$

Solution:

15. Let $T = \{(1,2),(1,3),(2,2),(2,3)\}$ and $S = \{(1,1),(1,3),(2,1),(3,2)\}$ are relations on the set $E = \{1, 2, 3\}$

(i) Find $T\ o\ S$, $\overline{T} \cap S$, $\overline{T}\ o\ \overline{S}$, $T^2\ o\ S^{-1}$

(ii) Find $\mathbf{M}_T$, $\mathbf{M}_S$, $\mathbf{M}_{T\cup S}$, $\mathbf{M}_{T\cap S}$, $\mathbf{M}_{T\circ S}$

Solution:

16. Let $R = \{(a,c),(b,a),(b,b)\}$ and $S = \{(a,b),(b,b),(c,a)\}$ are relations on the set $A = \{a, b, c\}$

(i) Find $\overline{R \cup S}$, $R^{-1} o S^{-1}$, $\overline{R} \cap S$, $R^2 o S$, $R \cap S^{-1}$, $R - S^{-1}$, $S o R$

(ii) Find $\mathbf{M}_R$, $\mathbf{M}_S$, $\mathbf{M}_{R \cup S}$, $\mathbf{M}_{R \cap S}$, $\mathbf{M}_{R \circ S}$

Solution:

17. Let R be the relation represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the matrix representing: a) R^{-1} b) $\bar{R}$ c) R^2

Solution:

18. Let R_1 and R_2 are relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices representing

a) $R_1 \cup R_2$ b) $R_1 \cap R_2$ c) $R_1 \circ R_2$ d) $R_2 \circ R_1$

Solution:

19. Let R be the relation represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Find the matrix representing:

a) R^2 b) R^3 c) R^4

Solution:

20. Let R be the relation from $A = \{1,2,3,4,5\}$ to $B = \{0,1,2,3\}$ defined as follows:

$a\ R\ b \Leftrightarrow a = b^2 + 1$

(i) List all ordered pairs of R

(ii) Represent the relation R by a matrix

(iii) Find the domain and image (range) of R .

21. Let R be a relation defined on the set $A = \{0,1,2,3,4,5,6,7\}$

$$x, y \in A \quad , \quad x\,R\,y \Leftrightarrow 2x - y = 4$$

(i) List all ordered pairs of R ?
(ii) Find the domain and the image of R ?
(iii) Represent R by the directed graph (diagraph) ?

Solution:

22. Let R be the relation from $A = \{0,1,2,3,4\}$ to $B = \{1,2,3,4,5\}$ defined as follows:

$$a\ R\ b \Leftrightarrow a > b$$

(i) List all ordered pairs of R

(ii) Represent the relation R by a matrix

(iii) Find the domain and image (range) of R .

23. Let R be a relation defined on the set $A = \{1,2,3,4\}$

$$x, y \in A \quad , \quad x\,R\,y \;\Leftrightarrow\; x^2 + y^2 \leq 13$$

(i) List all ordered pairs of R ?

(ii) Find the domain and the image of R ?

(iii) Represent R by the directed graph (diagraph) ?

(iv) Represent the relation R by a matrix

Solution: