King Saud University
Department of Mathematics

Final Exam in Math 151
Semester 2, 1439/40 H.

Q1. **(a)** Use induction to show that 2^{n+1} divides $(2n)! = 1\times2\times3\times\cdots\times(2n)$ for all integers $n \geq 2$. (3 pts)

(b) Let $R = \{(a,a),(a,b),(a,c),(b,a),(c,c)\}$ be a relation on $A = \{a,b,c\}$. Determine whether the relation R is reflexive, symmetric, antisymmetric, transitive. (4 pts)

(c) Let S be the equivalence relation on $B = \{1,2,3,4,5\}$ such that $1S3$, $3S4$, $2S5$ and $2\not S4$.

(i) List all ordered pairs of S. (2 pts)

(ii) Find the (distinct) equivalence classes of S. (1 pt)

Q2. **(a)** (i) Write $f(x,y,z) = (x\overline{y}+z)(\overline{x}+\overline{y})$ in CSP form. (2 pts)

(ii) Write $g(x,y,z) = \overline{xz+\overline{y}z}$ in CPS form. (2 pts)

(b) Let h be the Boolean function represented by the K-map below.

(i) Write h in MSP form. (2 pts)

(ii) Write h in MPS form. (2 pts)

(iii) Construct a minimal "*AND-OR*" circuit for h. (1 pt)

(iv) Construct a circuit for h using *NAND* gates only. (1 pt)

(v) Construct a circuit for h using *NOR* gates only. (1 pt)

	zw	$z\overline{w}$	$\overline{z}\,\overline{w}$	$\overline{z}w$
xy	1	0	1	1
$x\overline{y}$	0	0	0	0
$\overline{x}\,\overline{y}$	0	0	0	0
$\overline{x}y$	1	1	1	1

Q3. **(a)** Let G be a graph with 7 edges and vertices a,b,c,d,e,f whose respective degrees are $x, 2x, 2x, 2x, 3x, 4x$.

(i) Find x. (2 pts)

(ii) Can G be a tree? (Justify your answer.) (1 pt)

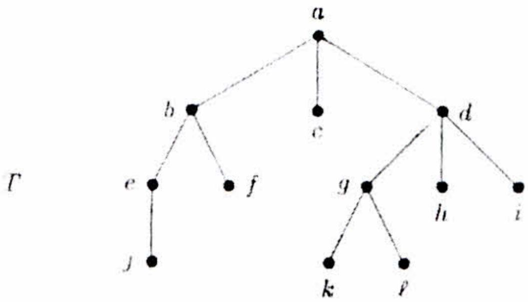
(b) (i) Give an example of a complete graph which is not complete bipartite. (1 pt)

(ii) Give an example of a complete bipartite graph which is not a complete graph. (1 pt)

Q4. **(a)** Form a binary search tree for the words *orange*, *blue*, *red*, *green*, *purple*, *black*, *pink* (using alphabetical order). (2 pts)

(b) Let T be the ordered rooted tree below.

(i) Find the *preorder* traversal of T. (2 pts)
(ii) Find the *inorder* traversal of T. (2 pts)
(iii) Find the *postorder* traversal of T. (2 pts)

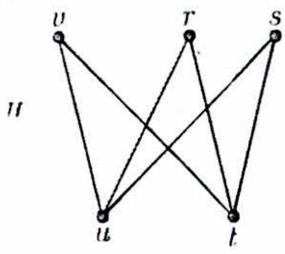


Q5. **(a)** Let E be the arithmetic expression $((x-2) \uparrow 3)/((y+4) * x))$.

(i) Represent E by an ordered rooted tree. (2 pts)
(ii) Write E in *prefix* notation. (1 pt)
(iii) Write E in *postfix* notation. (1 pt)

(b) For the graph H below, find a spanning tree with root r.

(i) using *depth-first* search; (1 pt)
(ii) using *breadth-first* search. (1 pt)



Semester II

(2019)

Answer Sheet Final Exam Math 151

Q1 (10 points)

(a) put $P(n):\ 2^{n+1} \mid (2n)!$

- Basis step: $n = 2$

(0,5)

$8 = 2^{2+1} \mid^{?} (2\times 2)! = 4! = 4\times 3\times 2 = 24$ ✓

So $P(2)$ is true.

- Inductive step: let $k \geq 3$. We assume that $P(k)$ is

(0,5) true ($2^{k+1} \mid (2k)! \Leftrightarrow (2k)! = a \cdot 2^{k+1}$)

Now we prove that $P(k+1)$ remains true. $2^{k+2} \mid^{?} (2k+2)!$

(2)

$$(2k+2)! = (2k+2)(2k+1)\underbrace{(2k)!}$$

$$= 2(k+1)(2k+1)\ a \cdot 2^{k+1} = \underbrace{a(k+1)(2k+1)}_{b\in\mathbb{N}}2^{k+2}$$

So $2^{k+2} \mid (2(k+1)!) \Leftrightarrow P(k+1)$ is true

we deduce that $P(n)$ is true for $n \geq 2$.

(b) $R = \{(a,a);(a,b);(a,c);(b,a);(c,c)\}$ on $A = \{a,b,c\}$

(1) • R is not reflexive because $I_A = \{(a,a),(b,b),(c,c)\} \not\subset R$

(1) • R is not symmetric because $(a,c)\in R$ but $(c,a)\notin R$

(1) • R is not antisymmetric because $(a,b)\in R$ and $(b,a)\in R$ $(a\neq b)$

(1) • R is not transitive because $(b,a)\in R$ & $(a,b)\in R$ but $(b,b)\notin R$ $\quad (R\circ R \not\subset R)$

(c)

(i) $S = \{(1,1);(2,2);(3,3);(4,4);(5,5);(1,3);(3,1);(3,4);(4,3);(2,5);(5,2);(1,4);(4,1)\}$

(2)

(ii) There are only 2 different classes

(1)

$[1] = \{1,3,4\} = [3] = [4]$

$[2] = \{2,5\} = [5]$

$|S|_B| = 2$

Q1 (11 points)

(a) (i) $f = \overline{(x\bar{y} + z)} (\bar{x} + \bar{y})$

$x\bar{x} = 0$

$\bar{y}\bar{y} = \bar{y}$

$= \underbrace{x\bar{x}\bar{y}}_{0} + x\bar{y}\bar{y} + \bar{x}z + \bar{y}z$

$= x\bar{y}(z+\bar{z}) + \bar{x}(y+\bar{y})z + (x+\bar{x})\bar{y}z$

$= x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \cancel{x\bar{y}z} + \cancel{\bar{x}\bar{y}z}$

2 $\boxed{CSP(f) = x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z}$

(ii) $g = \overline{(xz + \bar{y}z)}$

$CPS(g) = \overline{CSP(\bar{g})}$

$\bar{g} = xz + \bar{y}z = x(y+\bar{y})z + (x+\bar{x})\bar{y}z$

$\bar{g} = xyz + x\bar{y}z + x\bar{y}z + \bar{x}\bar{y}z$

2 $\boxed{CPS(g) = (\bar{x}+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(x+y+\bar{z})}$

(b)

	zw	$z\bar{w}$	$\bar{z}\bar{w}$	$\bar{z}w$
xy	1	0	1	1
$x\bar{y}$	0	0	0	0
$\bar{x}\bar{y}$	0	0	0	0
$\bar{x}y$	1	1	1	1

① ② ③

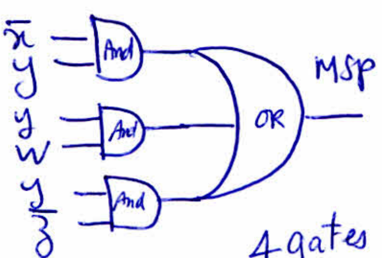
(i) $\boxed{MSP(h) = \bar{x}y + yw + y\bar{z}}$ 2

(ii) $MPS(h) = \overline{MSP(\bar{h})}$

$MSP(\bar{h}) = \bar{y} + xz\bar{w}$

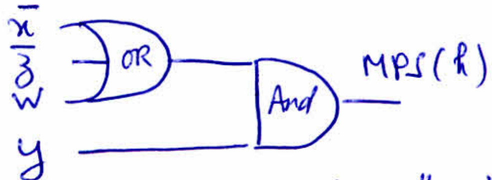
$\boxed{MPS(h) = y \cdot (\bar{x} + \bar{z} + w)}$ 2

(iii)



4 gates

2 gates

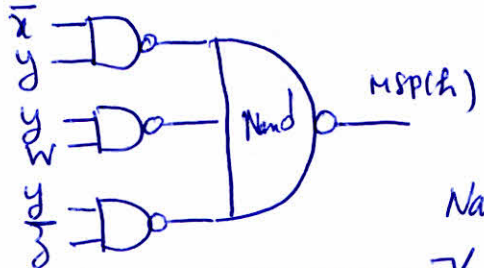


1

A minimal "And/Or" circuit for h

(iv) $MSP(h) = \left(\overline{\overline{x}y + yw + y\overline{z}}\right)' = \left[(\overline{\overline{x}y}) \cdot (\overline{yw}) \cdot (\overline{y\overline{z}})\right]'$

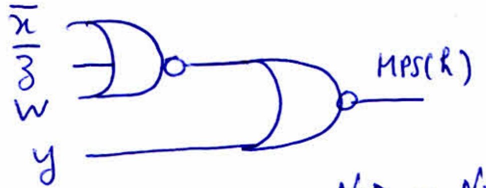
1



Nand - Network for h

(v) $MPS(h) = \left[\overline{y \cdot (\overline{x} + \overline{z} + w)}\right]' = \left[\overline{y} + (\overline{\overline{x} + \overline{z} + w})\right]'$

1



Nor - Network for h.

Q3 (5 points)

(a)

(i) $\sum_{i=1}^{6} deg(V_i) = 2|E(G)|$

2

$x + 2x + 2x + 2x + 3x + 4x = 2 \times 7$

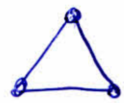
$14x = 14$

So $x = 1$

1 (ii) No, G is not a tree because we know if G is tree there exist at least 2 vertices of degree 1.

(b)

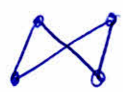
1 (i)



K_3

Complete graph but not complete bipartite

1 (ii)

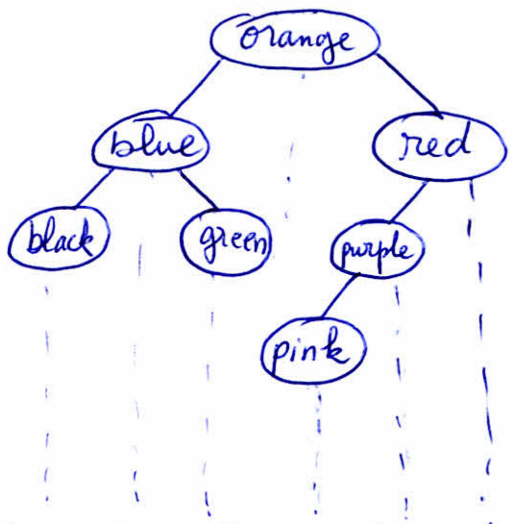


$K_{2,2}$

complete bipartite but not complete graph.

Q4 (8 pts)

(a)



2

black / blue / green / orange / pink / purple / red.

(b)

Root-left-right (i) Preorder:

a	b	e	j	f	c	d	g	k	l	h	i

2

left-Root-right (ii) inorder:

j	e	b	f	a	c	k	g	l	d	h	i

2

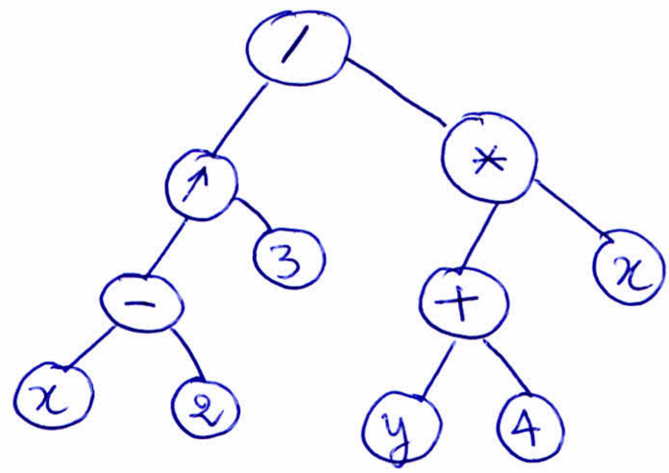
left-right-Root (iii) postorder:

j	e	f	b	c	k	l	g	h	i	d	a

2

Q5 (6 pts)

(a) (i)



2

E: $((x-2)\uparrow 3)/((y+4)*x)$

(ii) prefix:

/	↑	−	x	2	3	*	+	y	4	x

1

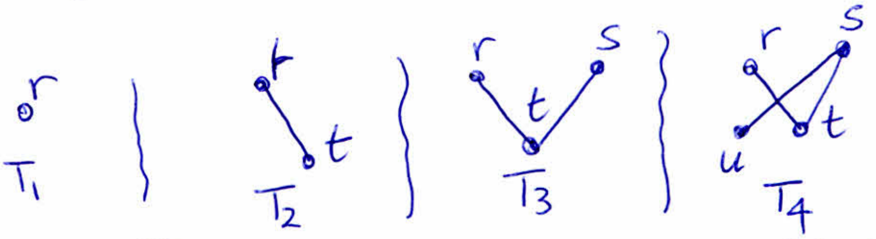
(iii) Postfix:

x	2	−	3	↑	y	4	+	x	*	/

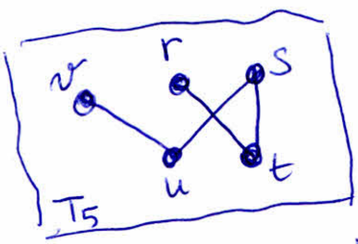
1

(b)

(i)

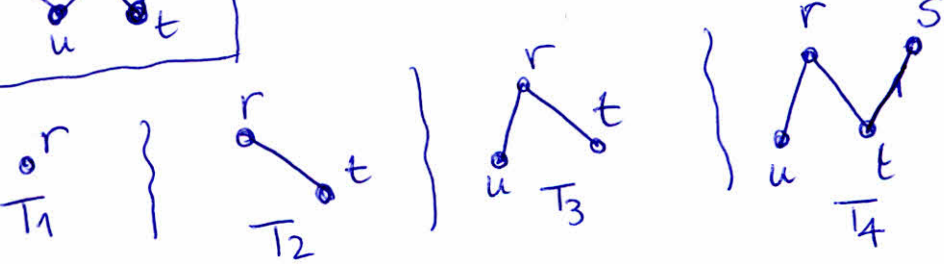


1

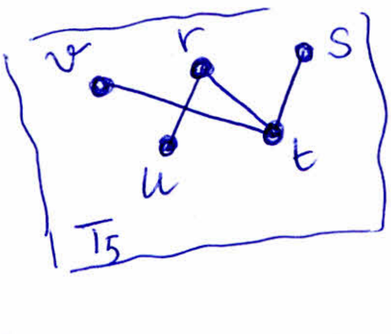


depth-first search rooted in r.

(ii)



1



breadth-first search rooted in r.