Q1. (a) Let R be the relation on $\mathbb{N} = \{1,2,3, \ldots\}$ such that $m\ R\ n$ if and only if $m - n > 1$.

Determine whether the relation R is reflexive, symmetric, antisymmetric, transitive. (4 pts)

(b) Let S be the relation on $\mathbb{Z} - \{0\}$ such that $a\ S\ b$ if and only if $ab > 0$.

(i) Show that S is an equivalence relation. (3 pts)

(ii) Find $[1]$ and $[-1]$. (2 pts)

(c) Find all (distinct) equivalence classes of the equivalence relation

$T = \{(a,a),(a,d),(b,b),(c,c),(c,e),(d,a),(d,d),(e,c),(e,e)\}$ on the set $A = \{a,b,c,d,e\}$.
(2 pts)

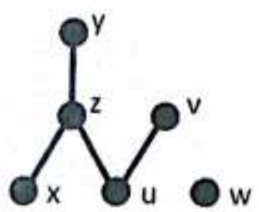
Q2. (a) Let $P = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3),(4,4)\}$ be a relation on $\{1,2,3,4\}$.

(i) Show that P is a partial order. (3 pts)

(ii) Draw the Hasse diagram for P. (1 pt)

(iii) Determine whether P is a total order. (1 pt)

(b) List all ordered pairs of the partial order Q on the set $B = \{u,v,w,x,y,z\}$, represented by the Hasse diagram below. (2 pts)



(c) Let S be the equivalence relation on $B = \{1,2,3,4,5\}$ such that $1S3$, $3S4$, $2S5$ and $2\not S4$.

(i) List all ordered pairs of S.

(ii) Find the (distinct) equivalence classes of S.

Q1. (a) Let R be the relation on $\mathbb{Z}^+$ such that $m\ R\ n$ if and only if mn is even. Determine whether the relation R is reflexive, symmetric, antisymmetric, transitive. (4 pts)

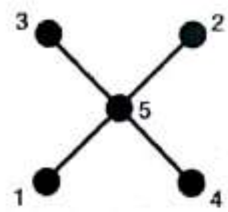
(b) Let S be the relation on $\mathbb{Z}$ such that $a\ S\ b$ if and only if $2 \mid (a^2 + b^2)$.

(i) Show that S is an equivalence relation. (3 pts)
(ii) Show that $[x] = [-x]$, for all integers x. (1 pt)
(iii) Determine whether $2 \in [-4]$. (1 pt)
(iv) Show that $[7] \cap [10] = \emptyset$. (1 pt)

Q2. (a) Let $A = \{2^m : m \in \{0,1,2,\cdots\}\}$. Define a relation T on A by: $2^m\ T\ 2^n \Leftrightarrow m \leq n$.

(i) Show that T is a total order. (4 pts)
(ii) Draw the Hasse diagram for T on the set $E = \{16, 8, 2, 64, 4\}$. (1 pt)

(b) Let P be the partial order on $E = \{1,2,3,4,5\}$, represented by the following Hasse diagram:



(i) List all ordered pairs of P. (2 pts)
(ii) Determine whether P is a total order. (1 pt)

Q1. (a) Let R be the relation on $A = \{1,2,3\}$ such that $m\ R\ n$ if and only if $m^2 \geq 2n$.

(i) List all ordered pairs of the relation R. (2 pts)
(i) Represent the relation R by a digraph. (1 pt)
(ii) Determine whether the relation R is reflexive, symmetric, antisymmetric, transitive. (4 pts)

(b) Let S be the relation on $\mathbb{Z}$ such that $a\ S\ b$ if and only if $a^2 - b^2 = a - b$.

(i) Show that S is an equivalence relation. (3 pts)
(ii) Find $[0]$ and [-1]. (2 pts)

Q2. (a) Let P be the relation on $\mathbb{Z}^+$ such that $x\ P\ y$ if and only if $x \mid (x + y)$.

(i) Show that P is a partial order. (3 pts)
(ii) Determine whether the relation P is a total order. (1 pt)
(iii) Draw the Hasse diagram for P on the subset $E = \{2,3,4,8\}$ of $\mathbb{Z}^+$. (2 pts)

Q2. (a) Let R be the relation on $\mathbb{Z}$ such that $x\,R\,y$ if and only if 4 divides $x + 3y$.

(i) Show that R is an equivalence relation. (3 pts)

(ii) Determine whether $-2 \in [6]$. (1 pt)

(b) Let $P = \{(1,1), (1,3), (2,1), (2,2), (2,3), (3,3), (4,1), (4,3), (4,4)\}$ be a partial order on the set $A = \{1,2,3,4\}$.

(i) Draw the Hasse diagram of P. (2 pts)

(ii) Determine whether P is a total order. (1 pt)