

Question	1	2	3	4	5	6	7	8	9	10
Answer	c	b	d	a	b	d	c	c	a	b

Choose the correct answer (write it in the table above):

1) The coordinate vector of  $w = (1, 2)$  relative to the basis  $S = \{(1, -2), (-3, 8)\}$  of  $\mathbb{R}^2$  is

- ☐ (a)  $(w)_S = (-2, 6)$ 
☐ (b)  $(w)_S = (-5, 14)$ 
☒ (c)  $(w)_S = (7, 2)$ 
☐ (d)  $(w)_S = (0, 0)$

2) If  $S = \{1 + x, 1 + x^2, x + x^2\}$  is a basis for  $\mathcal{P}_2$  and the coordinate vector of  $p(x) \in \mathcal{P}_2$  is  $(p(x))_S = (2, -1, -2)$ , then  $p(x)$  is

- ☐ (a)  $x + 3x^2$ 
☒ (b)  $1 - 3x^2$ 
☐ (c)  $2 + 2x + 2x^2$ 
☐ (d)  $1 - 2x^2$

\*3) If  $u = (-1, a, 2)$ ,  $v = (2, 3, -2)$  and  $w = (4, 0, 0)$ , then the set  $\{u, v, w\}$  is linearly independent if and only if

- ☐ (a)  $a \neq 3$ 
☐ (b)  $a = 3$ 
☐ (c)  $a = -3$ 
☒ (d)  $a \neq -3$

4) The vector space  $\mathbb{R}^2$  is spanned by

- ☒ (a)  $\{(-1, 0), (2, 4)\}$ 
☐ (b)  $\{(-1, -1), (\sqrt{2}, \sqrt{2})\}$ 
☐ (c)  $\{(2, 1)\}$ 
☐ (d)  $\{(6, -18), (-3, 9), (1, -3)\}$

5) The Wronskian of  $f_1 = 1, f_2 = x, f_3 = x^2, f_4 = x^3$  equals

$$\begin{vmatrix} 0 & 1 & 2x & 3x^2 \\ 0 & 0 & 2 & 6x \\ 0 & 0 & 0 & 6 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 2x & 3x^2 \\ 0 & 2 & 6x \\ 0 & 0 & 6 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 6x \\ 0 & 6 \end{vmatrix} = 12$$

- (a) 0      (b) 12      (c)  $x^6$       (d)  $3x^2$

6) Let  $S = \{(2, 0, -1), (4, 0, 7), (-1, 1, 4)\} \subset \mathbb{R}^3$ . Then

$$\begin{bmatrix} 2 & 4 & -1 \\ 0 & 0 & 1 \\ -1 & 7 & 4 \end{bmatrix}$$

$$- \begin{vmatrix} 2 & 4 \\ -1 & 7 \end{vmatrix} = -(14 - (-4)) = -18$$

- (a)  $S$  is linearly dependent      (b)  $S$  is linearly independent, but not a basis of  $\mathbb{R}^3$       (c)  $S$  is linearly dependent, but  $S$  spans  $\mathbb{R}^3$       (d)  $S$  is a basis of  $\mathbb{R}^3$

7) If the vectors  $(5, \frac{2}{3})$  and  $(7, x)$  are linearly dependent, then  $x$  equals

$$\frac{7}{5} \cdot \frac{2}{3} = \frac{14}{15}$$

- (a)  $\frac{7}{5}$       (b)  $\frac{5}{7}$       (c)  $\frac{14}{15}$       (d)  $\frac{15}{14}$

8) Let  $V = \mathbb{R}^3$ . Which of the following subsets of  $V$  is subspace of  $V$ ?

- (a)  $\{(a, 1, 1), a \in \mathbb{R}\}$       (b)  $\{(x, y, z) : 3x - 4y + 5z - 8 = 0\}$       (c)  $\{(x, y, z) : x = 3t, y = 4t, z = -t, t \in \mathbb{R}\}$       (d)  $\{(x, y, z) : x^2 = y^2\}$

9) Which of the following vectors span  $\mathbb{R}^3$ ?

$$\begin{bmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{bmatrix}$$

$$\begin{vmatrix} 4 & 8 \\ 2 & 8 \end{vmatrix} + \begin{vmatrix} 2 & 8 \\ 3 & 8 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = (32 - 16) + (16 - 24) + (4 - 12) = 16 + (-8) - 8 = 0$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix}$$

$$\det(A) = 0$$

- (a)  $(2, 2, 2), (0, 0, 3), (0, 1, 1)$       (b)  $(2, -1, 3), (4, 1, 2), (8, -1, 8)$       (c)  $(1, 1, 2), (1, 0, 1), (2, 1, 3)$       (d)  $(1, 2, 4), (2, 4, 8), (-1, 0, 0)$

10) A linear combination of the vectors  $v_1 = (1, 1, 0), v_2 = (0, -1, 1), v_3 = (1, 0, 1)$  is

- (a)  $(1, 1, 1)$       (b)  $(3, 2, 1)$       (c)  $(2, 2, 1)$       (d)  $(2, 1, 3)$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$-R_1 + R_2$

$\det(A) = 0$

$$k_1 v_1 + k_2 v_2 + k_3 v_3$$

$$k_1(1, 1, 0) + k_2(0, -1, 1) + k_3(1, 0, 1)$$

$$(k_1, k_1, 0) + (0, -k_2, k_2) + (k_3, 0, k_3)$$

$$k_1 + k_3 =$$

$$k_1 = k_2$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 3 & 2 & 2 \\ 1 & -1 & 0 & 1 & 2 & 1 & 2 \end{bmatrix}$$