جميع الأجوبة صحيحة عدا سؤال واحد):

If $W=\{(a+c, b+c, b-c, c)\ ;\ a,b,c \in R\}$ is a vector subspace in R^4, then its basis is

- $\{(0,0,1,0),\ (0,1,1,0),\ (1,1,-1,1)\}$
- $\{(1,0,0,0),\ (0,1,1,0),\ (1,1,-1,1)\}$
- $\{(0,1,0,0),\ (0,1,1,0),\ (1,1,-1,1)\}$
- $\{(0,0,0,1),\ (0,1,1,0),\ (1,1,-1,1)\}$

Let 0 be an eigenvalue of the matrix $\begin{bmatrix} 2 & 0 & a \\ 0 & 1 & 0 \\ -4 & 0 & 2 \end{bmatrix}$. Then:

- $a=1$
- $a=-4$
- $a=-1$
- $a=2$

Let $A^{-1}=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$, where A denotes the matrix of coefficients of the following linear system:

$$\begin{aligned} 3x_1 - 2x_2 - x_3 &= 1 \\ -4x_1 + x_2 - x_3 &= 1 \\ 2x_1 \qquad + x_3 &= 0. \end{aligned}$$

Then its solution set is equal to:

- $\{(3,\ 7,\ -6)\}$
- $\{(-5,\ 2,\ 4)\}$
- $\{(-2,\ 4,\ 5)\}$
- $\{(-5,\ 3,\ 7)\}$

Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation such that its matrix with respect to the bases $B = \{u_1 = (1,0,0), u_2 = (0,1,0), u_3 = (0,0,1)\}$ and $C = \{v_1 = (-1,1), v_2 = (2,0)\}$ is $\begin{pmatrix} 2 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$. Then:

- ○ T(x,y,z)=(-3x-4y+ z, x+2y+z)
- ○ T(x,y,z)=(2x+4y+ z,2x-y+z)
- ◉ T(x,y,z) = (-4x-3y+2z , 2x+y)
- ○ T(x,y,z)=(2x-4y+2z,2x+y-z)

Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T(x,y,z) = (2x+y+3z\ ,\ x+y+2z)$. If $(1,a,b) \in Ker(T)$, then:

- ◉ a = 1, b = -1
- ○ a=2, b=1
- ○ a=2, b=-1
- ○ a=-2, b= 1

Let $A = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ and $D = A^2 - 2\mathbf{I}$. Then D is equal to the matrix:

- ◉ $-\mathbf{I}$
- ○ $\mathbf{I}$
- ○ $-2\mathbf{I}$
- ○ A

Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(x,y) = (x+2y, 2x-y)$. Then the associated matrix of T with respect to the basis $B = \{(-1,2), (2,0)\}$ is:

- ○ $\begin{pmatrix} -1 & 2 \\ 2 & 0 \end{pmatrix}$
- ○ $\begin{pmatrix} -2 & 1/2 \\ 2 & 2 \end{pmatrix}$
- ◉ $\begin{pmatrix} -2 & 2 \\ 1/2 & 2 \end{pmatrix}$
- ○ $\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$

If u and v are two vectors in the inner product space $\mathbb{R}^n$ such that $||u||=3$ and $||v||=2$, then $<2u-3v\, , 2u+3v>$ is equal to:

- ○ -12
- ○ 12
- ○ 6
- ◉ 0

If $A = \begin{bmatrix} -1 & -2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix}$ then A^{-1} is equal to the matrix:

- ○ $\begin{bmatrix} 1 & -2 & -7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
- ○ $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 1 \end{bmatrix}$
- ◉ $\begin{bmatrix} -1 & 2 & -7 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

Consider the real matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & \mathbf{a} \\ 0 & 2 & 0 \\ 0 & \mathbf{a} & 3 \end{bmatrix}$. The set of values of **a** making the matrix **A** diagonalizable is equal to:

- ○ $\{1, 2, 3\}$
- ○ $\emptyset$
- ◉ $\mathbb{R}$
- ○ $\{0\}$

Let $-2, 0, 1, 3$ be the eigenvalues of 4×4 real matrix A. Which of the folowing statements is correct?

- ○ A is not diagonalizable but invertible.
- ○ A is diagonalizable and invertible.
- ◉ A is diagonalizable and not invertible.
- ○ A is neither diagonalizable nor invertible.

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation defined by $T(x,y) = (x+y, 2x-y)$. Then the matrix of T with respect to the basis $B = \{u_1 = (1,-1), u_2 = (0,1)\}$ is equal to:

- ○ $\begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix}$
- ○ $\begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$
- ◉ $\begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$
- ○ $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$

Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$. *Then the cofactor of the entry of A that lies in the second row and third column is:*

- ○ 4
- ◉ -2
- ○ 2
- ○ -4

If A is a square matrix of order 3 *such that* $\det(A) = 1$, *then* $\det(2A) =$

- ○ -2
- ○ -8
- ◉ 8
- ○ 2

Let $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ be an eigenvector of the matrix $A = \begin{bmatrix} a & -3a+b \\ 2a-b & b \end{bmatrix}$ corresponding to the eigenvalue 3. Then:

- ○ $a = 1, b = 2$
- ○ $a = 1, b = 3$
- ○ $a = 4, b = 1$
- ◉ $a = 1, b = 4$

The following system of linear equations has:

$$\begin{cases} w + 2x + y + z = 0 \\ -w + 3x - y + z = 0 \end{cases}$$

- ○ Unique solution
- ○ No solution
- ○ Only the trivial solution
- ○ Infinitely many solutions

Let $\mathbf{E_1}$ denote the eigenspace of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ corresponding to its eigenvalue **1**.

Then the dimension of $\mathbf{E_1}$ is equal to:

- ○ **3**
- ◉ **0**
- ○ **2**
- ○ **1**

Let $\mathbf{S} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be an orthonormal basis of an inner product space $\mathbf{V}$. If $\boldsymbol{v} \in \mathbf{V}$ satisfies $\langle \boldsymbol{v}, \boldsymbol{u}_1 \rangle = 2$, $\langle \boldsymbol{v}, \boldsymbol{u}_2 \rangle = 2$ and $\langle \boldsymbol{v}, \boldsymbol{u}_3 \rangle = -2$, then $||\boldsymbol{v}||$ is equal to:

- ○ $3\sqrt{2}$
- ◉ $2\sqrt{3}$
- ○ $4\sqrt{2}$
- ○ 2

If $[v]_C = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ *where C is the stadard basis and* $B = \{v_1 = (1,1,1)\,,\, v_2 = (0,1,0)\,,\, v_3 = (1,1,0)\}$ *is another basis of* R^3*, then* $[v]_B$ *is equal to:*

- ○ $[v]_B = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$
- ○ $[v]_B = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$
- ○ $[v]_B = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$
- ◉ $[v]_B = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ *be a linear transformation and* $v_1, v_2, v_3 \in \mathbb{R}^4$*. If* $T(v_1) = (1, -1, 2)$, $T(v_2) = (0,3,2)$ *and* $T(v_3) = (-3,1,2)$*, then* $T(v_1 + 2v_2 - 2v_3)$ *is equal to:*

- ○ (3,5,7)
- ◉ (7,3,2)
- ○ (3,-2,7)
- ○ (2,3,-5)