

Name: _____ ID: _____

Question I Let $\mathbf{u} = (-3, 2, 1)$, $\mathbf{v} = (4, 7, -3)$ and $\mathbf{w} = (5, -1, 2)$.

Find the components of

- (a) $-\mathbf{u} + (\mathbf{v} + 4\mathbf{w})$
 (b) The vector \mathbf{x} which satisfies the equation $\mathbf{u} + \mathbf{v} = 2\mathbf{x} + \mathbf{w}$

$$\begin{aligned} \text{a)} & -(-3, 2, 1) + ((4, 7, -3) + 4(5, -1, 2)) \\ & = (3, -2, -1) + ((4, 7, -3) + (20, -4, 8)) \\ & = (3, -2, -1) + (24, 3, 5) \\ & = (27, 1, 4) \end{aligned}$$

$$\begin{aligned} \text{b)} \quad 2\mathbf{x} + \mathbf{w} &= \mathbf{u} + \mathbf{v} \\ 2\mathbf{x} &= \mathbf{u} + \mathbf{v} - \mathbf{w} \\ \mathbf{x} &= \frac{\mathbf{u} + \mathbf{v} - \mathbf{w}}{2} \\ \mathbf{x} &= \frac{(-3, 2, 1) + (4, 7, -3) - (5, -1, 2)}{2} \end{aligned}$$

$$\mathbf{x} = (-2, 5, -2)$$

Question II: Determine whether the following is true or false.

Justify your answer.

- (i) $\mathbf{z} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ is a **unit** vector

(T)

$$||\mathbf{z}|| = \sqrt{(z_1)^2 + (z_2)^2 + (z_3)^2} = 1$$

if the norm
 \Rightarrow equals 1 the
 is called unit
 vector

$$\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + 0^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1 \quad \text{is unit vector}$$

(ii) If $\mathbf{u}_1 = (1, -3, 4)$ and $\mathbf{u}_2 = (0, 2, 5)$, then the distance between \mathbf{u}_1 and \mathbf{u}_2 is $d(\mathbf{u}_1, \mathbf{u}_2) = 6$ (F)

$$d(\mathbf{u}_1, \mathbf{u}_2) = \|\mathbf{u}_1 - \mathbf{u}_2\| = \sqrt{(-1)^2 + (-5)^2 + (-1)^2}$$

← يعني الفرق بين \mathbf{u}_1 و \mathbf{u}_2 كل واحد بما يقابله

$$\sqrt{(1-0)^2 + (-3-2)^2 + (4-5)^2} = 3\sqrt{5} = 5.19$$

$5.19 \neq 6$

(iii) If $\mathbf{v}_1 = (2, 0, -1)$ and $\mathbf{v}_2 = (3, 1, -2)$, then the dot product $\mathbf{v}_1 \cdot \mathbf{v}_2 = 8$ (T)

$$\mathbf{v}_1 \cdot \mathbf{v}_2 = v_{11}v_{21} + v_{12}v_{22} + v_{13}v_{23}$$

$$\mathbf{v}_1 \cdot \mathbf{v}_2 = (2 \times 3) + (0 \times 1) + (-1 \times -2) = 8$$

(iv) If $\mathbf{u} \cdot \mathbf{v} = 3$ and $\|\mathbf{u} + \mathbf{v}\| = 4$, then $\|\mathbf{u} - \mathbf{v}\| = 2$ (T)

قانون

$$\mathbf{u} \cdot \mathbf{v} = \frac{1}{4} \|\mathbf{u} + \mathbf{v}\|^2 - \frac{1}{4} \|\mathbf{u} - \mathbf{v}\|^2$$

$$3 = \frac{1}{4}(4)^2 - \frac{1}{4}(2)^2$$

$$3 = 3 \quad \checkmark$$

Good Luck ☺