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① $A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$

$\cos\theta = \dfrac{\langle A, B\rangle}{\|B\|\|A\|}$

$\langle A, B\rangle = \left\langle \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix} \right\rangle = \begin{bmatrix} -2 & 16 \\ 6 & 2 \end{bmatrix} = 0$

$\|A\|^2 = \langle A, A\rangle = \left\langle \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \right\rangle = \begin{bmatrix} 20 & 10 \\ 10 & 10 \end{bmatrix} = 30 = \sqrt{30}$

$\|B\|^2 \langle B, B\rangle = \left\langle \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}, \begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix} \right\rangle = \begin{bmatrix} 10 & -10 \\ -10 & 20 \end{bmatrix} = 30 = \sqrt{30}$

$\cos\theta = \dfrac{0}{\sqrt{30}\sqrt{30}} = 0$

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② $(a_1, a_2, a_3)\ v = (b_1, b_2, b_3)$

$\langle u, v\rangle = a_1 b_1 + a_2 b_1 >$

① $(u, v) = \langle (a_1, a_2, a_3), (b_1, b_2, b_3)\rangle = a_1 b_2 + a_2 b_1$

$(v, u) = \langle (b_1, b_2, b_3), (a_1, a_2, a_3)\rangle = b_1 a_2 + b_2 a_1$

② Let $w = (c_1, c_2, c_3)$

$\langle u + v, w\rangle = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3), (c_1, c_2, c_3)\rangle = c_2(a_1 + b_1) + c_1(a_2 + b_2) = \cancel{\ }$

$\langle u, w\rangle + \langle v, w\rangle = \langle (a_1, a_2, a_3), (c_1, c_2, c_3)\rangle + \langle (b_1, b_2, b_3), (c_1, c_2, c_3)\rangle = a_1 c_2 + a_2 c_1 + b_1 c_2 + b_2 c_1$

$c_2(a_1 + b) + c_1(a_2 + b_2)$

③ $\langle ku, v\rangle = \langle (ka_1, ka_2, ka_3), (b_1, b_2, b_3)\rangle = ka_1 b_2 + ka_2 b_1$

$k\langle u, v\rangle = k\langle (a_1, a_2, a_3), (b_1, b_2, b_3)\rangle = k(a_1 b_2 + a_2 b_1) = ka_1 b_2 + ka_2 b_1$

④ $\langle u, u\rangle \geq 0 = \langle (a_1, a_2, a_3), (a_1, a_2, a_3)\rangle = a_1 a_2 + a_2 a_1$

let $u = (1, -2, 3) \to \langle u, u\rangle = \langle (1, -2, 3), (1, -2, 3)\rangle = (1)(-2) + (-2)(1) = -4 < 0$

so it's <u>not</u> an inner product

(3) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$V_1 = (1,1,1)$, $V_2 = (0,1,1)$, $V_3 = (0,0,1)$

* The orthognal Basis.

$u_1 = v_1 = \begin{bmatrix}(1,1,1)\end{bmatrix}$

$u_2 = V_2 - \frac{\langle V_2, u_1 \rangle}{\|u_1\|^2} u_1 = (0,1,1) - \frac{2}{3}(1,1,1) =$

$= \begin{bmatrix}(\frac{-2}{3}, \frac{1}{3}, \frac{1}{3})\end{bmatrix}$

$u_3 = V_3 - \frac{\langle V_3, u_2 \rangle}{\|u_2\|^2} \cdot u_2 - \frac{\langle V_3, u_1 \rangle}{\|u_1\|} \cdot u_1$

$= (0,0,1) - \frac{1/3}{2/3}(\frac{-2}{3}, \frac{1}{3}, \frac{1}{3}) - \frac{1}{3}(1,1,1)$

$= \begin{bmatrix}(0, \frac{-1}{2}, \frac{1}{2})\end{bmatrix}$

$\langle V_2, u_1 \rangle = \langle (0,1,1), (1,1,1) \rangle = 2$

$\|u_2\|^2 = \langle 1,1,1 \rangle, (1,1,1) \rangle = 3$

$\langle V_3, u_2 \rangle = \langle (0,0,1), (\frac{-2}{3}, \frac{1}{3}, \frac{1}{3}) \rangle = \frac{1}{3}$

$\|u_2\|^2 = \langle (\frac{-2}{3}, \frac{1}{3}, \frac{1}{3}), (\frac{-2}{3}, \frac{1}{3}, \frac{1}{3}) \rangle$

$= \frac{2}{3}$

$\langle V_3, u_1 \rangle = \langle (0,0,1), (1,1,1) \rangle = 1$

$\|u_3\|^2 = \langle (0, \frac{-1}{2}, \frac{1}{2}), (0, \frac{-1}{2}, \frac{1}{2}) \rangle = \frac{1}{2}$

The orthonormal Basis

$w_1 = \frac{u_1}{\|u_1\|} = \frac{(1,1,1)}{\sqrt{3}}$

$w_2 = \frac{u_2}{\|u_2\|} = \frac{(\frac{-2}{3}, \frac{1}{3}, \frac{1}{3})}{\sqrt{\frac{2}{3}}}$

$w_3 = \frac{u_3}{\|u_3\|} = \frac{(0, \frac{-1}{2}, \frac{1}{2})}{\sqrt{\frac{1}{2}}}$