بسم الله الرحمن الرحيم

محاضرات في ريض 244 جامعة الملك سعود

MIDTEMRM MATH-244 مراجعة

تابع معنا اقوي مراجعات الميد تيرم لريض 244

عبدالله الحفني جوال ٠٥٨٣٤٢٢٢٠٠

مذكرة القمة

فى

(MATH-244)

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A If $A = \begin{vmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 0 & -3 \\ -6 & 7 & -3 \\ -3 & -9 & -2 \end{vmatrix}$, then find the values of a

x , y and z such that $xA^2 + yAB + z\,I = 0$

SOLUTION STEPS

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{matrix} 1 & 2 & 1 \\ 0 & -1 & 3 \\ 1 & 1 & 2 \end{matrix}$$

$$A^2 = \begin{bmatrix} 2 & 3 & 3 \\ 1 & 4 & 3 \\ 9 & 3 & 8 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ -6 & 7 & -3 \\ -3 & -9 & -2 \end{bmatrix} \begin{matrix} 1 & 2 & 1 \\ 0 & -1 & 3 \\ 1 & 1 & 2 \end{matrix} = \begin{bmatrix} -2 & -9 & -5 \\ 5 & -16 & -5 \\ -23 & 3 & -16 \end{bmatrix}$$

sub in $xA^2 + yAB + zI_3 = 0$

$$\begin{bmatrix} 2x & 3x & 3x \\ x & 4x & 3x \\ 9x & 3x & 8x \end{bmatrix} + \begin{bmatrix} -2y & -9y & -5y \\ 5y & -16y & -5y \\ -23y & 3y & -16y \end{bmatrix} + \begin{bmatrix} z & 0 & 0 \\ 0 & z & 0 \\ 0 & 0 & z \end{bmatrix} = 0$$

$$\begin{bmatrix} 2x-2y+z & 3x-9y & 3x-5y \\ x+5y & 4x-16y+z & 3x-5y \\ 9x-23y & 4x+3y & 8x-16y+z \end{bmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$2x - 2y + z = 0 --- \boxed{1}$

$3x - 9y = 0 ------ \boxed{2}$

$x + 5y -------- \boxed{3}$

from $Eq_{(2)}$, $Eq_{(3)}$

$x = 3y$

$x + 5y = 0 \Rightarrow 3y + 5y = 0$

$\boxed{x = 0}$, $\boxed{y = 0}$

from $Eq_{(1)}$ $\boxed{z = 0}$

Then $x = 0 , y = 0 , z = 0$

Question (2) **A** Find $adj\,(A)$ and A^{-1} for the matrix :

$$A=\begin{vmatrix} 1 & 0 & a \\ 2 & b & c \\ -1 & 1 & 1 \end{vmatrix},\ \text{where } ab+b+2a-c\neq 0$$

SOLUTION STEPS

$$c_{11}=b-c \qquad c_{12}=-2-c \qquad c_{13}=2+b$$

$$c_{21}=-a \qquad c_{22}=a+1 \qquad c_{23}=-1$$

$$c_{31}=-ab \qquad c_{32}=2a-c \qquad c_{33}=b$$

$$c=\begin{bmatrix} b-c & -2-c & 2+b \\ -a & a+1 & -1 \\ -ab & 2a-c & b \end{bmatrix}$$

$$c^T=\begin{bmatrix} b-c & -a & ab \\ -2-c & a+1 & 2a-c \\ 2+b & -1 & b \end{bmatrix}$$

$$adj\,(A)=c^T=\begin{bmatrix} b-c & -a & ab \\ -2-c & a+1 & 2a-c \\ 2+b & -1 & b \end{bmatrix}$$

$$|A|=c_{11}a_{11}+c_{12}a_{12}+c_{13}a_{13}=ab+b+2a-c \quad \neq 0$$

Rule: $A^{-1}=\dfrac{1}{|A|}\ adj\,(A)$

Then, $A^{-1}=\dfrac{1}{ab+b+2a-c}=\begin{bmatrix} b-c & -a & ab \\ -2-c & a+1 & 2a-c \\ 2+b & 1 & b \end{bmatrix}$

B Let $A\in M_{3\times 3}(\mathbb{R})$ with determinant $|A|=2$, Then find $\left|2\left(adj\,(A)\right)^{-1}+A\right|$

SOLUTION STEPS

since $A^{-1}=\dfrac{1}{|A|}\ adj\,(A)$

Then $adj\,(A)=|A|\,A^{-1}$ $\quad\boxed{\text{to}\ ^{-1}}$

$$\left(adj\,(A)\right)^{-1}=\left(|A|\,A^{-1}\right)^{-1}=\frac{1}{|A|}A=\frac{1}{2}A$$

Now sub in $\left|2\left(adj\,(A)\right)^{-1}+A\right|$

$$\left|\cancel{2}\cdot\frac{1}{\cancel{2}}A+A\right|=|2A|=2^3|A|=8\cdot 2=16$$

Question (3) **A** Let $A=\begin{vmatrix}1&-1&4&5\\-2&1&-11&-8\\-1&2&2&2\end{vmatrix}$ and $B=\begin{vmatrix}1&-1&3&2\\0&1&4&1\\1&0&8&6\end{vmatrix}$

Show that the matrces A and B are row equivalent to each other

SOLUTION STEPS

$$A=\begin{bmatrix}1&-1&4&5\\-2&1&-11&-8\\-1&2&2&2\end{bmatrix}\xrightarrow[R_1+R_3]{2R_1+R_2}\approx\begin{bmatrix}1&-1&4&5\\0&-1&-3&2\\0&1&6&7\end{bmatrix}\xrightarrow[-R_2+R_1]{R_2+R_3}\approx\begin{bmatrix}1&0&7&3\\0&-1&-3&2\\0&0&3&9\end{bmatrix}$$

$$\xrightarrow[\frac{R_3}{3}]{-R_2}\approx\begin{bmatrix}1&0&7&3\\0&1&3&-2\\0&0&1&3\end{bmatrix}\xrightarrow[-3R_3+R_2]{-1R_3+R_1}\approx\begin{bmatrix}1&0&0&-18\\0&1&0&-11\\0&0&1&3\end{bmatrix}$$

$$B=\begin{bmatrix}1&-1&3&2\\0&1&4&1\\1&0&8&6\end{bmatrix}\xrightarrow{-R_1+R_3}\approx\begin{bmatrix}1&-1&3&2\\0&1&4&1\\0&1&5&4\end{bmatrix}\xrightarrow[-R_2+R_3]{R_2+R_1}\begin{bmatrix}1&0&7&3\\0&1&4&1\\0&0&1&3\end{bmatrix}$$

$$\xrightarrow{-R_3+R_2}\begin{bmatrix}1&0&7&3\\0&1&3&-2\\0&0&1&3\end{bmatrix}\xrightarrow[-3R_3+R_2]{-1R_3+R_1}\approx\begin{bmatrix}1&0&0&-18\\0&1&0&-11\\0&0&1&3\end{bmatrix}$$

$$\text{R.R.E.F}(A)=\begin{bmatrix}1&0&0&-18\\0&1&0&-11\\0&0&1&3\end{bmatrix}=\text{R.R.E.F}(B)$$

Then , A and B are row equivalent to each other

B Compute the inverse matrix of tha matrix A, where $A=\begin{pmatrix}1&1&1&1\\0&1&1&1\\1&0&1&1\\1&1&0&1\end{pmatrix}$

SOLUTION STEPS

$$\left[\begin{array}{cccc|cccc}1&1&1&1&1&0&0&0\\0&1&1&1&0&1&0&0\\1&0&1&1&0&0&1&0\\1&1&0&1&0&0&0&1\end{array}\right]\xrightarrow[\substack{-1R_{23}\\-1R_{24}}]{-1R_{21}}\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&1&1&0&1&0&0\\1&-1&0&0&0&-1&1&0\\1&0&-1&0&0&-1&0&1\end{array}\right]\xrightarrow[-1R_{14}]{-1R_{13}}\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&1&1&0&1&0&0\\0&-1&0&0&-1&0&1&0\\0&0&-1&0&-1&0&0&1\end{array}\right]$$

$$\longrightarrow\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&-1&0&0&-1&0&1&0\\0&0&-1&0&-1&0&0&1\\0&1&1&1&0&1&0&0\end{array}\right]\longrightarrow\left[\begin{array}{cccc|cccc}1&0&0&0&1&-1&0&0\\0&1&0&0&1&0&-1&0\\0&0&1&0&1&0&0&-1\\0&0&0&1&-2&1&1&1\end{array}\right]$$

Then , $A^{-1}=\begin{pmatrix}1&-1&0&0\\1&0&-1&0\\1&0&0&-1\\-2&1&1&1\end{pmatrix}$

Question (4) Determine the values of α such that the following linear system:

$$x+2y-z=2$$
$$x-2y+3z=1 \qquad \text{has:}$$
$$x+2y-(\alpha^2-3)z=\alpha$$

$\boxed{i}$ No solution $\boxed{ii}$ Unique solution $\boxed{iii}$ Infinitely many solution

SOLUTION STEPS

Augmented Matrix is: $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 1 & -2 & 3 & 1 \\ 1 & 2 & 3-\alpha^2 & \alpha \end{array}\right] \xrightarrow[-R_1+R_3]{-R_1+R_2} \approx \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -4 & 4 & -1 \\ 0 & 0 & 4-\alpha^2 & \alpha-2 \end{array}\right]$

from $R_3 : (4-\alpha^2)z=(\alpha-2)$

$\boxed{i}$ If $\alpha=2$ Then the system has infinitely many solution

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -4 & 4 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \boxed{\begin{array}{l} x+2y-z=2 \\ -4x+4x=-1 \end{array}}$$

$y=z+\frac{1}{4} \Rightarrow y=t+\frac{1}{4}\ , t\in\mathbb{R}$ $\boxed{\text{put } z=t}$

$x=-t-\frac{3}{2}$

$\boxed{ii}$ If $\alpha=-2$ Then the system has no solution

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -4 & 4 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{iii}$ The system has unique solution when $\alpha\in\mathbb{R}-\{\pm 2\}$

Question (5) For which values of a will be following system

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

$\boxed{i}$ Infinitly many Solution. $\boxed{ii}$ No solution. $\boxed{iii}$ Exactly one solution.

SOLUTION STEPS

(Target) الهدف

نريد قيمة (a) التى تجعل للنظام
عدد لانهائى من الحلول
اومفيش حل او حل وحيد

Augmented Matrix is : $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array}\right]$

by Using $R.E.F$

$$\xrightarrow[-4R_1+R_3]{-3R_1+R_2} \approx \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right] \xrightarrow[-R_2+R_3]{R_2/_{-7}} \approx \left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & 1 & 2 & \frac{10}{7} \\ 0 & 0 & a^2-16 & a-4 \end{array}\right] \begin{array}{l} \to x+2y-3z=4 \\ \to y+2z=\frac{10}{7} \\ \to a^2-16z=a-4 \end{array}$$

توضيح

$a^2 - 16 = 0 \quad | \quad a - 4 = 0$

$a = \pm 4 \quad | \quad a = 4$

مشتركان في 4

Case. i

For $a = 4 \Rightarrow (0)z = 0$ Many solution

Let $z = t \;;\; t \in \mathbb{R}$

$y + 2t = \frac{10}{7} \longrightarrow y = \frac{10}{7} - 2t$

$x + 2y - 3z = 4 \longrightarrow x + \frac{20}{7} - 2t - 3t = 4 \Rightarrow x = 5t + \frac{8}{7}$

Case. ii

$a = -4 \Rightarrow 0z = -8 \Rightarrow 0 \neq -8$

No solution

Case. iii

Put $a \neq 4$, $a \neq -4$

For Ex. $a = 0 \Rightarrow -14z = -4 \Rightarrow z = \frac{1}{4}$

$y + 2z = \frac{10}{7} \Rightarrow y = \frac{10}{7} - \frac{1}{2} = \frac{13}{14}$

$x + 2y - 3z = 4$

Now :

$\boxed{i}$ at $a = 4$, Infinitely many solution

$\boxed{ii}$ at $a = -4$, No solution

$\boxed{iii}$ at $a \neq \pm 4$, Exactly one solution.(unique solution)

Question (6) **A** By using the Cramer's rule. solve the following system :

$$x+2y-z=2$$
$$x+3y+3z=2$$
$$x+3y+z=4$$

SOLUTION STEPS

The system equivalent

AX = B

$$A=\begin{bmatrix}1&2&-1\\1&3&3\\1&3&5\end{bmatrix},\ B=\begin{bmatrix}2\\2\\4\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\begin{array}{ccc}+&-&+\end{array}$$
$$|A|=\begin{vmatrix}1&2&-1\\1&3&3\\1&3&5\end{vmatrix}=(15\cdot 9)1-(5-3)2-(0)1=2$$

$$\begin{array}{ccc}+&-&+\end{array}$$
$$|A_1|=\begin{vmatrix}2&2&-1\\2&3&3\\4&3&5\end{vmatrix}=22 \qquad x=\frac{|A_1|}{|A|}=\frac{22}{2}=11$$

$$|A_2|=\begin{vmatrix}1&2&-1\\1&2&3\\1&4&5\end{vmatrix}=-8 \qquad y=\frac{|A_2|}{|A|}=\frac{-8}{2}=-4$$

$$|A_3|=\begin{vmatrix}1&2&2\\1&3&2\\1&3&4\end{vmatrix}=2 \qquad z=\frac{|A_3|}{|A|}=\frac{2}{2}=1$$

$$S.S=\{(11,-4,1)\}$$

B Compute the following determinant $A=\begin{vmatrix}-1&1&1&1\\1&-1&1&1\\1&1&-1&1\\1&1&1&-1\end{vmatrix}$

SOLUTION STEPS

$$=\begin{vmatrix}2&1&1&1\\2&-1&1&1\\2&1&-1&1\\2&1&1&-1\end{vmatrix}=2\begin{vmatrix}1&1&1&1\\1&-1&1&1\\1&1&-1&1\\1&1&1&-1\end{vmatrix}=2\begin{vmatrix}1&1&1&1\\0&-2&0&0\\0&0&-2&0\\0&0&0&-2\end{vmatrix}=-16$$

Question (7) Let $W=\{A\in M_{2\times2}(\mathbb{R}):AB=BA\}$, where $B=\begin{bmatrix}-1&1\\0&1\end{bmatrix}$, then

$\boxed{i}$ Show that W is a vector subspace of vector space $M_{2\times2}(\mathbb{R})$

$\boxed{ii}$ Find a basis and dimension of W.

SOLUTION STEPS

$\boxed{i}$ Let $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$; $a,b,c,d\in\mathbb{R}$

put $a=b=c=d=0$

$\Rightarrow A=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$\Rightarrow AB=\begin{bmatrix}0&0\\0&0\end{bmatrix}\begin{bmatrix}-1&1\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$BA=\begin{bmatrix}-1&1\\0&1\end{bmatrix}\begin{bmatrix}0&0\\0&0\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

Then $0\in W$ ----$\boxed{1}$

$\forall A_1\in W$ Then $A_1B=BA_1\cdots(*)$

$\forall A_2\in W$ Then $A_2B=BA_2\cdots(**)$

add $(*),(**)$

$\boxed{A_1B+A_2B=BA_1+BA_2}$

Now

$(A_1+A_2)B=A_1B+A_2B=\boxed{BA_1+BA_2}$

$A_1+A_2\in W$----$\boxed{2}$

$\forall A\in W$; $\alpha\in\mathbb{R}$

$(\alpha A)B=B(\alpha A)=\alpha BA=\alpha AB$

Then $\alpha A\in W$----$\boxed{3}$

From $\boxed{1}$, $\boxed{2}$, $\boxed{3}$

W is sub-space of $M_{2\times2}(\mathbb{R})$

$\boxed{ii}$ $\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}-1&1\\0&1\end{bmatrix}=\begin{bmatrix}-1&1\\0&1\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}$

$\begin{bmatrix}-a&a+b\\-c&c+d\end{bmatrix}=\begin{bmatrix}-a+c&-b+d\\c&d\end{bmatrix}$

$\cancel{-a}=\cancel{-a}+c$ | $a+b=-b+d$ | $c+d=d$

$c=0$ | $a+2b=d$ | $d=d$

put $d=t$; $t\in\mathbb{R}$

$a+2b=t$, put $b=r$

$a=-2r+t$

$\begin{bmatrix}2r+t&-r+t\\0&t\end{bmatrix}=r\begin{bmatrix}2&-1\\0&0\end{bmatrix}+t\begin{bmatrix}-1&1\\0&1\end{bmatrix}$

the basis of the set $=\left\{\begin{bmatrix}2&-1\\0&0\end{bmatrix},\begin{bmatrix}-1&1\\0&1\end{bmatrix}\right\}$

$\dim.(W)=2$

Question (8) **A** Show that $A=\left\{\begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}\in M_{2\times2}(\mathbb{R}):\alpha+\beta=\gamma-\delta\right\}$ is a vector subspace of $M_{2\times2}(\mathbb{R})$ Also find a basis and dimension of the vector spacewhere A.

SOLUTION STEPS

since $\alpha+\beta=\gamma-\delta$ then $\alpha+\beta-\gamma+\delta=0$

$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\Rightarrow 0+0-0+0=0$

so, $0\in \text{A}----\boxed{1}$

Let $u=\begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}$, $\forall u\in \text{A}\Rightarrow \alpha+\beta-\gamma+\delta=0$

Let $v=\begin{bmatrix}\alpha_1 & \beta_1\\ \gamma_1 & \delta_1\end{bmatrix}$, $\forall v\in \text{A}\Rightarrow \alpha_1+\beta_1-\gamma_1+\delta_1=0$

Now $u+v=\begin{bmatrix}\alpha+\alpha_1 & \beta+\beta_1\\ \gamma+\gamma_1 & \delta+\delta_1\end{bmatrix}$

$\Rightarrow(\alpha+\alpha_1)+(\beta+\beta_1)-(\gamma+\gamma_1)+(\delta+\delta_1)$

$=(\alpha+\beta-\gamma+\delta)+(\alpha_1+\beta_1-\gamma_1+\delta_1)=0+0=0$

then $u+v\in \text{A}----\boxed{2}$

$\forall u\in \text{A}, \text{K}\in\mathbb{R}$

$\text{K}u=\begin{bmatrix}K\alpha & K\beta\\ K\gamma & K\delta\end{bmatrix}\Rightarrow(K\alpha)+(K\beta)-(K\gamma)+K\delta$

$=K(\alpha+\beta-\gamma+\delta)=K(0)=0----\boxed{3}$

From $\boxed{1},\boxed{2},\boxed{3}$

A is sub-space of $\text{M}_{2\times2}(\mathbb{R})$

$\gamma=\alpha+\beta+\delta$

$\begin{bmatrix}\alpha & \beta\\ \alpha+\beta+\delta & \delta\end{bmatrix}=\alpha\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}+\beta\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}+\delta\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$

the abasis is the set $=\left\{\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix},\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}\right\}$

$\text{dim.(A)}=3$

B Let A,B be matrices of size (3,3) such that A is not invertible and $|B|=2$. Find $|A\,\text{adj}(A)+2B^{-1}|$.

SOLUTION STEPS

A is not invertible so $\det A=0$

$$|A\,\text{adj}(A)+2B^{-1}|=|\det AI_3+2B^{-1}|=|2B^{-1}|=2^3|B^{-1}|=\frac{2^3}{|B|}=\frac{8}{2}=4$$

Question (9) Show that $S=\left\{X\in M_{2\times2}(\mathbb{R}): X=-X^{T}\right\}$ is a subspace of the vector space $M_{2\times2}(\mathbb{R})$ and show further that the set $\left\{\begin{bmatrix}0&-1\\1&0\end{bmatrix}\right\}$ is a basis for S.

SOLUTION STEPS

$0\in M_{2\times2}(\mathbb{R}): 0=-0^{T}$

$0\in S ----\boxed{1}$

$\forall X_1, X_2\in S\Rightarrow\left\langle\begin{matrix}X_1=-X_1^{T}\\X_2=-X_2^{T}\end{matrix}\right.$

$X_1+X_2=-(X_1+X_2)^{T}=-(X_1^{T}+X_2^{T})$

$=-X_1^{T}+X_2^{T}=X_1+X_2$

$Now\ X_1+X_2\in S ----\boxed{2}$

$\forall\alpha\in\mathbb{R}, X\in S: X=-X^{T}$

$\alpha X=-(\alpha X)^{T}=-\alpha X^{T}=\alpha X$

$\alpha X\in S ----\boxed{3}$

From $\boxed{1}$, $\boxed{2}$, $\boxed{3}$

S is sub-space in $M_{2\times2}(\mathbb{R})$

Another technique

$\begin{bmatrix}a&c\\b&d\end{bmatrix}=\begin{bmatrix}-a&-b\\-c&-d\end{bmatrix}$

$a=-a$	$c=-b$	$d=-d$
$2a=0$	$b=t$	$2d=0$; $t\in\mathbb{R}$
$a=0$	$c=-t$	$d=0$

$\begin{bmatrix}0&-t\\t&0\end{bmatrix}=t\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

the abasis of the set $=\left\{\begin{bmatrix}0&-1\\1&0\end{bmatrix}\right\}$

put $X=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

$X^{T}=\begin{bmatrix}0&1\\-1&0\end{bmatrix}\Rightarrow -X^{T}=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

Then $\begin{bmatrix}0&-1\\1&0\end{bmatrix}$ is a basis for S

Question (10) **A** Determine whether $S = \left\{ X \in M_{2\times2}(\mathbb{R}) : X = X^T \right\}$ is a proper subspace of the vector space $M_{2\times2}(\mathbb{R})$.

SOLUTION STEPS

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$; $\forall a,b,c,d \in \mathbb{R}$

since $X = X^T$

Then $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$a = a \;\Big|\; b = c \;\Big|\; d = d$

find dim.

$$\begin{pmatrix} a & b \\ b & d \end{pmatrix} = a\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + d\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

is span and linearly indepent

a basis of the set $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

B Determine whether the set $w = \left\{ (x, y) \in \mathbb{R}^2 \; ; x^2 = y^2 \right\}$ is a sub-space or not.

SOLUTION STEPS

suppose $u = (1,-1) \in \mathbb{R}^2 \Rightarrow 1^2 = (-1)^2$

$v = (1,1) \in \mathbb{R}^2 \Rightarrow 1^2 = 1^2$

$u + v = (2,0) \Rightarrow (2)^2 \neq 0^2$

$u + v \notin w$ is not sub-space for $\mathbb{R}^2$

Question (11) **A** Find a basis of the vector space $\mathbb{R}^3$ which contains the set $\{(1,1,0)(1,-1,0)\}$

SOLUTION STEPS

using standar abasis algorithm

$$\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & e_1 & e_2 & e_3 \end{array} \\ \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array} \xrightarrow{-R_1+R_2} \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{R_2}{-2}} \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

The basis $=\{v_1, v_2, e_3\}$

so $,v_1, v_2, e_3$ are lin early in dependnt

The basis $=\{(1,1,0),(1,-1,0),(0,0,1)\}$

B Let F be the sub-space of $\mathbb{R}^3$ generated by the vectors

$v_1=(1,-1,2)$, $v_2=(0,1,-1)$, $v_3=(1,0,1)$, and $v_4=(1,1,0)$

Is the vector $v=(1,1,1)$ in F ? (Justify your answer)

SOLUTION STEPS

if $(1,1,1)\in F \Leftrightarrow F$ in some a,b,c,d in $\mathbb{R}$

$(1,1,1)=av_1+bv_2+cv_3+dv_4$

$$\Leftrightarrow \begin{cases} a+c+d=1 \\ -a+b+d=1 \\ 2a-b+c=1 \end{cases} \text{ is consistent.}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ -1 & 1 & 0 & 1 & 1 \\ 2 & -1 & 1 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 \\ 0 & -1 & -1 & -2 & -1 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right) \text{ no solution so } v\notin F$$

$Question\ (12)$ IF $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 9 \\ 8 & 9 & 7 \\ 6 & 1 & 5 \end{bmatrix}$, Then find :

1 $\boxed{i}$ Rank (A) $\boxed{ii}$ Nullity (A) $\boxed{iii}$ Nullity (A^T)

2 $\boxed{i}$ A basis for col(A) $\boxed{ii}$ A basis for Row(A) $\boxed{iii}$ A basis for Nullity (A)

SOLUTION STEPS

Question (13) IF ${}_SP_B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 9 & 5 \end{bmatrix}$ is the transition matrix from the basis $B = \{v_1, v_2, v_3\}$ of $\mathbb{R}^3$ to its standard $S = \{e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)\}$ then find $[v_2]_S$

SOLUTION STEPS

Question (14) For the Euclidean inner product space $\mathbb{R}^3$:

$\boxed{i}$ Find $\cos\theta$, where θ is the angle between the vectors $(1,0,1)$ and $(0,-1,1)$.

$\boxed{ii}$ Use the Gram-Schmidt process to obtain an orthonormal basis from the given basis $\{(1,0,0),(1,1,0),(1,1,1)\}$ for $\mathbb{R}^3$.

SOLUTION STEPS

Question (15) Let $B=\{v_1=(1,3,1), v_2=(1,5,-1), v_3=(1,0,-1)\}$

$C=\{(1,-1,0),(0,1,1),(-1,1,1)\}$ tow basis $\mathbb{R}^3$ Find :

$\boxed{i}$ ${}_{C}P_{B}$ $\boxed{ii}$ ${}_{B}P_{C}$ $\boxed{iii}$ $[v]_B$ $\boxed{iv}$ $[v]_C$ $\boxed{v}$ $[v_2]_C$

so, $v=(1,0,3)$

SOLUTION STEPS

Question (16) Let V be avect. space of dimensirn. 3 and

$B=\{u_1,u_2,u_3\}$, $C=\{v_1,v_2,v_3\}$ are basis for V such that

$u_1+v_2=v_1+v_3$, $u_2-v_1=2v_2+3v_3$, $u_3+2v_3=-2v_1+v_2$ Find :

$\boxed{i}$ ${}_C P_B$ $\boxed{ii}$ IF $v=-3u_1+u_2+2u_3$ find $[v]_B$ and $[v]_C$

SOLUTION STEPS

Question (17) Let V be inner product space and Let u and v be vectors in V suppose that $\|u\| = \sqrt{3}$, $\|v\| = 4$ and angle between u and v is $\frac{\pi}{6}$ we recall that $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Compute the following inner product $\langle u, v \rangle, \langle u + v, 2u - v \rangle$

SOLUTION STEPS

Question (18) $\boxed{A}$ Let w is a sub-space in $\mathbb{R}^3$ genrated by $u_1=(1,2,-1)$, $u_2=(-1,1,2)$

$\boxed{i}$ Show that $u=(1,5,0)$ belong to w .

$\boxed{ii}$ Show that $v=(1,2,-2)$ Not belong to w .

SOLUTION STEPS

suppoce that $u=\alpha_1 u_1+\alpha_2 u_2$

$(1,5,0)=\alpha_1(1,2,-1)+\alpha_2(-1,1,2)$

$$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 2 & 1 & 5 \\ -1 & 2 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 3 & 3 \\ 0 & 1 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

ملاحظة :- لكى ينتمى المتجة لفضاء جزئى
شرط يكون تركيب خطى

عدد لانهائى من الحلول
many infinity solution

حل وحيد
unique solution

$\alpha_2=1$, $\alpha_1-\alpha_2=1 \Rightarrow \alpha_1=2$

one solution u is belong to w

$v=(1,2,-2)=\alpha_1(1,2,-1)+\alpha_2(-1,1,2)$

$$\left(\begin{array}{cc|c} 1 & -1 & 1 \\ 2 & 1 & 2 \\ -1 & 2 & -2 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 1 & -1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{array}\right)$$

no solution

Then $v \notin w$

$\boxed{B}$ Let $S=\{x^2+3, -x^2+x+1, -x^2+2x-2, x+2\}$ so that S be sabset in $P_2(x)$

and let spanned space by S. show That $P_2(x)=w$.

SOLUTION STEPS

Question (19) Let E be the sub-space of $\mathbb{R}^4$ spanned by the following vectors :

$u_1 = (1,1,2,-1)$, $u_2 = (2,-2,1,3), u_3 = (3,0,5,1), u_4 = (-1,3,1,-4), u_5 = (1,2,4,-2)$

find a basis of contained in $\{u_1 , u_2, u_3, u_4, u_5\}$

SOLUTION STEPS

Question (20) Let $S = \left\{ \frac{1}{3}(-2,1,1) \ , \ \frac{1}{3}(1,2,-2) \ , \ \frac{1}{3}(2,1,2) \right\} \subseteq \mathbb{R}^3$

$\boxed{i}$ Show that is S on orthonormal basis in the inner product space (Euclidean)

$\boxed{ii}$ Find $[v]_S$ so $v = (3,2,-1)$

SOLUTION STEPS