

# Chapter 14

## Fluid Mechanics

14.1 Pressure

14.2 Variation of Pressure with Depth

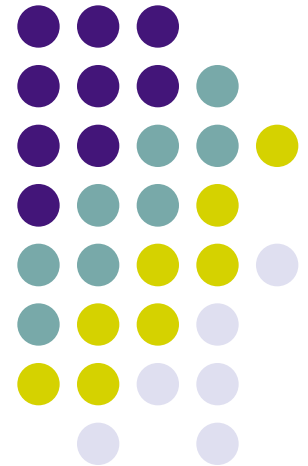
14.3 Pressure Measurements

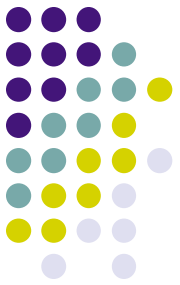
14.4 Buoyant Forces and Archimedes's Principle

14.5 Fluid Dynamics

14.6 Bernoulli's Equation

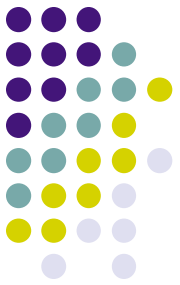
14.7 Other Applications of Fluid Dynamics





# States of Matter

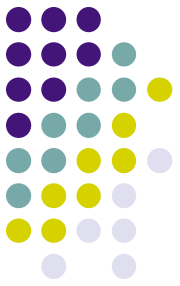
- Solid
  - Has a definite volume and shape
- Liquid
  - Has a definite volume but not a definite shape
- Gas
  - Has neither a definite volume nor shape



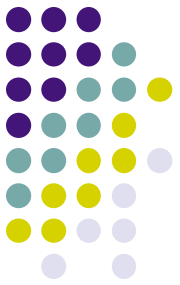
# Fluids

- A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container
- Both liquids and gases are fluids

# Statics and Dynamics with Fluids

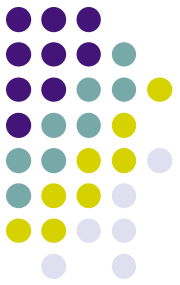


- Fluid Statics
  - Describes fluids at rest
- Fluid Dynamics
  - Describes fluids in motion
- The same physical principles that have applied to statics and dynamics up to this point will also apply to fluids



# Forces in Fluids

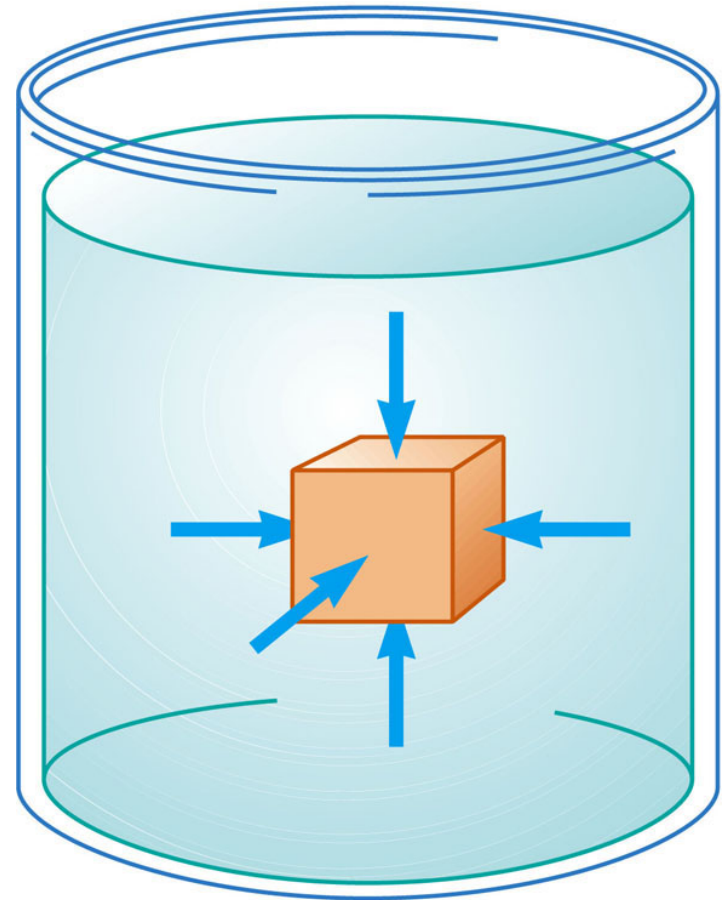
- Fluids do not sustain shearing stresses or tensile stresses
- The force exerted by a static fluid on an object is always perpendicular to the surfaces of the object



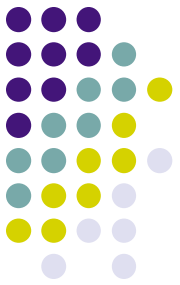
# Pressure

- The **pressure**  $P$  of the fluid at the level to which the device has been submerged is the ratio of the force to the area

$$P \equiv \frac{F}{A}$$

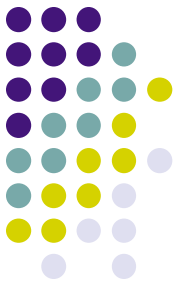


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# Pressure, cont

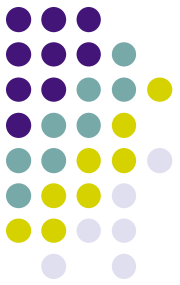
- Pressure is a scalar quantity
  - Because it is proportional to the magnitude of the force
- If the pressure varies over an area, evaluate  $dF$  on a surface of area  $dA$  as  $dF = P dA$
- Unit of pressure is **pascal** (Pa)  
 $1 \text{ Pa} = 1 \text{ N/m}^2$



# Pressure vs. Force

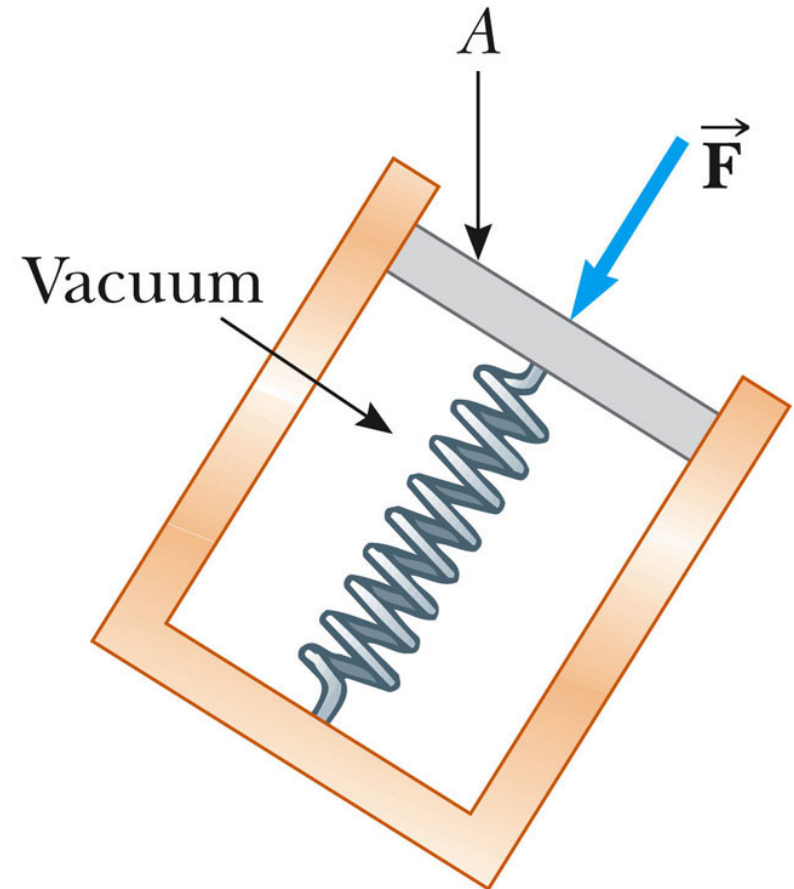
- Pressure is a scalar and force is a vector
- The direction of the force producing a pressure is perpendicular to the area of interest





# Measuring Pressure

- The spring is calibrated by a known force
- The force due to the fluid presses on the top of the piston and compresses the spring
- The force the fluid exerts on the piston is then measured



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### Example 14.1    The Water Bed

The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep.

**(A)** Find the weight of the water in the mattress.

#### SOLUTION

**Conceptualize** Think about carrying a jug of water and how heavy it is. Now imagine a sample of water the size of a water bed. We expect the weight to be relatively large.

**Categorize** This example is a substitution problem.

Find the volume of the water filling the mattress:

$$V = (2.00 \text{ m})(2.00 \text{ m})(0.300 \text{ m}) = 1.20 \text{ m}^3$$

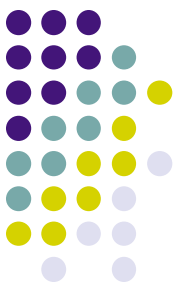
Use Equation 1.1 and the density of fresh water (see Table 14.1) to find the mass of the water bed:

$$M = \rho V = (1\,000 \text{ kg/m}^3)(1.20 \text{ m}^3) = 1.20 \times 10^3 \text{ kg}$$

Find the weight of the bed:

$$Mg = (1.20 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) = 1.18 \times 10^4 \text{ N}$$

which is approximately 2 650 lb. (A regular bed, including mattress, box spring, and metal frame, weighs approximately 300 lb.) Because this load is so great, it is best to place a water bed in the basement or on a sturdy, well-supported floor.



► 14.1 continued

**(B)** Find the pressure exerted by the water bed on the floor when the bed rests in its normal position. Assume the entire lower surface of the bed makes contact with the floor.

**SOLUTION**

When the water bed is in its normal position, the area in contact with the floor is  $4.00 \text{ m}^2$ . Use Equation 14.1 to find the pressure:

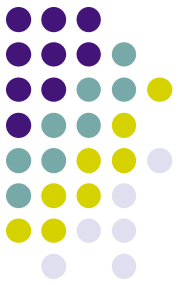
$$P = \frac{1.18 \times 10^4 \text{ N}}{4.00 \text{ m}^2} = 2.94 \times 10^3 \text{ Pa}$$

**WHAT IF?** What if the water bed is replaced by a 300-lb regular bed that is supported by four legs? Each leg has a circular cross section of radius 2.00 cm. What pressure does this bed exert on the floor?

**Answer** The weight of the regular bed is distributed over four circular cross sections at the bottom of the legs. Therefore, the pressure is

$$\begin{aligned} P &= \frac{F}{A} = \frac{mg}{4(\pi r^2)} = \frac{300 \text{ lb}}{4\pi(0.0200 \text{ m})^2} \left( \frac{1 \text{ N}}{0.225 \text{ lb}} \right) \\ &= 2.65 \times 10^5 \text{ Pa} \end{aligned}$$

This result is almost 100 times larger than the pressure due to the water bed! The weight of the regular bed, even though it is much less than the weight of the water bed, is applied over the very small area of the four legs. The high pressure on the floor at the feet of a regular bed could cause dents in wood floors or permanently crush carpet pile.



# Density Notes

- Density is defined as the mass per unit volume of the substance
- The values of density for a substance vary slightly with temperature since volume is temperature dependent
- The various densities indicate the average molecular spacing in a gas is much greater than that in a solid or liquid



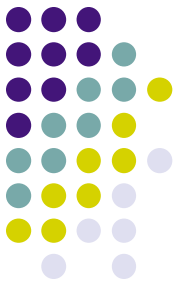
# Density Table

**TABLE 14.1**

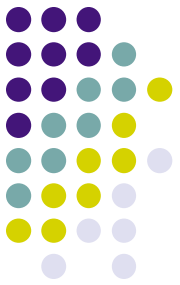
**Densities of Some Common Substances at Standard Temperature (0°C) and Pressure (Atmospheric)**

Substance	$\rho$ (kg/m <sup>3</sup> )	Substance	$\rho$ (kg/m <sup>3</sup> )
Air	1.29	Ice	$0.917 \times 10^3$
Aluminum	$2.70 \times 10^3$	Iron	$7.86 \times 10^3$
Benzene	$0.879 \times 10^3$	Lead	$11.3 \times 10^3$
Copper	$8.92 \times 10^3$	Mercury	$13.6 \times 10^3$
Ethyl alcohol	$0.806 \times 10^3$	Oak	$0.710 \times 10^3$
Fresh water	$1.00 \times 10^3$	Oxygen gas	1.43
Glycerin	$1.26 \times 10^3$	Pine	$0.373 \times 10^3$
Gold	$19.3 \times 10^3$	Platinum	$21.4 \times 10^3$
Helium gas	$1.79 \times 10^{-1}$	Seawater	$1.03 \times 10^3$
Hydrogen gas	$8.99 \times 10^{-2}$	Silver	$10.5 \times 10^3$

# Variation of Pressure with Depth

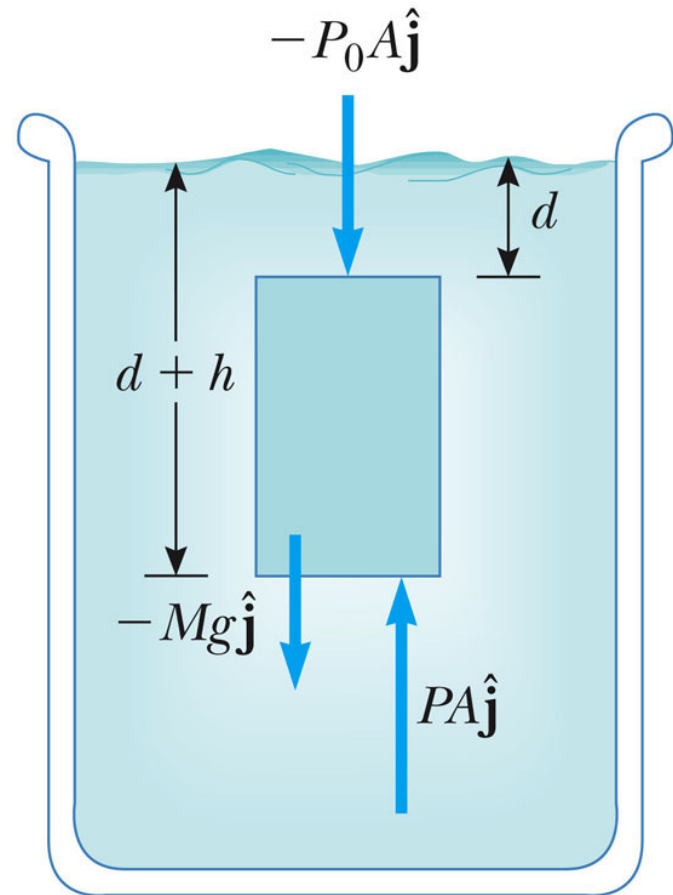


- Fluids have pressure that varies with depth
- If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium
- All points at the same depth must be at the same pressure
  - Otherwise, the fluid would not be in equilibrium

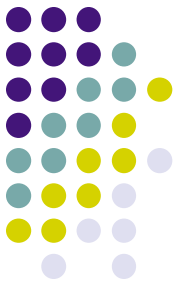


# Pressure and Depth

- Examine the darker region, a sample of liquid within a cylinder
  - It has a cross-sectional area  $A$
  - Extends from depth  $d$  to  $d + h$  below the surface
- Three external forces act on the region



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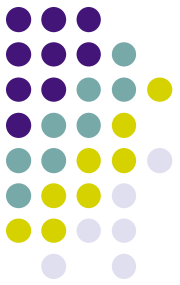
# Pressure and Depth, cont

- The liquid has a density of  $\rho$ 
  - Assume the density is the same throughout the fluid
  - This means it is an incompressible liquid
- The three forces are:
  - Downward force on the top,  $P_0A$
  - Upward on the bottom,  $PA$
  - Gravity acting downward,  $Mg$ 
    - The mass can be found from the density:

$$M = \rho V = \rho Ah$$

Section 14.2



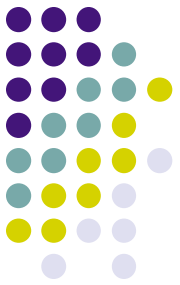


# Pressure and Depth, final

- Since the net force must be zero:

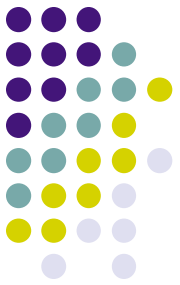
$$\sum \vec{F} = PA\hat{j} - P_o A\hat{j} - Mg\hat{j}$$

- This chooses upward as positive
- Solving for the pressure gives
  - $P = P_o + \rho gh$
- The pressure  $P$  at a depth  $h$  below a point in the liquid at which the pressure is  $P_o$  is greater by an amount  $\rho gh$



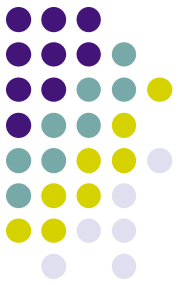
# Atmospheric Pressure

- If the liquid is open to the atmosphere, and  $P_0$  is the pressure at the surface of the liquid, then  $P_0$  is *atmospheric pressure*
- $P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$



# Pascal's Law

- The pressure in a fluid depends on depth and on the value of  $P_0$
- An increase in pressure at the surface must be transmitted to every other point in the fluid
- This is the basis of Pascal's law



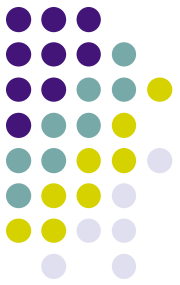
# Pascal's Law, cont

- ***A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container***

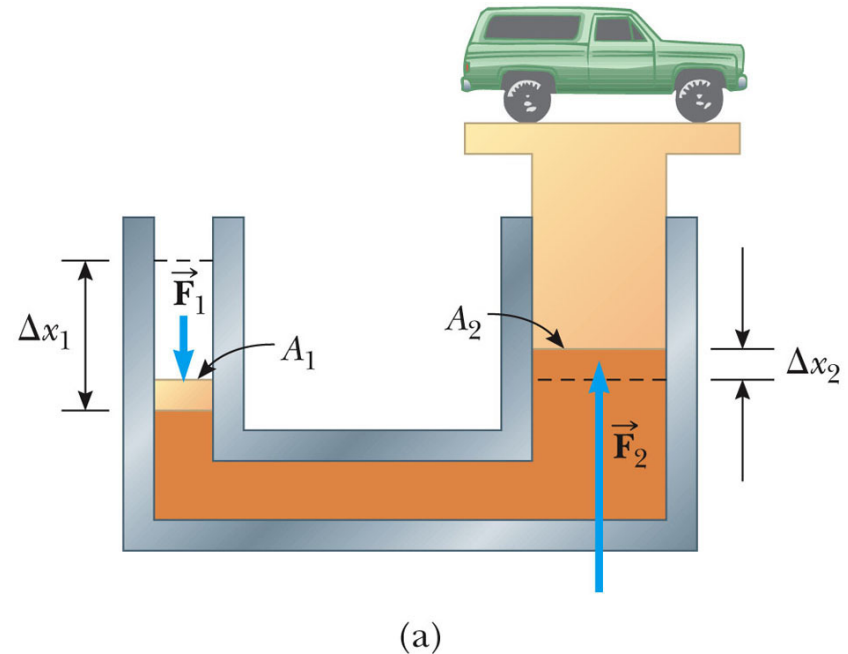
$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

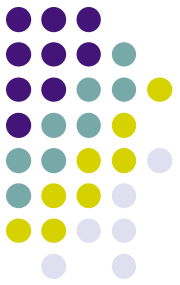
# Pascal's Law, Example



- Diagram of a hydraulic press (right)
- A large output force can be applied by means of a small input force
- The volume of liquid pushed down on the left must equal the volume pushed up on the right



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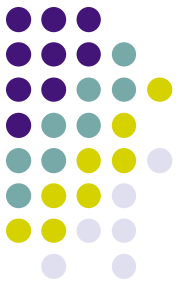
# Pascal's Law, Example cont.

- Since the volumes are equal,

$$A_1 \Delta x_1 = A_2 \Delta x_2$$

- Combining the equations,
  - $F_1 \Delta x_1 = F_2 \Delta x_2$  which means  $Work_1 = Work_2$
  - This is a consequence of Conservation of Energy

# Pascal's Law, Other Applications



- Hydraulic brakes
- Car lifts
- Hydraulic jacks
- Forklifts

**Example 14.2****The Car Lift**

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section of radius 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm.

**(A)** What force must the compressed air exert to lift a car weighing 13 300 N?

**SOLUTION**

**Conceptualize** Review the material just discussed about Pascal's law to understand the operation of a car lift.

**Categorize** This example is a substitution problem.

Solve  $F_1/A_1 = F_2/A_2$  for  $F_1$ :

$$\begin{aligned} F_1 &= \left( \frac{A_1}{A_2} \right) F_2 = \frac{\pi(5.00 \times 10^{-2} \text{ m})^2}{\pi(15.0 \times 10^{-2} \text{ m})^2} (1.33 \times 10^4 \text{ N}) \\ &= 1.48 \times 10^3 \text{ N} \end{aligned}$$

**(B)** What air pressure produces this force?

**SOLUTION**

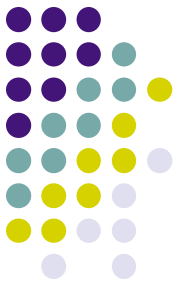
Use Equation 14.1 to find the air pressure that produces this force:

$$\begin{aligned} P &= \frac{F_1}{A_1} = \frac{1.48 \times 10^3 \text{ N}}{\pi(5.00 \times 10^{-2} \text{ m})^2} \\ &= 1.88 \times 10^5 \text{ Pa} \end{aligned}$$

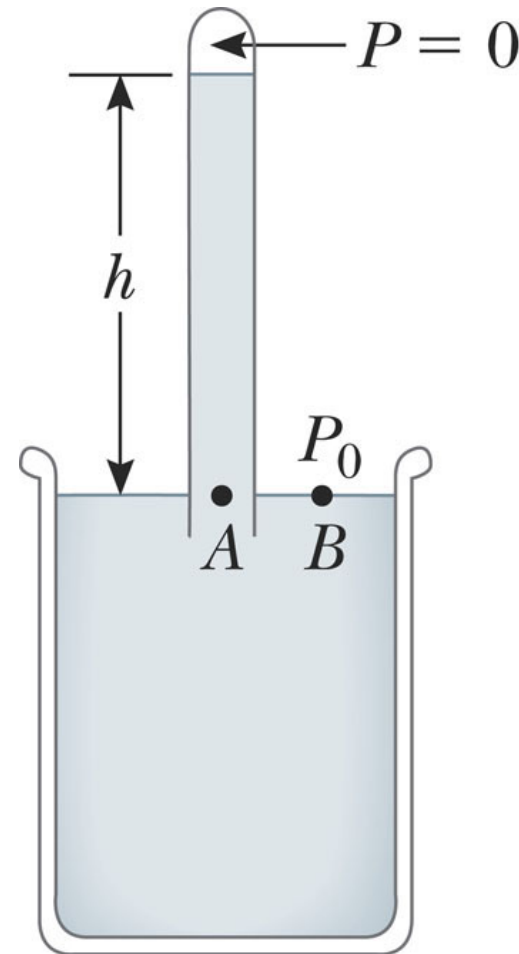
This pressure is approximately twice atmospheric pressure.



# Pressure Measurements: Barometer



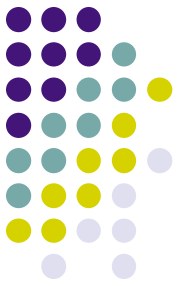
- Invented by Torricelli
- A long closed tube is filled with mercury and inverted in a dish of mercury
  - The closed end is nearly a vacuum
- Measures atmospheric pressure as  $P_o = \rho_{\text{Hg}}gh$
- One 1 atm = 0.760 m (of Hg)



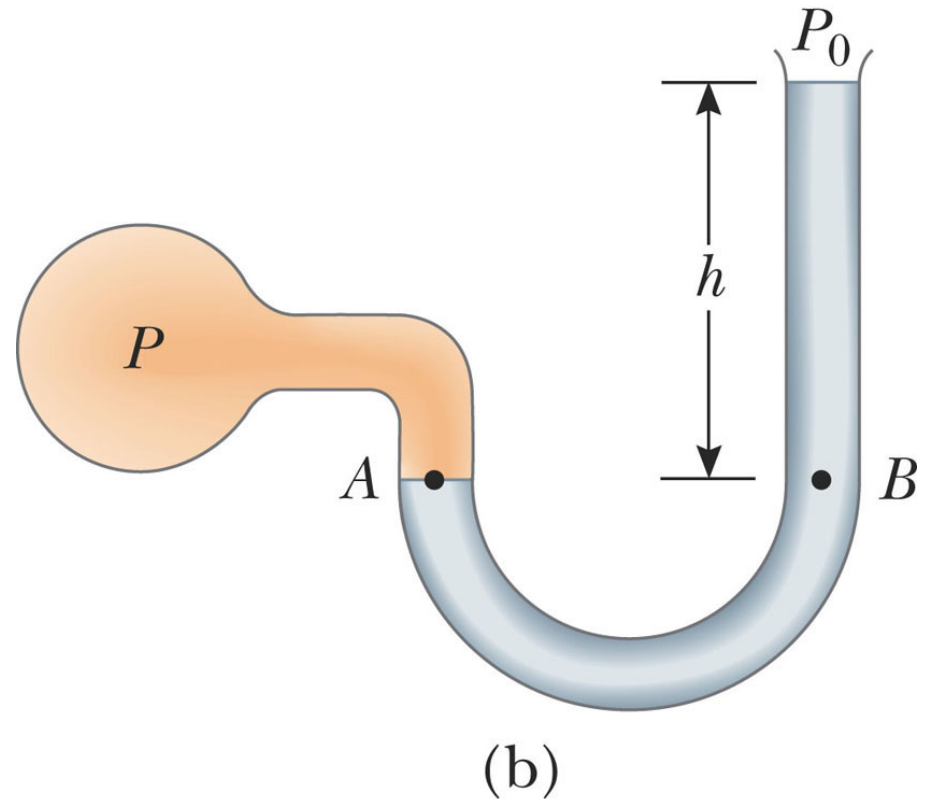
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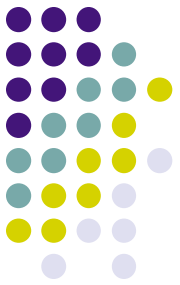
# Pressure Measurements: Manometer



- A device for measuring the pressure of a gas contained in a vessel
- One end of the U-shaped tube is open to the atmosphere
- The other end is connected to the pressure to be measured
- Pressure at  $B$  is  $P = P_0 + \rho gh$



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# Absolute vs. Gauge Pressure

- $P = P_0 + \rho gh$
- $P$  is the **absolute pressure**
- The **gauge pressure** is  $P - P_0$ 
  - This is also  $\rho gh$
  - This is what you measure in your tires

### Example 14.3

### A Pain in Your Ear

Estimate the force exerted on your eardrum due to the water when you are swimming at the bottom of a pool that is 5.0 m deep.

Use Equation 14.4 to find this pressure difference:

$$\begin{aligned} P_{\text{bot}} - P_0 &= \rho gh \\ &= (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.0 \text{ m}) = 4.9 \times 10^4 \text{ Pa} \end{aligned}$$

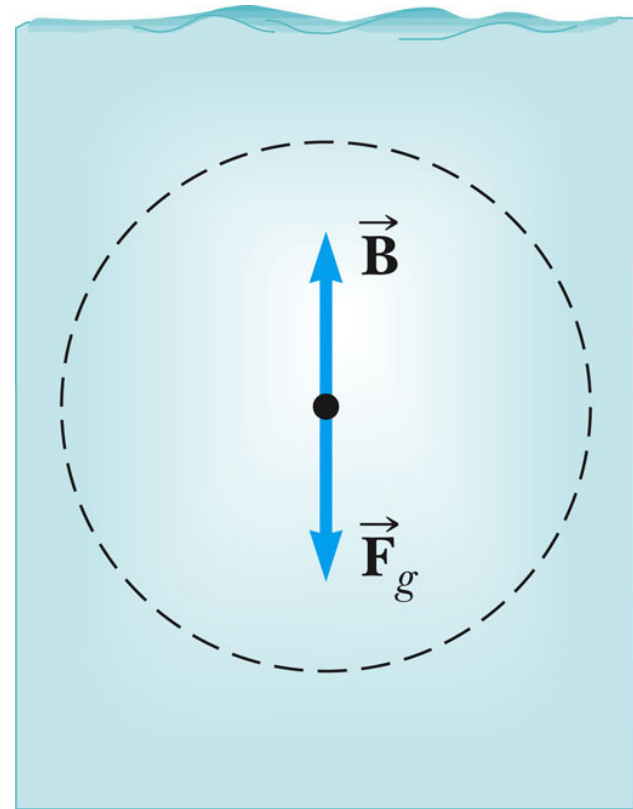
Use Equation 14.1 to find the magnitude of the net force on the ear:

$$F = (P_{\text{bot}} - P_0)A = (4.9 \times 10^4 \text{ Pa})(1 \times 10^{-4} \text{ m}^2) \approx 5 \text{ N}$$



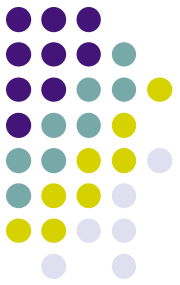
# Buoyant Force

- The **buoyant force** is the upward force exerted by a fluid on any immersed object
- The parcel is in equilibrium
- There must be an upward force to balance the downward gravitational force



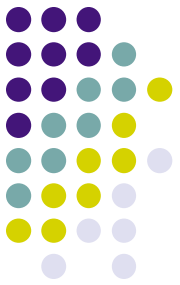
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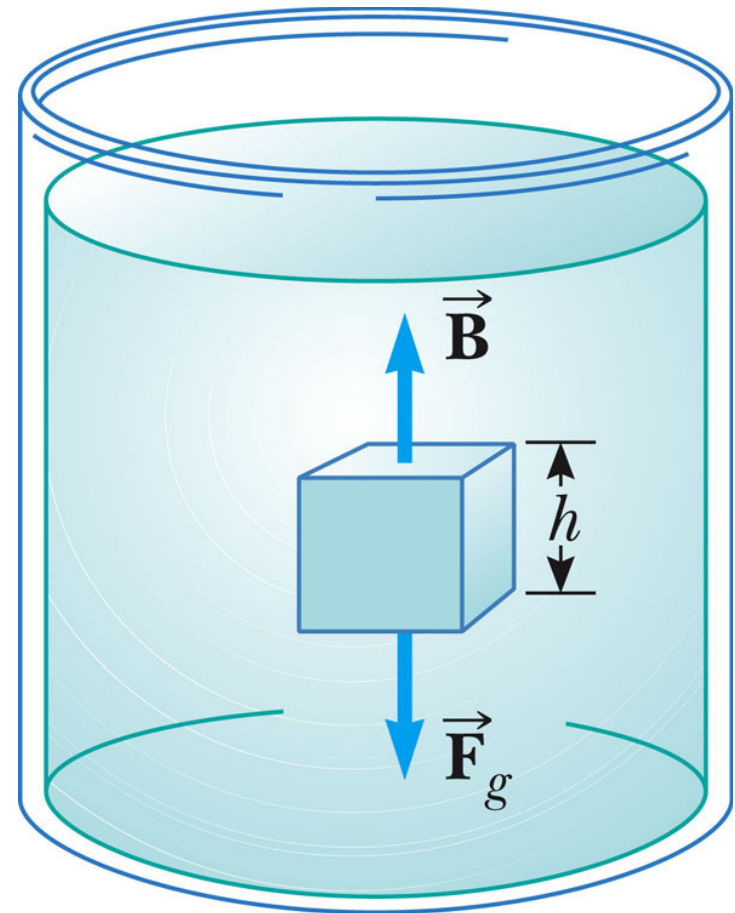
# Archimedes' s Principle

- The magnitude of the buoyant force always equals the weight of the fluid displaced by the object
  - This is called ***Archimedes' s Principle***



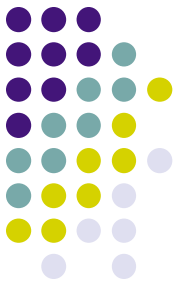
# Archimedes' s Principle, cont

- The pressure at the top of the cube causes a downward force of  $P_{top} A$
- The pressure at the bottom of the cube causes an upward force of  $P_{bot} A$
- $B = (P_{bot} - P_{top}) A$   
 $= \rho_{fluid} g V = Mg$



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# Archimedes's Principle: Totally Submerged Object



- An object is totally submerged in a fluid of density  $\rho_{\text{fluid}}$

- The upward buoyant force is

$$B = \rho_{\text{fluid}} g V = \rho_{\text{fluid}} g V_{\text{object}}$$

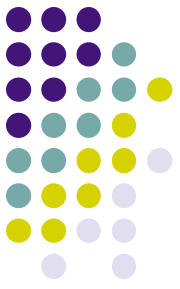
- The downward gravitational force is

$$F_g = Mg = \rho_{\text{obj}} g V_{\text{obj}}$$

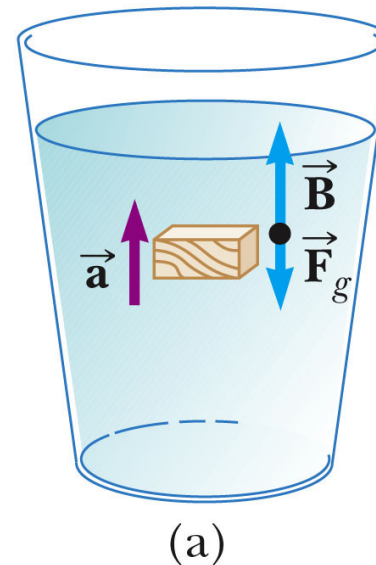
- The net force is  $B - F_g = (\rho_{\text{fluid}} - \rho_{\text{obj}}) g V_{\text{obj}}$



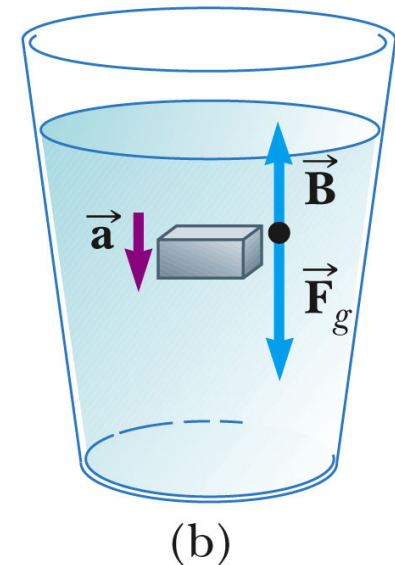
# Archimedes' s Principle: Totally Submerged Object, cont



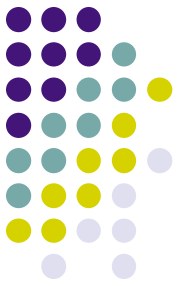
- If the density of the object is less than the density of the fluid, the unsupported object accelerates upward
- If the density of the object is more than the density of the fluid, the unsupported object sinks
- ***The direction of the motion of an object in a fluid is determined only by the densities of the fluid and the object***



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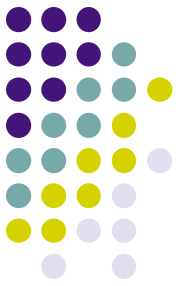
# Archimedes' s Principle: Floating Object



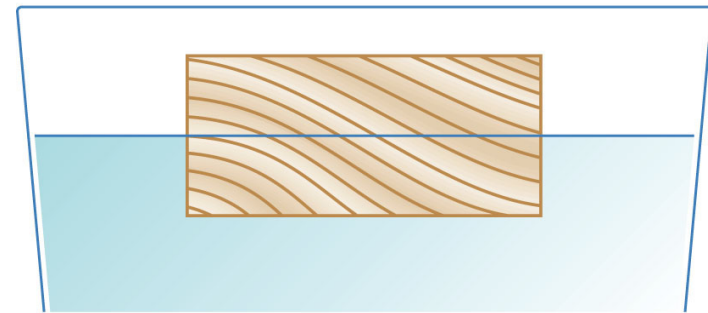
- The object is in static equilibrium
- The upward buoyant force is balanced by the downward force of gravity
- Volume of the fluid displaced corresponds to the volume of the object beneath the fluid level

$$\frac{V_{\text{fluid}}}{V_{\text{obj}}} = \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}}$$

# Archimedes' s Principle: Floating Object, cont



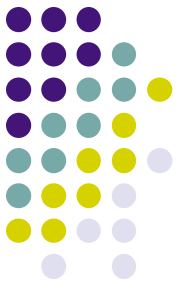
- The fraction of the volume of a floating object that is below the fluid surface is equal to the ratio of the density of the object to that of the fluid
- Use the active figure to vary the densities



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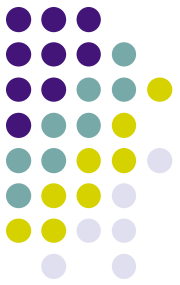


# Archimedes' s Principle, Crown Example

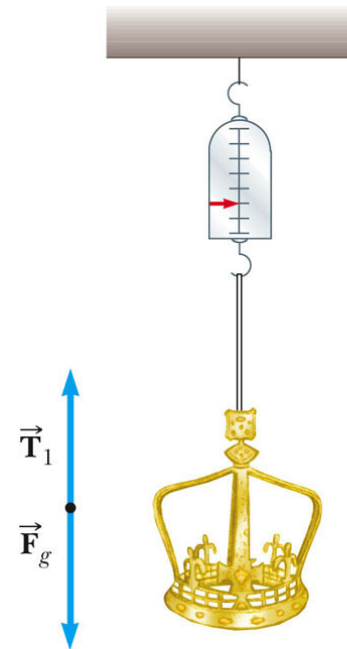


- Archimedes was (supposedly) asked, “Is the crown made of pure gold?”
- Crown' s weight in air = 7.84 N
- Weight in water (submerged) = 6.84 N
- Buoyant force will equal the apparent weight loss
  - Difference in scale readings will be the buoyant force

# Archimedes' s Principle, Crown Example, cont.

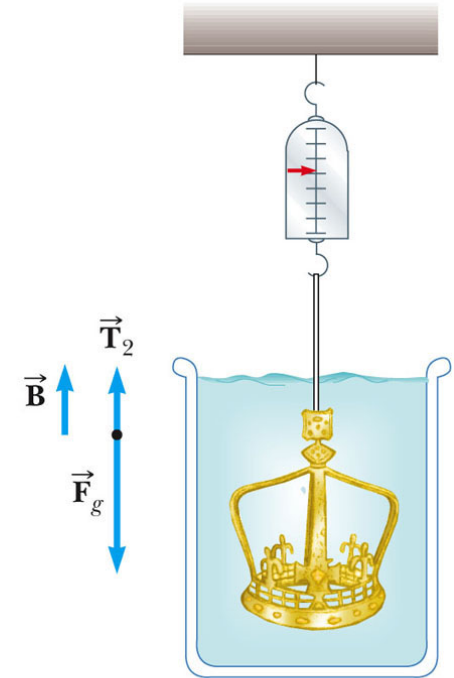


- $\Sigma F = B + T_2 - F_g = 0$
- $B = F_g - T_2$   
(Weight in air – “weight” in water)
- Archimedes' s principle says  $B = \rho g V$ 
  - Find  $V$
- Then to find the material of the crown,  $\rho_{\text{crown}} = m_{\text{crown in air}} / V$



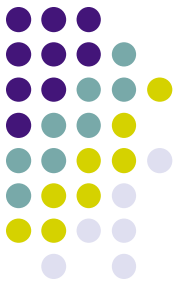
(a)

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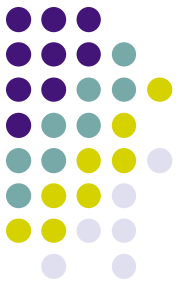
(b)

# Archimedes' s Principle, Iceberg Example



- What fraction of the iceberg is below water?
- The iceberg is only partially submerged and so  $V_{\text{seawater}} / V_{\text{ice}} = \rho_{\text{ice}} / \rho_{\text{seawater}}$  applies
- The fraction below the water will be the ratio of the volumes ( $V_{\text{seawater}} / V_{\text{ice}}$ )

# Archimedes' s Principle, Iceberg Example, cont

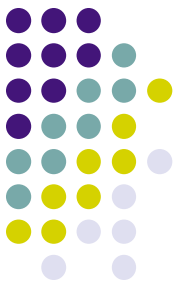


- $V_{\text{ice}}$  is the total volume of the iceberg
- $V_{\text{water}}$  is the volume of the water displaced
  - This will be equal to the volume of the iceberg submerged
- About 89% of the ice is below the water' s surface



(b)

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Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. According to legend, he solved this problem by weighing the crown first in air and then in water as shown in Figure 14.11. Suppose the scale read 7.84 N when the crown was in air and 6.84 N when it was in water. What should Archimedes have told the king?

### SOLUTION

**Conceptualize** Figure 14.11 helps us imagine what is happening in this example. Because of the buoyant force, the scale reading is smaller in Figure 14.11b than in Figure 14.11a.

**Categorize** This problem is an example of Case 1 discussed earlier because the crown is completely submerged. The scale reading is a measure of one of the forces on the crown, and the crown is stationary. Therefore, we can categorize the crown as a *particle in equilibrium*.

**Analyze** When the crown is suspended in air, the scale reads the true weight  $T_1 = F_g$  (neglecting the small buoyant force due to the surrounding air). When the crown is immersed in water, the buoyant force  $\vec{B}$  reduces the scale reading to an *apparent weight* of  $T_2 = F_g - B$ .

Apply the particle in equilibrium model to the crown in water:

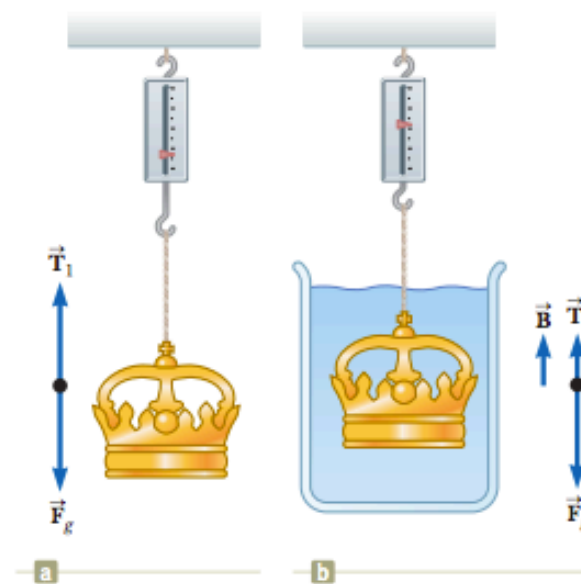
Solve for  $B$ :

Because this buoyant force is equal in magnitude to the weight of the displaced water,  $B = \rho_w g V_{\text{disp}}$ , where  $V_{\text{disp}}$  is the volume of the displaced water and  $\rho_w$  is its density. Also, the volume of the crown  $V_c$  is equal to the volume of the displaced water because the crown is completely submerged, so  $B = \rho_w g V_c$ .

Find the density of the crown from Equation 1.1:

Substitute numerical values:

Abeer Alghamdi



**Figure 14.11** (Example 14.5) (a) When the crown is suspended in air, the scale reads its true weight because  $T_1 = F_g$  (the buoyancy of air is negligible). (b) When the crown is immersed in water, the buoyant force  $\vec{B}$  changes the scale reading to a lower value  $T_2 = F_g - B$ .

$$\sum F = B + T_2 - F_g = 0$$

$$B = F_g - T_2$$

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{m_c g}{(B/\rho_w)} = \frac{m_c g \rho_w}{B} = \frac{m_c g \rho_w}{F_g - T_2}$$

$$\rho_c = \frac{(7.84 \text{ N})(1000 \text{ kg/m}^3)}{7.84 \text{ N} - 6.84 \text{ N}} = 7.84 \times 10^3 \text{ kg/m}^3$$



**WHAT IF?** Suppose the crown has the same weight but is indeed pure gold and not hollow. What would the scale reading be when the crown is immersed in water?

**Answer** Find the buoyant force on the crown:

$$B = \rho_w g V_w = \rho_w g V_c = \rho_w g \left( \frac{m_c}{\rho_c} \right) = \rho_w \left( \frac{m_c g}{\rho_c} \right)$$

Substitute numerical values:

$$B = (1.00 \times 10^3 \text{ kg/m}^3) \frac{7.84 \text{ N}}{19.3 \times 10^3 \text{ kg/m}^3} = 0.406 \text{ N}$$

Find the tension in the string hanging from the scale:

$$T_2 = F_g - B = 7.84 \text{ N} - 0.406 \text{ N} = 7.43 \text{ N}$$



## Example 14.6 A Titanic Surprise

An iceberg floating in seawater as shown in Figure 14.12a is extremely dangerous because most of the ice is below the surface. This hidden ice can damage a ship that is still a considerable distance from the visible ice. What fraction of the iceberg lies below the water level?

### SOLUTION

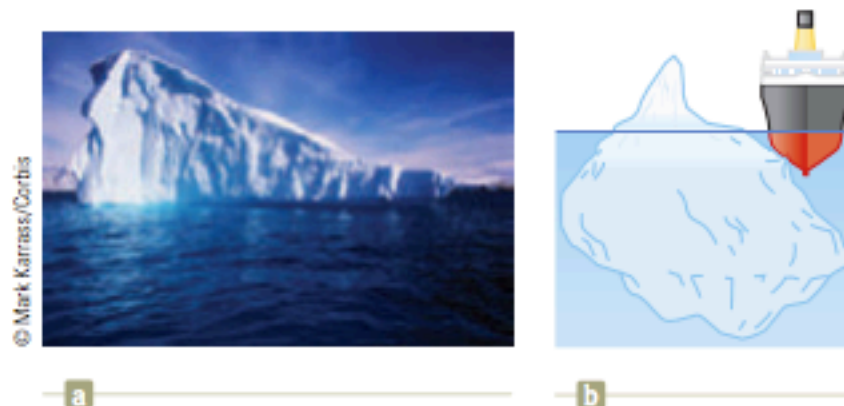
**Conceptualize** You are likely familiar with the phrase, “That’s only the tip of the iceberg.” The origin of this popular saying is that most of the volume of a floating iceberg is beneath the surface of the water (Fig. 14.12b).

**Categorize** This example corresponds to Case 2 because only part of the iceberg is underneath the water. It is also a simple substitution problem involving Equation 14.6.

Evaluate Equation 14.6 using the densities of ice and seawater (Table 14.1):

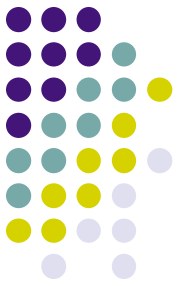
$$f = \frac{V_{\text{disp}}}{V_{\text{ice}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{seawater}}} = \frac{917 \text{ kg/m}^3}{1\,030 \text{ kg/m}^3} = 0.890 \text{ or } 89.0\%$$

Therefore, the visible fraction of ice above the water’s surface is about 11%. It is the unseen 89% below the water that represents the danger to a passing ship.



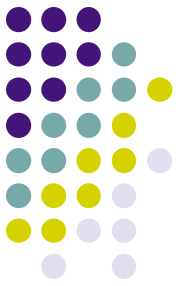
**Figure 14.12** (Example 14.6) (a) Much of the volume of this iceberg is beneath the water. (b) A ship can be damaged even when it is not near the visible ice.

# Types of Fluid Flow – Laminar



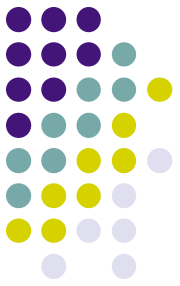
- Laminar flow
  - Steady flow
  - Each particle of the fluid follows a smooth path
  - The paths of the different particles never cross each other
  - Every given fluid particle arriving at a given point has the same velocity
  - The path taken by the particles is called a *streamline*

# Types of Fluid Flow – Turbulent

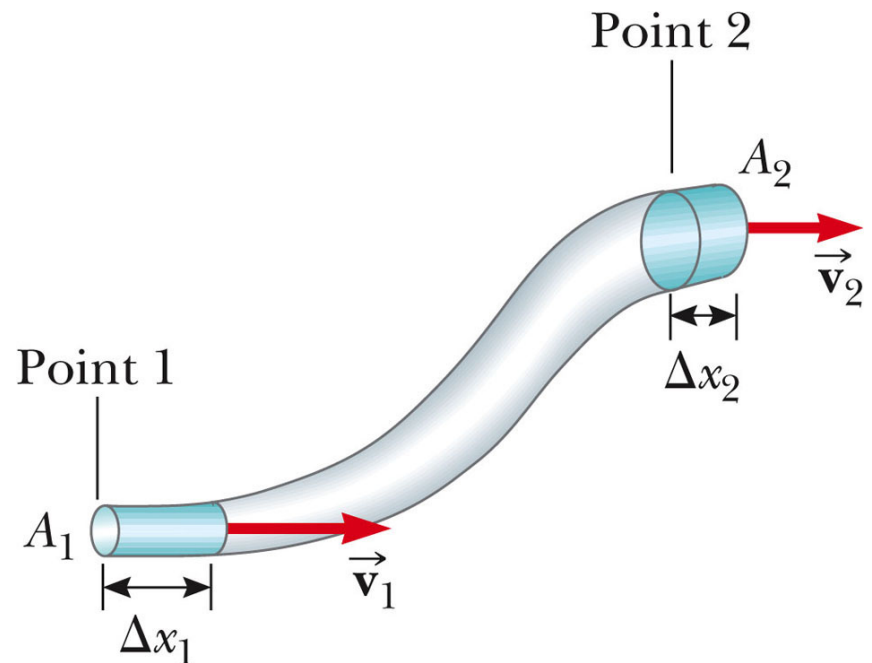


- An irregular flow characterized by small whirlpool-like regions
- Turbulent flow occurs when the particles go above some critical speed

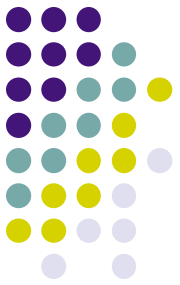
# Equation of Continuity



- Consider a fluid moving through a pipe of nonuniform size (diameter)
- The particles move along streamlines in steady flow
- The mass that crosses  $A_1$  in some time interval is the same as the mass that crosses  $A_2$  in that same time interval



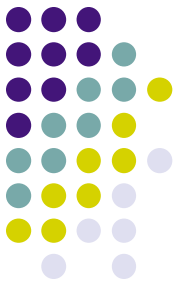
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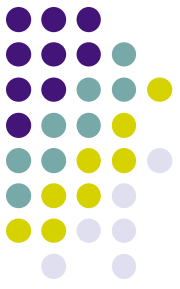
# Equation of Continuity, cont

- $m_1 = m_2$  or  $\rho A_1 v_1 = \rho A_2 v_2$
- Since the fluid is incompressible,  $\rho$  is a constant
- $A_1 v_1 = A_2 v_2$ 
  - This is called the **equation of continuity for fluids**
  - The product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid

# Equation of Continuity, Implications



- The speed is high where the tube is constricted (small  $A$ )
- The speed is low where the tube is wide (large  $A$ )
- The product,  $Av$ , is called the *volume flux* or the *flow rate*
- $Av = \text{constant}$  is equivalent to saying the volume that enters one end of the tube in a given time interval equals the volume leaving the other end in the same time
- $Q = Av = V/t$ 
  - If no leaks are present



### Example 14.7

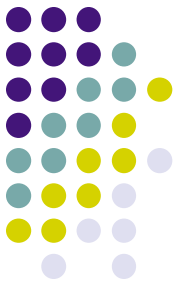
A gardener uses a water hose to fill a 30.0-L bucket. The gardener notes that it takes 1.00 min to fill the bucket. A nozzle with an opening of cross-sectional area 0.500 cm<sup>2</sup> is then attached to the hose. at what rate does water flow through the hose? and what the flow speed of water?

$$Q = V/t = (30 \times 10^{-3}) / 60 = 0.5 \times 10^{-3} \text{ m}^3/\text{s}$$

$$A = 0.5 \times 10^{-4} \text{ m}^2$$

$$Q = Av$$

$$v = Q/A = 0.5 \times 10^{-3} / 0.5 \times 10^{-4} = 10 \text{ m/s}$$



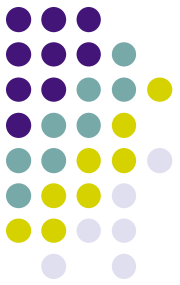
# Daniel Bernoulli

- 1700 – 1782
- Swiss physicist
- Published *Hydrodynamica*
  - Dealt with equilibrium, pressure and speeds in fluids
  - Also a beginning of the study of gasses with changing pressure and temperature



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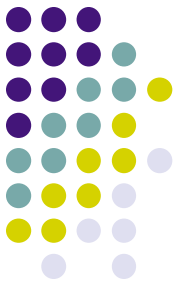




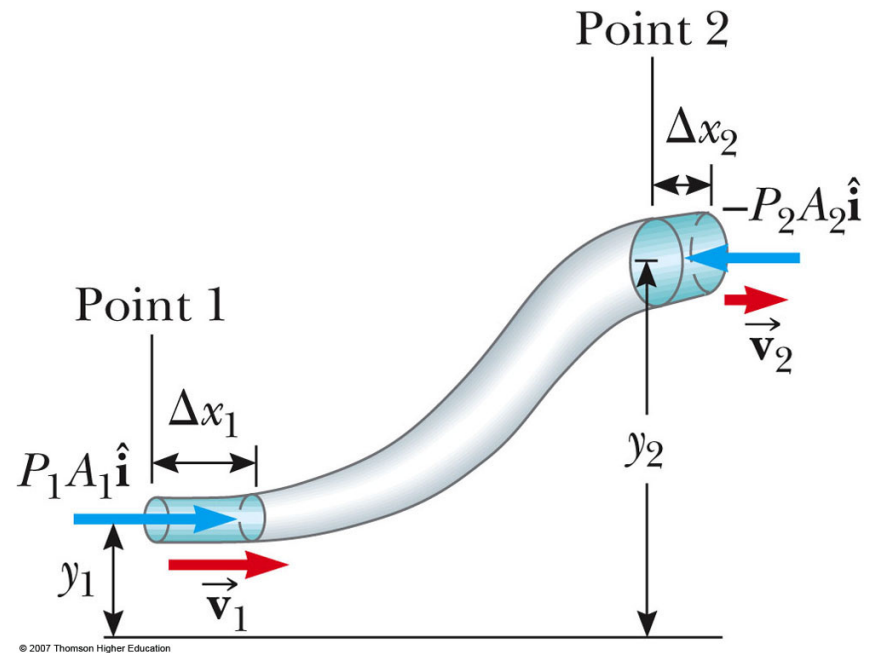
# Bernoulli's Equation

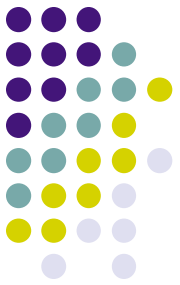
- As a fluid moves through a region where its speed and/or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes
- The relationship between fluid speed, pressure and elevation was first derived by Daniel Bernoulli

# Bernoulli's Equation, 2



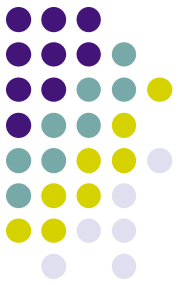
- Consider the two shaded segments
- The volumes of both segments are equal
- The net work done on the segment is  $W = (P_1 - P_2) V$
- Part of the work goes into changing the kinetic energy and some to changing the gravitational potential energy





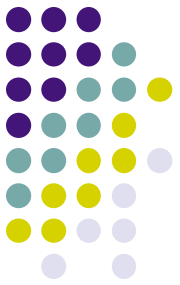
# Bernoulli's Equation, 3

- The change in kinetic energy:
  - $\Delta K = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$
  - There is no change in the kinetic energy of the unshaded portion since we are assuming streamline flow
  - The masses are the same since the volumes are the same



# Bernoulli's Equation, 4

- The change in gravitational potential energy:
  - $\Delta U = mgy_2 - mgy_1$
- The work also equals the change in energy
- Combining:
  - $(P_1 - P_2)V = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 + mgy_2 - mgy_1$



# Bernoulli' s Equation, 5

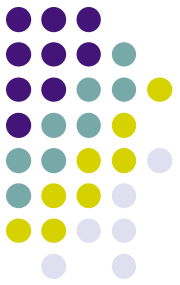
- Rearranging and expressing in terms of density:

$$P_1 + \frac{1}{2} \rho v_1^2 + mgy_1 = P_2 + \frac{1}{2} \rho v_2^2 + mgy_2$$

- This is Bernoulli' s Equation and is often expressed as

$$P + \frac{1}{2} \rho v^2 + \rho gy = \text{constant}$$

- When the fluid is at rest, this becomes  $P_1 - P_2 = \rho gh$  which is consistent with the pressure variation with depth we found earlier



# Bernoulli's Equation, Final

- The general behavior of pressure with speed is true even for gases
  - As the speed increases, the pressure decreases

## Example 14.8 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 14.19, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 of Figure 14.19a if the pressure difference  $P_1 - P_2$  is known.

### SOLUTION

**Conceptualize** Bernoulli's equation shows how the pressure of an ideal fluid decreases as its speed increases. Therefore, we should be able to calibrate a device to give us the fluid speed if we can measure pressure.

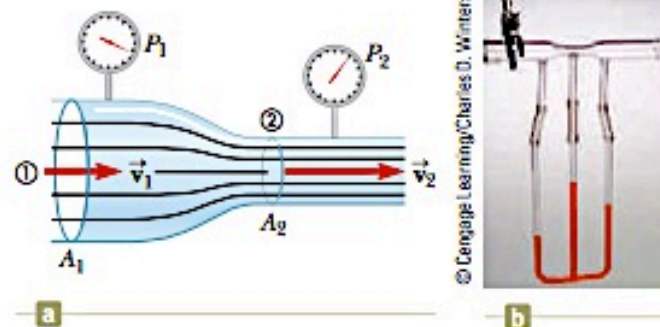
**Categorize** Because the problem states that the fluid is incompressible, we can categorize it as one in which we can use the equation of continuity for fluids and Bernoulli's equation.

**Analyze** Apply Equation 14.8 to points 1 and 2, noting that  $y_1 = y_2$  because the pipe is horizontal:

Solve the equation of continuity for  $v_1$ :

Substitute this expression into Equation (1):

Solve for  $v_2$ :



**Figure 14.19** (Example 14.8) (a) Pressure  $P_1$  is greater than pressure  $P_2$  because  $v_1 < v_2$ . This device can be used to measure the speed of fluid flow. (b) A Venturi tube, located at the top of the photograph. The higher level of fluid in the middle column shows that the pressure at the top of the column, which is in the constricted region of the Venturi tube, is lower.

$$(1) \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

$$P_1 + \frac{1}{2}\rho \left( \frac{A_2}{A_1} \right)^2 v_2^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

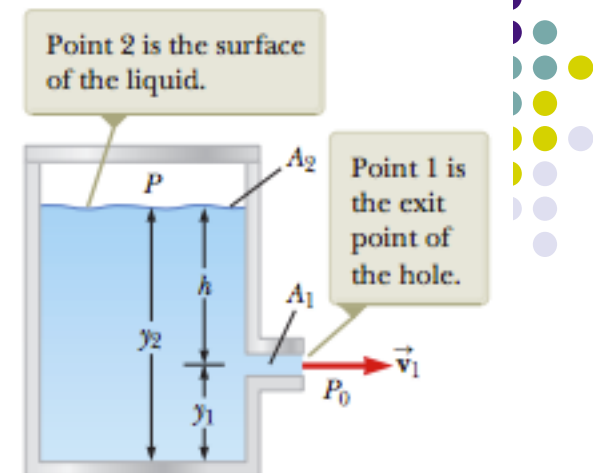


**Example 14.9****Torricelli's Law****AM**

An enclosed tank containing a liquid of density  $\rho$  has a hole in its side at a distance  $y_1$  from the tank's bottom (Fig. 14.20). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure  $P$ . Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance  $h$  above the hole.

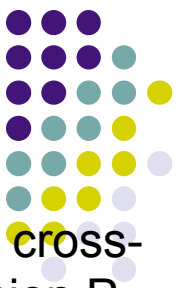
**SOLUTION**

**Conceptualize** Imagine that the tank is a fire extinguisher. When the hole is opened, liquid leaves the hole with a certain speed. If the pressure  $P$  at the top





# Home Work



1-Two drinking glasses having equal weights but different shapes and different cross-sectional areas are filled to the same level with water. According to the expression  $P = P_0 + \rho gh$ , the pressure is the same at the bottom of both glasses. In view of this, why does one weigh more than the other?

The weight depends upon the total volume of glass. The pressure depends only on the depth.

2-Lead has a greater density than iron, and both are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume?

Exactly the same. Buoyancy equals density of water times volume displaced.

## Section 14.1 Pressure

3-Calculate the mass of a solid iron sphere that has a diameter of 3.00 cm.

$$M = \rho_{\text{iron}} V = (7860 \text{ kg/m}^3) \left[ \frac{4}{3} \pi (0.0150 \text{ m})^3 \right]$$
$$M = \boxed{0.111 \text{ kg}}$$

4-A 50.0-kg woman balances on one heel of a pair of highheeled shoes. If the heel is circular and has a radius of 0.500 cm, what pressure does she exert on the floor?

$$P = \frac{F}{A} = \frac{50.0(9.80)}{\pi (0.500 \times 10^{-2})^2} = \boxed{6.24 \times 10^6 \text{ N/m}^2}$$

## Section 14.2 Variation of Pressure with Depth

5-. (a) Calculate the absolute pressure at an ocean depth of 1 000 m. Assume the density of seawater is 1 024 kg/m<sup>3</sup> and that the air above exerts a pressure of 101.3 kPa. (b) At this depth, what force must the frame around a circular submarine porthole having a diameter of 30.0 cm exert to counterbalance the force exerted by the water?

$$(a) \quad P = P_0 + \rho gh = 1.013 \times 10^5 \text{ Pa} + (1024 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1000 \text{ m})$$

$$P = \boxed{1.01 \times 10^7 \text{ Pa}}$$

(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$P_{\text{gauge}} = P - P_0 = \rho gh = 1.00 \times 10^7 \text{ Pa}$$

The resultant inward force on the porthole is then

$$F = P_{\text{gauge}} A = 1.00 \times 10^7 \text{ Pa} [\pi (0.150 \text{ m})^2] = \boxed{7.09 \times 10^5 \text{ N}}.$$

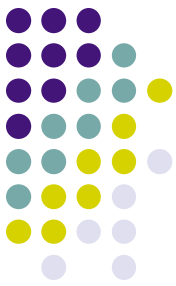
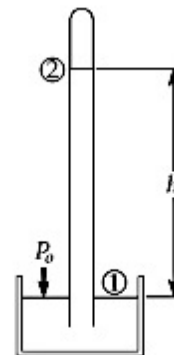
## 6-Section 14.3 Pressure Measurements

Blaise Pascal duplicated Torricelli's barometer using a alcohol, of density 984 kg/m<sup>3</sup>, as the working liquid (Fig. P14.17). What was the height  $h$  of the alcohol column for normal atmospheric pressure? Would you expect the vacuum above

tr  $P_0 = \rho gh$

$$h = \frac{P_0}{\rho g} = \frac{101.3 \times 10^3 \text{ Pa}}{(984 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.5 \text{ m}}$$

**No. Some alcohol and water will evaporate.** The equilibrium vapor pressures of alcohol and water are higher than the vapor pressure of mercury.



- Section 14.4 Buoyant Forces and Archimede's Principle
- 7-A piece of aluminum with mass 1.00 kg and density 2 700 kg/m<sup>3</sup> is suspended from a string and then completely immersed in a container of water (Figure P14.25) Calculate the tension in the string (a) before and (b) after the metal is immersed.

(a) Before the metal is immersed:

$$\sum F_y = T_1 - Mg = 0 \text{ or}$$

$$T_1 = Mg = (1.00 \text{ kg})(9.80 \text{ m/s}^2) \\ = \boxed{9.80 \text{ N}}$$

(b) After the metal is immersed:

$$\sum F_y = T_2 + B - Mg = 0 \text{ or}$$

$$T_2 = Mg - B = Mg - (\rho_w V)g$$

$$V = \frac{M}{\rho} = \frac{1.00 \text{ kg}}{2700 \text{ kg/m}^3}$$

Thus,

$$T_2 = Mg - B = 9.80 \text{ N} - (1000 \text{ kg/m}^3) \left( \frac{1.00 \text{ kg}}{2700 \text{ kg/m}^3} \right) (9.80 \text{ m/s}^2) = \boxed{6.17 \text{ N}}.$$

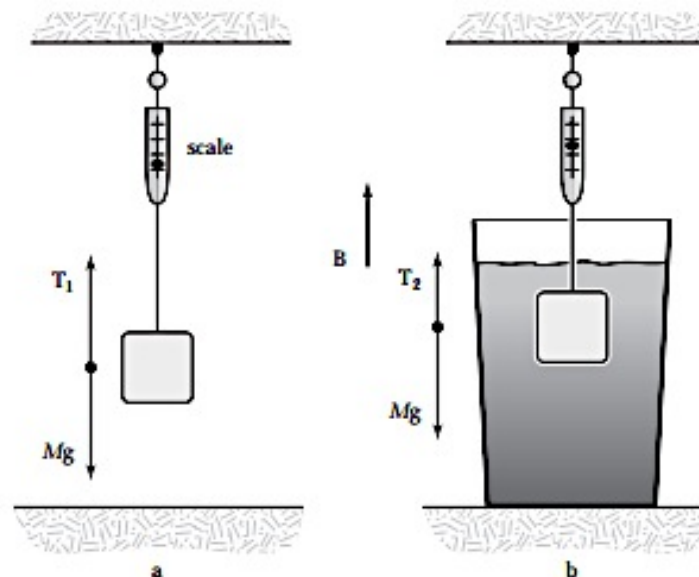


FIG. P14.25

- Section 14.5 Fluid Dynamics
- Section 14.6 Bernoulli's Equation

8- A horizontal pipe 10.0 cm in diameter has a smooth reduction to a pipe 5.00 cm in diameter. If the pressure of the water in the larger pipe is  $8.00 \times 10^4$  Pa and the pressure in the smaller pipe is  $6.00 \times 10^4$  Pa, at what rate does water flow through the pipes?

By Bernoulli's equation,

$$8.00 \times 10^4 \text{ N/m}^2 + \frac{1}{2}(1000)v^2 = 6.00 \times 10^4 \text{ N/m}^2 + \frac{1}{2}(1000)16v^2$$

$$2.00 \times 10^4 \text{ N/m}^2 = \frac{1}{2}(1000)15v^2$$

$$v = 1.63 \text{ m/s}$$

$$Q = Av = \pi(5 \times 10^{-2})^2 \times (1.63) = 0.0128 \text{ m}^3/\text{s}$$



FIG. P14.38

9-Water flows through a fire hose of diameter 6.35 cm at a rate of  $0.0120 \text{ m}^3/\text{s}$ . The fire hose ends in a nozzle of inner diameter 2.20 cm. What is the speed with which the water exits the nozzle?

$$\text{Flow rate } Q = 0.0120 \text{ m}^3/\text{s} = v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{Q}{A_2} = \frac{0.0120}{A_2} = \boxed{31.6 \text{ m/s}}$$

10.



A 14 cm inner diameter (i.d.) water main furnishes water (through intermediate pipes) to a 1.00 cm i.d. faucet pipe. If the average speed in the faucet pipe is 3.0 cm/s, what will be the average speed it causes in the water main?

The two flows are equal. From the continuity equation, we have

$$J = A_1 v_1 = A_2 v_2$$

Letting 1 be the faucet and 2 be the water main, we have

$$v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2} = (3.0 \text{ cm/s}) \left( \frac{1}{14} \right)^2 = 0.015 \text{ cm/s}$$



What volume of water will escape per minute from an open-top tank through an opening 3.0 cm in diameter that is 5.0 m below the water level in the tank? (See Fig. 14-1.)

We can use Bernoulli's equation, with 1 representing the top level and 2 the orifice. Then  $P_1 = P_2$  and  $h_1 = 5.0\text{ m}$ ,  $h_2 = 0$ .

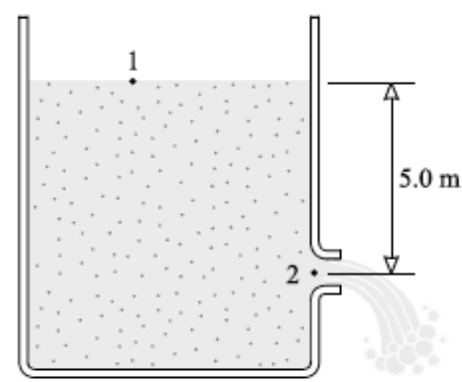


Fig. 14-1

$$P_1 + \frac{1}{2}\rho v_1^2 + h_1\rho g = P_2 + \frac{1}{2}\rho v_2^2 + h_2\rho g$$
$$\frac{1}{2}\rho v_1^2 + h_1\rho g = \frac{1}{2}\rho v_2^2 + h_2\rho g$$

If the tank is large,  $v_1$  can be approximated as zero. Then, solving for  $v_2$ , we obtain Torricelli's equation:

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.81\text{ m/s}^2)(5.0\text{ m})} = 9.9\text{ m/s}$$

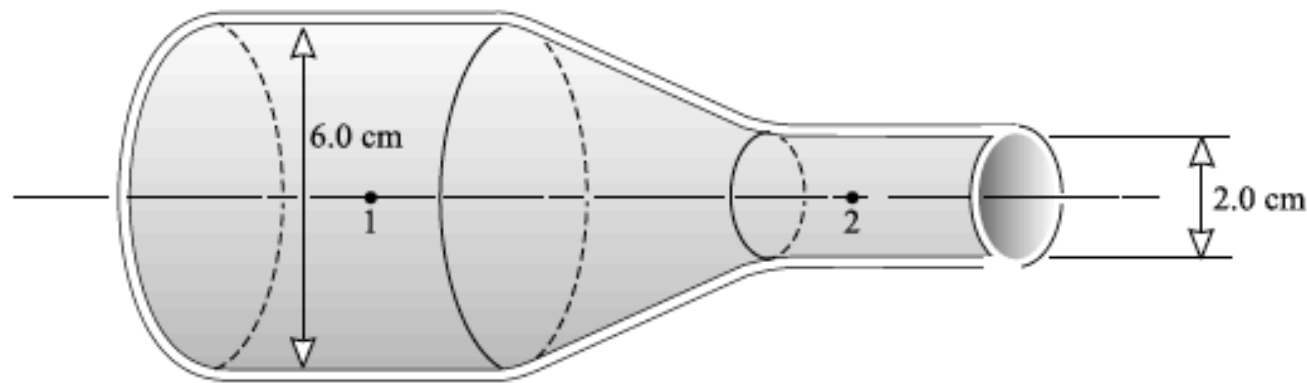
and the flow is given by

$$Q = v_2 A_2 = (9.9\text{ m/s})\pi(1.5 \times 10^{-2}\text{ m})^2 = 7.0 \times 10^{-3}\text{ m}^3/\text{s} = 0.42\text{ m}^3/\text{min}$$

12. A horizontal pipe has a constriction in it, as shown in Fig. 14-3. At point 1 the diameter is 6.0 cm, while at point 2 it is only 2.0 cm. At point 1,  $v_1 = 2.0$  m/s and  $P_1 = 180$  kPa. Calculate  $v_2$  and  $P_2$ .

We have two unknowns and will need two equations. Using Bernoulli's equation with  $h_1 = h_2$ , we have

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad \text{or} \quad P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = P_2$$



**Fig. 14-3**

Furthermore,  $v_1 = 2.0$  m/s, and the equation of continuity tells us that

$$v_2 = v_1 \frac{A_1}{A_2} = (2.0 \text{ m/s}) \left( \frac{r_1}{r_2} \right)^2 = (2.0 \text{ m/s})(9.0) = 18 \text{ m/s}$$

Substituting then gives

$$1.80 \times 10^5 \text{ N/m}^2 + \frac{1}{2}(1000 \text{ kg/m}^3)[(2.0 \text{ m/s})^2 - (18 \text{ m/s})^2] = P_2$$

from which  $P_2 = 0.20 \times 10^5 \text{ N/m}^2 = 20 \text{ kPa}$ .