EE2012 Project Report

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Q1.

An engineer is building a model to observe the distribution of the lifetime T for an electronic component. Supposed that the time *T* (in years) belongs to an exponential distribution ($f(T) = \lambda e^{(-\lambda T)}u(T)$), with an expected value of *t* years. For *t* = 3, 4, 5, I will report the following.

(a)

PDF: $f(T) = \lambda e^{(-\lambda T)}u(T)$

CDF: $F(T) = \int f(t)dT = 1 - e^{(-\lambda T)}$ (for T >=0)

(b)

Expected value for PDF is $E[t] = \int_0^\infty tf(t)dt = \int_0^\infty \lambda te^{-\lambda t}dt$

By integration by parts:

$\therefore u(T) = t, v'(T) = \lambda e^{-\lambda T}$

$\therefore u'(T) = 1, v(T) = -e^{-\lambda T}$

$\therefore E[T] = -te^{-\lambda T}|_0^\infty + \int_0^\infty e^{-\lambda T}dT$

$\therefore E[T] = -\infty \times e^{-\infty} - (-0 \times e^0) + (-\frac{1}{\lambda}e^{-\lambda T})|_0^\infty$

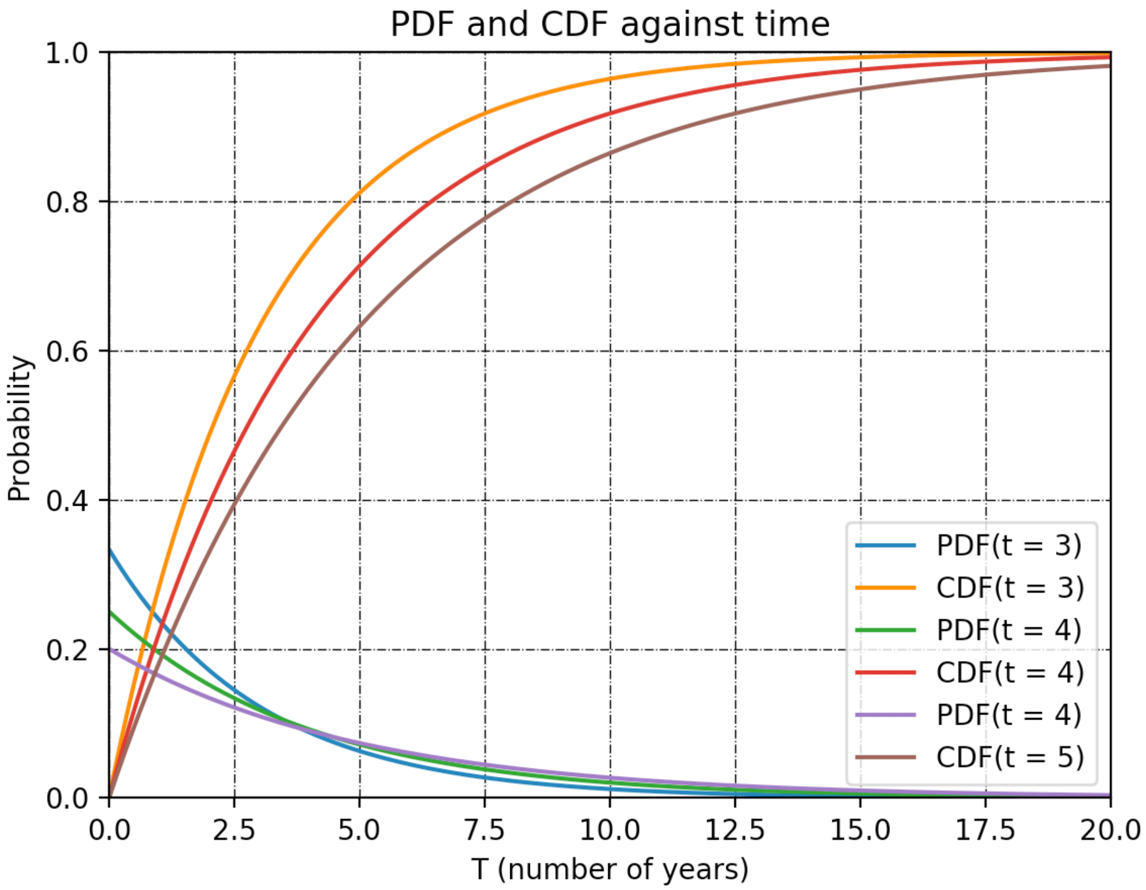
$\because e^{-\infty} = 0_\lambda \therefore E[T] = (-\frac{1}{\lambda}e^{-\lambda T})|_0^\infty = -\frac{1}{\lambda}e^{-\infty} - (-\frac{1}{\lambda}e^0)$

$\therefore E[T] = \frac{1}{\lambda} \therefore \lambda = \frac{1}{E[T]}$

Expected years value (years)	λ value
3	$\frac{1}{3}$
4	$\frac{1}{4}$

5	$\frac{1}{5}$

(c)



(d)

In order to compute the probability *p* of a component that lasts beyond 5 years, the follow mathematical equations are used:

$$p = 1 - CDF(t)$$

∵ the probability of a component lasts beyond t number of years is essentially the total probability - the total probability of a component that does not last till t number of years.

This is the probability of a component that lasts beyond 5 years:

If the expected lifetime is 3 years, the probability is 0.18887560283756188

If the expected lifetime is 4 years, the probability is 0.28650479686019015

If the expected lifetime is 5 years, the probability is 0.36787944117144233

(e)

When the expected lifetime of the component changes, the probability density function as well as the cumulative density function also changes. This will then change cumulative probability of a product last beyond a certain number of years. In plain terms, the increase in the expected lifetime of the component makes it more probable for the component to last beyond a fixed number of years.

Also, if we take reference to the above diagram, we can see that at 2.5 years, if the expected lifetime of the component is higher, it is less probable that the component will last less than 2.5 years.

Q2

(a)

Inverse transformation method takes uniform samples of a number p between 0 and 1, which is interpreted as a probability, and then returns the largest number from the domain of the distribution *P(X)* such that the following equation is satisfied: $P(-\infty < X < x) \leq p$. Then a histogram is plotted as the weight[1] of data against *x*. This histogram is understood as the probability density function of the data as the weight of the data represents the probability density of having X to be within a small range.

The actual process is separated into a few steps as outlined below:

- Step 1: Get the CDF inverse function.[2]
- Step 2: Generate a random number between 0 and 1.
- Step 3: Get the biggest *T* value (number of years) that will produce this cumulative probability through substituting the *p* value into the inverse function in step *1* to get the *T* value.

[1] Weight here refers to the percentage of this data against total number of data points

[2] The inverse function is required to map probability to the number of years of component life

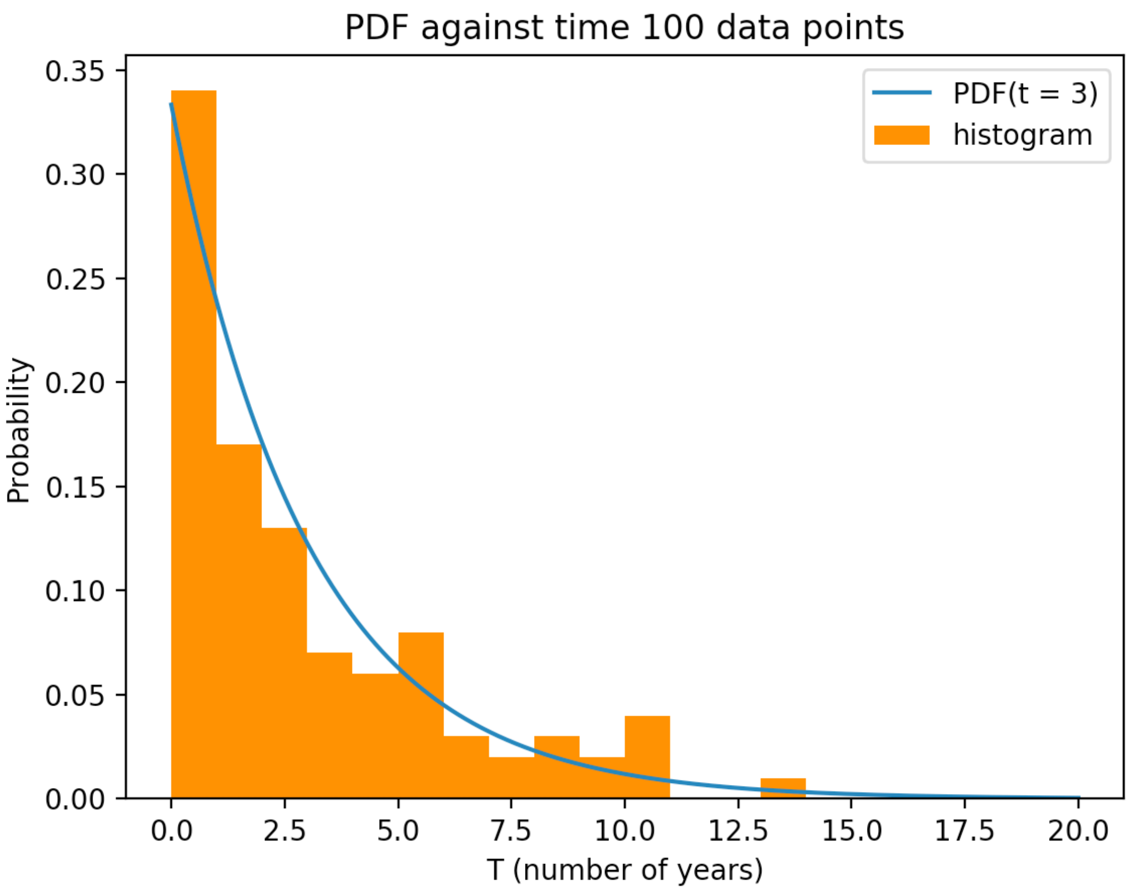
- Step 4: Repeat step 2 and 3 for enough number of times for a range of data points.
- Step 5: Plot the histogram of weight of data against T (years).
- Step 6: Analyse the histogram for trends

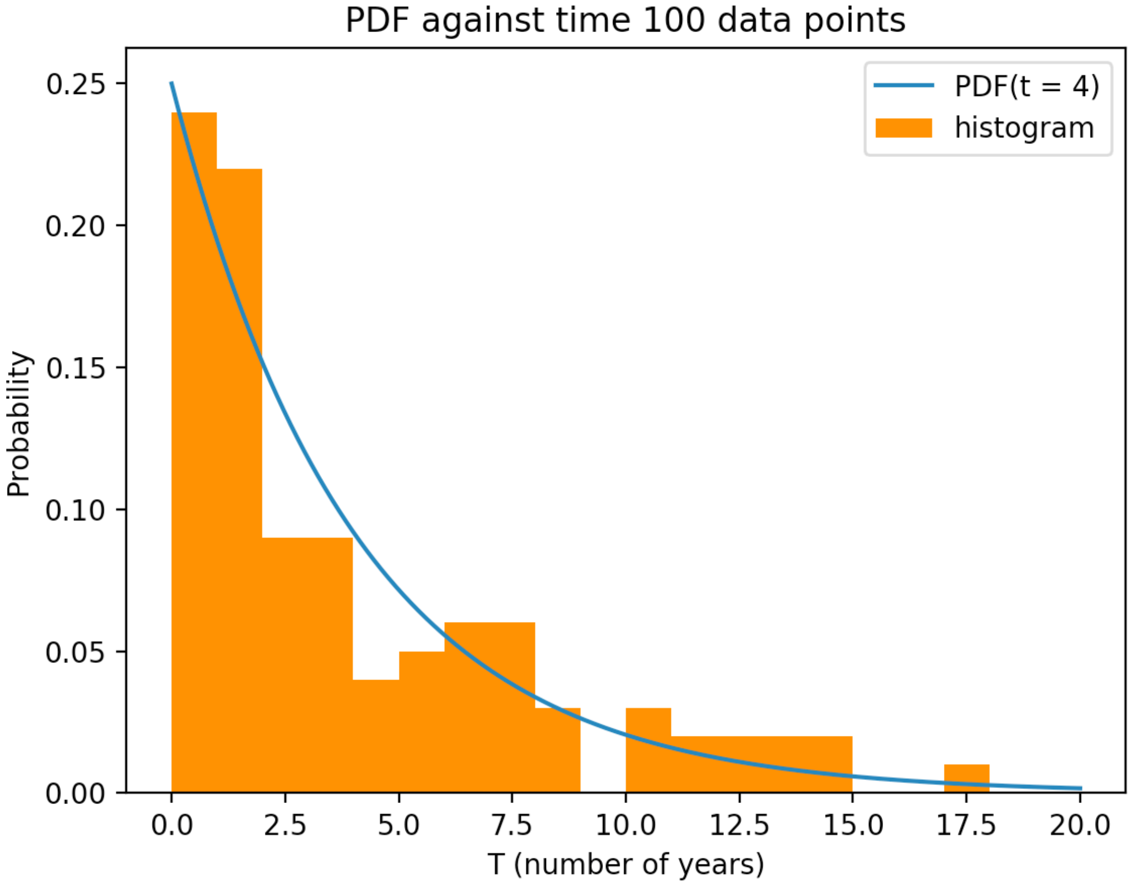
The inverse function is as follows:

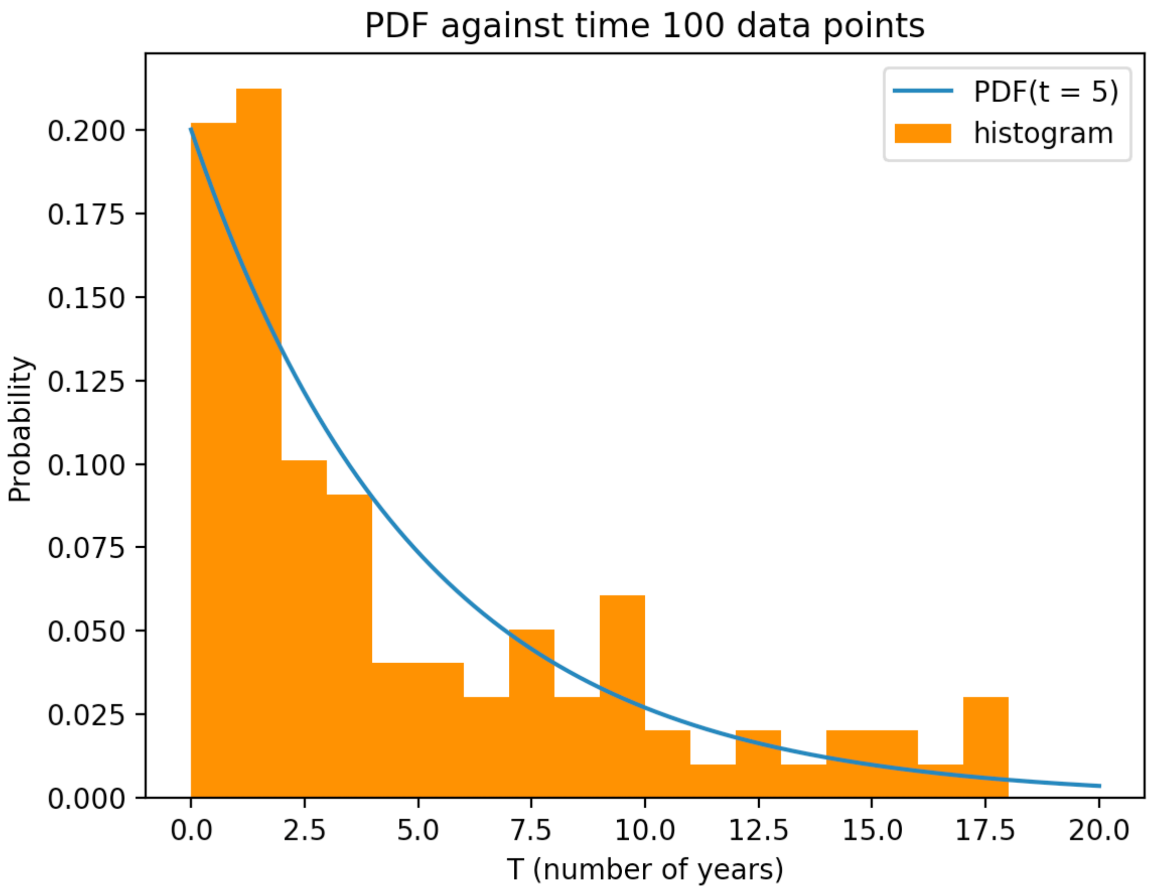
$$\because p = 1 - e^{-\lambda T} \quad \therefore T = \frac{1}{-\lambda} ln(1-p)$$

(b & c)

Python script (refer to the code) result:




PDF against time 100 data points
PDF(t = 4)
histogram
Probability
T (number of years)
0.25
0.20
0.15
0.10
0.05
0.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



(d)

I will first discuss the similarity and the difference between the histogram data and the PDF curve generated through the formula itself.

Similarity:

- The histogram generally follows the trend of the PDF curve.
- The histogram has a significantly more data around the lower T region

Difference:

- Some data on histogram does not follow the PDF curve very well, the weight may be either too high or too low.
- Data especially at the two extreme ends tend to disagree with the PDF curve trend about the probability density.

Overall, the results from the inverse transform method do follow the trend of PDF but does not follow very closely. This could be due to the fact that we have only generated 100 data points for the histogram. It is noted that if we generate more data points, the histogram will follow the trend more closely as a comparison graph is shown below: