No.

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Date

Tutorial 11

1. $1+0.3z^{-1}-0.1z^{-2}=0$ solve for ~~$z_1 = 0.2$~~ ~~$z_2 = -0.5$~~. $z_1^{-1}=\frac{1}{0.2}$ $z_2^{-1}=-\frac{1}{0.5}$

① ROC $|z|>0.5$ right-sided sequence.

suppose $X(z) = \frac{7}{(1-0.2z^{-1})(1+0.5z^{-1})} = \frac{\rho_1}{1-0.2z^{-1}} + \frac{\rho_2}{1+0.5z^{-1}}$

$$\rho_1 = \frac{7}{(1-0.2z^{-1})(1+0.5z^{-1})}\cdot(1-0.2z^{-1})\Big|_{z=0.2} = 2.$$

$$\rho_2 = \frac{7}{(1-0.2z^{-1})(1+0.5z^{-1})}(1+0.5z^{-1})\Big|_{z=-0.5} = 5 \quad \therefore X(z) = \frac{2}{1-0.2z^{-1}} + \frac{5}{1+0.5z^{-1}}$$

$\therefore x[n] = 2(0.2)^n\mu[n] + 5(-0.5)^n\mu[n]$

② ROC: $|z|<0.2$ left-sided sequence

$x[n] = -2(0.2)^n\mu[-n-1] - 5(-0.5)^n\mu[-n-1]$

③ ROC: $0.2<|z|<0.5$ two-side sequence.

$x[n] = 2(0.2)^n\mu[n] - 5(-0.5)^n\mu[-n-1]$

2. using long division

$$\begin{array}{r} 1+z^{-3}+z^{-6}+z^{-9}+\cdots \\ 1-z^{-3}\,\big)\overline{\,1\qquad\qquad\qquad} \\ 1-z^{-3} \\ \hline z^{-3} \\ z^{-3}-z^{-6} \\ \hline z^{-6} \\ z^{-6}-z^{-9} \\ \hline z^{-9} \end{array}$$

$\because |z|>1 \quad \therefore$ right-sided sequence.

$\therefore x[n] = \sum_{k=0}^{\infty}\delta[n-3k] \quad k=0,1,2\cdots$

3. $x[n]*h_1[n]*h_3[n] + (x[n]+x[n]*h_1[n])*h_2[n] = y[n]$

$\therefore x[n]*(h_1[n]*h_3[n] + (1+h_1[n])*h_2[n]) = y[n]$

$\therefore H(z) = H_1(z)H_3(z) + (1+H_1(z))H_2(z)$

$= (2.1+3.3z^{-1}+0.7z^{-2})(3.2+4.5z^{-1}+0.9z^{-2}) + (1+2.1+3.3z^{-1}+0.7z^{-2})(1.4-5.2z^{-1}+0.8z^{-2})$

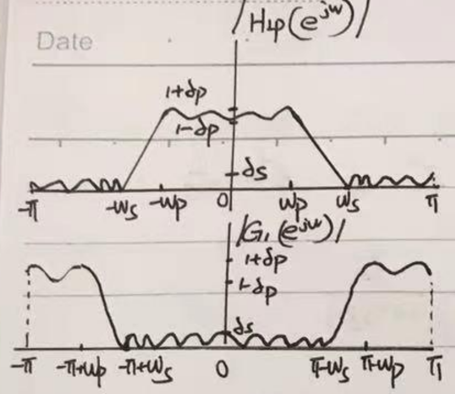
$= (6.72+20.01z^{-1}+18.98z^{-2}+6.12z^{-3}+0.63z^{-4}) + (4.34-11.5z^{-1}-13.7z^{-2}-z^{-3}+0.56z^{-4})$

$= 11.06+8.51z^{-1}+5.28z^{-2}+5.12z^{-3}+1.19z^{-4}$

4. $G_1(z) = H_{LP}(-z) = H_{LP}(e^{-j\pi}z)$

$G_1(e^{j\omega}) = H_{LP}(e^{-j\pi}e^{j\omega}) = H_{LP}(e^{j(\omega-\pi)})$

$|G_1(e^{j\omega})| = |H_{LP}(e^{-j\pi}e^{j\omega})| = |H_{LP}(e^{j(\omega-\pi)})| = |H_{LP}(e^{j(\pi-\omega)})|$



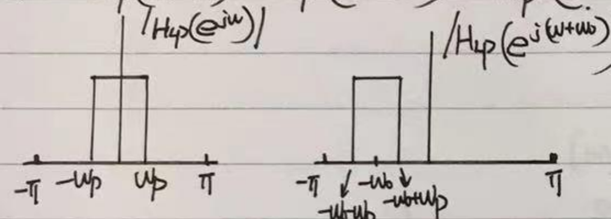
$G_1(z)$ is a highpass filter.

$G_1(z) = H_{LP}(e^{-j\pi}z) = \sum_n h[n](e^{-j\pi}z)^{-n} = \sum_n (-1)^n h[n] z^{-n}$.

$\therefore g_1[n] = (-1)^n h[n]$

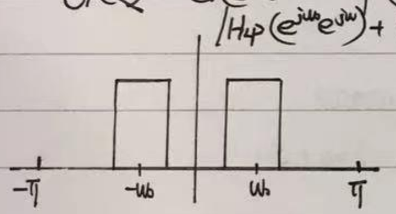
bandpass edge : $\pi-\omega_p$ bandstop edge : $\pi-\omega_s$ passband ripple : δ_p stopband ripple δ_s

5. $H_{LP}(e^{j\omega_0}z) = H_{LP}(e^{j\omega_0}e^{j\omega}) = H_{LP}(e^{j(\omega+\omega_0)})$



A bandpass filter

$G(z)$ $G(e^{j\omega}) = H_{LP}(e^{j(\omega+\omega_0)}) + H_{LP}(e^{j(\omega-\omega_0)})$



$G(e^{j\omega}) = \sum_n h_{LP}[n]e^{-jn\omega_0}e^{-jn\omega} + \sum_n h_{LP}[n]e^{jn\omega_0}e^{-jn\omega}$

$= 2\sum_n h_{LP}[n]\cos(n\omega_0)\, e^{-j\omega n}$.

$\therefore g[n] = 2h[n]\cos(\omega_0 n)$ is real coefficient bandpass filter.

center frequency ω_0. band width $2\omega_p$.

6. $H(z) = H_0(z)F_0(z) - H_0(-z)F_0(-z)$

to let $y[n]$ be delayed and scaled replica of input $x[n]$

$H(z) = z^{-k}$. $H_0(z) = 1+\alpha z^{-1}$ $\therefore (1+\alpha z^{-1})F_0(z) - (1-\alpha z^{-1})F_0(-z) = z^{-k}$

if $F_0(z) = a+bz^{-1}$ $(1+\alpha z^{-1})(a+bz^{-1}) - (1-\alpha z^{-1})(a-bz^{-1}) = z^{-k}$.

$\therefore 2a\alpha z^{-1} + 2bz^{-1} = z^{-k}$

if $k=1$. $2b+2a\alpha=1$ $\therefore b=\frac{1}{4}$ $a=\frac{1}{4\alpha}$ is one possible solution.

$F_0(z) = \frac{1}{4\alpha} + \frac{1}{4}z^{-1} = \frac{1}{4\alpha}(1+\alpha)z^{-1}$

No.

Date

7. $H(e^{j\omega}) = a_1 e^{j(k+1)\omega} + a_2 e^{jk\omega} + a_3 e^{j(k-1)\omega} + a_2 e^{j(k-2)\omega} + a_1 e^{j(k-3)\omega}$.

to have a real function of ω $H(e^{j\omega})$

$k+1 = -(k-3)$

$k = -(k-2)$

$k-1 = 0$ solve for $k=1$.

8. (a). $|z_1| = 0.6073$. not in unit circle ~~another~~ other 3 zeros are:

$z_4 = 0.1 + 0.599j$ $z_5 = \frac{1}{z_1^*} = 0.2711 - 1.6242j$ $z_6 = 0.2711 + 1.6242j$.

$|z_2| = 0.5$ not in unit circle ~~ano~~ other 3 zeros are:

$z_7 = -0.3 - 0.4j$ $z_8 = \frac{1}{z_2^*} = -1.2 + 1.6j$ $z_9 = -1.2 - 1.6j$.

z_3 is real not on unit circle $\therefore z_{10} = 0.5$

type III filter has odd number of zeros at 1. -1.

at least. $z_{11} = 1$ $z_{12} = -1$

(b). $H(z) = \prod_{k=1}^{12} (1 - z_k z^{-1}) = 1 - 0.2423z^{-1} + 1.0076z^{-2} - 6.5294z^{-3} + 1.3338z^{-4} - 17.2533z^{-5}$
$+ 17.2533z^{-7} - 1.3338z^{-8} + 6.5294z^{-9} - 1.0076z^{-10} + 0.2423z^{-11}$
$- z^{-12}$