Tutorial6

Tutorial 6. [illegible] 12011923

1.Solution: $H(e^{j\omega})=\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$

$=\sum_{k=-\infty}^{\infty}(0.4)^k u[k]\, e^{-j\omega k}=\sum_{k=0}^{\infty}(0.4)^k e^{-j\omega k}$

$=\sum_{k=0}^{\infty}(0.4e^{-j\omega})^k=\frac{1}{1-0.4e^{-j\omega}}$

$H(e^{j\pm\frac{\pi}{4}})=\frac{1}{1-0.4e^{\mp j\frac{\pi}{4}}}\approx 1.2972e^{\pm 0.3182}$

2.Solution (a) $H(e^{j\omega})=\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$

$=\sum_{k=0}^{4} h[k]e^{-j\omega k}=h[0]+h[1]e^{-j\omega}+h[2]e^{-j2\omega}+h[3]e^{-j3\omega}+h[4]e^{-j4\omega}$

$=h[0]+h[1]e^{-j\omega}+h[2]e^{-j2\omega}-h[1]e^{-j3\omega}-h[0]e^{-j4\omega}$

$\because h[n]=-h[4-n]$

$\therefore h[2]=-h[2]\Rightarrow h[2]=0$

$\Rightarrow H(e^{j\omega})=h[0]+h[1]e^{-j\omega}-h[1]e^{-j3\omega}-h[0]e^{-j4\omega}$

$=e^{-j2\omega}\{h[0]e^{j2\omega}-h[0]e^{-j2\omega}+h[1]e^{j\omega}-h[1]e^{-j\omega}\}$

$=je^{-j2\omega}\{2h[0]\sin(2\omega)+2h[1]\sin(\omega)\}$

$|H(e^{j\frac{\pi}{4}})|=2h[0]\sin(\frac{\pi}{2})+2h[1]\sin(\frac{\pi}{4})=0.5$

$|H(e^{j\frac{\pi}{2}})|=2h[0]\sin(\pi)+2h[1]\sin(\frac{\pi}{2})=1$

$\Rightarrow h[0]=\frac{1-\sqrt{2}}{4},\ h[1]=0.5$

$\Rightarrow H(e^{j\omega})=je^{-j2\omega}\{\frac{1-\sqrt{2}}{2}\sin(2\omega)+\sin(\omega)\}$

(b) The expression is given by

$H(e^{j\omega})=je^{-j2\omega}\{\frac{1-\sqrt{2}}{2}\sin(2\omega)+\sin(\omega)\}$.

(c) $H(e^{j\omega})=e^{j\frac{\pi}{2}}e^{-j2\omega}\{\frac{1-\sqrt{2}}{2}\sin(2\omega)+\sin(\omega)\}$

$=\{\frac{1-\sqrt{2}}{2}\sin(2\omega)+\sin(\omega)\}e^{j(\frac{\pi}{2}-2\omega)}$

$\rightarrow \theta(\omega)=\frac{\pi}{2}-2\omega$

$\rightarrow \tau_p(\omega)=-\frac{\theta(\omega)}{\omega}=-(\frac{\pi}{2\omega}-2)=2-\frac{\pi}{2\omega}$

$\tau_g(\omega)=-\frac{d[\theta(\omega)]}{d\omega}=2$.

3.Solution: (a) $H(e^{j\omega})=\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}=\sum_{k=0}^{3} h[k]e^{-j\omega k}$

$=h[0]+h[1]e^{-j\omega}+h[2]e^{-j2\omega}+h[3]e^{-j3\omega}$

$=h[0]+h[0]e^{-j3\omega}+h[1]e^{-j\omega}+h[1]e^{-j2\omega}$

$=e^{-j\frac{3}{2}\omega}\{h[0]e^{j\frac{3}{2}\omega}+h[0]e^{-j\frac{3}{2}\omega}+h[1]e^{j\frac{1}{2}\omega}+h[1]e^{-j\frac{1}{2}\omega}\}$

$=e^{-j\frac{3}{2}\omega}\{2h[0]\cos(\frac{3}{2}\omega)+2h[1]\cos(\frac{1}{2}\omega)\}$

$|H(e^{j\frac{\pi}{4}})|=2h[0]\cos(\frac{3}{8}\pi)+2h[1]\cos(\frac{1}{8}\pi)=1$

$|H(e^{j\frac{\pi}{2}})|=2h[0]\cos(\frac{3}{4}\pi)+2h[1]\cos(\frac{1}{4}\pi)=0.5$

$\Rightarrow h[0]=-0.132689,\ h[1]=0.486242$.

(b) The expression is given by

$H(e^{j\omega})=e^{-j\frac{3}{2}\omega}\{-0.265378\cos(\frac{3}{2}\omega)+0.972484\cos(\frac{1}{2}\omega)\}$.

(c) $\theta(\omega)=-\frac{3}{2}\omega$

$\Rightarrow \tau_p(\omega)=-\frac{\theta(\omega)}{\omega}=\frac{3}{2}$, $\tau_g(\omega)=-\frac{d\theta(\omega)}{d\omega}=\frac{3}{2}$.

4.Solution: (a) $H_A(e^{j\omega})=\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$

$=\sum_{n=-\infty}^{\infty}\{0.3\delta[n]-\delta[n-1]+0.3\delta[n-2]\}e^{-j\omega n}$

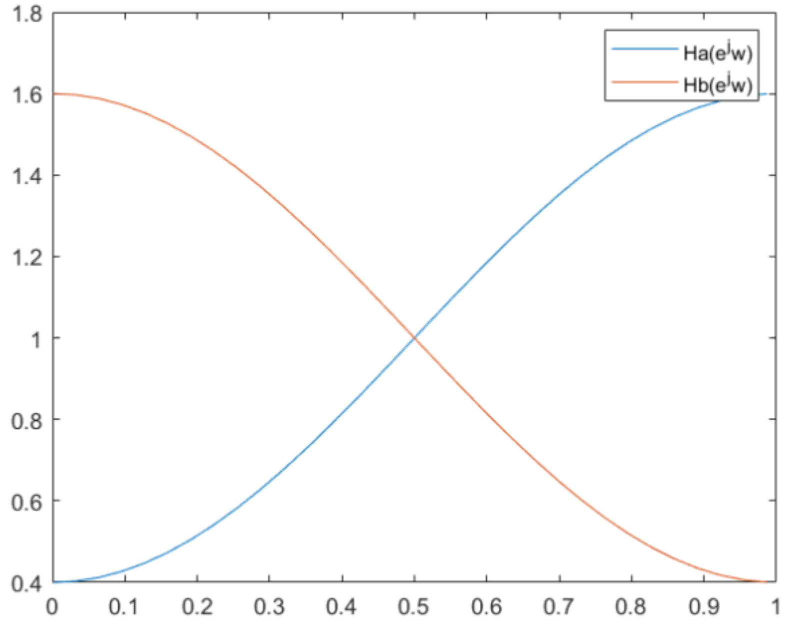
$=0.3-e^{-j\omega}+0.3e^{-j2\omega}$.

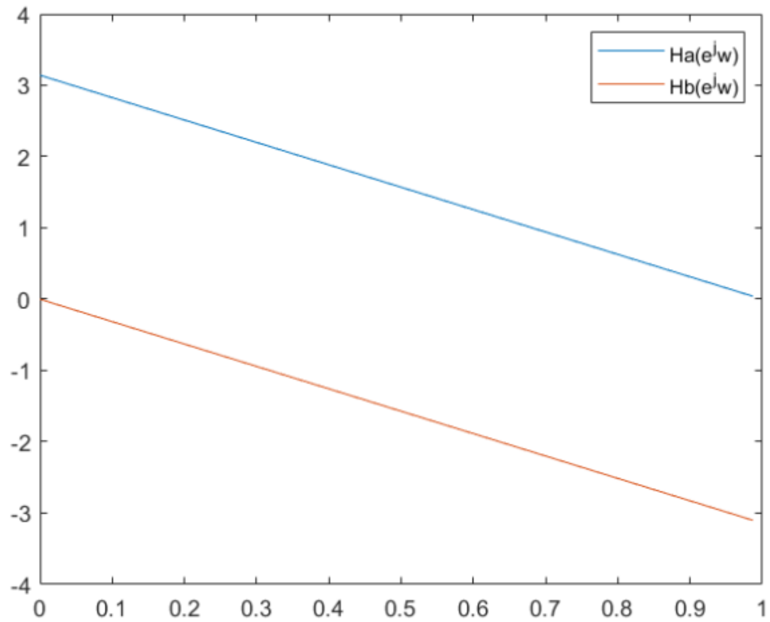
$H_B(e^{j\omega})=\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$

$=\sum_{n=-\infty}^{\infty}\{0.3\delta[n]+\delta[n-1]+0.3\delta[n-2]\}e^{-j\omega n}$

$=0.3+e^{-j\omega}+0.3e^{-j2\omega}$.

The plot of magnitude and phase response will be used by MATLAB.





Filter A is highpass filter and Filter B is lowpass filter. Both filter have linear phase, but filter A starts with π while filter B start with 0.

(b) $H_c(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}$

$= \sum_{k=-\infty}^{\infty} (-1)^n h_A[n] e^{-j\omega n}$

$= \sum_{k=-\infty}^{\infty} (e^{-j\pi})^n h_A[n] e^{-j\omega n}$

$= \sum_{k=-\infty}^{\infty} h_A[n] e^{-j(\omega+\pi)n}$

$= H_A(e^{j(\omega+\pi)})$

5. Solution: Rearrange the difference equation.

$y[n]+d_1 y[n-1]+d_2 y[n-2]+d_3 y[n-3]$

$= d_3 x[n]+d_2 x[n-1]+d_1 x[n-2]+x[n-3]$.

Take DTFT by both sides

$Y(e^{j\omega})+d_1 e^{-j\omega} Y(e^{j\omega})+d_2 e^{-j2\omega} Y(e^{j\omega})$

$+d_3 e^{-j3\omega} Y(e^{j\omega})$

$= d_3 X(e^{j\omega})+d_2 e^{-j\omega} X(e^{j\omega})+d_1 e^{-j2\omega} X(e^{j\omega})+e^{-j3\omega} X(e^{j\omega})$.

$\Rightarrow H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{d_3+d_2 e^{-j\omega}+d_1 e^{-j2\omega}+e^{-j3\omega}}{1+d_1 e^{-j\omega}+d_2 e^{-j2\omega}+d_3 e^{-j3\omega}}$.

$|H(e^{j\omega})|^2 = \frac{d_3+d_2 e^{-j\omega}+d_1 e^{-j2\omega}+e^{-j3\omega}}{1+d_1 e^{-j\omega}+d_2 e^{-j2\omega}+d_3 e^{-j3\omega}} \cdot \frac{d_3+d_2 e^{j\omega}+d_1 e^{j2\omega}+e^{j3\omega}}{1+d_1 e^{j\omega}+d_2 e^{j2\omega}+d_3 e^{j3\omega}}$

$= H(e^{j\omega}) \cdot H^*(e^{j\omega}) = \frac{d_3+d_2 e^{-j\omega}+d_1 e^{-j2\omega}+e^{-j3\omega}}{1+d_1 e^{-j\omega}+d_2 e^{-j2\omega}+d_3 e^{-j3\omega}} \cdot \frac{e^{-j3\omega}}{e^{-j3\omega}} \cdot \frac{d_3 e^{-j3\omega}+d_2 e^{-j2\omega}+d_1 e^{-j\omega}+1}{e^{-j3\omega}+d_1 e^{-j2\omega}+d_2 e^{-j\omega}+d_3} = 1$

$\Rightarrow |H(e^{j\omega})| = 1$. $\Rightarrow$ it has a unity magnitude response for all values of ω.

6. Solution: $H(e^{j\omega}) = |H(e^{j\omega})| e^{j\theta(\omega)}$

$\frac{dH(e^{j\omega})}{d\omega} = \frac{d|H(e^{j\omega})|}{d\omega} e^{j\theta(\omega)} + |H(e^{j\omega})| \frac{d[e^{j\theta(\omega)}]}{d\omega} = \frac{d|H(e^{j\omega})|}{d\omega} e^{j\theta(\omega)} + |H(e^{j\omega})| e^{j\theta(\omega)} \cdot j \cdot \frac{d\theta(\omega)}{d\omega}$.

$\Rightarrow \frac{j \frac{d[H(e^{j\omega})]}{d\omega}}{H(e^{j\omega})} = j \frac{d|H(e^{j\omega})|}{d\omega} \frac{1}{|H(e^{j\omega})|} + (-1) \frac{|H(e^{j\omega})| e^{j\theta(\omega)}}{|H(e^{j\omega})| e^{j\theta(\omega)}} \frac{d\theta(\omega)}{d\omega}$.

$\Rightarrow \mathrm{Re}\left\{ \frac{j \frac{dH(e^{j\omega})}{d\omega}}{H(e^{j\omega})} \right\} = -\frac{d\theta(\omega)}{d\omega} = \tau(\omega)$

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ACADEMIC YEAR 2022-2023 SEMESTER 1

7. Solution: when $u[n] = z^n$, then its output is

$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] = \sum_{k=-\infty}^{\infty} h[k] z^{n-k} = z^n \sum_{k=-\infty}^{\infty} h[k] z^{-k}$.

$\because$ z is a complex constant $\therefore z = e^{j\omega}$, $z^n = e^{j\omega n}$, ω is a constant.

$\Rightarrow y[n] = z^n \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k} = H(e^{j\omega}) z^n$

the input is z^n, the output is z^n multiplied by a constant, hence z^n is an eigenfunction of an LTI system

when $v[n] = z^n u[n]$, then its output is:

$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] = \sum_{k=-\infty}^{\infty} h[k] z^{n-k} u[n-k] = z^n \sum_{k=-\infty}^{n} h[k] z^{-k}$

$\because$ the value of $\sum_{k=-\infty}^{n} h[k] z^{-k}$ is dependent on n. $\therefore$ it is not an eigenfunction of an LTI system.