EE326 Digital Signal Processing

Tutorial Partial Answers

Dr. Yu Yajun,
Associate Professor
Department of Electrical & Electronic Engineering
Southern University of Science & Technology

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1. (a) $\{2, 0, -1, 6, -3, 2, 0\}, \quad -6 \le n \le 0,$
   (b) $\{32, 8, -28, -12, 0, 4, 4\}, \quad -3 \le n \le 3$
   (c) $\{1, 1, 0, -3, -7, 2, 8\}, \quad 0 \le n \le 6$
   (d) $\{8, 2, -7, -3, 0, 1, 1, 0, 2, 0, -1, 6, -3, 2, 0\}, \quad -8 \le n \le 6$
   (e) $\{-8, 4, -42, -18, 0\}, \quad -2 \le n \le 2$

2. Yes, LCM(N_1, N_2, N_3).

3. (b) Not fully recovered, not fully recovered

4. $\mu[n] + \mu[-n-1]$

5. (a) $N = 100$ for $K = 7$
   (b) $N = 25$ for $K = 3$
   (c) Not periodic.

6. T satisfies $\Omega_0 NT = 2\pi K$.
   $N = 2$ for K=3.

7. (a) $h[0](x[n] + x[n-6]) + h[1](x[n-1] + x[n-5])$
   $+h[2](x[n-2] + x[n-4]) + h[3]x[n-3]$

   (b) $p_0 x[n] + p_1 x[n-1] + p_2 x[n-2] - d_1 y[n-1] - d_2 y[n-2]$

   (c) $p_0 x[n] + p_1 x[n-1] + p_2 x[n-2] - d_1 y[n-1] - d_2 y[n-2]$

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1. (a) linear, not causal, BIBO stable, time-invariant.

   (b) non-linear, not causal, BIBO stable, not time-invariant.

   (c) non-linear, causal, not BIBO stable, time-invariant.

   (d) non-linear, not causal, BIBO stable, time-invariant.

2. time-invariant and noncausal.

3. linear, time-invariant and noncausal.

4. time-invariant when $y[-1]=0$.
   linear when $y[-1]=0$.

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1. (a) {16, 4, –22, 40, –5, –27, 9, –6, –1, 3, –1, 2, 0}, $-8 \le n \le 4$
   (b) {6, 12, –5, 16, 40, –8, 23, 22, 21, 0, 9, 2, 0}, $-5 \le n \le 7$
   (c) {24, 54, –17, –37, 41, 52, –19, –53, –24, 5, 12, 7, 1}, $-7 \le n \le 5$

2. $y[n - N_1 - N_2]$.

3. (a) $2N+2M+1$, $[-2M, 2N]$
   (b) $2N-2K+1$, $[2K, 2N]$
   (c) $M+2N-K+1$, $[-M+K, 2N]$
   (d) $N+M+L-R+1$, $[-L-M, N-R]$

4. $\left(\frac{1-a^{n+1}}{1-a}\right)\mu[n]$

5. (a) conjugate symmetric. (b) conjugate anti-symmetric. (c) conjugate symmetric.

7. $|\alpha| > 1$

8. $x[n] \circledast s[n] - x[n] \circledast s[n-1]$

9. $\left(\frac{1-(-a)^{n+1}}{1+a}\right)\mu[n]$

10. $x[n] \circledast h_T[n] \circledast \text{Inverse}(\delta[n] - h_5[n] \circledast h_T[n])$

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1. (a) $\frac{1-\alpha^2}{1-2\alpha\cos\omega+\alpha^2}$

(b) $\frac{A}{2}e^{j\varphi}\frac{1}{1-\alpha e^{-j\omega}e^{j\omega_0}}+\frac{A}{2}e^{-j\varphi}\frac{1}{1-\alpha e^{-j\omega}e^{-j\omega_0}}$

(c) $\frac{\alpha e^{-j\omega}}{\left(1-\alpha e^{-j\omega}\right)^2}-2\alpha^{-2}e^{j2\omega}-\alpha^{-1}e^{j\omega}$

2. $\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}2-j\\10\\-1+j\end{bmatrix}$

3. (a) $x[n]=\frac{1}{2\pi}$

(b) $x[n]=\begin{cases}1, & -N\le n\le -1\\0, & \text{otherwise}\end{cases}$

(c) $x[n]=\begin{cases}3, & n=0\\1, & 0<|n|\le N\\0, & \text{otherwise}\end{cases}$

(d) $x[n]=n\alpha^n\mu[n]$

4. (a) $\{1,4,6,4,1\}, -2\le n\le 2$
(b) $\{4,-4,1,8,-8,2,4,-4,1\}, 0\le n\le 8$

5. (a) -3; (b) -21; (c) -10π; (d) 526π; (e) 3142π

6. (a) $\left(1+e^{-j4\omega}\right)G_1\left(e^{j\omega}\right)$

(b) $G_1\left(e^{j\omega}\right)+e^{-j7\omega}G_1\left(e^{-j\omega}\right)$

(c) $e^{-j3\omega}G_1\left(e^{-j\omega}\right)+e^{-j4\omega}G_1\left(e^{j\omega}\right)$

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1. (a) $\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2k\pi)$

(b) $\frac{1}{2}\left(e^{j\varphi}\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2k\pi) + e^{-j\varphi}\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + \omega_0 + 2k\pi)\right)$

2. (a) $2\omega_c$; (b) ω_c; (c) $\frac{1}{3}\omega_c$; (d) $3\omega_c$; (e) ω_c.

3. 1062.5 Hz

4. 300Hz, 500Hz, -850Hz.

5. 150, 400, 925, 3600, or 8150, 16400, 925, 4000 Hz. The solution is not unique.

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1. $1.2972e^{\pm 0.3182}$
2.
(b) $je^{-j2\omega}(-0.2072\sin(2\omega)+\sin\omega)$

(c) $-\frac{\pi}{2\omega}+2,\ 2.$

3.

(b) $e^{-j\frac{3}{2}\omega}(-0.265378\cos\left(\frac{3}{2}\omega\right)+0.972484\cos\left(\frac{\omega}{2}\right))$

(c) $\frac{3}{2},\ \frac{3}{2}.$

4. (a) $H_A(e^{j\omega})=0.3-e^{-j\omega}+0.3e^{-j2\omega},\ H_B(e^{j\omega})=0.3+e^{-j\omega}+0.3e^{-j2\omega}$

(b) $H_C(e^{j\omega})=H_A(e^{j(\omega+\pi)}).$

5. $\frac{d_3+d_2e^{-j\omega}+d_1e^{-j2\omega}+e^{-j3\omega}}{1+d_1e^{-j\omega}+d_2e^{-j2\omega}+d_3e^{-j3\omega}},$

7. No

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1. (a) $\begin{cases} N/(2j), & k=1 \\ -N/(2j), & k=N-1 \\ 0, & \text{otherwise} \end{cases} \quad 0 \le k \le N-1$

(b) $\begin{cases} \frac{N}{2}, & k=0, \\ -\frac{N}{4}, & k=2, N-2, \\ 0, & \text{otherwise} \end{cases}$

(c) $\begin{cases} 3N/(8j), & k=1 \\ -N/(8j), & k=3 \\ N/(8j), & k=N-3 \\ -3N/(8j), & k=N-1 \\ 0, & otherwise \end{cases} \quad 0 \le k \le N-1$

2. $N^2 x[n]$

3. (a)

$G[2l] = X[l], \quad 0 \le l \le N-1$

$$G[2l+1] = \frac{1}{N}\sum_{m=0}^{N-1} X[m] \frac{sin\left(\frac{2m-1-2l}{2}\right)\pi}{sin\left(\frac{2m-1-2l}{2N}\right)\pi} \times e^{j\frac{N-1}{2N}(-2m+1+2l)\pi}$$

(b)

$H[2l] = X[l],$

$$H[2l+1] = -\frac{1}{N}\sum_{m=0}^{N-1} X[m] \frac{\sin\left(\frac{2m-1-2l}{2}\right)\pi}{\sin\left(\frac{2m-1-2l}{2N}\right)\pi} \times e^{j\frac{N-1}{2N}(-2m+1+2l)\pi}$$

4. $x[m]W_{3N}^{2m}$

5. $\frac{1.25-4.5e^{-j\omega}+1.75e^{-j2\omega}+5e^{-j3\omega}}{12.4+5.8e^{-j\omega}+5e^{-j2\omega}-6.2e^{-j3\omega}}$

6. (a) Can, (b) Cannot

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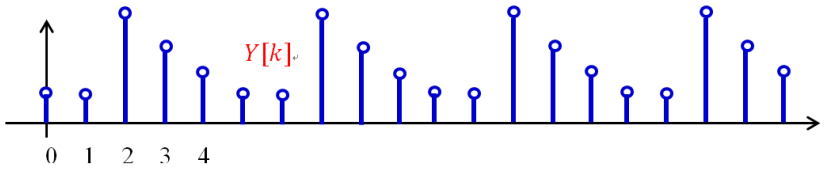
1. (a)If $0 \le k \le N-1, Y[k] = X[k]$; If $N \le k \le LN-1,\ Y[k] = X[\langle k \rangle_N]$

(b)



2. (a) -4 ; (b) -5

3. $\{3,\ 3,\ -9,\ 11\}$

4.

(b) $\{-34, 85, -9, -41, 89, 9\}$, for $0 \le n \le 5$

5. $X[k] = \left(1 + W_N^k\right) G[\langle k \rangle_{N/2}] + \left(1 - W_N^k\right) H[\langle k \rangle_{N/2}]$

7. $y[n] = \{-2,\ 1\}$, for $0 \le n \le 1$

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3. $\left\lfloor \frac{1}{2}\log_2 N \right\rfloor$, 3, 4, 4

4. $\frac{N}{2}(\log_2 N - 2) - \left(\frac{N}{2} - 2\right)$

5. 840 RMs, 3600 RMs, 1040 RMs or 780 RMs.

7. The first stage.

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1. (a) 1. $\{z:$ All values of $z\}$

   (b) $\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$. $\{z: |z| > |\alpha|\}$

   (c) $\frac{-\alpha^{-2}z^2}{1-\alpha^{-1}z}$. $\{z: |z| < |\alpha|\}$

   (d) $\frac{1-\alpha^2}{(1-\alpha z)(1-\alpha z^{-1})}$. $\left\{z: |\alpha| < |z| < \frac{1}{|\alpha|}\right\}$.

2. (a) $\{z: |z| > 0.2\}$
   (b) $\{z: |z| > 0.5\}$
   (c) $\{z: |z| < 0.5\}$
   (d) $\{z: |z| > 0.5\}$
   (e) $\{z: 0.2 < |z| < 0.5\}$
   (f) $\emptyset$

3. $\frac{1}{N}\frac{z^{N-1}\left(1-z^{-N}\right)^2}{(1-z^{-1})^2}$, $\{z: z \neq 0\}$. $\frac{1}{N}\frac{\sin^2\left(\frac{\omega N}{2}\right)}{\sin^2\left(\frac{\omega}{2}\right)}$

4. $\{-2.86, 17.4, -7.46, 8.26, 2.64, -8.04, -4.43, 0.93\}$

5. two real zeros at $z = -2.1$ and $z = 0.1$, and two real poles at $z = 1.1$ and $z = -0.6$. not stable.

6. $\frac{1-0.5\alpha z^{-1}+2\alpha^2 z^{-2}}{(1+0.9\alpha z^{-1})(1+0.4\alpha z^{-1})}$. $|\alpha| < 1.11$.

7. stable, stable. $G\left(e^{j\omega}\right) = H\left(e^{j\omega M}\right)$.

8. (a)
   (i) $\{z: |z| < 0.3\}$, a left-sided sequence
   (ii) $\{z: |z| > 5\}$, a right-sided sequence
   (iii) $\{z: 0.3 < |z| < 0.6\}$, a two-sided sequence
   (iv) $\{z: 0.6 < |z| < 5\}$, a two-sided sequence
   (v) $\emptyset$, a two-sided sequence

(b) exists if ROC is $0.6 < z < 5$.

(c) Stable if the ROC is $0.6 < z < 5$. cannot be both stable and causal.

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1. i) $\{z: |z| > 0.5\},\ 2(0.2)^n\mu[n] + 5(-0.5)^n\mu[n]$
   ii) $\{z: |z| < 0.2\},\ -2(0.2)^n\mu[-n-1] - 5(-0.5)^n\mu[-n-1]$
   iii) $\{z: 0.2 < |z| < 0.5\},\ 2(0.2)^n\mu[n] - 5(-0.5)^n\mu[-n-1]$

2. $x[n] = \sum_{n=0}^{\infty} \delta[n-3k], k = 0, 1, 2, \ldots$

3. $H(z) = 11.06 + 8.51z^{-1} + 5.28z^{-2} + 5.12z^{-3} + 1.19z^{-4}$

4.
   $G_1(z)$ highpass filter. $(-1)^n h[n]$. $\pi - \omega_p,\ \pi - \omega_s,\ \delta_p,\ \delta_s$.

5. bandpass filter.
   $2\omega_p$. $2h_{LP}[n]\cos(n\omega_0)$

6. One possible solution is $b = \frac{m}{4}, a = \frac{m}{4\alpha}$, where m is a scale factor

7. 1.

8. (a) $0.1 + j0.599,\ 0.2711 - j1.6242,\ 0.2711 + j1.6242;$
   $-0.3 - j0.4,\ -1.2 + j1.6,\ -1.2 - j1.6;$
   $0.5;$
   $1,\ -1.$

   (b) $1 - 0.2423z^{-1} + 1.0076z^{-2} - 6.5294z^{-3} + 1.3338z^{-4}$
   $-17.2533z^{-5} + 17.2533z^{-7} - 1.3338z^{-8} + 6.5294z^{-9} - 1.0076z^{-10} + 0.2423z^{-11} - z^{-12}$

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1. (1) 2, 0, 0, 0, or 0, 2, 0, 0, or 0, 0, 1, 0
   1, 0, 0, 0
   1, 1, 0, 0
   0, 1, 0, 0

2. $H(z) = \frac{0.1757(1+z^{-1})}{1-0.6486z^{-1}}$

3. (a) $\left|H_{BS}(e^{j\omega})\right|^2 = \left(\frac{1+\alpha}{2}\right)^2 \frac{2\cos 2\omega - 8\beta\cos\omega + 2 + 4\beta^2}{2\alpha\cos 2\omega - 2\beta(1+\alpha)^2\cos\omega + 1 + \alpha^2 + \beta^2(1+\alpha)^2}$

   (c) 1, 1

6. (a) $H_2(z)$
   $= 2.5(2 - 1.6z^{-1} + z^{-2})(1 + 1.6z^{-1} + z^{-2})(1 + z^{-1})(1 - 0.8z^{-1} + 0.5z^{-2})$

   (b) $H_3(z)$
   $= 2.5(1 - 1.6z^{-1} + 2z^{-2})(1 + 1.6z^{-1} + z^{-2}) \times (1 + z^{-1})(0.5 - 0.8z^{-1} + z^{-2})$

   (c) none.

7. $\frac{(1+0.81z^{-1})(1-0.62z^{-1})}{(5+2.2z^{-1})(-3.1+1z^{-1})}$,

8. $0 < K < 0.5$

9. $C(z) = \frac{0.3+0.1167z^{-1}-0.4533z^{-2}-1.0717z^{-3}-0.9338z^{-4}-0.4819z^{-5}-0.225z^{-6}}{z^{-1}+2.85z^{-2}+2.925z^{-3}+1.6875+0.5063z^{-5}}$

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2. (a) no (b)(c) 1. (d) No difference.

4. $h_{HP}[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}, & n = 0 \\ -\frac{\sin \omega_c n}{n\pi}, & n \neq 0 \end{cases}$

5. (a) 0.0273, 0.0035
(b) 0.131dB, 27.959dB

6. ${\delta_p}^2 + 2\delta_p$, ${\delta_s}^2$.

$(1 + \delta_p)^M - 1$, ${\delta_s}^M$

7. $\frac{-2s^2+30s}{3s^2+10s+125}$

8. (a) 10 rad/s.
(b) $\frac{0.3882(1-z^{-1})}{1+0.2235z^{-1}}$

9. (a) $\frac{(k+\sigma_0)+j\Omega_0}{(-k+\sigma_0)+j\Omega_0}$
(b) Yes
(c) $f_1(z) = f_2(-z)$.
(d) $\omega = -2\cot^{-1}\frac{\Omega}{k}$, or $\Omega = -k\cot\frac{\omega}{2}$
(e) a highpass digital filter.

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1. (a) 158.6951, 45.9425Hz
   (b) 0.6301Hz, 2.8950Hz

2. $\frac{0.0958+0.1916z^{-1}+0.0958z^{-2}}{1-1.0291z^{-1}+0.4592z^{-2}}$

3. $\hat{z}=\frac{a+z_k}{1+az_k}$;

4. $\frac{0.34(1-z^{-1})^2}{1-0.1842z^{-1}+0.1776z^{-2}}$

5. $\frac{0.3766-0.6803\hat{z}^{-1}+0.6803\hat{z}^{-2}-0.3766\hat{z}^{-3}}{1.3954+0.0705\hat{z}^{-1}+0.9783\hat{z}^{-2}+0.1892\hat{z}^{-3}}$

6. $-\sum_{n=1}^{M}\left|\frac{\sin n\omega_c}{n\pi}\left(1-\cos\frac{2\pi n}{2M+1}\right)\right|^2$

7. (a) $h[n]=\frac{\sin(0.705\pi n)}{n\pi}\left(0.54+0.46\cos\frac{2\pi n}{63}\right)$

   (b)

$$h_c[n]=\begin{cases}\left(1-\dfrac{0.5\pi}{\pi}\right)\left(0.5+0.5\cos\dfrac{2\pi(n-20)}{41}\right)=0.5, & \text{for } n=20\\ -\dfrac{\sin 0.5\pi(n-20)}{(n-20)\pi}\left(0.5+0.5\cos\dfrac{2\pi(n-20)}{41}\right), & \text{for } n\neq 20\end{cases}$$

   (c) $h_c[n]=\left(\frac{\sin(0.65\pi(n-23))}{(n-23)\pi}-\frac{\sin(0.325\pi(n-23))}{(n-23)\pi}\right)\left(0.54+0.46\cos\frac{2\pi(n-23)}{47}\right)$

   (d)

$$h_c[n]=\begin{cases}\left(\dfrac{0.33\pi}{\pi}+1-\dfrac{0.75\pi}{\pi}\right)\left(0.5+0.5\cos\dfrac{2\pi(n-32)}{65}\right)=0.58, & \\ \text{for } n=32 & \\ \dfrac{\sin 0.33\pi(n-32)-\sin 0.75\pi(n-32)}{(n-32)\pi}\left(0.5+0.5\cos\dfrac{2\pi(n-20)}{41}\right) & \\ \text{for } n\neq 32 & \end{cases}$$

8. (a) 33.73dB, 0.0206, (b)31.6dB, 0.0263,

9. 28.387dB, 0.038