EE323 Digital Signal Processing

Tutorial Questions

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EE323 DIGITAL SIGNAL PROCESSING
TUTORIAL 1

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1. Consider the following sequences:

$$x[n] = \{2, 0, -1, 6, -3, 2, 0\}, \qquad -3 \le n \le 3,$$
$$y[n] = \{8, 2, -7, -3, 0, 1, 1\}, \qquad -5 \le n \le 1,$$
$$w[n] = \{3, 6, -1, 2, 6, 6, 1\}, \qquad -2 \le n \le 4.$$

The sample values of each of the above sequence outside the ranges specified are all zeros. Generate the following sequences.
(a) $c[n] = x[n+3]$, (b) $d[n] = 4y[n-2]$,
(c) $e[n] = y[1-n]$, (d) $u[n] = x[n-3] + y[n+3]$,
(e) $v[n] = y[n-3] \cdot w[n+2]$.

2. Let $\tilde{x}_1[n], \tilde{x}_2[n], \tilde{x}_3[n]$ be periodic sequences with fundamental periods, N_1, N_2, and N_3, respectively. Is a linear combination of these three periodic sequences a periodic sequence? If it is, what is its fundamental period?

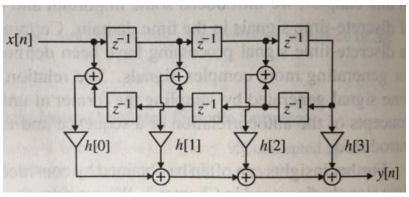
3. (a) Show that a causal real sequence $x[n]$ can be fully recovered from its even part for all $n \ge 0$, whereas it can be recovered from its odd part for all $n > 0$.
(b) Is it possible to fully recover a casual complex sequence $y[n]$ from its conjugate antisymmetric part? Can $y[n]$ be fully recovered from its conjugate symmetric part? Justify your answers.

4. Express the sequence $x[n] = 1$, $-\infty < n < \infty$, in term of the unit step sequence $u[n]$.

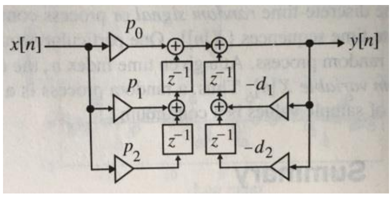
5. Determine the fundamental period of the following periodic sequence $x[n]=\cos(\omega_0 n)$ for the following values of the angular frequency ω_0:
(a) 0.14π, (b) 0.24π, (c) 0.75 (note: no π here!)

6. A continuous-time sinusoidal signal $x_a(t) = \cos(\Omega_0 t)$ is sampled at $t = nT$, $-\infty < n < \infty$, generating the discrete-time sequence $x[n] = x_a(nT) = \cos(\Omega_0 nT)$. For what values of T is $x[n]$ a periodic sequence? What is the fundamental period of $x[n]$ if $\Omega_0 = 18$ radians/second and $T = \frac{\pi}{6}$ seconds?

7. The following three schematics are operations developed using the three basic operations: addition, multiplication, and delaying. Develop the expression for $y[n]$ for each operation as a function of $x[n]$.



(a)



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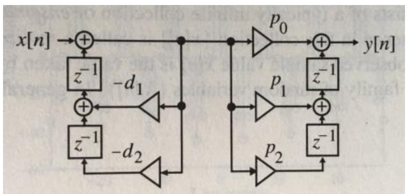

(c) (optional)

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1. For each of the following discrete-time systems, where $y[n]$ and $x[n]$ are, respectively, the output and input sequences, determine whether or not the system is (1) linear, (2) causal, (3)BIBO bounded, and (4) time-invariant. Show the necessary derivation.
   (a) $y[n] = x[n+3]$;
   (b) $y[n] = x[2-n]+\alpha$, where α is a nonzero constant;
   (c) $y[n] = \ln(1-|x[n]|)$;
   (d) $y[n] = \beta+\sum_{l=-1}^{3} x[n-l]$, where β is a nonzero constant.

2. Determine if the median filter defined by $y[n]= \mathrm{med}\{x[n-k], \ldots, x[n-1], x[n], x[n+1], \ldots, x[n+k]\}$ for a window length-$(2k+1)$ is a time-invariant system? Is it a causal system?

3. The second derivative $y[n]$ of a sequence $x[n]$ at time instant n is usually approximated by $y[n] = x[n+1]-2x[n]+x[n-1]$. If $y[n]$ and $x[n]$ denote the output and input of a discrete-time system, is the system linear? Is it time invariant? Is it causal?

4. Consider a causal discrete-time system characterized by a first-order linear, constant-coefficient difference equation given by
$$y[n] = ay[n-1]+bx[n], \quad \text{for } n \geq 0$$
where $y[n]$ and $x[n]$ are, respectively, the output and input sequences. Compute the expression for the output sample $y[n]$ in terms of the initial condition $y[-1]$ and the input samples.
   (a) Is the system time-invariant if $y[-1] = 1$? Is the system linear if $y[-1] = 1$?
   (b) Repeat part (a) if $y[-1] = 0$.

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1. Consider the following sequences:

$$x[n] = \{2, 0, -1, 6, -3, 2, 0\}, \qquad -3 \le n \le 3,$$
$$y[n] = \{8, 2, -7, -3, 0, 1, 1\}, \qquad -5 \le n \le 1,$$
$$w[n] = \{3, 6, -1, 2, 6, 6, 1\}, \qquad -2 \le n \le 4.$$

   Determine the following sequence by a linear convolution of the sequences given above:
   (a) $u[n] = x[n] \circledast y[n]$;
   (b) $v[n] = x[n] \circledast w[n]$;
   (c) $g[n] = w[n] \circledast y[n]$.

2. Let $y[n] = x_1[n] \circledast x_2[n]$, and $v[n] = x_1[n-N_1] \circledast x_2[n-N_2]$. Express $v[n]$ in terms of $y[n]$.

3. Consider the following finite-length sequences:
   (i) $\{h[n]\}, -M \le n \le N$,
   (ii) $\{g[n]\}, K \le n \le N$,
   (iii) $\{w[n]\}, -L \le n \le -R$,
   where M, N, K, L and R are positive integers with $K < N$ and $L > R$. Define
   (a) $y_1[n] = h[n] \circledast h[n]$,
   (b) $y_2[n] = g[n] \circledast g[n]$,
   (c) $y_3[n] = h[n] \circledast g[n]$,
   (d) $y_4[n] = h[n] \circledast w[n]$,
   What is the length of each of the convolved sequences? What is the range of the index n for which each of the above convolved sequences is defined?

4. Develop a closed-form expression for the convolution: $a^n \mu[n] \circledast \mu[n]$.

5. Consider two complex-valued sequences $h[n]$ and $g[n]$ expressed as a sum of their respective conjugate symmetric and conjugate anti-symmetric parts, i.e., $h[n]= h_{cs}[n]+ h_{ca}[n]$, and $g[n]= g_{cs}[n]+ g_{ca}[n]$. For each of the following sequences, determine if it is conjugate symmetric or conjugate antisymmetric.
   (a) $h_{cs}[n] \circledast g_{cs}[n]$ (b) $h_{ca}[n] \circledast g_{cs}[n]$ (c) $h_{ca}[n] \circledast g_{ca}[n]$

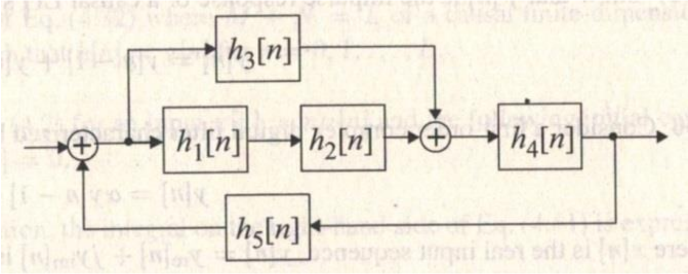
6. Show that the following sequences are absolutely summable, where $|\alpha|<1$
   (a) $x_1[n]= \alpha^n \mu[n-1]$ for $|\alpha|<1$, (b) $x_2[n]= n\alpha^n \mu[n-1]$ for $|\alpha|<1$, and (c) $x_3[n]= \mu[n]/((n+2)(n+3))$

7. An LTI discrete-time system is characterized by a left-side impulse response given by $h[n] = \alpha^n \mu[-n-1]$. Determine the range of the value of the constant α for which the system is BIBO stable.

8. Develop a general expression for the output $y[n]$ of an LTI discrete-time system in terms of its input $x[n]$ and the unit step response $s[n]$ of the system.

9. Determine the step response of an LTI discrete-time system characterized by an impulse response $h[n] = (-\alpha)^n \mu[n]$, $0 < \alpha < 1$.

10. Determine the expression for the impulse response of following LTI system.



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1. Determine the DTFT of the following sequences:
(a) two sided sequence $y[n] = \alpha^{|n|}, |\alpha| < 1$.
(b) causal sequence $x[n] = A\alpha^n \cos(\omega_0 n + \varphi)\mu[n]$, where $A, \alpha, \omega_0,$ and φ are real, and $|\alpha| < 1$.
(c) $x[n] = n\alpha^n \mu[n+2], |\alpha| < 1$

2. The values of the DTFT of the sequence $x[n] = a\delta[n] + b\delta[n-1] + c\delta[n-2]$ at the frequencies $\omega = \frac{3\pi}{2}, \omega = 3\pi,$ and $\omega = 6\pi$ are given by $3 - j$, 0, and 2, respectively. Determine the values of the samples a, b, c.

3. Determine the inverse DTFT of the following DTFTs:
(a) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k)$
(b) $X(e^{j\omega}) = \frac{e^{j\omega}(1-e^{j\omega N})}{1-e^{j\omega}}$
(c) $X(e^{j\omega}) = 1 + 2\sum_{l=0}^{N} \cos \omega l$
(d) $X(e^{j\omega}) = \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}, |\alpha| < 1$

4. Evaluate the linear convolution of each following sequences with itself using the DTFT-based method.
(a) $x[n] = \{1, 2, 1\}, -1 \le n \le 1,$
(b) $x[n] = \{-2, 1, 0, -1, 2\}, 0 \le n \le 4.$

5. Let $X(e^{j\omega})$ denote the DTFT of a length-9 sequence $x[n]$ given by

$$x[n] = \{3, 1, -5, -11, 0, -5, 3, 3, 8\}, -5 \le n \le 3$$

Evaluate the following function of $X(e^{j\omega})$ without computing the transform itself:

(a) $X(e^{j0})$, (b) $X(e^{j\pi})$, (c) $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$, (d) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2\, d\omega$, (e) $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega$.

6. Let $G_1(e^{j\omega})$ denote the DTFT of the sequence $g_1[n] = \{1, 4, 2, 3\}, 0 \le n \le 3$. Express the DTFT of the following sequences in terms of $G_1(e^{j\omega})$. Do not evaluate $G_1(e^{j\omega})$.
(a) $g_2[n] = \{1, 4, 2, 3, 1, 4, 2, 3\}, 0 \le n \le 7,$
(b) $g_3[n] = \{1, 4, 2, 3, 3, 2, 4, 1\}, 0 \le n \le 7,$
(c) $g_4[n] = \{3, 2, 4, 1, 1, 4, 2, 3\}, 0 \le n \le 7.$

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1. Compute the DTFT of the following sequences with ω_0 real, and discuss the convergence of the DTFT of the sequences for $-\infty < n < \infty$.
(a) $x[n] = e^{j\omega_0 n}$, (b) $x[n] = \cos(\omega_0 n + \varphi)$

2. The Nyquist frequency of a continuous-time signal $x_c(t)$ is ω_c. Determine the Nyquist frequency of each of the following continuous signals:

(a) $y_1(t) = x_c(t)x_c(t)$, (b) $y_2(t) = \int_{-\infty}^{\infty} x_c(t-\tau)\, x_c(t) d\tau$, (c) $y_3(t) = x_c(t/3)$,

(d) $y_4(t) = x_c(3t)$, (e) $y_5(t) = \frac{dx_c(t)}{dt}$

3 A 4.0 s long segment of a continuous-time signal is uniformly sampled without aliasing and generating a finite-length sequence containing 8500 samples. What is the highest frequency component that could be present in the continuous-time signal?

4 A continuous-time signal $x_c(t)$ is composed of a linear combination of complex sinusoidal signals of frequencies 300Hz, 500Hz, 1.2kHz, 2.15kHz, and 3.5kHz. The signal $x_c(t)$ is sampled at a 3.0kHz rate, and the sampled sequence is passed through an ideal lowpass filter with a cutoff frequency of 900Hz, generating a continuous-time signal $y_c(t)$. What are the frequency components present in the reconstructed signal $y_c(t)$?

5 A continuous-time signal $x_c(t)$ is composed of a linear combination of complex sinusoidal signals of frequencies F_1 Hz, F_2 Hz, F_3 Hz, and F_4 Hz. The signal $x_c(t)$ is sampled at an 8kHz rate, and the sampled sequence is passed through an ideal lowpass filter with a cutoff frequency of 3.5kHz, generating a continuous-time signal $y_c(t)$ composed of three sinusoidal signals of frequencies 150Hz, 400Hz, and 925Hz, respectively. What are the possible values of F_1, F_2, F_3, and F_4? Is your answer unique? If not, indicate another set of possible values of these frequencies.

6 Consider the system in the Figure 6.1, where the signal $x_a(t)$ has a band-limited spectrum $X_a(j\Omega)$, as sketched in Figure 6.2, and is being sampled at the Nyquist rate. The block of “Discrete time system” has an ideal lowpass filter with a frequency response $H(e^{j\omega})$, as shown in Figure 6.3, and has a cutoff frequency $\omega_c = \Omega_m T/2$, where T is the sampling period. Sketch as accurately as possible the spectrum $Y_a(j\omega)$ of the output continuous-time signal $y_a(t)$.

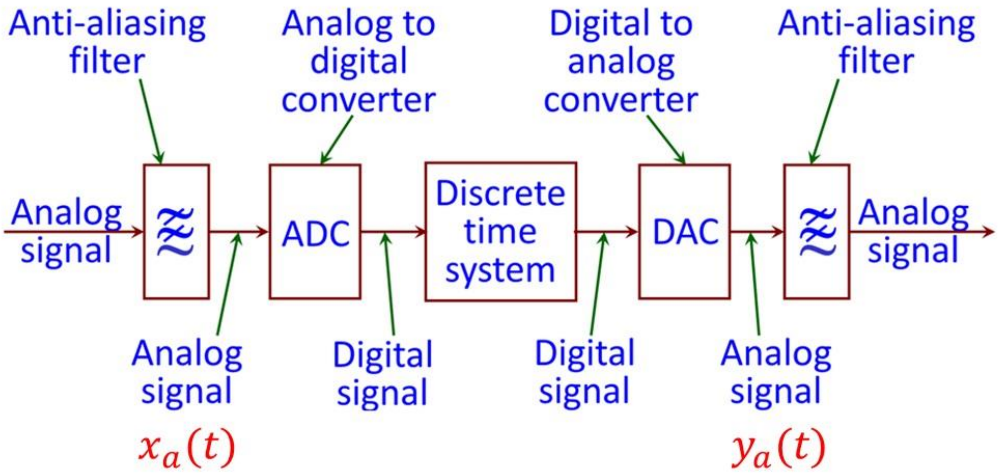

Figure 6.1

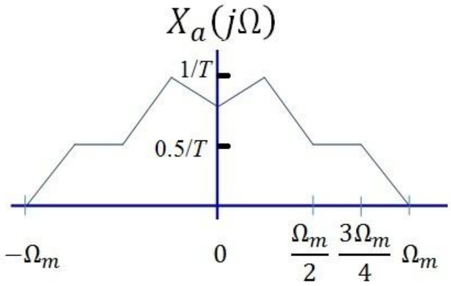

Figure 6.2

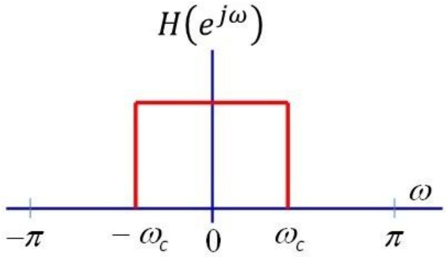

Figure 6.3

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1. Consider an LTI system discrete-time system with an impulse response $h[n] = (0.4)^n \mu[n]$. Determine the frequency response $H(e^{j\omega})$ of the system and evaluate its value at $\omega = \pm\frac{\pi}{4}$.

2. (a) Design a length-5 FIR bandpass filter with an anti-symmetric impulse response $h[n]$, i.e., $h[n] = -h[4-n], 0 \le n \le 4$, satisfying the following magnitude response values: $\left|H\left(e^{j\frac{\pi}{4}}\right)\right| = 0.5$ and $\left|H\left(e^{j\frac{\pi}{2}}\right)\right| = 1$.

(b) Determine the exact expression for the frequency response of the filter designed.

(c) Find the phase delay and group delay of the filter.

3. (a) Design a length-4 FIR bandpass filter with a symmetric impulse response $h[n]$, i.e., $h[n] = h[3-n], 0 \le n \le 3$, satisfying the following magnitude response values: $\left|H\left(e^{j\frac{\pi}{4}}\right)\right| = 1$ and $\left|H\left(e^{j\frac{\pi}{2}}\right)\right| = 0.5$.

(b) Determine the exact expression for the frequency response of the filter designed.

(c) Find the phase delay and group delay of the filter.

4. Consider the two LTI causal digital filters with impulse responses given by

$$h_A[n] = 0.3\delta[n] - \delta[n-1] + 0.3\delta[n-2], \qquad h_B[n] = 0.3\delta[n] + \delta[n-1] + 0.3\delta[n-2]$$

(a) Sketch the magnitude and phase response of the two filters and compare their characteristics.

(b) Let $h_A[n]$ be the impulse response of a causal digital filter with a frequency response $H_A(e^{j\omega})$. Define another digital filter whose impulse response $h_C[n]$ is given by

$$h_C[n] = (-1)^n h_A[n], \qquad \text{for all } n.$$

What is the relation between the frequency response $H_C(e^{j\omega})$ of this filter and the frequency response $H_A(e^{j\omega})$ of the prototype filter?

5. An LTI causal discrete-time system is characterized by the difference equation

$$y[n] = d_3 x[n] + d_2 x[n-1] + d_1 x[n-2] + x[n-3] - d_1 y[n-1] - d_2 y[n-2] - d_3 y[n-3],$$

where $y[n]$ and $x[n]$ are, respectively, the output and input sequences. Determine the expression for frequency response and show that it has a unity magnitude response for all values of ω.

6. Show that the group delay $\tau(\omega)$ of an LTI discrete-time system characterized by a frequency response $H(e^{j\omega})$ can be expressed as

$$\tau(\omega) = \mathrm{Re}\left\{\frac{j\dfrac{dH(e^{j\omega})}{d\omega}}{H(e^{j\omega})}\right\}$$

7. Show that the sequence $u[n] = z^n$, where z is a complex constant, is an eigenfunction of an LTI discrete-time system. Is the sequence $v[n] = z^n \mu[n]$ with z a complex constant also an eigenfunction of an LTI discrete-time system? Justify your answer.

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1. Determine the N-point DFTs of the following length-N sequences defined for $0 \le n \le N-1$.

(a) $x_a[n] = \sin\left(\frac{2\pi n}{N}\right)$, (b) $x_b[n] = \sin^2\left(\frac{2\pi n}{N}\right)$, (c) $x_c[n] = \sin^3\left(\frac{2\pi n}{N}\right)$

2. Let $x[n]$ be a length-N sequence with $X[k]$ donating its N-point DFT. We represent the DFT operation as $X[k] = \mathcal{F}\{x[n]\}$. Determine the sequence $y[n]$ obtained by applying the DFT operation 4 times to $x[n]$, i.e.,

$$y[n] = \mathcal{F}\left\{\mathcal{F}\left\{\mathcal{F}\{\mathcal{F}\{x[n]\}\}\right\}\right\}$$

3. Let $x[n], 0 \le n \le N-1$, be a length-N sequence with an N-point DFT given by $X[k], 0 \le k \le N-1$. Determine the 2N-point DFT of each of the following length-2N sequence in terms of $X[k]$.

(a) $g[n] = \begin{cases} x[n], & 0 \le n \le N-1 \\ 0, & N \le n \le 2N-1 \end{cases}$ (b) $h[n] = \begin{cases} 0, & 0 \le n \le N-1 \\ x[n-N], & N \le n \le 2N-1 \end{cases}$

4. Let $x[n], 0 \le n \le N-1$, be a length-N sequence with an N-point DFT given by $X[k], 0 \le k \le N-1$. Define a length-3N sequence $y[n]$ given by

$$y[n] = \begin{cases} x[n], & 0 \le n \le N-1 \\ 0, & N \le n \le 3N-1 \end{cases}$$

with $Y[k], 0 \le k \le 3N-1$, denoting its 3N-point DFT. Let $W[l] = Y[3l+2], 0 \le l \le N-1$, with $w[n], 0 \le n \le N-1$, denoting its N-point IDFT. Express $w[n]$ in terms of $x[n]$.

5. Consider a rational discrete-time Fourier transform $X(e^{j\omega})$ with real coefficients of the form of

$$X(e^{j\omega}) = \frac{P(e^{j\omega})}{D(e^{j\omega})} = \frac{p_0 + p_1 e^{-j\omega} + \cdots + p_{M-1} e^{-j\omega(M-1)}}{d_0 + d_1 e^{-j\omega} + \cdots + d_{N-1} e^{-j\omega(N-1)}}$$

Let $P[k]$ denote the M-point DFT of the numerator coefficients $\{p_i\}$ and $D[k]$ denote the N-point DFT of the denominator coefficients $\{d_i\}$. Determine the exact expressions of the DTFT $X(e^{j\omega})$ for $M = N = 4$, if the 4-point DFTs of its numerator and denominator coefficients are given by

$$P[k] = \{3.5, -0.5 - j9.5, 2.5, -0.5 + j9.5\}, D[k] = \{17, 7.4 + j12, 17.8, 7.4 - j12\}.$$

6. Let $X(e^{j\omega})$ denote the DTFT of the length-9 sequence $\{x[n]\} = \{1, -3, 4, -5, 7, -5, 4, -3, 1\}$.

(a) For the DFT sequence $X_1[k]$, obtained by sampling $X(e^{j\omega})$ at uniform intervals of $\pi/6$ starting from $\omega = 0$, determine the IDFT $x_1[n]$ of $X_1[k]$ without computing $X(e^{j\omega})$ and $X_1[k]$. Can you recover $x[n]$ from $x_1[n]$?

(b) For the DFT sequence $X_2[k]$, obtained by sampling $X(e^{j\omega})$ at uniform intervals of $\pi/4$ starting from $\omega = 0$, determine the IDFT $x_2[n]$ of $X_2[k]$ without computing $X(e^{j\omega})$ and $X_2[k]$. Can you recover $x[n]$ from $x_2[n]$?

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1. (a) Consider a length-N sequence $x[n], 0 \le n \le N-1$, with an N-point DFT $X[k], 0 \le k \le N-1$. Define a sequence $y[n]$ of length LN, $0 \le n \le NL-1$, given by

$$y[n] = \begin{cases} x[n/L], n = 0, L, 2L, \dots, (N-1)L, \\ 0, \qquad\qquad\qquad\qquad \text{otherwise}, \end{cases} \tag{8.1}$$

where L is a positive integer. Express the NL-point DFT $Y[k]$ of $y[n]$ in terms of $X[k]$.

(b) The 5-point DFT $X[k]$ of a length-5 sequence $x[n]$ is shown in Figure 8.1. Sketch the 20-point DFT $Y[k]$ of a length-20 sequence $y[n]$ generated using (8.1).

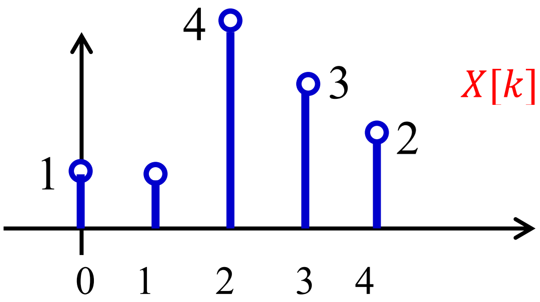

Figure 8.1

2. Consider the sequence $\{x[n]\} = \{2, -5, 6, -3, 4, -4, 0, -7, 8\}, -5 \le n \le 3$.

(a) Let $\{y[n]\}$ denote the sequence obtained by a left circular shift of $\{x[n]\}$ by 12 sample periods. Determine the value of the sample $y[-3]$.

(b) Let $\{z[n]\}$ denote the sequence obtained by a right circular shift of $\{x[n]\}$ by 15 sample periods. Determine the value of the sample $z[2]$.

3. Let $\{x[n]\} = \{-3, 2, -1, 4\}$, and $\{h[n]\} = \{1, 3, 2, -2\}$ be two length-4 sequences defined for $0 \le n \le 3$. Determine $y[n] = x[n] \, (4) \, h[n]$

4. (a) Let $g[n]$ and $h[n]$ be two sequences of length 6 each. If $y_L[n]$ and $y_C[n]$ denote the linear and 6-point circular convolutions of $g[n]$ and $h[n]$, respectively, develop a method to determine $y_C[n]$ in terms of $y_L[n]$.

(b) Consider the two length-6 sequences, $\{g[n]\} = \{3, -5, 2, 6, -1, 4\}$, and $\{h[n]\} = \{-2, 4, 7, -5, 4, 3\}$. Determine the $y_L[n]$ obtained by a linear convolution of $g[n]$ and $h[n]$. Using the method developed in Part (a), determine the sequence $y_C[n]$ given by the circular convolution of $g[n]$ and $h[n]$ from $y_L[n]$.

5. Denote $X[k], 0 \le k \le N-1$, the N-point DFT of sequence $x[n]$, with N even. Define two length-$\left(\frac{N}{2}\right)$ sequences given by: $g[n] = \frac{1}{2}(x[2n] + x[2n+1])$ and $h[n] = \frac{1}{2}(x[2n] - x[2n+1]), 0 \le n \le \frac{N}{2} - 1$. If $G[k]$ and $H[k], 0 \le k \le N/2 - 1$ are the $\left(\frac{N}{2}\right)$-point DFT of $g[n]$ and $h[n]$, respectively, determine $X[k]$ in terms of $G[k]$ and $X[k]$.

6. Let $x[n], 0 \leq n \leq N-1$, be a length-N sequence with an N-point DFT given by $X[k], 0 \leq k \leq N-1$.
(a) If $x[n]$ is a symmetric sequence satisfying the condition $x[n] = x[\langle N-1-n \rangle_N]$, show that $X[N/2] = 0$ for N even.
(b) If $x[n]$ is an antisymmetric sequence satisfying the condition $x[n] = -x[\langle N-1-n \rangle_N]$, show that $X[0] = 0$.
(c) If $x[n]$ is a sequence satisfying the condition $x[n] = -x[\langle n+M \rangle_N]$ with $N = 2M$, show that $X[2l] = 0$ for $l = 0, 1, \ldots, M-1$.

7. Let $x[n] = \{2, 1, 2\}, 0 \leq n \leq 2$ and $w[n] = \{-4, 0, -3, 2\}, 0 \leq n \leq 3$. If $w[n] = x[n] \circledast y[n]$, determine $y[n]$ using a DFT-based method.

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1. Develop an algorithm for the complex multiplication of two complex numbers using only three real multiplications and five real additions.

2. Develop the flow graph for the DIT FFT algorithm from that of Figure on Slide 311 of Lecture Notes 6 (L6) for the case $N = 8$ in which the input is in normal order and output is in the bit reversed order.

3. If M DFT samples of the N-point DFT of a length-N sequence are required with $M < N$, what is the smallest value of M for which the N-point FFT algorithm is computationally more efficient than a direct computation of the M DFT samples? What are the values of M for the following values of N: $N = 32, N = 64,$ and $N = 128$.

4. We know that the total number of complex multiplications $\mathcal{R}(v)$ needed in the implementation of the DIT and DIF FFT algorithms can be made smaller than $N/2 \log_2 N$ if the multiplications by ± 1 etc. can be avoid. Develop an exact expression for $\mathcal{R}(v)$ that includes only non-trivial multiplications by complex twiddle factors, i.e., excluding the multiplications with twiddle factors ± 1 and $\pm j$.

5. We wish to determine the sequence $y[n]$ generated by a linear convolution of a length-40 real sequence $x[n]$ and a length-21 real sequence $h[n]$. To this end, we can follow one of the following methods:
Method # 1. Direct computation of the linear convolution.
Method # 2. Computation of the linear convolution via a single circular convolution.
Method # 3. Computation of the linear convolution using FFT algorithm.
Determine the least number of real multiplications needed in each of the above methods. For the FFT algorithm, do not include in the count multiplications by $\pm 1, \pm j,$ and W_N^0.

6. We have known that the computation of DFT may be expressed in matrix form

$$\boldsymbol{X} = \mathbf{W}_N \boldsymbol{x}$$

where $\boldsymbol{X}$ and $\boldsymbol{x}$ are the DFT vector and input sample vector respectively, and $\mathbf{W}_N$ the constant matrix for DFT computation given on Slide 265 of L6.
(a) Show that the 8-point DIT FFT algorithm shown in Figure on Slide 308 of L6 is equivalent to expressing the DFT matrix as a product of four matrices as indicated below:

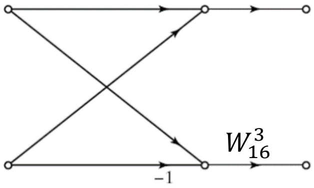
$$\mathbf{W}_N = \mathbf{V}_8 \mathbf{V}_4 \mathbf{V}_2 \mathbf{E} \tag{8.1}$$

Determine the matrices given above and show that multiplication by each matrix $\mathbf{V}_k, k = 8, 4, 2$, requires at most eight complex multiplications.
(<u>Hints: The matrix $\mathbf{E}$ is used for input sequence re-ordering, and $\mathbf{V}_2, \mathbf{V}_4, \mathbf{V}_8$ is used to realize the first, second and third stage butterflies, respectively.</u>)
(b) Since the DFT matrix $\mathbf{W}_N$ is its own transpose, i.e., $\mathbf{W}_N = \mathbf{W}_N^T$, another FFT algorithm is readily obtained by forming the transpose of the right-hand side of Eq. (8.1), resulting in a factorization of $\mathbf{W}_N$ given by

$$\mathbf{W}_N = \mathbf{E}^T \mathbf{V}_2^T \mathbf{V}_4^T \mathbf{V}_8^T$$

Show that the flow graph representation of the above factorization is precisely the 8-point DIF FFT algorithm of Figure on Slide 323 of L6.

7. The butterfly in the following figure was taken from a decimation-in-frequency FFT with N=16, where the input sequence was arranged in normal order. Note that a 16-point FFT will have four stages, indexed m=1, 2, 3, 4. Which of the four stages have butterflies of this form? Justify your answer.



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1. Determine the *z*-transform and the corresponding ROC of the following sequences:

(a) $g[n] = \delta[n]$
(b) $g[n] = n\alpha^n \mu[n]$
(c) $g[n] = -\alpha^n \mu[-n-2]$
(d) $g[n] = \alpha^{|n|}, |\alpha| < 1$

2. Determine the ROC of the *z*-transform of the following sequences:

(a) $x_1[n] = (0.2)^n \mu[n+1]$
(b) $x_2[n] = -(0.5)^n \mu[n-6]$
(c) $x_3[n] = (-0.5)^n \mu[-n-3]$
(d) $y_1[n] = x_1[n] + x_2[n]$
(e) $y_2[n] = x_1[n] + x_3[n]$
(f) $y_3[n] = x_2[n] + x_3[n]$

3. Determine the *z*-transform of the sequence

$$y[n] = \begin{cases} 1 - \frac{|n|}{N}, & , -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

and its ROC, where *N* is even. Show that the ROC includes the unit circle for the transform. Evaluate the z-transform on the unit circle to obtain the DTFT of the sequence.

4. Let $X(z)$ denote the *z*-transform of the length-10 sequence $x[n]$ defined for $0 \le n \le 9$

$$\{x[n]\} = \{6.29, 8.11, -7.46, 8.26, 2.64, -8.04, -4.43, 0.93, -9.15, 9.29\}$$

Let $X_0[k]$ represent the sample of $X(z)$ evaluated on the unit circle at eight equally spaced points given by $z = e^{j\frac{2\pi k}{8}}, 0 \le k \le 7$, i.e.,

$$X_0[k] = X(z)|_{z=e^{j\frac{2\pi k}{8}}}, 0 \le k \le 7$$

Determine the 8-point IDFT $x_0[n]$ of $X_0[k]$ without computing the latter function.

5. A causal IIR system has an input-output relation given by

$$y[n] = x[n] + 2x[n-1] - 0.21x[n-2] + 0.5y[n-1] + 0.66y[n-2]$$

Determine the z-transform transfer function of the system. Plot its poles and zeros. Determine if the system is stable or not?

6. Consider the causal stable transfer function

$$G(z) = \frac{1 - 0.5z^{-1} + 2z^{-2}}{(1 + 0.9z^{-1})(1 + 0.4z^{-1})}.$$

Develop a transfer function $H(z)$ by scaling the complex variable z by a constant α, i.e., $H(z) = G\left(\frac{z}{\alpha}\right)$.

Determine the range of values of α for which $H(z)$ remains stable.

7. A causal first-order transfer function is given by

$$H(z) = \frac{1+\beta z^{-1}}{1+\alpha z^{-1}},$$

Where $-1 < \alpha < 0, 1 < \beta < 2$. Determine the locations of the poles and zeros of $H(z)$ and $G(z) = H(z^M)$ where M is a positive integer. Check the stability of $H(z)$ and $G(z)$. What is the relation between the frequency response of these two transfer functions?

8. The transfer function of an LTI discrete-time system is given by

$$H(z) = \frac{3(z+1.8)(z-4)}{(z+0.3)(z-0.6)(z+5)}$$

(a) What are the possible ROCs associated with $H(z)$? What kind of sequences, for example, left-sided, right-sided, two-sided or finite-length sequences, is for each ROC?

(b) Does the frequency response $H\left(e^{j\omega}\right)$ of the system exist? Justify your answer.

(c) Can the system be stable? If it is stable can it be causal?

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1. The z-transform $X(z) = \frac{7}{1+0.3z^{-1}-0.1z^{-2}}$ has three non-empty ROCs. Evaluate their respective inverse z-transforms corresponding to each ROC.

2. Use power series expansion to determine the inverse z-transform of $X(z) = \frac{1}{1-z^{-3}}, |z| > 1$.

3. Consider the digital filter structure of Figure 1, where $H_1(z) = 2.1 + 3.3z^{-1} + 0.7z^{-2}$, $H_2(z) = 1.4 - 5.2z^{-1} + 0.8z^{-2}$, $H_3(z) = 3.2 + 4.5z^{-1} + 0.9z^{-2}$. Determine the transfer function $H(z)$ of the composite filter.

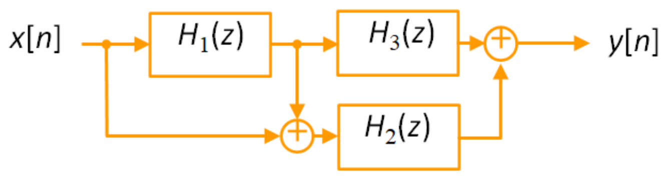

Figure 1

4. Let $H_{LP}(z)$ denote the transfer function of a real coefficient lowpass filter with a passband edge at ω_p, stopband edge at ω_s, passband ripple of δ_p, and stopband ripple of δ_s. Sketch the magnitude response of $G_1(z) = H_{LP}(-z)$, for $-\pi \le \omega \le \pi$. What type of filter is $G_1(z)$? Determine its impulse response $g_1[n]$ in terms of the impulse response $h_{LP}[n]$ of $H_{LP}(z)$. Determine the bandedge and ripple of $G_1(z)$ in terms of that of $H_{LP}(z)$.

5. Let $H_{LP}(z)$ denote the transfer function of an ideal real-coefficient lowpass filter having a cutoff frequency of ω_p, with $\omega_p < \frac{\pi}{2}$. Consider the complex coefficient transfer function $H_{LP}(e^{j\omega_0}z)$, where $\omega_p < \omega_0 < \pi - \omega_p$. Sketch its magnitude response for $-\pi \le \omega \le \pi$. What type of filter does it represent? Now consider the transfer function $G(z) = H_{LP}(e^{j\omega_0}z) + H_{LP}(e^{-j\omega_0}z)$. Sketch its magnitude response for $-\pi \le \omega \le \pi$. Show that $G(z)$ is a real-coefficient bandpass filter with a passband centered at ω_0. Determine the width of its passband in terms of ω_p and its impulse response $g[n]$ in terms of the impulse response of $h_{LP}[n]$ of the parent lowpass filter.

6. Consider the discrete-time system of Figure 2. For $H_0(z) = 1 + \alpha z^{-1}$, find a suitable $F_0(z)$ so that the output $y[n]$ is a delayed and scaled replica of the input.

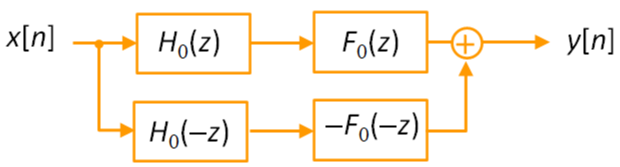

Figure 2

7. A causal FIR LTI discrete-time system is described by the difference equation

$$y[n] = a_1 x[n+k+1] + a_2 x[n+k] + a_3[n+k-1] + a_2[n+k-2] + a_1 x[n+k-3]$$

where $y[n]$ and $x[n]$ denote, respectively, the output and the input sequence. Determine the expression for its frequency response $H(e^{j\omega})$. For what value of the constant *k* will the system have a frequency response $H(e^{j\omega})$ that is real function of ω.

8. A Type 3 real-coefficient FIR with a transfer function $H(z)$ has the following zeros: $z_1 = 0.1 - j0.599$, $z_2 = -0.3 + j0.4$, $z_3 = 2$.

(a) Determine the location of the remaining zeros of $H(z)$ having the lowest order.

(b) Determine the transfer function $H(z)$ of the filter.

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1. The general form of the transfer function $H(z)$ of a linear-phase FIR filter with a real-valued impulse response is given by

$$H(z) = (1+z^{-1})^{N_1}(1-z^{-1})^{N_2}\prod_{i=1}^{N_3}(1+\alpha_i z^{-1}+z^{-2})\prod_{i=1}^{N_4}(1+\beta_i z^{-1}+\gamma_i z^{-2}+\beta_i z^{-3}+z^{-4})$$

What are the values of the constants $N_1, N_2, N_3,$ and N_4 for the lowest-order Type I, Type II, Type III, and Type IV linear-phase FIR filters, respectively.

2. Design a first-order lowpass IIR digital filter with normalized 3-dB cutoff frequency 0.42 rad/samples.

3. A bandstop IIR digital filter can be generated by a second-order transfer functions given by

$$H_{BS}(z) = \frac{1+\alpha}{2}\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}, |\alpha|<1, |\beta|<1$$

(a) Determine the squared-magnitude response of the bandstop IIR filter.
(b) Show that the notch frequency ω_0, at which the magnitude response is 0, is given by $\omega_0 = \cos^{-1}\beta$.
(c) Determine the magnitude response at $\omega = 0$ and $\omega = \pi$.
(d) It is known that the maximum magnitude response of the filter is 1. Show that the 3-dB notch bandwidth of the bandstop filter is given by $B_w = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$.

4. Based on the results obtained in Question 3, design a bandstop filter with notch frequency at 0.35π, and a 3-dB notch bandwidth of 0.15π.

5. Show that the following M^{th}-order complex coefficient transfer function is that of a causal allpass filter.

$$A_M(z) = \frac{d_M^* + d_{M-1}^* z^{-1} + \cdots + d_1^* z^{-M+1} + z^{-M}}{1 + d_1 z^{-1} + \cdots + d_{M-1} z^{-M+1} + d_M z^{-M}}$$

6. The transfer function of a Type 2 linear phase FIR filter is given by

$$H_1(z) = 2.5(1-1.6z^{-1}+2z^{-2})(1+1.6z^{-1}+z^{-2})(1+z^{-1})(1-0.8z^{-1}+0.5z^{-2})$$

(a) Determine the transfer function $H_2(z)$ of a minimum-phase FIR filter having the same magnitude as that of $H_1(z)$.
(b) Determine the transfer function $H_3(z)$ of a maximum-phase FIR filter having the same magnitude as that of $H_1(z)$.
(c) How many other length-8 FIR filter exist that have the same magnitude response as that of $H_1(z)$?

7. A typical transmission channel is characterized by a causal transfer function

$$H(z) = \frac{(2.2+5z^{-1})(1-3.1z^{-1})}{(1+0.81z^{-1})(1-0.62z^{-1})}$$

In order to correct for the magnitude distortion introduced by the channel on a signal passing through it, we wish to connect a causal stable digital filter characterized by a transfer function $G(z)$ at the receiving end.

Determine $G(z)$.

8. Figure 1 shows a typical closed-loop discrete-time feedback control system in which $G(z)$ is the plant and $C(z)$ is the compensator. If $G(z) = \frac{z^{-2}}{1+1.5z^{-1}+0.5z^{-2}}$ and $C(z) = K$, determine the range of values of K for which the overall structure is stable.

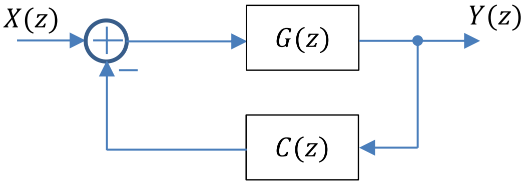

Figure 1

9. In the closed-loop discrete-time feedback control system of Figure 1, the plant transfer function is given by

$$G(z) = \frac{1.2 + 1.8z^{-1}}{1 + 0.7z^{-1} + 0.8z^{-2}}$$

Determine the transfer function $C(z)$ of the compensator so that the overall closed-loop transfer function of the feedback system is

$$H(z) = \frac{z^{-1} + 1.35z^{-2} + 0.9z^{-3} + 0.3375z^{-4}}{0.3 + 0.5z^{-1} + 0.505z^{-2} + 0.375z^{-3} + 0.21z^{-4}}.$$

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1. Develop the transposed form structure of a length-8 Type IV linear phase FIR filter making use of the coefficient symmetry.

2. Analyze the digital structure of Figure 1, and determine its transfer function $H(z) = \frac{Y(z)}{X(z)}$.

(a) Is this a canonic structure?
(b) What should be the value of the multiplier coefficient K so that $H(z)$ has a unity gain at $\omega = 0$?
(c) What should be the value of the multiplier coefficient K so that $H(z)$ has a unity gain at $\omega = \pi$?
(d) Is there a difference between these two values of K ? If not, why not?

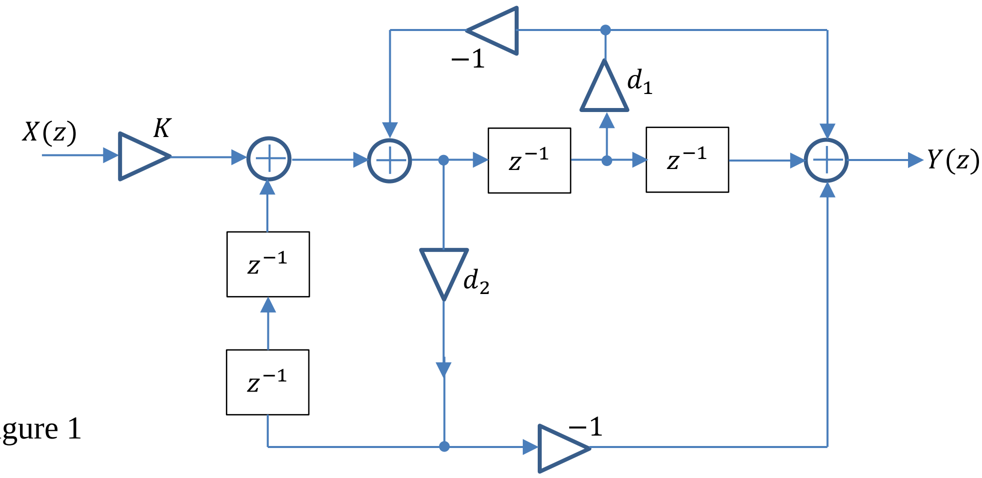

Figure 1

3. Develop a canonic direct-form realization of the transfer function

$$H(z) = \frac{3 + 4.5z^{-2} - 2.9z^{-3}}{1 + 2.2z^{-1} - 0.81z^{-3} + 5.1z^{-4}},$$

and then determine its transposed configuration.

4. Derive the impulse response coefficients $h_{HP}[n]$ of the ideal highpass digital filter with the zero-phase frequency response

$$H_{HP}(e^{j\omega}) = \begin{cases} 0, & |\omega| < \omega_c \\ 1, & \omega_c \le |\omega| \le \pi. \end{cases}$$

5. (a) Determine the peak ripple values δ_p and δ_s for the peak passband ripple $\alpha_p = 0.24\text{dB}$ and minimum stopband attenuation $\alpha_s = 49\text{dB}$.
(b) Determine the peak passband ripple α_p and minimum stopband attenuation α_s in dB for the peak ripple value $\delta_p = 0.015$, and $\delta_s = 0.04$.

6. Let $G(z)$ be the transfer function of a lowpass digital filter with a passband edge at ω_p, stopband edge at ω_s, passband ripple of δ_p, and stopband ripple of δ_s, as indicated in figure on Slide 3 of Lecture Notes 10. Consider a cascade of two identical filters with a transfer function $G(z)$. What

are the passband and stopband ripples of the cascade at passband and stopband, respectively? Generalize the results for a cascade of M identical sections.

7. The causal IIR digital transfer function

$$G_a(z) = \frac{4(z^2 + z - 2)}{10z^2 + 4z + 6}$$

was designed using bilinear transformation with k=5. Determine its prototype causal analog transfer function.

8. A first-order analog Butterworth highpass filter has an s-Transform transfer function $H_a(s) = \frac{s}{s+10}$.

(a) Determine the 3-dB cutoff frequency of the analog filter.

(b) Use bilinear transformation to transform the analog filter into a highpass digital filter transfer function with 250 Hz sampling frequency and 80 Hz 3-dB cutoff frequency.

9. Another bilinear transformation that can be used to design digital filters from an analog filter is given by

$$s = k\left(\frac{1 + z^{-1}}{1 - z^{-1}}\right)$$

(a) Develop the mapping of a point $s = \sigma_0 + j\Omega_0$ in the s-plane to a point z in the z-plane.

(b) Does this mapping have all the desirable properties indicated on Slide 14 of lecture notes 10?

(c) What is the relation of the above linear transformation to the bilinear transformation given on slide 16 of the lecture notes 10?

(d) Express the normalized digital angular frequency ω as a function of the normalized analog angular frequency Ω.

(e) If $H_a(s)$ is a causal analog lowpass transfer function, what is the type of the digital transfer function $G(z)$ that is obtained by the above bilinear transformation?

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1. To design a lowpass digital filter with $\omega_p = 0.24\pi, \omega_s = 0.68\pi, \alpha_p = 1\text{dB}$, and $\alpha_s = 24\text{dB}$ using bilinear transformation $s \to k\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$, we have to first design a prototype lowpass analog filter.

(a) If the lowpass analog filter has a passband edge $F_p = 10\text{Hz}$, determine the value of k, and the stopband edge F_s of the analog prototype filter.

(b) Using k =10 in the bilinear transformation, determine F_p and F_s of the analog prototype filter.

2. A chebyshev lowpass analog filter meeting the analog specification of question 1(b) is given by

$$H_a(s) = \frac{15.4035s^{-2}}{1 + 4.3463s^{-1} + 17.2830s^{-2}}$$

Use bilinear transformation to transform the analog filter into the lowpass digital filter.

3. Let $H_{LP}(z)$ be an IIR lowpass transfer function with a zero (pole) at $z = z_k$. Let $H_D(\hat{z})$ denote the lowpass transfer function obtained by lowpass-to-lowpass transformation given by $z^{-1} = \frac{\hat{z}^{-1}-\alpha}{1-\alpha\hat{z}^{-1}}$, which moves the zero (pole) at $z = z_k$ of $H_{LP}(z)$ to a new location $\hat{z} = \hat{z}_k$. Express $\hat{z}_k$ in terms of z_k. If $H_{LP}(z)$ has a zero at $z = -1$, show that $H_D(\hat{z})$ also has a zero at $z = -1$.

4. A second-order lowpass IIR digital filter with a 3-dB cutoff frequency at $\omega_c = 0.55\pi$ has a transfer function

$$G_{LP}(z) = \frac{0.34(1 + z^{-1})^2}{1 + 0.1842z^{-1} + 0.1776z^{-2}}$$

Design a second-order highpass filter $H_{HP}(z)$ with a 3-dB cutoff frequency at $\hat{\omega}_c = 0.45\pi$ by the lowpass-to-highpass spectral transformation.

5. A third-order elliptic highpass filter with a passband edge at $\omega_p = 0.52\pi$ has a transfer function

$$G_{HP}(z) = \frac{0.2397(1 - 1.5858z^{-1} + 1.5858z^{-2} - z^{-3})}{1 + 0.3272z^{-1} + 0.7459z^{-2} + 0.179z^{-3}}.$$

Design a highpass filter $H_{HP}(z)$ with a passband edge at $\hat{\omega}_p = 0.48\pi$ by transforming the above highpass transfer function using the lowpass-to-lowpass spectral transformation.

6. Let $h_d[n], -\infty < n < \infty$, denote the impulse response samples of an ideal zero-phase lowpass filter with a frequency response $H_d(e^{j\omega})$. It has been shown that the frequency response $H(e^{j\omega})$ of the zero-phase FIR filter $h[n], -M < n < M$, obtained by multiplying $h_d[n]$ with a rectangular window $w_R[n], -M < n < M$, has the least integral-squared error $E_R = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H\left(e^{j\omega}\right) - H_d\left(e^{j\omega}\right)\right|^2 d\omega$. Let E_{Hann} denote the integral-squared error if a length-$(2M+1)$ Hanning window is used to develop the FIR filter. Determine an expression for the excess error $E_{excess} = E_R - E_{Hann}$.

7. Design causal FIR filters with the smallest length meeting the following specification using the approach based on fixed window function.

(a) Lowpass filter, $\omega_p = 0.65\pi, \omega_s = 0.76\pi, \delta_p = 0.002, \delta_s = 0.004$.
(b) Highpass filter, $\omega_p = 0.58\pi,\ \omega_s = 0.42\pi,\ \delta_p = 0.008,\ \delta_s = 0.01$.
(c) bandpass filter, $\omega_{p1} = 0.4\pi, \omega_{p2} = 0.55\pi, \omega_{s1} = 0.25\pi, \omega_{s2} = 0.75\pi, \delta_p = 0.02, \delta_{s1} = 0.006$, $\delta_{s2} = 0.008$, where δ_{s1} and δ_{s2} are, respectively, the ripple in the lower and upper stopbands.
(d) bandstop filter $\omega_{p1} = 0.33\pi, \omega_{p2} = 0.8\pi, \omega_{s1} = 0.5\pi, \omega_{s2} = 0.7\pi, \delta_{p1} = 0.04, \delta_{p2} = 0.04$, $\delta_s = 0.03$, where δ_{p1} and δ_{p2} are, respectively, the ripple in the lower and upper passbands.

8. A lowpass FIR filter of order $N = 71$ is to be designed with a transition band given by $\omega_s - \omega_p = 0.04\pi$ with minimax criteria. Determine the approximate value of the stopband attenuation α_s in dB and the corresponding stopband ripple δ_s of the designed filter if the filter order is estimated using each of the following formulas: (a) Kaiser's formula, (b) Bellanger's formula. Assume the passband and stopband ripples to be the same.

9. Repeat Problem 8 if the filter is designed using the Kaiser's window-based method.