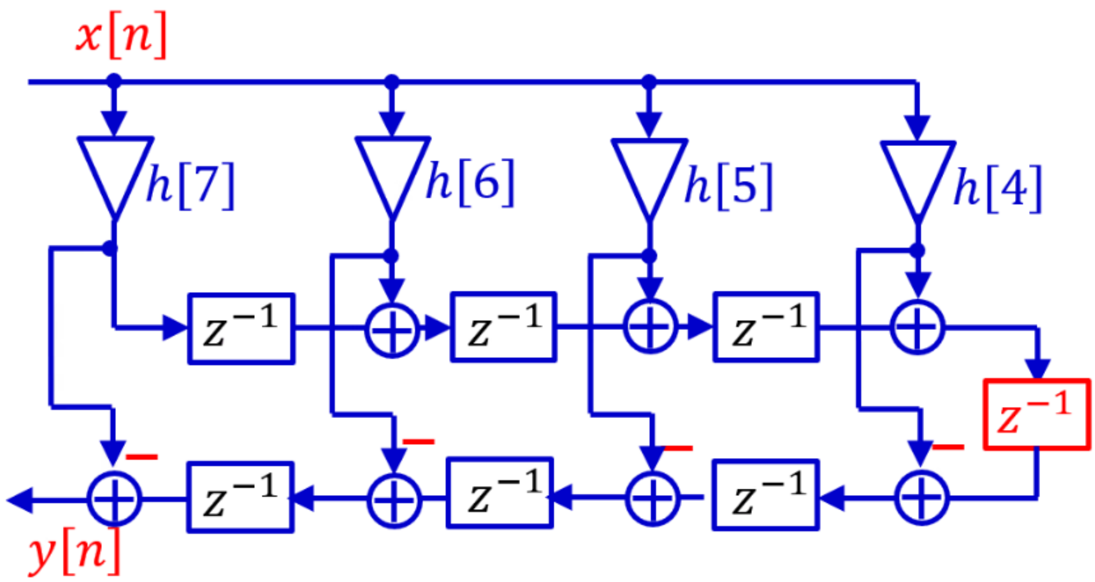
Tutorial13

1. Develop the transposed form structure of a length-8 Type IV linear phase FIR filter making use of the coefficient symmetry.

A: Anti-symmetrical impulse response:

$$h[n] = -h[7-n], 0 \leq n \leq 7$$



2. Analyze the digital structure of Figure 1, and determine its transfer function $H(z) = Y(z)/X(z)$.

(a) Is this a canonic structure?

(b) What should be the value of the multiplier coefficient K so that $H(z)$ has a unity gain at $\omega = 0$?

(c) What should be the value of the multiplier coefficient K so that $H(z)$ has a unity gain at $\omega = \pi$?

(d) Is there a difference between these two values of K ? If not, why not?

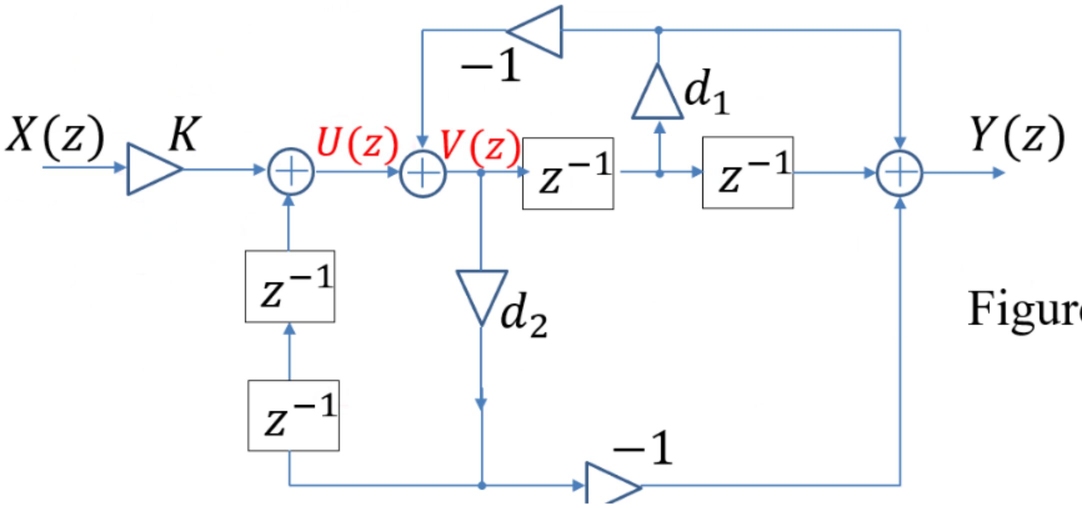


Figure 1

A: (a) From the structure, we have: $U(z) = KX(z) + d_2V(z)z^{-2}$

$$V(z) = -d_1V(z)z^{-1} + U(z)$$

$$Y(z) = d_1V(z)z^{-1} + V(z)z^{-2} - d_2V(z)$$

Thus, we have

$$V(z) = \frac{KX(z)}{1 + d_1z^{-1} - d_2z^{-2}} \Rightarrow Y(z)$$

$$= \frac{KX(z)(-d_2 + d_1z^{-1} + z^{-2})}{1 + d_1z^{-1} - d_2z^{-2}}$$

Therefore, the transfer function is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{K(-d_2 + d_1z^{-1} + z^{-2})}{1 + d_1z^{-1} - d_2z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{K(-d_2 + d_1z^{-1} + z^{-2})}{1 + d_1z^{-1} - d_2z^{-2}}$$

The structure is a 2nd-order filter, but using 4 delays (>2). Therefore the structure is not a canonic structure.

(b)(c) K should be equal to 1 to have a unity gain at $\omega = 0$ and $\omega = \pi$.

(d) There is no difference for the value of K, because the structure has a constant magnitude for all ω, as

$$\left|H(e^{j\omega})\right|^2 = H(z)H(z^{-1})\Big|_{z=e^{j\omega}} = K^2$$

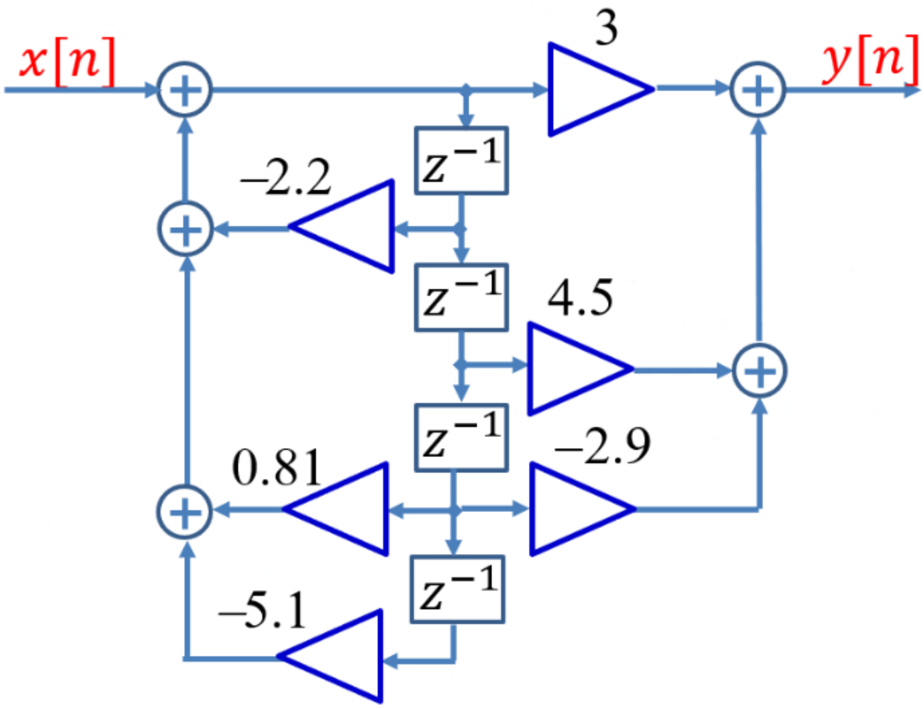
when $K = 1$, the filter is an allpass filter

3. Develop a canonic direct-form realization of the transfer function

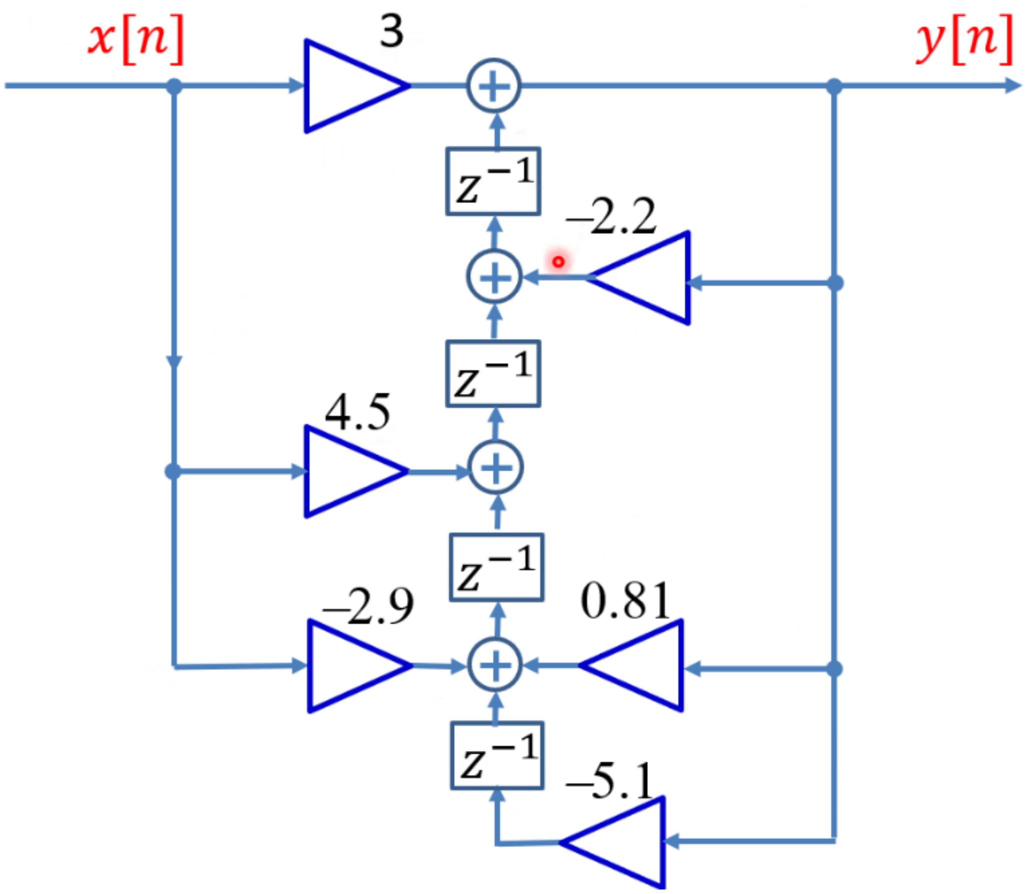
$$H(z) = \frac{3 + 4.5z^{-2} - 2.9z^{-3}}{1 + 2.2z^{-1} - 0.81z^{-3} + 5.1z^{-4}},$$

and then determine its transposed configuration.

A:



The transposed structure is given by



5. (a) Determine the peak ripple values δ_p and δ_s for the peak passband ripple $\alpha_p = 0.24\text{dB}$ and minimum stopband attenuation $\alpha_s = 49\text{dB}$.

(b) Determine the peak passband ripple α_p and minimum stopband attenuation α_s in dB for the peak ripple value $\delta_p = 0.015$, and $\delta_s = 0.04$.

A: (a) : $\delta_p = 1 - 10^{-\frac{\alpha_p}{20}} = 1 - 10^{-0.012} = 0.0273$

$$\delta_s = 10^{-\frac{\alpha_s}{20}} = 10^{-2.45} = 0.0035$$

(b) $\alpha_p = -20\log_{10}(1-\delta_p) = -20\log_{10} 0.985 = 0.131\text{dB}$

$\alpha_s = -20\log_{10}(\delta_s) = -20\log_{10} 0.04 = 27.959\text{dB}$

6. Let $G(z)$ be the transfer function of a lowpass digital filter with a passband edge at ω_p, stopband edge at ω_s, passband ripple of δ_p, and stopband ripple of δ_s, as indicated in figure on Slide 3 of Lecture Notes 10. Consider a cascade of two identical filters with a transfer function $G(z)$. What are the passband and stopband ripples of the cascade at passband and stopband, respectively? Generalize the results for a cascade of M identical sections.

A: $H(z) = G^2(z)$, or equivalently $H(e^{j\omega}) = G^2(e^{j\omega})$.

Thus, $|H(e^{j\omega})| = |G^2(e^{j\omega})| = |G(e^{j\omega})|^2$

Let δ_p and δ_s be the passband and stopband ripples of $G(e^{j\omega})$, respectively. Also, let $\delta_{p,2}$ and $\delta_{s,2}$ be the passband and stopband ripples of $H(e^{j\omega})$, respectively.

Thus, the maximum and minimum values in passband of $H(e^{j\omega})$ are $(1+\delta_p)^2$ and $(1-\delta_p)^2$, respectively. So the maximum ripple in the passband is

$$\delta_{p,2} = \max\left((1+\delta_p)^2 - 1, 1 - (1-\delta_p)^2\right) = \delta_p{}^2 + 2\delta_p$$

The maximum ripple in the stopband is $\delta_{s,2} = \delta_s{}^2$.

For a cascade of M sections, $\delta_{p,M} = (1+\delta_p)^M - 1$, $\delta_{s,M} = \delta_s{}^M$

7. The causal IIR digital transfer function

$$G_a(z) = \frac{4(z^2 + z - 2)}{10z^2 + 4z + 6}$$

was designed using bilinear transformation with *k*=5. Determine its prototype causal analog transfer function.

Bilinear Transformation

$$s = k\left(\frac{1-z^{-1}}{1+z^{-1}}\right), k > 0, \qquad \text{or } z = \frac{k+s}{k-s}$$

A: Substitute $z = \frac{k+s}{k-s} = \frac{5+s}{5-s}$ into $G_a(z)$, we have

$$H_a(s) = \frac{4\left(\left(\frac{5+s}{5-s}\right)^2 + \frac{5+s}{5-s} - 2\right)}{10\left(\frac{5+s}{5-s}\right)^2 + 4\left(\frac{5+s}{5-s}\right) + 6} = \frac{-2s^2 + 30s}{3s^2 + 10s + 125}$$

8. A first-order analog Butterworth highpass filter has an *s*-Transform transfer function $H_a(s) = s/(s + 10)$.

(a) Determine the 3-dB cutoff frequency of the analog filter.

(b) Use bilinear transformation to transform the analog filter into a highpass digital filter transfer function with 250 Hz sampling frequency and 80 Hz 3-dB cutoff frequency.

A: (a) $H_a(j\Omega) = \frac{j\Omega}{j\Omega+10} \Longrightarrow |H_a(j\Omega)|^2$

$= \frac{\Omega^2}{\Omega^2+100} \xrightarrow{\Omega\to\infty} |H_a(j\Omega)|^2 = 1$, i.e., the passband gain is 1.

3-dB cutoff frequency is the frequency where $|H_a(j\Omega)|^2 = \frac{1}{2}$. Therefore, $\Omega_c = 10$ rad/s.

(b) Since the analog filter has a response $H_a(j0) = 0$, and $H_a(j\infty) = 1$, it's a highpass filter.

The cutoff frequency of the digital filter is $\omega_c = \frac{80}{250} \times 2\pi = 0.64\pi$, mapping $\Omega_c = 10$ rad/s to $\omega_c = 0.64\pi$

Therefore (page 512),

$$10 = k \tan\left(\frac{0.64\pi}{2}\right)$$

Then we have $k = 6.3462$ and

$$H(z) = \frac{6.3462 \dfrac{1 - z^{-1}}{1 + z^{-1}}}{6.3462 \dfrac{1 - z^{-1}}{1 + z^{-1}} + 10} = \frac{0.3882(1 - z^{-1})}{1 + 0.2235z^{-1}}$$

9. Another bilinear transformation that can be used to design digital filters from an analog filter is given by

$$s = k\left(\frac{1+z^{-1}}{1-z^{-1}}\right)$$

Bilinear Transformation

$$s = k\left(\frac{1-z^{-1}}{1+z^{-1}}\right), k > 0, \qquad \text{or } z = \frac{k+s}{k-s}$$

(a) Develop the mapping of a point $s = \sigma_0 + j\Omega_0$ in the s-plane to a point z in the z-plane.

(b) Does this mapping have all the desirable properties indicated on Slide 14 of lecture notes 10?

(c) What is the relation of the above linear transformation to the bilinear transformation given on slide 16 of the lecture notes 10?

(d) Express the normalized digital angular frequency ω as a function of the normalized analog angular frequency Ω.

(e) If $H_a(s)$ is a causal analog lowpass transfer function, what is the type of the digital transfer function $G(z)$ that is obtained by the above bilinear transformation?

A: (a) $s = k\left(\frac{1+z^{-1}}{1-z^{-1}}\right) \Rightarrow z = \frac{s+k}{s-k}$. For $s = \sigma_0 + j\Omega_0$,

$$z = \frac{(k+\sigma_0) + j\Omega_0}{(-k+\sigma_0) + j\Omega_0}$$

(b) $|z|^2 = \frac{(k+\sigma_0)^2 + {\Omega_0}^2}{(-k+\sigma_0)^2 + {\Omega_0}^2}$. For point on imaginary axis of s-plane, i.e., $\sigma_0 = 0$, we have $|z|^2 = \frac{(k)^2 + {\Omega_0}^2}{(-k)^2 + {\Omega_0}^2} = 1$, i.e., on unit circle of z-plane.

For point on left half s-plane, i.e., $\sigma_0 < 0$, we have $|z|^2 = \frac{(k+\sigma_0)^2 + {\Omega_0}^2}{(-k+\sigma_0)^2 + {\Omega_0}^2} < 1$, i.e., inside the unit circle of z-plane.

Therefore, this mapping has all the desirable properties on Slide 14 of lecture notes 10.

(c) Let $s = f_1(z) = k\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ be the bilinear transformation given on Slide 16, and $s = f_2(z) = k\left(\frac{1+z^{-1}}{1-z^{-1}}\right)$ the transformation above. $f_1(z) = f_2(-z)$.

(d) When $j\Omega = k\frac{1+e^{-j\omega}}{1-e^{-j\omega}} = k\frac{e^{\frac{j\omega}{2}}+e^{-\frac{j\omega}{2}}}{e^{\frac{j\omega}{2}}-e^{-\frac{j\omega}{2}}} = -jk\cot\frac{\omega}{2}$,

Therefore, $\omega = -2\cot^{-1}\frac{\Omega}{k}$, or $\Omega = -k\cot\frac{\omega}{2}$

(e) When $\Omega = 0 \rightarrow \omega = \pm\pi$, and when $\Omega = \pm\infty \rightarrow \omega = 0$. So, a lowpass analog filter is mapped to a highpass digital filter.