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Tutorial 12.

1. Type I: Symmetrical impulse response. length odd. order even

zeros or even at $z=1$ or $z=-1$.

for lowest-order Type I: $N_1=2 \quad N_2=0 \quad N_3=0 \quad N_4=0$

$N_1=0 \quad N_2=2 \quad N_3=0 \quad N_4=0$

$N_1=0 \quad N_2=0 \quad N_3=1 \quad N_4=0$

Type II: odd zeros at $z=-1$, zero or even zeros at $z=1$

for lowest-order Type II: $N_1=1 \quad N_2=0 \quad N_3=0 \quad N_4=0$

Type III: odd zeros at $z=-1$ and $z=1$

for lowest-order Type III: $N_1=1 \quad N_2=1 \quad N_3=0 \quad N_4=0$

Type IV: odd zeros at $z=1$. even or zeros at $z=-1$

for lowest-order Type IV: $N_1=0 \quad N_2=1 \quad N_3=0 \quad N_4=0$

2. first-order IIR filter

$H(z)=\frac{1-\alpha}{2}\cdot\frac{1+z^{-1}}{1-\alpha z^{-1}}, \ \alpha|\alpha|<1$ let $|H_{LP}(e^{j\omega})|^2=\frac{1}{2}$ solve for $\alpha=\frac{1-\sin\omega_c}{\cos\omega_c}=\cancel{\frac{1-\sin\omega_c}{}}\frac{1-\sin 0.42}{\cos 0.42}$

$=0.6486.$

$\therefore H(z)=0.1757\cdot\frac{1+z^{-1}}{1-0.6486z^{-1}}$

3. (a) $|H_{BS}(e^{j\omega})|^2=|H_{BS}(z)H_{BS}(z^{-1})|_{z=e^{j\omega}}=\left(\frac{1+\alpha}{2}\right)^2\cdot\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}}\,\frac{1-2\beta z+z^2}{1-\beta(1+\alpha)z+\alpha z^2}\Bigg|_{z=e^{j\omega}}$

$=\left(\frac{1+\alpha}{2}\right)^2\cdot\frac{(z^{-2}+z^2)-4\beta(z^{-1}+z)+2+4\beta^2}{\alpha(z^{-2}+z^2)-\beta(1+\alpha)^2(z^{-1}+z)+1+\alpha^2+\beta^2(1+\alpha)^2}\Bigg|_{z=e^{j\omega}}=\left(\frac{1+\alpha}{2}\right)^2\frac{2\cos2\omega-8\beta\cos\omega+2+4\beta^2}{2\alpha\cos2\omega-2\beta(1+\alpha)^2\cos\omega+1+\alpha^2+\beta^2(1+\alpha)^2}$

(b) let $|H_{BS}(e^{j\omega})|^2=0 \quad 2\cos2\omega-8\beta\cos\omega+2+4\beta^2=0.$

$\cos2\omega-4\beta\cos\omega+1+2\beta^2=0$

$\therefore(\cos\omega)^2-2\beta\cos\omega+\beta^2=0 \qquad \cos\omega=\frac{2\beta\pm\sqrt{4\beta^2-4\beta^2}}{2}=\beta \qquad \therefore \omega_0=\cos^{-1}\beta$

(c). $|H_{BS}(e^{j0})|^2=\left(\frac{1+\alpha}{2}\right)^2\cdot\frac{2-8\beta+4\beta^2+2}{2\alpha-2\beta(1+\alpha)^2+1+\alpha^2+\beta^2(1+\alpha)^2}=\frac{4+4\beta^2-8\beta}{4\cdot\left(-2\beta+\beta^2+\frac{2\alpha+1+\alpha^2}{(1+\alpha)^2}\right)}=1$

$\therefore$ magnitude response at $\omega=0$ is 1

$|H_{BS}(e^{j\pi})|^2=\left(\frac{1+\alpha}{2}\right)^2\cdot\frac{2+8\beta+2+4\beta^2}{2\alpha+2\beta(1+\alpha)^2+1+\alpha^2+\beta^2(1+\alpha)^2}=1 \quad \therefore$ magnitude response at $\omega=\pi$ is 1 (d). let $|H_{BS}(e^{jw})|^2 = \frac{1}{2}$.

~~sole~~ solve for $2(1+\alpha^2)\cos^2 w - 2(1+\alpha)^2\beta\cos w + (1+\alpha)^2\beta^2 - (1-\alpha)^2 = 0$.

$\therefore \cos w_1 + \cos w_2 = -\frac{b}{a} = \frac{(1+\alpha)^2\beta}{(1+\alpha^2)}$

$\cos w_1 \cos w_2 = \frac{c}{a} = \frac{1}{2}\frac{(1+\alpha)^2\beta^2 - (1-\alpha)^2}{(1+\alpha^2)}$

$\cos(w_{c_2} - w_{c_1}) = \cos w_{c_2}\cos w_{c_1} + \sin w_{c_2}\sin w_{c_1}$. $\sin w_{c_1}\sin w_{c_2} = \sqrt{(1-\cos w_{c_1}^2)(1-\cos w_{c_2}^2)}$

solve for $\cos(w_{c_2} - w_{c_1}) = \frac{2\alpha}{(1+\alpha^2)}$

$\therefore W_{c_2} - W_{c_1} = B_W = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$

4. $w_0 = \cos^{-1}\beta$ $\therefore \beta = \cos w_0 = \cos \cancel{0.3\pi}\, 0.35\pi = 0.4540$

$B_W = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$ $\therefore \frac{2\alpha}{1+\alpha^2} = \cos B_W = \cos 0.15\pi = 0.8910$ solve for $\alpha_1 = 1.6319$ (not stable)

$\alpha_2 = 0.6128$

$\therefore H_{BS}(z) = \frac{1+\alpha}{2}\cdot\frac{1-2\beta z^{-1} + z^{-2}}{1-\beta(1+\alpha)z^{-1} + \alpha z^{-2}} = \frac{0.8064(1-0.908z^{-1} + z^{-2})}{1-0.7322z^{-1} + 0.6128z^{-2}}$

5. $\because$ complex coefficient $\therefore |A_M(z)|^2 = A_M(z)A_M^*(z)$

$= \frac{d_M^* + d_{M-1}^* z^{-1} + \cdots + z^{-M}}{1 + d_1 z^{-1} + \cdots + d_M z^{-M}} \cdot \frac{d_M + d_{M-1} z + \cdots + z^M}{1 + d_1^* z + \cdots + d_M^* z^M}$

$= \frac{d_M^* + d_{M-1}^* z^{-1} + \cdots + z^{-M}}{1 + d_1 z^{-1} + \cdots + d_M z^{-M}} \cdot \frac{z^M}{z^M} \cdot \frac{(d_M z^{-M} + \cdots + d_1 z^{-1} + 1)}{(d_M^* + d_{M-1}^* z^{-1} + \cdots + z^{-M})} = 1.$

$\therefore A_M(z)$ is an all-pass filter.

6. (a) zeros of $H_1(z)$ are:

$1-1.6z^{-1}+2z^{-2}=0 \Rightarrow z_1=0.8+\sqrt{1.36}\,j \quad z_2=0.8-\sqrt{1.36}\,j \Rightarrow$ Outside unit circle.

$1+1.6z^{-1}+z^{-2}=0 \Rightarrow z_3=-0.8+0.6j \quad z_4=-0.8-0.6j.$

$1+z^{-1}=0 \Rightarrow z_5=-1$

$1-0.8z^{-1}+0.5z^{-2}=0 \Rightarrow z_6=0.4+\sqrt{0.34}\,j \quad z_7=0.4-\sqrt{0.34}\,j. \Rightarrow$ Inside unit circle

$\therefore H_1(z)=2.5(1+1.6z^{-1}+z^{-2})(1+z^{-1})(1-0.8z^{-1}+0.5z^{-2})(2-1.6z^{-1}+z^{-2})\cdot\dfrac{1-1.6z^{-1}+2z^{-2}}{2-1.6z^{-1}+z^{-2}}$

$=H_2(z)\cdot\dfrac{1-1.6z^{-1}+2z^{-2}}{2-1.6z^{-1}+z^{-2}}$ $\quad\dfrac{1-1.6z^{-1}+2z^{-2}}{2-1.6z^{-1}+z^{-2}}$ is an all-pass filter.

Also, it can be shown that $H_1(z)H_1(z^{-1})=H_2(z)H_2(z^{-1})$ $\therefore H_1(z), H_2(z)$ has same magnitude.

$\therefore H_2(z)=2.5(2-1.6z^{-1}+z^{-2})(1+z^{-1})(1-0.8z^{-1}+0.5z^{-2})(1+1.6z^{-1}+z^{-2})$

(b). $H_1(z)=2.5(1-1.6z^{-1}+2z^{-2})(1+1.6z^{-1}+z^{-2})(1+z^{-1})(z^{-2}-0.8z^{-1}+0.5)\cdot\dfrac{1-0.8z^{-1}+0.5z^{-2}}{z^{-2}-0.8z^{-1}+0.5}$

$=H_3(z)\cdot\dfrac{1-0.8z^{-1}+0.5z^{-2}}{z^{-2}-0.8z^{-1}+0.5}$ $\quad\dfrac{1-0.8z^{-1}+0.5z^{-2}}{z^{-2}-0.8z^{-1}+0.5}$ is an all pass filter

$\therefore H_1(z).\ H_3(z)$ has same magnitude.

$H_3(z)=2.5(1-1.6z^{-1}+2z^{-2})(1+1.6z^{-1}+z^{-2})(1+z^{-1})(z^{-2}-0.8z^{-1}+0.5)$

(c) Because there are three combinations having the same magnitude response, the other zeros are all on unit circle. $\therefore$ there are no other length-8 FIR filter having same magnitude response.

7. $H(z)G(z)$ is an all-pass filter.

if its form is $\dfrac{(z^{-1}+0.81)(z^{-1}-0.62)}{(2.2+5z^{-1})(1-3.1z^{-1})}=G(z)$. $\quad G(z)H(z)=\dfrac{(z^{-1}+0.81)(z^{-1}-0.62)}{(1+0.81z^{-1})(1-0.62z^{-1})}$

for $G(z)$, its poles are $z_1=\frac{-5}{2.2}<-1 \quad z_2=3.1>1$ not causal.

$\therefore G(z)=\dfrac{(1+0.81z^{-1})(1-0.62z^{-1})}{(5+2.2z^{-1})(-3.1+z^{-1})}$

$H(z)G(z)=\dfrac{(2.2+5z^{-1})(1-3.1z^{-1})}{(5+2.2z^{-1})(-3.1+z^{-1})}$ is an all pass filter.

$\therefore G(z)=\dfrac{(1+0.81z^{-1})(1-0.62z^{-1})}{(5+2.2z^{-1})(-3.1+z^{-1})}$

8. $Y(z) = \left(X(z) - kY(z)\right)G(z) \qquad \therefore Y(z)\left(1+kG(z)\right) = X(z)G(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{G(z)}{1+kG(z)} = \frac{\frac{z^{-2}}{1+1.5z^{-1}+0.5z^{-2}}}{1+k\cdot\frac{z^{-2}}{1+1.5z^{-1}+0.5z^{-2}}} = \frac{z^{-2}}{1+1.5z^{-1}+(0.5+k)z^{-2}}$$

poles: $z = \frac{-1.5 \pm \sqrt{2.25-4\cdot 0.5-4k}}{2} = -0.75 \pm \frac{\sqrt{0.25-4k}}{2}$

let poles all inside unit circle: $\cancel{\Rightarrow k \quad 0.25-4k \geq 0 \quad 0<k\leq 0.0625}$

$\cancel{-0.75-\frac{\sqrt{0.25-4k}}{2} > -1 \Rightarrow k>0}$

if poles are real $\quad 0.25-4k \geq 0 \Rightarrow k \leq \frac{1}{16} \qquad -0.75+\frac{\sqrt{0.25-4k}}{2} < 1$

$-0.75-\frac{\sqrt{0.25-4k}}{2} > -1 \Rightarrow k>0 \qquad \therefore 0<k\leq\frac{1}{16}$

if poles are complex $\quad 0.25-4k<0 \Rightarrow k>\frac{1}{16}$

$\cancel{\frac{\sqrt{0.25-4k}}{2} < 1 \Rightarrow k<\frac{17}{16} \quad \therefore \frac{1}{16}<k<\frac{17}{16}}$

$\cancel{\therefore 0<k<\frac{17}{16}}$

~~9~~ $\quad 0.75^2 + \left(\frac{\sqrt{4k-0.25}}{2}\right)^2 < 1 \Rightarrow k<0.5 \qquad \therefore 0<k<0.5$

9. According to 8. $\quad H(z) = \frac{Y(z)}{X(z)} = \frac{G(z)}{1+G(z)C(z)} \qquad \therefore C(z) = \frac{G(z)-H(z)}{G(z)H(z)}$

$$\therefore C(z) = \frac{1.2+0.4667z^{-1}-1.8133z^{-2}-4.2867z^{-3}-3.735z^{-4}-1.9275z^{-5}-0.9z^{-6}}{1+2.3667z^{-1}+3.65z^{-2}+3.7617z^{-3}+2.9217z^{-4}+1.49z^{-5}+0.56z^{-6}}$$

or
$$C(z) = \frac{0.3+0.1167z^{-1}-0.4533z^{-2}-1.0717z^{-3}-0.9338z^{-4}-0.4819z^{-5}-0.225z^{-6}}{z^{-1}+2.85z^{-2}+2.925z^{-3}+1.6875+0.5063z^{-5}}$$