1191214 杨鸿嘉

Date. /

Tutorial 6.

$|0.4 \cdot e^{-jw}| < 1$

1. Solution: $H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n] e^{-jwn} = \sum_{n=-\infty}^{\infty} (0.4)^n u[n] e^{-jwn} = \sum_{n=0}^{\infty} (0.4)^n e^{-jwn} = \frac{1}{1-0.4e^{-jw}}$

at $w = \pm\frac{\pi}{4}$ $\quad H(e^{jw}) = \frac{1}{1-0.4e^{\pm j\frac{\pi}{4}}} = \frac{1}{1-0.4(\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2})} = \frac{1}{\pm j0.2\sqrt{2} + 1 - 0.2\sqrt{2}}$

$= 1.2972e^{\pm 0.3757}$

$w = \frac{\pi}{4}$ $\quad H(e^{jw}) = 1.2972e^{0.3757}$

$w = -\frac{\pi}{4}$ $\quad H(e^{jw}) = 1.2972e^{-0.3757}$

2. Solution: (a). $H(e^{jw}) = h[0] + h[1]e^{-jw} + h[2]e^{-j2w} + h[3]e^{-j3w} + h[4]e^{-j4w}$

$= h[0] + h[1]e^{-jw} + -h[1]e^{-j3w} - h[0]e^{-j4w}$

$= h[0](1-e^{-j4w}) + h[1](e^{-jw} - e^{-j3w})$

$= e^{-j2w}[h[0](e^{j2w} - e^{-j2w}) + h[1](e^{+jw} - e^{-jw})]$

$= 2je^{-j2w}(h[0]\sin(2w) + h[1]\sin(w))$

$|H(e^{j\frac{\pi}{4}})| = |2h[0]\sin(\frac{\pi}{2}) + 2h[1]\sin(\frac{\pi}{4})| = |2h[0] + \sqrt{2}h[1]| = 0.5$

$|H(e^{j\frac{\pi}{2}})| = |2h[0]\sin(\pi) + 2h[1]\sin(\frac{\pi}{2})| = |2h[1]| = 1.$

solve for
$h_1[0] = \frac{1}{4} - \frac{\sqrt{2}}{4}$ $\quad h_1[1] = \frac{1}{2}$
$h_2[0] = -\frac{1}{4} - \frac{\sqrt{2}}{4}$ $\quad h_2[1] = \frac{1}{2}$
$h_3[0] = \frac{1}{4} + \frac{\sqrt{2}}{4}$ $\quad h_3[1] = -\frac{1}{2}$
$h_4[0] = \frac{\sqrt{2}}{4} - \frac{1}{4}$ $\quad h_4[1] = -\frac{1}{2}$

(b) $\therefore$ $H(e^{jw})_1 = je^{-j2w}(-0.207\sin(2w) + 1\sin(w))$

$H(e^{jw})_2 = je^{-j2w}(-1.2071\sin(2w) + \sin(w))$

$H(e^{jw})_3 = je^{-j2w}(1.2071\sin(2w) - \sin(w))$

$H(e^{jw})_4 = je^{-j2w}(0.207\sin(2w) - \sin(w))$ $\quad$ four possible results

2. (c) $H(e^{jw}) = j\,[\cos(2w) - j\sin(2w)]\,(-0.207\sin(2w) + \sin(w))$

$= [\sin(2w) + j\cos(2w)]\,(-0.207\sin(2w) + \sin(w))$

$\theta(w) = \arctan\left(\frac{\cos(2w)}{\sin(2w)}\right) = \frac{\pi}{2} - 2w.$

phase delay: $\tau_p(w_0) = -\frac{\theta(w_0)}{w_0} = 2 - \frac{\pi}{2w}$

group delay: $\tau_g(w) = -\frac{d\theta(w)}{dw} = 2.$

3. (a) $H(e^{jw}) = h[0] + h[1]e^{-jw} + h[2]e^{-j2w} + h[3]e^{-j3w}.$

$= h[0]\,(1+e^{-j3w}) + h[1]\,(e^{-jw} + e^{-j2w})$

$= e^{-j3w/2}\,(h[0]e^{j3w/2} + h[1]e^{jw/2} + h[1]e^{-jw/2} + h[0]e^{-j3w/2})$

$= e^{-j3w/2}\,(2h[0]\cos(\frac{3w}{2}) + 2h[1]\cos(\frac{w}{2}))$

$|H(e^{j\frac{\pi}{4}})| = |2h[0]\cos(\frac{3}{8}\pi) + 2h[1]\cos(\frac{\pi}{4})| = 1$

$|H(e^{j\frac{\pi}{2}})| = |2h[0]\cos(\frac{3}{4}\pi) + 2h[1]\cos(\frac{\pi}{4})| = 0.5.$

solve for
$$\begin{cases} h[0]_1 = 0.1327 & h[1]_1 = 0.4862. \\ h[0]_2 = 0.6327 & h[1]_2 = 0.2791 \\ h[0]_3 = -0.6327 & h[1]_3 = -0.2791 \\ h[0]_4 = -0.1327 & h[1]_4 = -0.4862. \end{cases}$$

(b) $\therefore H(e^{jw})_1 = e^{-j\frac{3}{2}w}\,[0.2654\cos(\frac{3}{2}w) + 0.9725\cos(\frac{w}{2})]$

$H(e^{jw})_2 = e^{-j\frac{3}{2}w}\,[1.2654\cos(\frac{3}{2}w) + 0.5583\cos(\frac{w}{2})]$

$H(e^{jw})_3 = e^{-j\frac{3}{2}w}\,[-1.2654\cos(\frac{3}{2}w) - 0.5583\cos(\frac{w}{2})]$

$H(e^{jw})_4 = e^{-j\frac{3}{2}w}\,[-0.2654\cos(\frac{3}{2}w) - 0.9725\cos(\frac{w}{2})]$ four possible results.

(c) $\theta(w) = -\frac{3}{2}w$ phase delay: $\tau_p(w_0) = -\frac{\theta(w_0)}{w_0} = \frac{3}{2}$

group delay: $\tau_g(w) = -\frac{d\theta(w)}{dw} = \frac{3}{2}$

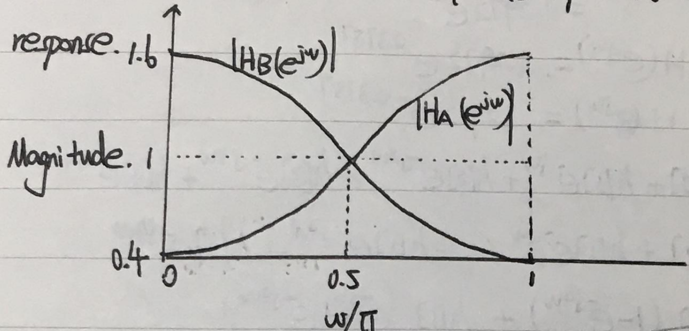
4. (a) $\delta[n] \xleftrightarrow{F} 1$ ∴ $H_A(e^{jw}) = 0.3(1+e^{-j2w}) - e^{-jw}$ $\quad H_B(e^{jw}) = 0.3(1+e^{-j2w}) + e^{-jw}$

$$H_A(e^{jw}) = e^{-jw}(0.3e^{jw} + 0.3e^{-jw} - 1) = e^{-jw}(0.6\cos w - 1)$$

$$H_B(e^{jw}) = e^{-jw}(0.3e^{jw} + 0.3e^{-jw}) + e^{-jw} = e^{-jw}(0.6\cos w + 1)$$

$|H_A(e^{jw})| = |0.6\cos w - 1| = 1 - 0.6\cos w \qquad |H_B(e^{jw})| = 0.6\cos w + 1$

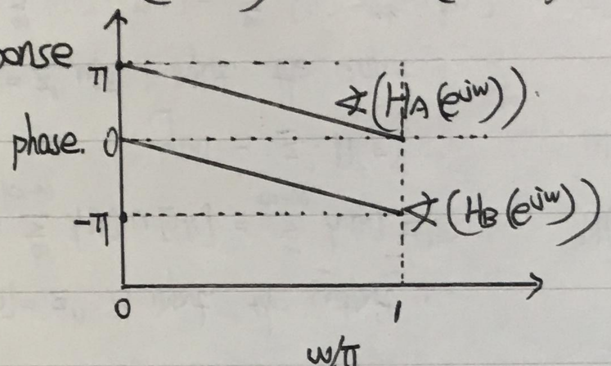
∴ magnitude response.



phase: $H_A(e^{jw}) = e^{-jw}(0.6\cos w - 1) = e^{-j(w+\pi)}(1 - 0.6\cos w)$ ∴ $\measuredangle(H_A(e^{jw})) = -w - \pi$.

$H_B(e^{jw}) = e^{-jw}(0.6\cos w + 1)$ ∴ $\measuredangle(H_B(e^{jw})) = -w$

∴ phase response



A is a highpass filter while B is lowpass. there magnitude is symmetry to y=1.
there phase response is different by π (in) phase

(b) n can only be 0, 1, 2.

$$H_C(e^{jw}) = \sum_{n=-\infty}^{\infty} h_C[n] e^{-jwn} = \sum_{n=0}^{3} (-1)^n h_A[n] e^{-jwn} = \sum_{n=0}^{3} (e^{-j\pi})^n h_A[n] e^{-jwn} = \sum_{n=0}^{3} h_A[n] e^{-j(w+\pi)n}$$

∴ $h_C[n] = e^{-j\pi n} \cdot h_A[n]$ ∴ $H_C(e^{jw}) = H_A(e^{j(w+\pi)})$

5. Taking Fourier Transform for both sides:

$$Y(e^{jw}) = d_3 X(e^{jw}) + d_2 e^{-jw} X(e^{jw}) + d_1 e^{-2jw} X(e^{jw}) + e^{-3jw} X(e^{jw}) + d_1 e^{-jw} Y(e^{jw})$$
$$+ d_2 e^{-2jw} Y(e^{jw}) + d_3 e^{-3jw} Y(e^{jw})$$

$$\therefore H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{d_3 + d_2 e^{-jw} + d_1 e^{-2jw} + e^{-3jw}}{1 + \cancel{d_1 e^{-}}\ d_1 e^{-jw} + d_2 e^{-2jw} + d_3 e^{-3jw}}$$

$$= e^{-3jw} \cdot \frac{d_3 e^{3jw} + d_2 e^{2jw} + d_1 e^{jw} + 1}{d_3 e^{-3jw} + d_2 e^{-2jw} + d_1 e^{-jw} + 1}$$

$$= e^{-3jw} \cdot \frac{d_3 \cos(3w) + d_2 \cos(2w) + d_1 \cos(w) + 1 + (d_3 \sin(3w) + d_2 \sin(2w) + d_1 \sin(w) + 1) j}{d_3 \cos(3w) + d_2 \cos(2w) + d_1 \cos(w) + 1 - (d_3 \sin(3w) + d_2 \sin(2w) + d_1 \sin(w) + 1) j}$$

$$\therefore |H(e^{jw})| = |e^{-3jw}| \cdot | \swarrow | = 1$$

$\therefore$ This LTI system has a unity magnitude response for all values of w

6. $H(e^{jw}) = |H(e^{jw})| e^{j\theta(w)}$ group delay $= \tau_g(w) = -\frac{d\theta(w)}{dw}$

$$\frac{dH(e^{jw})}{dw} = \frac{d|H(e^{jw})|}{dw} e^{j\theta(w)} + e^{j\theta(w)} \cdot j \cdot \frac{d\theta(w)}{dw} \cdot |H(e^{jw})|$$

$$\therefore \frac{j\frac{dH(e^{jw})}{dw}}{H(e^{jw})} = \frac{j\frac{d|H(e^{jw})|}{dw}}{H(e^{jw})} - \frac{d\theta(w)}{dw} \qquad \therefore Re\left\{\frac{j\frac{dH(e^{jw})}{dw}}{H(e^{jw})}\right\} = -\frac{d\theta(w)}{dw}$$

$$\therefore \tau(w) = Re\left\{\frac{j\frac{dH(e^{jw})}{dw}}{H(e^{jw})}\right\}$$

7. ①. if $u[n] = z^n$ is input, the output is:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k] = \sum_{n=-\infty}^{\infty} h[k]\, z^{n-k} \qquad H(z) = \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}$$

$\therefore y[n] = z^n \cdot H(z)$ $\therefore u[n] = z^n$ is an eigenfunction of the system

②. if $v[n] = z^n \mu[n]$ is input, the output is:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, v[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, z^{n-k} \mu[n-k] = z^n \mu[n-k]\, H(z)$$

$\therefore v[n] = z^n \mu[n]$ is not an eigenfunction of system.