1. A causal IIR discrete-time system is characterized by the input-output relation

$$y[n] = x[n] - \alpha y[n-R], \qquad 0 < \alpha < 1,$$

where $y[n]$ and $x[n]$ denote, respectively, the output and the input sequences.

1) Determine the expression for the frequency response $H(e^{j\omega})$ of the system.

2) Determine the maximum and the minimum values of its magnitude response.

3) How many peaks and dips of the magnitude response occur in the range $-\pi < \omega \leq \pi$, for R=4?

4) What are the locations of the peaks and the dips?

1.

1) From $y[n] = x[n] - \alpha y[n-R]$, we have

$$Y(e^{j\omega}) = X(e^{j\omega}) - \alpha Y(e^{j\omega})e^{-jR\omega}$$

i.e.,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 + \alpha e^{-jR\omega}}$$

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2) Since $0 < \alpha < 1$ and is real,

when $R\omega = 2k\pi$ for integer k, $e^{-jR\omega} = 1$, $|H(e^{j\omega})|$ has a minimum value $\frac{1}{1+\alpha}$.

When $R\omega = (2k+1)\pi$ for integer k, $e^{-jR\omega} = -1$, $|H(e^{j\omega})|$ has a maximum value $\frac{1}{1-\alpha}$.

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3) 4) When $R = 4$, in $-\pi < \omega \leq \pi$,

4 peaks: $\omega = \frac{(2k+1)\pi}{4} = -\frac{3\pi}{4}$ or $-\frac{\pi}{4}$ or $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ for $k = -2, -1, 0, 1$

4 dips: $\omega = \frac{2k\pi}{4} = -\frac{\pi}{2}$ or 0 or $\frac{\pi}{2}$ or π for $k = -1, 0, 1, 2$

2. Consider the five-point sequence

$$x[n] = 6\delta[n] + 5\delta[n-1] + 4\delta[n-2] + 3\delta[n-3] + 2\delta[n-4]$$

(a) Write $x[\langle -n \rangle_5]$ and $x[\langle n-8 \rangle_5]$, for $n = 0, 1, 2, \ldots, 4$.

(b) Determine $X[k]$, the five-point DFT of $x[n]$. Express your answer in terms of $W_5 = e^{-j2\pi/5}$.

(c) Plot the sequence $w[n]$, $n = 0, 1, 2, \ldots, 4$, that is obtained by computing the inverse five-point DFT of $W[k] = W_5^{-2k} X[k]$.

(d) Use any convenient method to evaluate the five-point circular convolution of $x[n]$ with the sequence $h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$. Sketch the result.

(e) If we convolve the given $x[n]$ with the given $h[n]$ by N-point circular convolution, how should N be chosen so that the result of the circular convolution is identical to the result of linear convolution? That is choose N so that

$$y_p[n] = x[n] \circledN h[n] = \sum_{m=0}^{N-1} x[m] h[(n-m)_N]$$

$$= x[n] \circledast h[n] = \sum_{m=-\infty}^{\infty} x[m] h[n-m] \quad \text{for } 0 \leq n \leq N-1$$

2.

(a) $x[\langle -n \rangle_5] = \{6,2,3,4,5\}$ for n = 0, 1, 2, …, 4.

$x[\langle n-8 \rangle_5] = \{4,3,2,6,5\}$ for n = 0, 1, 2, …, 4.

(b) $X[k] = \sum_{n=0}^{4} x[n] W_5^{kn}$

$X[0] = 6 + 5 + 4 + 3 + 2 = 20$

$X[1] = 6W_5^0 + 5W_5^1 + 4W_5^2 + 3W_5^3 + 2W_5^4$

$X[2] = 6W_5^0 + 5W_5^2 + 4W_5^4 + 3W_5^6 + 2W_5^8$

$X[3] = 6W_5^0 + 5W_5^3 + 4W_5^6 + 3W_5^9 + 2W_5^{12}$

$X[4] = 6W_5^0 + 5W_5^4 + 4W_5^8 + 3W_5^{12} + 2W_5^{16}$

Question 2

(c) $W[k] = W_5^{-2k}X[k] = \sum_{n=0}^{4} x[n]W_5^{(n-2)k} = \sum_{n=0}^{4} x[\langle r+2\rangle_5]W_5^{rk}$

$w[n] = x[\langle n+2\rangle_5] = \{4,3,2,6,5\}, \quad (0 \le n \le 4)$

(d) $h_{zp}[n] = \{1,1,1,0,0\}, \ (0 \le n \le 4)$

$y[n] = x[n] \ 5 \ h[n] = \{11,13,15,12,9\}, \quad (0 \le n \le 4)$

(e) $N = Len\,(x[n]) + Len\,(h[n]) - 1 = 7$

3. Let $x[n], 0 \le n \le N-1$, be a length-N real sequence with an N-point DFT $X[k]$, $0 \le k \le N-1$. Show that $X[\langle N-k\rangle_N] = X^*[k]$.

when $0 < k \le N-1$

$$X[\langle N-k\rangle_N] = X[N-k] = \sum_{n=0}^{N-1} x[n]W_N^{(N-k)n} = \sum_{n=0}^{N-1} x[n]W_N^{Nn}W_N^{-kn}$$

$$= \sum_{n=0}^{N-1} x[n]W_N^{-kn} = X^*[k]$$

When $k = 0$

$$X[\langle N-k\rangle_N] = X[0] = \sum_{n=0}^{N-1} x[n]W_N^0 = X[0] = X^*[0]$$

Therefore, for all $0 \le k \le N-1$

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$$X[\langle N-k\rangle_N] = X^*[k]$$

4. A causal LTI system has impulse response $h[n]$, for which the z-transform is

$$H(z)=\frac{1-z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

(a) Plot the poles and zeros of $H(z)$.
(b) Indicate the ROC of $H(z)$.
(c) Is the system stable? Explain.
(d) Find the z-transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n]=-\frac{1}{3}\left(-\frac{1}{4}\right)^{n}\mu[n]+\frac{4}{3}(2)^{n}\mu[-n-1].$$

(e) Find the impulse response $h[n]$ of the system. (25 marks)

(a) poles: $\frac{1}{3},-\frac{1}{4}$

zeros: 1

(b) ROC: $\left\{z:|z|>\frac{1}{3}\right\}\cap\left\{z:|z|>\frac{1}{4}\right\}=\left\{z:|z|>\frac{1}{3}\right\}$

(c) The system is stable because no pole is outside the unit circle.

(d) $Y(z)=-\frac{1}{3}\cdot\frac{1}{1+\frac{1}{4}z^{-1}}-\frac{4}{3}\cdot\frac{1}{1-2z^{-1}}=\frac{-\frac{5}{3}+\frac{1}{3}z^{-1}}{\left(1+\frac{1}{4}z^{-1}\right)(1-2z^{-1})}$

$$X(z)=\frac{Y(z)}{H(z)}=\frac{\left(\frac{-\frac{5}{3}+\frac{1}{3}z^{-1}}{\left(1+\frac{1}{4}z^{-1}\right)(1-2z^{-1})}\right)}{\frac{1-z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}}=\frac{\left(-\frac{5}{3}+\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}{(1-2z^{-1})(1-z^{-1})}$$

$$=\frac{-\frac{1}{9}z^{-2}+\frac{8}{9}z^{-1}-\frac{5}{3}}{(1-2z^{-1})(1-z^{-1})}$$

(e) $H(z)=\frac{a}{1-\frac{1}{3}z^{-1}}+\frac{b}{1+\frac{1}{4}z^{-1}}=\frac{-\frac{8}{7}}{1-\frac{1}{3}z^{-1}}+\frac{\frac{15}{7}}{1+\frac{1}{4}z^{-1}}$

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So, for a causal system, $h[n]=-\frac{8}{7}\left(\frac{1}{3}\right)^{n}\mu[n]+\frac{15}{7}\left(-\frac{1}{4}\right)^{n}\mu[n]$

5. $X(e^{j\omega})$ is the DTFT of the discrete-time signal

$$x[n] = \left(\frac{1}{2}\right)^n \mu[n]$$

Find a length-4 sequence $g[n]$ whose four-point DFT $G[k]$ is identical to samples of the DTFT of $x[n]$ at $\omega_k = 2\pi k/4$, for $k = 0, 1, .., 3$, i.e.,

$$g[n] = 0 \ \text{ for } n < 0, \text{and } n > 3$$

and

$$G[k] = X(e^{j2\pi k/4}) \ \text{ for } k = 0, 1, .., 3.$$

(25 marks)

$$x[n] \xrightarrow{\text{DTFT}} X(e^{j\omega}) \xrightarrow[\text{4 points}]{\text{Sample}} G[k] \xrightarrow[\text{IDFT}]{\text{4-point}} g[n]$$

(5 marks could be given to such expression if the final answer is not correct. No marks are deducted if the final answer is correct and this part is missing)

$$g[n] = \sum_{m=-\infty}^{\infty} x[n+4m], \ \ 0 \le n \le 3$$

$$= \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\} + \frac{1}{16}\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\} + \frac{1}{16^2}\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\} + \frac{1}{16^3}\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\} + \cdots$$

$$= \frac{1 - \left(\frac{1}{16}\right)^{\infty}}{1 - \frac{1}{16}}\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\} = \frac{16}{15}\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\} = \left\{\frac{16}{15}, \frac{8}{15}, \frac{4}{15}, \frac{2}{15}\right\}$$