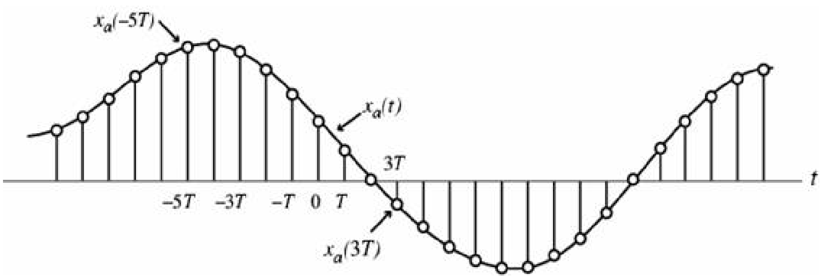
Lecture 2
Time Domain Representation of Discrete Time Signals

Discrete Time Signal

- Samples of a Continuous Time (CT) Signal

$$x[n] = x_a(nT),\ n = \ldots, -1, 0, 1, 2, \ldots$$



- The spacing T between two consecutive samples is called the sampling interval or sampling period

倒数

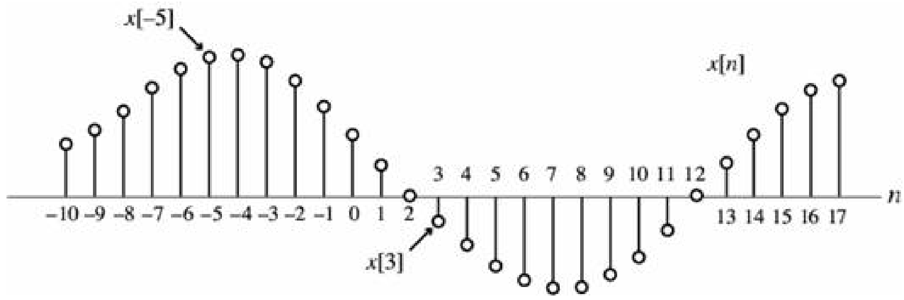
- Reciprocal of sampling interval T, denoted as F_T , is called the sampling frequency:

$$F_T = 1/T$$

Discrete Time Signal

本质离散的 → 人为产生的

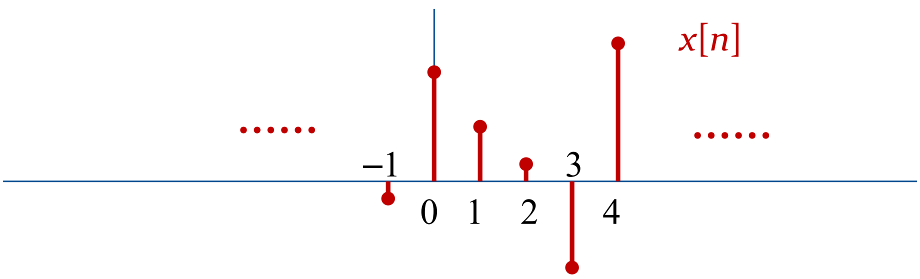
- Or, inherently discrete



- Examples?

Discrete Time Signal

- Signals represented as sequences of numbers, called **samples.**
- Sample value of a typical signal or sequence denoted as $x[n]$ with n being an integer in the range $-\infty \le n \le \infty$.
- $x[n]$ is called the n^{th} sample of the sequence.

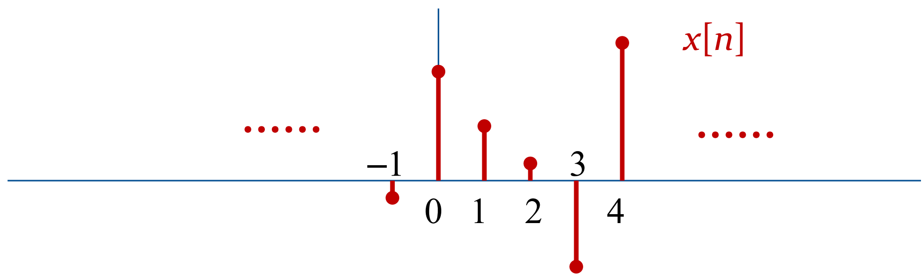


Discrete Time Signal

- $\{x[n]\}$ defined only for integer values of n and undefined for non-integer values of n.
- Discrete-time signal may also be written as a sequence of numbers inside braces:

$$\{x[n]\} = \{\ldots, -0.2, 2.2, 1.1, 0.2, -1.9, 2.9, \ldots\}$$

placed under the sample at time index $n = 0$



Real and Complex Sequences

- $\{x[n]\}$ is a real sequence, if $x[n]$ is real for all values of n, otherwise, $\{x[n]\}$ is a complex sequence

- A complex sequence $\{x[n]\}$ can be written as

$$\{x[n]\} = \{x_{re}[n]\} + j\,\{x_{im}[n]\}$$

- Its complex conjugate is

$$\{x^*[n]\} = \{x_{re}[n]\} - j\,\{x_{im}[n]\}$$

Example:

- $\{x[n]\} = \{\cos 0.25n\}$ is a real sequence, while $\{y[n]\} = \{e^{j0.3n}\}$ is a complex sequence

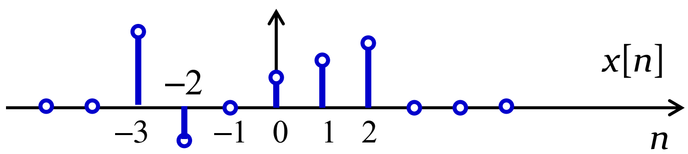
- We can write

$$\{y[n]\} = \{\cos 0.3n + j\sin 0.3n\} = \{\cos 0.3n\} + j\{\sin 0.3n\}$$

where $\{y_{re}[n]\} = \{\cos 0.3n\}$ and $y_{im}[n] = \{\sin 0.3n\}$

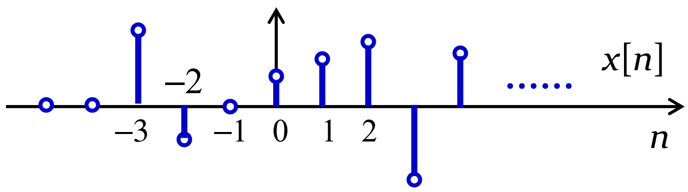
Length of a Discrete-Time Signal

- **Finite Length** (also called finite duration or finite extent)
  - Defined only for a finite time interval: $N_1 \le n \le N_2$, where $-\infty < N_1$ and $N_2 < \infty$ with $N_1 \le N_2$



  - Length of the above finite-length sequence is $N = N_2 - N_1 + 1$
  - Example: $x[n]=n^2$, $-3 \le n \le 4$ is a finite-length sequence of length $4 - (-3) + 1 = 8$

- **Infinite Length**
  - A right-sided sequence $\{x[n]\}$ has zero-valued samples for $n < N_1$



后续会解释 因果序列?

    - If $N_1 \geq 0$, a right-sided sequence is usually called a causal sequence.
  - A left-sided sequence $\{x[n]\}$ has zero-valued samples for $n > N_2$.
    - If $N_2 \leq 0$, a left-sided sequence is usually called an anti-causal sequence.

反因果序列

  - A general two-sided sequence is defined for all values of n in the range $-\infty < n < \infty$
    - Example: $\{y[n]\} = \{\cos 0.4n\}$ is a general infinite-length sequence

Operations on Sequences

- A **discrete-time system** operates on one (or more) sequence, called the **input sequence**, according some prescribed rules and develops another one (or more) sequence, called the **output sequence**, with more desirable properties

Operations on Sequences

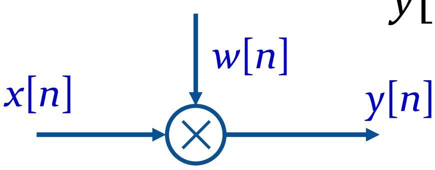
- For example, the input may be a signal corrupted with additive noise
- Discrete-time system is designed to generate an output by removing the noise component from the input
- In most cases, the operation defining a particular discrete-time system is composed of some **elementary operations**

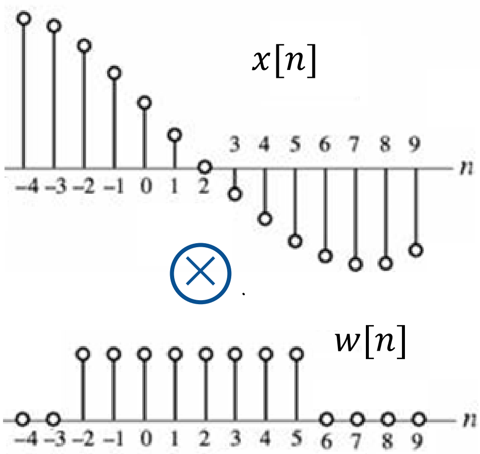
Elementary Operations

- **Product (modulation)** operation:

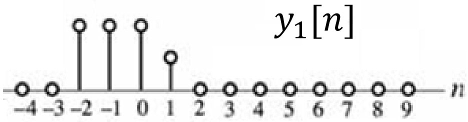
Modulator

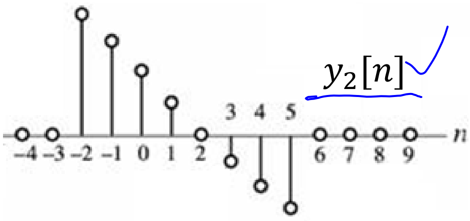
$$y[n] = x[n] \cdot w[n]$$





- An application is in forming a finite-length sequence from an infinite-length sequence by multiplying the latter with a finite-length sequence called a **window sequence**
- Process called **windowing**



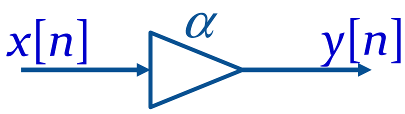


Which one ??

Elementary Operations

- **Multiplication operation:**

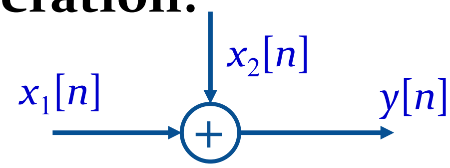
  Multiplier



$$y[n] = \alpha x[n]$$

- **Addition operation:**

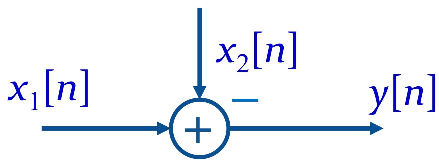
  Adder



$$y[n] = x_1[n] + x_2[n]$$

- **Subtraction operation:**

  Subtractor



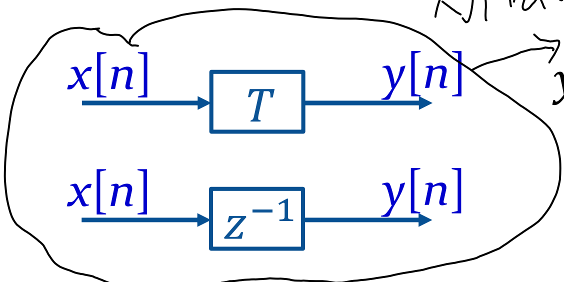
$$y[n] = x_1[n] - x_2[n]$$

Elementary Operations

- **Time-shifting Operation:** $y[n] = x[n - n_0]$, where n_0 is an integer
- If $n_0 > 0$, it is delaying operation

  Unit Delay

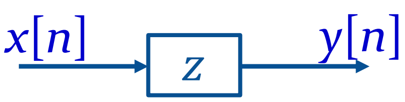
为什么可以这样表示(后续会解释)



$$y[n] = x[n - 1]$$

- If $n_0 < 0$, it is an advance operation

  Unit Advance

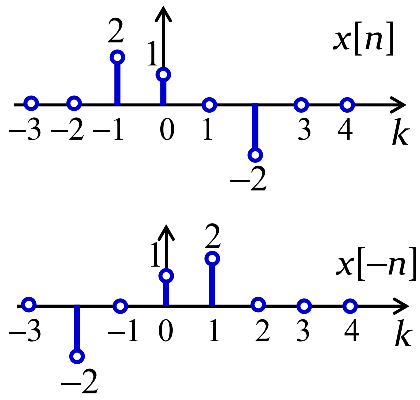


$$y[n] = x[n + 1]$$

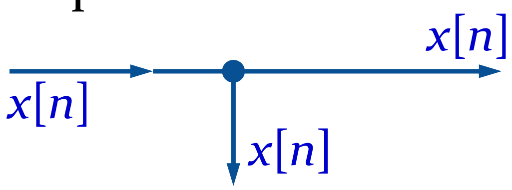
Elementary Operations

- **Time-reversal (folding)** **operation**:

$$y[n] = x[-n]$$



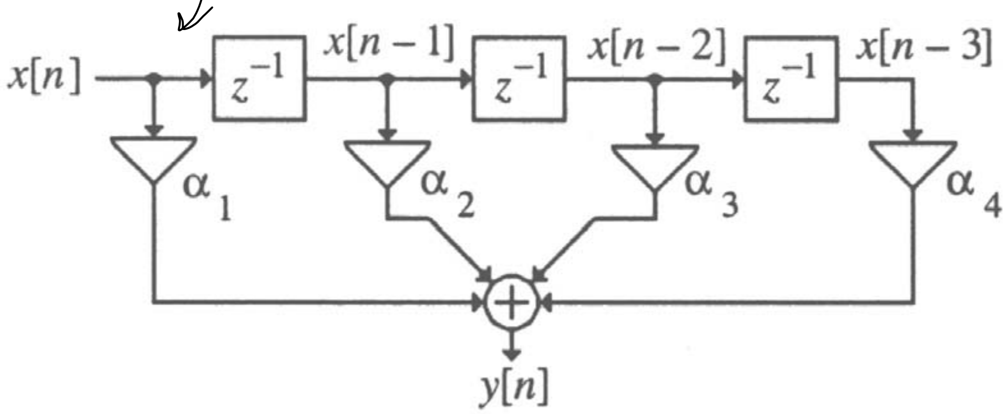
- **Branching operation:** Used to provide multiple copies of a sequence



延迟可用寄存器实现

Combinations of Basic Operations

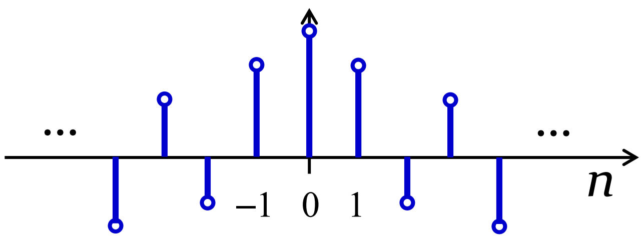
- Example: 数字滤波器 4种 branching



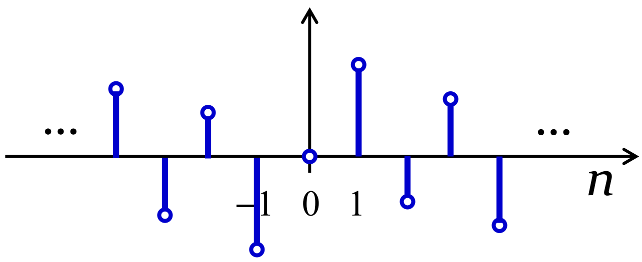
$$y[n] = \alpha_1 x[n] + \alpha_2 x[n-1] + \alpha_3 x[n-2] + \alpha_4 x[n-3]$$

Classification of Sequences

- Based on Symmetry
  - Conjugate-symmetric: $x[n] = x^*[-n]$ → 实部关于y轴对称，虚部关于原点对称
    - Even sequence: a real conjugate-symmetric sequence
  - Conjugate-antisymmetric: $x[n] = -x^*[-n]$ → 实部关于原点对称，虚部关于y轴对称
    - Odd sequence: a real conjugate-antisymmetric sequence



Even sequence



Odd sequence

how to prove it.

- Any complex sequence $x[n]$ can be expressed as

$$x[n] = x_{cs}[n] + x_{ca}[n],$$

where

细节理解.

$$\begin{cases} x_{cs}[n] = \frac{1}{2}(x[n] + x^*[-n]), \\ x_{ca}[n] = \frac{1}{2}(x[n] - x^*[-n]), \end{cases}$$

Conjugate Symmetric part

Conjugate anti-symmetric part

- Any real sequence $x[n]$ can be expressed as

$$x[n] = x_{ev}[n] + x_{od}[n],$$

where

$$x_{ev}[n] = \frac{1}{2}(x[n] + x[-n]),$$ Even part

$$x_{od}[n] = \frac{1}{2}(x[n] - x[-n]),$$ Odd part

Example 1: Generation of Symmetric Parts of a Complex Sequence

- Sequence: $\{g[n]\} = \{0, 1+j4, -2+j3, \underset{\uparrow}{4-j2}, -5-j6, -j2, 3\}$
- A: We form

$$\{g^*[n]\} = \{0, 1-j4, -2-j3, \underset{\uparrow}{4+j2}, -5+j6, j2, 3\}, \text{ and}$$

$$\{g^*[-n]\} = \{3, j2, -5+j6, \underset{\uparrow}{4+j2}, -2-j3, 1-j4, 0\},$$

- Thus:

$$g_{cs}[n] = \frac{1}{2}(g[n] + g^*[-n])$$

$$= \{1.5, 0.5 + j3, -3.5 + j4.5, \underset{\uparrow}{4}, -3.5 - j4.5, 0.5 - j3, 1.5\}$$

$$g_{ca}[n] = \frac{1}{2}(g[n] - g^*[-n])$$

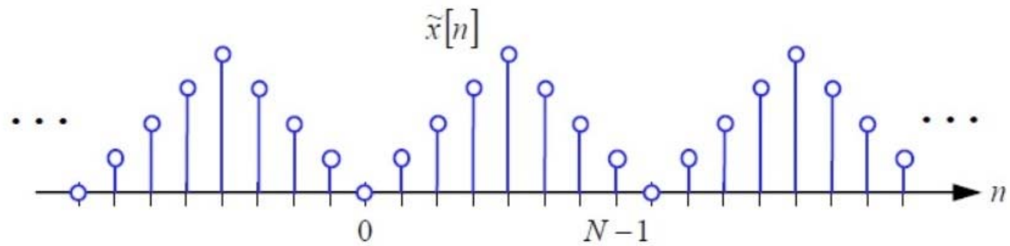
$$= \{-1.5, 0.5 + j, 1.5 - j1.5, \underset{\uparrow}{-2j}, -1.5 - j1.5, -0.5 + j, 1.5\}$$

Classification of Sequences

- Base on Periodicity
  - A sequence $\tilde{x}[n]$ satisfying → 无限长度序列

$$\tilde{x}[n] = \tilde{x}[n + kN] \quad \text{for all } n$$

is called a periodic sequence with a period N.



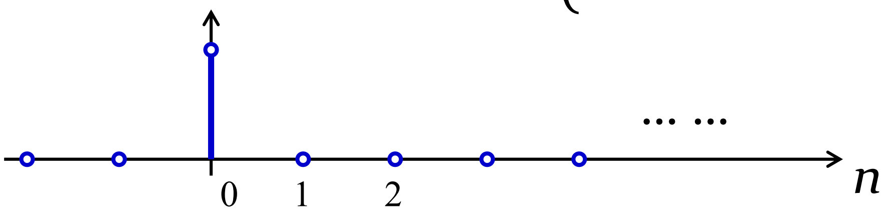
  - The fundamental period N_f of a periodic signal is the smallest value of N for which the above equation holds.

Basic Sequences

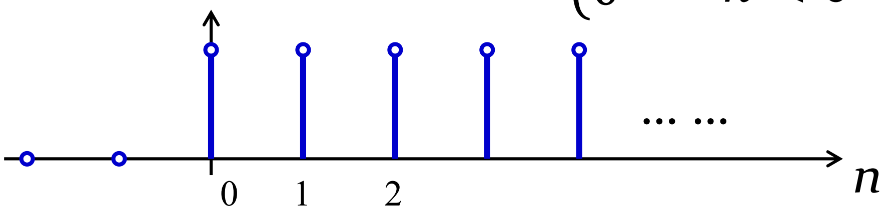
$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

- Unit impulse $\qquad \delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$

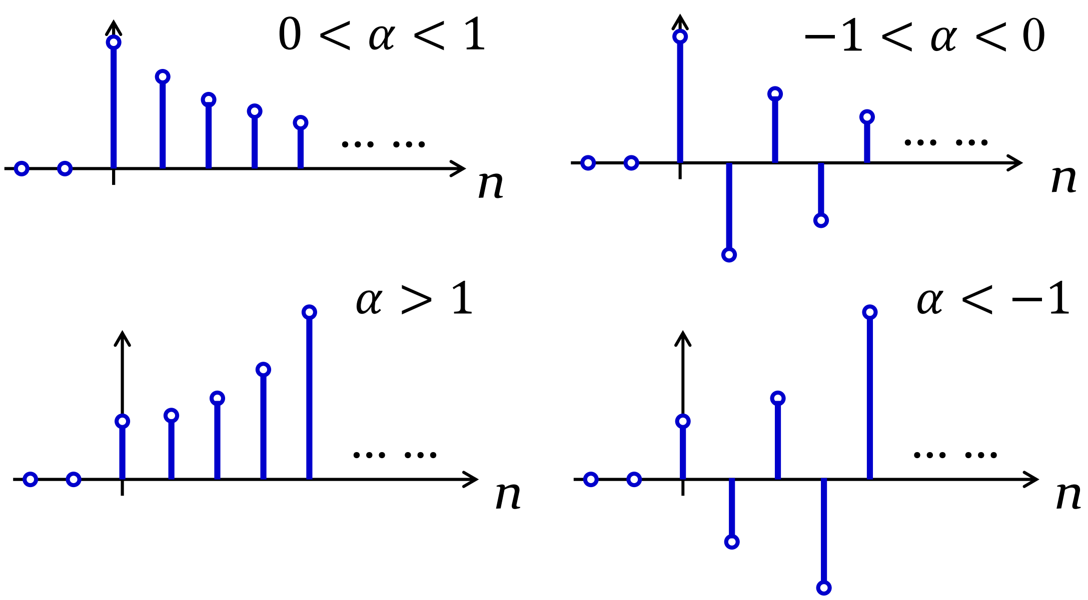


- Unit Step $\qquad \mu[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$



Basic Sequences

- Exponential $\qquad x[n] = \begin{cases} A\alpha^n & n \geq 0 \\ 0 & n < 0 \end{cases}$

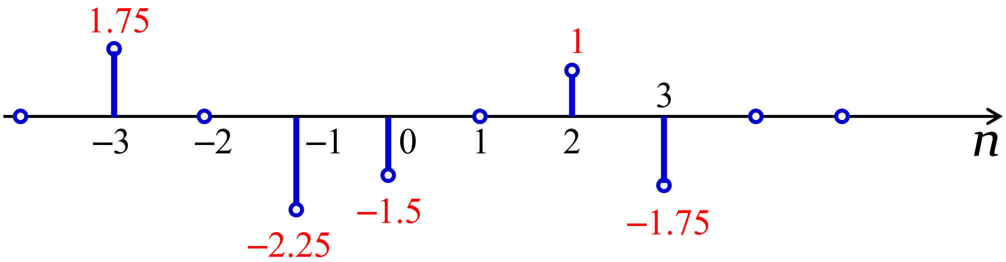


Bounded

Unbounded

Representing an arbitrary sequence

- as a weighted sum of unit impulse and its delayed versions.



$$x[n] = 1.75\delta[n+3] - 2.25\delta[n+1] - 1.5\delta[n] + \delta[n-2] - 1.75\delta[n-3]$$

- A general form

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Discrete Sinusoids

$$x[n] = A\cos(\omega_0 n + \varphi)$$

or $x[n] = Ae^{j\omega_0 n + j\varphi}$

- Q: Period or not? $x[n] = x[n+N]$ for N integer.

Discrete Sinusoids

$$x[n] = A\cos(\omega_0 n + \varphi)$$

or $x[n] = Ae^{j\omega_0 n + j\varphi}$

- Q: Period or not? $x[n] = x[n+N]$ for N integer.
- A: Yes only if ω_0/π is rational (Different from CT!)

- To find fundamental period N
  - Find smallest integers K and N, satisfying:

$$\omega_0 N = 2\pi K$$

Discrete Sinusoids

- Example:

$$\cos\left(\frac{5}{7}\pi n\right) \qquad N = 14 \quad (K = 5)$$

$$\cos\left(\frac{1}{5}\pi n\right) \qquad N = 10 \quad (K = 1)$$

$$\cos\left(\frac{5}{7}\pi n\right) + \cos\left(\frac{1}{5}\pi n\right) \Rightarrow N = \mathrm{SCM}(14, 10) = 70$$

Discrete Sinusoids

- Another difference:
- Q: Which one is a higher frequency signal for $\sin(\omega_0 n)$?

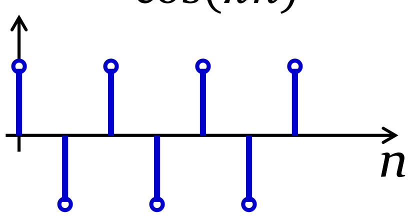
$$\omega_0 = \pi \qquad \text{or} \qquad \omega_0 = \frac{3}{2}\pi$$

Discrete Sinusoids

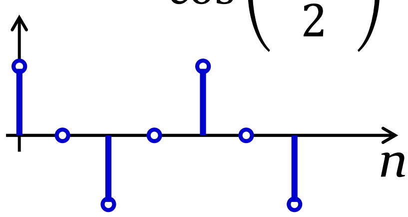
- Another difference:
- Q: Which one is a higher frequency signal?

$$\omega_0 = \pi \qquad \text{or} \qquad \omega_0 = \frac{3}{2}\pi$$

$\cos(\pi n)$



$\cos\left(\frac{3\pi n}{2}\right) = \cos\left(\frac{\pi n}{2}\right)$



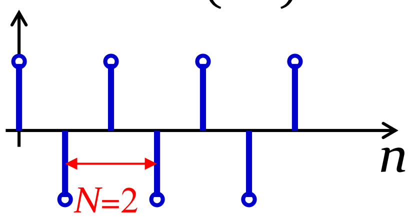
Discrete Sinusoids

- Another difference:
- Q: Which one is a higher frequency signal?

$$\omega_0 = \pi \qquad \text{or} \qquad \omega_0 = \frac{3}{2}\pi$$

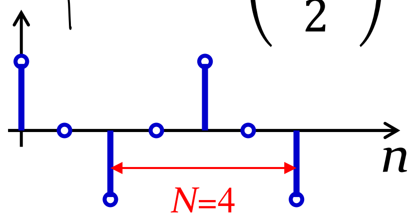
- A: $\omega_0 = \pi$

$\cos(\pi n)$



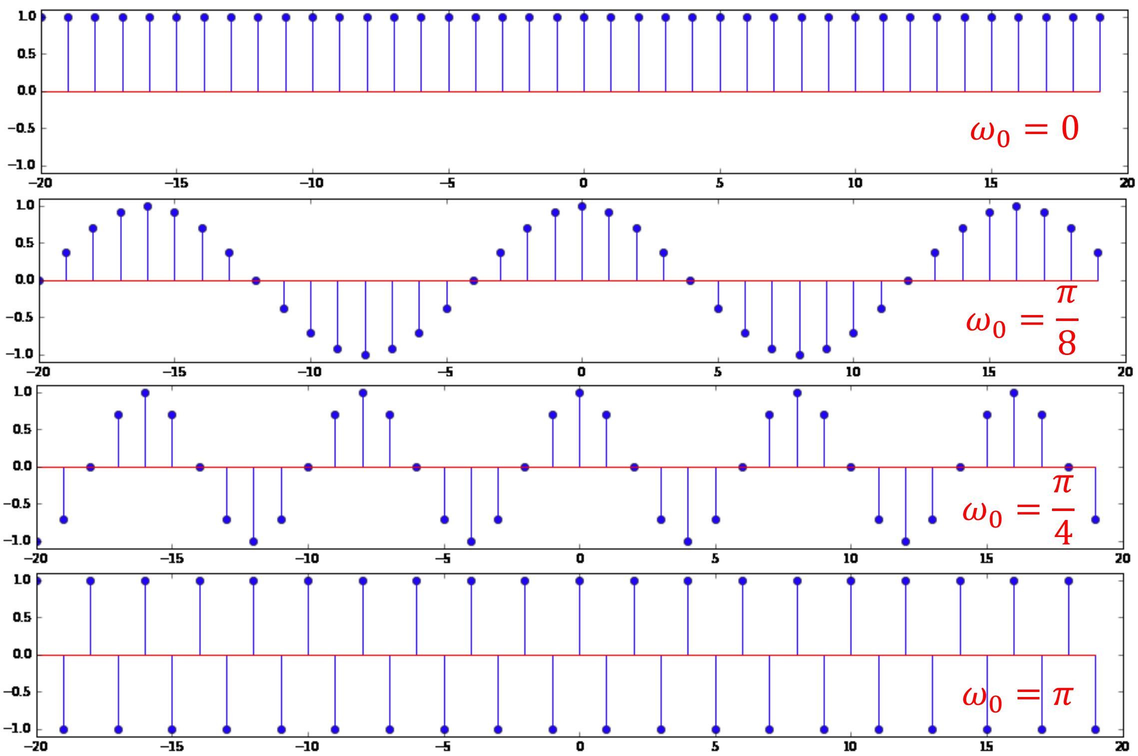
周期更长，频率更低

$$\cos\left(\frac{3\pi n}{2}\right) = \cos\left(\frac{\pi n}{2}\right)$$



Discrete Sinusoids

$\cos(\omega_0 n)$

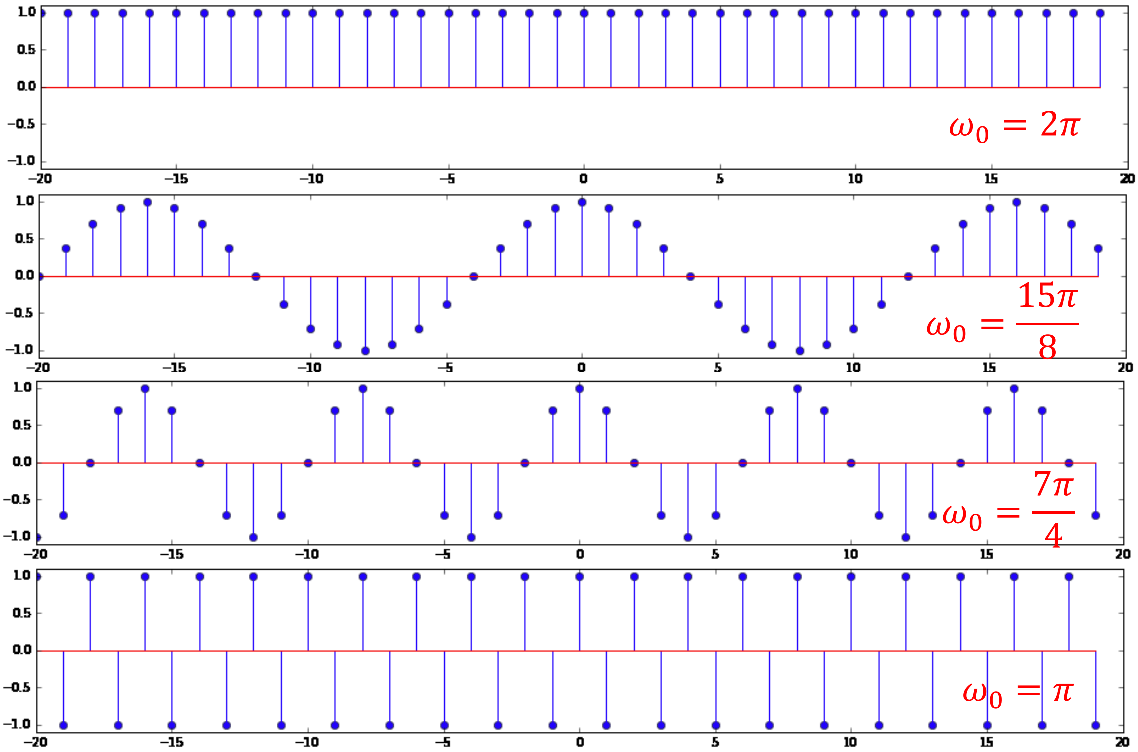


conclusion: $T=\frac{2\pi}{\omega_0}$, $f=\frac{1}{T}=\frac{\omega_0}{2\pi}$, $\omega_0=(2k+1)\pi$, 当频率最高

对ω_0也是一个周期函数

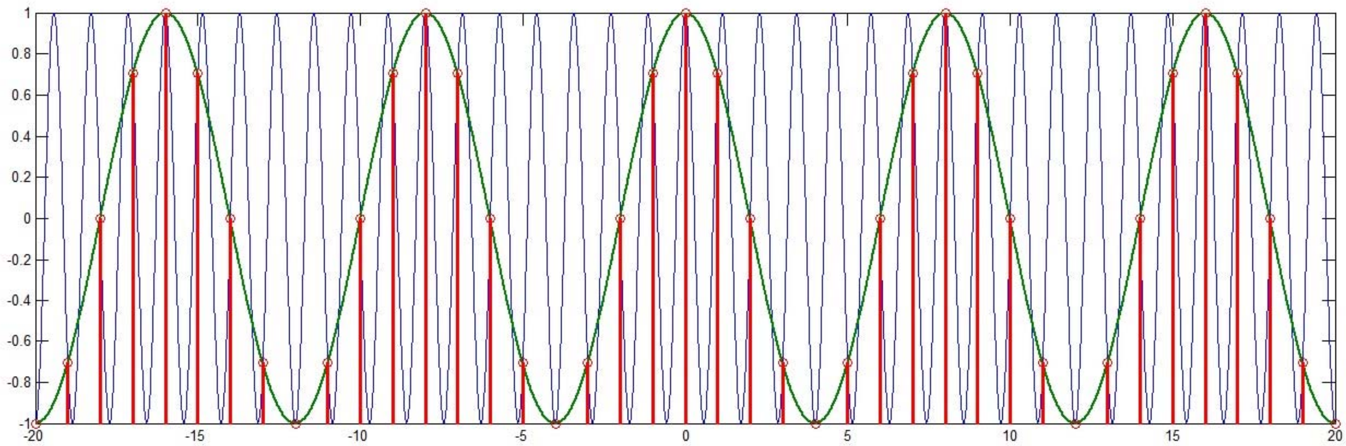
Discrete Sinusoids

$\cos(\omega_0 n)$



$\cos\left(\frac{7}{4}\pi t\right)$ $\cos\left(\frac{7\pi}{4}n\right)$ and $\cos\left(\frac{\pi}{4}n\right)$

$\cos\left(\frac{1}{4}\pi t\right)$



单选题 1分 设置

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Is $x[n] = A\cos(\omega_0 n + \varphi)$ periodic or not?

A Yes

B No

C It depends on ω_0

D It depends on φ

提交

单选题 1分 设置

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Determine the fundamental frequency of $\cos\left(\frac{5}{7}\pi n\right) + \cos\left(\frac{1}{5}\pi n\right)$

A 14

B 10

C 2

D 70

提交

单选题 0.5分

设置

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For $\cos(\omega_0 n)$, which one is a higher frequency signal?

A $\omega_0 = \pi$

B $\omega_0 = \frac{3}{2}\pi$

提交