Date.

11911214 杨鸿嘉

Tutorial 8

(1) (a). $Y[k]=\sum_{n=0}^{NL-1} y[n] W_{NL}^{nk} = \sum_{n=0}^{N-1} x[n] W_{NL}^{nLk} = \sum_{n=0}^{N-1} x[n] W_N^{nk}$.

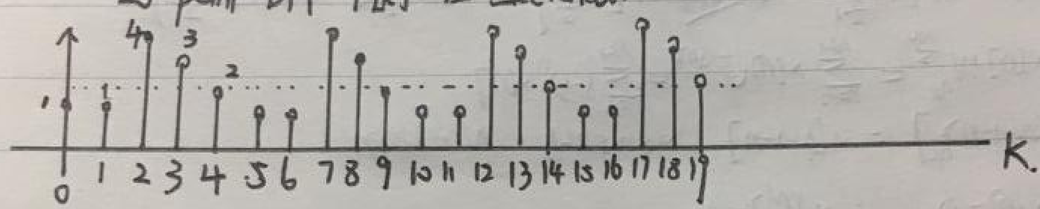
if $0\le k\le N-1$. $Y[k]=X[k]$

if $N\le k\le LN-1$. let $k=lN+m$. $1\le l\le N-1$.

$m=\langle k\rangle_N$. $Y[k]=Y[lN+m]=\sum_{n=0}^{N-1} x[n]W_N^{n(lN+m)}=\sum_{n=0}^{N-1}x[n]W_N^{nm}=X[m]=X[\langle k\rangle_N]$

(b). $Y[k]=X[\langle k\rangle_5]$ $0\le k\le 19$ $Y[k]$ will repeat $X[k]$ 4 times

$\therefore$ 20-point DFT $Y[k]$ is sketched:



2.(a) $\because \{y[n]\}=\{x[\langle n+12\rangle_9]\}$ $\therefore y[-3]=x[\langle -3+12\rangle_9]=x[0]=-4$

(b). $\because \{z[n]\}=\{x[\langle n-15\rangle_9]\}$ $\therefore z[2]=x[\langle 2-15\rangle_9]=x[4]=-5$

3.

$\{h[\langle -n\rangle_4]\}=\{1,-2,2,3\}$

$\therefore y[0]=-3\cdot 1+2\cdot(-2)+(-1)\cdot 2+4\cdot 3=3$

$y[1]=(-3)\cdot 3+2\cdot 1+(-1)(-2)+4\cdot 2=3$

$y[2]=(-3)\cdot 2+2\cdot 3+(-1)\cdot 1+4\cdot(-2)=-9$

$y[3]=(-3)\cdot(-2)+2\cdot 2+(-1)\cdot 3+4\cdot 1=11$ $\therefore \{y[n]\}=\{3,3,-9,11\}$

4. (a). $y_L[n] = \sum_{k=0}^{5} g[k]h[n-k] \quad 0 \le n \le 10 \qquad y_C[n] = \sum_{k=0}^{5} g[k]h[\langle n-k \rangle_6]$

$y_C[0] = g[0]h[0] + g[1]h[5] + g[2]h[4] + g[3]h[3] + g[4]h[2] + g[5]h[1] = y_L[0] + y_L[6]$

$y_C[1] = g[0]h[1] + g[1]h[0] + g[2]h[5] + g[3]h[4] + g[4]h[3] + g[5]h[2] = y_L[1] + y_L[7]$

$y_C[2] = y_L[2] + y_L[8]$

$y_C[3] = y_L[3] + y_L[9]$

$y_C[4] = y_L[4] + y_L[10]$

$y_C[5] = y_L[5]$

(b). $y_L[0] = 3\cdot(-2) = -6 \qquad y_L[1] = 3\cdot4 + (-5)\cdot(-2) = 22.$

$y_L[2] = g[0]h[2] + g[1]h[1] + g[2]h[0] = 3\cdot7 + (-5)\cdot4 + 2\cdot(-2) = -3$

$y_L[3] = 3\cdot(-5) + (-5)\cdot7 + 2\cdot4 + 6\cdot(-2) = -54$

$y_L[4] = 3\cdot4 + (-5)\cdot(-5) + 2\cdot7 + 6\cdot4 + (-1)\cdot(-2) = 77$

$y_L[5] = 3\cdot3 + (-5)\cdot4 + 2\cdot(-5) + 6\cdot7 + (-1)\cdot4 + 4\cdot(-2) = 9$

$y_L[6] = -5\cdot3 + 2\cdot4 + 6\cdot(-5) + (-1)\cdot7 + 4\cdot4 = -28$

$y_L[7] = 2\cdot3 + 6\cdot4 + (-1)\cdot(-5) + 4\cdot7 = 63$

$y_L[8] = 6\cdot3 + (-1)\cdot4 + 4\cdot(-5) = -6$

$y_L[9] = (-1)\cdot3 + 4\cdot4 = 13 \qquad y_L[10] = 12$

$\therefore y_C[0] = -6 - 28 = -34 \qquad y_C[1] = 22 + 63 = 85$

$y_C[2] = -3 - 6 = -9 \qquad y_C[3] = -54 + 13 = -41$

$y_C[4] = 77 + 12 = 89 \qquad y_C[5] = 9$

5. $g[n] = \frac{1}{2}(x[2n] + x[2n+1])$ $\quad h[n] = \frac{1}{2}(x[2n] - x[2n+1])$ $\quad 0 \le n \le \frac{N}{2}-1$.

we can solve for $x[2n] = g[n] + h[n]$ $\quad x[2n+1] = g[n] - h[n]$

$$\therefore X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} = \sum_{r=0}^{\frac{N}{2}-1} \left(x[2r] W_N^{2rk} + x[2r+1] W_N^{(2r+1)k}\right) = \sum_{r=0}^{\frac{N}{2}-1}\left(x[2r] W_{\frac{N}{2}}^{rk} + x[2r+1] W_N^{k} W_{\frac{N}{2}}^{rk}\right)$$

$$= \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_{\frac{N}{2}}^{rk} + W_N^k \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_{\frac{N}{2}}^{rk} = \sum_{r=0}^{\frac{N}{2}-1} [g[r] + h[r]] W_{\frac{N}{2}}^{rk} + W_N^k \sum_{r=0}^{\frac{N}{2}-1} (g[r] - h[r]) W_{\frac{N}{2}}^{rk}$$

$$= (1 + W_N^k) \sum_{r=0}^{\frac{N}{2}-1} g[r] W_{\frac{N}{2}}^{rk} + (1 - W_N^k) \sum_{r=0}^{\frac{N}{2}-1} h[r] W_{\frac{N}{2}}^{rk}$$

$$= (1 + W_N^k)\, G[\langle k \rangle_{\frac{N}{2}}] + \overset{(1-W_N^k)}{\cancel{(1+W_N^k)}}\, H[\langle k \rangle_{\frac{N}{2}}] \quad \cancel{0 \le k < \tfrac{N}{2}} \quad 0 \le k \le \frac{N}{2}-1$$

6. (a) $X[\frac{N}{2}] = \sum_{n=0}^{N-1} x[n] W_N^{\frac{nN}{2}} = \sum_{n=0}^{N-1} x[n] e^{-j\pi n} = \sum_{n=0}^{N-1} (-1)^n x[n]$

$\because x[n] = x[\langle N-1-n \rangle_N] = x[N-1-n] \quad 0 \le n \le N-1.$ N is even

$\therefore \sum_{n=0}^{N-1} (-1)^n x[n] = 0 \quad \therefore X[\frac{N}{2}] = 0$

(b) $X[0] = \sum_{n=0}^{N-1} x[n] W_N^0 = \sum_{n=0}^{N-1} x[n]$ $\quad \because x[n] = -x[\langle N-1-n \rangle_N] = -x[N-1-n] \quad 0 \le n \le N-1$

$\therefore X[0] = 0$

(c). $N = 2M$. $\quad x[n] = -x[\langle n+M \rangle_N]$

$$X[2l] = \sum_{n=0}^{N-1} x[n] W_N^{n2l} = \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{n2l} + \sum_{n=\frac{N}{2}}^{N-1} x[n] W_N^{n2l} = \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{n2l} + \sum_{n=0}^{\frac{N}{2}-1} x[n+\tfrac{N}{2}] W_N^{(n+\frac{N}{2})2l}.$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{n2l} + \sum_{n=0}^{\frac{N}{2}-1} x[n+\tfrac{N}{2}] W_N^{n2l} = \sum_{n=0}^{M-1} (x[n] + x[n+M]) W_{2M}^{n2l}$$

$\therefore X[2l] = 0$ for $l = 0, 1, \dots, M-1$

7. $w[n] = x[n] \circledast y[n]$. $\quad \therefore y[n]$ has length $: 4 - 3 + 1 = 2$.

suppose $y[0] = y_1 \quad y[1] = y_2$

$w[0] = \sum_{m=0}^{3} x[m]\, h[\langle 0-m \rangle_N] = 2y_1 = -4 \quad \therefore y_1 = -2$

$w[1] = \sum_{m=0}^{3} x[m]\, h[\langle 1-m \rangle_N] = 2y_2 + y_1 = 0 \quad \therefore y_2 = 1$

$\therefore y[n] = \{-2, 1\} \quad 0 \le n \le 1$