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Tutorial 3

1、 $x[n] = \{2, 0, -1, \underset{\uparrow}{6}, -3, 2, 0\}$ $y[n] = \{8, 2, -7, -3, 0, \underset{\uparrow}{1}, 1\}$ $w[n] = \{3, 6, \underset{\uparrow}{-1}, 2, 6, 6, 1\}$

(a) $u[n] = x[n] \circledast y[n]$ $-8 \le n \le 4$

$u[-8] = x[-3]\,y[-5] = 16.$ $u[-7] = x[-3]\,y[-4] + x[-2]\,y[-5] = 4+0 = 4.$

$u[-6] = x[-3]\,y[-3] + x[-2]\,y[-4] + x[-1]\,y[-5] = -14+0+(-8) = -22.$

$u[-5] = x[-3]\,y[-2] + x[-2]\,y[-3] + x[-1]\,y[-4] + x[0]\,y[-5] = -6+0-2+48 = \cancel{38}.40$

$u[-4] = x[-3]\,y[-1] + x[-2]\,y[-2] + x[-1]\,y[-3] + x[0]\,y[-4] + x[1]\,y[-5] = 0+0+7+12-\overset{24}{\cancel{15}} = -5$

$u[-3] = x[-3]\,y[0] + x[-2]\,y[-1] + x[-1]\,y[-2] + x[0]\,y[-3] + x[1]\,y[-4] + x[2]\,y[-5] = 2+0+3-42-6+16 = -27.$

$u[-2] = x[-3]\,y[1] + x[-2]\,y[0] + x[-1]\,y[-1] + x[0]\,y[-2] + x[1]\,y[-3] + x[2]\,y[-4] + x[3]\,y[-5] = 9$

$u[-1] = \cancel{x[-3]\,y[2]} + x[-2]\,y[1] + x[-1]\,y[0] + x[0]\,y[-1] + x[1]\,y[-2] + x[2]\,y[-3] + x[3]\,y[-4] = -6.$

$u[0] = x[-1]\,y[1] + x[0]\,y[0] + x[1]\,y[-1] + x[2]\,y[-2] + x[3]\,y[-3] = -1+6-6 = -1$

$u[1] = x[0]\,y[1] + x[1]\,y[0] + x[2]\,y[-1] + x[3]\,y[-2] = 6+\cancel{8}-3+0 = 3.$

$u[2] = x[1]\,y[1] + x[2]\,y[0] + x[3]\,y[-1] = -3+2+0 = -1.$ $u[3] = x[2]\,y[1] + x[3]\,y[0] = 2$ $u[4] = x[3]\,y[1] = 0.$

$\therefore u[n] = \{16, 4, -22, 40, -5, -27, 9, -6, \underset{\uparrow}{-1}, 3, -1, 2, 0\}$ $-8 \le n \le 4.$

(b). $v[n] = x[n] \circledast w[n]$ $-5 \le n \le 7$

n	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
x[n]			2	0	-1	6	-3	2	0				
w[n]				3	6	-1	2	6	6	1			
	6	12	-2	4	12	12	2.						
		0	0	0	0	0	0	0					
			-3	-6	1	-2	-6	-6	-1				
				18	36	-6	12	36	36	6			
					-9	-18	3	-6	-18	-18	-3		
						6	12	-2	4	12	12	2	
							0	0	0	0	0	0	0
v[n]	6	12	-5	16	40	-8	23	22	21	0	9	2	0

$\therefore v[n] = \{6, 12, -5, 16, 40, \underset{\uparrow}{-8}, 23, 22, 21, 0, 9, 2, 0\}$ $-5 \le n \le 7$

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$g[n] = w[n] \circledast y[n] \quad -7 \leq n \leq 5$

(c).

n	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
$w[n]$						3	6	-1	2	6	6	1	
$y[n]$			8	2	-7	-3	0	1	1				
	24	6	-21	-9	0	3	3						
		48	12	-42	-18	0	6	6					
			-8	-2	7	3	0	-1	-1				
				16	4	-14	-6	0	2	2			
					48	12	-42	-18	0	6	6		
						48	12	-42	-18	0	6	6	
							8	2	-7	-3	0	1	1
$g[n]$	24	54	-17	-37	41	~~47~~ 52.	-19	-53	-24	5	12	7	1

$\therefore g[n] = \{24, 54, -17, -37, 41, 52, -19, \underset{\uparrow}{-53}, -24, 5, 12, 7, 1\} \quad -7 \leq n \leq 5.$

2. $v[n] = \sum_{k=-\infty}^{\infty} x_1[k-N_1]\, x_2[n-k-N_2]$ let $k - N_1 = m$

$= \sum_{m=-\infty}^{\infty} x_1[m]\, x_2[n-(m+N_1)-N_2] = \sum_{m=-\infty}^{\infty} x_1[m]\, x_2[n-N_1-N_2-m]$

$= y[n-N_1-N_2] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-N_1-N_2-k] \quad \therefore v[n] = y[n-N_1-N_2]$

3. (a) length : $(M+N+1)2-1 = 2M+2N+1$

range : $[-2M, 2N]$

(b) length : ~~$(K+N+1)2-1 = 2K+2N+1$~~ $(N-K+1)2-1 = 2N-2K+1$

range : $[2K, 2N]$

(c) length : $(M+N+1+\ N-K+1)-1 = M+2N-K+1$

range : $[K-M, 2N]$

(d) length : $(M+N+1+L-R+1)-1 = M+N+L-R+1$

range : $[-M-L, N-R]$

4. $\alpha^n \mu[n] \circledast \mu[n] = \sum_{k=-\infty}^{\infty} \alpha^k \mu[k] \cdot \mu[n-k] \quad k \geq 0,\ n-k \geq 0 \quad k \leq n$

$= \sum_{k=0}^{n} \alpha^k = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right)\mu[n]$

5. $h[n] = h_{cs}[n] + h_{ca}[n] \quad g[n] = g_{cs}[n] + g_{ca}[n] \quad h_{cs}[n] = h_{cs}^*[-n] \quad h_{ca}[n] = -h_{ca}^*[-n]$

(a) let $h_{cs}[n] \circledast g_{cs}[n] = \sum_{k=-\infty}^{\infty} h_{cs}[k]\, g_{cs}[n-k] = y[n]$

$$y^*[-n] = \sum_{k=-\infty}^{\infty} h_{cs}^*[k]\, g_{cs}^*[-n-k] = \sum_{k=-\infty}^{\infty} h_{cs}^*[-k]\, g_{cs}^*[-n+k] = \sum_{k=-\infty}^{\infty} h_{cs}^*[-k]\, g_{cs}^*[-(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} h_{cs}[k]\, g_{cs}[n-k] = y[n] \quad \therefore\ h_{cs}[n] \circledast g_{cs}[n] \text{ is conjugate symmetric}$$

(b) let $h_{ca}[n] \circledast g_{cs}[n] = \sum_{k=-\infty}^{\infty} h_{ca}[k]\, g_{cs}[n-k] = y[n]$

$$y^*[-n] = \sum_{k=-\infty}^{\infty} h_{ca}^*[k]\, g_{cs}^*[-n-k] = \sum_{k=-\infty}^{\infty} h_{ca}^*[-k]\, g_{cs}^*[-n+k] = \sum_{k=-\infty}^{\infty} h_{ca}^*[-k]\, g_{cs}^*[-(n-k)]$$

$$= -\sum_{k=-\infty}^{\infty} h_{ca}[k]\, g_{cs}[n-k] = -y[n] \quad \therefore\ h_{ca}[n] \circledast g_{cs}[n] \text{ is conjugate antisymmetric.}$$

(c). let $h_{ca}[n] \circledast g_{ca}[n] = \sum_{k=-\infty}^{\infty} h_{ca}[k]\, g_{ca}[n-k] = y[n]$

$$y^*[-n] = \sum_{k=-\infty}^{\infty} h_{ca}^*[k]\, g_{ca}^*[-n-k] = \sum_{k=-\infty}^{\infty} h_{ca}^*[-k]\, g_{ca}^*[-(n-k)] = \sum_{k=-\infty}^{\infty} h_{ca}[k]\, g_{ca}[n-k] = y[n]$$

$\therefore\ h_{ca}[n] \circledast g_{ca}[n]$ is conjugate symmetric.

6. (a). $x_1[n] = \alpha^n \mu[n-1]$ $\quad \sum_{n=1}^{\infty} |\alpha^n| = \sum_{n=1}^{\infty} |\alpha|^n = \frac{|\alpha|}{1-|\alpha|} < \infty \quad |\alpha|<1$ absolutely summable.

(b) $x_2[n] = n\alpha^n \mu[n-1]$ $\quad \sum_{n=1}^{\infty} |n\alpha^n| = \sum_{n=1}^{\infty} n|\alpha|^n = \frac{|\alpha|}{(1-|\alpha|)^2} < \infty \quad |\alpha|<1.$ absolutely summable.

(c). $x_3[n] = \frac{\mu[n]}{(n+2)(n+3)}$ $\quad \sum_{n=0}^{n} \frac{1}{(n+2)(n+3)} = \sum_{n=0}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+3}\right) = \frac12 - \frac13 + \frac13 - \frac14 \cdots + \frac{1}{n+2} - \frac{1}{n+3} = \frac12 - \frac{1}{n+3}$

$= \frac12 < \infty$

$\therefore$ absolutely summable

7. $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} |\alpha^k \mu[-k-1]|$ $\quad -k-1 \geq 0 \quad k \leq -1$

$$= \sum_{k=-\infty}^{-1} |\alpha^k| = \sum_{k=1}^{\infty} |\alpha^{-k}| = \sum_{k=1}^{\infty} \left|\frac{1}{\alpha}\right|^k = \frac{|\frac{1}{\alpha}|}{1-|\frac{1}{\alpha}|} < \infty \text{ if } \left|\frac{1}{\alpha}\right| < 1 \quad \therefore |\alpha| > 1$$

8. $x[n] = \sum_{k=-\infty}^{\infty} x[k](\mu[n-k] - \mu[n-k-1]) = \sum_{k=-\infty}^{\infty} x[k]\mu[n-k] - \sum_{k=-\infty}^{\infty} x[k]\mu[n-k-1]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]s[n-k] - \sum_{k=-\infty}^{\infty} x[k]s[n-k-1] = x[n] \circledast s[n] - \overset{x[n]}{\cancel{x[n-1]}} \circledast s[n-1]$$

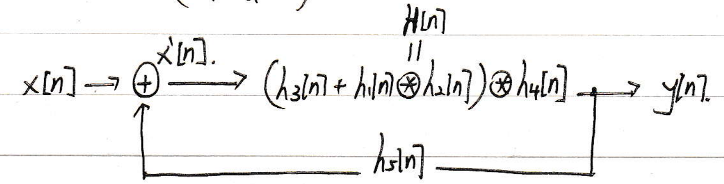
$$\therefore\ y[n] = x[n] \circledast s[n] - x[n] \circledast s[n-1]$$

9. $s[n] = h[n] \circledast \mu[n] = (-\alpha)^n \mu[n] \circledast \mu[n] = \sum_{k=-\infty}^{\infty} (-\alpha)^k \mu[k]\mu[n-k] = \begin{cases} \sum_{k=0}^{n} (-\alpha)^k & n \geq 0. \\ 0 & n<0 \end{cases}$

$$= \begin{cases} \frac{1-(-\alpha)^{n+1}}{1-(-\alpha)} & n \geq 0 \\ 0 & n<0 \end{cases}$$

$$\therefore\ s[n] = \left(\frac{1-(-\alpha)^{n+1}}{1+\alpha}\right)\mu[n]$$

10.



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$x'[n] = x[n] + h_5[n] \circledast y[n]$ $\quad$ $x'[n] \circledast H[n] = y[n]$.

$\therefore \; y[n] = H[n] \circledast \left(x[n] + h_5[n] \circledast y[n] \right)$

$= H[n] \circledast x[n] + H[n] \circledast h_5[n] \circledast y[n]$.

$\therefore \; H[n] \circledast x[n] = \left(\delta[n] - H[n] \circledast h_5[n] \right) \circledast y[n]$.

$\therefore \; y[n] = \text{Inverse}\left(\delta[n] - H[n] \circledast h_5[n] \right) \circledast H[n] \circledast x[n]$.

$\therefore$ impulse response: $\text{Inverse}\left(\delta[n] - H[n] \circledast h_5[n] \right) \circledast H[n]$

where. $H[n] = \left(h_1[n] \circledast h_2[n] + h_3[n] \right) \circledast h_4[n]$