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$x(a+b)$ 先b后a

$e^{j\omega t}$ 均周期；$e^{j\omega n}$ 仅$\frac{\omega_0}{2\pi}$有理才周期

ω不同，信号不同；ω差2π相同

ω越大振荡越快；ω接近π振荡快

$T=\begin{cases}\omega_0=0, 无定义\\ \omega_0\neq0, \frac{2\pi}{\omega_0}\end{cases}$ $N=\begin{cases}\omega_0=0, 无定义\\ \omega_0\neq0, m\frac{2\pi}{\omega_0}\end{cases}$

成谐波关系 specify 指定 频率 frequency response

$\phi_k(t)=e^{jk\omega_0 t}$

$\phi_k[n]=e^{jk\frac{2\pi}{N}n}, N, T$

With memory: 仅取决于该时刻输入

causality: 取决于现在及过去的输入

linearity: $y(ax(t)+bx(t)) = ay_1(t)+by_2(t)$

time invariance: $y(t)=x(\frac{t}{3})$，令 $x_2(t)=x_1(t-t_0)$，$y_2(t)=x_2(\frac{t}{3})=x_1(\frac{t}{3}-t_0)$ $\neq y_1(t-t_0)=x(\frac{t-t_0}{3})$

卷积 unit impulse response

$y[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]$

$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$

$x(t)=x(t)*\delta(t)$

$\delta(t)=\delta(t)*\delta(t)$

$x(t)*\delta(t-t_0)=x(t-t_0)$

$\int_{-\infty}^{+\infty}\delta(\tau)d\tau=1$

$\frac{d}{dt}[x(t)*h(t)]=x(t)*\frac{d}{dt}h(t)=\frac{d}{dt}x(t)*h(t)$

$\int_{-\infty}^{t}[x(\tau)*h(\tau)]d\tau=x(t)*\int_{-\infty}^{t}h(\tau)d\tau=[\int_{-\infty}^{t}x(\tau)d\tau]*h(t)$

$x(t)*h(t)=y(t)\Rightarrow x(t-t_1)*h(t-t_2)=y(t-t_1-t_2)$

$\delta(t)=\frac{du(t)}{dt}$, $\delta[n]=u[n]-u[n-1]$

$u(t)=\int_{-\infty}^{t}\delta(\tau)d\tau=\int_{\infty}^{0}\delta(t-m)(-dm)=\int_{0}^{\infty}\delta(t-m)dm$

$u[n]=\sum_{m=-\infty}^{n}\delta[m]=\sum_{k=0}^{\infty}\delta[n-k]$

卷积性质

记忆：对$t\neq0$，$h(t)=0$，$y(t)=h(0)x(t)$；$n\neq0$，$h[n]=0$，$y[n]=h[0]x[n]$

因果：$k>n$，$h[n-k]=0$ ∴ $n<0$，$h[n]=0$；$y[n]=\sum_{k=0}^{\infty}h[k]x[n-k]$；$t<0$，$h(t)=0$，$y(t)=\int_{0}^{+\infty}h(\tau)x(t-\tau)d\tau$

稳定：$\sum_{k=-\infty}^{\infty}|h[k]|<\infty$，$\int_{-\infty}^{+\infty}|h(\tau)|d\tau<\infty$

傅里叶级数

$x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}=\sum_{k=-\infty}^{+\infty}a_k e^{jk\frac{2\pi}{T}t}$ synthesis

$a_k=\frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt=\frac{1}{T}\int_T x(t)e^{-jk\frac{2\pi}{T}t}dt$ analysis 可能周期

$x[n]=\sum_{k=\langle N\rangle}a_k e^{jk\omega_0 n}=\sum_{k=\langle N\rangle}a_k e^{jk\frac{2\pi}{N}n}$ 分为周期

$a_k=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk\omega_0 n}=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-jk\frac{2\pi}{N}n}$ $a_k=a_{k+N}$

傅级数性质

$x(t)\overset{FS}{\leftrightarrow}a_k$，$y(t)\overset{FS}{\leftrightarrow}b_k$，$\omega_0=\frac{2\pi}{T}$ | $x[n]\overset{FS}{\leftrightarrow}a_k$，$y[n]\overset{FS}{\leftrightarrow}b_k$，$\omega_0=\frac{2\pi}{N}$

CT		DT	
$Ax(t)+By(t)$	Aa_k+Bb_k	$Ax[n]+By[n]$	Aa_k+Bb_k
$x(t-t_0)$	$a_k e^{-jk\omega_0 t_0}$	$x[n-n_0]$	$a_k e^{-jk\omega_0 n_0}$
$e^{jM\omega_0 t}x(t)$	a_{k-M}	$e^{jM\omega_0 n}x[n]$	a_{k-M}
$x^*(t)$	a^*_{-k}	$x^*[n]$	a^*_{-k}
$x(-t)$	a_{-k}	$x[-n]$	a_{-k}
$x(\alpha t),\alpha>0,T=\frac{T}{\alpha}$	a_k	$x_{(m)}[n]$ 周期mN 是m倍数	$\frac{1}{m}a_k$，$T=mN$
$\int_T x(\tau)y(t-\tau)d\tau$	Ta_kb_k	$\sum_{r=\langle N\rangle}x[r]y[n-r]$	Na_kb_k
$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty}a_lb_{k-l}$	$x[n]y[n]$	$\sum_{l=\langle N\rangle}a_lb_{k-l}$
$\frac{dx(t)}{dt}$	$jk\omega_0 a_k$	$x[n]-x[n-1]$	$(1-e^{-jk\omega_0})a_k$
$\int_{-\infty}^{t}x(t)dt$ 仅当$a_0=0$才有限	$(\frac{1}{jk\omega_0})a_k$	$\sum_{k=-\infty}^{n}x[k]$ 仅当$a_0=0$有限	$(\frac{1}{1-e^{-jk\omega_0}})a_k$

$\frac{1}{T}\int_T|x(t)|^2dt=\sum_{k=-\infty}^{+\infty}|a_k|^2$　　$\frac{1}{N}\sum_{n=\langle N\rangle}|x[n]|^2=\sum_{k=\langle N\rangle}|a_k|^2$

实信号共轭对称性

$x(t)$实：$a_k=a^*_{-k}$，$Re\{a_k\}=Re\{a_{-k}\}$，$Im\{a_k\}=-Im\{a_{-k}\}$，$|a_k|=|a_{-k}|$，$\measuredangle a_k=-\measuredangle a_{-k}$

$x(t)$实偶 $\Rightarrow a_k$实偶

$x(t)$实奇 $\Rightarrow a_k$纯虚奇

$x(t)$实 $\Rightarrow Ev\{x(t)\}\overset{FS}{\leftrightarrow}Re\{a_k\}$，$Od\{x(t)\}\overset{FS}{\leftrightarrow}jIm\{a_k\}$

频率响应

$H(j\omega)=\int_{-\infty}^{+\infty}h(t)e^{-j\omega t}dt$ 不连周期

$x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}$

$y(t)=\sum_{k=-\infty}^{+\infty}a_kH(jk\omega_0)e^{jk\omega_0 t}$

$H(e^{j\omega})=\sum_{n=-\infty}^{+\infty}h[n]e^{-j\omega n}$ 周期

$x[n]=\sum_{k=\langle N\rangle}a_k e^{jk\frac{2\pi}{N}n}$

$y[n]=\sum_{k=\langle N\rangle}a_k H(e^{j\frac{2\pi k}{N}})e^{jk\frac{2\pi}{N}n}$

$x(t)=e^{st}$　$y(t)=H(s)e^{st}$

eigenfunction　eigenvalue, 常数

傅变

$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega$　(对$x(t)=2\pi\delta(t-1)$，$x(t)=\delta(t-\alpha)$)

$X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$ 不连周期

周期：$X(j\omega)=\sum_{k=-\infty}^{+\infty}2\pi a_k\delta(\omega-k\omega_0)$

$x[n]=\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})e^{j\omega n}d\omega$

$X(e^{j\omega})=\sum_{n=-\infty}^{+\infty}x[n]e^{-j\omega n}$ 周期2π

周期：$X(e^{j\omega})=\sum_{k=-\infty}^{+\infty}2\pi a_k\delta(\omega-\frac{2\pi k}{N})$

傅变性质

CT		DT	
$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$	$ax[n]+by[n]$	$aX(e^{j\omega})+bY(e^{j\omega})$
$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
$x^*(t)$	$X^*(-j\omega)$	$x^*[n]$	$X^*(e^{-j\omega})$
$x(-t)$	$X(-j\omega)$	$x[-n]$	$X(e^{-j\omega})$
$x(at)$	$\frac{1}{\lvert a\rvert}X(\frac{j\omega}{a})$	$x_{(k)}[n]=\begin{cases}x[\frac{n}{k}], n为k倍数\\ 0, n不为k倍数\end{cases}$	$X(e^{jk\omega})$
$x(t)*y(t)$	$X(j\omega)Y(j\omega)$	$x[n]*y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi}X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$	$x[n]-x[n-1]$	$(1-e^{-j\omega})X(e^{j\omega})$
$\int_{-\infty}^{t}x(t)dt$	$\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$	$\sum_{k=-\infty}^{n}x[k]$	$\frac{1}{1-e^{-j\omega}}X(e^{j\omega})+\pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta(\omega-2\pi k)$
$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$	$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$

$\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$　　$\sum_{n=-\infty}^{+\infty}|x[n]|^2=\frac{1}{2\pi}\int_{2\pi}|X(e^{j\omega})|^2d\omega$

傅变实信号共轭对称性

$x(t)$实：$X(j\omega)=X^*(-j\omega)$；$Re\{X(j\omega)\}$偶；$Im\{X(j\omega)\}$奇；$|X(j\omega)|$偶；$\measuredangle X(j\omega)$奇

$x(t)$实偶 $\rightarrow X(j\omega)$实偶

$x(t)$实奇 $\rightarrow X(j\omega)$纯虚奇

$Ev\{x(t)\}\overset{F}{\leftrightarrow}Re\{X(j\omega)\}$，$Od\{x(t)\}\overset{F}{\leftrightarrow}jIm\{X(j\omega)\}$

$z=x+jy=re^{j\theta}=r\angle\theta$，$r=\sqrt{x^2+y^2}=|z|$，$\theta=\measuredangle z=\arctan(\frac{y}{x})$，$z^*=x-jy=re^{-j\theta}$

$z=r\angle\theta$，$e^{j\pi}=-1$，$\frac{1}{z}=\frac{1}{r}\angle-\theta$，$z_1z_2=r_1r_2\angle\theta_1+\theta_2$，$\frac{z_1}{z_2}=\frac{r_1}{r_2}\angle\theta_1-\theta_2$

实函数奇偶分解

$f(x)=Ev\{f(x)\}+Od\{f(x)\}$，$\frac{f(x)+f(-x)}{2}$，$\frac{f(x)-f(-x)}{2}$

$\sum_{n=0}^{N-1}\alpha^n=\begin{cases}N & \alpha=1\\ \frac{1-\alpha^N}{1-\alpha} & \alpha\neq1\end{cases}$

$\sum_{n=0}^{\infty}n\alpha^n=\frac{\alpha}{(1-\alpha)^2}, |\alpha|<1$　$\sum_{n=0}^{\infty}\alpha^n=\frac{1}{1-\alpha}, |\alpha|<1$　$\sum_{n=k}^{\infty}\alpha^n=\frac{\alpha^k}{1-\alpha}, |\alpha|<1$

$e^{j\omega t}=\cos\omega t+j\sin\omega t$

$\sin\omega t=\frac{1}{2j}(e^{j\omega t}-e^{-j\omega t})$

$\cos\omega t=\frac{1}{2}(e^{j\omega t}+e^{-j\omega t})$

$\sin(a+b)=\frac{1}{2j}(e^{ja}e^{jb}-e^{-ja}e^{-jb})$

$\sin(a-b)=\frac{1}{2j}(e^{ja}e^{-jb}-e^{-ja}e^{jb})$

$\cos(a+b)=\frac{1}{2}(e^{ja}e^{jb}+e^{-ja}e^{-jb})$

$\cos(a-b)=\frac{1}{2}(e^{ja}e^{-jb}+e^{-ja}e^{jb})$

$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$

$S_aC_b=\frac{1}{2}[S(a+b)+S(a-b)]$

$C_aS_b=\frac{1}{2}[S(a+b)-S(a-b)]$

$C_aC_b=\frac{1}{2}[C(a+b)+C(a-b)]$

$S_aS_b=-\frac{1}{2}[C(a+b)-C(a-b)]$

$S_x+S_y=2S\frac{x+y}{2}C\frac{x-y}{2}$

$S_x-S_y=2C\frac{x+y}{2}S\frac{x-y}{2}$

$C_x+C_y=2C\frac{x+y}{2}C\frac{x-y}{2}$

$C_x-C_y=-2S\frac{x+y}{2}S\frac{x-y}{2}$

even 偶　odd 奇

$\sin(a+b)=S_aC_b+C_aS_b$

$\cos(a+b)=C_aC_b-S_aS_b$

$\text{sinc}(\theta)=\frac{\sin\pi\theta}{\pi\theta}$

基本傅变对

信号	傅变	傅级数 a_k
$\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}$	$2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(\omega-k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$a_1=1$，$a_k=0(k\neq1)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$a_1=a_{-1}=\frac{1}{2}$，$a_k=0$(其余)
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$a_1=-a_{-1}=\frac{1}{2j}$，$a_k=0$
1	$2\pi\delta(\omega)$	$a_0=1$，$a_k=0$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta(\omega-\frac{2\pi k}{T})$	$a_k=\frac{1}{T}$ 所有k
$x(t)=\begin{cases}1 & \lvert t\rvert<T_1\\ 0 & T_1<\lvert t\rvert\leq\frac{T}{2}\end{cases}$，$x(t+T)=x(t)$	$\sum_{k=-\infty}^{+\infty}\frac{2\sin k\omega_0 T_1}{k}\delta(\omega-k\omega_0)$	$a_k=\frac{\omega_0 T_1}{\pi}\text{sinc}(\frac{k\omega_0 T_1}{\pi})=\frac{\sin k\omega_0 T_1}{k\pi}$，$a_0=\frac{2T_1}{T}$
$x(t)=\begin{cases}1 & \lvert t\rvert<T_1\\ 0 & \lvert t\rvert>T_1\end{cases}$	$\frac{2\sin\omega T_1}{\omega}$（$u(t+T_1)-u(t-T_1)$）	
$\frac{\sin Wt}{\pi t}$	$X(j\omega)=\begin{cases}1 & \lvert\omega\rvert<W\\ 0 & \lvert\omega\rvert>W\end{cases}$（$u(\omega+W)-u(\omega-W)$）	
$\delta(t)$	1	
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$	
$\delta(t-t_0)$	$e^{-j\omega t_0}$	
$e^{-at}u(t)$, $Re\{a\}>0$	$\frac{1}{a+j\omega}$	
$te^{-at}u(t)$, $Re\{a\}>0$	$\frac{1}{(a+j\omega)^2}$	
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $Re\{a\}>0$	$\frac{1}{(a+j\omega)^n}$	

DT信号	傅变	傅级数 a_k
$\sum_{k=\langle N\rangle}a_k e^{jk\frac{2\pi}{N}n}$	$2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(\omega-\frac{2\pi k}{N})$	a_k
$e^{j\omega_0 n}$	$2\pi\sum_{l=-\infty}^{+\infty}\delta(\omega-\omega_0-2\pi l)$	若$\frac{\omega_0}{2\pi}$有理，$a_k=\begin{cases}1 & k=m, m\pm N, m\pm2N\ldots\\ 0 & 其他\end{cases}$
$\cos\omega_0 n$	$\pi\sum_{l=-\infty}^{+\infty}\{\delta(\omega-\omega_0-2\pi l)+\delta(\omega+\omega_0-2\pi l)\}$	若$\frac{\omega_0}{2\pi}$有理，$a_k=\begin{cases}\frac{1}{2} & k=\pm m, \pm m\pm N, \pm m\pm2N\ldots\\ 0 & 其他\end{cases}$
$\sin\omega_0 n$	$\frac{\pi}{j}\sum_{l=-\infty}^{+\infty}\{\delta(\omega-\omega_0-2\pi l)-\delta(\omega+\omega_0-2\pi l)\}$	若$\frac{\omega_0}{2\pi}$有理，$a_k=\begin{cases}\frac{1}{2j} & k=r, r\pm N, r\pm2N\ldots\\ -\frac{1}{2j} & k=-r, -r\pm N, -r\pm2N\ldots\\ 0 & 其他\end{cases}$
1	$2\pi\sum_{l=-\infty}^{+\infty}\delta(\omega-2\pi l)$	$a_k=\begin{cases}1 & k=0,\pm N,\pm2N\ldots\\ 0 & 其他\end{cases}$
$x[n]=\begin{cases}1 & \lvert n\rvert\leq N_1\\ 0 & N_1<\lvert n\rvert\leq\frac{N}{2}\end{cases}$，$x[n+N]=x[n]$	$2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(\omega-\frac{2\pi k}{N})$	$a_k=\frac{\sin[(2\pi k/N)(N_1+\frac{1}{2})]}{N\sin[\frac{2\pi k}{2N}]}$, $k\neq0,\pm N,\pm2N\ldots$；$a_k=\frac{2N_1+1}{N}$, $k=0,\pm N,\pm2N\ldots$
$\sum_{k=-\infty}^{+\infty}\delta[n-kN]$	$\frac{2\pi}{N}\sum_{k=-\infty}^{+\infty}\delta(\omega-\frac{2\pi k}{N})$	$a_k=\frac{1}{N}$
$\alpha^n u[n]$, $\lvert\alpha\rvert<1$	$\frac{1}{1-\alpha e^{-j\omega}}$	
$x[n]=\begin{cases}1 & \lvert n\rvert\leq N_1\\ 0 & \lvert n\rvert>N_1\end{cases}$	$\frac{\sin[\omega(N_1+\frac{1}{2})]}{\sin(\frac{\omega}{2})}$	
$\frac{\sin Wn}{\pi n}=\frac{W}{\pi}\text{sinc}(\frac{Wn}{\pi})$, $0<W<\pi$	$X(\omega)=\begin{cases}1 & 0\leq\lvert\omega\rvert\leq W\\ 0 & W<\lvert\omega\rvert\leq\pi\end{cases}$ 周期2π	
$\delta[n]$	1	
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{+\infty}\pi\delta(\omega-2\pi k)$	
$\delta[n-n_0]$	$e^{-j\omega n_0}$	
$(n+1)\alpha^n u[n]$, $\lvert\alpha\rvert<1$	$\frac{1}{(1-\alpha e^{-j\omega})^2}$	
$\frac{(n+r-1)!}{n!(r-1)!}\alpha^n u[n]$, $\lvert\alpha\rvert<1$	$\frac{1}{(1-\alpha e^{-j\omega})^r}$	

傅变 频率响应函数

$y(t)=x(t)*h(t)\overset{F}{\leftrightarrow}Y(j\omega)=H(j\omega)X(j\omega)$

$\sum_{k=0}^{N}a_k\frac{d^ky(t)}{dt^k}=\sum_{k=0}^{M}b_k\frac{d^kx(t)}{dt^k}\Rightarrow H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}=\frac{\sum_{k=0}^{M}b_k(j\omega)^k}{\sum_{k=0}^{N}a_k(j\omega)^k}$

$\sum_{k=0}^{N}a_ky[n-k]=\sum_{k=0}^{M}b_kx[n-k]\Rightarrow H(e^{j\omega})=\frac{\sum_{k=0}^{M}b_ke^{-jk\omega}}{\sum_{k=0}^{N}a_ke^{-jk\omega}}$

Impulse-train sampling 冲激串采样

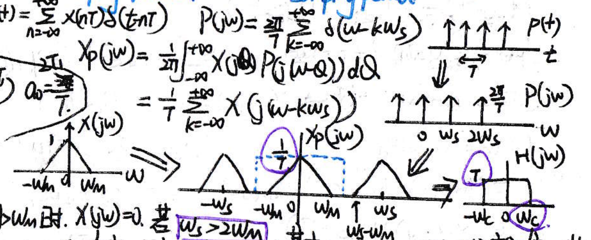
$x_p(t)=x(t)p(t)$，$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$ sampling function

$\omega_s=\frac{2\pi}{T}$ sampling frequency，T sampling period

$x_p(t)=\sum_{n=-\infty}^{+\infty}x(nT)\delta(t-nT)$

$P(j\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta(\omega-k\omega_s)$

$X_p(j\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)P(j(\omega-\theta))d\theta=\frac{1}{T}\sum_{k=-\infty}^{+\infty}X(j(\omega-k\omega_s))$



Sampling theorem

$x(t)$是一带限信号，$|\omega|>\omega_M$时，$X(j\omega)=0$。若$\omega_s>2\omega_M$，其中$\omega_s=\frac{2\pi}{T}$，那么$x(t)$就唯一地由其样本$x(nT)$，$n=0,\pm1,\pm2\ldots$所确定。已知这些样本值，我们能用如下办法重建$x(t)$：产生一个周期冲激串，其冲激幅度就是这些样本值，然后将该冲激串通过一个增益为T，截止频率大于ω_M而小于$\omega_s-\omega_M$的理想低通滤波器，该滤波器输出就是$x(t)$

Nyquist rate：$2\omega_M$

Signal reconstruction：interpolation 内插

$h(t)=\frac{\omega_cT\sin(\omega_ct)}{\pi\omega_ct}$　band limit 带限

$x_r(t)=x_p(t)*h(t)=[\sum_{n=-\infty}^{+\infty}x(nT)\delta(t-nT)]*h(t)=\sum_{n=-\infty}^{+\infty}x(nT)h(t-nT)$

$=\sum_{n=-\infty}^{+\infty}x(nT)\frac{\omega_cT}{\pi}\frac{\sin(\omega_c(t-nT))}{\omega_c(t-nT)}$　band-limited interpolation 带限内插

Zero-Order Hold 零阶保持（先传输前中保持形）

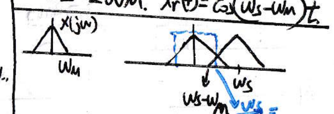
$H_0(j\omega)=e^{-j\omega T/2}[\frac{2\sin(\frac{\omega T}{2})}{\omega}]$

$H_r(j\omega)=\frac{e^{j\omega T/2}H(j\omega)}{\frac{2\sin(\omega T/2)}{\omega}}$

$H(j\omega)=H_r(j\omega)H_0(j\omega)$

Undersampling：Aliasing 欠采样 混叠

当 $\omega_s<2\omega_M$，$x_r(t)=\cos(\omega_s-\omega_M)t$



$h(t)=e^{-4|t|}$　$H(j\omega)=\int_{-\infty}^{0}e^{4t}e^{-j\omega t}dt+\int_{0}^{\infty}e^{-4t}e^{-j\omega t}dt$

$h[n]=e^{-4|n|}$　$H(e^{j\omega})=\sum_{-\infty}^{-1}+\sum_{0}^{\infty}$

$e^{-3|t|}\sin t=e^{-3t}\sin t\,u(t)+e^{3t}\sin t\,u(-t)$

block diagram

DT：$x_1[n]\rightarrow\oplus\rightarrow y[n]=x_1[n]+x_2[n]$；$x[n]\overset{a}{\rightarrow}y[n]=ax[n]$；$x[n]\rightarrow\boxed{D}\rightarrow y[n]=x[n-1]$

CT：$x_1(t)\rightarrow\oplus\rightarrow x_1(t)+x_2(t)$；$x(t)\overset{a}{\rightarrow}ax(t)$；$x(t)\rightarrow\boxed{D}\rightarrow\frac{dx(t)}{dt}$；$x(t)\rightarrow\boxed{I}\rightarrow\int_{-\infty}^{t}x(\tau)d\tau$

$\frac{dy(t)}{dt}+ay(t)=bx(t)$

$y(t)=-\frac{1}{a}\frac{dy(t)}{dt}+\frac{b}{a}x(t)$

$\frac{1}{(a+j\omega)(b+j\omega)}=\frac{1}{b-a}(\frac{1}{a+j\omega}-\frac{1}{b+j\omega})$

$\sum_{n=-\infty}^{\infty}(-1)^n\delta(t-n)=[\delta(t)-\delta(t-1)]_{0\sim2周期}$

$x_c(t) \to \otimes \to x_p(t) \to$ [冲激串到离散时间序列转换] $\to x_d[n]=x_c(nT) \to$ [$h_d[n]$, $H_D(e^{j\omega})$] $\to y_d[n]=y_c(nT) \to$ [离散时间序列到冲激串转换] $\to y_p(t) \to$ [$H(j\omega)$] $\to y_c(t)$ $\quad \omega_s=\frac{2\pi}{T}$

$p(t)=\sum_{n=-\infty}^{+\infty}\delta(t-nT)$

continuous-to-discrete time conversion C/D转换 (ADC) D/C转换 DAC $\frac{\pi}{T}=\frac{\omega_s}{2}$

$x_p(t)=\sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT) \to X_p(j\omega)=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\omega nT}$

$x_d[n]=x_c(nT) \to X_d(e^{j\Omega})=\sum_{n=-\infty}^{\infty}x_c(nT)e^{-j\Omega n}$ 常数 $\quad X_d(e^{j\Omega})=X_p(j\frac{\Omega}{T})$

又 $X_p(j\omega)=\frac{1}{T}\sum_{k=-\infty}^{+\infty}X_c(j(\omega-k\omega_s))$ $\quad \therefore X_d(e^{j\Omega})=\frac{1}{T}\sum_{k=-\infty}^{+\infty}X_c(j(\frac{\Omega}{T}-k\omega_s))$. $\quad y_d[n]=x_d[n]*h_d[n]$

$Y_d(e^{j\Omega})=X_d(e^{j\Omega})H_D(e^{j\Omega})$

$y_p(t)=\sum_{n=-\infty}^{\infty}y_d[n]\delta(t-nT) \Rightarrow Y_p(j\omega)=\sum_{n=-\infty}^{\infty}y_d[n]e^{-j\omega nT}=Y_d(e^{j\Omega T})$

$y_c(t)=y_p(t)*h_{lp}(t) \Rightarrow Y_c(j\omega)=Y_p(j\omega)\cdot H_{lp}(j\omega)$

$\therefore Y_c(j\omega)=Y_p(j\omega)H_{lp}(j\omega)=Y_d(e^{j\omega T})H_{lp}(j\omega)=X_d(e^{j\omega T})H_D(e^{j\omega T})H_{lp}(j\omega)$

$=X_p(j\omega)H_D(e^{j\omega T})H_{lp}(j\omega)=\frac{1}{T}\sum_{k=-\infty}^{+\infty}X_c(j(\omega-k\omega_s))H_D(e^{j\omega T})H_{lp}(j\omega)$

$=X_c(j\omega)H_D(e^{j\omega T})$

$=X_c(j\omega)\tilde{H}_D(e^{j\omega T})$ $\quad \tilde{H}_D(e^{j\omega T})=\begin{cases}H_D(e^{j\omega T}) & |\Omega|<\frac{\omega_s}{2}\\ 0 & 其他\end{cases}$

$H_d(e^{j\Omega})$, $\tilde{H}_D(e^{j\omega T})$, $Y_c(j\omega)$, $X_p(j\omega)$, $X_d(e^{j\Omega})$, $Y_d(e^{j\Omega})$, $Y_p(j\omega)$, $X_c(j\omega)$ $\quad \omega_s T=2\pi$

离散时间采样

$p[n]=\sum_{k=-\infty}^{+\infty}\delta[n-kN]$

$x_p[n]=x[n]p[n]=\sum_{k=-\infty}^{+\infty}x[kN]\delta[n-kN]$

$P(e^{j\omega})=\frac{2\pi}{N}\sum_{k=-\infty}^{+\infty}\delta(\omega-k\omega_s)$ $\quad \omega_s=\frac{2\pi}{N}$

$X_p(e^{j\omega})=\frac{1}{2\pi}\int_{2\pi}P(e^{j\theta})X(e^{j(\omega-\theta)})d\theta$

$=\frac{1}{N}\sum_{k=0}^{N-1}X(e^{j(\omega-k\omega_s)})$

$H(e^{j\omega})$ 也是周期的 $\quad X_r(e^{j\omega})$

$h[n]=\frac{N\omega_c}{\pi}\frac{\sin\omega_c n}{\omega_c n}$

$x_r[n]=x_p[n]*h[n]=\sum_{k=-\infty}^{+\infty}x[kN]\frac{N\omega_c}{\pi}\frac{\sin\omega_c(n-kN)}{\omega_c(n-kN)}$

decimation 抽取 downsampling 减采样

$x_b[n]=x_p[nN]=x[nN]$

$X_b(e^{j\omega})=\sum_{k=-\infty}^{+\infty}x_b[k]e^{-j\omega k}=\sum_{k=-\infty}^{+\infty}x_p[kN]e^{-j\omega k}$. 令 $n=kN$

$=\sum_{n=-\infty}^{+\infty}x_p[n]e^{-j\omega\frac{n}{N}}=X_p(e^{j\omega/N})$

interpolation — upsampling 增采样

Amplitude modulation 幅度调制

$y(t)=x(t)c(t)$ — carrier signal; $x(t)$: modulating signal

transmitted 发送 unscrambled 解密

sinusoidal amplitude modulation 正弦幅度调制

$x(t)\to\otimes\to y(t)\to\otimes\to w(t)\to$ [$H(j\omega)$] $\to x(t)$ ($\cos\omega_c t$, $\cos\omega_c t$)

$C(j\omega)=\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]$

$Y(j\omega)=\frac{1}{2}[X(j(\omega-\omega_c))+X(j(\omega+\omega_c))]$ 带通输出功率 接收机昂贵 $\omega_M<\omega_c$

$w(t)=y(t)\cos\omega_c t$ $\quad \frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2\pi}\int_{-\infty}^{+\infty}\ldots=(\frac{1}{2}+\frac{1}{2})\frac{1}{2}=\frac{1}{2}$

Synchronous Demodulation 同步解调 $\quad \omega_M<\omega_{co}<2\omega_c-\omega_M$

Asynchronous Demodulation 非同步解调

包络检测器 envelope detection. 输出功率大, 接收机廉价.

两个假设. $x(t)$为正, $x(t)$比ω_c变化慢得多

$x(t)\to\otimes\to\oplus\to y(t)=(A+x(t))\cos\omega_c t$ $\quad |x(t)|<k \Rightarrow A>k$ $\quad \frac{k}{A}$ modulation index

Single-Sideband AM (SSB) 单边带正弦幅度调制

upper sideband 上边带, lower sideband 下边带. 频谱利用率上了, 复杂性上了

$y(t)$ 保留下边带. 要保留上边带 则由改回减

$H(j\omega)=\begin{cases}-j & \omega>0\\ j & \omega<0\end{cases}$

$Y_1(j\omega)=\frac{1}{2}[X_1(j(\omega-\omega_c))+X(j(\omega+\omega_c))]+\ldots$

$X_p(j\omega)=-jX_+(j\omega)+jX_-(j\omega)$

$Y_2(j\omega)=X_p(j\omega)*\frac{\pi}{2\pi j}[\delta(\omega-\omega_c)-\delta(\omega+\omega_c)]$

$=\frac{1}{2}X_+(j(\omega-\omega_c))+\frac{1}{2}X(j(\omega-\omega_c))+\frac{1}{2}X_+(j(\omega+\omega_c))-\frac{1}{2}X(j(\omega+\omega_c))$

$Y_1(j\omega)+Y_2(j\omega)=X_-(j(\omega-\omega_c))+X_+(j(\omega+\omega_c))$

双 AM-DSB/WC → 载波存在; 单SSB SC → 载波抑制

Frequency-Division Multiplexing (FDM) 频分多路复用

用正弦幅度调制将各信号在频率上搬移, 使其频谱不再重叠, 在同一宽带信道上传输这些信号.

收信号: 带通+解调.

AM with Pulse-Train Carrier 脉冲串载波调制

$y(t)=x(t)c(t)$. $c(t)$: T, $\omega_c=\frac{2\pi}{T}$. $a_k=\frac{\sin(k\omega_c\frac{\Delta}{2})}{\pi k}$

$C(j\omega)=2\pi\sum_{k=-\infty}^{+\infty}a_k\delta(\omega-k\omega_c)$ $\quad Y(j\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)C(j(\omega-\theta))d\theta$

$=\sum_{k=-\infty}^{+\infty}a_kX(j(\omega-k\omega_c))$

$\omega_c>2\omega_M$. 可用 $\omega_M<\omega_{co}<\omega_c-\omega_M$ 恢复

Pulse-amplitude Modulation (PAM) 脉冲幅度调制

以样本 $x(nT)$ 调制一脉冲序列幅度

Time-Division Multiplexing (TDM) 时分多路复用

每一路信号被安排在一组持续期Δ的时隙. 数每T秒重复一次.

$T\downarrow$. 路数$\uparrow$ $\quad T\uparrow$: $\frac{2\pi}{T}>2\omega_M$. 不能增$\uparrow$; $\Delta\downarrow$: 信号占带宽$\uparrow$. 通讯不便

intersymbol interference (ISI) 码间干扰

信道是低通滤波器

$a_0=\frac{1}{T}\int_{-T_1}^{T_1}dt=\frac{2T_1}{T}$ $\quad a_k=\frac{\sin(k\omega_0T_1)}{k\pi}$ $(\omega_0T=2\pi)$

$g(t)=x(t-1)-\frac{1}{2}$. 取 $T=4$, $T_1=1$

$g(t)\xleftrightarrow{FS}\begin{cases}a_ke^{-jk\pi/2}, & k\neq0\\ a_0-\frac{1}{2}, & k=0\end{cases} \Rightarrow b_k=\begin{cases}\frac{\sin(\frac{\pi k}{2})}{k\pi}e^{-jk\pi/2}, & k\neq0\\ 0 & k=0\end{cases}$

$x(t)=\frac{\sin t\cdot\sin(\frac{t}{2})}{\pi t^2}$ 傅变.

$=\pi\cdot\frac{\sin t}{\pi t}\cdot\frac{\sin(\frac{t}{2})}{\pi t}$

$X(j\omega)=\pi\cdot\frac{1}{2\pi}F(\frac{\sin t}{\pi t})*F(\frac{\sin\frac{t}{2}}{\pi t})$

$X_1(j\omega)=\begin{cases}1 & |\omega|<1\\ 0 & |\omega|>1\end{cases}$ $\quad X_2(j\omega)=\begin{cases}1 & |\omega|<\frac{1}{2}\\ 0 & |\omega|>\frac{1}{2}\end{cases}$

$\frac{1}{2}\int_{-\infty}^{\infty}[u(\tau+1)-u(\tau-1)][u(\omega-\tau+\frac{1}{2})-u(\omega-\tau-\frac{1}{2})]d\tau$

$\int_{-1}^{\omega+\frac{1}{2}}d\tau-\int_{-1}^{\omega-\frac{1}{2}}d\tau-\int_{1}^{\omega+\frac{1}{2}}d\tau+\int_{1}^{\omega-\frac{1}{2}}d\tau$

$(\omega>-\frac{3}{2})\quad(\omega>-\frac{1}{2})\quad(\omega>\frac{1}{2})\quad(\omega>\frac{3}{2})$

$\frac{1}{2}\sum_{k=0}^{\infty}(\frac{1}{3})^k(\frac{1}{4})^{n+k}u[n+k+4]+\sum_{k=0}^{\infty}(\frac{1}{3})^k(\frac{1}{4})^{n-k}u[n-k+3]$ 下限包含n 上限包含n

$k\geq-n-4$, $k\leq n+3$

$=\begin{cases}\frac{1}{2}\sum_{-n-4}^{\infty}\square, & n<-4\\ \frac{1}{2}\sum_{0}^{\infty}\square, & n>-4\end{cases}$ $\quad =\sum_{0}^{n+3}\square, n\geq-3$

$Y(e^{j\omega})=\frac{1-\frac{1}{4}e^{-j\omega}}{(1+\frac{1}{2}e^{-j\omega})^2}=\frac{1}{(\ )^2}-\frac{1}{4}e^{-j\omega}\frac{1}{(\ )^2}$

$(n+1)(-\frac{1}{2})^nu[n]\xleftrightarrow{FT}\frac{1}{(\ )^2}$

$\therefore -\frac{1}{4}n(-\frac{1}{2})^{n-1}u[n-1]\xleftrightarrow{FT}-\frac{1}{4}e^{-j\omega}\frac{1}{(\ )^2}$

求傅变

$x[n]=(\frac{1}{3})^{-n}u[n-1]$

令 $x_1[n]=(\frac{1}{3})^nu[n-1]$ $=\frac{1}{3}(\frac{1}{3})^{n-1}u[n-1]$

$X(e^{j\omega})=\frac{1}{3}e^{-j\omega}\frac{1}{1-\frac{1}{3}e^{-j\omega}}$ $\quad X(e^{j\omega})=X_1(e^{-j\omega})$

$x[n]=(\frac{1}{3})^{|n|}u[n-2]=(\frac{1}{3})^nu[n-2]$

令 $x_1[n]=(\frac{1}{3})^nu[n-2]=\frac{1}{9}(\frac{1}{3})^{n-2}u[n-2]$

$x[n]=2^n\sin(\frac{\pi}{4}n)u[-n]$

令 $x_1[n]=2^{-n}\sin(-\frac{\pi}{4}n)u[n]=-(\frac{1}{2})^n\sin(\frac{\pi}{4}n)u[n]$

$=-\frac{1}{2j}(\frac{1}{2}e^{j\frac{\pi}{4}})^nu[n]+\frac{1}{2j}(\frac{1}{2}e^{-j\frac{\pi}{4}})^nu[n]$

$X(e^{j\omega})=\ldots$

$x[n]=\begin{cases}n & -3\leq n\leq3\\ 0 & 其他\end{cases}$

$x[n]=-3\delta[n+3]-2\delta[n+2]-\delta[n+1]+\delta[n-1]+2\delta[n-2]+3\delta[n-3]$

$X(e^{j\omega})=\ldots$

$x[n]=(\frac{1}{2})^{|n|}\cos(\frac{\pi}{8}(n-1))$

$x[n]=\frac{1}{2}(\frac{1}{2})^n\cos(\frac{\pi}{8}n-\frac{\pi}{8})u[n]+(\frac{1}{2})^{-n}\cos(\frac{\pi}{8}n+\frac{\pi}{8})u[-n-1]$

$x_1[n]=(\frac{1}{2})^{|n|}=(\frac{1}{2})^nu[n]+(\frac{1}{2})^{-n}u[-n]$

$x_2[n]=\cos(\frac{\pi}{8}n-\frac{\pi}{8})$ $\quad X(e^{j\omega})=\ldots$

$X_2(e^{j\omega})=\pi e^{-j\frac{\pi}{8}}\delta(\omega-\frac{\pi}{8})+\pi e^{j\frac{\pi}{8}}\delta(\omega+\frac{\pi}{8})$ $\quad -\pi<\omega\leq\pi$

$X(e^{j\omega})=\frac{1}{2\pi}\int_{-\pi}^{\pi}X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$

$=\frac{\frac{3}{8}e^{-j\frac{\pi}{8}}}{5-4\cos(\omega-\frac{\pi}{8})}+\frac{\frac{3}{8}e^{j\frac{\pi}{8}}}{5-4\cos(\omega+\frac{\pi}{8})}$

$H(e^{j\omega})=\frac{1}{1-\frac{1}{2}e^{-j\omega}}$ $\quad x[n]=(-1)^n$ $\quad T=2$ $\quad a_0=0$, $a_1=1$

$X(e^{j\omega})=2\pi\sum_{k=-\infty}^{\infty}\delta[\omega-(2k+1)\pi]$

$Y(e^{j\omega})=2\pi\sum_{k=-\infty}^{\infty}\frac{1}{1-\frac{1}{2}e^{-j(2k+1)\pi}}\delta[\omega-(2k+1)\pi]$

$=\frac{4\pi}{3}\sum_{k=-\infty}^{\infty}\delta[\omega-(2k+1)\pi]$

$y[n]=\frac{2}{3}(-1)^n$

$Y(e^{j\omega})$ $\quad y[n]=(\frac{\sin\frac{\pi}{4}n}{\pi n})^2*(\frac{\sin\omega_c n}{\pi n})$.

$|\omega_c|\leq\pi$. 对 ω_c 限制. 使 $y[n]=$

$X_1(e^{j\omega})=u(\omega+\frac{\pi}{4})-u(\omega-\frac{\pi}{4})$ $\quad -\pi<\omega\leq\pi$

$(x_1[n])^2=\frac{1}{2\pi}X_1(e^{j\omega})*X_1(e^{j\omega})$

$=\begin{cases}-\frac{1}{2\pi}|\omega|+\frac{1}{4} & |\omega|\leq\frac{\pi}{2}\\ 0 & \frac{\pi}{2}<|\omega|\leq\pi\end{cases}$

$Y(e^{j\omega})=\frac{1}{2\pi}X_1(e^{j\omega})*X_1(e^{j\omega})\cdot X_2(e^{j\omega})$

$\pi\geq|\omega_c|\geq\frac{\pi}{2}$

线性常系数差分方程

$y[n]-\frac{1}{2}y[n-1]=x[n]$

$h[n]-\frac{1}{2}h[n-1]=\delta[n]$

由初始松弛. $h[n]=0$, $n<0$

$h[0]=\frac{1}{2}h[-1]+\delta[0]=1$

$h[1]=\frac{1}{2}h[0]+\delta[1]=\frac{1}{2}+0=\frac{1}{2}$

$h[2]=\frac{1}{2}h[1]+\delta[2]=(\frac{1}{2})^2$

$h[n]=(\frac{1}{2})^nu[n]$

$(\delta[n]+\delta[n-1])*(\delta[n]+\delta[n-1])$

$=\delta[n]+2\delta[n-1]+\delta[n-2]$

$\int_{-\infty}^{\infty}\omega X(j\omega)d\omega=0$

$\frac{dx(t)}{dt}=\frac{1}{2\pi}\int_{-\infty}^{+\infty}j\omega X(j\omega)e^{j\omega t}d\omega$

$\frac{dx(t)}{dt}\Big|_{t=0}=0$