Tutorial Questions （Week 5）

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Outline

- Review
- Basic Problems with Answers 2.20
- Basic Problems 2.29
- Advanced Problems 2.40, 2.43, 2.47
- Q&A

Prior knowledge review

- Basic knowledge on signal computation
- Exponential signals: Euler’s relation, periodic, integral
- CT/DT unit impulse/step function
- System Properties

    1. Memoryless or with memory
    2. Causality
    3. Invertibility
    4. Stability
    5. Time-invariance
    6. Linearity

Basic knowledge review

- CT/DT LTI systems
- Convolution operation procedure
  1. **Figure computation** based on “Flip-slide-multiply-sum/integral”
  2. Some known or **typical convolution results**
  3. **Properties** of convolution
- Unit impulse response and properties of LTI systems
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Example 1

- $x[n] = a^n u[n]$
- $h[n] = u[n]$
- $y[n] = x[n] * h[n]$?

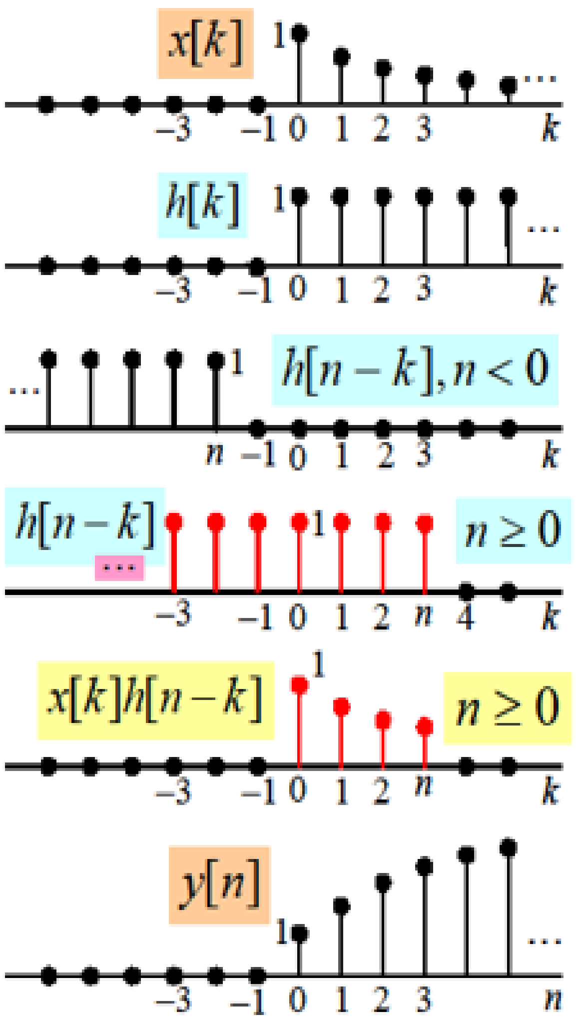
$$y[n] = \begin{cases} \dfrac{1-a^{n+1}}{1-a}, & n \geq 0 \\ 0, \quad n < 0 \end{cases}$$

Flip

Slide

Multiply

Sum

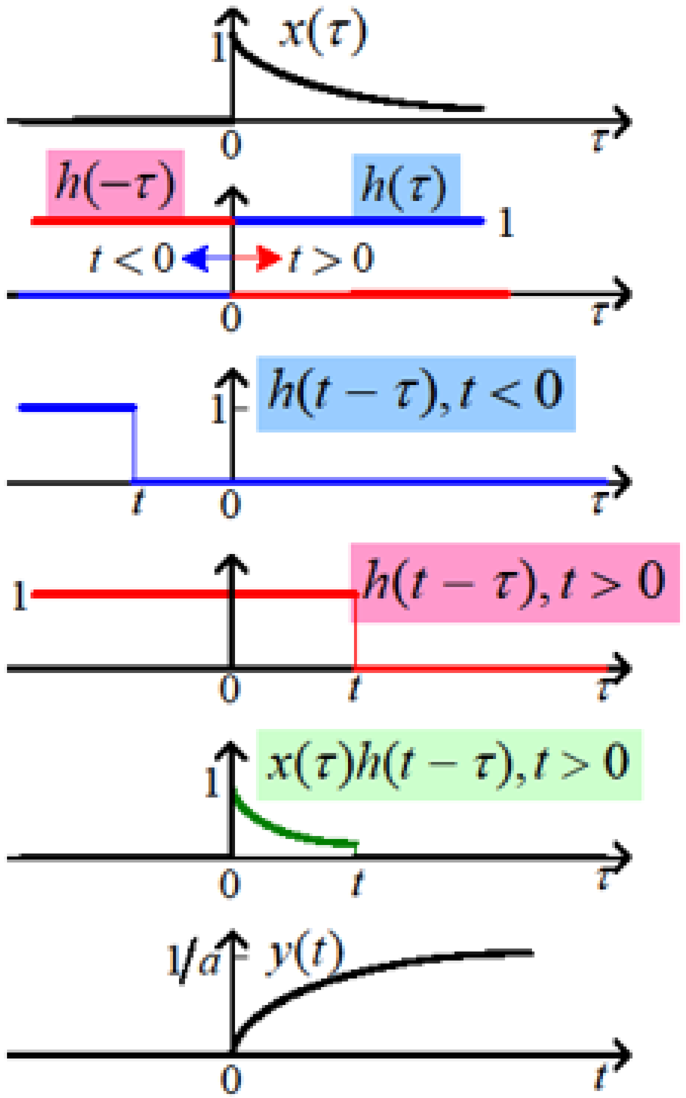


$$h[n-k]$$

$$x[k]h[n-k] = \begin{cases} a^k, & 0 \leq k \leq n \\ 0, & k < 0, k > n \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$$

Example 2



$$x(t)=e^{-at}u(t)$$
$$h(t)=u(t)$$
$$y(t)=x(t)*h(t)$$

$$y(t)=\begin{cases}\dfrac{1-e^{-at}}{a}, & t\geq 0\\ 0, & t<0\end{cases} \Rightarrow \frac{1-e^{-at}}{a}u(t)$$

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表 3.4　基本信号的卷积表

连续时间卷积积分			离散时间卷积和		
$x(t)$	$h(t)$	$x(t)*h(t)$	$x[n]$	$h[n]$	$x[n]*h[n]$
$x(t)$	$\delta(t)$	$x(t)$	$x[n]$	$\delta[n]$	$x[n]$
$x(t)$	$u(t)$	$\int_{-\infty}^{t} x(\tau)\mathrm{d}\tau$	$x[n]$	$u[n]$	$\sum_{k=-\infty}^{n} x[k]$
$x(t)$	$\delta'(t)$	$x'(t)$	$x[n]$	$\Delta\delta[n]$	$x[n]-x[n-1]$
$u(t)$	$u(t)$	$tu(t)$	$u[n]$	$u[n]$	$(n+1)u[n]$
$\mathrm{e}^{-at}u(t)$	$u(t)$	$\frac{1-\mathrm{e}^{-at}}{a}u(t)$	$a^n u[n]$	$u[n]$	$\frac{1-a^{n+1}}{1-a}u[n]$
$\sin(\omega t)u(t)$	$u(t)$	$\frac{1-\cos(\omega t)}{\omega}u(t)$	$\sin(\Omega n)u[n]$	$u[n]$	
$\cos(\omega t)u(t)$	$u(t)$	$\frac{\sin(\omega t)}{\omega}u(t)$	$\cos(\Omega n)u[n]$	$u[n]$	
$\mathrm{e}^{-at}u(t)$	$\mathrm{e}^{-at}u(t)$	$t\mathrm{e}^{-at}u(t)$	$a^n u[n]$	$a^n u[n]$	$(n+1)a^n u[n]$
$\mathrm{e}^{-at}u(t)$	$\mathrm{e}^{-bt}u(t)$	$\frac{\mathrm{e}^{-at}-\mathrm{e}^{-bt}}{b-a}u(t)$	$a^n u[n]$	$b^n u[n]$	$\frac{b^{n+1}-a^{n+1}}{b-a}u[n]$

说明：表 3.4 中空着的卷积和运算结果，感兴趣的读者可自行补上。

[1] 徐守时等著，中国科学技术大学出版社，信号与系统·第三版，第78页

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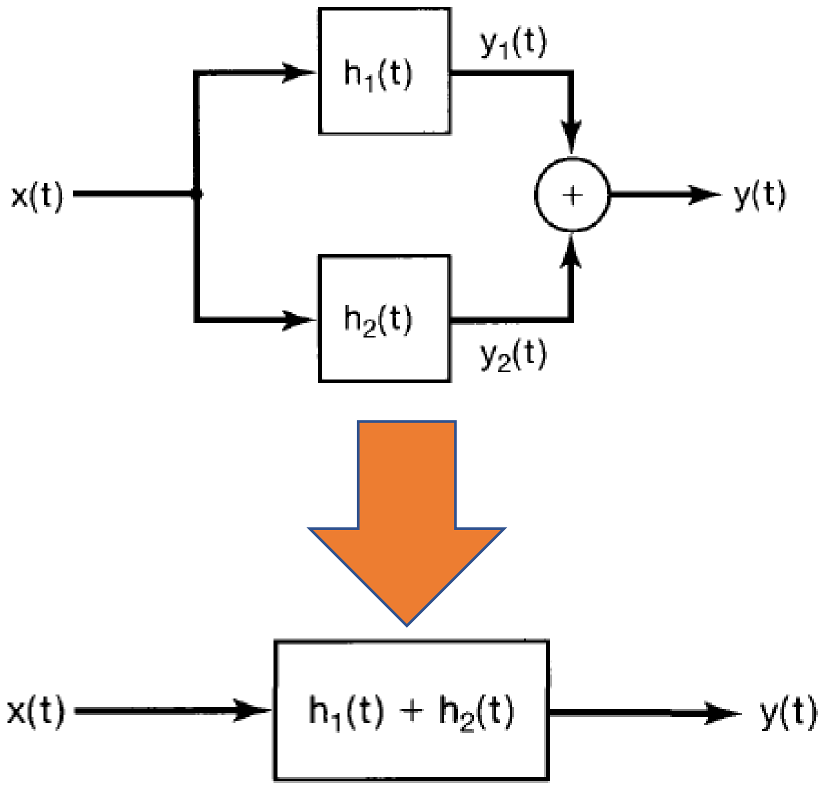
Commutative property

$$x(t)*h(t)=h(t)*x(t)$$

$$\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{\infty}h(\tau)x(t-\tau)d\tau$$

$$x[n]*h[n]=h[n]*x[n]$$

$$\sum_{m=-\infty}^{\infty}x[m]h[n-m]=\sum_{m=-\infty}^{\infty}h[m]x[n-m]$$



Distributive property

$$x(t)*\left[h_1(t)+h_2(t)\right]=x(t)*h_1(t)+x(t)*h_2(t)$$

$$x[n]*\left\{h_1[n]+h_2[n]\right\}=x[n]*h_1[n]+x[n]*h_2[n]$$

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Associative property

$$\left[x(t)*h_1(t)\right]*h_2(t)=x(t)*\left[h_1(t)*h_2(t)\right]$$

$$\left\{x[n]*h_1[n]\right\}*h_2[n]=x[n]*\left\{h_1[n]*h_2[n]\right\}$$

Time-invariant property (Collect the time shift)

$$y(t)=x(t)*h(t)$$

$$x(t)*h\left(t-t_0\right)=y\left(t-t_0\right)$$

$$\boxed{x\left(t-t_1\right)*h\left(t-t_2\right)=y\left(t-t_1-t_2\right)}$$

$$x[n]*h[n]=y[n]$$

$$x[n]*h[n-m]=y[n-m]$$

$$\boxed{x\left[n-m_1\right]*h\left[n-m_2\right]=y\left[n-m_1-m_2\right]}$$

Properties of convolution

Difference property

$$\boxed{\frac{d}{dt}\left[x(t)*h(t)\right]=x(t)*\frac{dh(t)}{dt}=\frac{dx(t)}{dt}*h(t)=\frac{dy(t)}{dt}}$$

$$\nabla\left\{x[n]*h[n]\right\}=\nabla x[n]*h[n]=x[n]*\nabla h[n]=\nabla y[n]$$

Integral property

$$\boxed{\int_{-\infty}^{t}\left[x(\tau)*h(\tau)\right]d\tau=x(t)*\int_{-\infty}^{t}h(\tau)d\tau=\int_{-\infty}^{t}x(\tau)d\tau*h(t)=\int_{-\infty}^{t}y(\tau)d\tau}$$

$$\sum_{k=-\infty}^{n}\left\{x[k]*h[k]\right\}=x[n]*\left\{\sum_{k=-\infty}^{n}h[k]\right\}=\left\{\sum_{k=-\infty}^{n}x[k]\right\}*h[n]=\sum_{k=-\infty}^{n}y[k]$$

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- For unit impulse/step signal
- More unit impulse/step signals, more simple

$$x(t) * \delta(t) = x(t)$$

$$x(t) * \delta\left(t - t_0\right) = x\left(t - t_0\right)$$

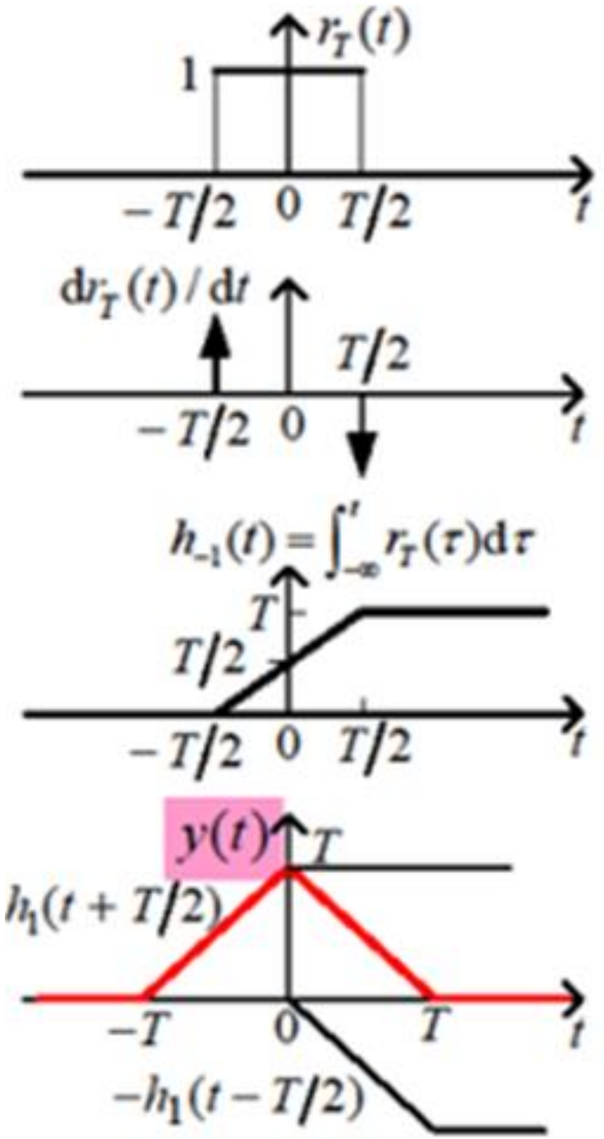
$$x(t) * u(t) = \int_{-\infty}^{t} x(\tau) d\tau$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

$$x[n] * \delta[n] = x[n]$$

$$x[n] * \delta[n-m] = x[n-m]$$

$$x[n] * u[n] = \sum_{m=-\infty}^{n} x[m]$$



$$y(t)=r_T(t)^{*}r_T(t)=\frac{d}{dt}r_T(t)^{*}\int_{-\infty}^{t}r_T(\tau)d\tau$$

$$h_{-1}(t)=\int_{-\infty}^{t}r_T(\tau)d\tau$$

$$y(t)=[\delta(t+T/2)-\delta(t-T/2)]*h_{-1}(t)$$
$$=h_{-1}(t+T/2)-h_{-1}(t-T/2)$$

1. **Figure computation** based on "Flip-slide-multiply-sum/integral"
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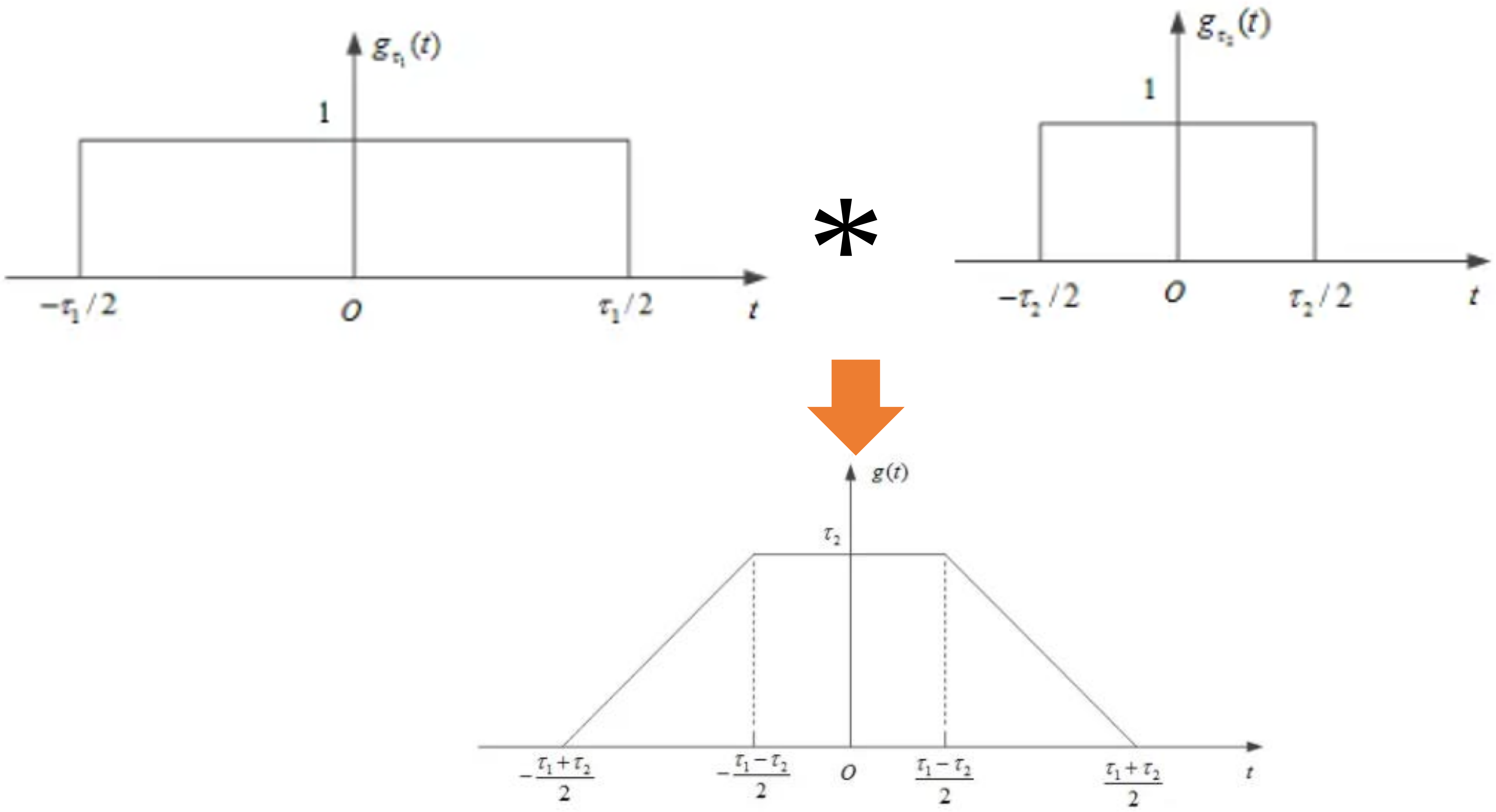

[2] 细节部分参考：https://www.bilibili.com/read/cv9118127

Basic knowledge review

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System properties:

- With memory or memoryless

  $y(n)=f\ (\ x(n)\ )$

- Invertible

  For a system $x \rightarrow y$, if $x1 \neq x2$, then $y1 \neq y2$

- Causal

  … up to that time n …

- Stable (BIBO)

  either prove the system is stable, or find a specific counterexample

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LTI System properties:

- With memory or memoryless
  - A linear, time-invariant, causal system is memoryless only

    if $h[n] = K\delta[n]$ $\quad h(t) = K\delta(t)$

    $y[n] = Kx[n]$ $\quad y(t) = Kx(t)$

    if k=1 further, they are identity systems

$$y[n] = x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] = x[n] * \delta[n]$$

$$y(t) = x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t) * \delta(t)$$

如果LTI可逆，一定能求 $\boxed{h_1(t) * h_2(t) = \delta(t)}$

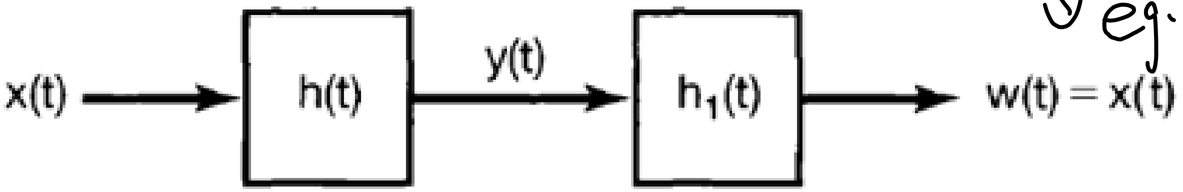
但不是所有的LTI都可逆

eg: $h(t) = \dfrac{\sin W_c t}{\pi t}$

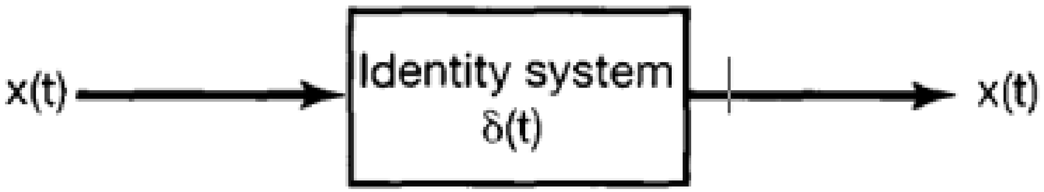
LTI System properties:

- Invertible

前提：逆系统存在



(a)



(b)

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LTI System properties:

- Causal

Causality: CT LTI system is causal $\leftrightarrow$ $h(t) = 0$, at $t < 0$

- This is because that the input unit impulse function $\delta(t)=0$ at $t<0$

$t - \tau \geq 0$, or $\tau \leq t$

As a result:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau$$

$y(t)$ only depends on $x(\tau < t)$.

LTI System properties:

- Stable (BIBO)

BIBO Stability: CT LTI system is stable $\leftrightarrow \int_{-\infty}^{+\infty} |h(\tau)|\, d\tau < \infty$

→ Sufficient condition: For $|x(t)| \le x_{\max} < \infty$.

Cauchy-Schwarz Inequation

$$|y(t)| = \left|\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau\right| \le x_{\max}\left|\int_{-\infty}^{+\infty} h(t-\tau)d\tau\right| < \infty.$$

→ Necessary condition: Suppose $\int_{-\infty}^{+\infty} |h(\tau)| d\tau = \infty$

Contradiction Case

Let $x(t) = h^*(-t)/|h^*(-t)|$, then $|x(t)| \equiv 1$ bounded

But $y(0) = \int_{-\infty}^{+\infty} x(\tau)h(-\tau)d\tau = \int_{-\infty}^{+\infty} \frac{h^*(-\tau)h(-\tau)}{|h(-\tau)|} d\tau = \int_{-\infty}^{+\infty} |h(-\tau)| d\tau = \infty$

2.20. Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} u_0(t)\cos(t)\,dt$

(b) $\int_0^5 \sin(2\pi t)\,\delta(t+3)\,dt$ $=0$

(c) $\int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)\,d\tau$

$$u_{-k}(t) = \frac{t^{k-1}}{(k-1)!}u(t).$$

With this notation, $u_k(t)$ for $k > 0$ denotes the impulse response of a cascade of k differentiators, $u_0(t)$ is the impulse response of the identity system, and, for $k < 0$, $u_k(t)$ is the impulse response of a cascade of $|k|$ integrators. Furthermore, since a differentiator is the inverse system of an integrator,

$$u(t) * u_1(t) = \delta(t),$$

or, in our alternative notation,

$$u_{-1}(t) * u_1(t) = u_0(t). \tag{2.161}$$

2.20. (a)

$$\int_{-\infty}^{\infty} u_0(t)\cos(t)dt = \int_{-\infty}^{\infty} \delta(t)dt = 1$$

(b)

$$\int_{0}^{5} \sin(2\pi t)\delta(t+3)dt = \sin(6\pi) = 0$$

(c) In order to evaluate the integral

$$\int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)d\tau,$$

consider the signal

$$x(t) = \cos(2\pi t)[u(t+5) - u(t-5)].$$

We know that

$$\begin{aligned}\frac{dx(t)}{dt} &= u_1(t) * x(t) = \int_{-\infty}^{\infty} u_1(t-\tau)x(\tau)d\tau \\ &= \int_{-5}^{5} u_1(t-\tau)\cos(2\pi\tau)d\tau\end{aligned}$$

Now,

$$\left.\frac{dx(t)}{dt}\right|_{t=1} = \int_{-5}^{5} u_1(1-\tau)\cos(2\pi\tau)d\tau$$

which is the desired integral. We now evaluate the value of the integral as

$$\left.\frac{dx(t)}{dt}\right|_{t=1} = \sin(2\pi t)|_{t=1} = 0.$$

$$\int_{-5}^{5} \delta'(1-\tau)\cos 2\pi\tau \, d\tau$$

$$\xrightarrow[\tau=1-m]{1-\tau=m,\ d\tau=-dm} -\int_{6}^{-4} \delta'(m)\cos 2\pi(1-m)\, dm$$

$$= \int_{-4}^{6} \cos 2\pi m \, d\,\delta(m)$$

$$= \cos 2\pi m \cdot \delta(m)\Big|_{-4}^{6} - \int_{-4}^{6} (-2\pi \sin 2\pi m)\,\delta(m)\, dm$$

$$= 0 + 0 \cdot \int_{-4}^{6} \delta(m)\, dm = 0$$

2.29. The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

(a) $h(t) = e^{-4t}u(t-2)$
(b) $h(t) = e^{-6t}u(3-t)$
(c) $h(t) = e^{-2t}u(t+50)$
(d) $h(t) = e^{2t}u(-1-t)$

(a) Causal because $h(t) = 0$ for $t < 0$. Stable because $\int_{-\infty}^{\infty} |h(t)|dt = e^{-8}/4 < \infty$.

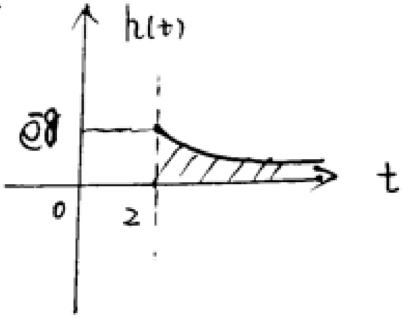
(b) Not causal because $h(t) \neq 0$ for $t < 0$. Unstable because $\int_{-\infty}^{\infty} |h(t)| = \infty$.

(c) Not causal because $h(t) \neq 0$ for $t < 0$. a Stable because $\int_{-\infty}^{\infty} |h(t)|dt = e^{100}/2 < \infty$.

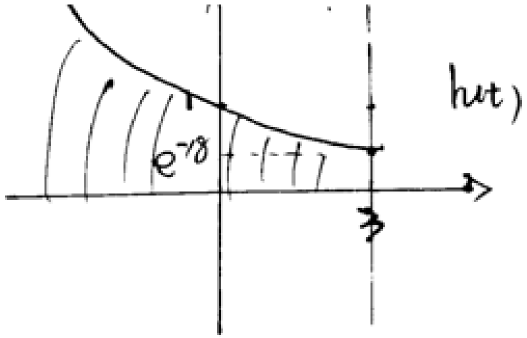
(d) Not causal because $h(t) \neq 0$ for $t < 0$. Stable because $\int_{-\infty}^{\infty} |h(t)|dt = e^{-2}/2 < \infty$.

2.29

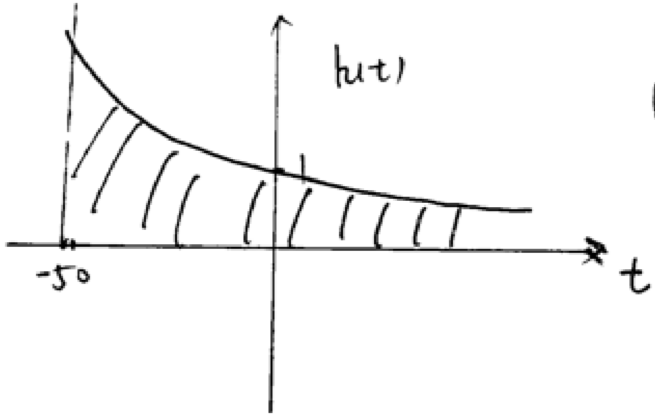
(a)



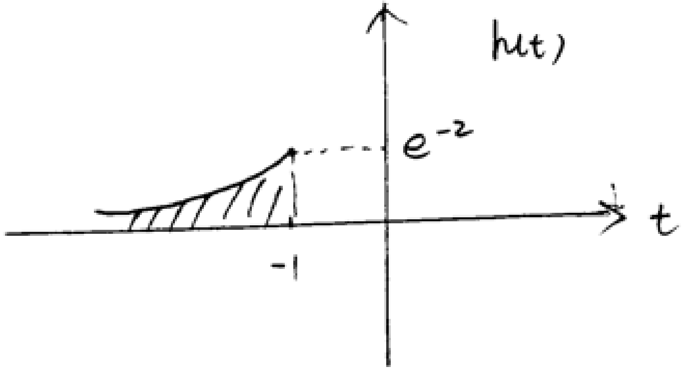
(b)



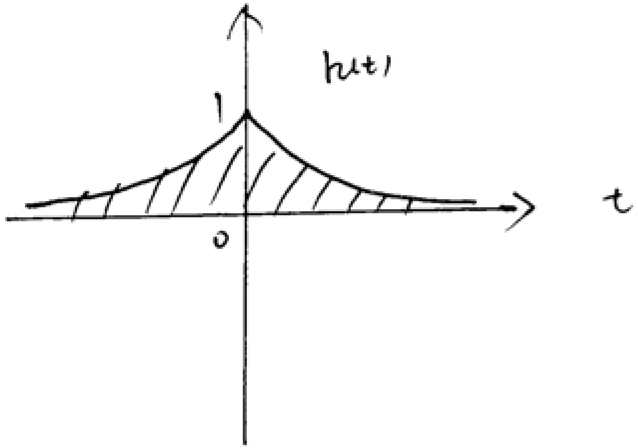
(c)



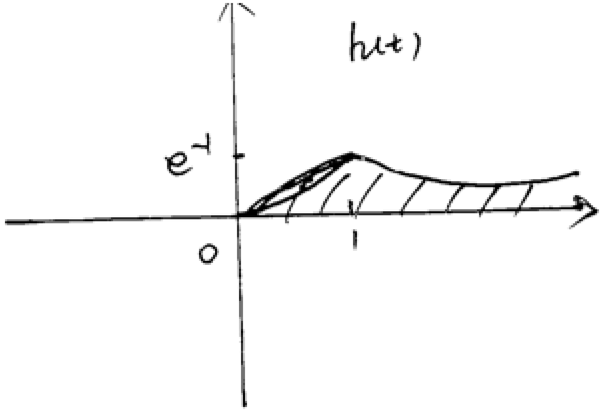
(d)



(e)



(f)



(g)

$$\begin{cases} h_1(t) = 2e^{-t}u(t) \\ h_2(t) = -e^{(t-100)/100}u(t). \\ h(t) = h_1(t) + h_2(t). \end{cases}$$

(e) $h(t) = e^{-6|t|}$

(f) $h(t) = te^{-t}u(t)$

(g) $h(t) = (2e^{-t} - e^{(t-100)/100})u(t)$

(e) Not causal because $h(t) \neq 0$ for $t < 0$. Stable because $\int_{-\infty}^{\infty} |h(t)|dt = 1/3 < \infty$.

(f) Causal because $h(t) = 0$ for $t < 0$. Stable because $\int_{-\infty}^{\infty} |h(t)|dt = 1 < \infty$.

(g) Causal because $h(t) = 0$ for $t < 0$. Unstable because $\int_{-\infty}^{\infty} |h(t)|dt = \infty$.

2.40. **(a)** Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau. = \int_{-\infty}^{\infty} e^{-(t-\tau)} u(t-\tau) x(\tau-2) d\tau$$

$$= x(t-2) * e^{-t} u(t)$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response of the system when the input $x(t)$ is as shown in Figure P2.40.

$$= x(t) * e^{-(t-2)} u(t-2)$$

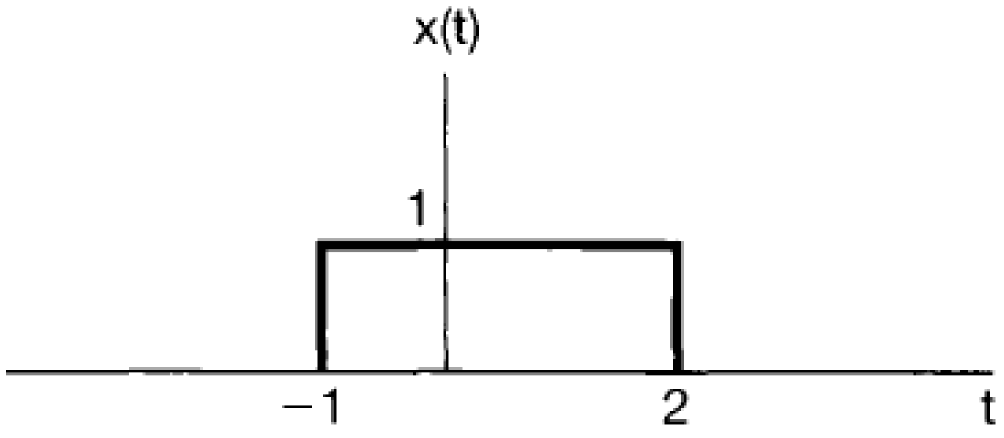


Figure P2.40

2.40 / a)

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)\, d\tau$$

Please note that $x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau$

$$\therefore\ y(t) = \int_{-\infty}^{+\infty} e^{-(t-\tau)} u(t-\tau)\, x(\tau-2)\, d\tau$$

$$= x(t-2) * \boxed{e^{-t} u(t)} \Longrightarrow \text{Note that it is not } h(t)\,!$$

$$= x(t) * e^{-(t-2)} u(t-2)$$

LTI & $h(t) = e^{-(t-2)} u(t-2) = e^{-t} u(t) * \delta(t-2)$

b) $x(t) = u(t+1) - u(t-2) = u(t) * [\delta(t+1) - \delta(t-2)]$

$$\therefore\ y(t) = u(t) * [\delta(t+1) - \delta(t-2)] * e^{-t}u(t) * \delta(t-2)$$

$$= \underline{u(t) * e^{-t}u(t)} * [\delta(t-1) - \delta(t-4)]$$

Note $e^{-at}u(t) * u(t) = \dfrac{1-e^{-at}}{a}u(t)$

$$= (1-e^{-t})\,u(t) * [\delta(t-1) - \delta(t-4)]$$

$$= [1-e^{-(t-1)}]\,u(t-1) - [1-e^{-(t-4)}]\,u(t-4)$$

2.43. One of the important properties of convolution, in both continuous and discrete time, is the associativity property. In this problem, we will check and illustrate this property.

(a) Prove the equality

$$[x(t) * h(t)] * g(t) = x(t) * [h(t) * g(t)] \qquad \text{(P2.43–1)}$$

by showing that both sides of eq. (P2.43–1) equal

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(\tau)h(\sigma)g(t-\tau-\sigma)\,d\tau\,d\sigma.$$

[3]更多卷积性质证明请参考: https://zhuanlan.zhihu.com/p/150737244

2.43. (a) We first have

$$
\begin{aligned}
[x(t) * h(t)] * g(t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(\sigma' - \tau)g(t - \sigma')d\tau d\sigma' \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(\sigma)g(t - \sigma - \tau)d\tau d\sigma
\end{aligned}
$$

Also,

$$
\begin{aligned}
x(t) * [h(t) * g(t)] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t - \sigma')h(\tau)g(\sigma' - \tau)d\sigma' d\tau \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\sigma)h(\tau)g(t - \tau - \sigma)d\tau d\sigma \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(\sigma)g(t - \sigma - \tau)d\tau d\sigma
\end{aligned}
$$

The equality is proved.

①

$$[x(t) * h(t)] * g(t) = \int_{-\infty}^{+\infty} \Big(\underbrace{\int_{-\infty}^{+\infty} x(\tau)\, h(\sigma'-\tau)\, d\tau}_{f(\sigma')} \Big)\, g(t-\sigma')\, d\sigma'$$

$$= \int_{-\infty}^{+\infty} x(\tau) \cdot \Big(\int_{-\infty}^{+\infty} h(\sigma'-\tau)\, g(t-\sigma')\, d\sigma' \Big)\, d\tau$$

$$\overset{\sigma'-\tau=\sigma}{\underset{\sigma'=\sigma+\tau,\ d\sigma'=d\sigma}{=\!=}} \int_{-\infty}^{+\infty} x(\tau) \cdot \Big(\int_{-\infty}^{+\infty} h(\sigma)\, g(t-\sigma-\tau)\, d\sigma \Big)\, d\tau$$

$$= \int_{-\infty}^{+\infty} x(\tau)\, h(\sigma)\, g(t-\sigma-\tau)\, d\tau\, d\sigma$$

② $$x(t) * [h(t) * g(t)] = \int_{-\infty}^{+\infty} x(t-\sigma') \cdot \Big(\underbrace{\int_{-\infty}^{+\infty} h(\tau) g(\sigma'-\tau) d\tau}_{f(\sigma')}\Big) d\sigma'$$

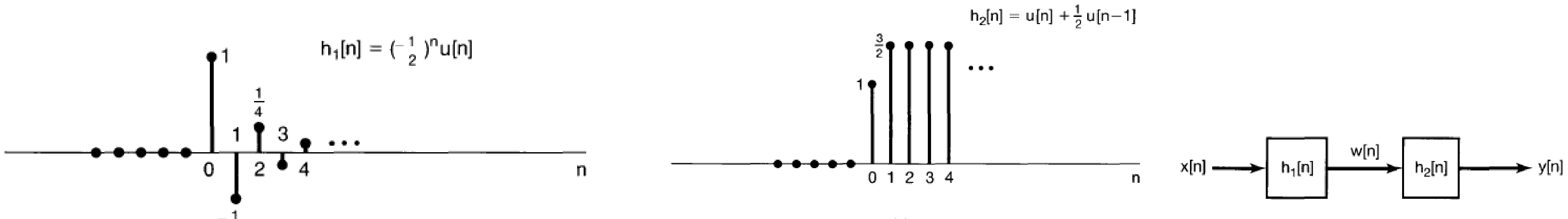
$$= \int_{-\infty}^{+\infty} h(\tau) \cdot \Big(\int_{-\infty}^{+\infty} x(t-\sigma') \cdot g(\sigma'-\tau) d\sigma'\Big) d\tau$$

$$\underset{\sigma' = t-\sigma}{\overset{t-\sigma' = \sigma,\ d\sigma' = -d\sigma}{=\!=\!=}} \int_{-\infty}^{+\infty} h(\tau) \cdot \Big(\int_{-\infty}^{+\infty} x(\sigma) g(t-\sigma-\tau) d\sigma\Big) d\tau$$

$$= \int_{-\infty}^{+\infty} x(\sigma) h(\tau) g(t-\sigma-\tau) d\tau d\sigma$$

Based on ① = ②, we complete the proof.

(b) Consider two LTI systems with the unit sample responses $h_1[n]$ and $h_2[n]$ shown in Figure P2.43(a). These two systems are cascaded as shown in Figure P2.43(b). Let $x[n] = u[n]$.



(a) (b)

(i) Compute $y[n]$ by first computing $w[n] = x[n] * h_1[n]$ and then computing $y[n] = w[n] * h_2[n]$; that is, $y[n] = [x[n] * h_1[n]] * h_2[n]$.

(ii) Now find $y[n]$ by first convolving $h_1[n]$ and $h_2[n]$ to obtain $g[n] = h_1[n] * h_2[n]$ and then convolving $x[n]$ with $g[n]$ to obtain $y[n] = x[n] * [h_1[n] * h_2[n]]$.

The answers to (i) and (ii) should be identical, illustrating the associativity property of discrete-time convolution.

(b) (i) We first have

$$w[n] = u[n] * h_1[n] = \sum_{k=0}^{n} \left(-\frac{1}{2}\right)^k = \frac{2}{3}\left[1 - (-\frac{1}{2})^{n+1}\right] u[n].$$

Now,

$$y[n] = w[n] * h_2[n] = (n+1)u[n].$$

(ii) We first have

$$g[n] = h_1[n] * h_2[n] = \sum_{k=0}^{n} \left(-\frac{1}{2}\right)^k + \frac{1}{2}\sum_{k=0}^{n-1}(-\frac{1}{2})^k = u[n]$$

Now,

$$y[n] = u[n] * g[n] = u[n] * u[n] = (n+1)u[n].$$

The same result was obtained in both parts (i) and (ii).

(c) Consider the cascade of two LTI systems as in Figure P2.43(b), where in this case

$$h_1[n] = \sin 8n$$

and

$$h_2[n] = a^n u[n], \quad |a| < 1,$$

and where the input is

$$x[n] = \delta[n] - a\delta[n-1].$$

Determine the output $y[n]$. (*Hint:* The use of the associative and commutative properties of convolution should greatly facilitate the solution.)

(c) Note that

$$x[n] * (h_2[n] * h_1[n]) = (x[n] * h_2[n]) * h_1[n].$$

Also note that

$$x[n] * h_2[n] = \alpha^n u[n] - \alpha^n u[n-1] = \delta[n].$$

Therefore,

$$x[n] * h_1[n] * h_2[n] = \delta[n] * \sin 8n = \sin 8n.$$

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2.47. We are given a certain linear time-invariant system with impulse response $h_0(t)$. We are told that when the input is $x_0(t)$ the output is $y_0(t)$, which is sketched in Figure P2.47. We are then given the following set of inputs to linear time-invariant systems with the indicated impulse responses:

	Input x(t)	*Impulse response h(t)*
(a)	$x(t) = 2x_0(t)$	$h(t) = h_0(t)$
(b)	$x(t) = x_0(t) - x_0(t-2)$	$h(t) = h_0(t)$
(c)	$x(t) = x_0(t-2)$	$h(t) = h_0(t+1)$
(d)	$x(t) = x_0(-t)$	$h(t) = h_0(t)$
(e)	$x(t) = x_0(-t)$	$h(t) = h_0(-t)$
(f)	$x(t) = x_0'(t)$	$h(t) = h_0'(t)$

(a) $y(t) = 2y_0(t)$.
(b) $y(t) = y_0(t) - y_0(t-2)$.
(c) $y(t) = y_0(t-1)$.
(d) Not enough information.
(e) $y(t) = y_0(-t)$.
(f) $y(t) = y_0''(t)$.

[Here $x_0'(t)$ and $h_0'(t)$ denote the first derivatives of $x_0(t)$ and $h_0(t)$, respectively.]

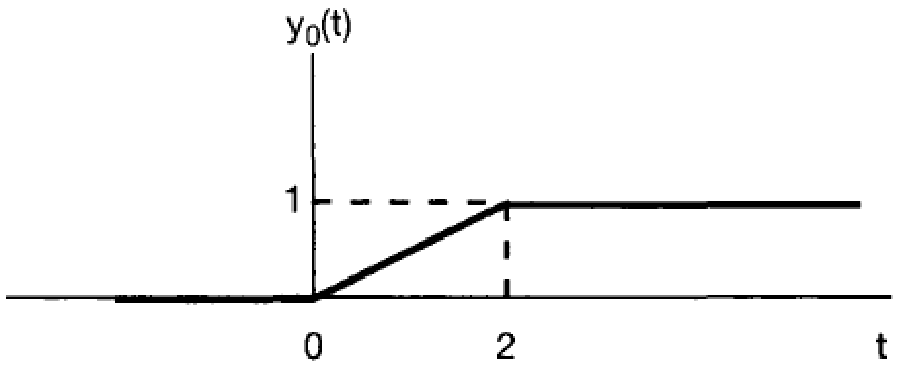

Figure P2.47

The signals for all parts of this problem are plotted in the Figure S2.47.

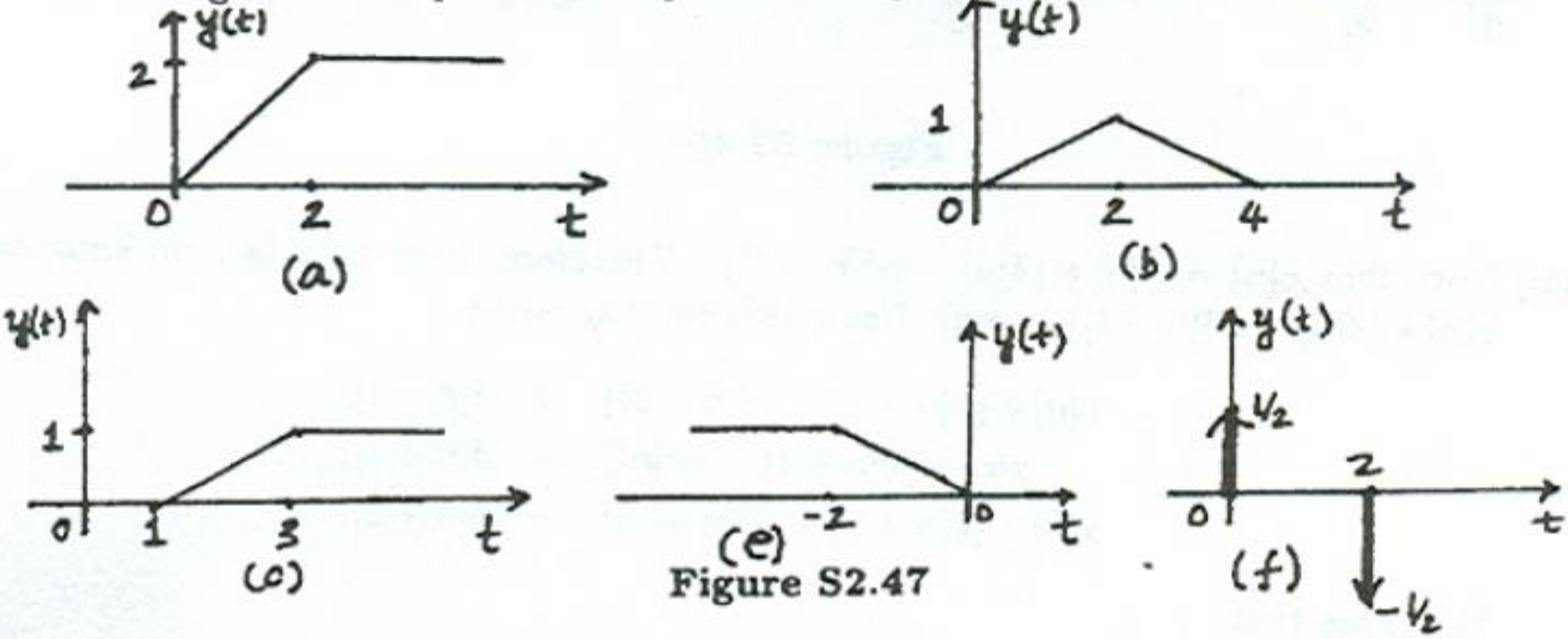

Figure S2.47

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Thanks for Your Attendance

Q&A

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