Signals and Systems

Southern University of Science and Technology

Autumn 2021

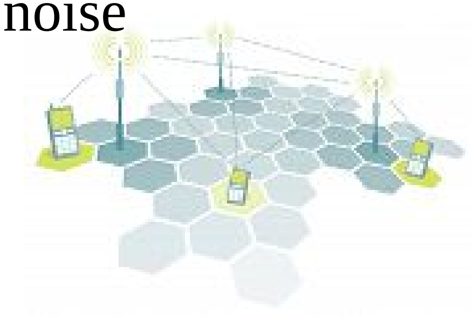
Signals and Systems

- The course is about using mathematical techniques to help analyze and synthesize systems which process signals.
  - Signals are variables that carry information.
  - Systems process input signals to produce output signals.

Slides partly extracted from “Signals and Systems”, Lecture Notes by Prof. Qing Hu, MIT, 2004, and Prof. Linshan Lee, NTU, 2009

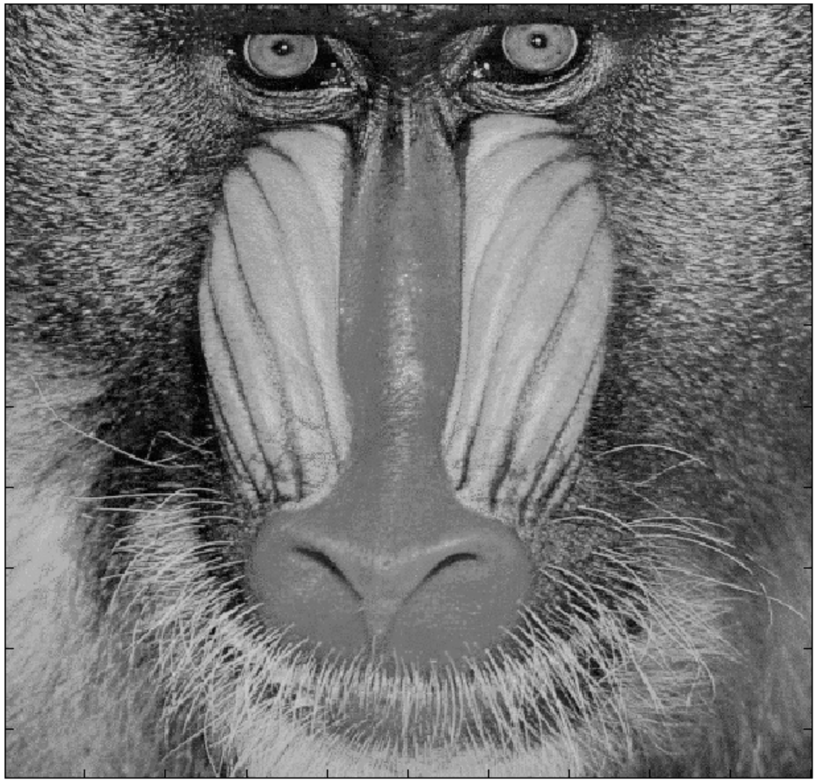
Typical examples of signals/systems
1) Communication systems

Message signal (information) speech, text, image, video → **Transmitter** → s(t) Transmitted signal → **Channel** (← noise) → r(t) Received signal → **Receiver** → Estimate of message signal (information)



Typical examples of signals/systems
2) Image

- Unblurred Image & No Noise



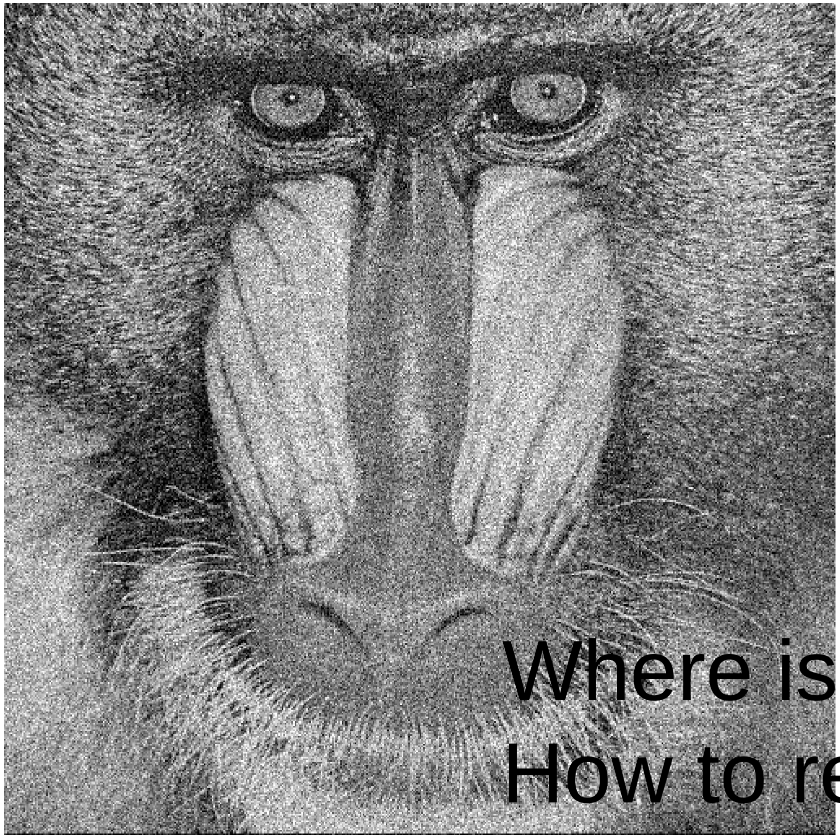
Typical examples of signals/systems
2) Image

- Blurred Image (bad focus)

Why blurred?
How to recover?

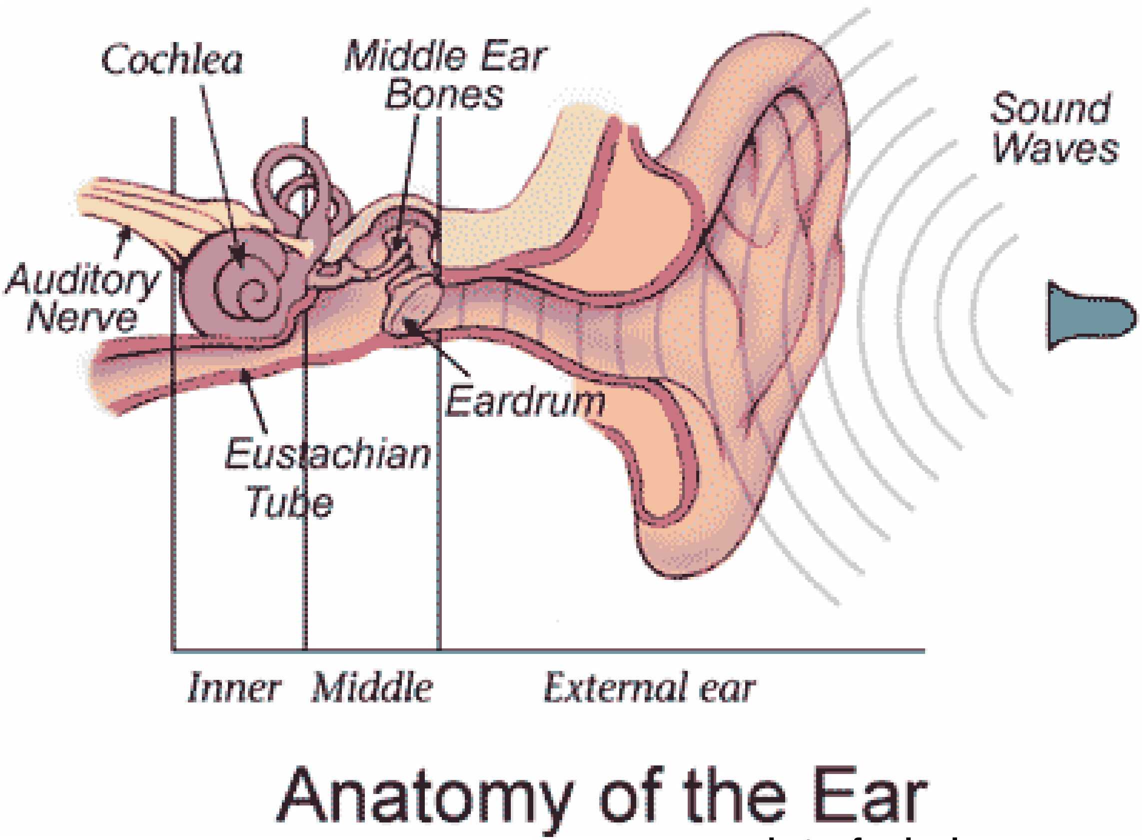
Typical examples of signals/systems
2) Image

- Unblurred Image – With Noise



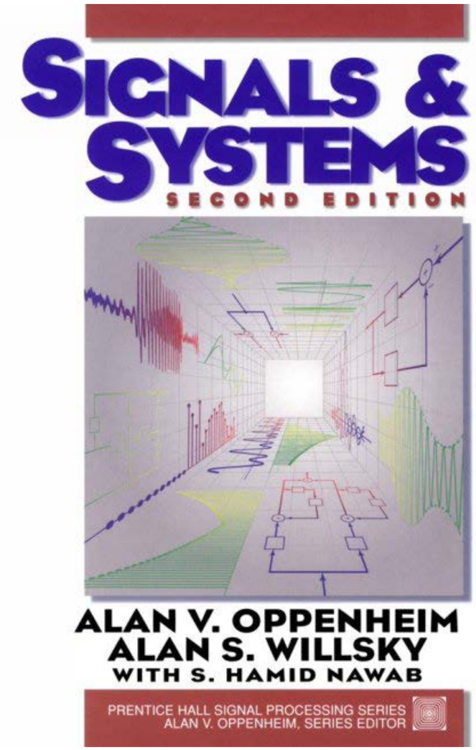
Where is noise added?
How to remove noise?

Typical examples of signals/systems
3) Human organ systems



med.stanford.edu

- "Signals and Systems", Oppenheim, Willsky and Nawab, 2nd Edition, 1997, Prentice-Hall.
- This course only teach Chapters 1 to 8.
  - Two weeks for one chapter
  - Middle-term exam for Chapters 1 to 4
  - Quiz at weeks 4 and 12
    - 平时成绩: 10%
    - 期中考试: 30%
    - 期末考试: 30%
    - 实验: 30%



Class Schedules (cont.)

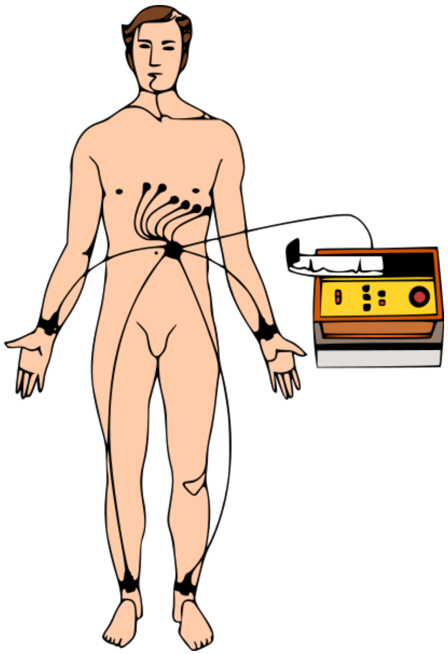
1. **Lab Session (start from week 1)**
2. **Tutorials (attend one)**
   - **Monday, Tuesday, Wednesday, Thursday，21:00-22:00**
   - **Room 111, Teaching Building I**
3. **Tutorial: Every week (no for week 1)**
4. **Assignment: Every week (no for week 1)**
   - **Submit assignment in the Blackboard system on Friday of the next week.**
   - **Late submission will have 20% reduction each day for the assignment score.**

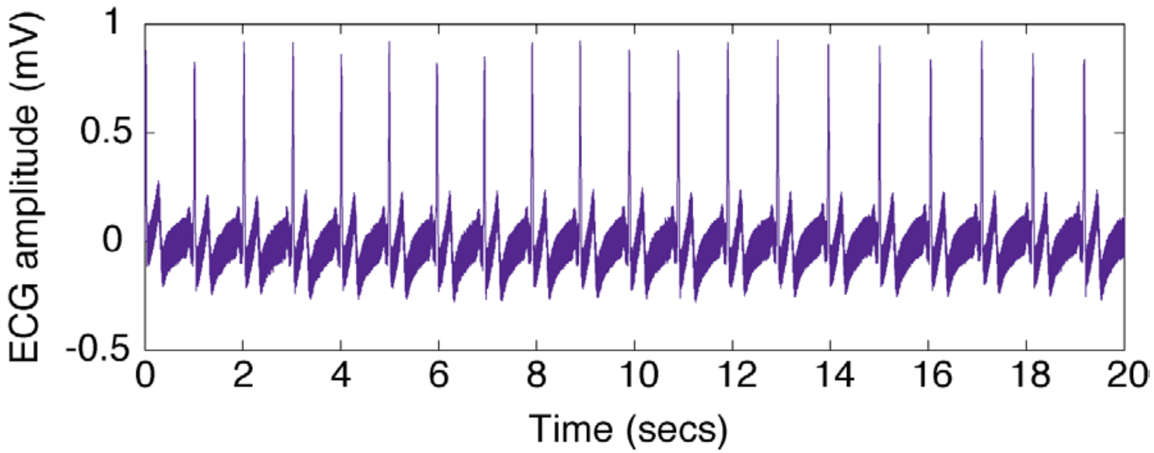
Examples of Signals: Physical Meaning

- Electrical signals – voltages and currents in a circuit
- Acoustic signals – audio or speech signals, vibration intensity of sound source
- Video signals – intensity variations in an image (e.g. a CT scan)
- Biological signals – sequence of bases in a gene (e.g., '…ATGGCTGA…')
- We will treat noise as unwanted signals.

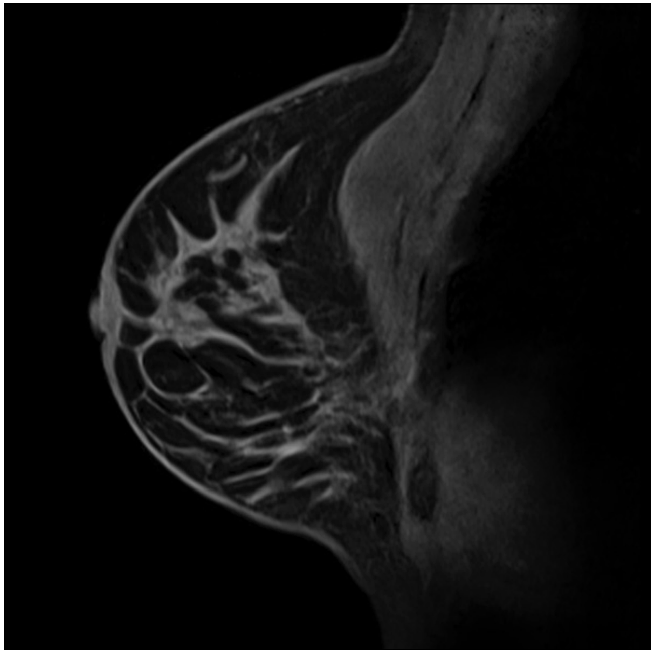
Signal Classification:
1) Type of Independent Variable

- Time is often the independent variable. Example: the electrical activity of the heart recorded with chest electrodes — the electrocardiogram (ECG).





The variables can also be spatial

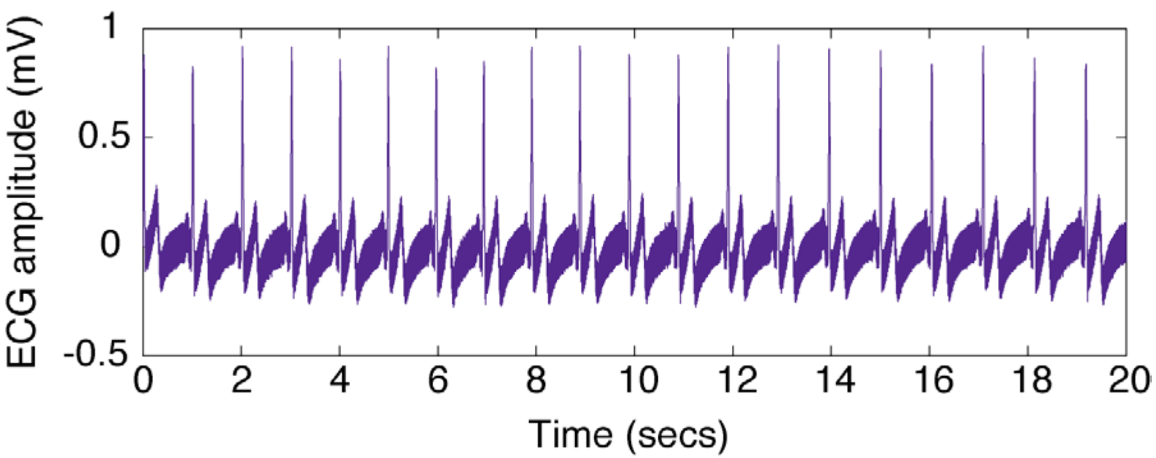


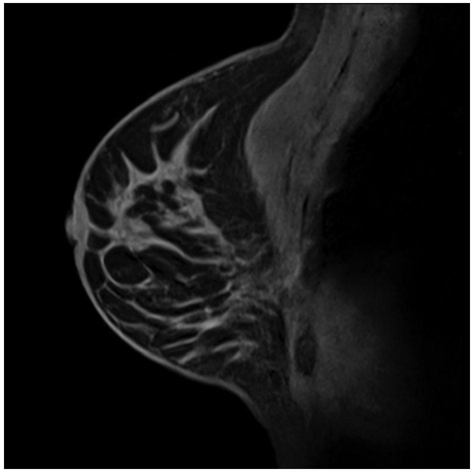
- e.g. Breast MRI

In this example, the signal is the intensity as a function of the spatial variables *x* and *y*.

Independent Variable Dimensionality

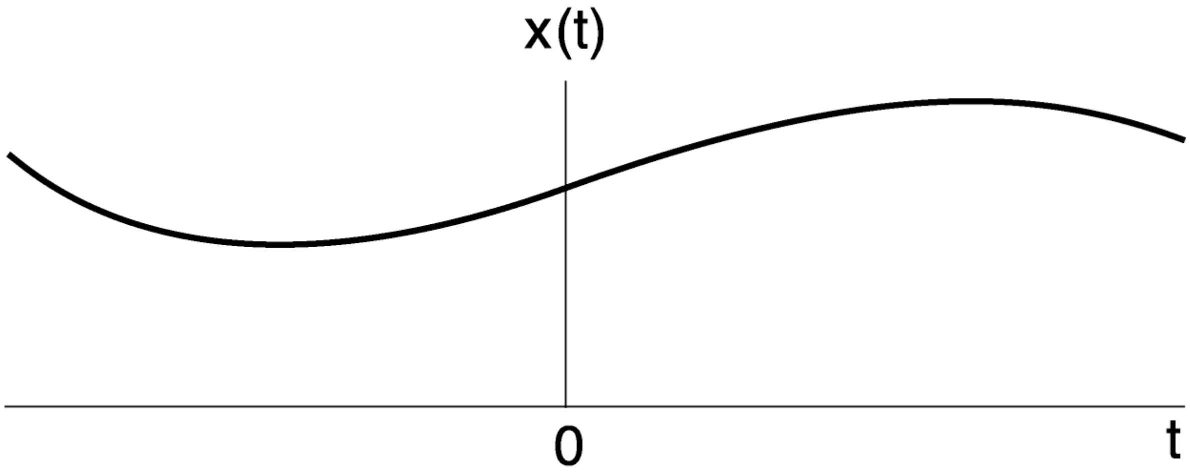
- An independent variable can be 1-D (*t* in the ECG), 2-D (*x*, *y* in an image), or 3-D (*x*, *y*, *t* in an video).





- We focus on 1-D for mathematical simplicity but the results can be extended to 2-D or even higher dimensions.
- Also, we will use a generic time ***t*** for the independent variable, whether it is time or space.

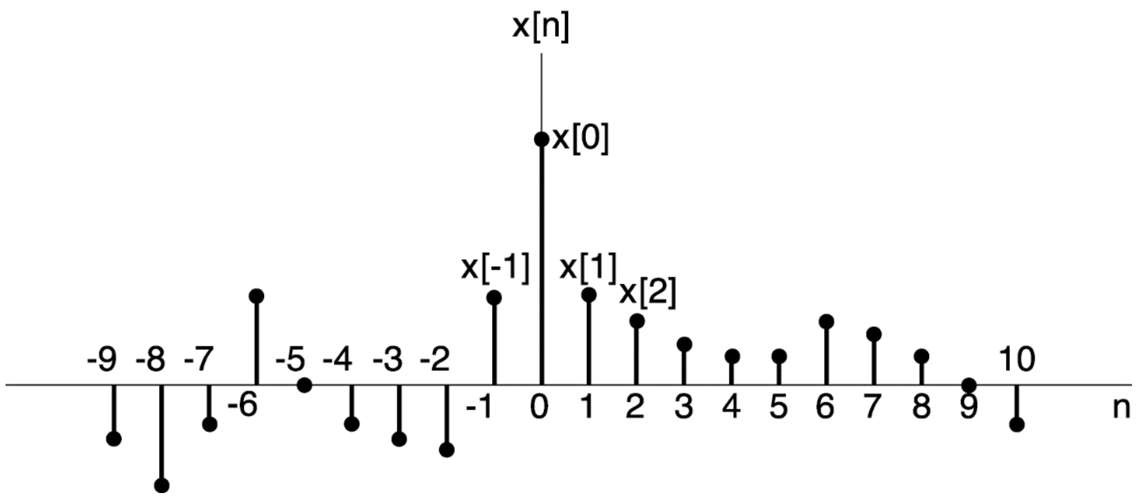
Signal Classification:
2) Continuous-time (CT) Signals



- Most of the signals in the physical world are CT signals, since the time scale is infinitesimally fine, so are the spatial scales. E.g. voltage & current, pressure, temperature, velocity, etc.

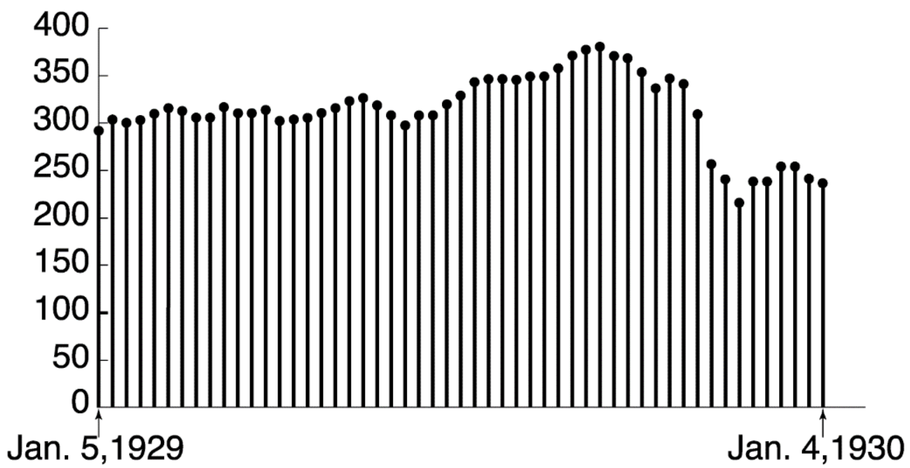
Discrete-time (DT) Signals

- $x[n]$, n — integer, time varies discretely



- Examples of DT signals in nature:
  - DNA base sequence
  - Population of the *n*th generation of certain species
  - …
- Notation: $x(t)$ — CT, $x[n]$ — DT

Many Human-made Signals are DT

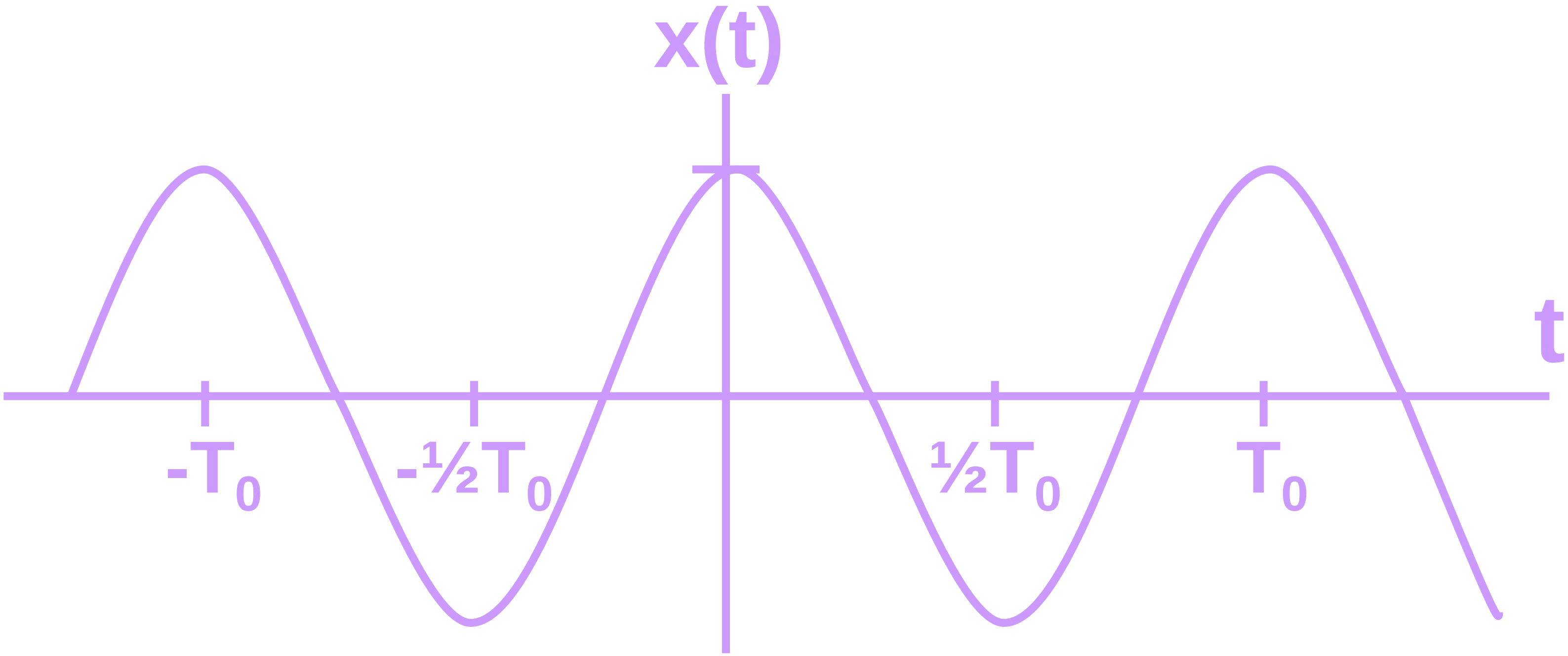


Weekly Dow-Jones industrial average

Digital image

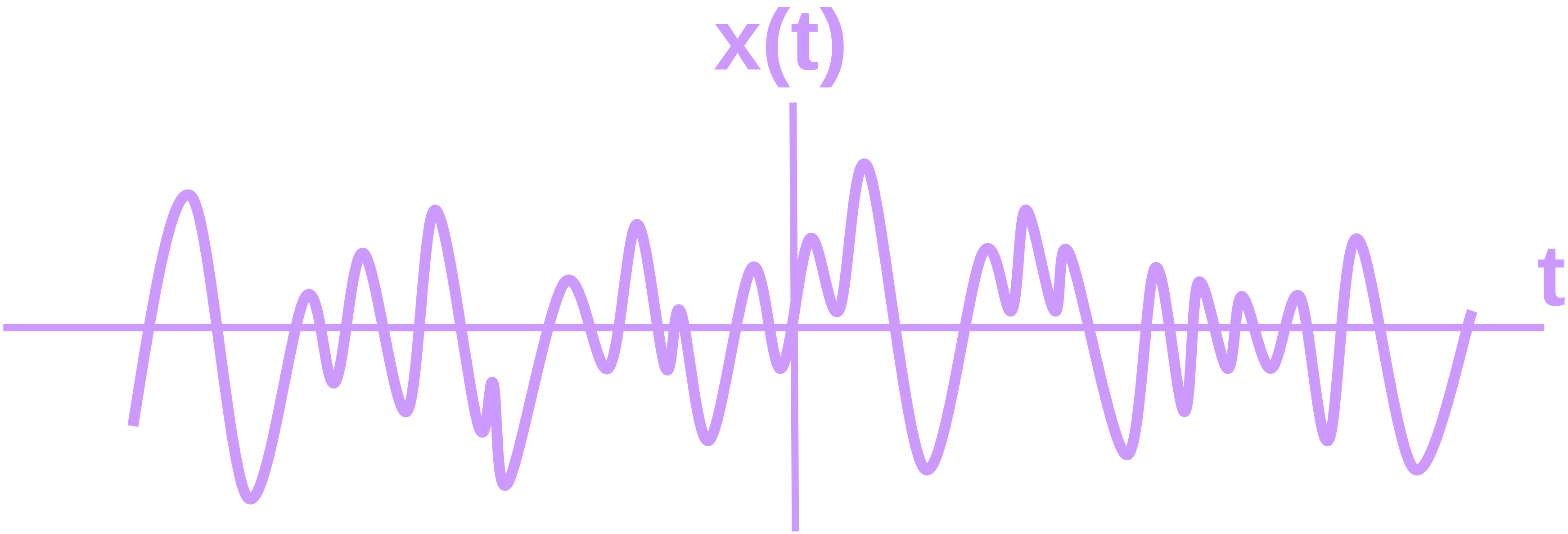
- Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

Signal Classification:
3) Deterministic Signal



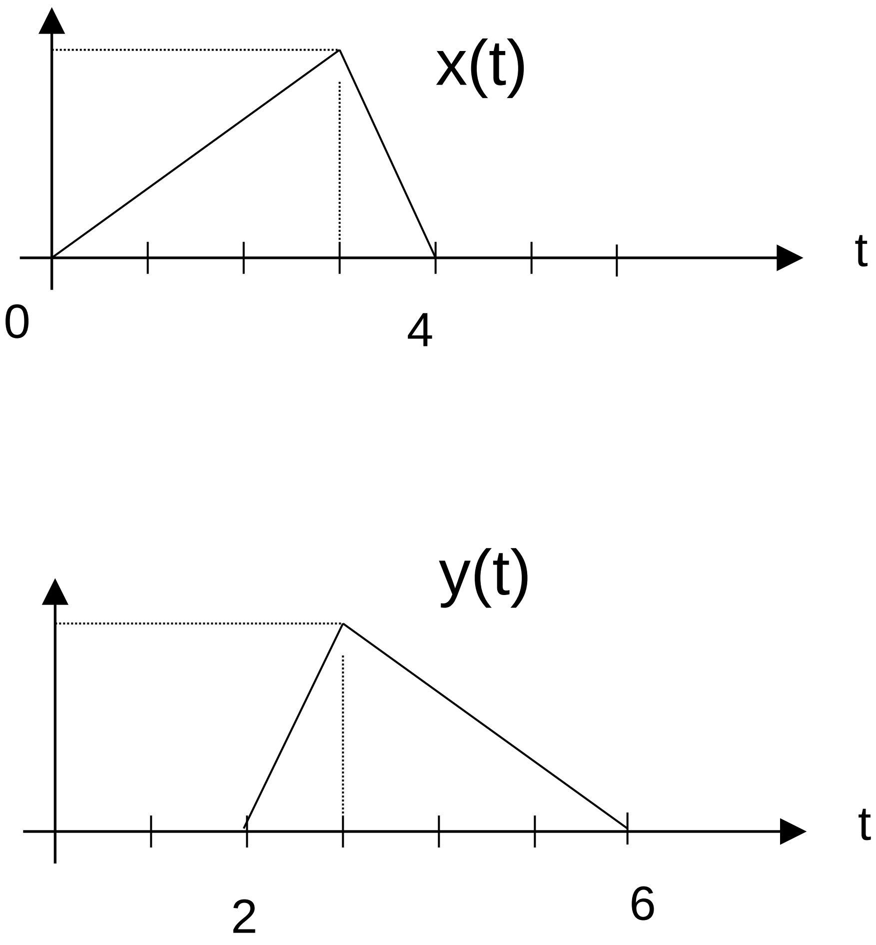
- Each value of the signal is fixed, and can be determined by a mathematical expression, rule, or table.
- Future values of the signal can be calculated from past values with complete confidence.

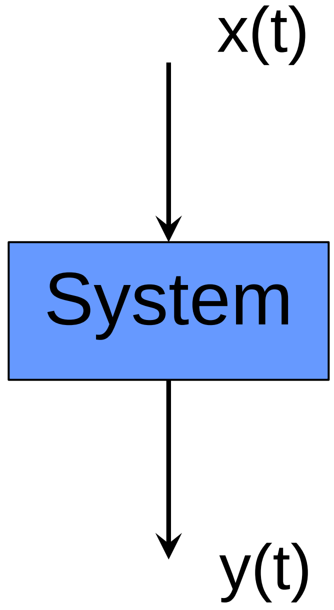
Random Signal



- Having a lot of uncertainty about its behaviour.
- Future values cannot be accurately predicted, and can usually only be guessed based on the averages of sets of signals.

Transformation of a Signal





Transformation of a Signal

- Time Shift

$$x(t) \rightarrow x(t-t_0) \quad , \quad x[n] \rightarrow x[n-n_0]$$

- Time Reversal

$$x(t) \rightarrow x(-t) \quad , \quad x[n] \rightarrow x[-n]$$

- Time Scaling

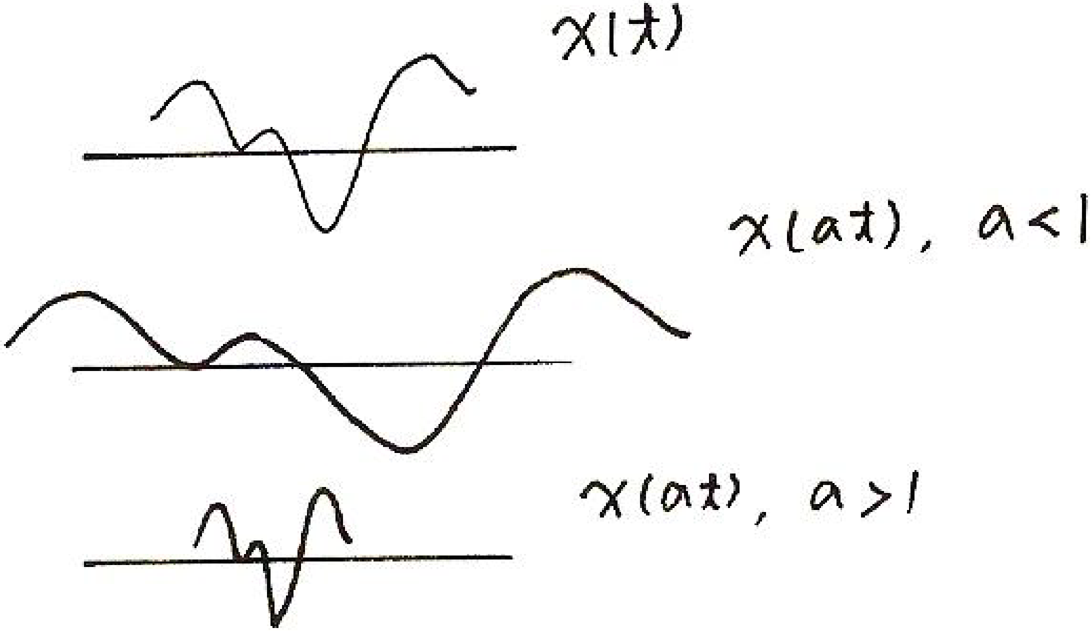
$$x(t) \rightarrow x(at) \quad , \quad x[n] \rightarrow ?$$

- Combination

$$x(t) \rightarrow x(at+b) \quad , \quad x[n] \rightarrow ?$$

Transformation of a Signal

Time Scaling



Transformation of a Signal

Combination

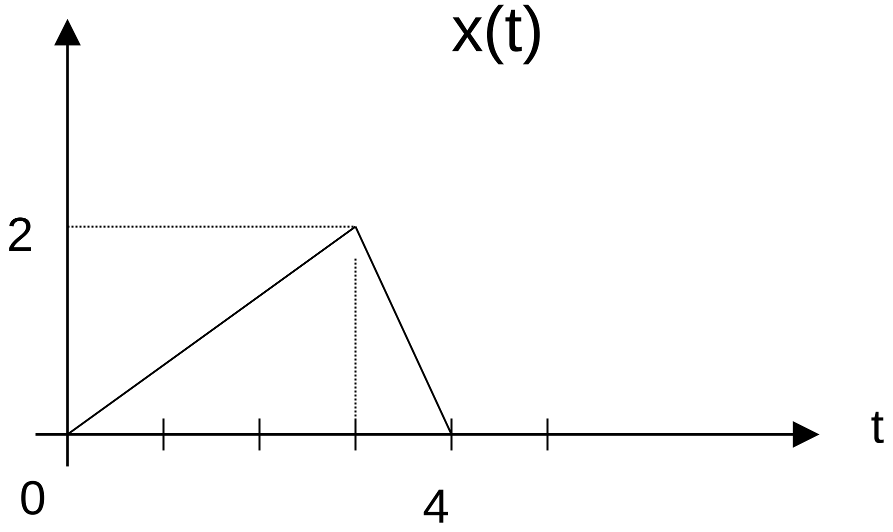
$x(t) \to x(at+b)$

1) Linearly stretched if $|a| < 1$, and linearly compressed if $|a| > 1$
2) Revered in time if $a < 0$
3) Shifted in time if $b \neq 0$

Suggested steps:

- First delay or advance x(t) with b, i.e. (3).
- Then scaling/reversing the resulting signal with factor a, i.e. (1) and (2).

Class problem



x(-2t+2) ?

Signals with Symmetry

- Periodic Signals
  - CT $x(t) = x(t + T)$, T : *period*

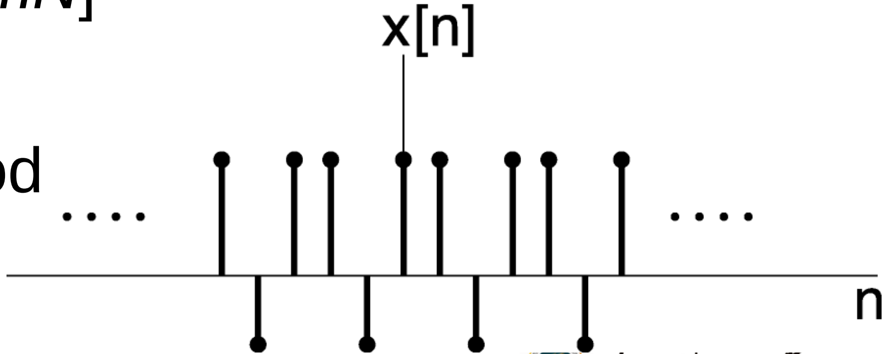
    $x(t) = x(t + mT)$, *m: integer*

    T_0: Fundamental period, the smallest positive value of T

  - DT $x[n] = x[n + N]=x[n + mN]$

    N: period

    N_0: fundamental period



- Aperiodic: NO period

Signals with Symmetry (cont.)

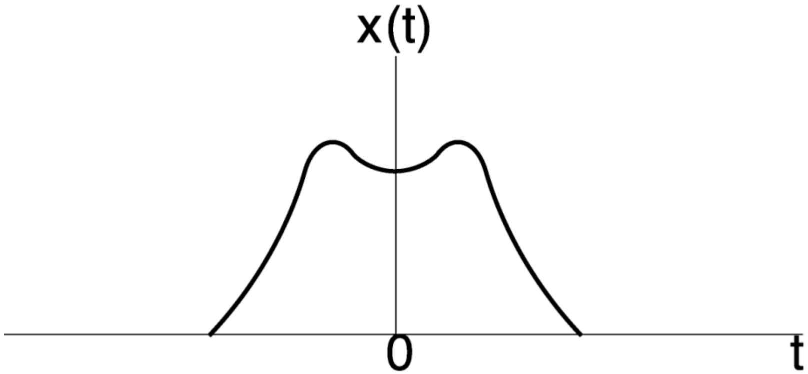
True or false

- If a signal is periodic, its duration is infinity from $(-\infty, +\infty)$.

- The period of continuous signal must be integer.

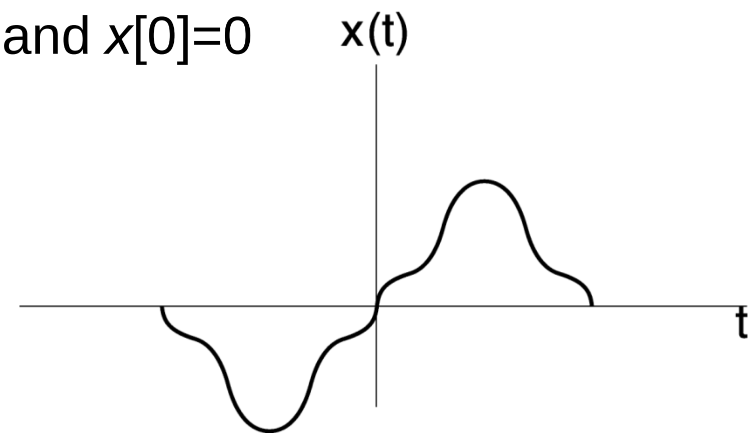
Signals with Symmetry (cont.)

- Even and Odd Signals
  - Even $x(t) = x(-t)$ *or* $x[n] = x[-n]$



Example: cos(t)

  - Odd $x(t) = -\,x(-t)$ *or* $x[n] = -\,x[-n]$
    - $x(0)=0$, and $x[0]=0$



Example: sin(t)

Signals with Symmetry (cont.)

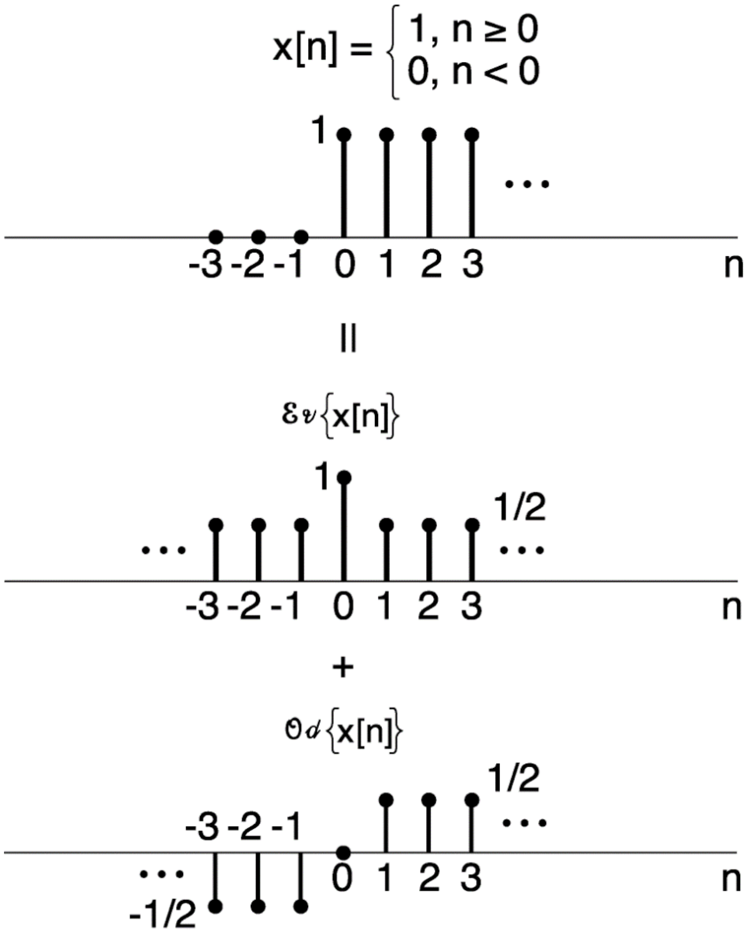
- Any signals can be expressed as a sum of *Even* and *Odd* signals. That is:

$x(t) = x_{even}(t) + x_{odd}(t)$,

where:

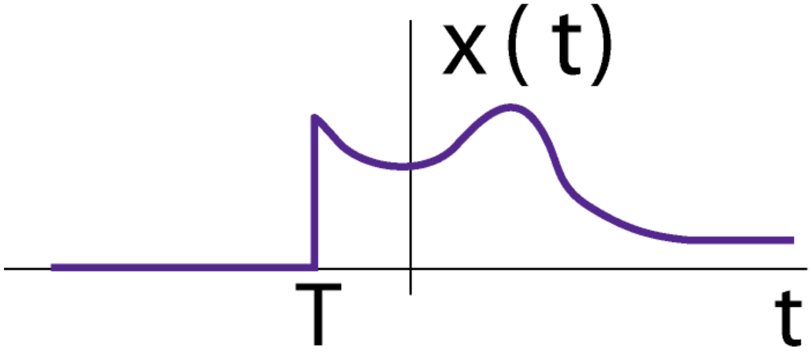
$x_{even}(t) = [x(t) + x(-t)]/2$,

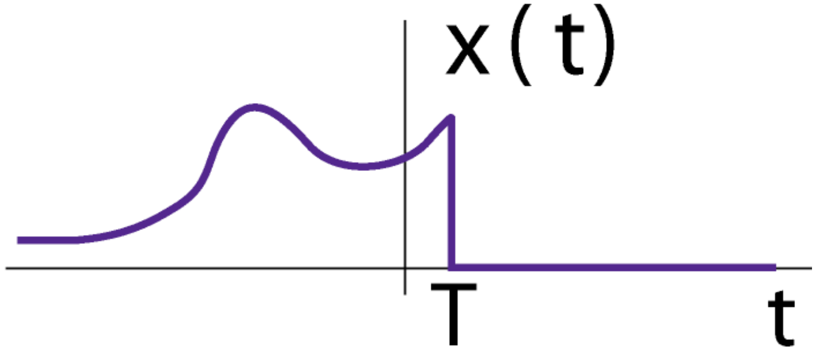
$x_{odd}(t) = [x(t) - x(-t)]/2$.



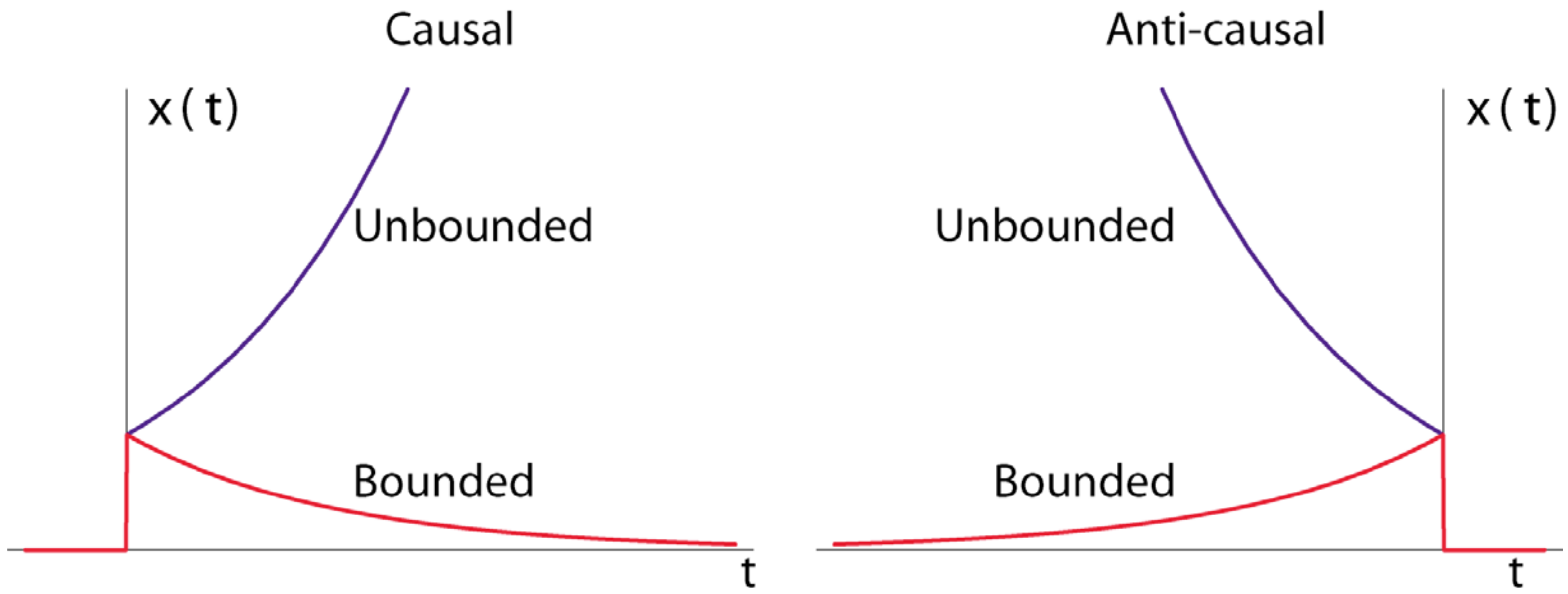
Right- and Left-Sided Signals

- A right-sided signal is zero for $t < T$, and
- A left-sided signal is zero for $t > T$, where T can be positive or negative.





Bounded and Unbounded Signals



- Whether the output signal of a *system* is bounded or unbounded determines the stability of the system.

Exponential Signals

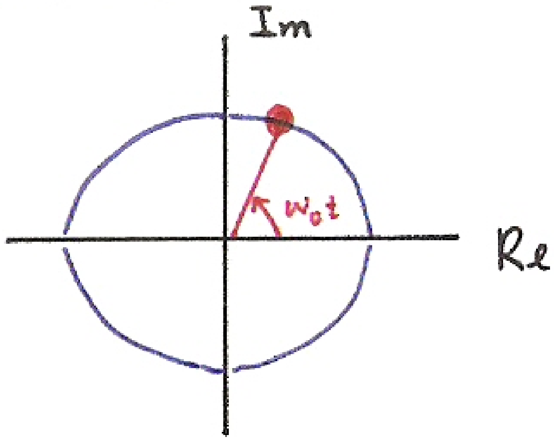
- A very important class of signals is presented as:
  - CT signals of the form $x(t) = e^{st}$
  - DT signals of the form $x[n] = z^n = e^{ßn}$, $z=e^{ß}$

    where *s* and z are complex numbers.
- For both *exponential* CT and DT signals, *x* is a complex quantity and has:
  - a real and imaginary part [i.e., *Cartesian form*], or equivalently
  - a magnitude and a phase angle [i.e., *polar form*].
- We will use whichever form that is convenient.

Periodic Complex Exponential/Sinusoidal Signals

when $s=j\omega_0$

$$x(t) = e^{j\omega_0 t}$$

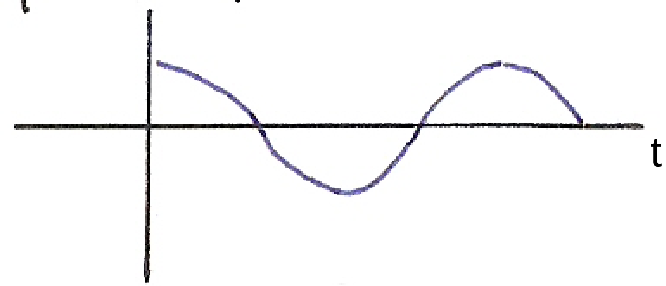


Euler's relation

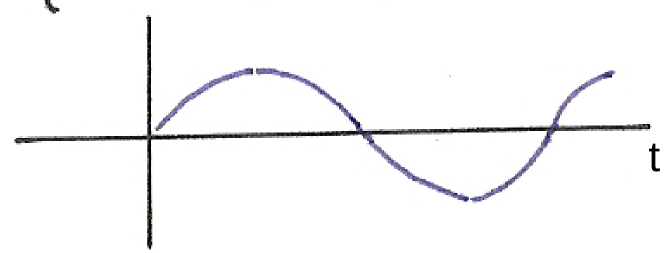
$$e^{jx} = \cos x + j\sin x$$

$j\omega_0 t$ is defined as phase

$$Re\{e^{j\omega_0 t}\} = \cos\omega_0 t$$



$$Im\{e^{j\omega_0 t}\} = \sin\omega_0 t$$

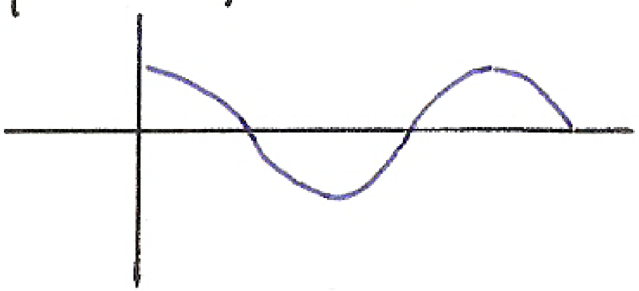


Real and imaginary parts are periodic signals with the same period, but out of phase (90^o phase difference)

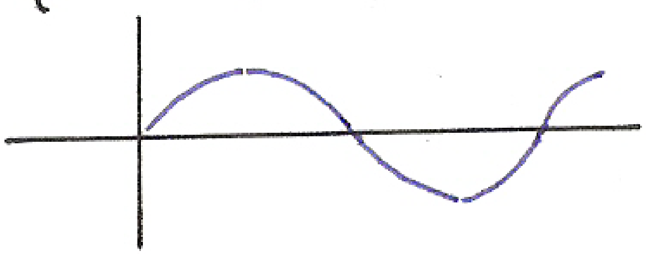
Periodic Complex Exponential/Sinusoidal Signals

$$x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

$\mathrm{Re}\{e^{jw_0t}\} = \cos w_0 t$



$\mathrm{Im}\{e^{jw_0t}\} = \sin w_0 t$



-Fundamental frequency: ω_0

-Fundamental period: $T_0 = \dfrac{2\pi}{\omega_0}$

-In CT, $e^{j\omega_0 t}$ **always periodic**

-Distinct signals for distinct values of

-Rapid variation with large ω_0

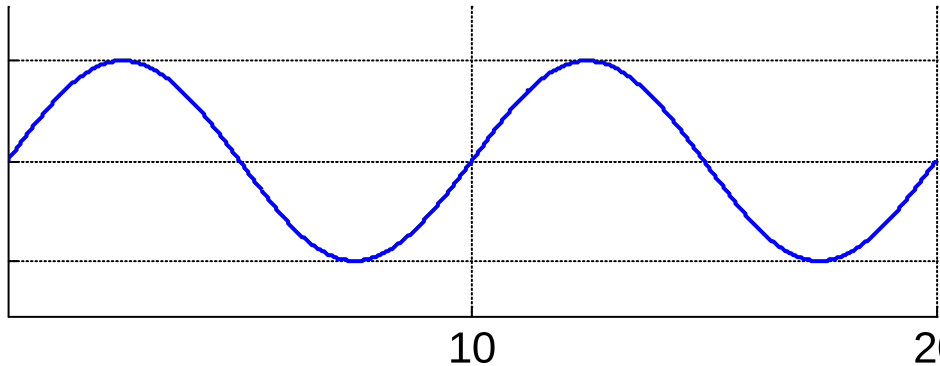
Periodic Complex Exponential/Sinusoidal Signals (cont.)

- To express sinusoidal by periodic exponentials, e.g.,

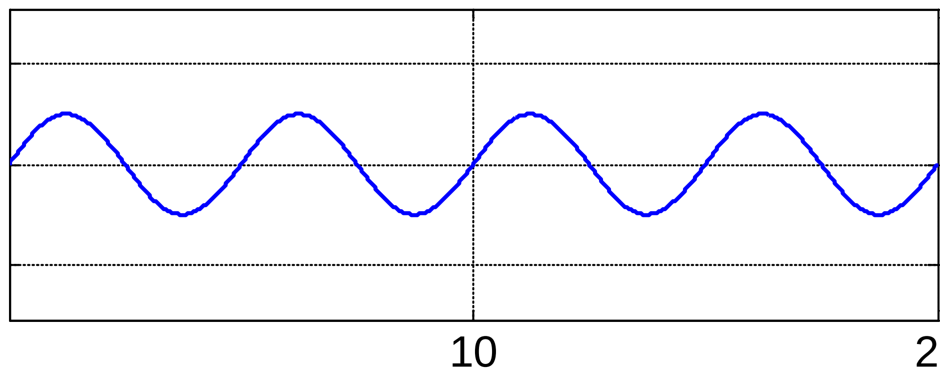
$$\cos(x) = \mathrm{Re}(e^{jx}) = (e^{jx} + e^{-jx})/2$$

$$\sin(x) = \mathrm{Im}(e^{jx}) = (e^{jx} - e^{-jx})/2j$$

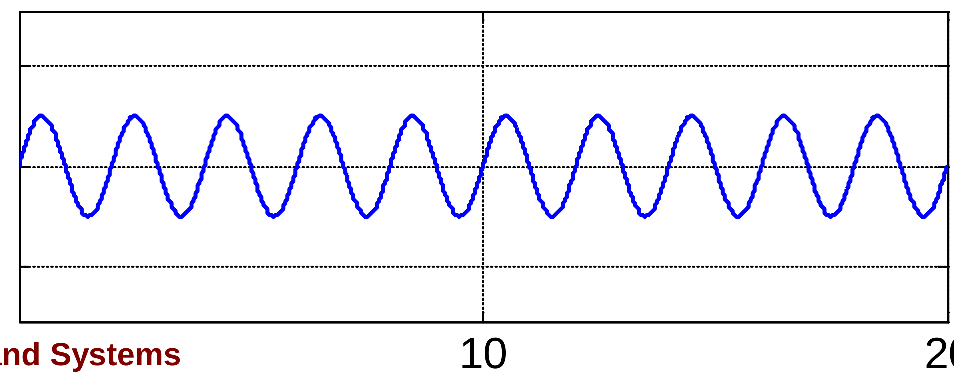
Harmonically Related Signal Sets



T_0=10 ms
ω_0 =2π×100 Hz



T_1=5 ms
ω_1 =2π×200 Hz



T_2=2 ms
ω_2 =2π×500 Hz

Harmonically Related Signal Sets (cont.)

- A set of periodic exponentials which have a common period T_0.

$$\{\phi_k(t) = e^{jk\omega_0 t}, k = 0, \pm 1, \pm 2, \dots\}$$

must be integer multiple

$$\text{fundamental frequency } |k\omega_0|$$

$$\text{fundamental period } \quad T_k = \frac{2\pi}{|k\omega_0|} = \frac{T_0}{|k|}, \quad T_0 = \frac{2\pi}{\omega_0}$$

- The *k*th harmonic $\phi_k(t)$ is periodic with period T_0, as it goes through $|k|$ of its fundamental periods T_k in duration of length T_0.

Summary of week 1

- Meaning of signals and systems
- How to describe signals?
- Transformation of a signal
- Signal properties
- Periodic complex exponential signal
  - Harmonically related signal set