Notes

- Assignments
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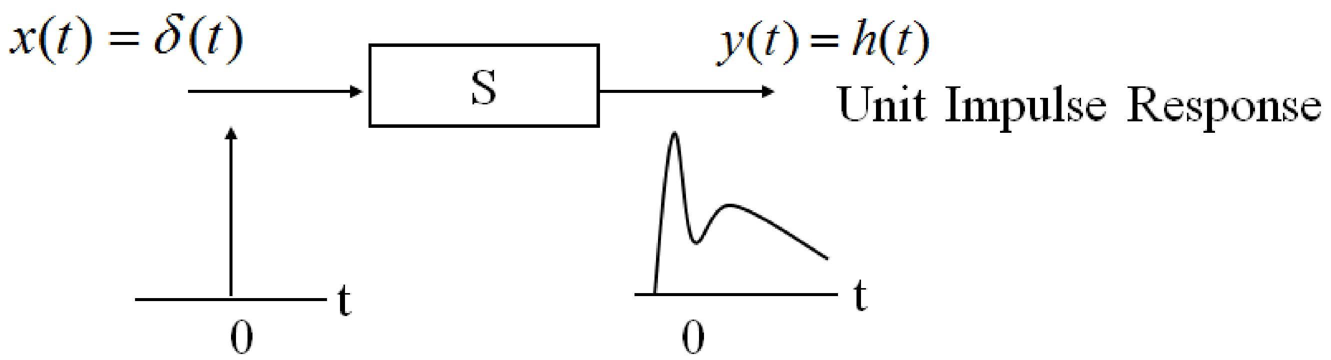
Review for Chapter 2

- **Why to introduce unit impulse response?**



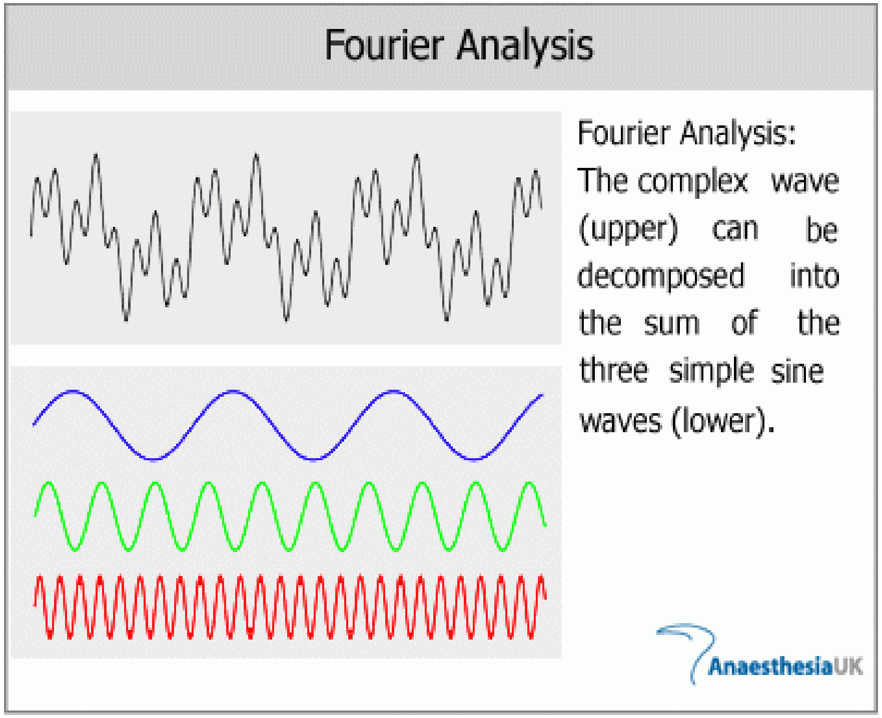
- **Why to introduce convolution?**
  - **(DT or CT) Signal can be represented by a linear combination of unit impulse function**
  - **When it goes through the system, the output is computed via convolution of input signal and unit impulse response**

Chapter 3

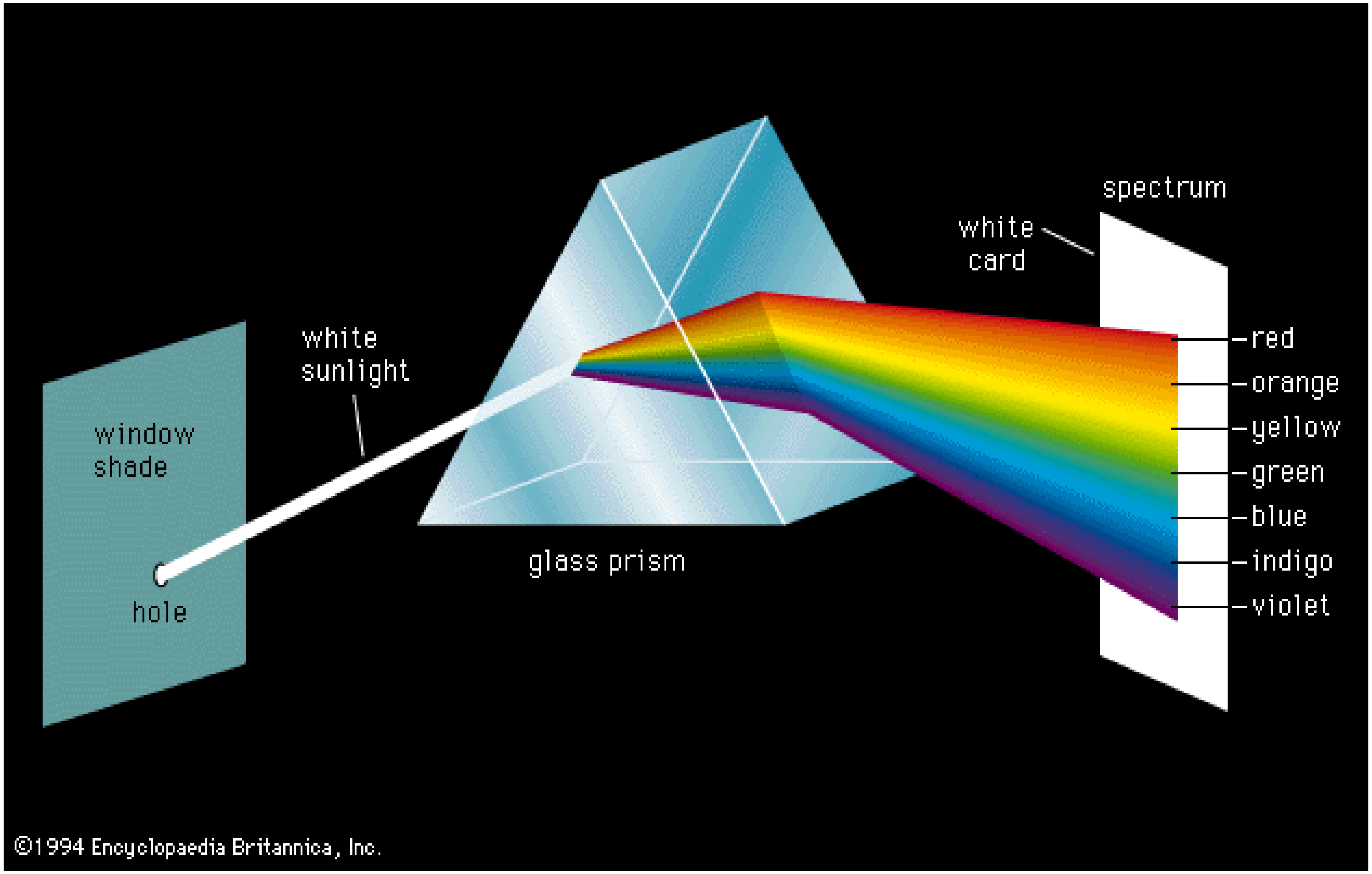
Fourier Series Representation of Periodic Signals

Fourier Series – A Time-Frequency Analysis Perspective

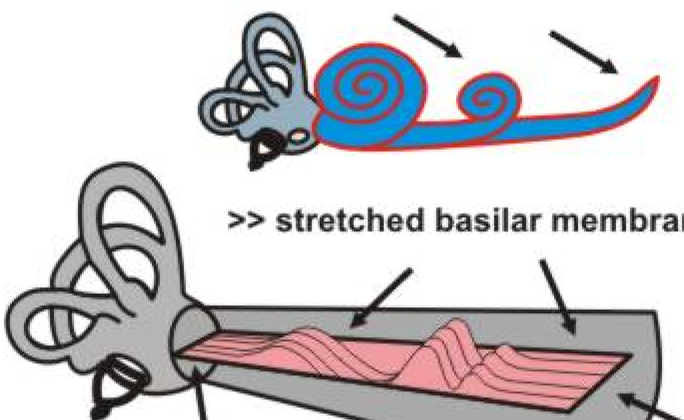
Joseph Fourier



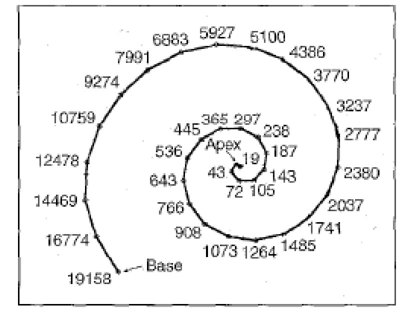
A Useful Analogy



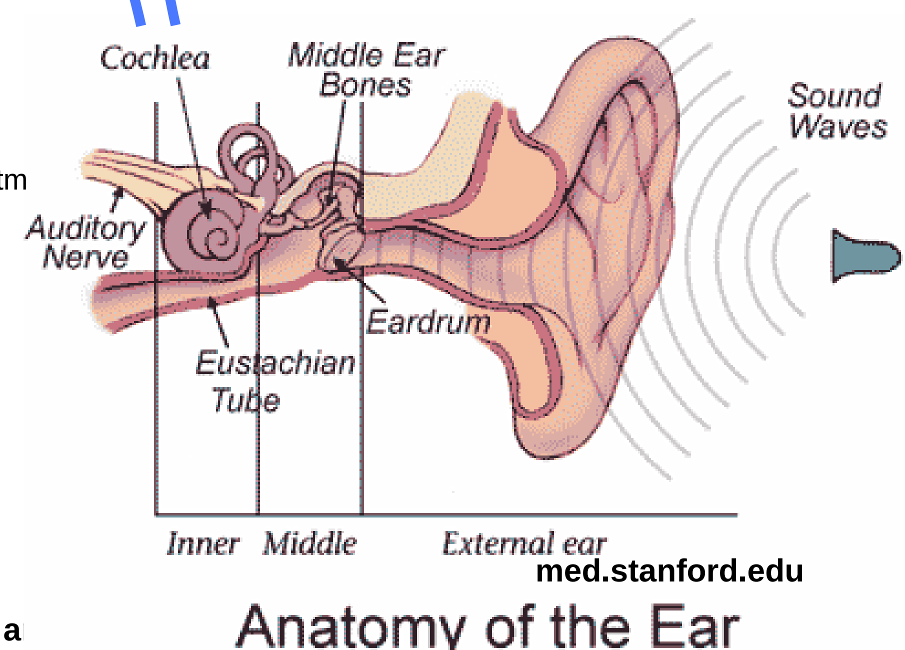
A Physiological Analogy



http://www.pc.rhul.ac.uk/staff/J.Zanker/PS1061/L5/PS1061_5.htm



Loizou, 1999



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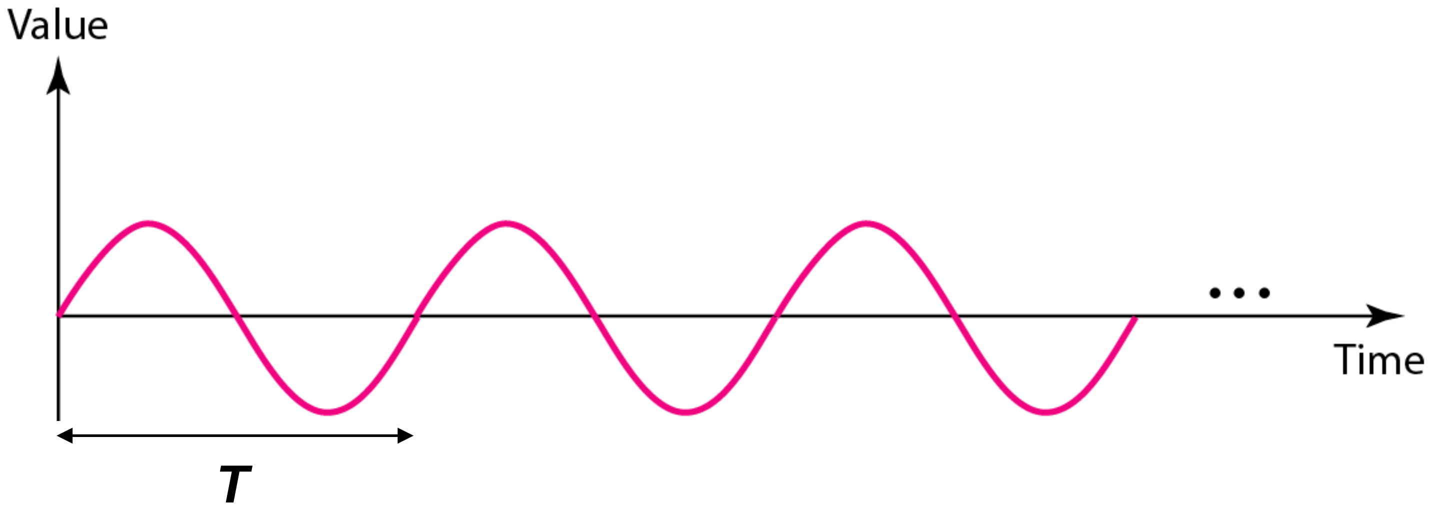
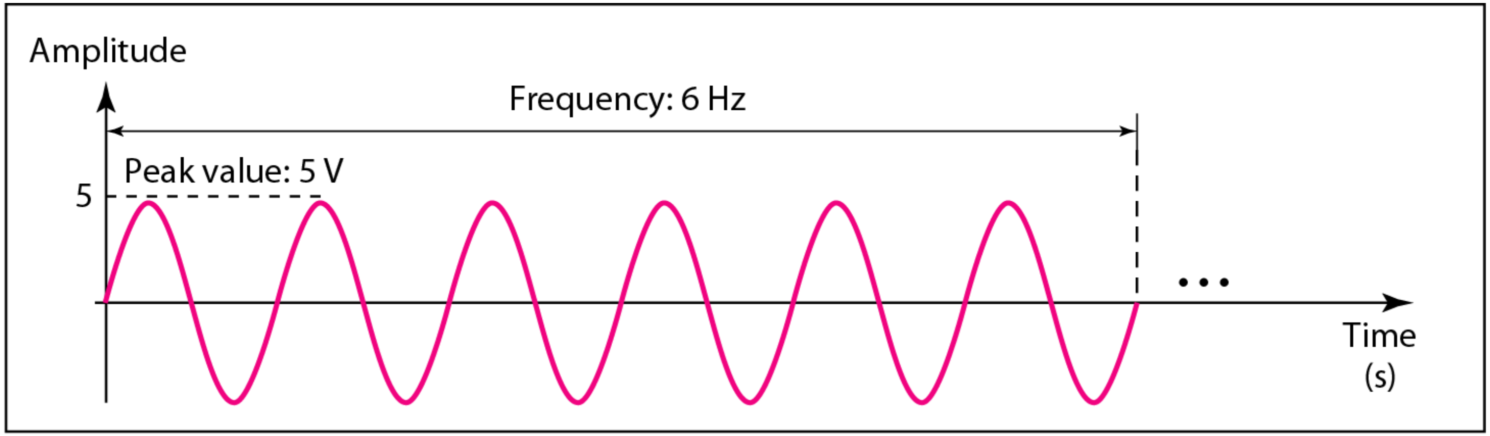
Note

Frequency and period are the inverse of each other.

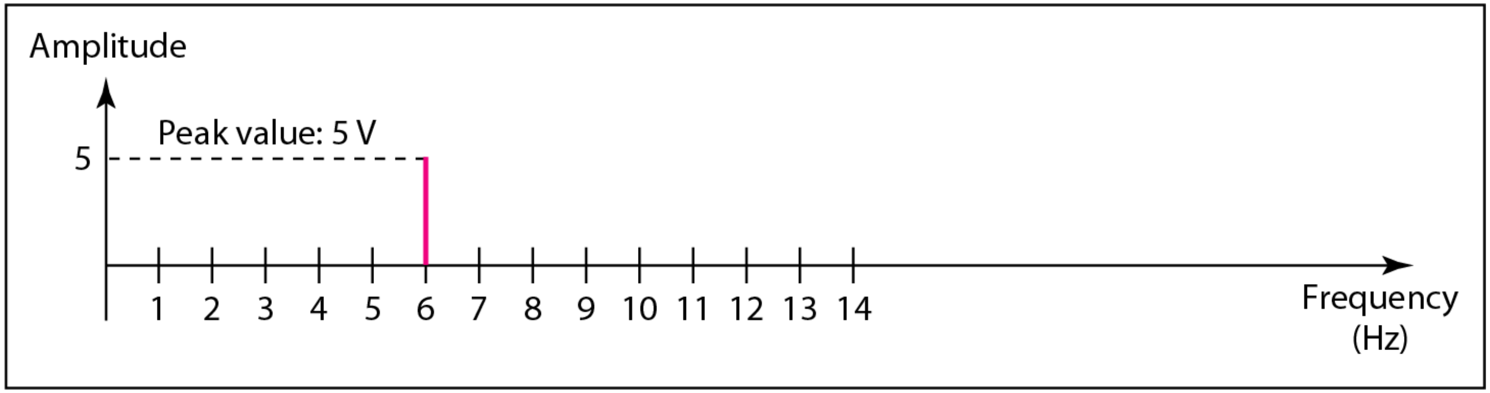
$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}$$



The time-domain and frequency-domain plots of a sine wave



a. A sine wave in the time domain (peak value: 5 V, frequency: 6 Hz)



b. The same sine wave in the frequency domain (peak value: 5 V, frequency: 6 Hz)

Example

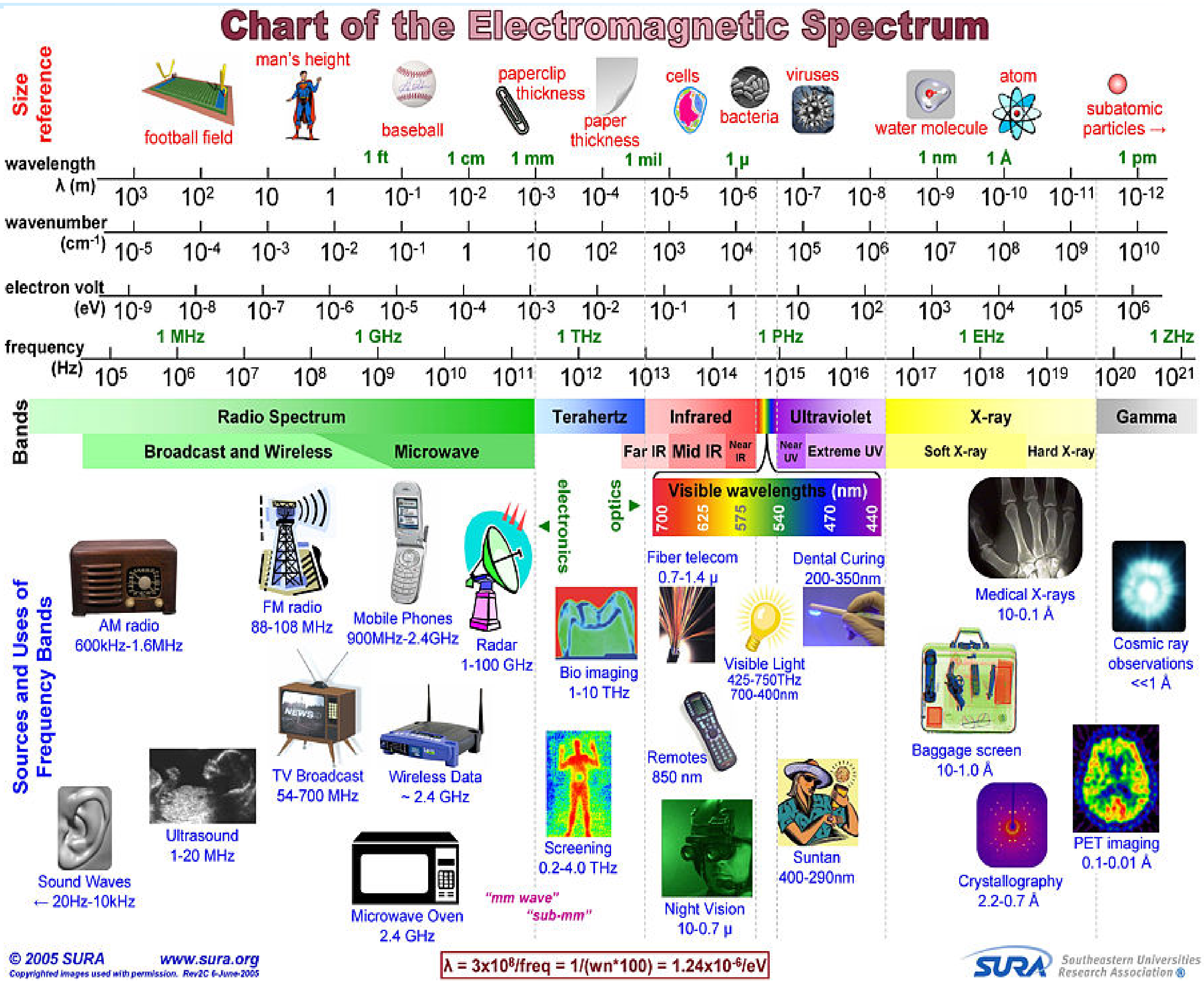


Chart of the Electromagnetic Spectrum
Size reference
football field
man's height
baseball
paperclip thickness
paper thickness
cells
bacteria
viruses
water molecule
atom
subatomic particles →
wavelength λ (m)
1 ft
1 cm
1 mm
1 mil
1 μ
1 nm
1 Å
1 pm
10^{3} 10^{2} 10 1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12}
wavenumber (cm^{-1})
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10}
electron volt (eV)
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^{2} 10^{3} 10^{4} 10^{5} 10^{6}
frequency (Hz)
1 MHz
1 GHz
1 THz
1 PHz
1 EHz
1 ZHz
10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19} 10^{20} 10^{21}
Bands
Radio Spectrum
Terahertz
Infrared
Ultraviolet
X-ray
Gamma
Broadcast and Wireless
Microwave
Far IR
Mid IR
Near IR
Near UV
Extreme UV
Soft X-ray
Hard X-ray
electronics
optics
Visible wavelengths (nm)
700 625 575 540 470 440
Sources and Uses of Frequency Bands
AM radio 600kHz-1.6MHz
FM radio 88-108 MHz
Mobile Phones 900MHz-2.4GHz
Radar 1-100 GHz
Bio imaging 1-10 THz
Fiber telecom 0.7-1.4 μ
Dental Curing 200-350nm
Medical X-rays 10-0.1 Å
Cosmic ray observations <<1 Å
Visible Light 425-750THz 700-400nm
TV Broadcast 54-700 MHz
Wireless Data ~ 2.4 GHz
Screening 0.2-4.0 THz
Remotes 850 nm
Baggage screen 10-1.0 Å
Sound Waves ← 20Hz-10kHz
Ultrasound 1-20 MHz
Microwave Oven 2.4 GHz
"mm wave"
"sub-mm"
Night Vision 10-0.7 μ
Suntan 400-290nm
Crystallography 2.2-0.7 Å
PET imaging 0.1-0.01 Å
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Copyrighted images used with permission. Rev2C 6-June-2005
λ = 3x10⁸/freq = 1/(wn*100) = 1.24x10⁻⁶/eV
SURA Southeastern Universities Research Association

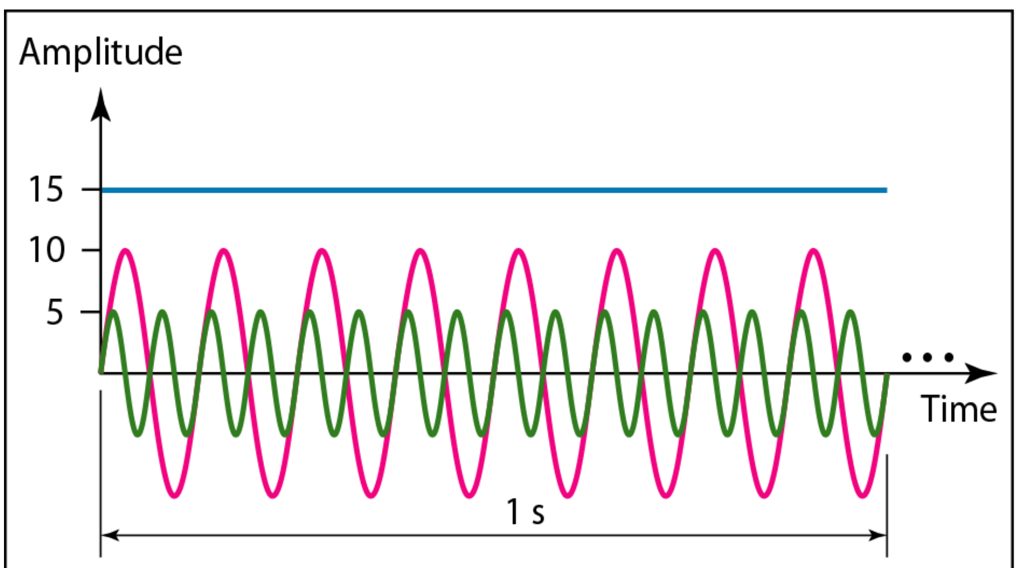
Note

A complete sine wave in the time domain can be represented by one single spike in the frequency domain.

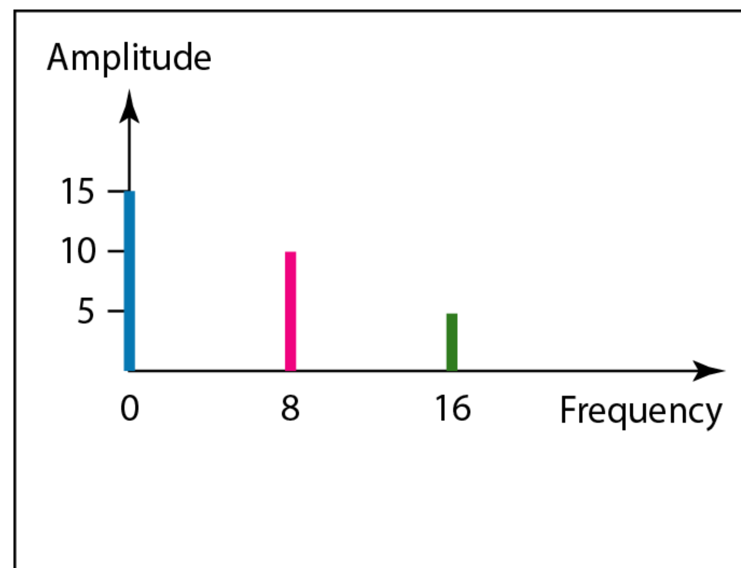
Example

The frequency domain is more compact and useful when we are dealing with more than one sine wave.

The time domain and frequency domain of three sine waves



a. Time-domain representation of three sine waves with frequencies 0, 8, and 16



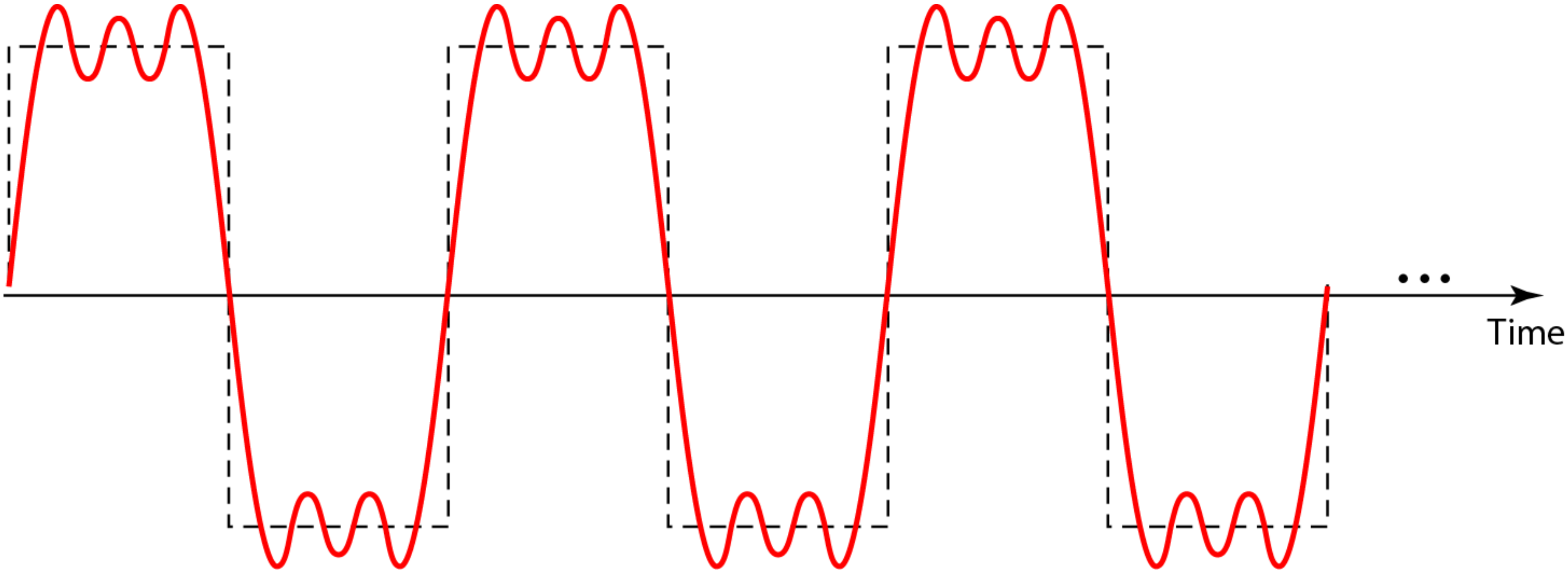
b. Frequency-domain representation of the same three signals

Composite Signals and Periodicity

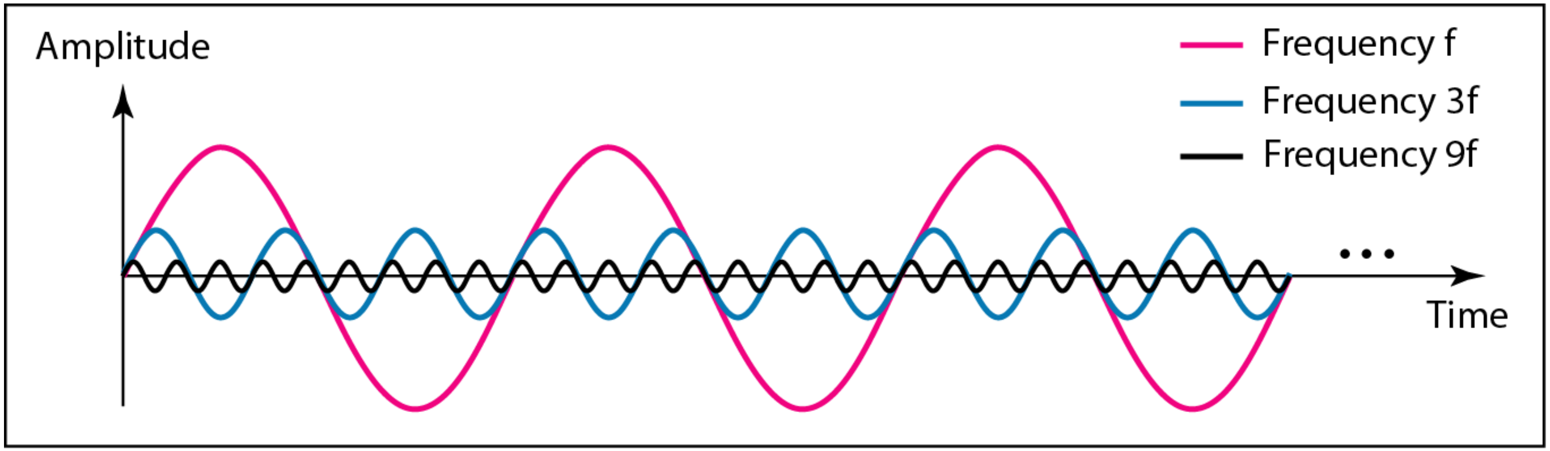
- If the composite signal is periodic, the decomposition gives a series of signals with discrete frequencies.

- If the composite signal is aperiodic, the decomposition gives a combination of sine waves with continuous frequencies.

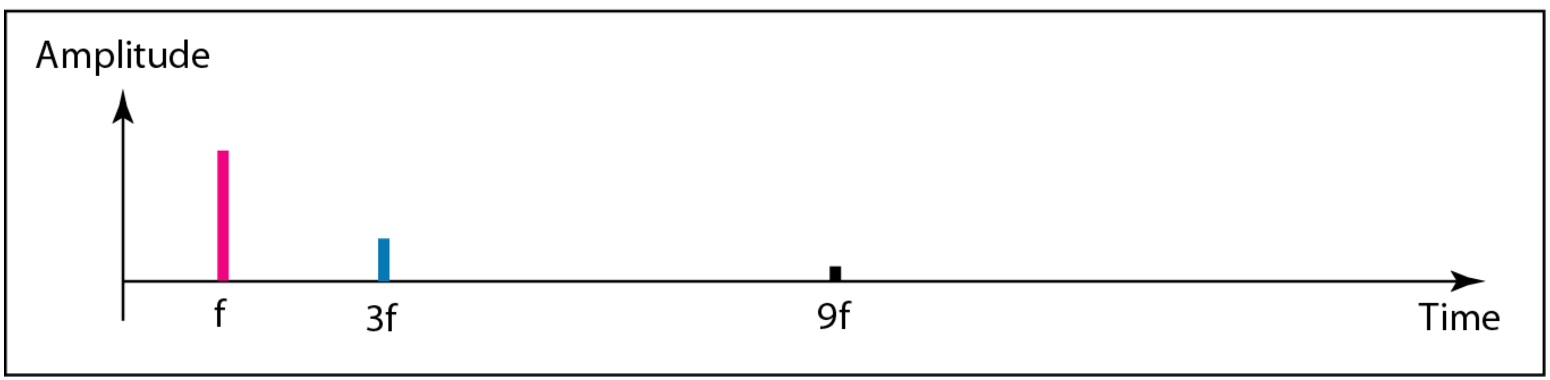
A composite periodic signal



Decomposition of a composite periodic signal in the time and frequency domains

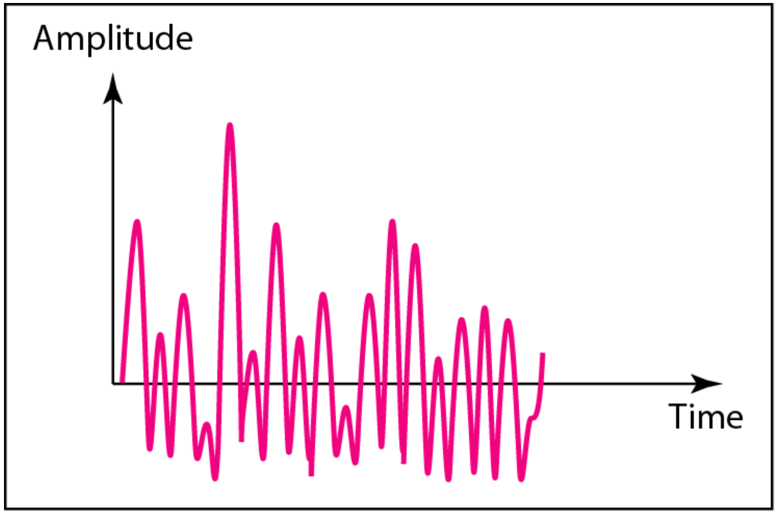


a. Time-domain decomposition of a composite signal

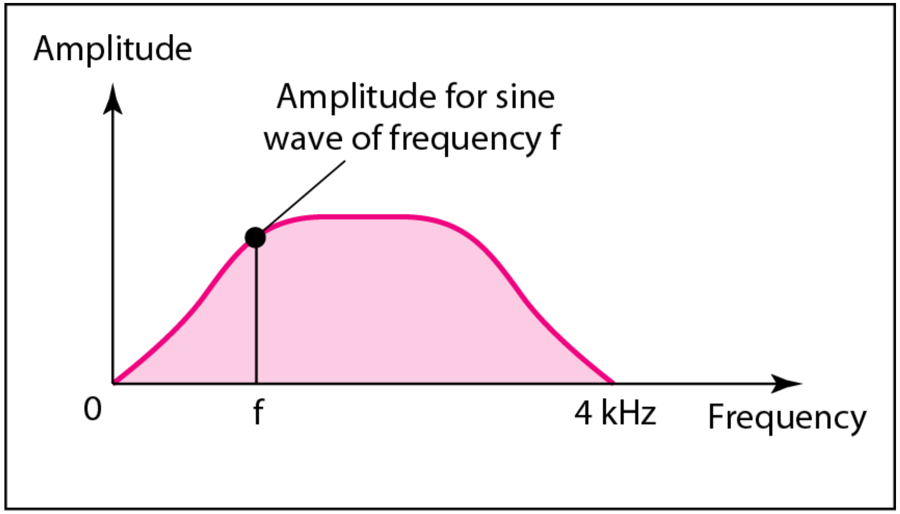


b. Frequency-domain decomposition of the composite signal

The time and frequency domains of an aperiodic signal



a. Time domain



b. Frequency domain

What is the difference between periodic and aperiodic signals?

Fourier Analysis

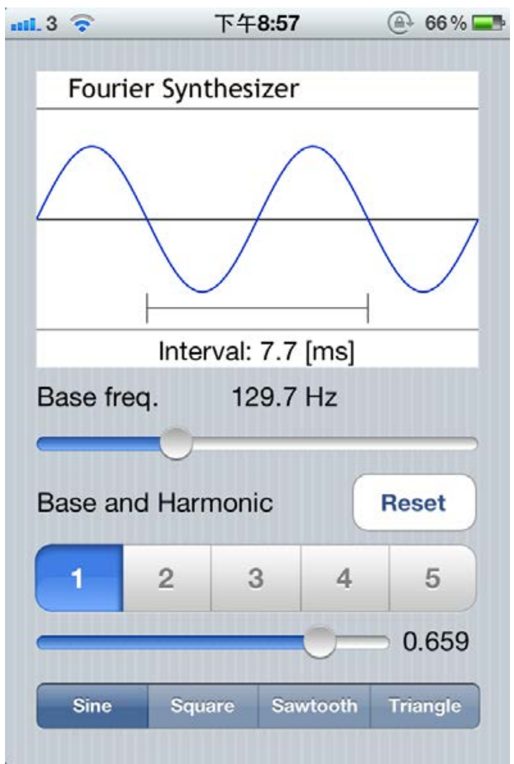
Note

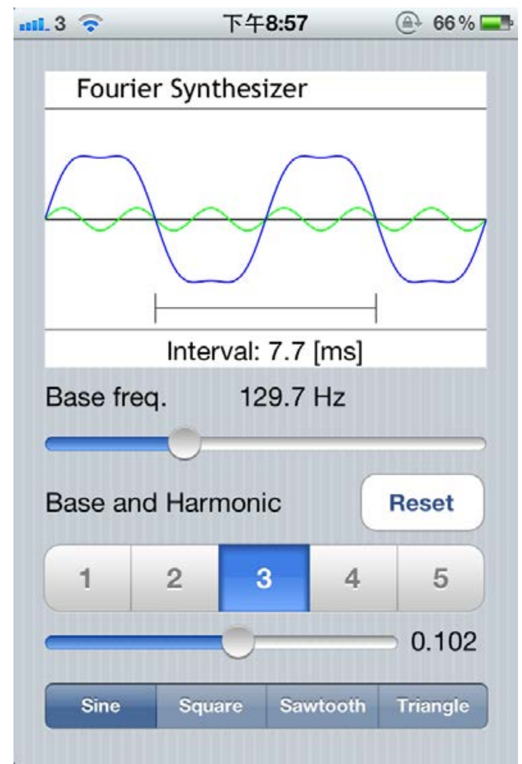
Fourier analysis is a tool that changes a time domain signal to a frequency domain signal and vice versa.

Apps

Fourier synthesizer:

Use pure-tone and/or harmonics to synthesize periodic signal





Fourier Series – An LTI System Analysis Perspective

How to represent signals in the study of LTI systems?

In chapter 2, based on unit impulse function and superposition property, we have:

1) The set of basic signals can be used to construct a broad and useful class of signals.

2) The response of an LTI system to each signal should be simple enough in structure, to provide us with a convenient representation for the response of the system to any signal constructed as a linear combination of the basic signals.

Any more basic function?

Eigenfunctions ϕ_k(t) and Their Properties

- (Focus on CT systems now, but results apply to DT systems as well.)

$$\phi_k(t) \longrightarrow \boxed{\text{System}} \longrightarrow \underbrace{\lambda_k}_{\text{eigenvalue}}\ \underbrace{\phi_k(t)}_{\text{eigenfunction}}$$

Eigenfunction in → same function out with a "gain"

- From the superposition property of LTI systems:

$$x(t)=\sum_k a_k \phi_k(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t)=\sum_k \lambda_k a_k \phi_k(t)$$

Now the task of finding response of LTI systems is to determine λ_k.
The solution is simple, general, and insightful.

Two Key Questions

1. What are the eigenfunctions of a general LTI system?

2. What kinds of signals can be expressed as superpositions of these eigenfunctions?

Various Eigenfunctions

Ex. #1: Identity system

$$x(t) \longrightarrow \boxed{\delta(t)} \longrightarrow x(t) * \delta(t) = x(t)$$

Any function is an eigenfunction for this LTI system.

Ex. #2: A delay

$$x(t) \longrightarrow \boxed{\delta(t-T)} \longrightarrow x(t-T)$$

Any periodic function $x(t) = x(t+T)$ is an eigenfunction.

Ex. #3: $h(t)$ even

$$\cos \omega t \longrightarrow \boxed{h(t) = h(-t)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\cos[\omega(t-\tau)]d\tau = \int_{-\infty}^{\infty} h(\tau)[\cos\omega t \cdot \cos\omega\tau + \sin\omega t \cdot \sin\omega\tau)]d\tau$$

$$= \cos\omega t \underbrace{\int_{-\infty}^{\infty} h(\tau)\cos\omega\tau d\tau}_{H(j\omega)}$$ — $\cos\omega t$ is an eigenfunction

Complex Exponentials - The *Only* Eigenfunctions of *Any* LTI Systems

$$x(t) = e^{st} \longrightarrow \boxed{h(t)} \longrightarrow$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau) e^{s(t-\tau)} d\tau$$

$$= \left[\int_{-\infty}^{+\infty} h(\tau) e^{-s\tau} d\tau \right] e^{st}$$

$$= \underbrace{H(s)}_{\text{eigenvalue}} \underbrace{e^{st}}_{\text{eigenfunction}}$$

$$\boxed{y(t) = x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau}$$

$$x[n] = z^n \longrightarrow \boxed{h[n]} \longrightarrow$$

$$y[n] = \sum_{m=-\infty}^{\infty} h[m] z^{n-m}$$

$$= \left[\sum_{m=-\infty}^{\infty} h[m] z^{-m} \right] z^n$$

$$= \underbrace{H(z)}_{\text{eigenvalue}} \underbrace{z^n}_{\text{eigenfunction}}$$

System Functions *H*(*s*) or *H*(*z*)

CT:

$$x(t) \xrightarrow{e^{st}} \boxed{h(t)} \xrightarrow{H(s)e^{st}} y(t)$$

$$H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}\,dt$$

$$x(t) = \sum a_k e^{s_k t} \longrightarrow y(t) = \sum H(s_k) a_k e^{s_k t}$$

DT:

$$x[n] \xrightarrow{z^n} \boxed{h[n]} \xrightarrow{H(z)z^n} y[n]$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

$$x[n] = \sum a_k z_k^n \longrightarrow y[n] = \sum H(z_k) a_k z_k^n$$

Question 2: What kinds of signals can we represent as "sums" of complex exponentials?

For Now: Focus on restricted sets of complex exponentials

CT: $s = j\omega$ - purely imaginary ,

i.e. signals of the form $e^{j\omega t}$

DT: $z = e^{j\omega}$, i.e. signals of the form $e^{j\omega n}$

$$\Downarrow$$

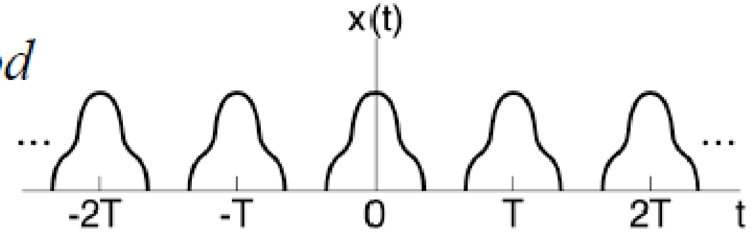
CT & DT Fourier Series and Transforms

periodic aperiodic

Fourier Series Representation of CT Periodic Signals

$x(t) = x(t+T)$ for all t

- smallest such T is the *fundamental period*
- $\omega_o = \dfrac{2\pi}{T}$ is the *fundamental frequency*



$e^{j\omega t}$ periodic with period $T \quad \Leftrightarrow \quad \omega = k\omega_o$

$\Downarrow$

Linear combination of harmonically related complex exponentials

$$\boxed{x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk2\pi t/T}}$$

-periodic with period T
- $\{a_k\}$ are the *Fourier (series) coefficients*
- $k = 0$ DC
- $k = \pm 1$ first harmonic
- $k = \pm 2$ second harmonic

Question: How do we find the Fourier coefficients?

Let's first take a detour by studying a three-dimensional vector:

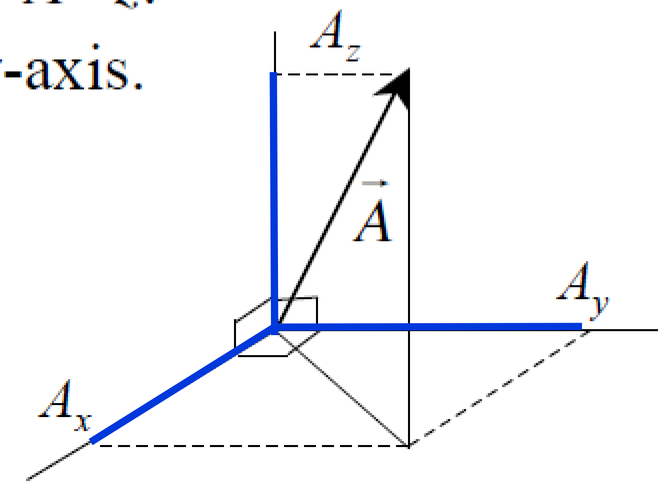
$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z},$$

$$\hat{x}, \hat{y}, \text{ and } \hat{z} \text{ are unit vectors.}$$

How do we find the coefficients A_x, A_y, and A_z?

Easy, $A_x = \vec{A} \bullet \hat{x},\ A_y = \vec{A} \bullet \hat{y},\ and\ A_z = \vec{A} \bullet \hat{z}.$

Project the vector onto the x-, y-, and z-axis.



Why does it work this way?

Orthogonality:

$$\boxed{\hat{y} \bullet \hat{x} = \hat{z} \bullet \hat{y} = \hat{x} \bullet \hat{z} \equiv 0}$$

Back to Fourier Series

Now let's imagine that $x(t)$ is a vector in a space of ∞-dimensions and $e^{jk\omega_o t}$ $(k = 0, \pm 1, \pm 2, \ldots)$ is a unit vector in this space. (Such a space does exist, it is called Hilbert space, just nobody lives there.) Then,

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_o t}, \quad \text{with the analogy}$$

$$\boxed{\begin{aligned} x(t) &\Leftrightarrow \vec{A} \\ e^{jk\omega_o t} &\Leftrightarrow \hat{x}, \hat{y}, \hat{z} \\ a_k &\Leftrightarrow A_x, A_y, A_z \end{aligned}}$$

Now all we need to do is to "project" $x(t)$ onto the $e^{jk\omega_o t} - axis$, then we will obtain the coefficients a_k. How to do that? Again, need **orthogonality.**

Obtaining Fourier Series Coe nts

Orthogonality in the Hilbert space:

$$\frac{1}{T}\int_T e^{jk\omega_o t}\cdot e^{-jn\omega_o t}dt = \frac{1}{T}\int_T e^{j(k-n)\omega_o t}dt = \begin{cases} 1, & k=n \\ 0, & k\neq n \end{cases} = \delta[k-n]$$

($\int_T$ = integral over *any* interval of length T, and the operation of

$$\frac{1}{T}\int_T e^{-jn\omega_o t}dt\cdot \quad \text{is to take an "inner product" with } e^{jn\omega_o t})$$

Now if we "project" $x(t)$ onto $e^{jn\omega_o t}$ by taking the operation:

$$\frac{1}{T}\int_T x(t)\cdot e^{-jn\omega_o t}dt = \frac{1}{T}\int_T \sum_{k=-\infty}^{+\infty} a_k e^{j(k-n)\omega_o t}\, dt,$$

then only one term (a_n) will be nonzero. That is:

$$\frac{1}{T}\int_T x(t)\cdot e^{-jn\omega_o t}dt = a_n.$$

Finally…

$$\boxed{\frac{1}{T}\int_T x(t)\cdot e^{-jn\omega_o t}\,dt = a_n\,.}$$

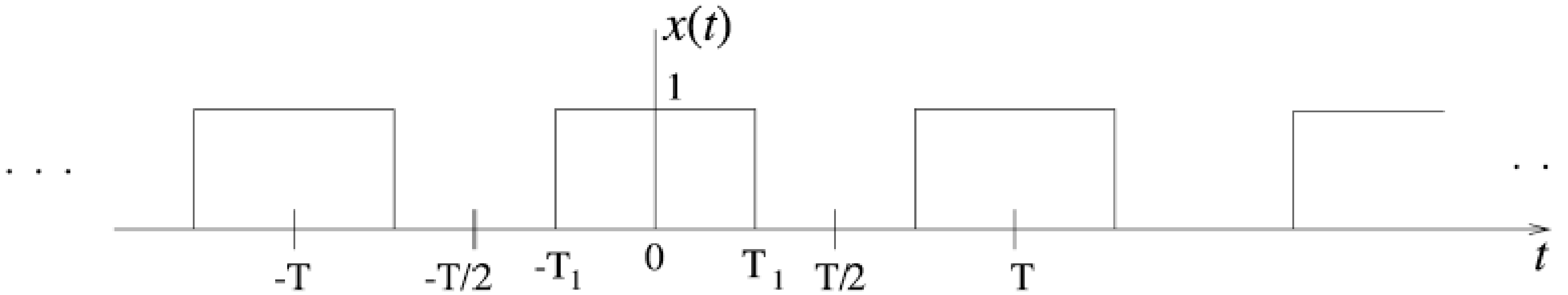
$$\Downarrow$$

CT Fourier Series Pair $\qquad (\omega_o = 2\pi / T)$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_o t} \qquad \text{(Synthesis equation)}$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_o t}\,dt \qquad \text{(Analysis equation)}$$

Example 3.5: Periodic Square Wave



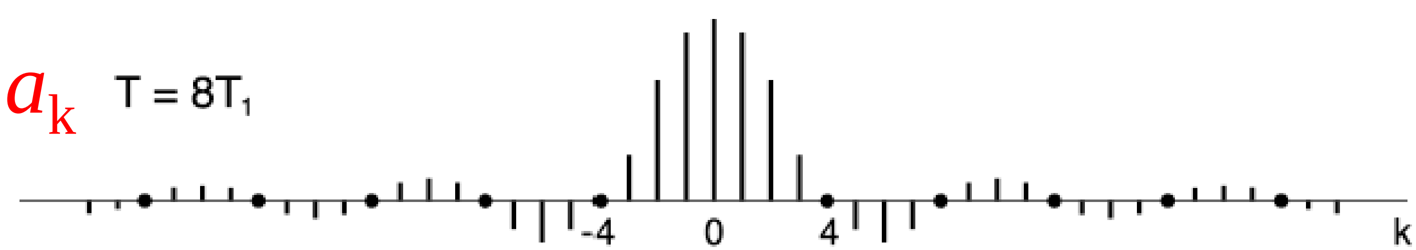
$$a_o = \frac{1}{T}\int_{-T/2}^{T/2} x(t)dt = \frac{2T_1}{T}$$

$k \neq 0$

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}dt$$

$(\omega_o = \frac{2\pi}{T})$:

$$= -\frac{1}{jk\omega_o T} e^{-jk\omega_0 t}\Big|_{-T_1}^{T_1} = \frac{\sin(k\omega_o T_1)}{k\pi}$$



Convergence of CT Fourier Series

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \quad ?$$

- One useful notion for engineers: there is no *energy* in the difference

$$e(t) = x(t) - \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

$$\int_T |e(t)|^2 \, dt = 0$$

(just need $x(t)$ to have finite energy per period) $\int_T |x(t)|^2 \, dt < \infty$

Dirichlet Conditions

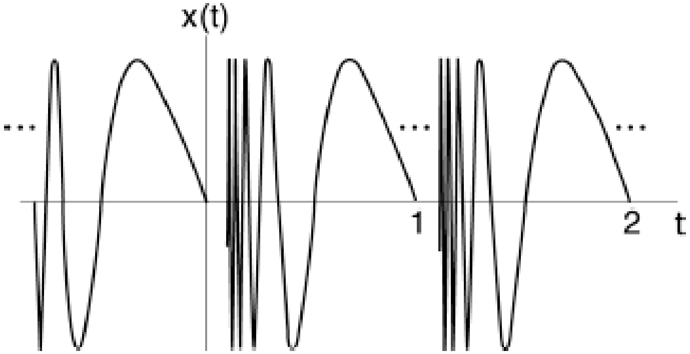
Condition 1. $x(t)$ is *absolutely integrable* over one period, i. e.

$$\int_T |x(t)|\,dt < \infty$$

Condition 2. In a finite time interval, $x(t)$ has a *finite* number of maxima and minima.

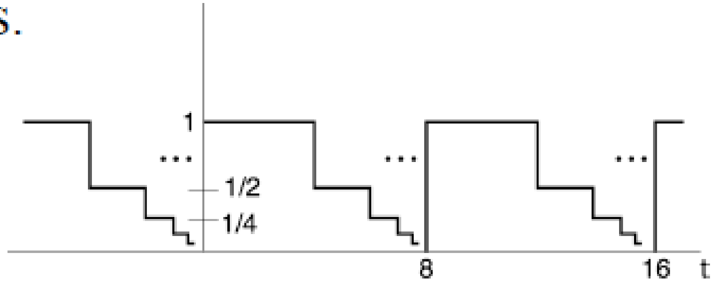
Ex. An example that violates Condition 2.

$$x(t) = \sin(2\pi / t) \quad 0 < t \le 1$$



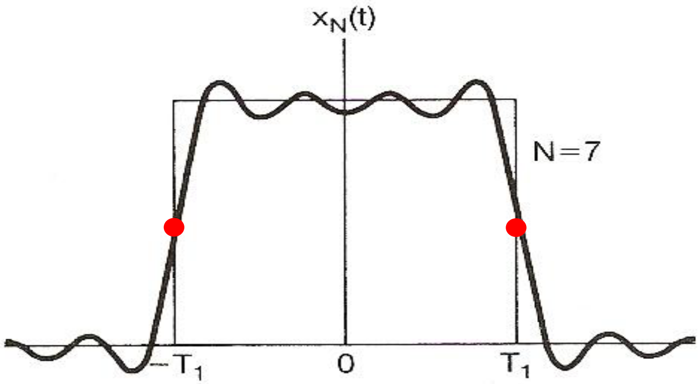
Condition 3. In a finite time interval, $x(t)$ has only a *finite* number of discontinuities.

Ex. An example that violates Condition 3.

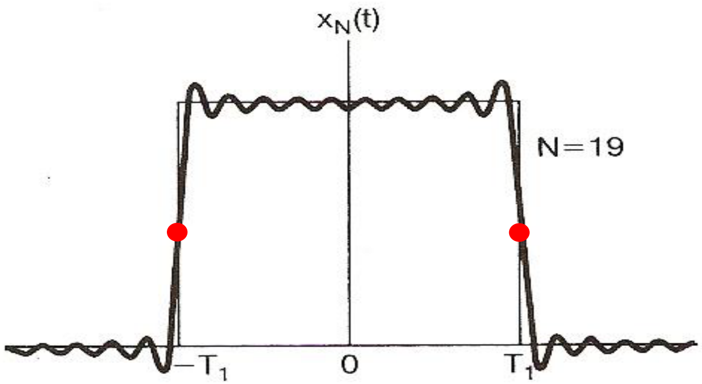


Dirichlet Conditions (cont.)

- How can the Fourier series (composed of continuous *sine* and *cos* functions) for the square wave (with many *discontinuities*) possibly make sense?
  - The Fourier series = $x(t)$ at points where $x(t)$ is continuous
  - The Fourier series = "midpoint" at points of discontinuity



(c)



(d)

$$x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$$

– As $N \to \infty$, $x_N(t)$ exhibits *Gibbs'* phenomenon at points of discontinuity

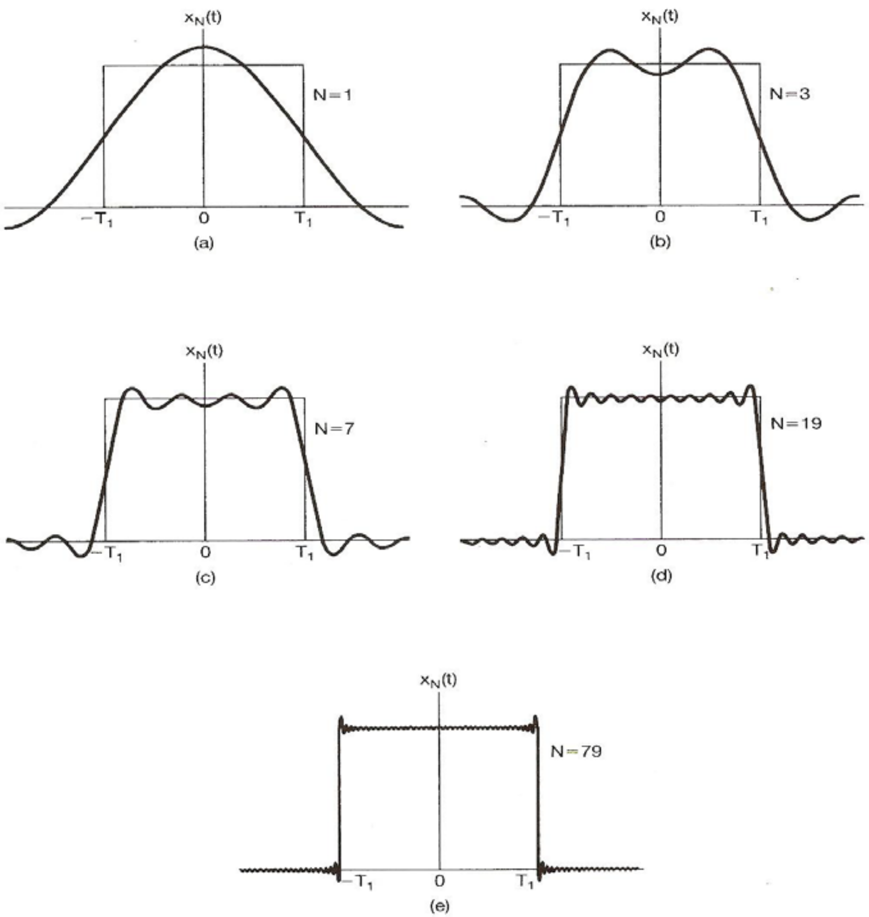


Figure 3.9 Convergence of the Fourier series representation of a square wave: an illustration of the Gibbs phenomenon. Here, we have depicted the finite series approximation $x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$ for several values of N.

- **Gibbs Phenomenon:** **The partial sum in the vicinity of the discontinuity exhibits ripples whose amplitude does not seem to decrease with increasing *N***

CT Fourier Series Pairs

$$(\omega_o = \frac{2\pi}{T})$$

Review:

$$\boxed{x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{j2\pi kt/T} \qquad a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt}$$

Skip it in future
for shorthand

$$x(t) \xleftrightarrow{FS} a_k$$

(A Few) Properties of CT Fourier Series

- Linearity $x(t) \longleftrightarrow a_k,\ y(t) \longleftrightarrow b_k, \Rightarrow \alpha x(t) + \beta y(t) \longleftrightarrow \alpha a_k + \beta b_k$
- Conjugate Symmetry

$$x(t) \quad real \quad \Rightarrow \quad a_{-k} = a_k^{\,*}$$

Proof:

$$a_{-k} = \frac{1}{T}\int_T x(t)e^{jk\omega_o t}dt = \left[\frac{1}{T}\int_T x^*(t)e^{-jk\omega_o t}dt\right]^* = a_k^*$$

$$\Downarrow \quad a_k = \mathrm{Re}\{a_k\} + j\,\mathrm{Im}\{a_k\}$$

$$\boxed{\mathrm{Re}\{a_{-k}\} + j\,\mathrm{Im}\{a_{-k}\}} \qquad = |a_k|\, e^{j\angle a_k} \qquad \boxed{\mathrm{Re}\{a_k\} - j\,\mathrm{Im}\{a_k\}}$$

$$\mathrm{Re}\{a_k\} \text{ is even},\quad \mathrm{Im}\{a_k\} \text{ is odd}$$

or $\quad |a_k|$ is even, $\quad \angle a_k$ is odd

- Time shift

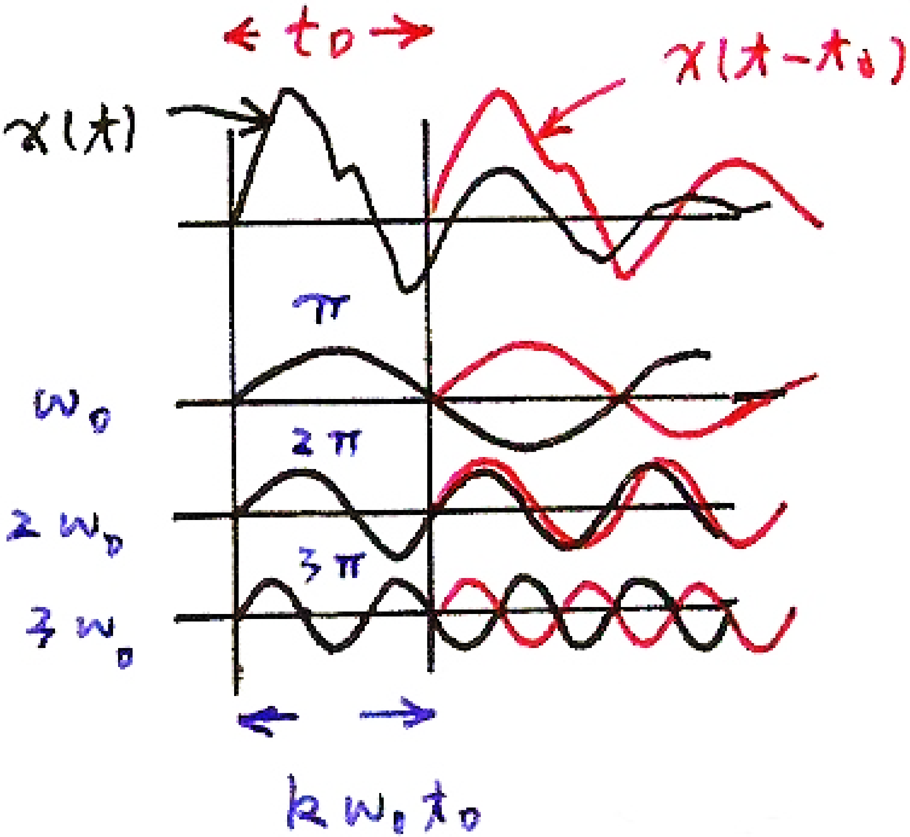
$$x(t) \longleftrightarrow a_k$$

$$x(t-t_o) \longleftrightarrow a_k e^{-jk\omega_o t_o} = a_k e^{-jk2\pi t_o/T}$$

Introduce a linear phase shift $\propto t_o$

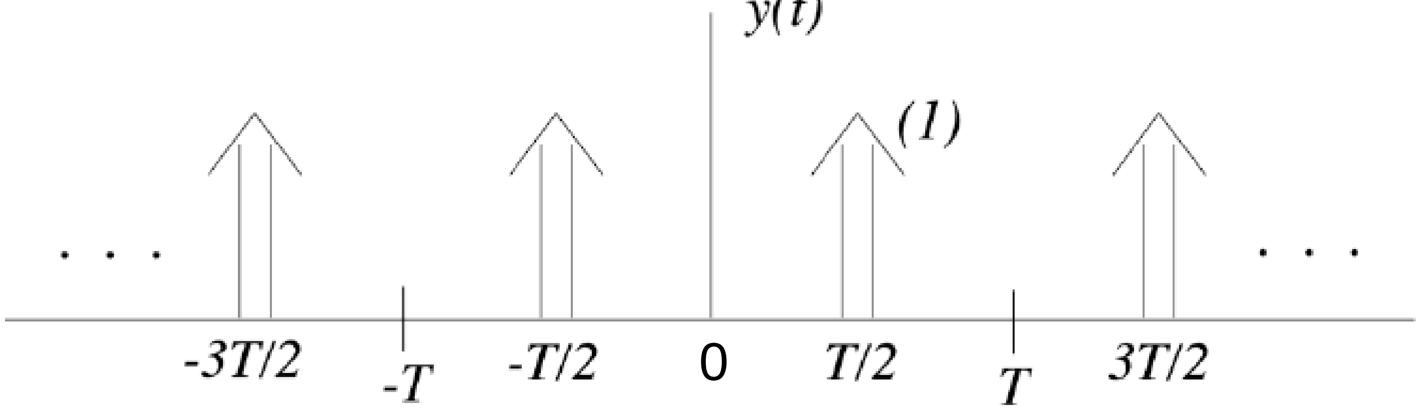
Time Shift

$$a_k \, e^{jk\omega_0(t-t_0)} = \boxed{e^{-j\underline{k}\omega_0 t_0} \, a_k} \, e^{jk\omega_0 t}$$



Example: Shift by Half Period

$$y(t) = x(t - T/2) \xleftrightarrow{e^{-jk\omega_o T/2} = e^{-jk\pi}} a_k e^{-jk\pi} = (-1)^k a_k$$



$$y(t) \longleftrightarrow (-1)^k a_k \qquad \left(a_k = \frac{1}{T} = \text{F.C. of } \sum_{n=-\infty}^{+\infty} \delta(t - nT)\right)$$

$$\|$$

$$\frac{(-1)^k}{T}$$

- Time Reversal

$$x(-t) \xleftrightarrow{FS} a_{-k}$$

the effect of sign change for $x(t)$ and a_k are identical

Example: $x(t)$: … $a_{-2}\ a_{-1}\ a_0\ a_1\ a_2$ …

$x(-t)$: … $a_2\ a_1\ a_0\ a_{-1}\ a_{-2}$ …

- Time Scaling

α : positive real number

$x(\alpha t)$: periodic with period T/α and fundamental frequency αw_0

$$x(\alpha t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(\alpha w_0)t}$$

a_k unchanged, but $x(\alpha t)$ and each harmonic component are different

- **Multiplication Property**

$$x(t) \longleftrightarrow a_k\ ,\ y(t) \longleftrightarrow b_k \text{ (Both } x(t) \text{ and } y(t) \text{ are}$$

$$\Downarrow \qquad \text{periodic with the same period } T)$$

$$x(t) \cdot y(t) \longleftrightarrow c_k = \sum_{l=-\infty}^{+\infty} a_l b_{k-l} = a_k * b_k$$

Proof: $\underbrace{\sum_l a_l e^{jl\omega_o t}}_{x(t)} \cdot \underbrace{\sum_m b_m e^{jm\omega_o t}}_{y(t)} = \sum_{l,m} a_l b_m e^{j(l+m)\omega_o t} \xrightarrow{l+m=k} \sum_k \underbrace{\left[\sum_l a_l b_{k-l}\right]}_{c_k} e^{jk\omega_o t}$

- **Parseval Relation**

$$\underbrace{\frac{1}{T}\int_T |x(t)|^2\,dt}_{\text{Average signal power}} = \sum_{k=-\infty}^{+\infty} \underbrace{|a_k|^2}_{\substack{\text{Power in the} \\ k_{th}\text{ harmonic}}}$$

Power is the same whether measured in the time-domain or the frequency-domain

More

Note:
$x(t)$ is periodic with
fundamental frequency ω_0

$$x(t) \longleftrightarrow a_k$$

- **Frequency shifting**

$$e^{jM\omega_0 t} x(t) \longleftrightarrow a_{k-M}$$

- **Differentiation**

$$\frac{dx}{dt} \longleftrightarrow jk\omega_0 a_k$$

- **Integration**

$$\int_{-\infty}^{t} x(t)dt \longleftrightarrow (\frac{1}{jk\omega_0})a_k$$

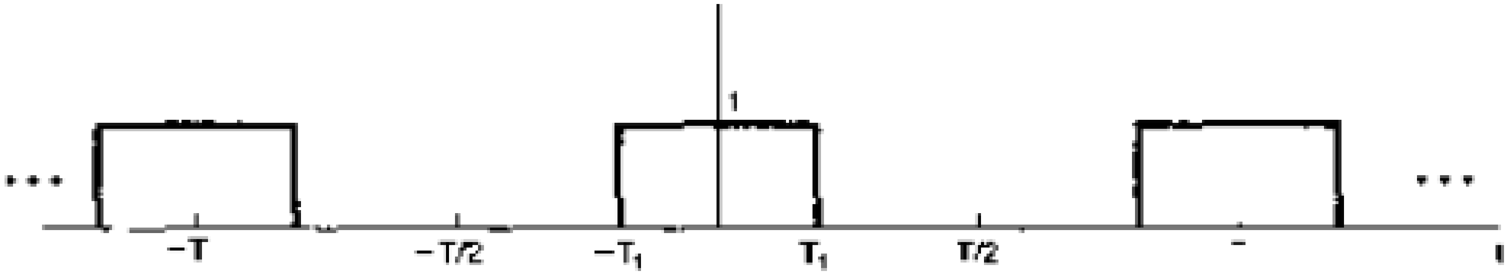
TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l}x(t)\\ y(t)\end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t)$, $\alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

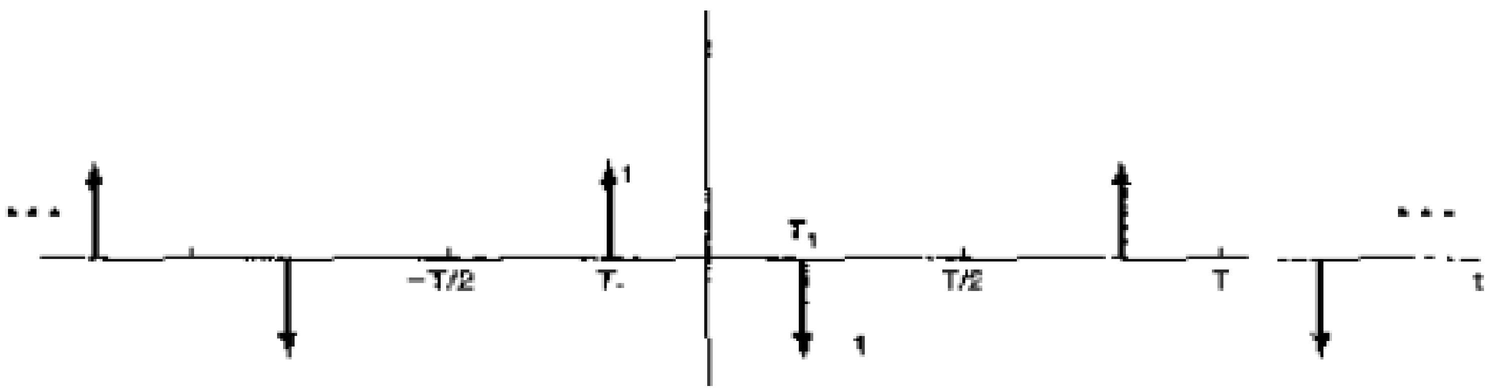
Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Example 3.8



1. by definition



2. using properties of time shift, differentiation, and integration

Summary

- Difference between periodic and aperiodic signals
- Eigenfunction and eigenvalue
- Fourier series for periodic signal
  - Analysis and synthesis equations
  - Properties
  - More: Gibbs phenomenon, Dirichlet conditions