边复习

Tutorial Week 14

8.34 8.35 8.38

8.34. In discussing amplitude modulation systems, modulation and demodulation were carried out through the use of a multiplier. Since multipliers are often difficult to implement, many practical systems use a nonlinear element. In this problem, we illustrate the basic concept.

In Figure P8.34, we show one such nonlinear system for amplitude modulation. The system consists of squaring the *sum* of the modulating signal and the carrier and then bandpass filtering to obtain the amplitude-modulated signal.

Assume that $x(t)$ is band limited, so that $X(j\omega) = 0$, $|\omega| > \omega_M$. Determine the bandpass filter parameters A, ω_l, and ω_h such that $y(t)$ is an amplitude-modulated version of $x(t)$ [i.e., such that $y(t) = x(t)\cos\omega_c t$]. Specify the necessary constraints, if any, on ω_c and ω_M.

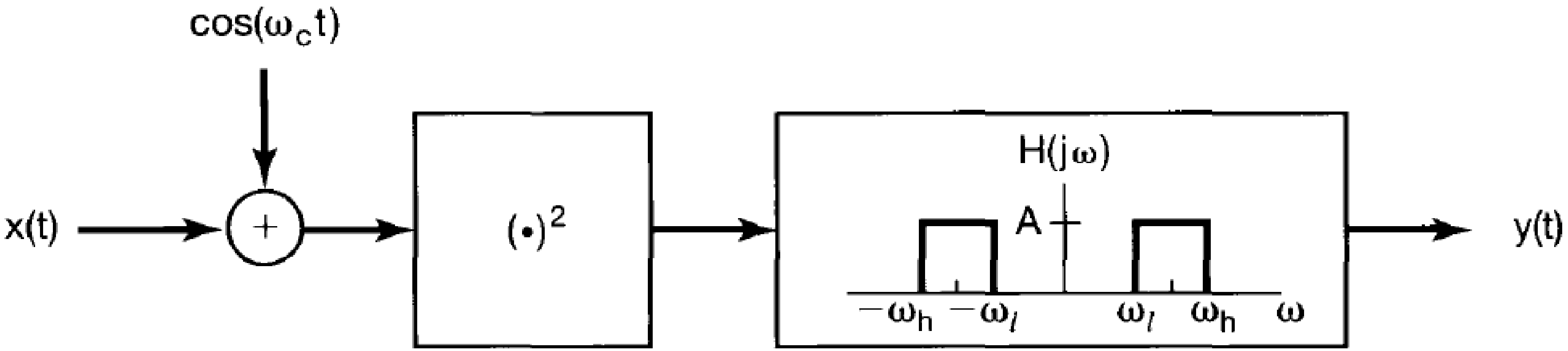


Figure P8.34

8.34. The output of the squarer is

$$r(t) = [x(t) + \cos(\omega_c t)]^2 = x^2(t) + \cos^2(\omega_c t) + 2x(t)\cos(\omega_c t).$$

The bandpass filter should reject $x^2(t) + \cos^2(\omega_c t)$ and multiply the remainder by 1/2. Therefore, $A = 1/2$. Since the spectral contribution of $2x(t)\cos(\omega_c t)$ is in the range $\omega_c - \omega_M \le |\omega| \le \omega_c + \omega_M$, we require $\omega_l = \omega_c - \omega_M$ and $\omega_h = \omega_c + \omega_M$.

Note that (i) the spectral contribution of $x^2(t)$ is in the range $|\omega| \le 2\omega_M$ and (ii) the spectral contribution of $\cos^2(\omega_c t)$ is at $\omega = 0$ and $\omega = \pm 2\omega_c$. Therefore, we need to ensure that

$$\omega_l > 2\omega_M \quad \Rightarrow \quad \omega_M < \omega_c/3.$$

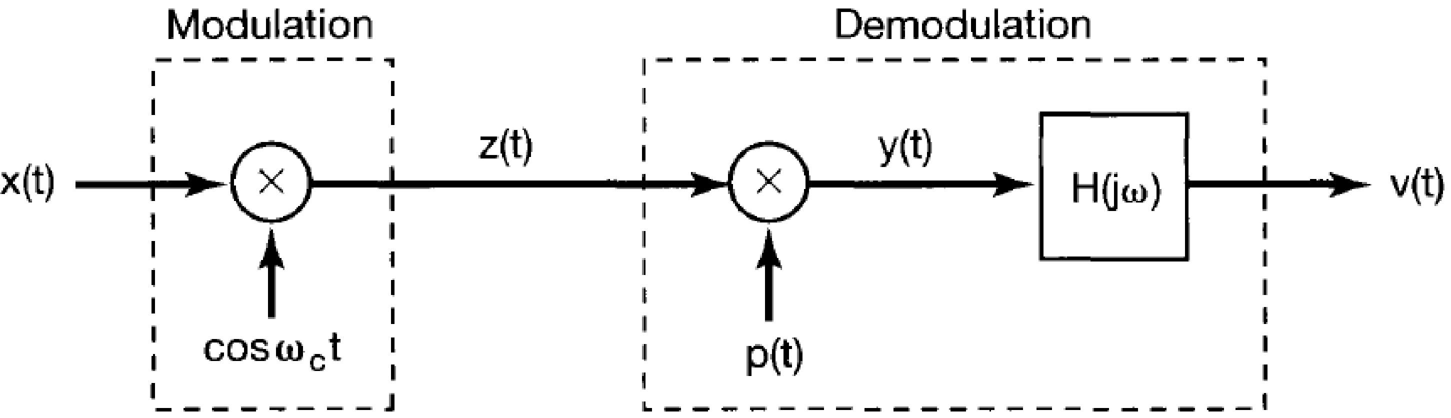
We also need to ensure that

$$\omega_h < 2\omega_c \quad \Rightarrow \quad \omega_M < \omega_c.$$

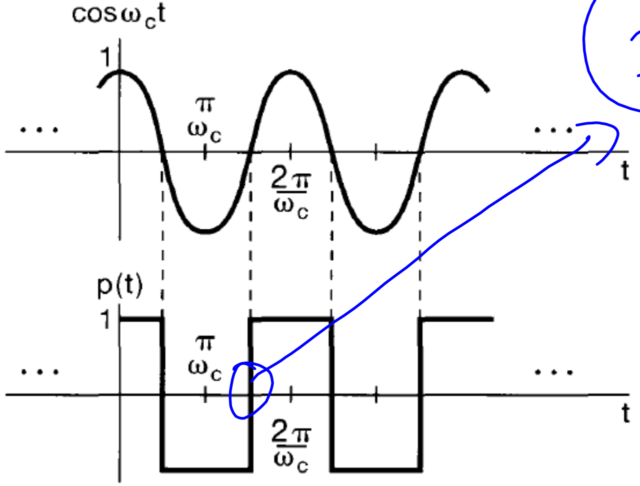
8.35. The modulation-demodulation scheme proposed in this problem is similar to sinusoidal amplitude modulation, except that the demodulation is done with a square wave with the same zero-crossings as $\cos \omega_c t$. The system is shown in Figure P8.35(a); the relation between $\cos \omega_c t$ and $p(t)$ is shown in Figure P8.35(b). Let the input signal $x(t)$ be a band-limited signal with maximum frequency $\omega_M < \omega_c$, as shown in Figure P8.35(c).

(a) Sketch and dimension the real and imaginary parts of $Z(j\omega)$, $P(j\omega)$, and $Y(j\omega)$, the Fourier transforms of $z(t)$, $p(t)$, and $y(t)$, respectively.

(b) Sketch and dimension a filter $H(j\omega)$ so that $v(t) = x(t)$.

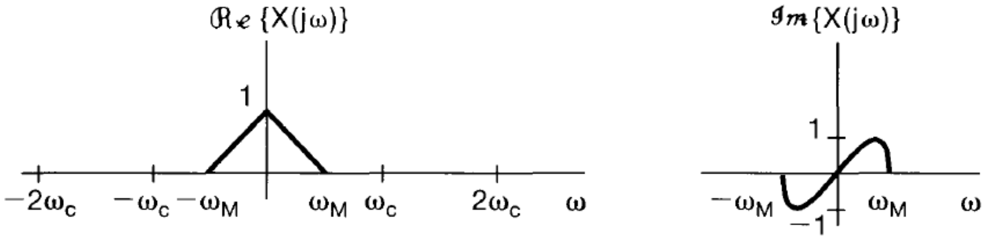


(a)



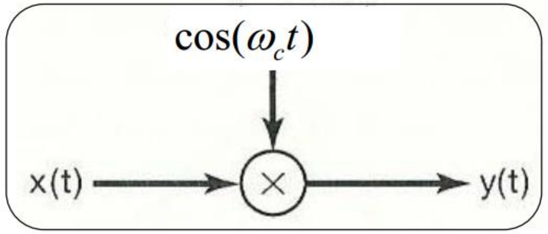
$\frac{3\pi}{2\omega_c}$

(b)



(c)

$$y(t) = x(t)c(t) = x(t)\cos(\omega_c t)$$



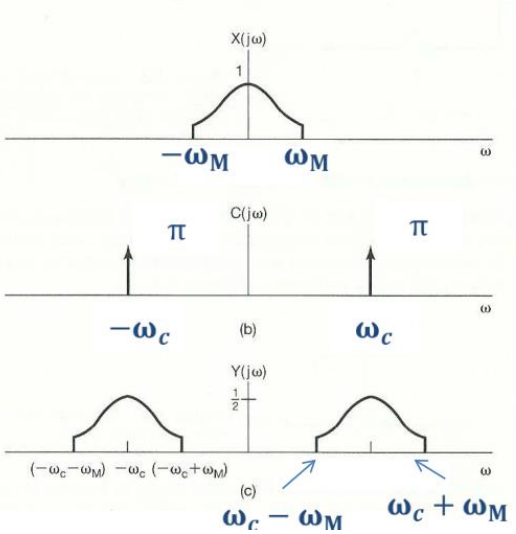


TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$

8.1.2 Amplitude Modulation with a Sinusoidal Carrier

In many situations, using a sinusoidal carrier of the form of eq. (8.2) is often simpler than and equally as effective as using a complex exponential carrier. In effect, using a sinusoidal carrier corresponds to retaining only the real or imaginary part of the output of Figure 8.2. A system that uses a sinusoidal carrier is depicted in Figure 8.3.

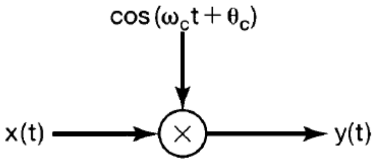


Figure 8.3 Amplitude modulation with a sinusoidal carrier.

The effect of amplitude modulation with a sinusoidal carrier in the form of eq. (8.2) can be analyzed in a manner identical to that in the preceding subsection. Again, for convenience we choose $\theta_c = 0$. In this case, the spectrum of the carrier signal is

$$C(j\omega) = \pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)], \tag{8.9}$$

and thus, from eq. (8.4),

$$Y(j\omega) = \frac{1}{2}[X(j\omega - j\omega_c) + X(j\omega + j\omega_c)]. \tag{8.10}$$

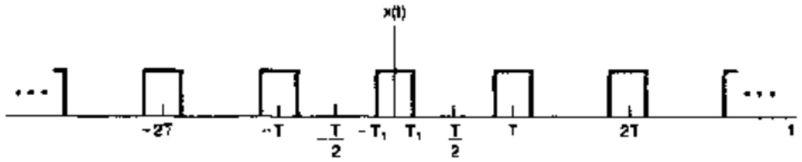
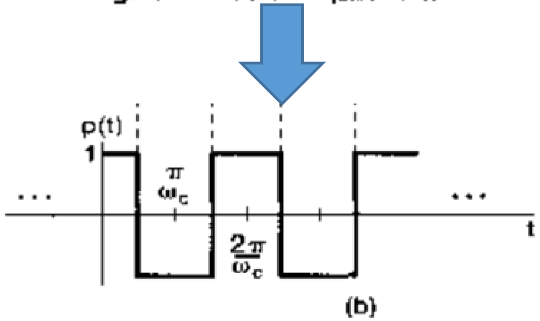


Figure 3.6 Periodic square wave.



8.35. (a) Since $Z(j\omega) = \frac{1}{2}X(j(\omega - \omega_c)) + \frac{1}{2}X(j(\omega + \omega_c))$, it is as shown in Figure S8.35.

The Fourier series coefficients of $p(t)$ can be shown to be $a_k = 4\sin(k\pi/2)/(2\pi k)$ for $k \neq 0$ and zero for $k = 0$. Therefore,

$$P(j\omega) = \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{4\sin(k\pi/2)}{k}\delta(\omega - k\omega_0).$$

This is as shown in Figure S8.35.

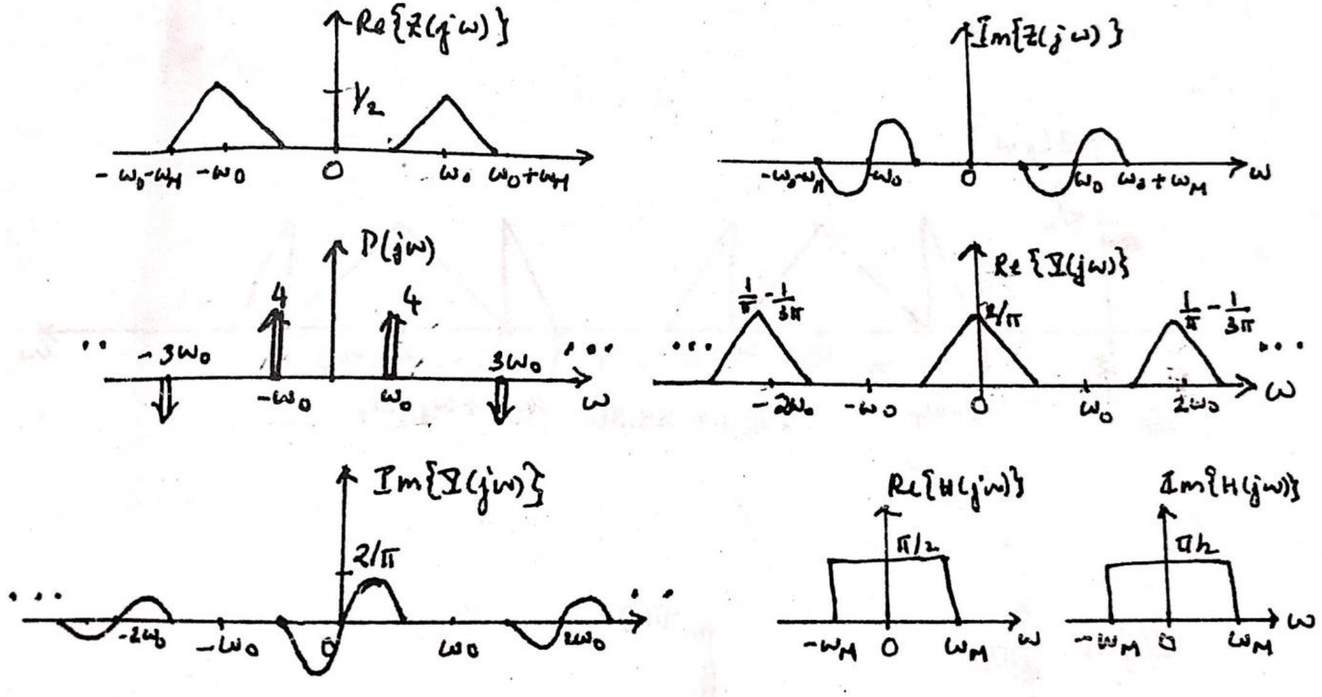
Since $y(t) = z(t)p(t)$,

$$Y(j\omega) = \frac{1}{2\pi}\left[Z(j\omega) * P(j\omega)\right].$$

Therefore, $Y(j\omega)$ is as shown in Figure S8.35.

(b) From the last figure in the previous part, it is clear that we require $H(j\omega)$ to be as shown in Figure S8.35 to ensure that $v(t) = x(t)$.

$$x[n] * \delta[n-n_0] = x[n-n_0]$$



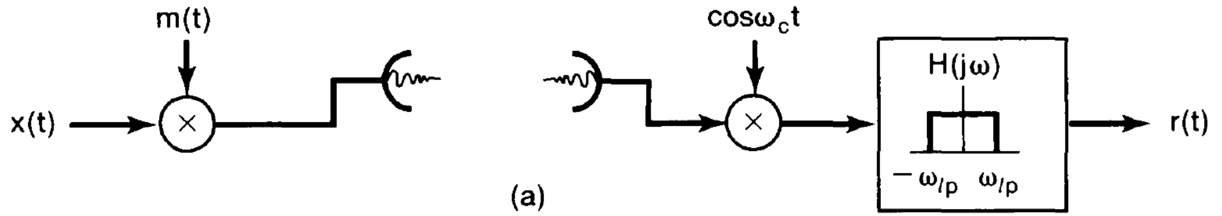
8.38. In Figure P8.38(a), a communication system is shown that transmits a band-limited signal $x(t)$ as periodic bursts of high-frequency energy. Assume that $X(j\omega) = 0$ for $|\omega| > \omega_M$. Two possible choices, $m_1(t)$ and $m_2(t)$, are considered for the modulating signal $m(t)$. $m_1(t)$ is a periodic train of sinusoidal pulses, each of duration D, as shown in Figure P8.38(b). That is,

$$m_1(t) = \sum_{k=-\infty}^{\infty} p(t - kT),$$

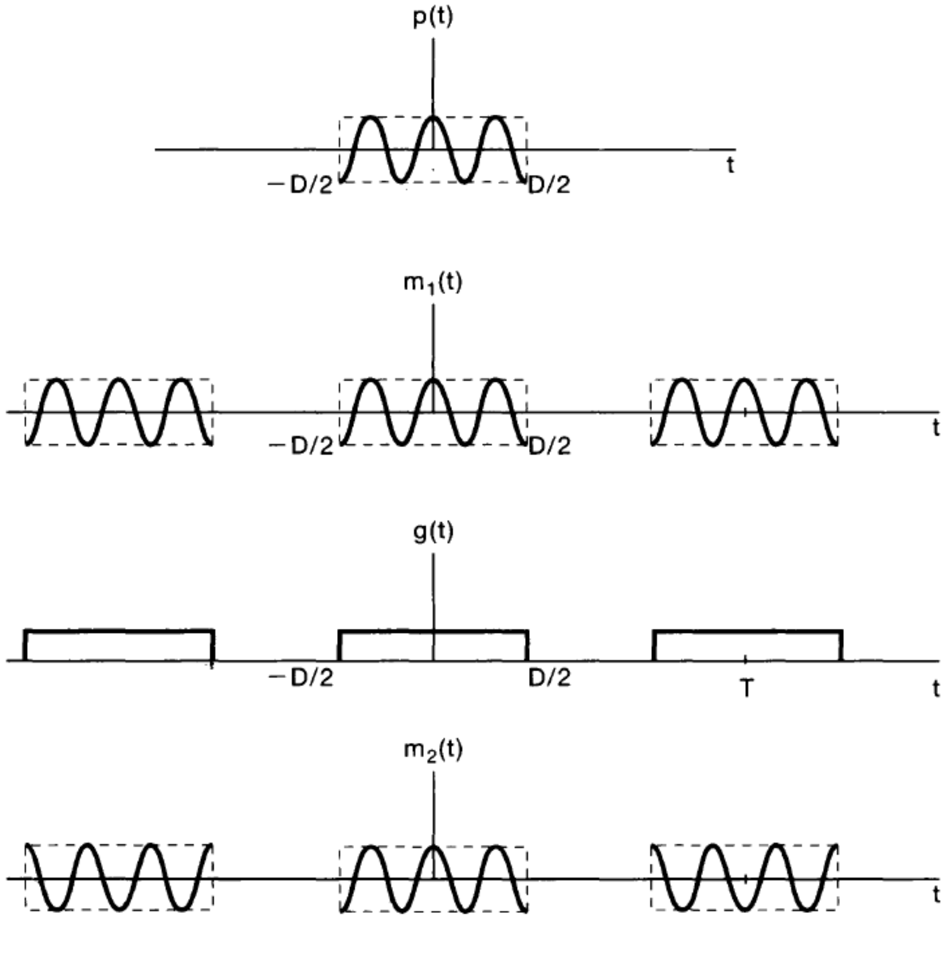
where

$$p(t) = \begin{cases} \cos \omega_c t, & |t| < (D/2) \\ 0, & |t| > (D/2) \end{cases}.$$

$m_2(t)$ is $\cos \omega_c t$ periodically blanked or gated; that is, $m_2(t) = g(t) \cos \omega_c t$, where $g(t)$ is as shown in Figure P8.38(b).



(a)



(b)

The following relationships between the parameters T, D, ω_c, and ω_M are assumed:

$$D < T,$$

$$\omega_c >> \frac{2\pi}{D},$$

$$\frac{2\pi}{T} > 2\omega_M.$$

Also, assume that $[\sin(x)]/x$ is negligible for $x >> 1$.

Determine whether, for some choice of ω_{lp}, either $m_1(t)$ or $m_2(t)$ will result in a demodulated signal $x(t)$. For each case in which your answer is yes, determine an acceptable range for ω_{lp}.

8.38. One of the key issues to note in this problem is that the structure of the demodulator is that of a synchronous demodulator. Therefore, the input signal to the demodulator has to have a replica of $X(j\omega)$ centered around ω_c. Only then will the demodulator be successful in recovering $x(t)$.

Case 1:
$P(j\omega)$ is given by

$$P(j\omega) = \frac{\sin[(\omega - \omega_c)D/2]}{\omega} + \frac{\sin[(\omega + \omega_c)D/2]}{\omega}.$$

$M_1(j\omega)$ will consist of impulses which occur at intervals of $2\pi/T$ weighted by $P(j\omega)$. Furthermore, note that if $y_1(t) = x(t)m_1(t)$, then we have

$$Y_1(j\omega) = \frac{1}{2\pi}\left[X(j\omega) * M_1(j\omega)\right].$$

Therefore, $Y_1(j\omega)$ will consist of weighted replicas of $X(j\omega)$ which occur every $2\pi/T$. Note that unless ω_c is a multiple of $2\pi/T$, $M_1(j\omega) = 0$ for $\omega = \pm\omega_c$. If $2\pi/T$ is arbitrary, (i.e., it is not specified to be a multiple of ω_c) $Y_1(j\omega)$ has no replicas of $X(j\omega)$ centered around ω_c. Since $y_1(t)$ constitutes the input to the demodulator, the signal $r(t)$ at the output of the demodulator will not be proportional to $x(t)$.

Case 2:
In this case,

$$M_2(j\omega) = \frac{1}{2}G(j(\omega - \omega_c)) + \frac{1}{2}G(j(\omega + \omega_c)),$$

where

$$G(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\sin(k\pi D/T)}{k}\delta(\omega - 2\pi k/T).$$

Clearly, $M_2(j\omega)$ has equal-valued impulses at $\pm\omega_c$. Therefore, the Fourier transform $Y_2(j\omega)$ of the signal $y_2(t) = x(t)m_2(t)$ has replicas of $X(j\omega)$ at $\pm\omega_c$. These replicas do not alias with other replicas of $X(j\omega)$ in $Y_2(j\omega)$ because $2\pi/T > 2\omega_M$. Thus, when demodulation is performed on $y_2(t)$, then $r(t)$ can be made proportional to $x(t)$ provided $2\omega_M < \omega_{lp} < 2\pi/T$.