Tutorial Problems (Week 7)

- Basic Problems with Answers 3.11
- Basic problems 3.30,3.37
- Advanced Problems 3.49

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3.11. Suppose we are given the following information about a signal $x[n]$:

1. $x[n]$ is a real and even signal.
2. $x[n]$ has period $N = 10$ and Fourier coefficients a_k.
3. $a_{11} = 5$.
4. $\frac{1}{10} \sum_{n=0}^{9} |x[n]|^2 = 50$.

Show that $x[n] = A\cos(Bn + C)$, and specify numerical values for the constants A, B, and C.

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

3.11 Since the Fourier series coefficients repeat every N=10,we have $a_1 = a_{11} = 5$ Furthermore ,since x[n] is real and even , a_k is also real and even .Therefore $a_1 = a_{-1} = 5$ We are also given that

$$\frac{1}{10}\sum_{n=0}^{9}\left|x[n]\right|^2 = 50$$

Using Parseval's relation ,

$$\sum_{k=<N>}|a_k|^2 = 50$$

$$\sum_{k=-1}^{8}|a_k|^2 = 50$$

$$|a_{-1}|^2 + |a_1|^2 + a_0^2 + \sum_{k=2}^{8}|a_k|^2 = 50$$

$$a_0^2 + \sum_{k=2}^{8}|a_k|^2 = 0$$

Therefore $a_k = 0$ for k=2 ,….8, Now using the synthesis eq. (3.94) , we have

$$x[n] = \sum_{k=<N>} a_k e^{j\frac{2\pi}{N}kn} = \sum_{k=-1}^{8} a_k e^{j\frac{2\pi}{10}kn}$$

$$= 5e^{j\frac{2\pi}{10}n} + 5e^{-j\frac{2\pi}{10}n} = 10\cos(\frac{\pi}{5}n)$$

3.30. Consider the following three discrete-time signals with a fundamental period of 6:

$$x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right), \qquad y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right), \qquad z[n] = x[n]y[n].$$

(a) Determine the Fourier series coefficients of $x[n]$.
(b) Determine the Fourier series coefficients of $y[n]$.
(c) Use the results of parts (a) and (b), along with the multiplication property of the discrete-time Fourier series, to determine the Fourier series coefficients of $z[n] = x[n]y[n]$.
(d) Determine the Fourier series coefficients of $z[n]$ through direct evaluation, and compare your result with that of part (c).

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\omega_0 n} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk(2\pi/N)n}$$

Multiplication $\quad x[n]y[n] \quad \sum_{l=\langle N\rangle} a_l b_{k-l}$

3.30.(a)The nonzero FS coefficients of x(t) are a0=1,a1=a_{-1}=1/2

(b)The nonzero FS coefficient FS coefficient of x(t) are b1= $b^*_{-1} = e^{-j\pi/4}/2$

(c)Using the multiplication property, we know that

$$z[n]=x[n]y[n] \xleftrightarrow{FS} c_k=\sum_{l=-1}^{1} a_l b_{k-l}$$

This implies that the nonzero Fourier series coefficients of z[n] are c0=cos(π /4)/2, $c_1=c^*_{-1}=e^{-j\pi/4}/2, c_2=c^*_{-2}=e^{-j\pi/4}/4$

(d) We have

n

$$z[n]=\sin(\frac{2\pi}{n}+\frac{\pi}{4})+\sin(\frac{2\pi}{6}n+\frac{\pi}{4})\cos(\frac{2\pi}{6}n)$$

6

$$=\sin(\frac{2\pi}{6}n+\frac{\pi}{4})+\frac{1}{2}[\sin(\frac{4\pi}{6}n+\frac{\pi}{4})+\sin(\frac{\pi}{4})]$$

This implies that the nonzero Fourier series coefficients of z[n] are c0=cos(π /4)/2,
$c_1=c^*_{-1}=e^{-j\pi/4}/2.\ c_2=c^*_{-2}=e^{-j\pi/4}/4$

积化和差公式

$$\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

3.37. Consider a discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{|n|}.$$

Find the Fourier series representation of the output $y[n]$ for each of the following inputs:

(a) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k]$

(b) $x[n]$ is periodic with period 6 and

$$x[n] = \begin{cases} 1, & n = 0, \pm 1 \\ 0, & n = \pm 2, \pm 3 \end{cases}$$

$$h[n] = \alpha^n u[n] \quad , \quad |\alpha| < 1$$

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n} = \frac{1}{1 - \alpha e^{-j\omega}}$$

3.37 The frequency response of the system may be easily shown to be

$$H(e^{jw})=\frac{1}{1-\frac{1}{2}e^{-jw}}-\frac{1}{1-2e^{-jw}}$$

(a) the Fourier series coefficients of x[n] are

$$a_k=1/4, \text{ for all k}$$

Also, N=4. Therefore, the Fourier series coefficients of y[n] are

$$b_k=a_kH(e^{j2k\pi/N})=\frac{1}{4}\left[\frac{1}{1-\frac{1}{2}e^{-j\pi k/2}}-\frac{1}{1-2e^{-j\pi k/2}}\right].$$

(b) In this case, the Fourier series coefficients of x[n] are

$$a_k=\frac{1}{6}[1+2\cos(k\pi/3)], \qquad \text{for all} k.$$

Also N=6 . Therefore, the Fourier series coefficients of y[n] are

$$b_k=a_kH(e^{j2k\pi/N})=\frac{1}{6}[1+2\cos(k\pi/3)]\left[\frac{1}{1-\frac{1}{2}e^{-j\pi k/3}}-\frac{1}{1-2e^{-j\pi k/3}}\right]$$

3.49. Let $x[n]$ be a periodic sequence with period N and Fourier series representation

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}. \tag{P3.49–1}$$

(a) Suppose that N is even and that $x[n]$ in eq. (P3.49–1) satisfies

$$x[n] = -x\left[n + \frac{N}{2}\right] \text{ for all } n.$$

Show that $a_k = 0$ for all even integers k.

(b) Suppose that N is divisible by 4. Show that if

$$x[n] = -x\left[n + \frac{N}{4}\right] \text{ for all } n,$$

then $a_k = 0$ for every value of k that is a multiple of 4.

(c) More generally, suppose that N is divisible by an integer M. Show that if

$$\sum_{r=0}^{(N/M)-1} x\left[n + r\frac{N}{M}\right] = 0 \text{ for all } n,$$

then $a_k = 0$ for every value of k that is a multiple of M.

3.49 (a) The FS coefficients are given by

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\rho nk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\rho nk}{N}} + \frac{1}{N}\sum_{n=N/2}^{N-1} x[n]e^{-j\frac{2\rho nk}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N/2-1} x[n]e^{-j\frac{2\rho nk}{N}} - \frac{e^{-j\pi k}}{N}\sum_{n=0}^{(N/2)-1} x[n]e^{-j\frac{2\rho nk}{N}}$$

$$= 0,$$

For k even

$\rho \rightarrow \pi$

(b) By adopting an approach similar to part (a),We may show that

$$a_k = \frac{1}{N}\left[\sum_{n=0}^{\frac{N}{4}-1}\left\{1 - e^{-j\pi k/2} + e^{-j\pi k} - e^{-3j\pi k/2}\right\}x[n]e^{\frac{-j2nk\pi}{N}}\right]$$

$$= 0$$

For k=4r, $r \ni \tau$

(c) If N/M is an integer, We may generalize the approach of part (a)to show that

$$a_k = \frac{1}{N}\left[\sum_{n=0}^{B-1}\left\{1 - e^{-j\pi r} + e^{-j\pi 4r} \cdots + e^{-j\pi 2(M-1)r}\right\}x[n]e^{\frac{-j2nk\pi}{N}}\right]$$

Where B=N/M and r=k/M. form the above equation ,it is clear that

$$a_k = 0, \qquad \text{if } k=rM,\ r \ni \tau$$

summary

- 离散时间傅里叶级数对
- 离散时间傅里叶级数性质（乘法、帕塞瓦尔）
- 离散时间LTI系统的输出响应
- 离散时间傅里叶级数系数的求解