APPENDIX

A. Formula and Definition

1. Euler Expansion: $e^{j\pm x} = \cos x \pm j \sin x$.
2. Continuous-time Convolution: $x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$
3. Discrete-time Convolution: $x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$
4. Continuous-time Fourier Series: $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}, a_k = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$
5. Discrete-time Fourier Series: $x[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}, a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk(2\pi/N)n}$
6. Continuous-time Fourier Transform: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega, X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$
7. Discrete-time Fourier Transform: $x[n] = \frac{1}{2\pi}\int_{2\pi} X\left(e^{j\omega}\right)e^{j\omega n}d\omega,\ X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
8. CT Periodic Signal Fourier Transform: $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t},\ X(jw) = 2\pi\sum_{k-\infty}^{+\infty} a_k\delta\left(\omega - k\omega_0\right)$
9. CT Parseval Theorem: $E_x = \int_{-\infty}^{\infty}|x(t)|^2dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega < \infty$

B. Convolution Result

1. $(u(t+T)-u(t-T)) * (u(t+T)-u(t-T)) = (2T - \frac{|t|}{2T})(u(t+2T)-u(t-2T))$ (Triangular wave!)
2. $e^{-at}u(t) * u(t) = \frac{1-e^{-at}}{a}u(t)$

C. Fourier Transform

1. $\delta\left(t-t_0\right) \xrightarrow{\mathcal{F}} e^{j\omega t_0}$ $\quad u(t) \xrightarrow{\mathcal{F}} \frac{1}{j\omega} + \pi\delta(\omega)$ $\quad e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi\delta\left(\omega-\omega_0\right)$
2. $\cos\omega_0 t \xrightarrow{\mathcal{F}} \pi\left[\delta\left(\omega-\omega_0\right)+\delta\left(\omega+\omega_0\right)\right]$ $\quad \sin\omega_0 t \xrightarrow{\mathcal{F}} \frac{\pi}{j}\left[\delta\left(\omega-\omega_0\right)-\delta\left(\omega+\omega_0\right)\right]$
3. $x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases} \xrightarrow{\mathcal{F}} \frac{2\sin\omega T_1}{\omega}$ $\quad \frac{\sin Wt}{\pi t} \xrightarrow{\mathcal{F}} X(j\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$
4. $e^{-at}u(t), \mathrm{Re}\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{a+j\omega}$ $\quad te^{-at}u(t), \mathrm{Re}\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{(a+j\omega)^2}$
5. $\frac{t^{n-1}}{(n-1)!}e^{-at}u(t), \mathrm{Re}\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{(a+j\omega)^n}$
6. $\sum_{n=-\infty}^{+\infty}\delta(t-nT) \xrightarrow{\mathcal{F}} \frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\omega - \frac{2\pi k}{T}\right)$
7. $\delta\left[n-n_0\right] \xrightarrow{\mathcal{F}} e^{j\omega n_0}$ $\quad e^{j\omega_0 n} \xrightarrow{\mathcal{F}} 2\pi\delta\left(\omega-\omega_0\right)$
8. $a^n u[n], |a| < 1 \xrightarrow{\mathcal{F}} \frac{1}{1-ae^{-j\omega}}$ $\quad (n+1)a^n u[n], |a| < 1 \xrightarrow{\mathcal{F}} \frac{1}{(1-ae^{-j\omega})^2}$

D. Properties of Fourier Transform

1. CT Time Shifting and Frequency Shifting: $x(t-t_0) \xrightarrow{\mathcal{F}} e^{-j\omega t_0}X(j\omega)$ $\quad e^{j\omega_0 t}x(t) \xrightarrow{\mathcal{F}} X\left(j\left(\omega-\omega_0\right)\right)$
2. CT Time and Frequency Scaling: $x(at) \xrightarrow{\mathcal{F}} \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$
3. CT Differentiation and Integration: $\frac{d}{dt}x(t) \xrightarrow{\mathcal{F}} j\omega X(j\omega)$ $\quad \int_{-\infty}^{t} x(t)dt \xrightarrow{\mathcal{F}} \frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4. DT Tmie Shifting and Frequency Shifting: $x[n-n_0] \xrightarrow{\mathcal{F}} e^{-j\omega n_0}X(j\omega)$ $\quad e^{j\omega_0 n}x[n] \xrightarrow{\mathcal{F}} X\left(j\left(\omega-\omega_0\right)\right)$
5. DT Difference: $x[n]-x[n-1] \xrightarrow{\mathcal{F}} (1-e^{jw})X(jw)$ $\quad nx[n] \xrightarrow{\mathcal{F}} j\frac{dX(e^{jw})}{dw}$