- Homework: 5.39, 5.48 in the attachment
- Tutorial: Second homework problem of Chapter 7B, 8.44, 8.46

1月5日 16:30 — 18:30 Final Exam No Cheating Paper

5.39 Twenty-four voice signals are sampled uniformly and are then time-division multiplexed, using PAM. The PAM signal is reconstructed from flat-topped pulses with 1-μs duration. The multiplexing operation provides for synchronization by adding an extra pulse of sufficient amplitude and also 1-μs duration. The highest frequency component of each voice signal is 3.4 kHz.

(a) Assuming a sampling rate of 8 kHz, calculate the spacing between successive pulses of the multiplexed signal.

(b) Repeat your calculation, assuming the use of Nyquist rate sampling.

5.48 Consider a multiplex system in which four input signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$ are respectively multiplied by the carrier waves

$$[\cos(\omega_a t) + \cos(\omega_b t)],$$
$$[\cos(\omega_a t + \alpha_1) + \cos(\omega_b t + \beta_1)],$$
$$[\cos(\omega_a t + \alpha_2) + \cos(\omega_b t + \beta_2)],$$

and

$$[\cos(\omega_a t + \alpha_3) + \cos(\omega_b t + \beta_3)]$$

S&S Solu

and the resulting DSB-SC signals are summed and then transmitted over a common channel. In the receiver, demodulation is achieved by multiplying the sum of the DSB-SC signals by the four carrier waves separately and then using filtering to remove the unwanted components. Determine the conditions that the phase angles $\alpha_1, \alpha_2, \alpha_3$ and $\beta_1, \beta_2, \beta_3$ must satisfy in order that the output of the kth demodulator be $m_k(t)$, where $k = 1, 2, 3, 4$.

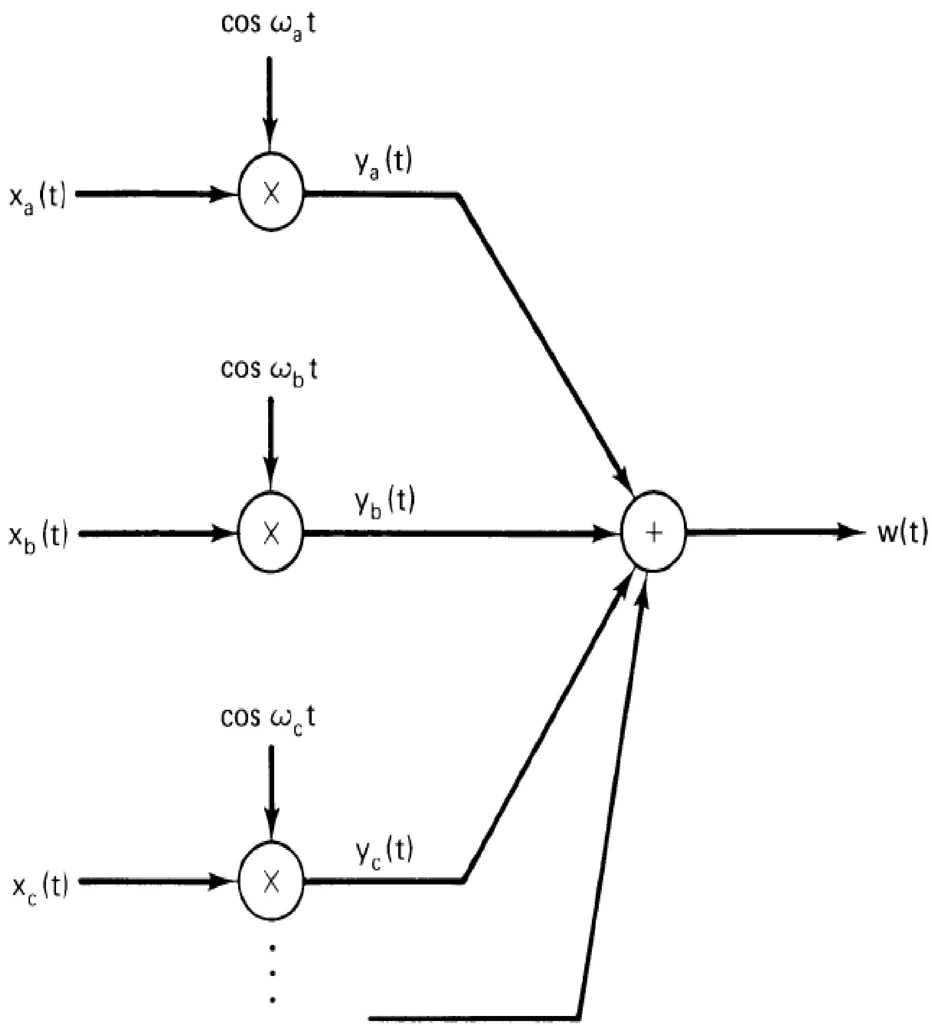
Summary

- Meaning of amplitude modulation
  - with a complex exponential carrier
  - with a sinusoidal carrier
- Demodulation for sinusoidal AM
  - Synchronous demodulation
  - Asynchronous demodulation, and its two important assumptions
- Frequency-division multiplexing (FDM)

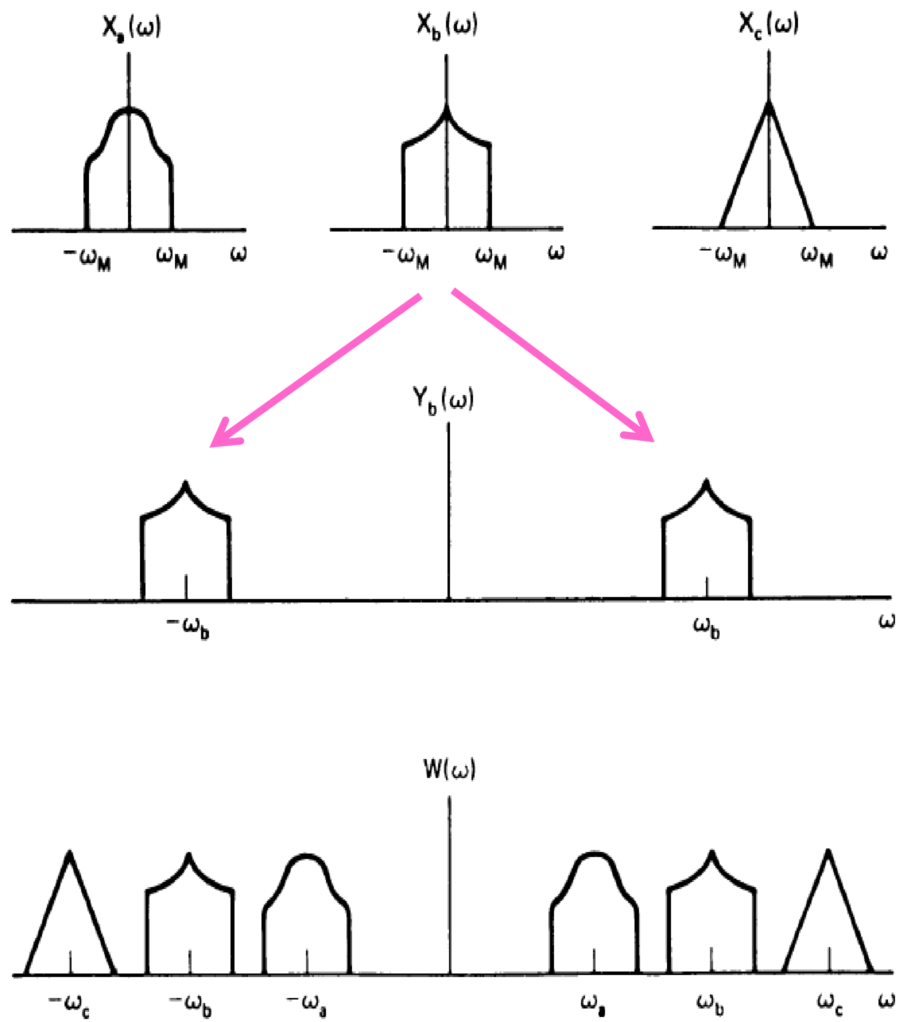
Frequency-division multiplexing (FDM)

- Systems for transmitting signals provide more bandwidth than is required for one signal.
  - E.g., speech signal → 20 ~ 20 kHz
    microwave channel → 300 MHz ~ 300 GHz
    satellite link → a few hundred MHz ~ 40 GHz
    (more in Fig. 8.18)
- Different modulating signals (e.g., speech), which are overlapping in frequency, can have their spectra shifted (e.g., by sinusoidal AM) without overlapping, so they can be transmitted simultaneously over a single wideband channel. 同时地

FDM (cont.)



cos $\omega_a t$
$x_a(t)$
$y_a(t)$
cos $\omega_b t$
$x_b(t)$
$y_b(t)$
cos $\omega_c t$
$x_c(t)$
$y_c(t)$
w(t)

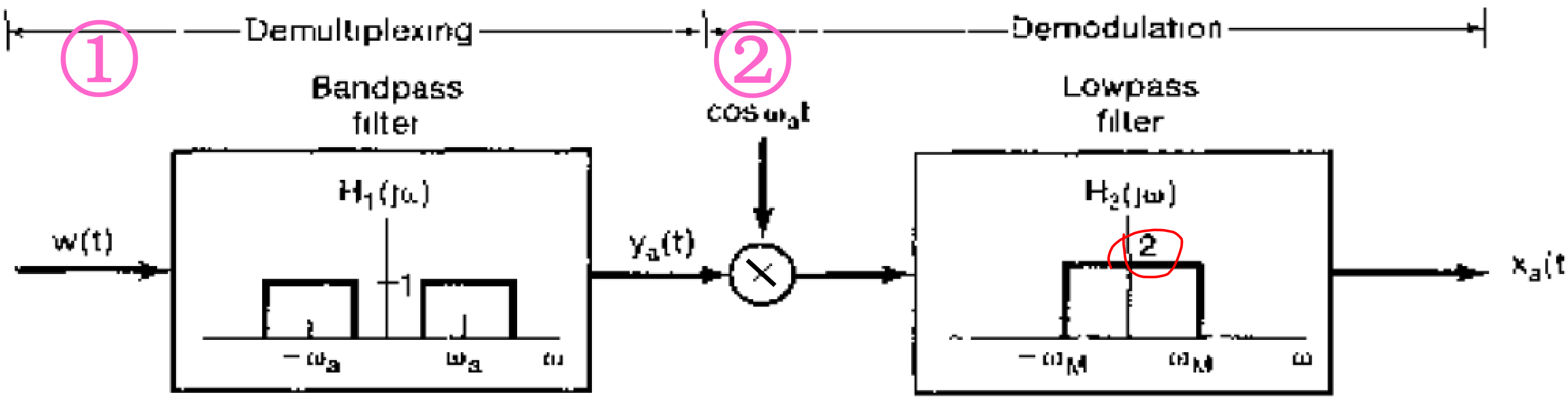


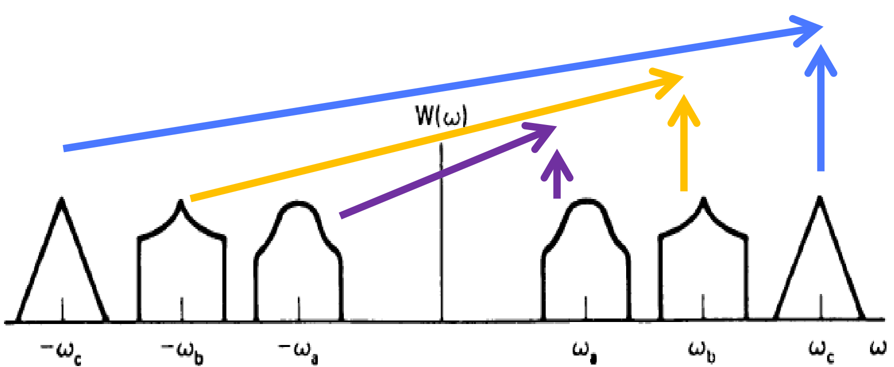
$X_a(\omega)$
$-\omega_M$ ω_M ω
$X_b(\omega)$
$-\omega_M$ ω_M ω
$X_c(\omega)$
$-\omega_M$ ω_M ω
$Y_b(\omega)$
$-\omega_b$ ω_b ω
$W(\omega)$
$-\omega_c$ $-\omega_b$ $-\omega_a$ ω_a ω_b ω_c ω

Review
FDM demultiplexing and demodulation

① bandpass filtering to have the modulated signal from one channel

② demodulation to recover the modulating signal



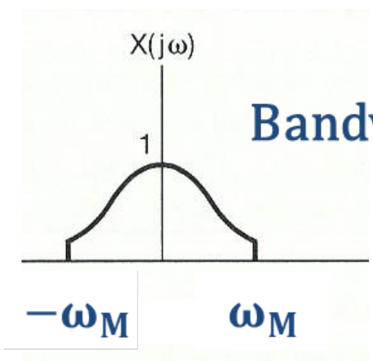


Occupy twice the original bandwidth

→ insufficient use of bandwidth

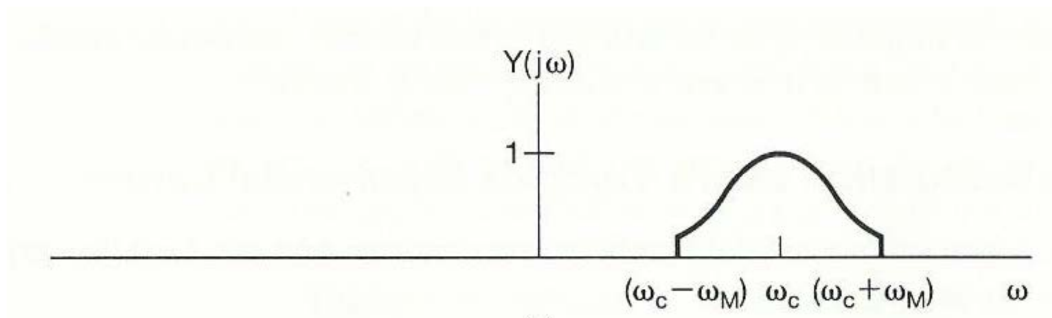
Single-sideband (SSB) sinusoidal AM

Occupied bandwidth:

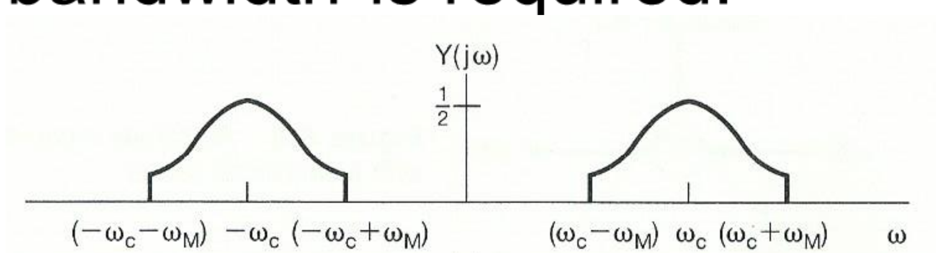


Bandwidth: $2\ \omega_M$

- With exponential carrier, the bandwidth is still $2\ \omega_M$

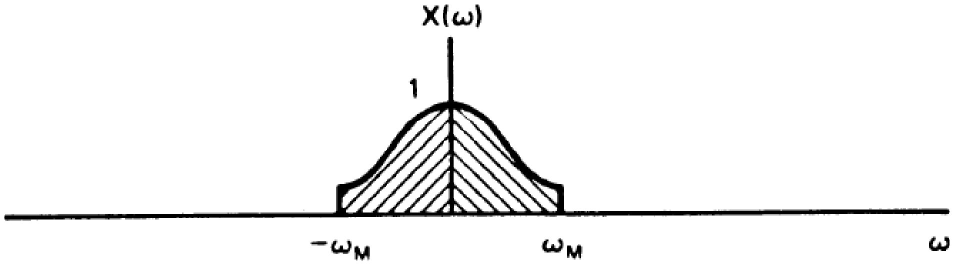


- With sinusoidal carrier, twice bandwidth is required.



SSB sinusoidal AM (cont.)

x(t)

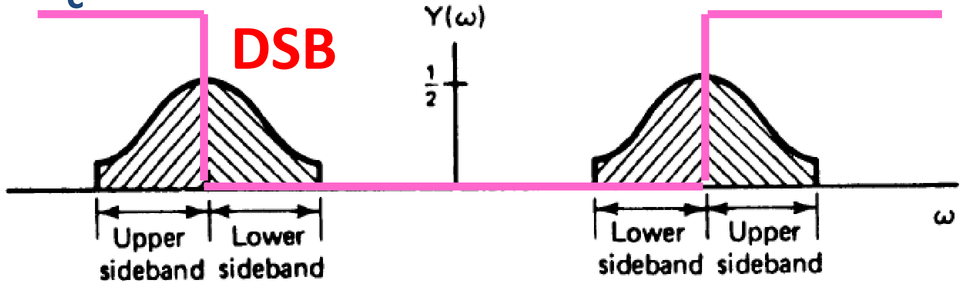


$x(t)\cos\omega_c t$

Double-sideband

DSB

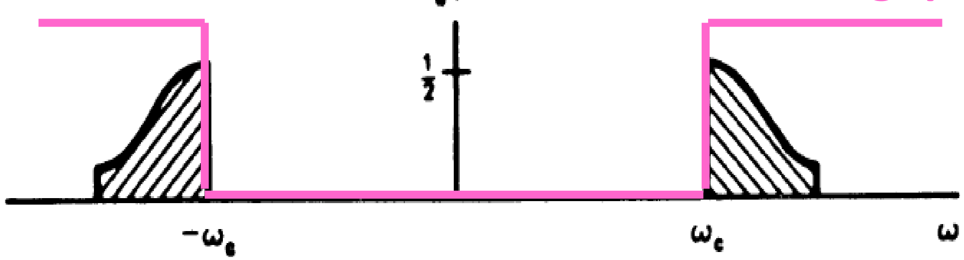
Ideal Highpass filter



<u>Observation:</u> x(t) can be recovered if two upper (or lower) sidebands are retained.

SSB

Ideal Highpass filter



Higher power and spectrum efficiency

Sinusoidal AM: Summary

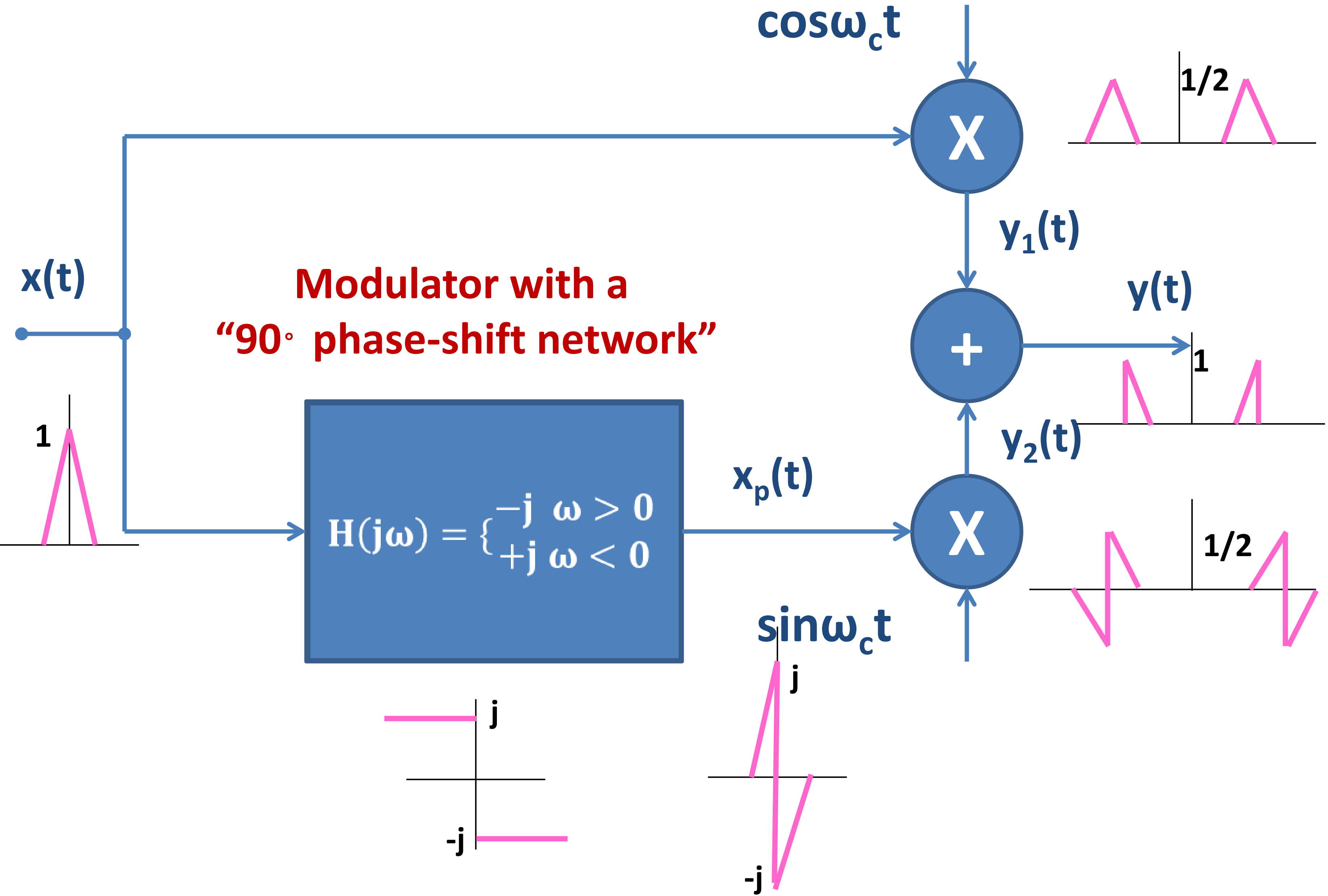
- Four types of sinusoidal AM
  - AM-DSB/SC: $y(t) = x(t) cos \omega_c t$
  - AM-DSB/WC: $y(t) = (x(t) + A) cos \omega_c t$
  - AM-SSB/SC: AM-DSB/SC + ideal highpass/lowpass filter
  - AM-SSB/WC: AM-SSB/SC + $A cos \omega_c t$

WC: with carrier

AM-DSB/WC

$$y(t) \simeq \cos \omega_c t - m(\sin \omega_m t)(\sin \omega_c t).$$

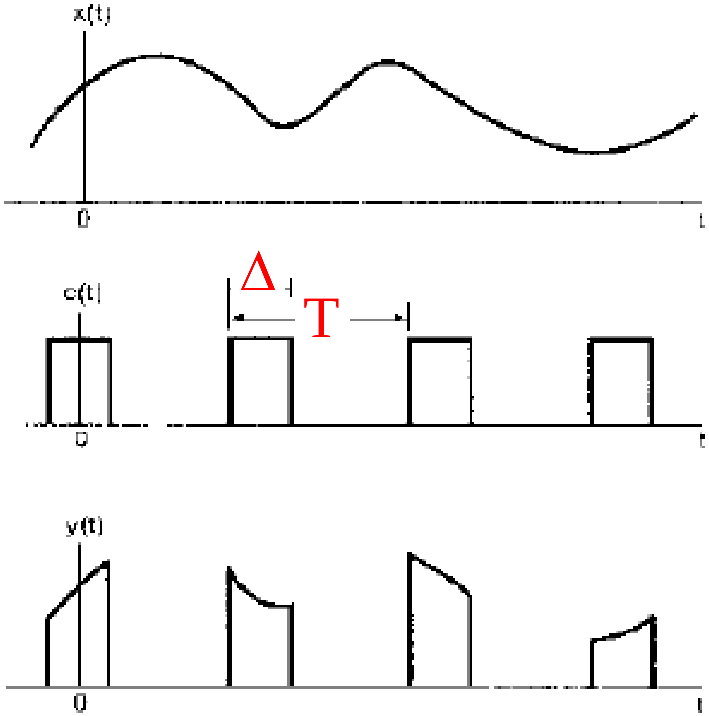
SSB sinusoidal AM (cont.)



Amplitude modulation with a pulse train

- Carrier signal could be a sinusoidal signal, or a pulse train.

Figure 8.23



$$X(j\omega)$$

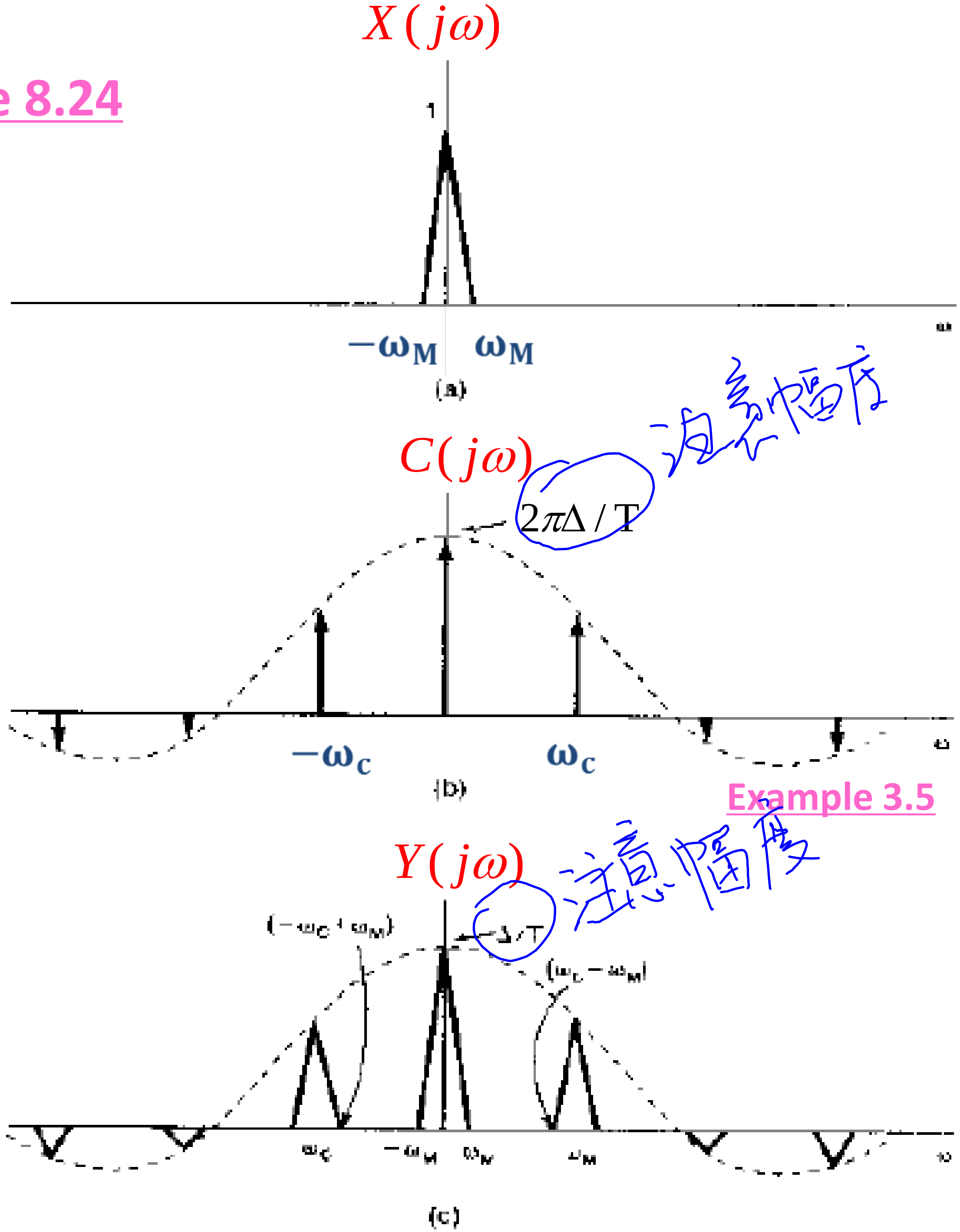
$$C(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_c)$$

$$a_k = \frac{\sin(k\omega_c \Delta / 2)}{\pi k}$$

Read Example 3.5

$$Y(j\omega) = \sum_{k=-\infty}^{\infty} a_k X(j(\omega - k\omega_c))$$

Figure 8.24



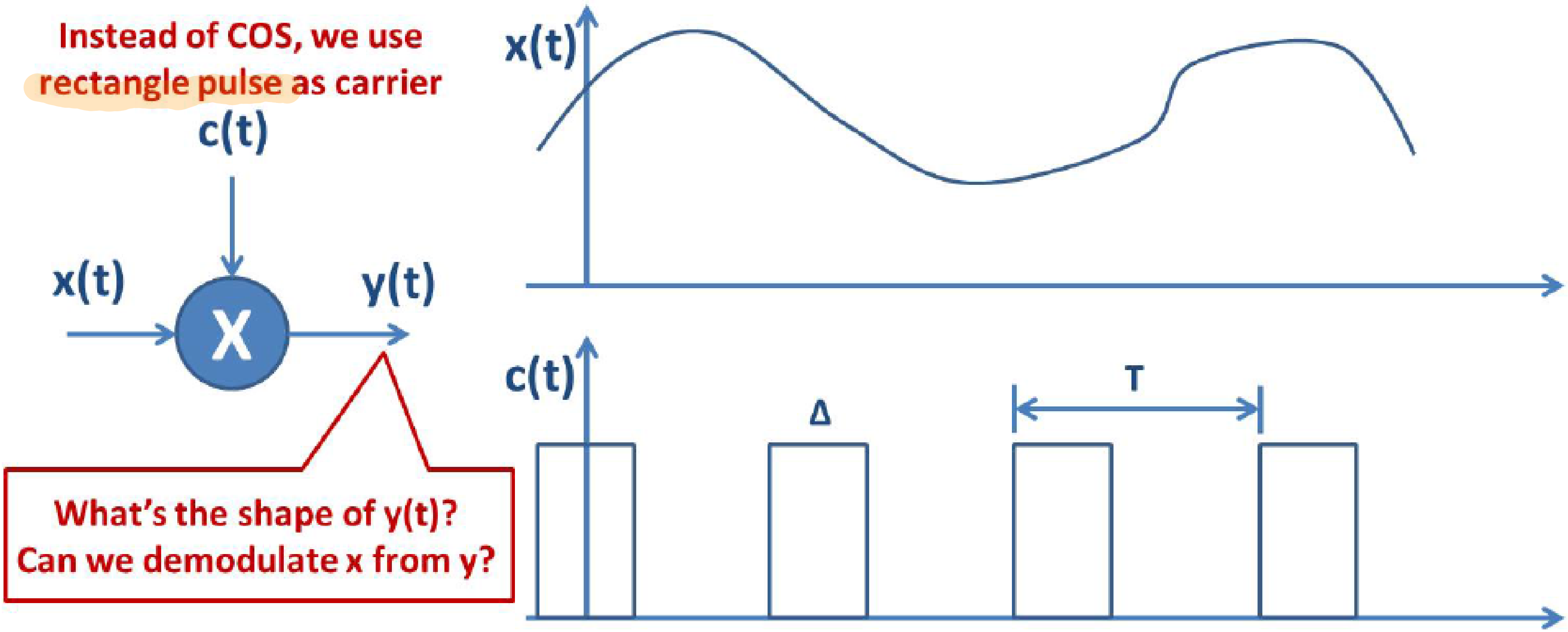
Example 3.5

Only affected by T

When $\omega_c > 2\omega_M$, $X(j\,\omega)$ can be recovered by lowpass filtering.

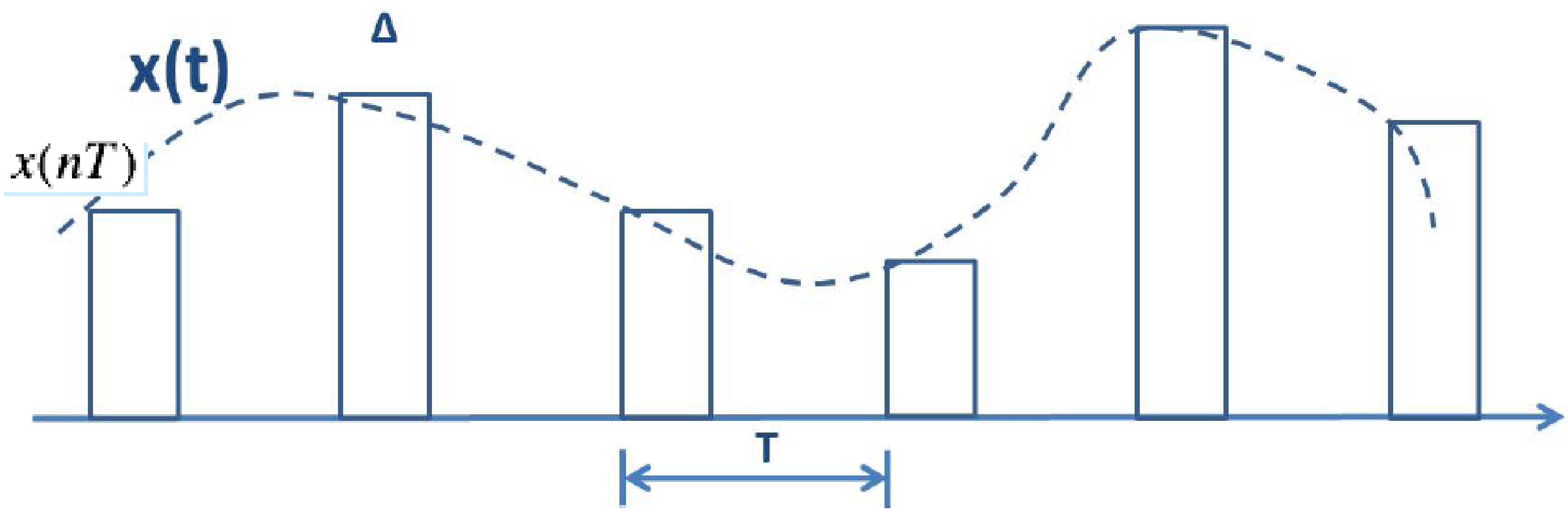
<u>Note:</u> similar to the condition in sampling.

AM with Pulse-Train Carrier



$$C(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{2 sin k\omega_0 \frac{\Delta}{2}}{k} \delta(\omega - k\omega_0) \quad where \quad \omega_0 = 2\pi / T$$

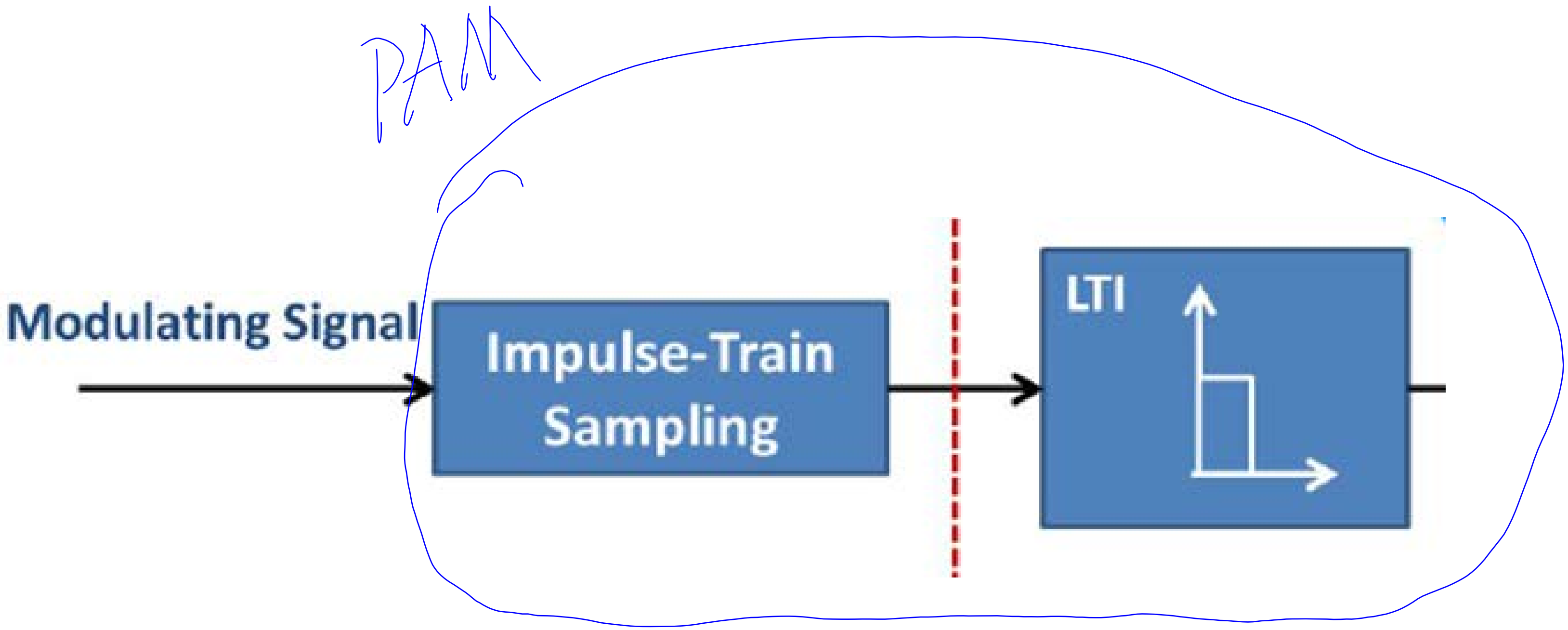
Pulse-Amplitude Modulation (PAM)



采样 + 保持

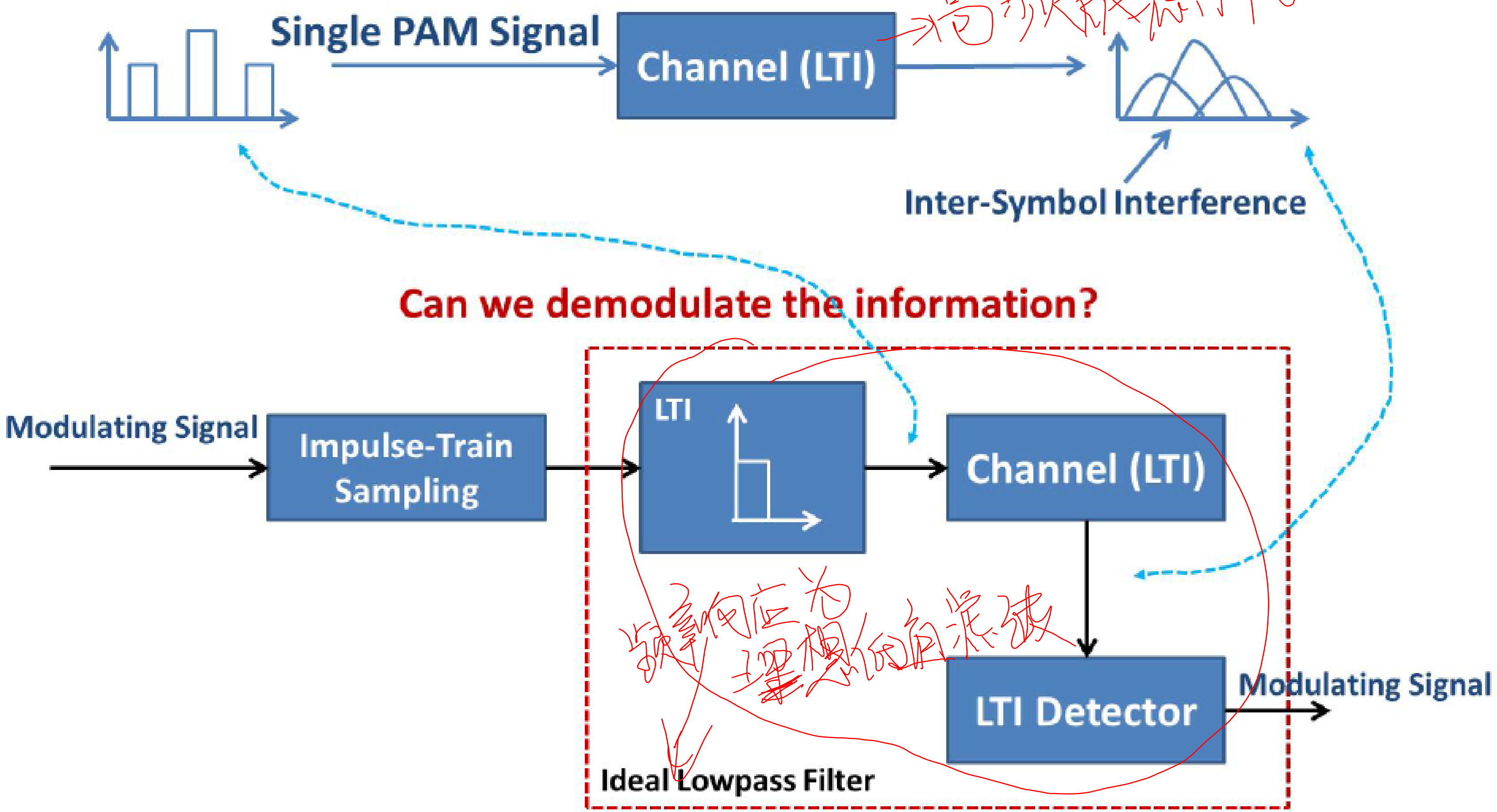
- Pulse-Amplitude Modulation (PAM): sample with period T and hold for duration Δ
- Questions:
  - Without channel distortion, how to demodulate?

Cont.



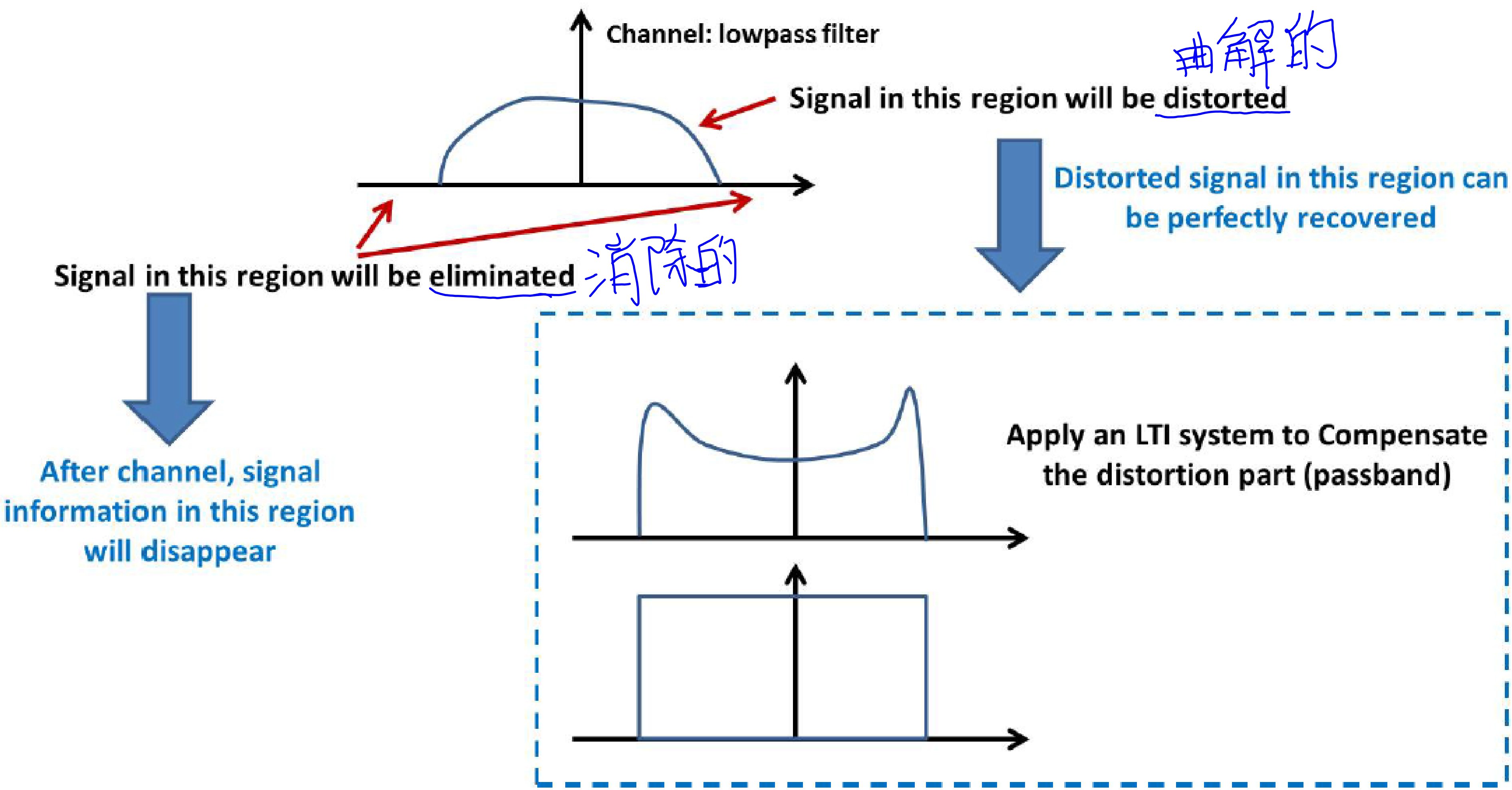
Inter-Symbol Interference (ISI) 码间串扰

- Communication channel is usually a low-pass filter (基于这个前提条件分析ISI)
- Receiver recovers the modulating signal with an LTI system



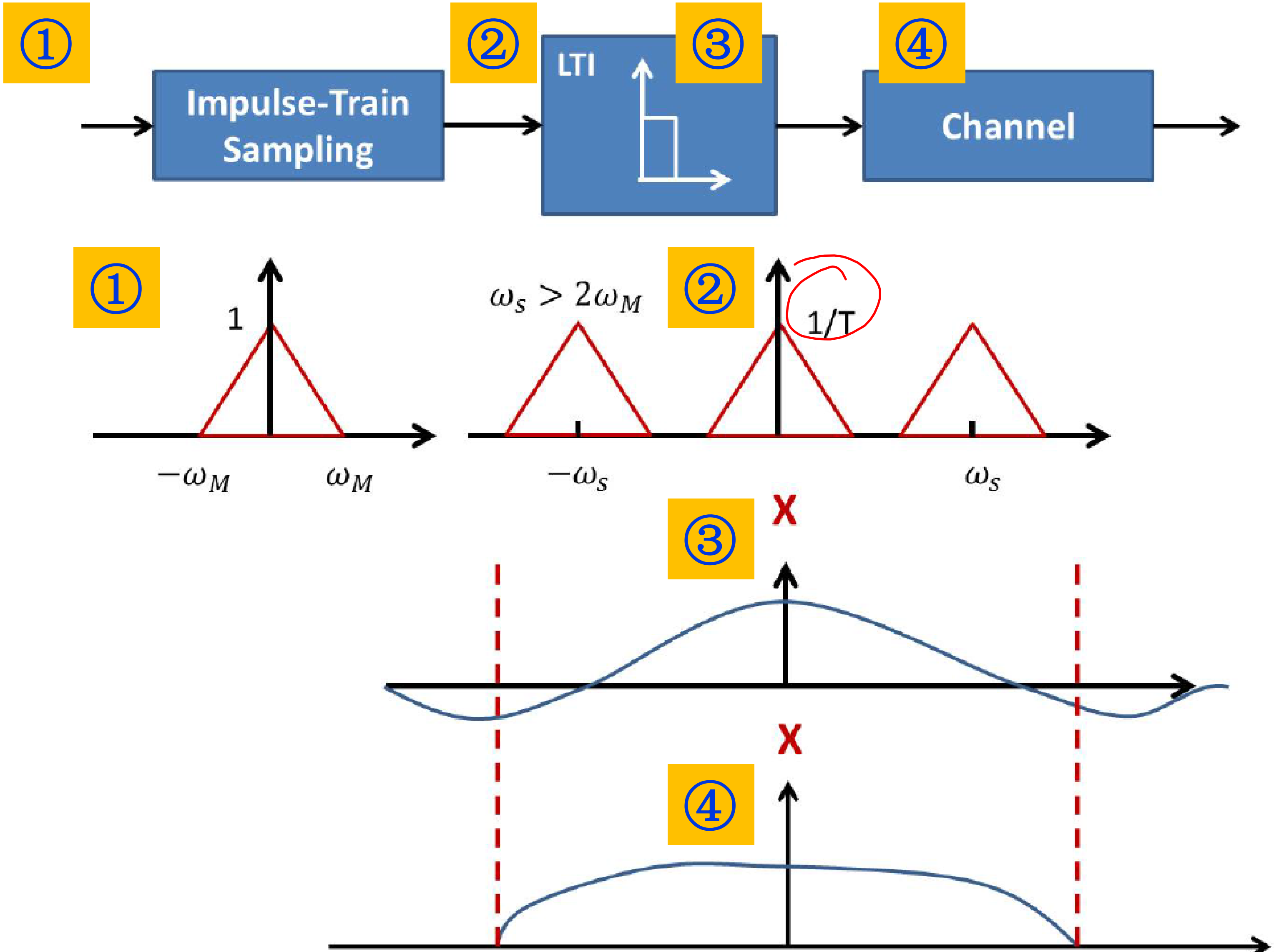
Against ISI (1)

- What's the cause of ISI?



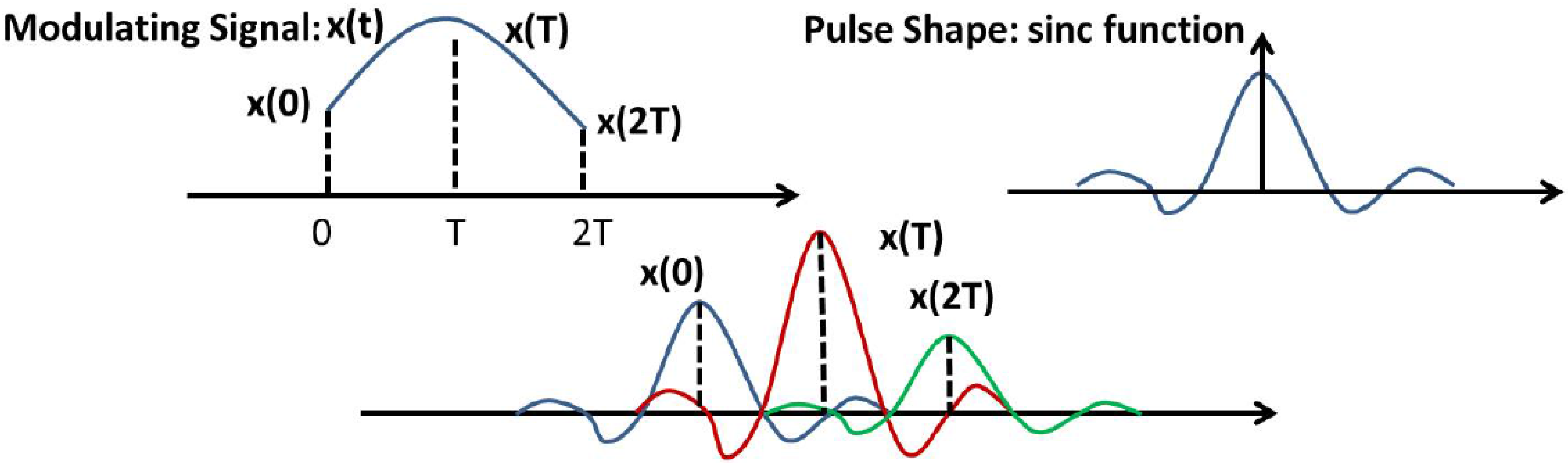
Against ISI (2)

- When using rectangular wave in PAM, the bandwidth is infinity

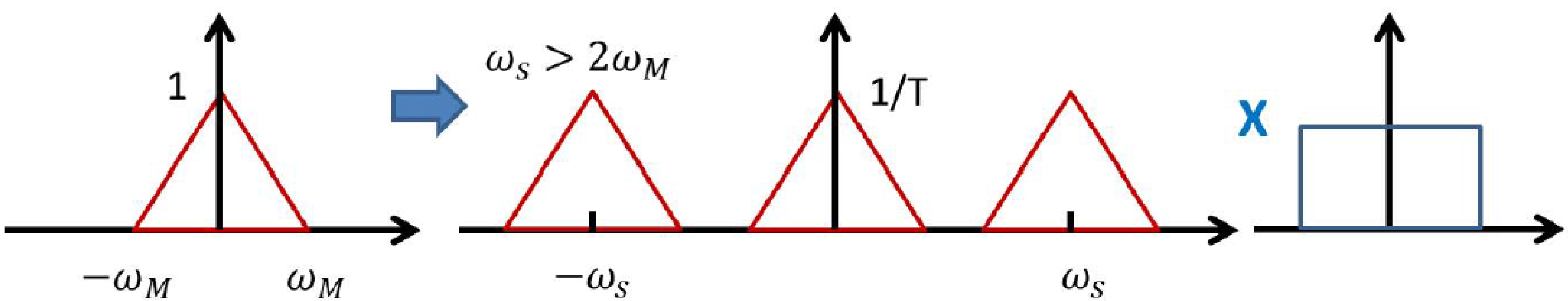


Against ISI (2) (Cont.)

- To avoid ISI, PAM signal should use band-limited pulse. E.g., sinc function



1. Receiver can perfectly recover this signal
2. Sampled values are kept in the peaks of pulses



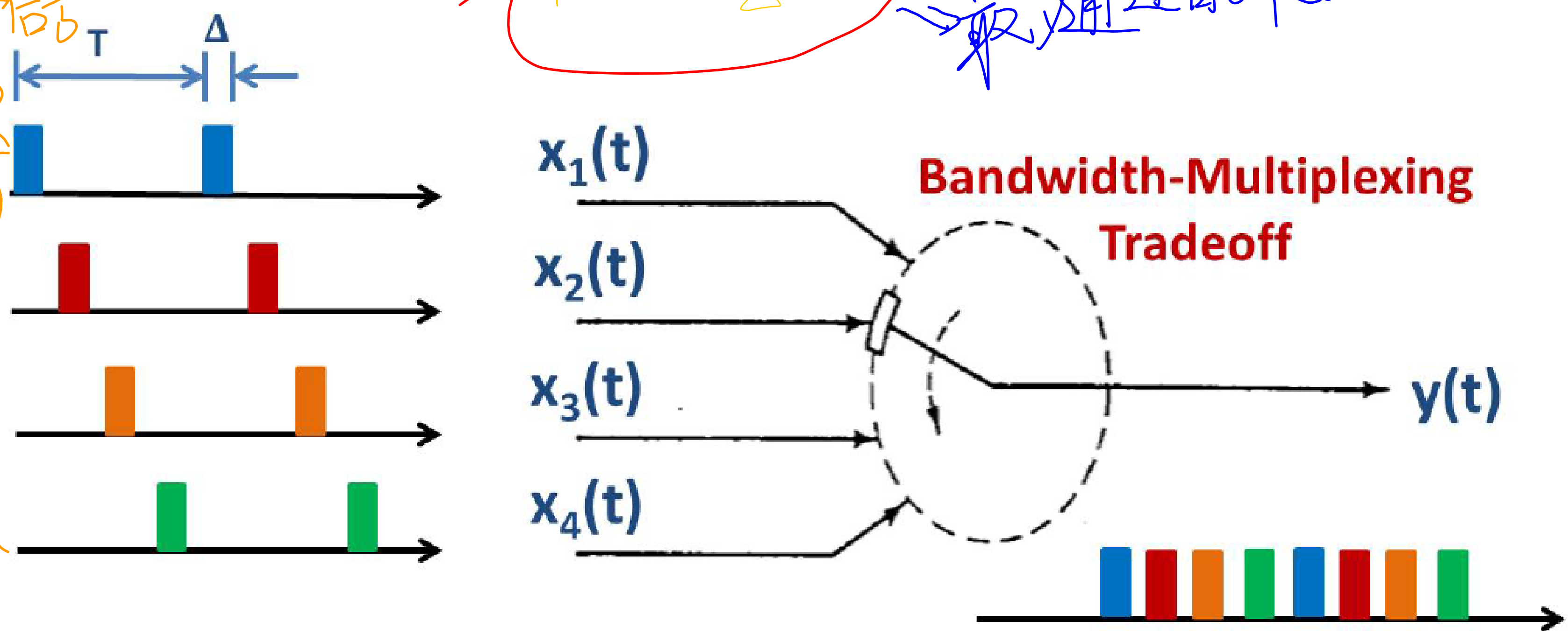
Summary on PAM

- Modulation $\leftrightarrow$ Sampling (zero-order hold); Demodulation $\leftrightarrow$ Recovery
- In additional to rectangular wave, other pulses can be used for PAM
- Demodulation without channel distortion
- Demodulation with channel distortion (ISI)
  - Recover modulating signal: make sure pulse shaping $+$ channel $+$ receiver's detector $=$ ideal low-pass filter
  - Recover modulated signal: use band-limited wave in PAM. E.g., sinc wave

Time-Division Multiplexing (TDM)

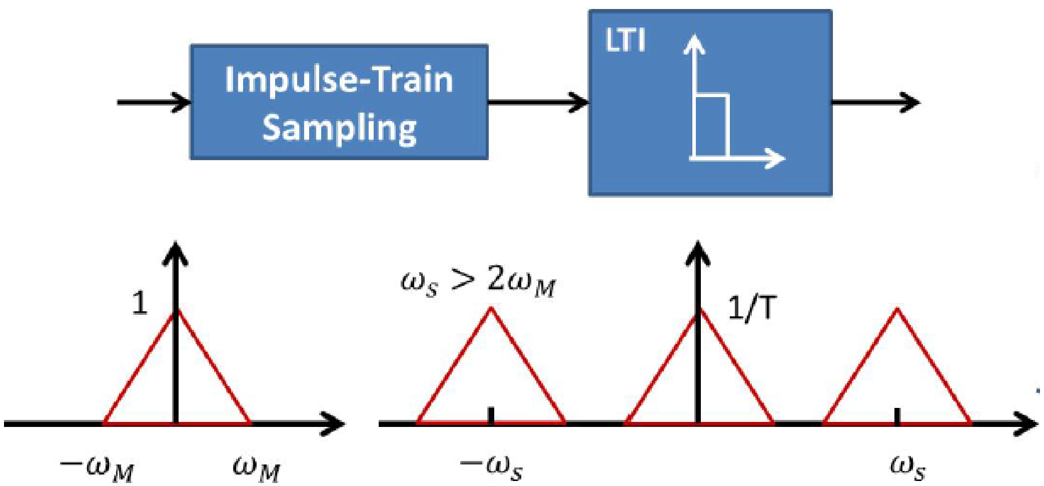
AM signals with pulse-train carrier or PAM signals can be multiplexed in time domain

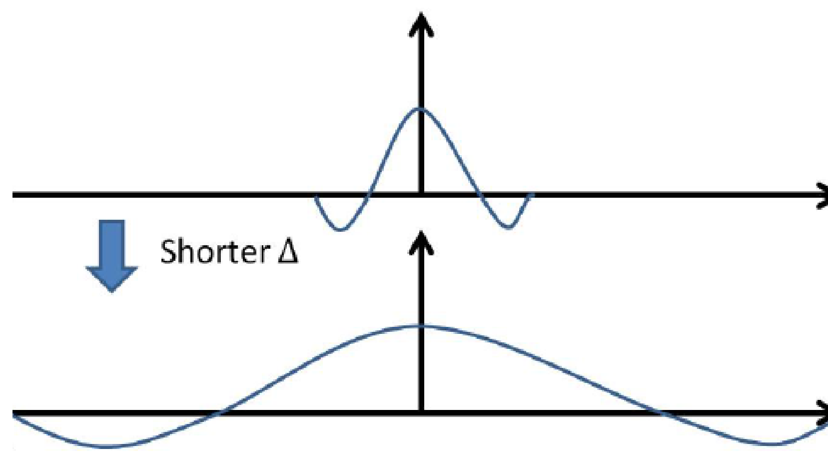
max = $\frac{T}{\Delta}$ → 最多通道的个数



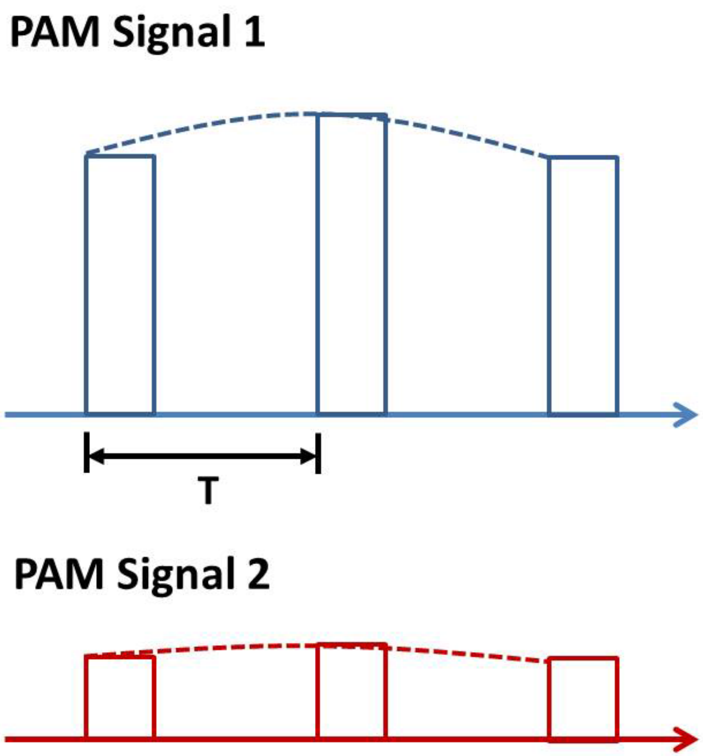
Cont.

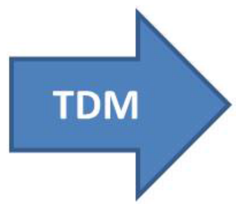
- The number of multiplexed signals is determined by T and Δ
- Can T be as large as we want?
  - T is the sampling period
  - Let ω_M be the bandwidth of modulating signal
  - $\frac{2\pi}{T} > 2\omega_M \Rightarrow T < \frac{\pi}{\omega_M}$
- Can Δ be as short as we want?
  - Shorter pulse $\Rightarrow$ larger bandwidth consumption

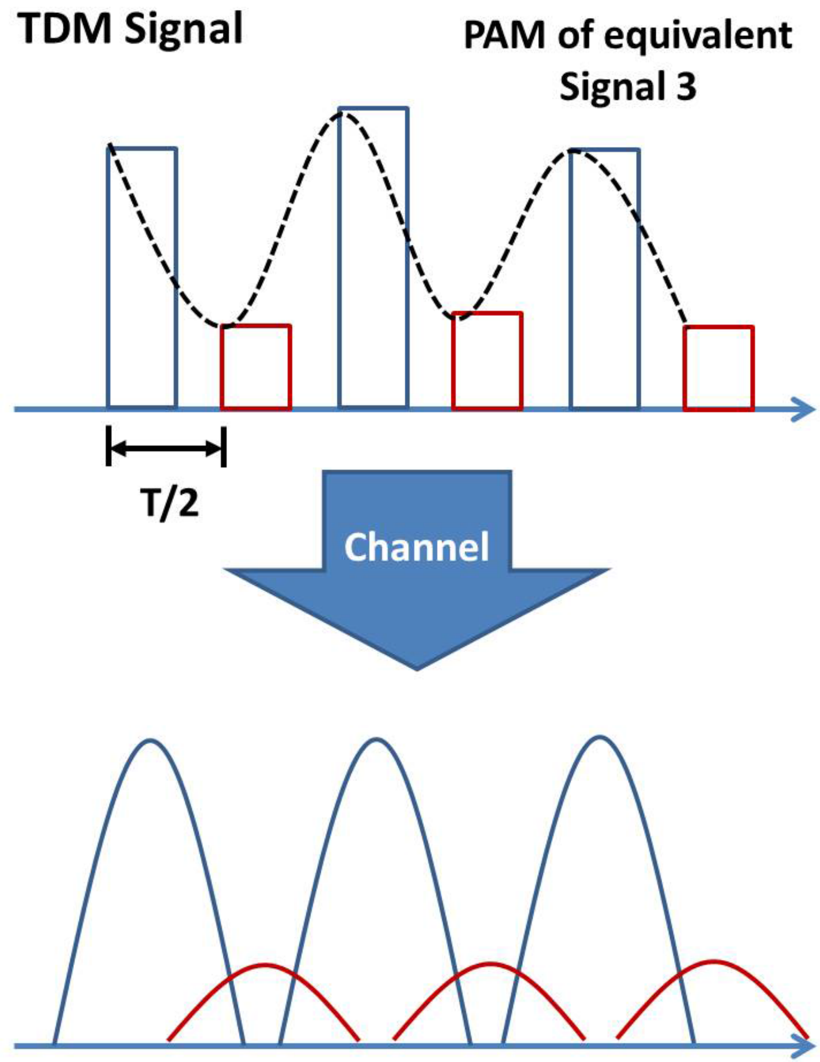




ISI in TDM





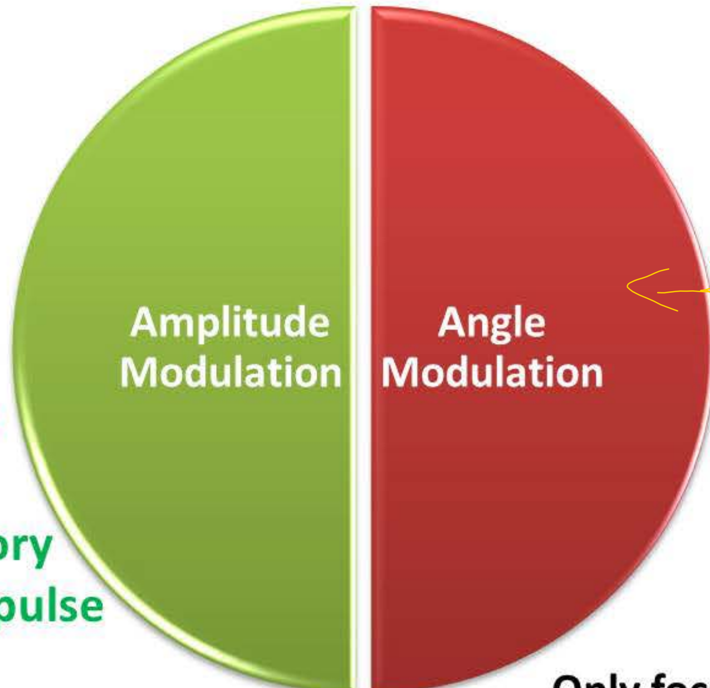


Similar to PAM, we have two approaches to handle ISI

What is more ...

Amplitude of carrier contains information

Angle of carrier contains information



- Sinusoid carrier
  - FDM
- Pulse train carrier
- PAM
  - Demod. without ISI
  - Demod. With ISI
    1. Sampling theory
    2. Band-limited pulse
  - TDM

- Phase modulation
- Frequency modulation
  - Narrowband
  - Wideband

Only focus on some basic properties
The demodulator will be introduced in next semester

Angle modulation 幅度不变

$$c(t) = A\cos(\omega_c t + \theta_c) = A\cos\theta(t)$$

$\theta(t) = \omega_c t + \theta_c$ and where ω_c is the frequency and θ_c the phase of the carrier.

Angle modulation: use modulating signal to change or vary the angle θ(t)

instantaneous frequency $\omega_i(t) = \dfrac{d\theta(t)}{dt}$

$$y(t) = A\cos[\omega_c t + \theta_c(t)] \qquad \theta_c(t) = \theta_0 + k_p x(t)$$

Phase modulation: the phase of y(t) is varying with modulating signal

---

$$y(t) = A\cos\theta(t) \qquad \frac{d\theta(t)}{dt} = \omega_c + k_f x(t)$$

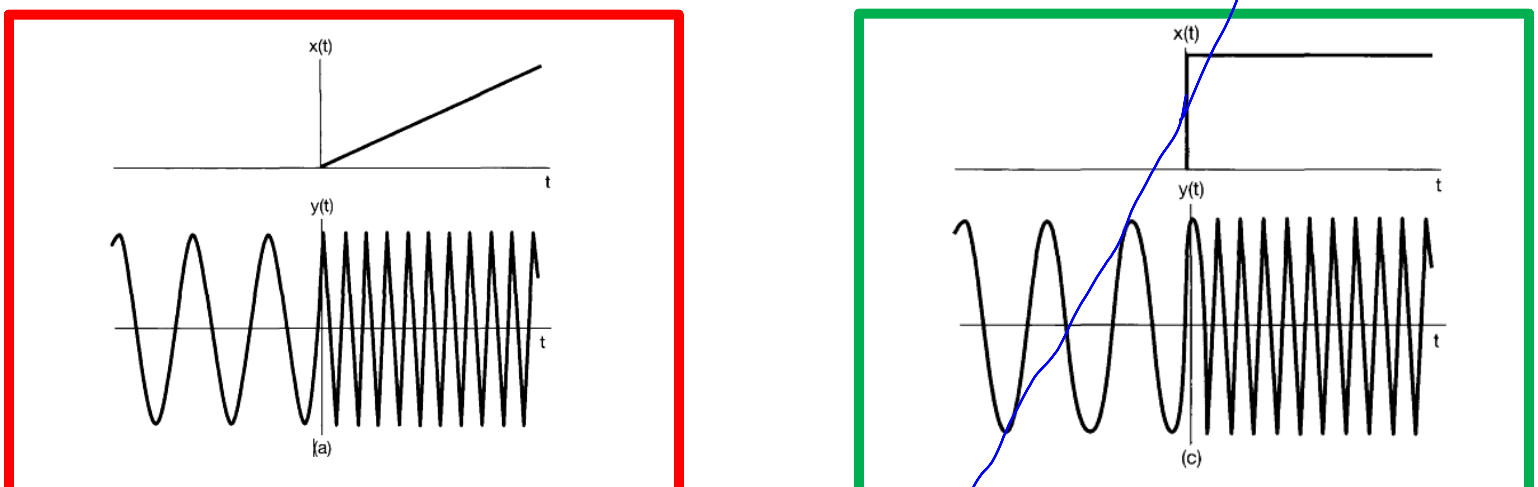
Frequency modulation: the derivative of the angle is varying with modulating signal

$$\boxed{\frac{d\theta(t)}{dt} = \omega_c + k_p \frac{dx(t)}{dt}}$$

$$\boxed{\frac{d\theta(t)}{dt} = \omega_c + k_f x(t)}$$

- Phase modulating with *x(t)* is identical to frequency modulating with the derivative of *x(t)*.
- Likewise, frequency modulating with *x(t)* is identical to phase modulating with the integral of *x(t)*.

Fig. 8.32



Narrowband frequency modulation

$$x(t) = A\cos\omega_m t$$

$$\omega_i(t) = \omega_c + k_f A\cos\omega_m t,$$

which varies sinusoidally between $\omega_c + k_f A$ and $\omega_c - k_f A$.

With $\Delta\omega = k_f A$, we have $\omega_i(t) = \omega_c + \Delta\omega\cos\omega_m t$,

$$y(t) = \cos[\omega_c t + \int k_f x(t)dt] = \cos\left(\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t + \theta_0\right),$$

where θ_0 is a constant of integration. For convenience we will choose $\theta_0 = 0$, so that

$$y(t) = \cos\left[\omega_c t + \frac{\Delta\omega}{\omega_m}\sin\omega_m t\right].$$

Modulation index for frequency modulation $m = \Delta\omega/\omega_m$

Cont.

- The properties of FM systems tend to be different, depending on whether the modulation index *m* is small or large.

$$y(t) = \cos(\omega_c t + m \sin \omega_m t)$$

or

$$y(t) = \cos \omega_c t \cos(m \sin \omega_m t) - \sin \omega_c t \sin(m \sin \omega_m t).$$

When m is sufficiently small ($\ll \pi/2$), we can make the approximations

$$\cos(m \sin \omega_m t) \simeq 1,$$
$$\sin(m \sin \omega_m t) \simeq m \sin \omega_m t,$$

$$y(t) \simeq \cos \omega_c t - m(\sin \omega_m t)(\sin \omega_c t).$$

$$\boxed{y(t) = x(t)c(t) = x(t)\cos(\omega_c t)}$$

Similar to AM-DSB/WC

Cont.

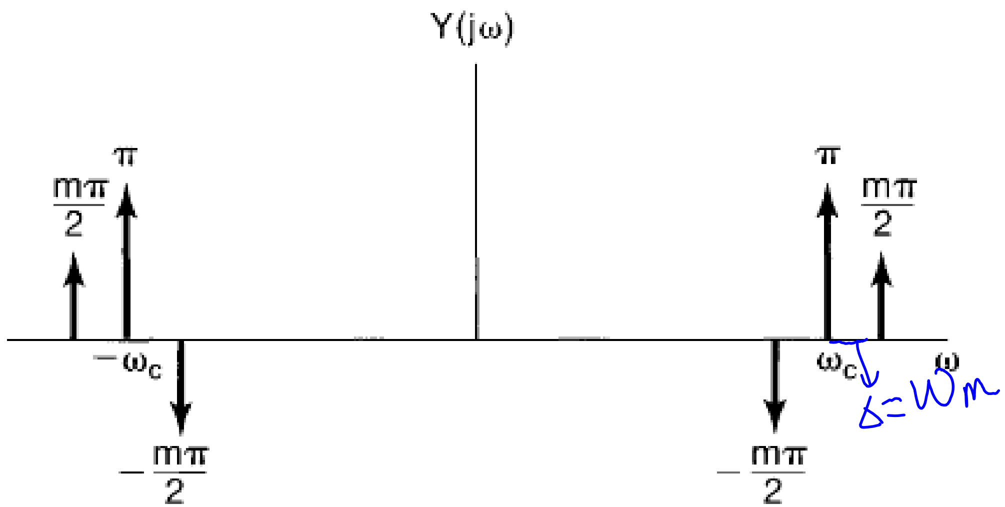


Figure 8.33 Approximate spectrum for narrowband FM.

- The bandwidth of the sidebands is equal to the bandwidth of the modulating signal.
  - ✓ The bandwidth of the sidebands is independent of the modulation index *m* (i.e., it depends only on the bandwidth of the modulating signal, not on its amplitude).

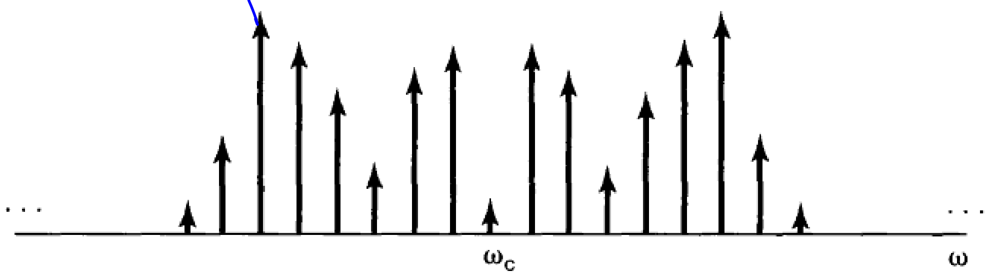
Wideband Frequency Modulation

$$y(t) = \cos\omega_c t \underline{\cos(m\sin\omega_m t)} - \sin\omega_c t \underline{\sin(m\sin\omega_m t)}.$$

- The terms $\cos[m\sin\omega_m t]$ and $\sin[m\sin\omega_m t]$ are periodic signals with fundamental frequency ω_m.
- Thus, the Fourier transform of each of these signals is an impulse train with impulses at integer multiples of ω_m and amplitudes proportional to the Fourier series coefficients.
- The coefficients for these two periodic signals 贝塞尔函数 involve a class of functions referred to as Bessel functions of the first kind.
  - The first term corresponds to a sinusoidal carrier of the form $\cos\omega_c t$ amplitude modulated by the periodic signal $\cos[m\sin\omega_m t]$
  - The second term to a sinusoidal carrier $\sin\omega_c t$ amplitude modulated by the periodic signal $\sin[m\sin\omega_m t]$

FT of $\cos\omega_c t \cos(m \sin \omega_m t)$

Cont.

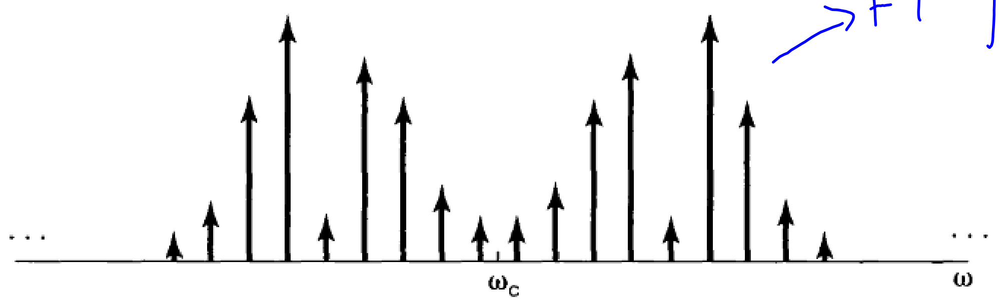


(a)

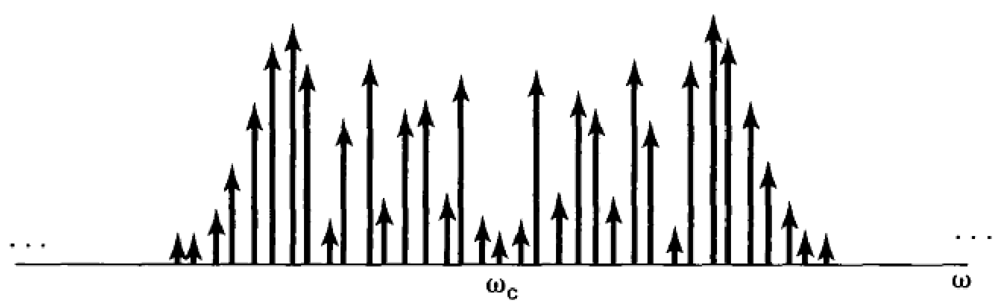
bandwidth

$$B \simeq 2k_f A = 2\Delta\omega.$$

FT of $\sin\omega_c t \sin(m \sin \omega_m t)$



(b)



(c)

Figure 8.35 Magnitude of spectrum of wideband frequency modulation with $m = 12$: (a) magnitude of spectrum of $\cos \omega_c t \cos[m \sin \omega_m t]$; (b) magnitude of spectrum of $\sin \omega_c t \sin[m \sin \omega_m t]$; (c) combined spectral magnitude of $\cos[\omega_c t + m \sin \omega_m t]$.

Cont.

- For wideband FM, since we assume that *m* is large, the bandwidth of the modulated signal is much larger than the bandwidth of the modulating signal.
- In contrast to the narrowband case, the bandwidth of the transmitted signal in wideband FM is directly proportional to amplitude *A* of the modulating signal and the gain factor k_f.