Lab 3. Fourier Series Representation of Periodic Signals

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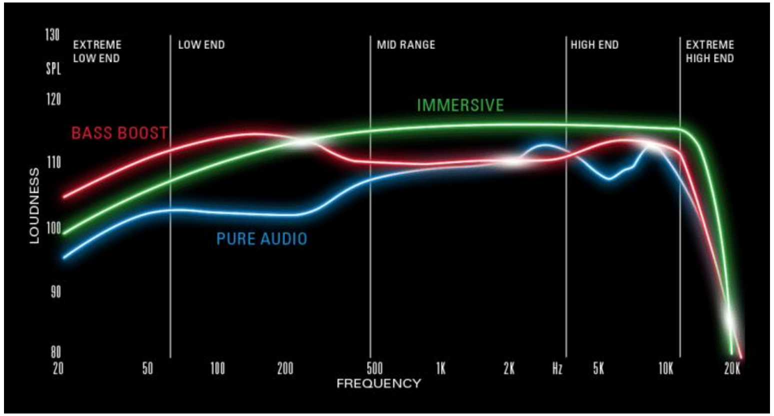
Overview

- In this tutorial, you will learn
  1. How to calculate the frequency response of DT LTI system (frequency domain)



  2. How to calculate the output of CT LTI system via Matlab (time domain)
  3. How to calculate the DTFS of signal via Matlab

Calculating the frequency response of DT LTI system

Complex Exponentials

- The *Only* Eigenfunctions of *Any* LTI Systems

$$x(t) = e^{st} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{+\infty} h(\tau) e^{s(t-\tau)} d\tau$$

$$= \left[\int_{-\infty}^{+\infty} h(\tau) e^{-s\tau} d\tau\right] e^{st}$$

$$= \underbrace{H(s)}_{\text{eigenvalue}} \underbrace{e^{st}}_{\text{eigenfunction}}$$

$s = j\omega$ - purely imaginary,
i.e. signals of the form $e^{j\omega t}$

For each s (each ω), eigenvalue eigenfunction

$H(s)$ *or* $H(e^{j\omega})$ – frequency response

DT LTI System

$$e^{j\omega n}$$

$$H(e^{j\omega})e^{j\omega n}$$

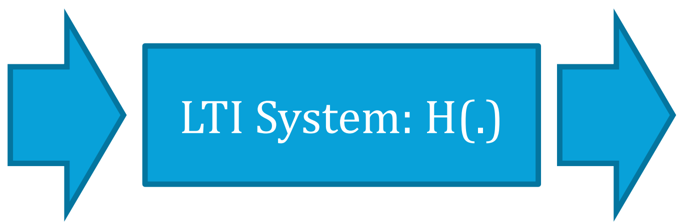
$$a_0 e^{j0\underline{\left(\frac{2\pi}{N}\right)}n}$$

$$a_1 e^{j1\left(\frac{2\pi}{N}\right)n}$$

$$a_2 e^{j2\left(\frac{2\pi}{N}\right)n}$$

$$\dots$$

$$a_{N-1} e^{j(N-1)\left(\frac{2\pi}{N}\right)n}$$



$$a_0 H(e^{j0\underline{\left(\frac{2\pi}{N}\right)}})e^{j0\underline{\left(\frac{2\pi}{N}\right)}n}$$

$$a_1 H(e^{j1\left(\frac{2\pi}{N}\right)})e^{j1\left(\frac{2\pi}{N}\right)n}$$

$$a_2 H(e^{j2\left(\frac{2\pi}{N}\right)})e^{j2\left(\frac{2\pi}{N}\right)n}$$

$$\dots$$

$$a_{N-1} H(e^{j(N-1)\left(\frac{2\pi}{N}\right)})e^{j(N-1)\left(\frac{2\pi}{N}\right)n}$$

LTI System by Difference Equation

- Reading assignment: textbook 2.4

- Causal DT LTI system can be specified by a linear constant-coefficient difference equation

$$\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

  - $A = [a_0, a_1, \dots a_K]$
  - $B = [b_0, b_1, \dots b_M]$

- For example:
  - $y[n]=0.5x[n]+x[n-1]+2x[n-2]$; $h[n]=?$ Finite Impulse Response (FIR)
  - $y[n]-0.8y[n-1]=2x[n]$; $h[n]=?$ Infinite Impulse Response (IIR)

- Causal DT LTI system is uniquely specified by two vectors $A=[a_0\ a_1\ a_2\ \ldots\ a_K]$ and $B=[b_0\ b_1\ \ldots\ b_M]$
  - A=[1] B=[0.5 1 2]
  - A=[1 -0.8] B=[2]

Frequency Response: freqz()

- freqz(): generate system frequency response

代表离散时间信号

freqs() 代表连续时间信号

```
[H omega] = freqz(b, a, N);
```

$$H(e^{j\omega_k}) \quad \omega_k = \left(\frac{\pi}{N}\right)k, 0 \le k \le N-1$$

N指把该区间分成N个点

```
[H omega] = freqz(b, a, N, 'whole');
```

计算（-pi,pi)区间内的值

$$H(e^{j\omega_k}) \quad \omega_k = \left(\frac{2\pi}{N}\right)k, 0 \le k \le N-1$$

Example

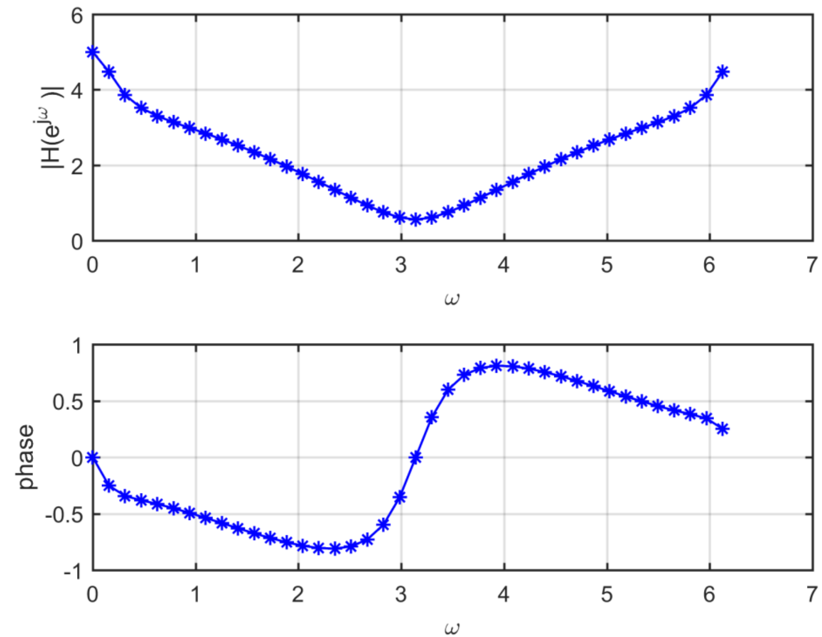
- Consider LTI System: y[n]-0.8y[n-1]=2x[n]-x[n-2]
  - Define the vector of coefficients:

```
A=[1 -0.8];
B=[2 0 -1];
```

  - Plot the frequency response:

```
[H omega] = freqz(B, A, 40, 'whole');
subplot(211);plot(omega, abs(H), '*-');
xlabel('\omega');                     幅频响应
ylabel('|H(e^{j\omega} )|'); grid;
subplot(212);plot(omega, angle(H), '*-');
xlabel ('\omega');                    相频响应
ylabel('phase'); grid;
```

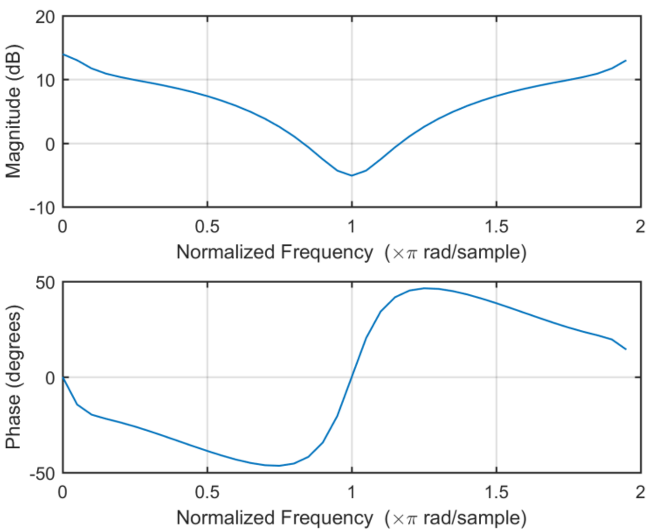
$H\left(e^{j0\left(\frac{2\pi}{N}\right)}\right)$ …… $H\left(e^{j(N-1)\left(\frac{2\pi}{N}\right)}\right)$

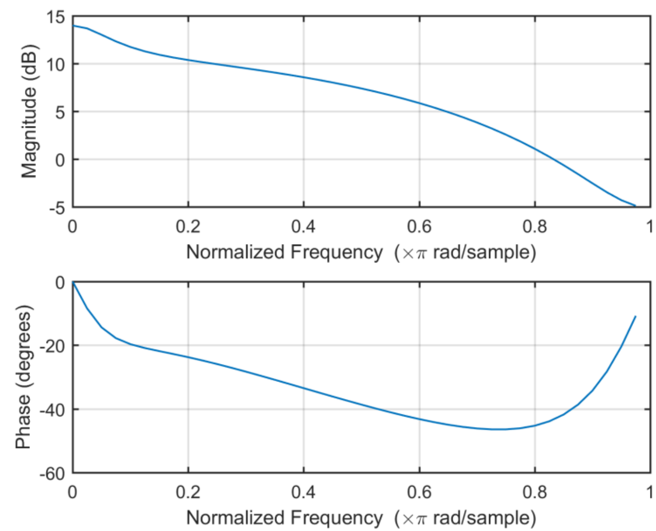


Search 'Greek Letters and Special Characters' in Matlab Help Documents

```
A=[1 -0.8];
B=[2 0 -1];
figure;
freqz(B,A,40, 'whole')
```

```
A=[1 -0.8];
B=[2 0 -1];
figure;
freqz(B,A,40)
```





Time to Call the Roll

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逃课没戏了!高校老师研发人脸识别无人机课堂点名

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A⁺ A⁻

原标题：就问你怕不怕！川大老师研发人脸识别无人机课堂点名

“上课的学生不知道有这节目，Spurise!”因独特的教学方式，学术圈网红教授，四川大学计算机系主任，魏骁勇又“作妖”了。这次搞了个升级版基于人脸识别的上课点名——无人机点名+巡堂。除了增加教学的趣味和参与度，也让学生在参与的过程中学会发现和解决新问题。

从利用徒手劈砖技能讲解物理原理，到“刷脸神器”打考勤，“刷女神器”帮联谊。魏骁勇教授6年来，一直变着花样的带给课堂惊喜。5月23日下午，在四川大学江安校区的教室里，魏

Calculating the output of CT LTI system

CT LTI System by Differential Equation

- Reading assignment: textbook 2.4.1
- Causal CT LTI system can be specified by a linear constant-coefficient differential equation

$$\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

- Coefficient vectors:
  - $A = [a_K, a_{K-1}, \dots a_0]$
  - $B = [b_M, b_{M-1}, \dots b_0]$

最高阶微分系数在前，与DT相反

- CT LTI system specified by differential equation
  - $\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$
  - $A = [a_K, a_{K-1}, \ldots a_0]$
  - $B = [b_M, b_{M-1}, \ldots b_0]$
- DT LTI system specified by difference equation
  - $\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$
  - $A = [a_0, a_1, \ldots a_K]$
  - $B = [b_0, b_1, \ldots b_M]$

Multiple Choice(single)

Points: 1

设置

Coefficient vector A and B of system 0.3y(t)+dy(t)/dt = 3x(t) are:

A. A=[1, 0.3]; B=3;

B. A=[0.3, 1]; B=3;

C. A=3; B = [1, 0.3];

D. A=3; B = [0.3, 1];

$$\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

$$A = [a_K, a_{K-1}, \dots a_0]$$

$$B = [b_M, b_{M-1}, \dots b_0]$$

提交

- Differential Equation and Transfer Function

$$\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^m x(t)}{dt^m}$$

$$A = [a_K, a_{K-1}, \dots a_0]$$
$$B = [b_M, b_{M-1}, \dots b_0]$$

$$\sum_{k=0}^{K} a_k s^k Y(s) = \sum_{m=0}^{M} b_m s^m X(s)$$

Y(s)和X（s)跟k没关系

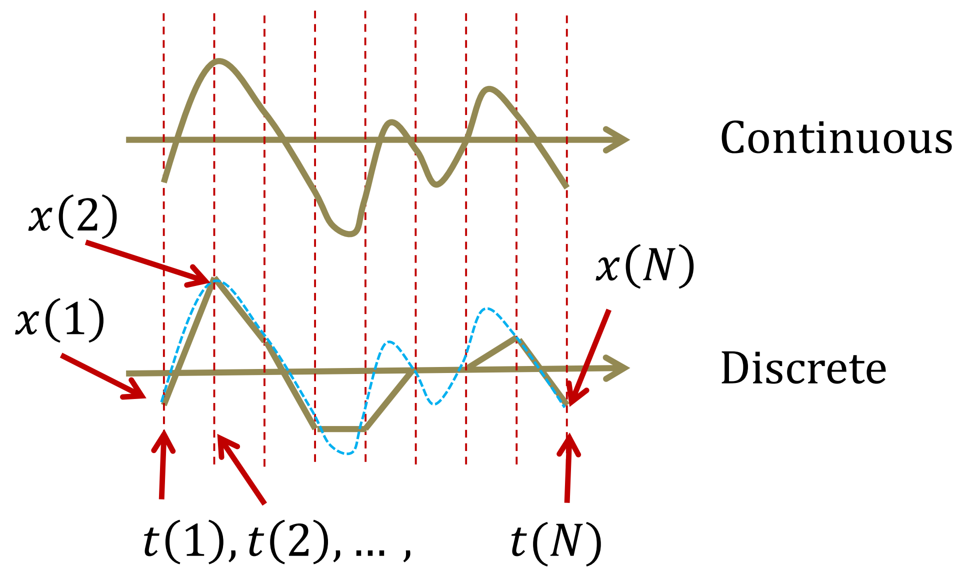
$$\boxed{s = j\omega}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{m=0}^{M} b_m s^m}{\sum_{k=0}^{K} a_k s^k}$$

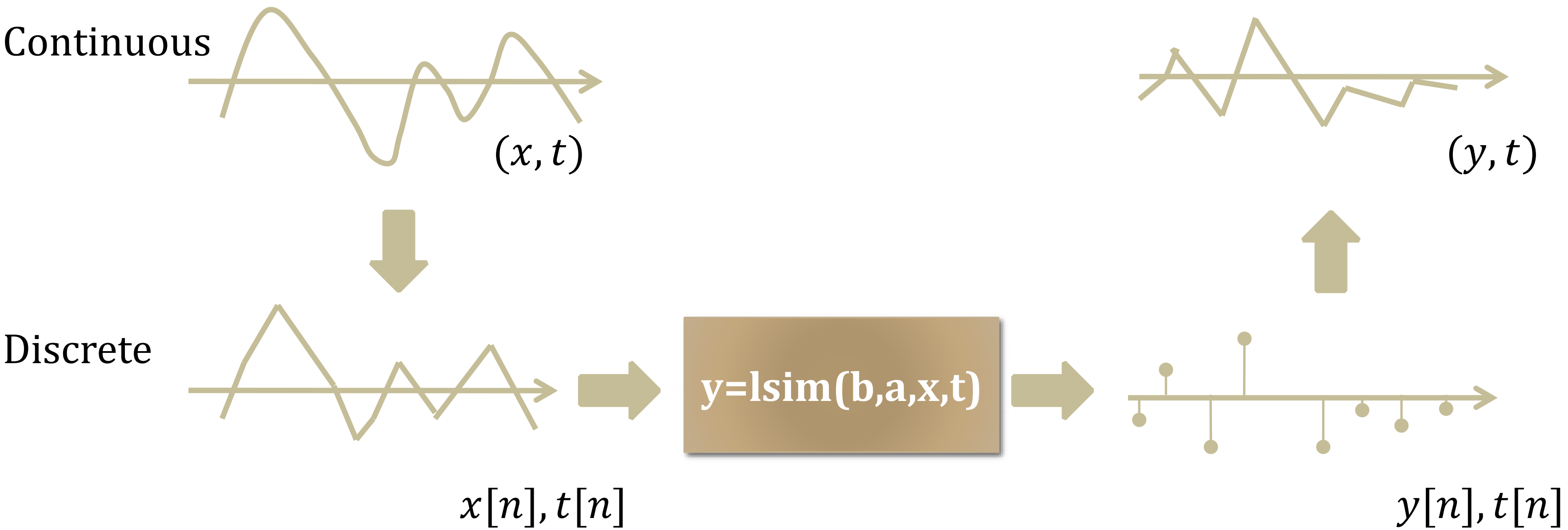
System function
or Transfer function

$$H(s) = \frac{B(s)}{A(s)} = \frac{b_M s^M + b_{M-1} s^{M-1} + \cdots + b_1 s^1 + b_0}{a_K s^K + a_{K-1} s^{K-1} + \cdots + a_1 s^1 + a_0}$$

- How to 模仿 simulate CT systems via Matlab?
  - lsim(): 生成 generate sampled output according to sampled input signal and CT system function:
    - Syntax: lsim(B, A, x, t) t表示x中每个点的采样时间
  - Coefficient vector:
    - $A = [a_K, a_{K-1}, \dots a_0]$
    - $B = [b_M, b_{M-1}, \dots b_0]$
  - Sampled input signal
    - Vector of sampling time: t
    - Vector of sampled value: x



- lsim(B,A,x,t)



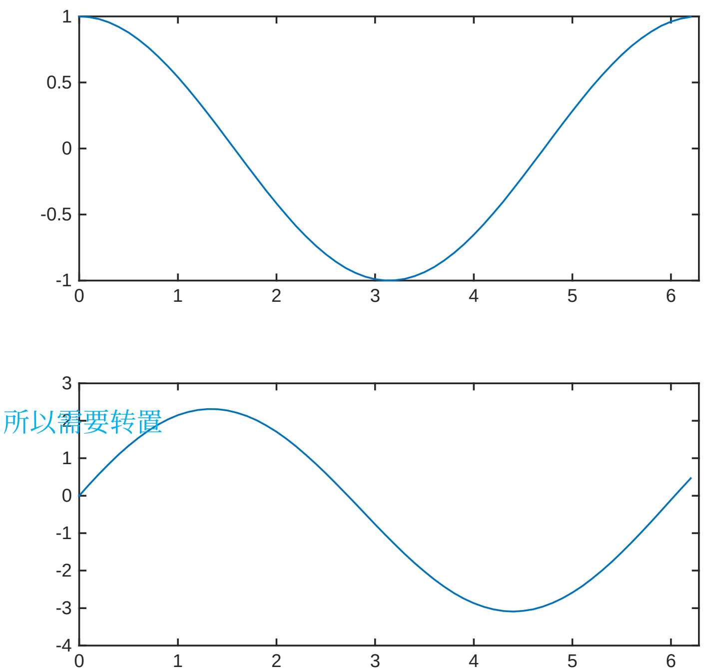
- Consider LTI System: 0.3y(t)+dy(t)/dt = 3x(t)

```
A=[1 0.3];
B=3;
```

- Sample the input signal x=cos(t):

```
t=0:0.1:2*pi;
x=cos(t);
y=lsim(B,A,x,t)';
subplot(2,1,1), plot(t,x);
xlim([0 2*pi]);
subplot(2,1,2), plot(t,y);
xlim([0 2*pi]);
```

lsim输出的是列向量，t和x是行向量，所以需要转置



Lab Assignment 3 – part (a)

- Read tutorial 3.1, 3.2, 3.3 (and 2.3) by yourself
- 3.8 part (a)(b) & 3.9
- Submission: TBA in next week

Several parts of this exercise require you to generate vectors which should be purely real, but have very small imaginary parts due to roundoff errors. Use *real* to remove these residual imaginary parts from these vectors.

Hints – A correction

- 3.9(c)

Advanced Problems

(c). Analytically calculate the CTFS for the square wave **x2**. You may find it helpful to first find a relationship between the signal $x_2(t)$ and the signal $s(t)$ defined in Eq. (3.9). Use the ten lowest frequency nonzero CTFS coefficients of **x2** to create the first 5 harmonic components individually. For example if you have the positive and

$$s(t) = \begin{cases} 1, & |t| < T/4, \\ 0, & T/4 \le |t| \le T/2 \end{cases} \tag{3.9}$$

Eq. (3.9) is provided in section 3.7

CTFS coefficients a_k given by

$$a_k = \frac{\sin(\pi k/2)}{\pi k}.$$

Example 3.5 in your textbook by Oppenheim – 2nd Edition

Hint: Properties of CTFS – Linearity, Time Shifting, ……

Poll

1 answer(s) at most

设置

A correction and a hint for assignment 3.9 were just mentioned, did you hear? Need a repeat?

- A Got it. No need 听到了，不用再讲
- B What? What?? 啊？什么??
- C Heard it, but need a repeat
  听到了，但是需要再听一遍才懂

提交