南方科技大学
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Tutorial Questions （Week 3）

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Outline

- Basic knowledge review
- Basic Problems with Answers 1.15, 1.18
- Basic Problems 1.29, 1.31
- Advanced Problems 1.33, 1.42
- Q&A

Basic knowledge review

- Basic knowledge on signal computation
- Exponential signals: Euler's relation, periodic, integral
- CT/DT unit impulse/step function
- System Properties
  1. Memoryless or with memory
  2. Causality
  3. Invertibility
  4. Stability
  5. Time-invariance
  6. Linearity

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1.15. Consider a system S with input $x[n]$ and output $y[n]$. This system is obtained through a series interconnection of a system S_1 followed by a system S_2. The input-output relationships for S_1 and S_2 are

$$S_1: \qquad y_1[n] = 2x_1[n] + 4x_1[n-1],$$

$$S_2: \qquad y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3],$$

where $x_1[n]$ and $x_2[n]$ denote input signals.

(a) Determine the input-output relationship for system S.

(b) Does the input-output relationship of system S change if the order in which S_1 and S_2 are connected in series is reversed (i.e., if S_2 follows S_1)?

1.15. (a) The signal $x_2[n]$, which is the input to S_2, is the same as $y_1[n]$. Therefore,

$$\begin{aligned} y_2[n] &= x_2[n-2] + \frac{1}{2}x_2[n-3] \\ &= y_1[n-2] + \frac{1}{2}y_1[n-3] \\ &= 2x_1[n-2] + 4x_1[n-3] + \frac{1}{2}(2x_1[n-3] + 4x_1[n-4]) \\ &= 2x_1[n-2] + 5x_1[n-3] + 2x_1[n-4] \end{aligned}$$

The input-output relationship for S is

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

(b) The input-output relationship does not change if the order in which S_1 and S_2 are connected in series is reversed. We can easily prove this by assuming that S_1 follows S_2. In this case, the signal $x_1[n]$, which is the input to S_1, is the same as $y_2[n]$. Therefore,

$$\begin{aligned} y_1[n] &= 2x_1[n] + 4x_1[n-1] \\ &= 2y_2[n] + 4y_2[n-1] \\ &= 2(x_2[n-2] + \frac{1}{2}x_2[n-3]) + 4(x_2[n-3] + \frac{1}{2}x_2[n-4]) \\ &= 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4] \end{aligned}$$

The input-output relationship for S is once again

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

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1.18. Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k],$$

where n_0 is a finite positive integer.

(a) Is this system linear?

(a) Is this system time-invariant?

(c) If $x[n]$ is known to be bounded by a finite integer B (i.e., $|x[n]| < B$ for all n), it can be shown that $y[n]$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B and n_0.

1.29. **(a)** Show that the discrete-time system whose input $x[n]$ and output $y[n]$ are related by $y[n] = \Re e\{x[n]\}$ is additive. Does this system remain additive if its input-output relationship is changed to $y[n] = \Re e\{e^{j\pi n/4}x[n]\}$? (Do not assume that $x[n]$ is real in this problem.)

(b) In the text, we discussed the fact that the property of linearity for a system is equivalent to the system possessing both the additivity property and homogeneity property. Determine whether each of the systems defined below is additive and/or homogeneous. Justify your answers by providing a proof for each property if it holds or a counterexample if it does not.

(i) $y(t) = \frac{1}{x(t)}\left[\frac{dx(t)}{dt}\right]^2$ (ii) $y[n] = \begin{cases} \frac{x[n]x[n-2]}{x[n-1]}, & x[n-1] \neq 0 \\ 0, & x[n-1] = 0 \end{cases}$

1.29. (a) Consider two inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \mathcal{R}e\{x_1[n]\} \quad \text{and} \quad x_2[n] \xrightarrow{S} y_2[n] = \mathcal{R}e\{x_2[n]\}.$$

Now consider a third input $x_3[n] = x_1[n] + x_2[n]$. The corresponding system output will be

$$\begin{aligned} y_3[n] &= \mathcal{R}e\{x_3[n]\} \\ &= \mathcal{R}e\{x_1[n] + x_2[n]\} \\ &= \mathcal{R}e\{x_1[n]\} + \mathcal{R}e\{x_2[n] \\ &= y_1[n] + y_2[n] \end{aligned}$$

Therefore, we may conclude that the system is additive.

Let us now assume that the input-output relationship is changed to $y[n] = \mathcal{R}e\{e^{j\pi/4}x[n]\}$. Also, consider two inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \mathcal{R}e\{e^{j\pi/4}x_1[n]\}$$

and

$$x_2[n] \xrightarrow{S} y_2[n] = \mathcal{R}e\{e^{j\pi/4}x_2[n]\}.$$

Now consider a third input $x_3[n] = x_1[n] + x_2[n]$. The corresponding system output will be

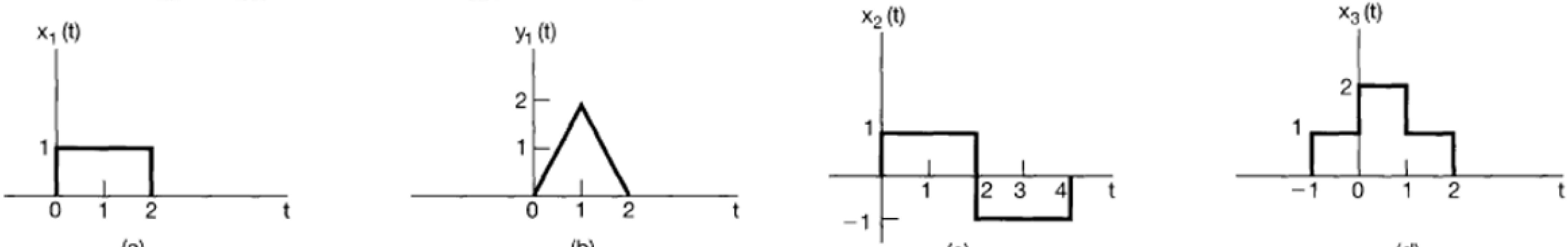
$$\begin{aligned} y_3[n] &= \mathcal{R}e\{e^{j\pi/4}x_3[n]\} \\ &= \cos(\pi n/4)\mathcal{R}e\{x_3[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_3[n]\} \\ &\quad + \cos(\pi n/4)\mathcal{R}e\{x_1[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_1[n]\} \\ &+ \cos(\pi n/4)\mathcal{R}e\{x_2[n]\} - \sin(\pi n/4)\mathcal{I}m\{x_2[n]\} \\ &= \mathcal{R}e\{e^{j\pi/4}x_1[n]\} + \mathcal{R}e\{e^{j\pi/4}x_2[n]\} \\ &= y_1[n] + y_2[n] \end{aligned}$$

Therefore, we may conclude that the system is additive.

1.31. In this problem, we illustrate one of the most important consequences of the properties of linearity and time invariance. Specifically, once we know the response of a linear system or a linear time-invariant (LTI) system to a single input or the responses to several inputs, we can directly compute the responses to many other input signals. Much of the remainder of this book deals with a thorough exploitation of this fact in order to develop results and techniques for analyzing and synthesizing LTI systems.

(a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure P1.31(a) is the signal $y_1(t)$ illustrated in Figure P1.31(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure P1.31(c).

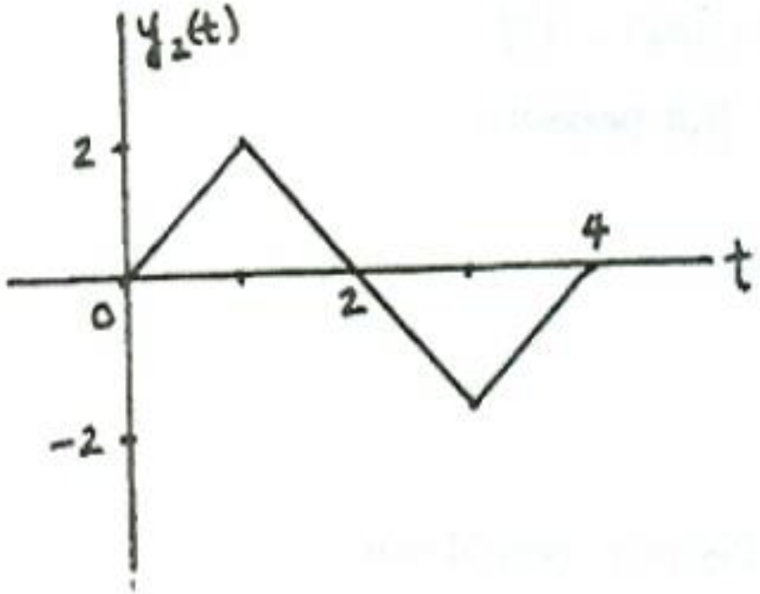
(b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure P1.31(d).

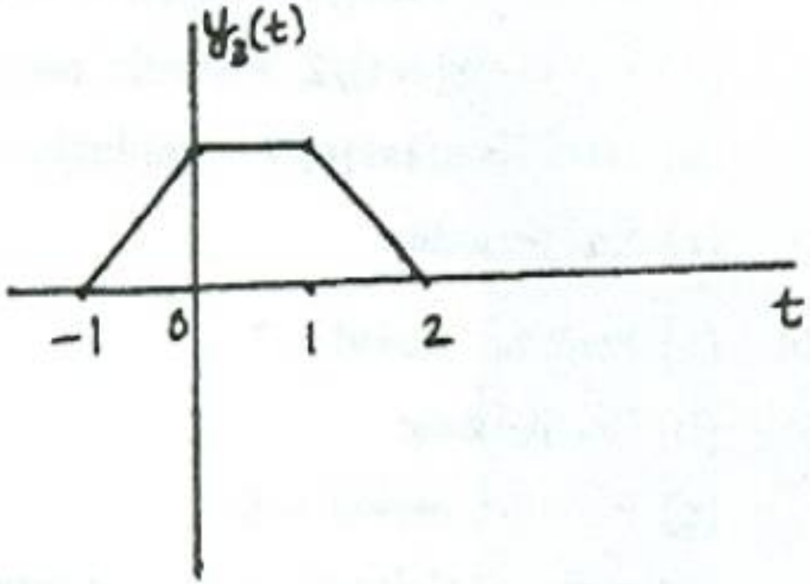


(a) (b) (c) (d)

1.31. (a) Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$. This is as shown in Figure S1.31.

(b) Note that $x_3(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity we get $y_3(t) = y_1(t) + y_1(t+1)$. This is as shown in Figure S1.31.





How about x(t)=u(t)-u(t-1)?

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1.33. Let $x[n]$ be a discrete-time signal, and let

$$y_1[n] = x[2n] \text{ and } y_2[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}.$$

The signals $y_1[n]$ and $y_2[n]$ respectively represent in some sense the speeded up and slowed down versions of $x[n]$. However, it should be noted that the discrete-time notions of speeded up and slowed down have subtle differences with respect to their continuous-time counterparts. Consider the following statements:

(1) If $x[n]$ is periodic, then $y_1[n]$ is periodic.
(2) If $y_1[n]$ is periodic, then $x[n]$ is periodic.
(3) If $x[n]$ is periodic, then $y_2[n]$ is periodic.
(4) If $y_2[n]$ is periodic, then $x[n]$ is periodic.

For each of these statements, determine whether it is true, and if so, determine the relationship between the fundamental periods of the two signals considered in the statement. If the statement is not true, produce a counterexample to it.

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1.42. **(a)** Is the following statement true or false?

The series interconnection of two linear time-invariant systems is itself a linear, time-invariant system.

Justify your answer.

(b) Is the following statement true or false?

The series interconnection of two nonlinear systems is itself nonlinear.

Justify your answer.

(c) Consider three systems with the following input-output relationships:

System 1: $$y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases},$$

System 2: $$y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2],$$

System 3: $$y[n] = x[2n].$$

Suppose that these systems are connected in series as depicted in Figure P1.42. Find the input-output relationship for the overall interconnected system. Is this system linear? Is it time invariant?

1.42. (a) Consider two systems S_1 and S_2 connected in series. Assume that if $x_1(t)$ and $x_2(t)$ are the inputs to S_1, then $y_1(t)$ and $y_2(t)$ are the outputs, respectively. Also, assume that if $y_1(t)$ and $y_2(t)$ are the inputs to S_2, then $z_1(t)$ and $z_2(t)$ are the outputs, respectively. Since S_1 is linear, we may write

$$ax_1(t) + bx_2(t) \xrightarrow{S_1} ay_1(t) + by_2(t),$$

where a and b are constants. Since S_2 is also linear, we may write

$$ay_1(t) + by_2(t) \xrightarrow{S_2} az_1(t) + bz_2(t),$$

We may therefore conclude that

$$ax_1(t) + bx_2(t) \xrightarrow{S_1,S_2} az_1(t) + bz_2(t).$$

Therefore, the series combination of S_1 and S_2 is linear.

Since S_1 is time invariant, we may write

$$x_1(t-T_0) \xrightarrow{S_1} y_1(t-T_0)$$

and

$$y_1(t-T_0) \xrightarrow{S_2} z_1(t-T_0).$$

Therefore,

$$x_1(t-T_0) \xrightarrow{S_1,S_2} z_1(t-T_0).$$

Therefore, the series combination of S_1 and S_2 is time invariant.

(b) False. Let $y(t) = x(t) + 1$ and $z(t) = y(t) - 1$. These correspond to two nonlinear systems. If these systems are connected in series, then $z(t) = x(t)$ which is a linear system.

(c) Let us name the output of system 1 as $w[n]$ and the output of system 2 as $z[n]$. Then,

$$\begin{aligned} y[n] &= z[2n] = w[2n] + \frac{1}{2}w[2n-1] + \frac{1}{4}w[2n-2] \\ &= x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2] \end{aligned}$$

The overall system is linear and time-invariant.

- **1教111，周一至周四**
- **21:00-22:00, in 27/28/29/30 Sep**
- Review
- Basic Problems with Answers 2.3, 2.7, 2.13
- Basic Problems 2.24, 2.26

Thanks for Your Attendance

Q&A

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