Lab 2. Linear Time-Invariant Systems

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Fall 2021

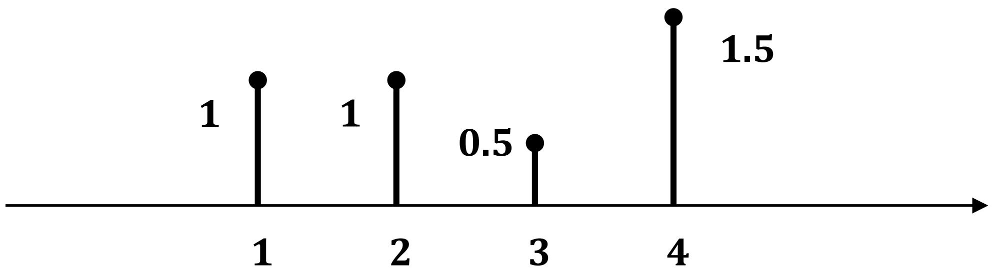
Overview

- In Lab 2, you will
  1. Verify the property of convolution
  2. Verify the property of LTI systems
  3. Design a LTI system for echo cancellation

- In this tutorial, you will learn
  1. How to calculate convolution via Matlab
  2. How to generate output of DT LTI systems
  3. How to design a wireless signal detector

How to Represent a DT Signal?

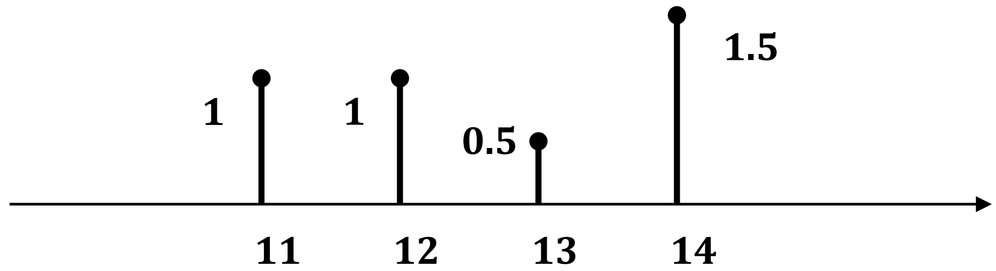
- DT signal:



- Matlab code:

```
x=[1 1 0.5 1.5]
stem(x)
```

- Another DT signal:



- Matlab code:

```
• x=[1 1 0.5 1.5]
• stem([11 12 13 14], x)
```

```
• x=[1 1 0.5 1.5]
• nx=[11 12 13 14]
• stem(nx, x)
```

Signal value

Non-zero region of time index

Calculating convolution via Matlab

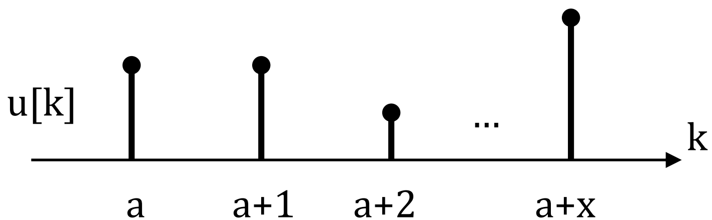
DT Signal Convolution

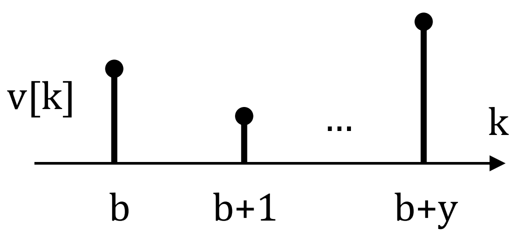
- $w[n] = u[n] * v[n] = \sum_{k=-\infty}^{+\infty} u[k]v[n-k]$
  - Non-zero interval of DT signal u: $nu = a{:}(a+x)$
  - Non-zero interval of DT signal v: $nv = b{:}(b+y)$
  - Non-zero interval of $w[n]$: $nw = (?){:}(??)$

- n-b=a ⇒ n=a+b
- n-b-y=a+x ⇒ n=a+b+y+x

Hence,

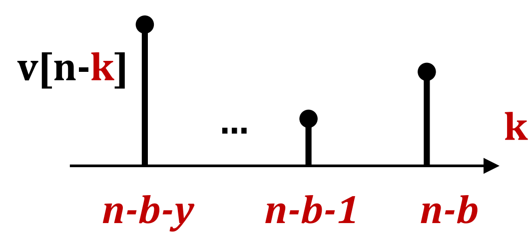
- nw=[a+b:a+b+x+y]
- ? = a+b = nu(1) + nv(1)
- ?? = a+b+x+y = nu(end) + nv(end)

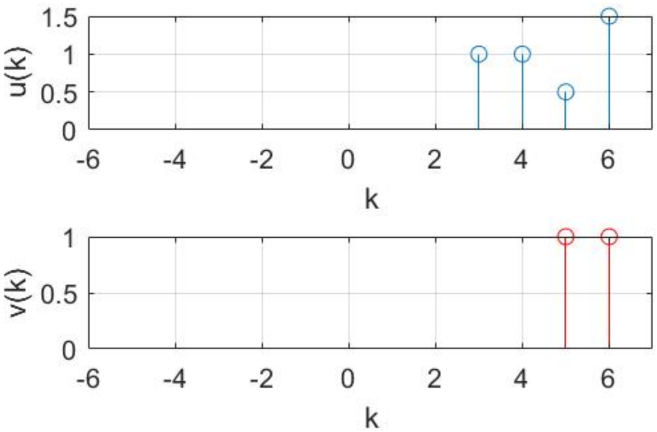




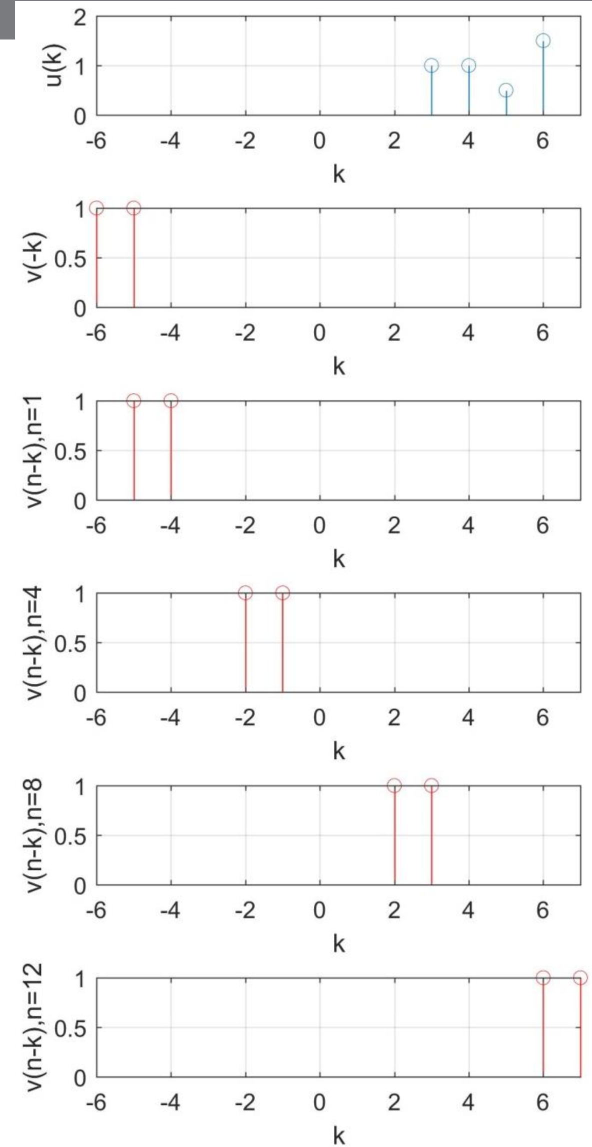
Flip w.r.t the origin

Shift to the right by *n*





$$w[n] = u[n] * v[n] = ?$$

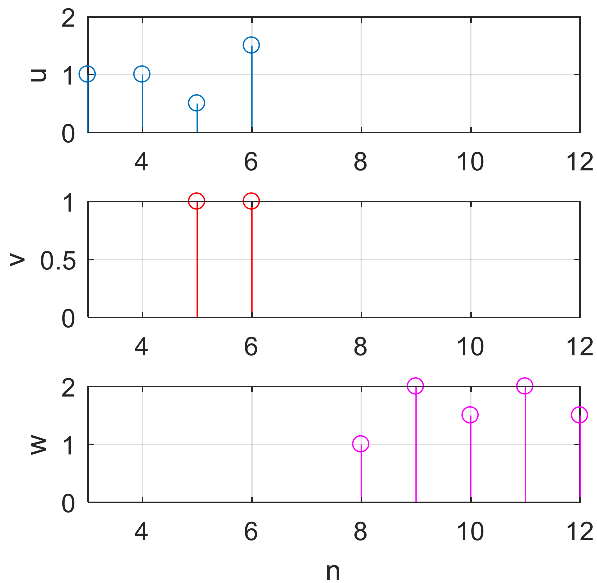


Calculating Convolution by conv

- Define $u, nu\ and\ v, nv$
- Calculate the signal values of $w[n]$ by w=conv(u,v)
- Calculate the non-zero interval of $w[n]$ by

$$nw = nu(1) + nv(1): nu(end) + nv(end)$$

```
• u=[1 1 0.5 1.5]
• nu=3:6
• v=[1 1]
• nv=5:6
• w=conv(u,v)
• nw=nu(1)+nv(1):nu(end)+nv(end)
• subplot(3,1,1), stem(nu,u), axis([3,12,ylim])
• subplot(3,1,2), stem(nv,v) , axis([3,12,ylim])
• subplot(3,1,3), stem(nw,w) , axis([3,12,ylim])
```



Generating output of DT LTI systems via Matlab

LTI System by Difference Equation

- Reading assignment: ***textbook*** 2.4 Causal LTI systems described by differential and difference equations.(2.4.2)

- A Causal DT LTI system specified by a linear constant-coefficient difference equation

$$\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

- For example:
  - y[n]=0.5x[n]+x[n-1]+2x[n-2]; h[n]=? Finite Impulse Response (FIR) 有限冲击响应
  - y[n]-0.8y[n-1]=2x[n]; h[n]=? Infinite Impulse Response (IIR) 无限冲激响应

- A Causal DT LTI system is uniquely specified by two vectors A=[a_0 a_1 a_2 … a_K] and B=[b_0 b_1 … b_M]
  - A=[1], B=[0.5, 1, 2]

    for $y[n] = 0.5x[n] + x[n-1] + 2x[n-2]$

  - A=[1, −0.8], B=[2]

    for $y[n] - 0.8y[n-1] = 2x[n]$

$$\boxed{\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]}$$

Open Question

Points: 10

设置

What's the coefficient vector A and B for the systems:

1. y[n] = 0.8x[n-1] + 0.6x[n-2] + 0.4x[n-3]
2. y[n] - 0.8y[n-2] = 0.6x[n] + 0.4x[n-3]

$$\boxed{\sum_{k=0}^{K} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]}$$

正常使用主观题需2.0以上版本雨课堂

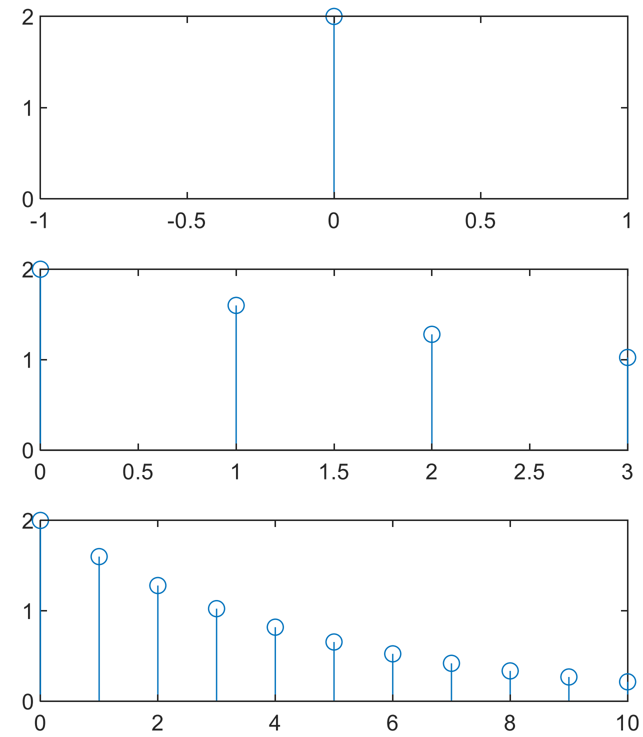
作答

Calculating Output Signal by filter 滤波器

- Syntax: y=filter(B, A, x)
- System: specify by the coefficient vector A and B
- x and y share the same range of time indices 两者时间范围相同
  - Output signal y may be truncated 若卷积得到的输出比输入要长，长的部分会被切掉

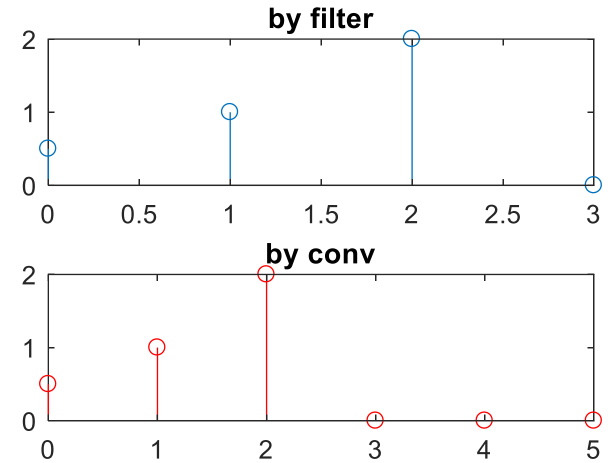
- $y[n] - 0.8y[n-1] = 2x[n]$

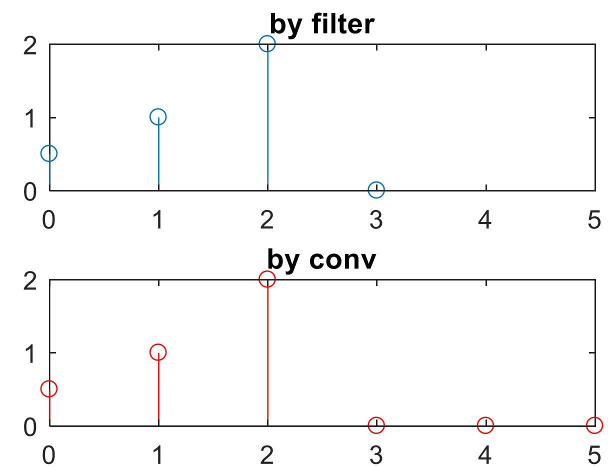
```
• A = [1 -0.8];
• B = 2;
• X1 = [1];  % Input signal is δ[n]
• Y1 = filter(B, A, X1); Y1n = 0;
• X2 = [1 0 0 0];
• Y2 = filter(B, A, X2); Y2n = 0:3;
• X3 = [1 0 0 0 0 0 0 0 0 0 0];
• Y3 = filter(B, A, X3); Y3n = 0:10;
• subplot(3,1,1), stem(Y1n,Y1)
• subplot(3,1,2), stem(Y2n,Y2)
• subplot(3,1,3), stem(Y3n,Y3)
```



- $y[n] = 0.5x[n] + x[n-1] + 2x[n-2]$

- A = [1];
- B = [0.5,1,2]; % h = B, hn = 0:2
- X = [1 0 0 0]; % Xn = 0:3
- Y1 = filter(B, A, X); % Y1n =Xn = 0:3
- Y1n = 0:3;
- Y2 = conv(X, B); % Y2n = (0+0): (2+3)=0:5 有点疑问
- Y2n = 0:5
- subplot(3,1,1), stem(Y1n, Y1)
- subplot(3,1,2), stem(Y2n, Y2, 'r')





Summary

	conv	filter
Scenario 场景	• Finite signal • Finite impulse response	• Finite signal • Infinite/finite impulse response
Remarks	• nw=nu(1)+nv(1): nu(end)+nv(end)	• x and y share the same range of time indices • Truncated output

Lab Assignment 2 – part (a)

- Read tutorial 2.1, 2.2 by yourself
- 2.4, 2.5
- Submission: TBD