4.50

Consider interpolating a signal $x[n]$ by repeating each value q times, as depicted in Fig.P4.50. That is, we define $x_0 = x\left[floor\left(\frac{n}{q}\right)\right]$, where $floor(z)$ is the greatest integer less than or equal to z. Let $x_z[n]$ be derived from $x[n]$ by inserting $q-1$ zeros between each value of $x[n]$; that is,

$$x_z[n] = \begin{cases} x\left[\frac{n}{q}\right], \frac{n}{q} \text{ interger} \\ 0, \quad \text{otherwise} \end{cases}.$$

We may now write $x_0[n] = x_z[n] * b_0[n]$, where

$$b_0[n] = \begin{cases} 1, 0 \le n \le q-1 \\ 0, \quad \text{otherwise} \end{cases}.$$

Note that this is the discrete-time analog of the zero-order hold. The interpolation process is completed by passing $x_0[n]$ through a filter with frequency response $H(e^{j\Omega})$.

a) Express $X_0(e^{j\Omega})$ in terms of $X(e^{j\Omega})$ and $H_0(e^{j\Omega})$. Sketch

$$\left|X_0(e^{j\Omega})\right| \text{ if } x[n] = \frac{\sin\left(\frac{3\pi}{4}n\right)}{\pi n}.$$

b) Assume that $X(e^{j\Omega})$ is as shown in Fig.P4.49. Specify the constrains on $H(e^{j\Omega})$ so that ideal interpolation is obtained for the following cases:

i. $q = 2, W = \frac{3\pi}{4}$

ii. $q = 4, W = \frac{3\pi}{4}$

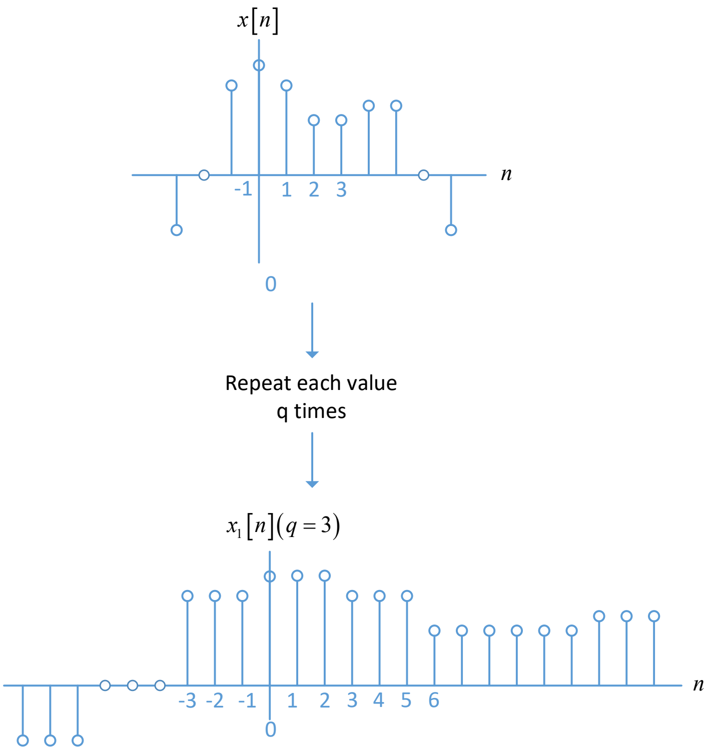

FIGURE P4.50

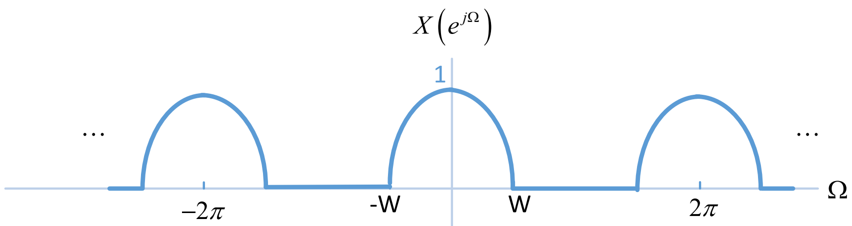

FIGURE P4.49

4.51

The system shown in Fig.P4.51 is used to implement a band-pass filter. The frequency response of discrete-time filter is

$$H\left(e^{j\Omega}\right)=\begin{cases}1, & \Omega_a \le |\Omega| \le \Omega_b \\ 0, & \text{otherwise}\end{cases}$$

on $-\pi<\Omega<\pi$. Find the sampling interval $T_s, \Omega_a, \Omega_b, W_1, W_2, W_3,$ and W_4 so that the equivalent continuous-time frequency response $G(j\omega)$ satisfies

$0.9<|G(j\omega)|<1.1$, for $100\pi<\omega<200\pi$

$G(j\omega)=0$ elsewhere

In solving this problem, choose W_1 and W_3 as small as possible, and choose $T_s, W_2,$ and W_4 as large as possible.

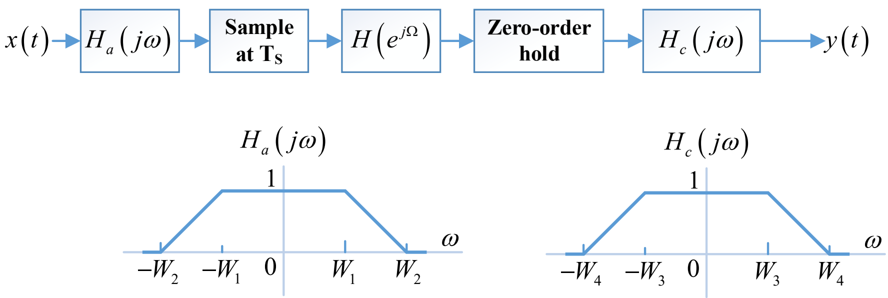

FIGURE P4.51