Tutorial Problems (Week 15)

Advanced Problems 8.44 8.46

8.44. In this problem, we explore an *equalization* method used to avoid intersymbol interference caused in PAM systems by the channel having nonlinear phase over its bandwidth.

When a PAM pulse with zero-crossings at integer multiples of the symbol spacing T_1 is passed through a channel with nonlinear phase, the received pulse may no longer have zero-crossings at times that are integer multiples of T_1. Therefore, in order to avoid intersymbol interference, the received pulse is passed through a *zero-forcing equalizer*, which forces the pulse to have zero-crossings at integer multiples of T_1. This equalizer generates a new pulse $y(t)$ by summing up weighted and shifted versions of the received pulse $x(t)$. The pulse $y(t)$ is given by

$$y(t) = \sum_{l=-N}^{N} a_l x(t - lT_1), \tag{P8.44–1}$$

where the a_l are all real and are chosen such that

$$y(kT_1) = \begin{cases} 1, & k = 0 \\ 0, & k = \pm 1, \pm 2, \pm 3, \ldots, \pm N \end{cases}.$$

(a) Show that the equalizer is a filter and determine its impulse response.

(b) To illustrate the selection of the weights a_l, let us consider an example. If $x(0T_1) = 0.0$, $x(-T_1) = 0.2$, $x(T_1) = -0.2$, and $x(kT_1) = 0$ for $|k| > 1$, determine the values of a_0, a_1, and a_{-1} such that $y(\pm T_1) = 0$.

PAM: *pulse-amplitude modulation* （脉冲幅度调制）

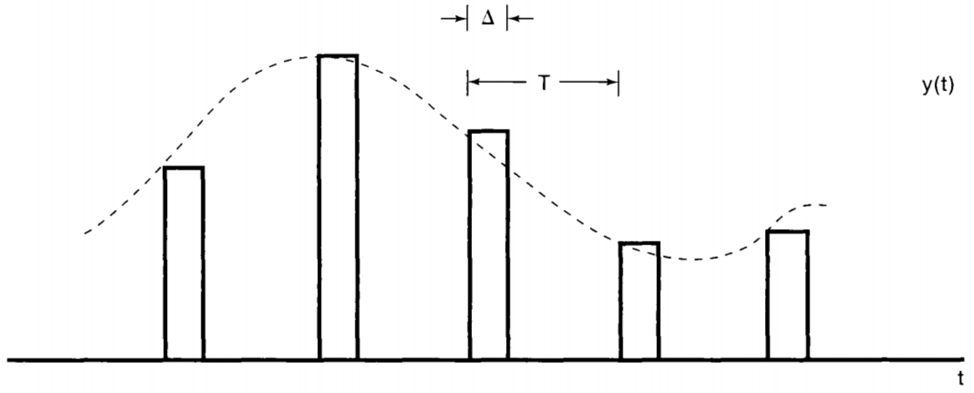


Figure 8.26 Transmitted waveform for a single PAM channel. The dotted curve represents the signal $x(t)$.

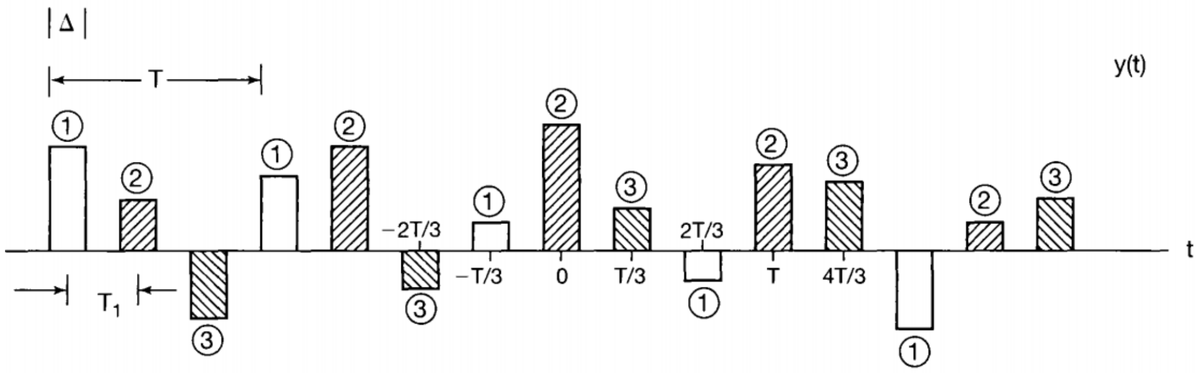
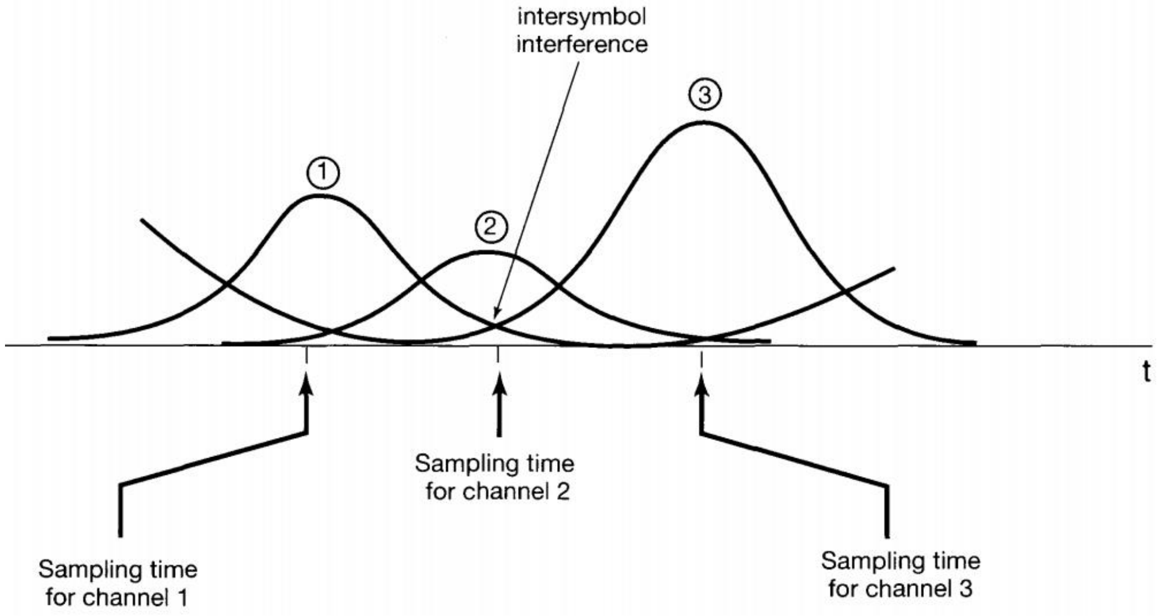


Figure 8.27 Transmitted waveform with three time-multiplexed PAM channels. The pulses associated with each channel are distinguished by shading, as well as by the channel number above each pulse. Here, the intersymbol spacing is $T_1 = T/3$.

Intersymbol interference (码间干扰)



8.44. (a) We may write $y(t)$ as

$$y(t) = x(t) * \sum_{l=-N}^{N} a_l \delta(t - lT_1).$$

Therefore, $y(t)$ is obtained by passing $x(t)$ through a filter with impulse response $h(t) = \sum_{l=-N}^{N} a_l \delta(t - lT_1)$.

(b) Using eq. (P8.44-1), we obtain the following three simultaneous equations

$$y(0) = a_{-1}x(T_1) + a_0 x(0) + a_1 x(-T_1),$$

$$y(T_1) = a_{-1}x(2T_1) + a_0 x(T_1) + a_1 x(0),$$

and

$$y(-T_1) = a_{-1}x(0) + a_0 x(-T_1) + a_1 x(-2T_1).$$

Substituting the given values for $x(t)$ and $y(t)$ and solving, we obtain

$$a_0 = 0, \qquad a_1 - a_{-1} = 5$$

8.46. Consider the complex exponential function of time,

$$s(t) = e^{j\theta(t)}, \tag{P8.46–1}$$

where $\theta(t) = \omega_0 t^2/2$.

Since the instantaneous frequency $\omega_i = d\theta/dt$ is also a function of time, the signal $s(t)$ may be regarded as an FM signal. In particular, since the signal sweeps linearly through the frequency spectrum with time, it is often called a frequency "chirp" or "chirp signal."

(a) Determine the instantaneous frequency.

(b) Determine and sketch the magnitude and phase of the Fourier transform of the "chirp signal." To evaluate the Fourier transform integral, you may find it helpful to complete the square in the exponent in the integrand and to use the relation

$$\int_{-\infty}^{+\infty} e^{jz^2} dz = \sqrt{\frac{\pi}{2}}(1 + j).$$

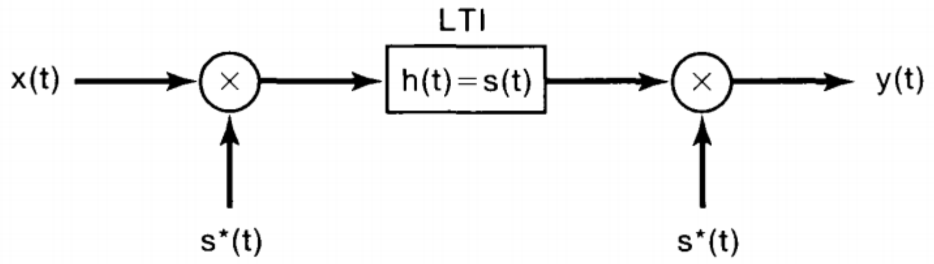


Figure P8.46

(c) Consider the system in Figure P8.46, in which $s(t)$ is the "chirp signal" in eq. (P8.46–1). Show that $y(t) = X(j\omega_0 t)$, where $X(j\omega)$ is the Fourier transform of $x(t)$.

(*Note:* The system in Figure P8.46 is referred to as the *"chirp" transform algorithm* and is often used in practice to obtain the Fourier transform of a signal.)

8.46. (a) The instantaneous frequency is

$$\boxed{\omega_i(t) = \frac{d\theta(t)}{dt}} = \frac{d}{dt}\left[\frac{\omega_0 t^2}{2}\right] = \omega_0 t.$$

(b) We have

magnitude-phase representation

$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}, \quad \sphericalangle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

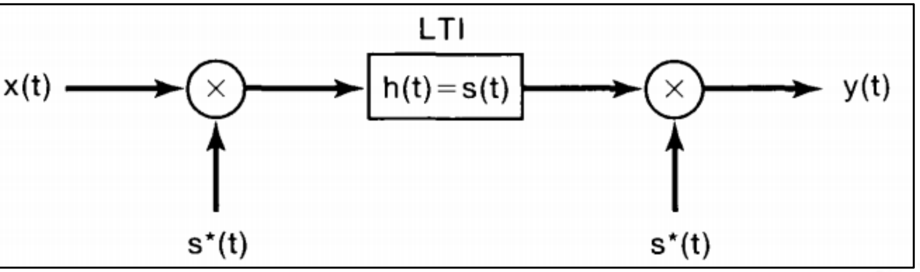
$$\begin{aligned} S(j\omega) &= \int_{-\infty}^{\infty} e^{j\omega_0 t^2/2} e^{-j\omega t} dt \\ &= e^{j\omega^2/(2\omega_0)} \int_{-\infty}^{\infty} e^{j[\sqrt{\omega_0/2}(t-\omega/\omega_0)]^2} dt \\ &= \sqrt{\frac{\pi}{\omega_0}}(1+j)e^{j\omega^2/(2\omega_0)} \end{aligned}$$

$$\int_{-\infty}^{+\infty} e^{jz^2} dz = \sqrt{\frac{\pi}{2}}(1+j).$$

(c) We have

$$X(j\omega_0 t) = \int_{-\infty}^{\infty} x(\tau) e^{-j\omega_0 t\tau} d\tau.$$

But $t\tau = \frac{1}{2}[t^2+\tau^2-(t-\tau)^2]$. Therefore,



$$\begin{aligned} X(j\omega_0 t) &= \int_{-\infty}^{\infty} x(\tau) e^{-j\omega_0 \tau^2/2} e^{-j\omega_0 t^2/2} e^{j\omega_0 (t-\tau)^2/2} d\tau \\ &= e^{-j\omega_0 t^2/2} \int_{-\infty}^{\infty} x(\tau) e^{-j\omega_0 \tau^2/2} e^{j\omega_0 (t-\tau)^2/2} d\tau \end{aligned}$$

Let $g(\tau) = x(\tau)e^{-j\omega_0\tau^2/2}$. Then

$$X(j\omega_0 t) = e^{-j\omega_0 t^2/2}[g(t) * e^{j\omega_0 t^2/2}].$$

This is exactly what Figure P8.46 implements.