Tutorial Questions (Week 2)

习题课内容

Basic Problems with Answers 1.15, 1.18

Basic Problems 1.29, 1.31

Advanced Problems 1.33, 1.42

Assignments for Chapter 1

- 1.20
- 1.21 (c) (f)
- 1.24 (a)
- 1.26
- 1.27 (a) (f)
- 1.41

1.15. Consider a system S with input $x[n]$ and output $y[n]$. This system is obtained through a series interconnection of a system S_1 followed by a system S_2. The input-output relationships for S_1 and S_2 are

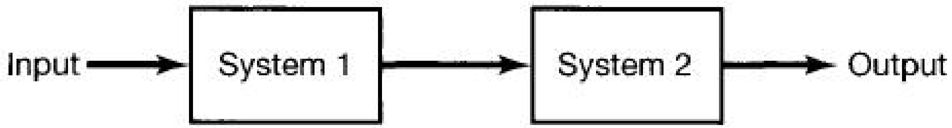
$$S_1: \qquad y_1[n] = 2x_1[n] + 4x_1[n-1],$$

$$S_2: \qquad y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3],$$

where $x_1[n]$ and $x_2[n]$ denote input signals.

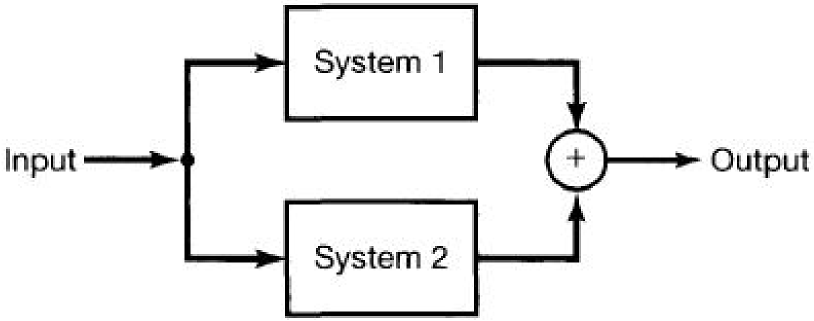
(a) Determine the input-output relationship for system S.

(b) Does the input-output relationship of system S change if the order in which S_1 and S_2 are connected in series is reversed (i.e., if S_2 follows S_1)?



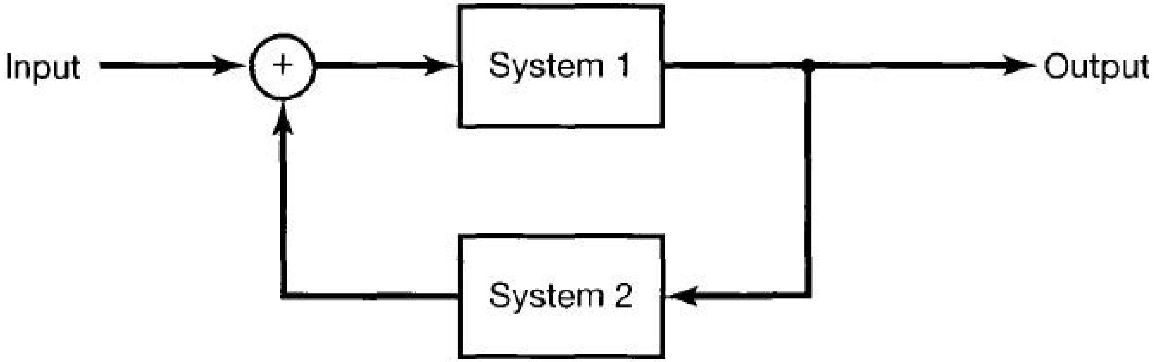
(a)

Series interconnection 级联



(b)

Parallel interconnection 并联



Feedback interconnection 反馈

1.15 (a) The signal $x_2[n]$, which is the input to S_2, is the same as $y_1[n]$. Therefore ,

$$y_2[n]= x_2[n-2]+\frac{1}{2}\ x_2[n-3]$$

$$= y_1[n-2]+\ \frac{1}{2}\ y_1[n-3]$$

$$=2x_1\ [n-2] +4x_1\ [n-3] +\frac{1}{2}(\ 2x_1[n-3]+ 4x_1[n-4])$$

$$=2x_1[n-2]+ 5x_1[n-3] + 2x_1[n-4]$$

The input-output relationship for S is

$$y[n]=2x[n-2]+ 5x\ [n-3] + 2x\ [n-4]$$

(b) The input-output relationship does not change if the order in which S_1 and S_2 are connected series reversed. . We can easily prove this assuming that S_1 follows S_2. In this case , the signal $x_1[n]$, which is the input to S_1 is the same as $y_2[n]$.

Therefore

$$
\begin{aligned}
y_1[n] &= 2x_1[n] + 4x_1[n-1] \\
&= 2y_2[n] + 4y_2[n-1] \\
&= 2\left(x_2[n-2] + \frac{1}{2}x_2[n-3]\right) + 4\left(x_2[n-3] + \frac{1}{2}x_2[n-4]\right) \\
&= 2x_2[n-2] + 5x_2[n-3] + 2x_2[n-4]
\end{aligned}
$$

The input-output relationship for S is once again

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

1.18. Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k],$$

where n_0 is a finite positive integer.

(a) Is this system linear?

(a) Is this system time-invariant?

(c) If $x[n]$ is known to be bounded by a finite integer B (i.e., $|x[n]| < B$ for all n), it can be shown that $y[n]$ is bounded by a finite number C. We conclude that the given system is stable. Express C in terms of B and n_0.

1.18.(a) Consider two arbitrary inputs $x_1[n]$and $x_2[n]$.

$$x_1[n] \rightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$x_2[n] \rightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Let $x_3[n]$ be a linear combination of $x_1[n]$ and $x_2[n]$. That is :

$$x_3[n]= ax_1[n]+b\, x_2[n]$$

where a and b are arbitrary scalars. If $x_3[n]$ is the input to the given system, then the corresponding output

$y_3[n]$ is $\qquad y_3[n]= \sum_{k=n-n_0}^{n+n_0} x_3[k]$

$$= \sum_{k=n-n_0}^{n+n_0} (ax_1[k]+bx_2[k]) = a \sum_{k=n-n_0}^{n+n_0} x_1[k] + b \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= ay_1[n]+b\, y_2[n]$$

Therefore the system is linear.

(b) Consider an arbitrary input $x_1[n]$.Let

$$y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

be the corresponding output .Consider a second input $x_2[n]$ obtained by shifting $x_1[n]$ in time:

$$x_2[n] = x_1[n-n_1]$$

The output corresponding to this input is

$$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{k=n-n_0}^{n+n_0} x_1[k-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$$

Also note that $$y_1[n-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k].$$

Therefore , $$y_2[n] = y_1[n-n_1]$$

This implies that the system is time-invariant.

(c) If $|x[n]| < B$, then $$y[n] \le \sum_{k=n-n_0}^{n+n_0} B = (2n_0+1)B.$$

Therefore C=(2n0+1)B

1.29. **(a)** Show that the discrete-time system whose input $x[n]$ and output $y[n]$ are related by $y[n] = \mathcal{R}e\{x[n]\}$ is additive. Does this system remain additive if its input-output relationship is changed to $y[n] = \mathcal{R}e\{e^{j\pi n/4}x[n]\}$? (Do not assume that $x[n]$ is real in this problem.)

(b) In the text, we discussed the fact that the property of linearity for a system is equivalent to the system possessing both the additivity property and homogeneity property. Determine whether each of the systems defined below is additive and/or homogeneous. Justify your answers by providing a proof for each property if it holds or a counterexample if it does not.

可加性
齐次性

(i) $y(t) = \frac{1}{x(t)}\left[\frac{dx(t)}{dt}\right]^2$ (ii) $y[n] = \begin{cases} \frac{x[n]x[n-2]}{x[n-1]}, & x[n-1] \neq 0 \\ 0, & x[n-1] = 0 \end{cases}$

1.29 (a) Consider two inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \Re_e\{x_1[n]\}. \text{ and } x_2[n] \xrightarrow{S} y_2[n] = \Re_e\{x_1[n]\}.$$

Now consider a third input x_3 [n]= x_2 [n]+ x_1 [n]. The corresponding system output

Will be

$$\begin{aligned} y_3[n] &= \Re_e\{x_3[n]\} \\ &= \Re_e\{x_1[n] + x_2[n]\} \\ &= \Re_e\{x_1[n]\} + \Re_e\{x_2[n]\} \\ &= y_1[n] + y_2[n] \end{aligned}$$

therefore, we may conclude that the system is additive

Let us now assume that inputs to the system such that

$$x_1[n] \xrightarrow{S} y_1[n] = \Re_e\{e^{j\pi/4} x_1[n]\}.$$

and

$$x_2[n] \xrightarrow{S} y_2[n] = \Re_e\{e^{j\pi/4} x_2[n]\}.$$

Now consider a third input x_3 [n]= x_2 [n]+ x_1 [n]. The corresponding system output

Will be

$$
\begin{aligned}
y_3[n] &= \Re_e\left\{e^{j\pi/4}x_3[n]\right\} \\
&= \cos(\pi n/4)\Re_e\left\{x_3[n]\right\} - \sin(\pi n/4)\mathrm{I}_m\left\{x_3[n]\right\} \\
&= \cos(\pi n/4)\Re_e\left\{x_1[n]\right\} - \sin(\pi n/4)\mathrm{I}_m\left\{x_1[n]\right\} \\
&+ \cos(\pi n/4)\Re_e\left\{x_2[n]\right\} - \sin(\pi n/4)\mathrm{I}_m\left\{x_2[n]\right\} \\
&= \Re_e\left\{e^{j\pi/4}x_1[n]\right\} + \Re_e\left\{e^{j\pi/4}x_2[n]\right\} \\
&= y_1[n] + y_2[n]
\end{aligned}
$$

therefore, we may conclude that the system is additive

(b) (i) Consider two inputs to the system such that

$$x_1(t) \xrightarrow{S} y_1(t) = \frac{1}{x_1(t)}\left[\frac{dx_1(t)}{dt}\right]^2 \quad \text{and} \quad x_2(t) \xrightarrow{S} y_2(t) = \frac{1}{x_1(t)}\left[\frac{dx_2(t)}{dt}\right]^2$$

Now consider a third input x_3[t]= x_2[t]+ x_1[t]. The corresponding system output Will be

$$y_3(t) = \frac{1}{x_3(t)}\left[\frac{dx_3(t)}{dt}\right]^2$$

$$= \frac{1}{x_1(t)+x_1(t)}\left[\frac{d\left[x_1(t)+x_1(t)\right]}{dt}\right]^2$$

$$\neq y_1(t)+y_2(t)$$

therefore, we may conclude that the system is not additive

Now consider a third input x_4 [t]= a x_1 [t]. The corresponding system output Will be

$$y_4(t) = \frac{1}{x_4(t)}\left[\frac{dx_4(t)}{dt}\right]^2$$

$$= \frac{1}{ax_1(t)}\left[\frac{d\left[ax_1(t)\right]}{dt}\right]^2$$

$$= \frac{a}{x_1(t)}\left[\frac{dx_1(t)}{dt}\right]^2$$

$$= ay_1(t)$$

Therefore, the system is homogeneous.

$$\text{(ii)}\quad y[n] = \begin{cases} \frac{x[n]x[n-2]}{x[n-1]}, & x[n-1] \neq 0 \\ 0, & x[n-1] = 0 \end{cases}$$

(ii) This system is not additive. Consider the fowling example .Let $x_1[n]$ =2 δ [n+2]+ 2 δ [n+1]+2 δ [n] and x_2 [n]= δ [n+1]+ 2 δ [n+1]+ 3 δ [n]. The corresponding outputs evaluated at n=0 are

$$y_1[0] = 2\, and \quad y_2[0] = 3/2$$

Now consider a third input x_3 [n]= x_2 [n]+ x_1 [n].= 3 δ [n+2]+4 δ [n+1]+5 δ [n]
The corresponding outputs evaluated at n=0 is $y_3[0]$=15/4. Gnarly, $y_3[0] \neq y_1[0] + y_2[0]$.This

$$y_4[n] = \begin{cases} \dfrac{x_4[n]x_4[n-2]}{x_4[n-1]}, x_4[n-1] \neq 0 \\ \qquad\qquad\qquad otherwise \\ 0, \end{cases}$$

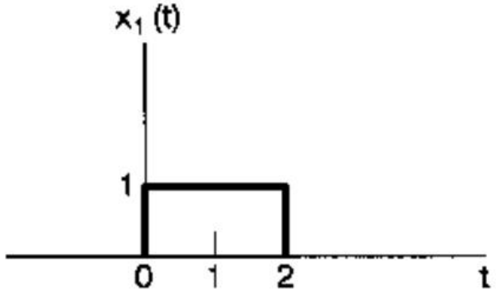
$$y_5[n] = \begin{cases} a\dfrac{x_4[n]x_4[n-2]}{x_4[n-1]}, x_4[n-1] \neq 0 \\ \qquad\qquad\qquad otherwise \\ 0, \end{cases} = a y_4[n]$$

Therefore, the system is homogenous.

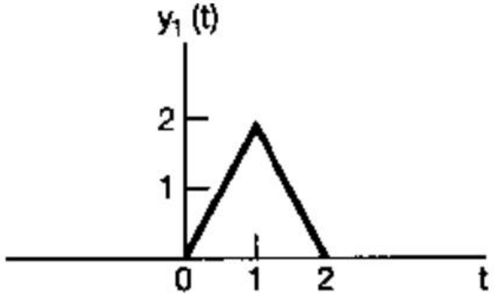
1.31 在本题中将要说明线性时不变性质的最重要结果之一，即一旦知道了一个线性系统或线性时不变(LTI)系统对某单一输入的响应，或者对若干个输入的响应，就能直接计算出对许多其它输入信号的响应。本书剩下的绝大部分都是利用这一点来建立分析与综合 LTI 系统的一些结果和方法的。

(a) 考虑一个 LTI 系统，它对示于图 P 1.31(a)的信号 $x_1(t)$的响应 $y_1(t)$示于图 P 1.31(b)中，确定并画出该系统对示于图 P 1.31 (c)的信号 $x_2(t)$的响应。

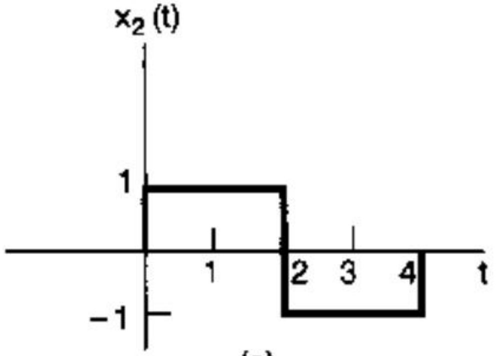
(b) 确定并画出上述(a)中的系统对示于图 P 1.31(d)的信号 $x_3(t)$的响应。



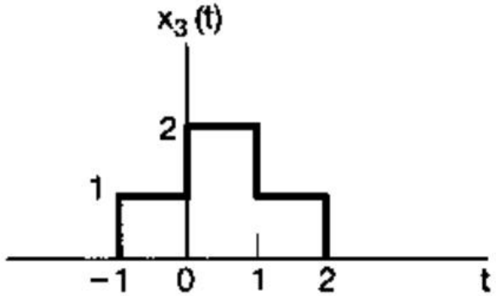
(a)



(b)



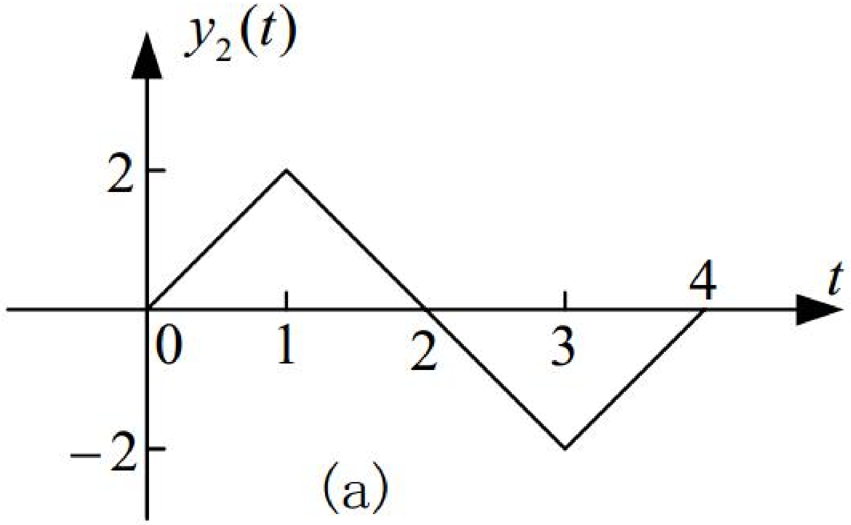
(c)



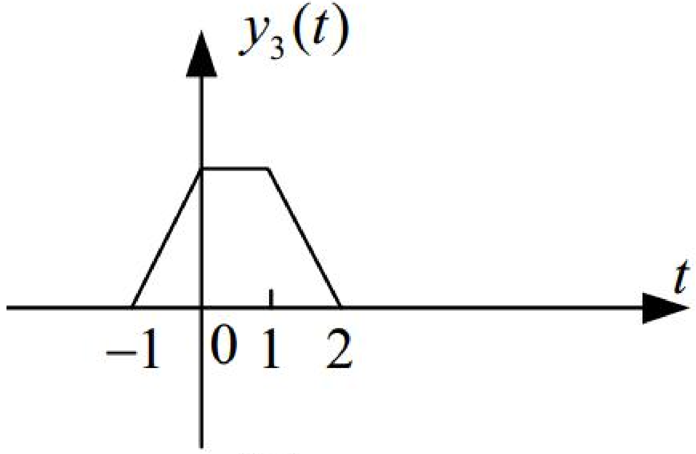
(d)

1.31 (a) Note that $x_2[t] = x_1[t] - x_1[t-2]$. Therefore, using linearity we get $y_2(t) = y_1(t) - y_1(t-2)$.this is shown in Figure S1.31

(b)Note that x3 (t)= x1 [t]+ x1 [t+1]. .Therefore, using linearity we get $y_3(t) = y_1(t+1) + y_1(t)$. this is shown in Figure S1.31



(a)



(b)

1.33 设 $x[n]$是一离散时间信号,并令

$$y_1[n]=x[2n] \text{和} y_2[n]=\begin{cases}x[n/2], & n \text{为偶}\\ 0, & n \text{为奇}\end{cases}$$

信号 $y_1[n]$和 $y_2[n]$分别代表 $x[n]$一种加速和减慢的形式。然而,应该注意在离散时间下的加速和减慢与连续时间下相比有一些细微的差别。考虑以下说法:

(1) 若 $x[n]$是周期的,$y_1[n]$也是周期的。

(2) 若 $y_1[n]$是周期的,$x[n]$也是周期的。

(3) 若 $x[n]$是周期的,$y_2[n]$也是周期的。

(4) 若 $y_2[n]$是周期的,$x[n]$也是周期的。

对以上每一种说法判断是否对。若对,确定这两个信号基波周期之间的关系;若不对,给出一个反例。

1)正确。设 $x(n)$ 的周期为 N 。如果 N 为偶数，则 $y_1(n)$ 的周期为 $N/2$；如果 N 为

奇数，则必须有 $2N_0=2N$，才能保证周期性，此时 $y_1(n)$ 的周期为 $N_0=N$ 。

(2)False. $y_1[n]$ periodic does no imply x[n] is periodic i.e. Let x[n] = g[n]+h[n] where

$$g[n]=\begin{cases}1, & n\ even\\ 0, & n\ odd\end{cases}\quad and \qquad h[n]=\begin{cases}0, & n\ even\\ (1/2)^n, & n\ odd\end{cases}$$

Then $y_1[n]=x[2n]$ is periodic but x[n] is clearly not periodic.

3)正确。若 $x(n)$ 的周期为 N，则 $y_2(n)$ 的周期为 $2N$ 。

4)正确。若 $y_2(n)$ 的周期为 N，则 N 只能是偶数。$x(n)$ 的周期为 $N/2$ 。

1.42 (a) 下列说法是对还是错？说明理由。

两个线性时不变系统的级联还是一个线性时不变系统。

(b) 下列说法是对还是错？说明理由。

两个非线性系统的级联还是非线性的。

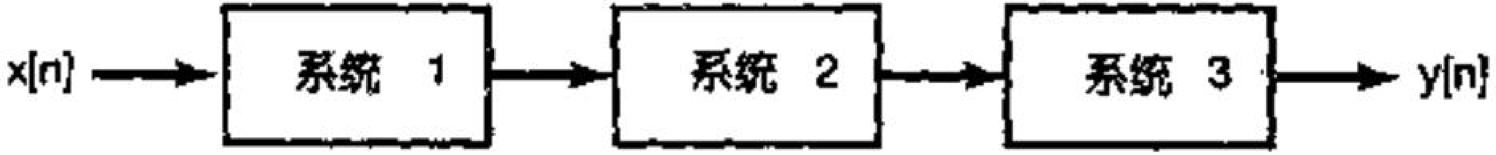
(c) 考虑具有下列输入-输出关系的三个系统：

系统 1： $y[n]=\begin{cases} x[n/2], & n \text{ 为偶} \\ 0, & n \text{ 为奇} \end{cases}$

系统 2： $y[n]=x[n]+\frac{1}{2}x[n-1]+\frac{1}{4}x[n-2]$

系统 3： $y[n]=x[2n]$

假设这三个系统按图 P1.42 级联，求整个系统的输入-输出关系。它是线性的吗？是时不变的吗？



1.42.(a) Consider two system S_1 and S_2 connected in series .Assume that if $x_1(t)$ and $x_2(t)$ are the inputs to S_1..then $y_1(t)$ and $y_2(t)$ are the outputs.respectively .Also,assume that if $y_1(t)$ and $y_2(t)$ are the input to S_2 ,then $z_1(t)$ and $z_2(t)$ are the outputs, respectively . Since S_1 is linear ,we may write

$$ax_1(t)+bx_2(t)\xrightarrow{s_1}ay_1(t)+by_2(t),$$

where a and b are constants. Since S_2 is also linear ,we may write

$$ay_1(t)+by_2(t)\xrightarrow{s_2}az_1(t)+bz_2(t),$$

We may therefore conclude that

$$a x_1(t)+b x_2(t)\xrightarrow{s_1 s_2}a z_1(t)+b z_2(t)$$

Therefore ,the series combination of S_1 and S_2 is linear.
Since S_1 is time invariant, we may write

$$x_1(t-T_0)\xrightarrow{s_1}y_1(t-T_0)$$

and

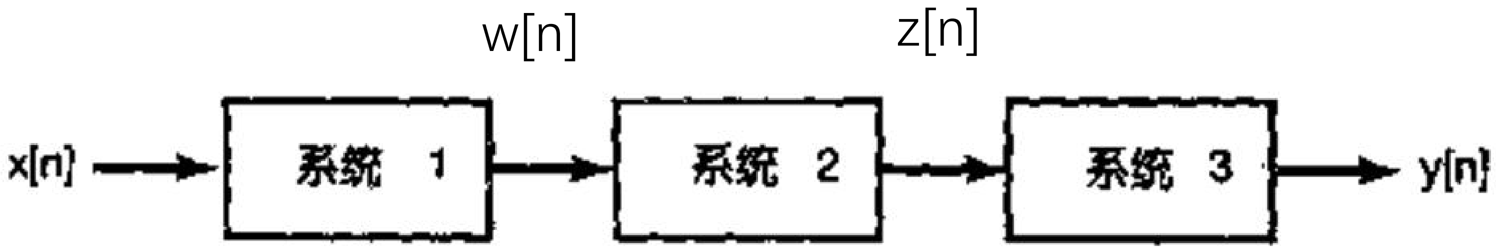
$$y_1(t-T_0)\xrightarrow{s_2}z_1(t-T_0)$$

Therefore,

$$x_1(t-T_0)\xrightarrow{s_1 s_2}z_1(t-T_0)$$

Therefore, the series combination of S_1 and S_2 is time invariant.

(b) False, Let y(t)=x(t)+1 and z(t)=y(t)-1.These corresponds to two nonlinear systems. If these systems are connected in series ,then z(t)=x(t) which is a linear system.



(c) Let us name the output of system 1 as w[n] and the output of system 2 as z[n] .Then

$$y[n] = z[2n] = w[2n] + \frac{1}{2} w[2n-1] + \frac{1}{4} w[2n-2]$$

$$= x[n] + \qquad \frac{1}{4} x[n-2]$$

The overall system is linear and time-invariant.

总结

- 基本的系统性质：线性（可加性和齐次性）、时不变、稳定性
- 线性时不变系统的输入-输出关系
- 信号的周期性
- 系统的级联

信号与系统视频课

- 麻省理工学院公开课：信号与系统：模拟与数字信号处理
- 讲师：Alan V. Oppenheim（奥本海姆）
- 网易公开课
- http://open.163.com/special/opencourse/signals.html