**Lab 4：The Continuous-Time Fourier Transform**

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| **Introduction**  4.2  A large class of signals can be represented using the continuous-time Fourier transform (CTFT) . In this exercise we have used MATLAB to compute numerical approximations to the CTFT integral. To store in X samples of X(jwk) ordered such that X(k + 1) is the CTFT evaluated at -n/r + (2k/Nτ) for 0 <k < N-1, use X=fftshift(tau\*fft(x)).  For this exercise, we have approximated the CTFT of x(t) = exp(-2|t|) using the function fft  and a truncated version of x(t). we have seen that for sufficiently small T, and computed  an accurate numerical approximation to X(jw).  4.5  In this exercise, we have learned how to find analytic expressions for the impulse responses of stable LTI systems whose inputs and outputs satisfy linear constant-coefficient differential equations. The frequency response of systems of this form can be written as ratios of polynomials in (jw). MATLAB represents these polynomials as a vector of coefficients of the polynomial in decreasing powers of the dependent variable, jw. For example, the polynomial G(jw) = 4(jw)^3 - 5(jw)^2 + 2(jw) - 7 would be represented in MATLAB by the vector G= [4 -5 2 -71. MATLAB contains several functions for manipulating polynomials in this format. One very useful function is residue, which computes the partial fraction expansion of a function consisting of a ratio of polynomials. In this exercise, we have learned to convert the differential equation relating the input and output of a stable, continuoustime, LTI system into vectors representing the polynomials appearing in the numerator and denominator of the frequency response. Then, we have used residue to process the frequency response so that the impulse response may be easily determined from the partial fraction expansion.  4.6  This exercise will explore amplitude modulation of Morse code messages. Here is what is known. The signal x(t) is of the form x(t) = m1(t) cos(2f1 t) + m2(t) cos(2f2 t) + m3(t) sin(2f1 t), where fl and f2 are given by the variables f 1 and f 2, respectively. It is also known that each of the signals m1(t) , m2(t) and m3 (t) correspond to a single letter of the alphabet which has been encoded using International Morse Code.  **Lab results & Analysis**：  4.2      (a)  We can easy find that the Fourier transform for g(t) is 1/(2+jw), so the Fourier transform for x(t) is 1/(2+jw)+ 1/(2-jw). And the figure of X(jw) shows below.  (b)    As the above figure shows, the ylable is vector y and xlable is n while w= for 0, w= w= for .  (c)      The above figure shows the picture of Y(jw), and we can find that in low frequency, the picture is the same as the picture in question(b).  (d)    The above figure shows the picture of Y(jw) while the xlable is w and ylable is Y(jw).  (e)    We use the formular X(jw)=Y(jw)exp(j5w) to calculate X, and the magnitude of X shows in above figure.  (f)    As the above figures show, the first line is the magnitude and angle of X which is calculated by X(jw)=Y(jw)exp(j5w); the second line is the magnitude and angle of X(jw) which is given in (a).    Plot the magnitude on a logarithmic scale, and at higher frequencies the approximation is not as good as at lower frequencies. And the reason is that since we have approximated x(t) with samples x(n), our approximation will be better for frequency components of the signal that do not vary much over time intervals of length T.  (g)    As the above figures show, the first line is the magnitude and angle of X which is calculated by X(jw)=Y(jw)exp(j5w); the second line is the magnitude and angle of X(jw) which is given in (a);the third line is the the magnitude and angle of Y. and only in low frequency does the result fit our prediction that the difference value <Y-<X=5w,it should be a line in frequency domain. （the reason why the picture of <Y is like the above figure shows is that the maximum of the angle is 2.）  4.5    A  Vector a1=[1 3/2 1/2],b1=[1 -2];  b      As the above figure shows, the magnitudes and angles of H1 and H1(jw) have been plot, H1 is the Fourier transform of the frequency response which was calculated by r1 and p1,while H1(jw) was calculated by a1 and b1. And we can see that the picture is the same for the two functions.  c    The above figure is the picture of h1(t), and we can easily find that h1(t) are absolutely integrable, because the values in every point in finite and as t increases, h1 goes to 0 very quickly. (in fact, it’s an exponential damping, surely h1(t) are absolutely integrable.)  d  The vector a2=[1 7 16 12], and b2=[3 10 5];  e      As the above figure shows, the magnitudes and angles of H2 and H2(jw) have been plot, H2 is the Fourier transform of the frequency response which was calculated by r2 and p2, while H2(jw) was calculated by a2 and b2. And we can see that the picture is the same for the two functions.  f    The inverse CTFT of a term of the form is , and the above figure is the picture of h2(t), and we can easily find that h2(t) are absolutely integrable, because the values in every point in finite and as t increases, h2 goes to 0 very quickly. (in fact, it’s an exponential damping, surely h2(t) are absolutely integrable.)  g  the vector a3=[1 -4], and b3=[-4];  h    As the above figure shows, the magnitudes and angles of H3 and H3(jw) have been plot, H3 is the Fourier transform of the frequency response which was calculated by r3 and p3, while H3(jw) was calculated by a3 and b3. And we can see that the picture is the same for the two functions.  i    the above figure is the picture of h3(t), and we can easily find that h3(t) are absolutely integrable, because the values in every point in finite and as t increases, h3 goes to 0 very quickly. (in fact, it’s an exponential damping, surely h3(t) are absolutely integrable.) h3(t) is obvious causal, because h3(t) has the factor u(t) so that it’s all zero in t<0, so the system is causal.  4.6                      The signal should be .  The reason why signal should be . is interpreted as below.  Assume there is , then  Let and go through this filter respectively.  For i=2, we obtain and , they are opposite;  For i=4, we obtain and , they are the same.  Use Matlab to plot them.    They are opposite, so we should replace as . The signal should be    Let go through this filter, then we obtain , which corresponds to .  Use MATLAB to plotas below.  Code: ­ ∙ ∙  Letter: D    The Fourier transform of is .  Let go through this filter, then we obtain , which corresponds to .    Code: · · ·  Letter: S  The Fourier transform of is .  Let go through this filter, then we obtain , which corresponds to .  And has been figured, use MATLAB to plotas below.    Code: · - - ·  Letter: P  The future of technology lies in “DSP”. | |
| **Experience:**  1.when using function fft, the results is not so accurate at high frequency domain(the reason is showed in 4.2)  2.when using the function residue, remember to check whether it has same roots. | |
| **Score** |  |

clear

% a

w1=-20:1:20;

x=1./(2+j\*w1)+1./(2-j\*w1);

plot(w1,abs(x),'g'),xlabel('w'),ylabel('X(jw)'),grid;

% b

tau=0.01;

T=10;

t=[0:tau:T-tau];

N=T/tau;

n=t/tau;

yt=exp(-2\*abs(t-5));

y=0;

k=[0:N/2,-N/2+1:1];

for i=t

y=y+tau\*exp(-2\*abs(i-5))\*exp(-j\*2\*pi\*(tau/T).\*k\*i);

end

plot(k,abs(y),'r'),xlabel('k'),ylabel('y'),grid;

% c

Y=tau\*fft(yt);

plot(0:N-1,abs(fftshift(Y)), 'r'),xlabel('n'),ylabel('Y(jw)'),grid;

% d

w=-(pi/tau)+(0:N-1)\*(2\*pi/(N\*tau));

plot(w,abs(fftshift(Y)), 'r'),xlabel('w'),ylabel('Y(jw)'),grid;

% e

X=Y.\*exp(j\*5\*w);

plot(w,abs(fftshift(X)), 'r'),xlabel('w'),ylabel('X(jw)'),grid;

% f

subplot(2,2,1), plot(w,abs(fftshift(X))),xlabel('w'),ylabel('|X|'),grid;

subplot(2,2,2), plot(w,angle(fftshift(X))),xlabel('w'),ylabel('<X'),grid;

subplot(2,2,3), plot(w,abs(1./(2+j\*w)+1./(2-j\*w)), 'r'),xlabel('w'),ylabel('|X(jw)|'),grid;

subplot(2,2,4), plot(w,angle(1./(2+j\*w)+1./(2-j\*w)), 'r'),xlabel('w'),ylabel('<X(jw)'),grid;

subplot(2,1,1), semilogy(w,abs(fftshift(X)), 'r'),title({'X';'(logarithmic scale)'}),xlabel('w'),ylabel('X'),grid;

subplot(2,1,2), semilogy(w,abs(1./(2+j\*w)+1./(2-j\*w)),'g'),title({'X(jw)';'(logarithmic scale)'}),xlabel('w'),ylabel('X(jw)'),grid;

% g

subplot(3,2,1), plot(w,abs(fftshift(X))),xlabel('w'),ylabel('|X|'),grid;

subplot(3,2,2), plot(w,angle(fftshift(X))),xlabel('w'),ylabel('<X'),grid;

subplot(3,2,3), plot(w,abs(1./(2+j\*w)+1./(2-j\*w)), 'r'),xlabel('w'),ylabel('|X(jw)|'),grid;

subplot(3,2,4), plot(w,angle(1./(2+j\*w)+1./(2-j\*w)), 'r'),xlabel('w'),ylabel('<X(jw)'),grid;

subplot(3,2,5), plot(w,abs(fftshift(Y)), 'g'),xlabel('w'),ylabel('|Y|'),grid;

subplot(3,2,6), plot(w,angle(fftshift(Y)), 'g'),xlabel('w'),ylabel('<Y'),grid;

**4.5**

clear

% a

a1=[1 3/2 1/2];

b1=[1 -2];

% b

[r1,p1]=residue(b1,a1);

H1=0;

w=[-10:1:10];

for i=[1:length(r1)]

H1=H1+r1(i)./(j.\*w-p1(i));

end

subplot(2,2,1), stem(w,abs(H1),'r'),xlabel('w'),ylabel('|H1|'),grid;

subplot(2,2,2), stem(w,angle(H1),'r'),xlabel('w'),ylabel('<H1'),grid;

H1jw=(j.\*w-2)./((j.\*w).^2+3/2\*j.\*w+1/2);

subplot(2,2,3), stem(w,abs(H1jw),'r'),xlabel('w'),ylabel('|H1(jw)|'),grid;

subplot(2,2,4), stem(w,angle(H1jw),'r'),xlabel('w'),ylabel('<H1(jw)'),grid;

% c

h1=0;

t=[0:0.5:15];

for i=[1:length(r1)]

h1=h1+r1(i).\*exp(p1(i).\*t);

end

stem(t,h1,'r'),xlabel('t'),ylabel('h1'),grid;

% d

a2=[1 7 16 12];

b2=[3 10 5];

% e

[r2,p2]=residue(b2,a2);

H2=r2(1)./(j.\*w-p2(1))+r2(2)./(j.\*w-p2(2))+r2(3)./((j.\*w-p2(3)).^2);

subplot(2,2,1), stem(w,abs(H2),'r'),xlabel('w'),ylabel('|H2|'),grid;

subplot(2,2,2), stem(w,angle(H2),'r'),xlabel('w'),ylabel('<H2'),grid;

H2jw=(3\*(j.\*w).^2+10.\*(j.\*w)+5)./((j.\*w).^3+7\*(j.\*w).^2+16\*(j.\*w)+12);

subplot(2,2,3), stem(w,abs(H2jw),'r'),xlabel('w'),ylabel('|H2(jw)|'),grid;

subplot(2,2,4), stem(w,angle(H2jw),'r'),xlabel('w'),ylabel('<H2(jw)'),grid;

% f

h2=r2(1).\*exp(p2(1).\*t)+r2(2).\*exp(p2(2).\*t)+r2(3).\*exp(p2(3).\*t).\*t;

stem(t,h2,'r'),xlabel('t'),ylabel('h2'),grid;

% g

a3=[1 -4];

b3=[-4];

% h

[r3,p3]=residue(b3,a3);

H3=r3./(j.\*w-p3);

subplot(2,2,1), stem(w,abs(H3),'r'),xlabel('w'),ylabel('|H3|'),grid;

subplot(2,2,2), stem(w,angle(H3),'r'),xlabel('w'),ylabel('<H3'),grid;

H3jw=(-4)./(j.\*w-4);

subplot(2,2,3), stem(w,abs(H3jw),'r'),xlabel('w'),ylabel('|H3(jw)|'),grid;

subplot(2,2,4), stem(w,angle(H3jw),'r'),xlabel('w'),ylabel('<H3(jw)'),grid;

% i

h3=r3.\*exp(p3.\*t);

stem(t,h2,'r'),xlabel('t'),ylabel('h2'),grid;

**4.6**

**z=[dash dot dash dot];**

**plot(t,z),xlabel('t'),ylabel('z(t)');**

**w=linspace(-40\*pi,40\*pi);**

**H=freqs(bf,af,w);**

**subplot(2,1,1),plot(w,abs(H)),xlabel('\omega'),ylabel('abs(X)');**

**subplot(2,1,2),plot(w,angle(H)),xlabel('\omega'),ylabel('angle(X)');**

**ydash=lsim(bf,af,dash,t(1:length(dash)));**

**ydot=lsim(bf,af,dot,t(1:length(dot)));**

**subplot(2,1,1),plot(t(1:length(dash)),ydash),xlabel('t'),ylabel('y(t)');**

**hold on;**

**plot(t(1:length(dash)),dash);**

**legend('ydash','dash');**

**subplot(2,1,2),plot(t(1:length(dot)),ydot),xlabel('t'),ylabel('y(t)');**

**hold on;**

**plot(t(1:length(dot)),dot);**

**legend('ydot','dot');**

**y=dash.\*cos(2\*pi\*f1\*t(1:length(dash)));**

**y0=lsim(bf,af,y,t(1:length(dash)));**

**subplot(2,1,1),plot(t(1:length(dash)),y),xlabel('t'),ylabel('y(t)');**

**subplot(2,1,2),plot(t(1:length(dash)),y0),xlabel('t'),ylabel('y\_0(t)');**

**xx=x.\*cos(2\*pi\*f1\*t(1:length(x)));**

**m1=2\*lsim(bf,af,xx,t);**

**plot(t,m1),xlabel('t'),ylabel('m\_1(t)');%-.. D**

**m2=4i\*lsim(bf,af,exp(1i\*2\*pi\*(f1-f2)\*t).\*xx,t);**

**plot(t,m2),xlabel('t'),ylabel('m\_2(t)');%...S**

**m3=4i\*(lsim(bf,af,exp(-1i\*4\*pi\*f1\*t).\*xx,t)-0.25\*m1);**

**plot(t,m3),xlabel('t'),ylabel('m\_3(t)');%.--.P**

**m4=4i\*lsim(bf,af,exp(1i\*2\*pi\*(f2-f1)\*t).\*xx,t);**

**subplot(2,1,1),plot(t,m2),xlabel('t'),ylabel('y\_1(t) after filter');**

**subplot(2,1,2),plot(t,m4),xlabel('t'),ylabel('y\_2(t) after filter');**